Dissipation in tunneling: fluctuation relations and Maxwell's demon

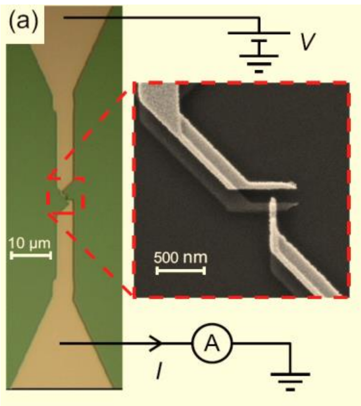
Jukka Pekola
O. V. Lounasmaa Laboratory, Aalto University, Helsinki

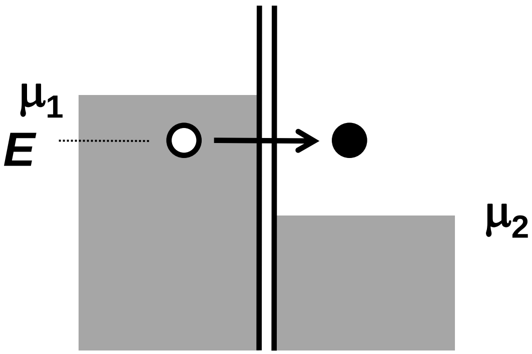
1. **Fluctuation relations in electronic circuits**
2. **Maxwell’s demon**
3. **Statistics of dissipation in qubits**

Collaborators: Olli-Pentti Saira, Youngsoo Yoon, Mikko Möttönen, Paolo Solinas, Jonne Koski, Aki Kutvonen, Tapio Ala-Nissila

Dmitri Averin (SUNY), Alexander Shnirman (KIT), Takahiro Sagawa (Univ. Tokyo), Frank Hekking (Grenoble)

Dissipation in electron tunneling





Dissipation generated by a tunneling event in a junction biased at voltage *V*

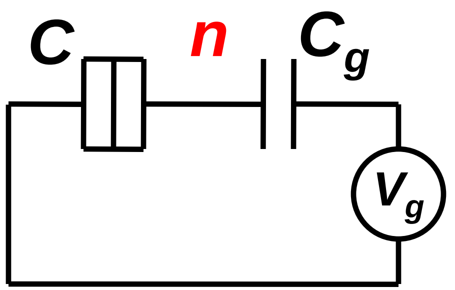
$$\Delta Q = (\mu_1 - E) + (E - \mu_2) = \mu_1 - \mu_2 = eV$$

ΔQ is first distributed to the electron system, then typically to the lattice by electron-phonon scattering

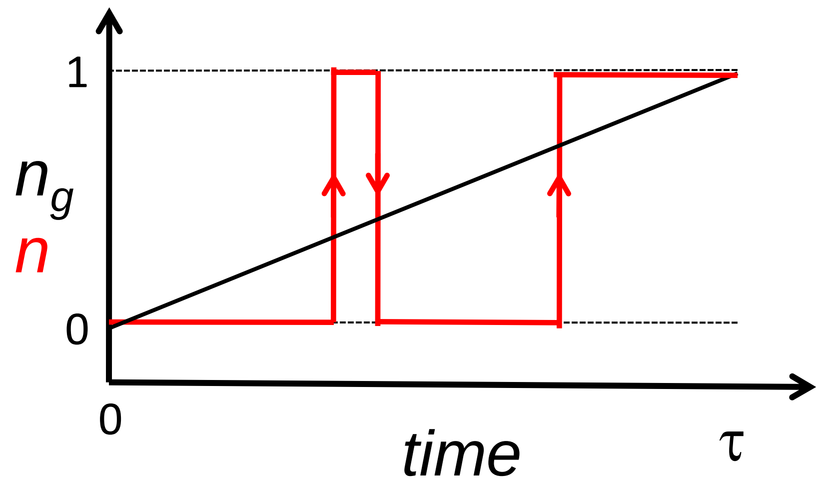
For average current *I* through the junction, the total average power dissipated is naturally

$$P = (I/e)\Delta Q = IV$$

Dissipation in driven single-electron transitions

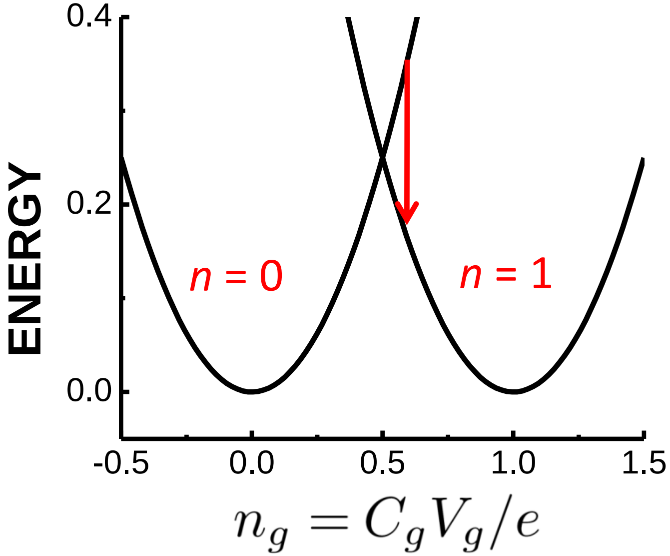


Single-electron box



$$Q_i = E_0(n_{g,i}) - E_1(n_{g,i}) = 2E_C\delta n_{g,i}$$

$$E_C = \frac{e^2}{2(C+C_g)} \qquad \delta n_{g,i} = n_{g,i} - 1/2$$



The total dissipated heat in a ramp:

$$Q = 2E_C \sum_i \pm\delta n_{g,i}$$

D. Averin and J. P., EPL **96**, 67004 (2011).

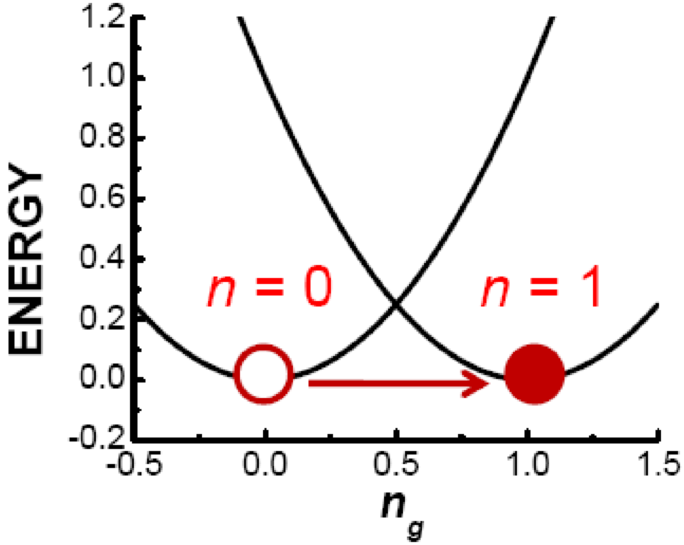
Work done by the gate

In general:

$$W_{\rm th} = \int dt \frac{\partial H}{\partial t} = \int dt \dot{\lambda} \frac{\partial H}{\partial \lambda}$$

For a SEB box: (for the gate sweep 0 -> 1)

$$W_{\rm th} - \Delta F = E_C \left(1 - n_i - n_f\right) + Q$$

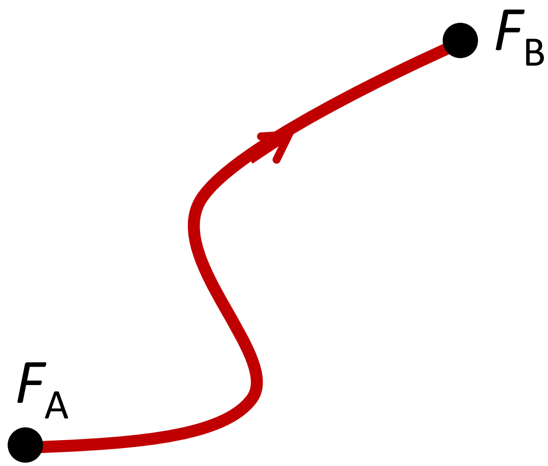


When the system is in the preferred charge state at the ends of the sweep,

$$W_{\rm th} - \Delta F = Q$$

J. P. and O.-P. Saira, JLTP 169, 70 (2012)

Crooks and Jarzynski fluctuation relations



Systems **driven** by control parameter(s), starting in equilibrium

$$W_d = W - \Delta F$$ "dissipated work"

C. Jarzynski 1997 $$\langle e^{-\beta W_d} \rangle = 1$$ $$\langle W \rangle \geq \Delta F$$

G. Crooks 1999 $$p_F(W_d)/p_R(-W_d) = e^{\beta W_d}$$

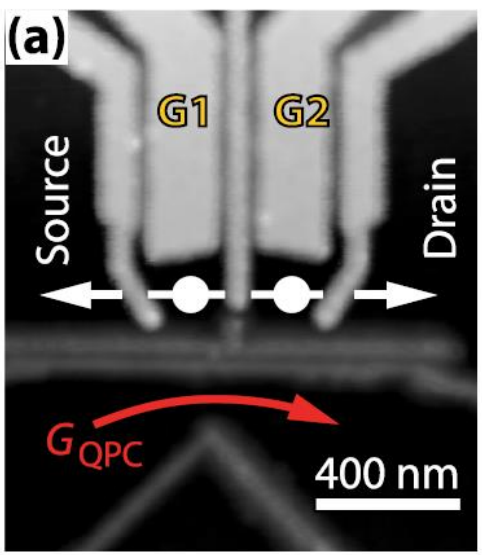
Fluctuation relations in steady-state

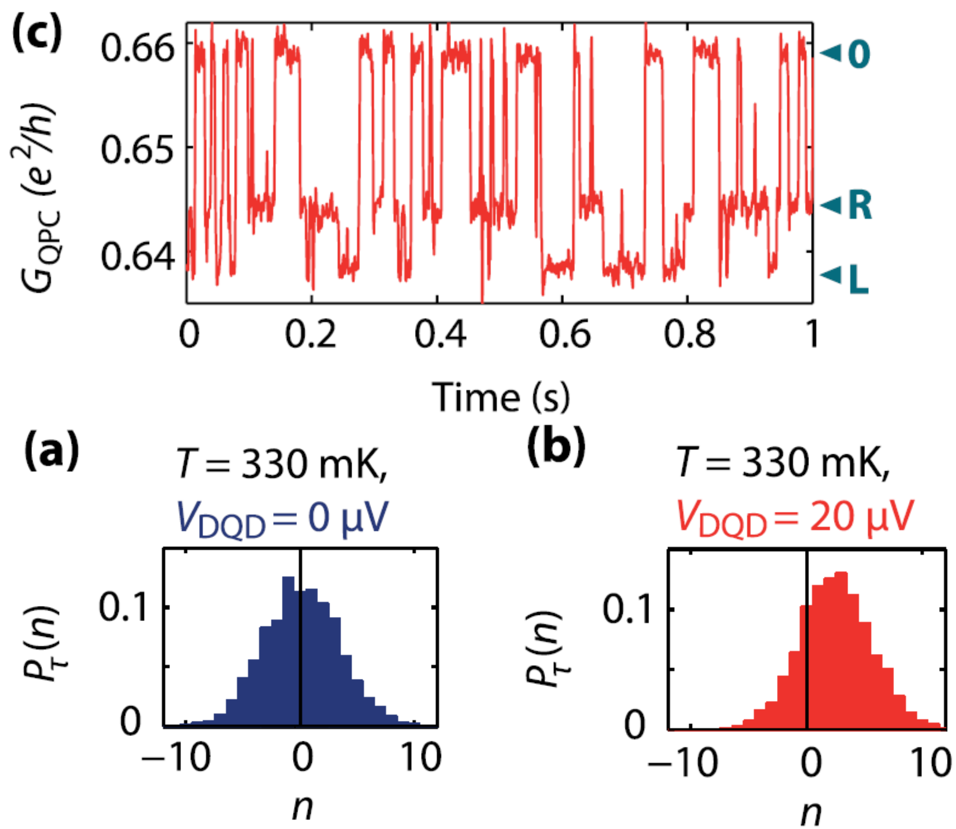
$$\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = e^{\Delta S/k_B}$$

Evans et al. 1993

Double quantum dot circuit
B. Küng et al., PRX **2**, 011001 (2012)
Utsumi et al.

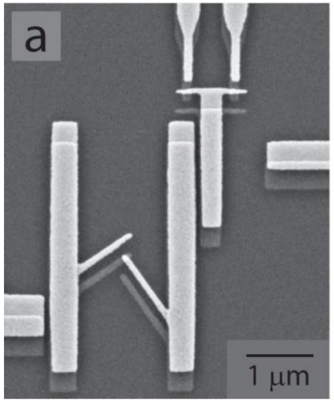
$$\frac{P_\tau(n)}{P_\tau(-n)} = e^{neV_{DQD}/k_B T}$$

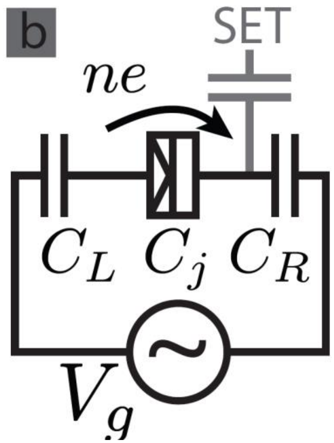


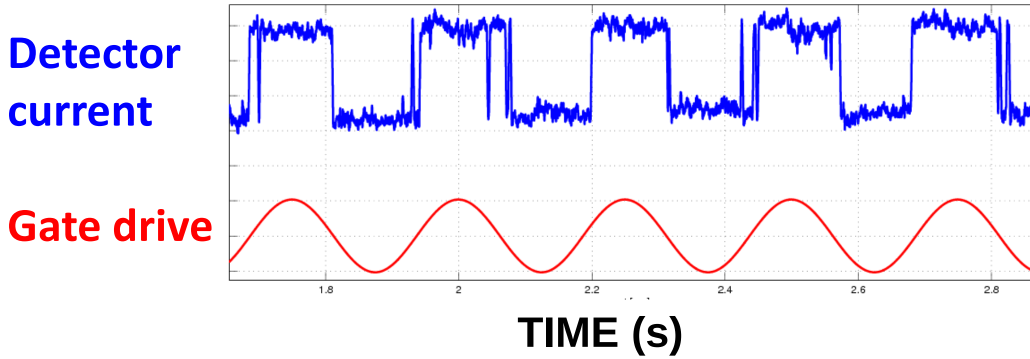


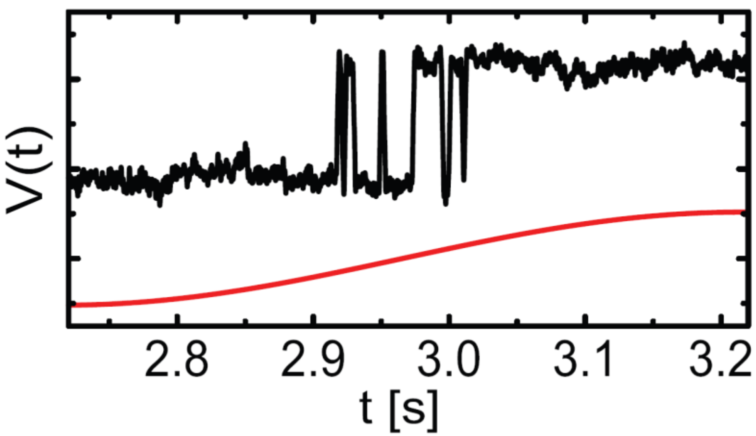
Experiment on a single-electron box

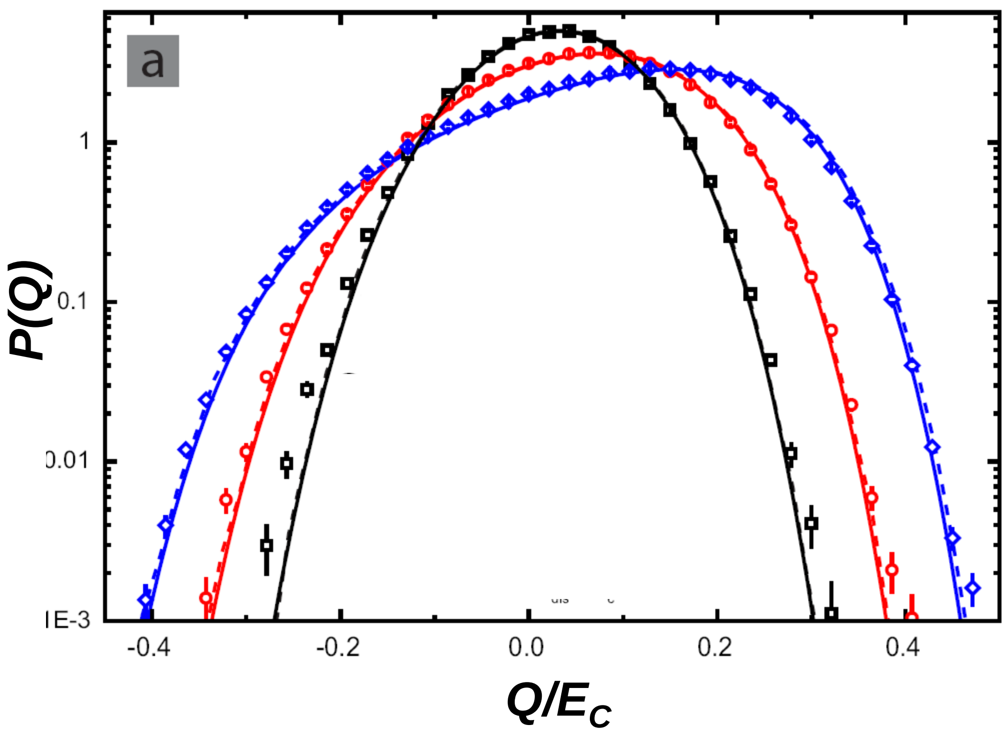
O.-P. Saira et al., PRL 109, 180601 (2012).





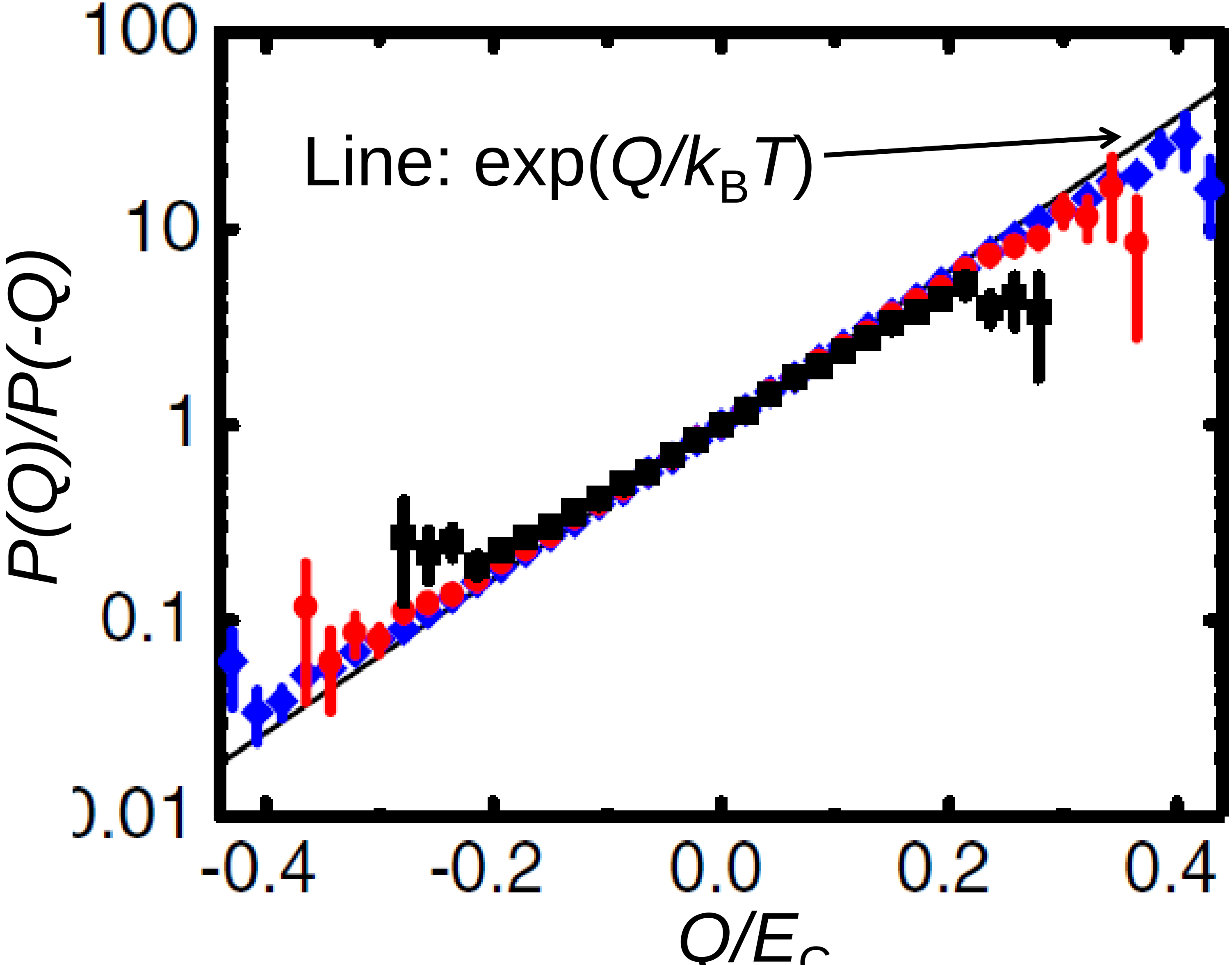






$$\langle e^{-\beta(W-\Delta F)}\rangle = 1.03 \pm 0.03$$

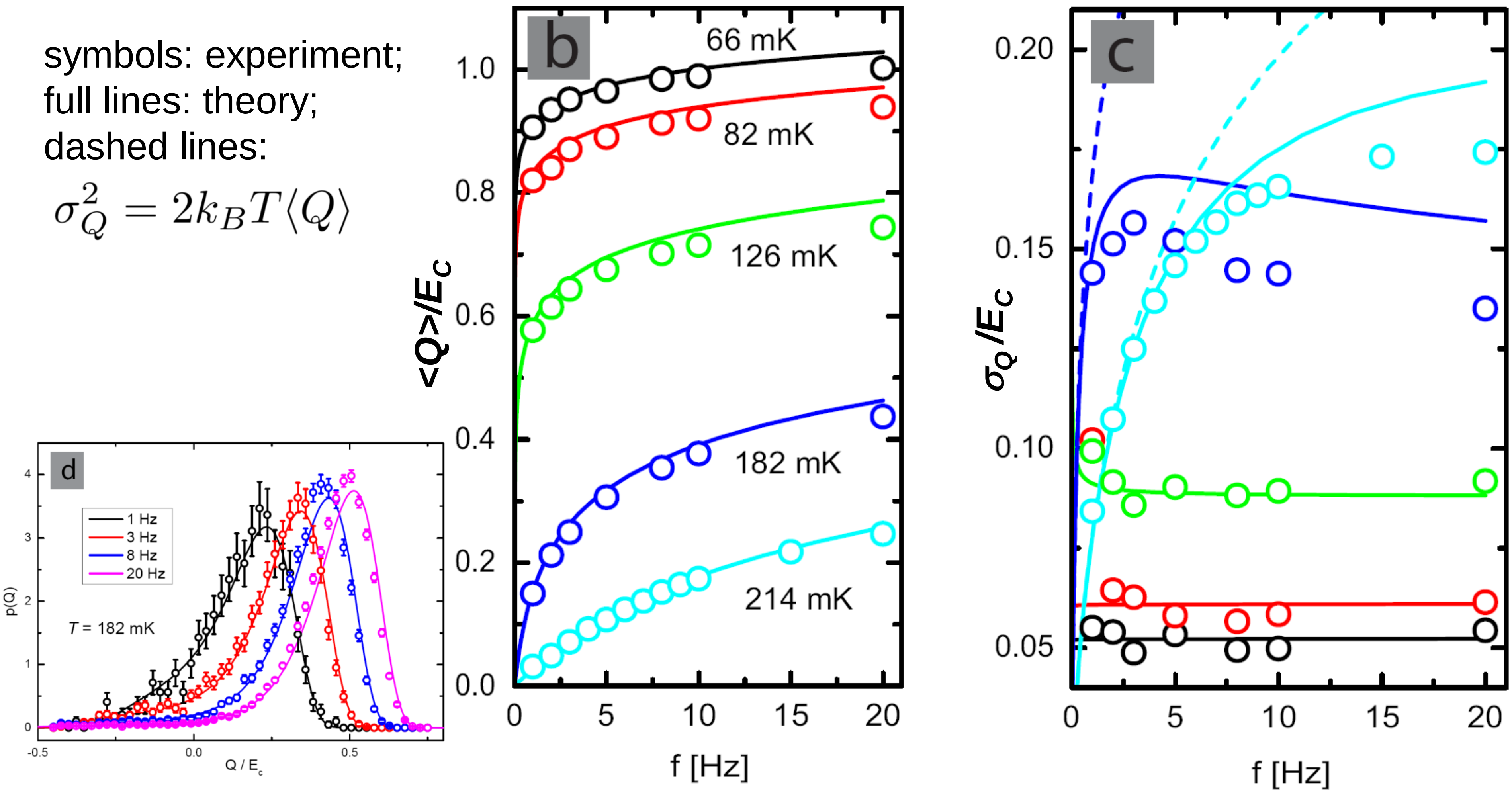
Crooks relation



Measurements of the heat (= work) distribution at various frequencies and temperatures

symbols: experiment;
full lines: theory;
dashed lines:

$$\sigma_Q^2 = 2k_B T\langle Q\rangle$$



Fluctuations under more general conditions (e.g. no single heat bath)

stochastic entropy $\Delta s_{\rm tot} = \Delta s + \Delta s_{\rm m}$

$$\Delta s = -\ln(\frac{p_1(t)}{p_0(0)})$$

$$\Delta s_{\rm m} = \sum_i \mp \ln(\Gamma_-(\tau_i)/\Gamma_+(\tau_i))$$

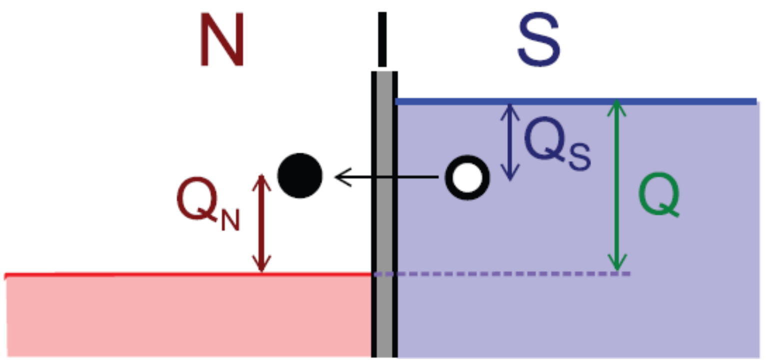
$$\langle e^{-\Delta s_{\rm tot}} \rangle = 1$$

U. Seifert, PRL **95**, 040602 (2005)

thermodynamic entropy

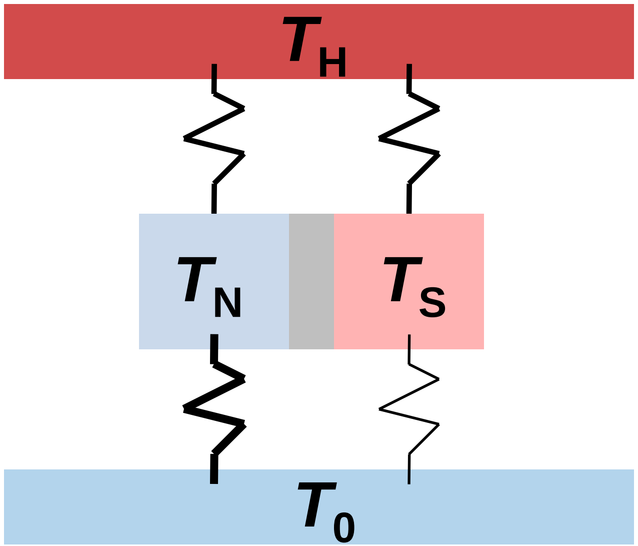
$$k_B \Delta S = Q_S/T_S + Q_N/T_N$$

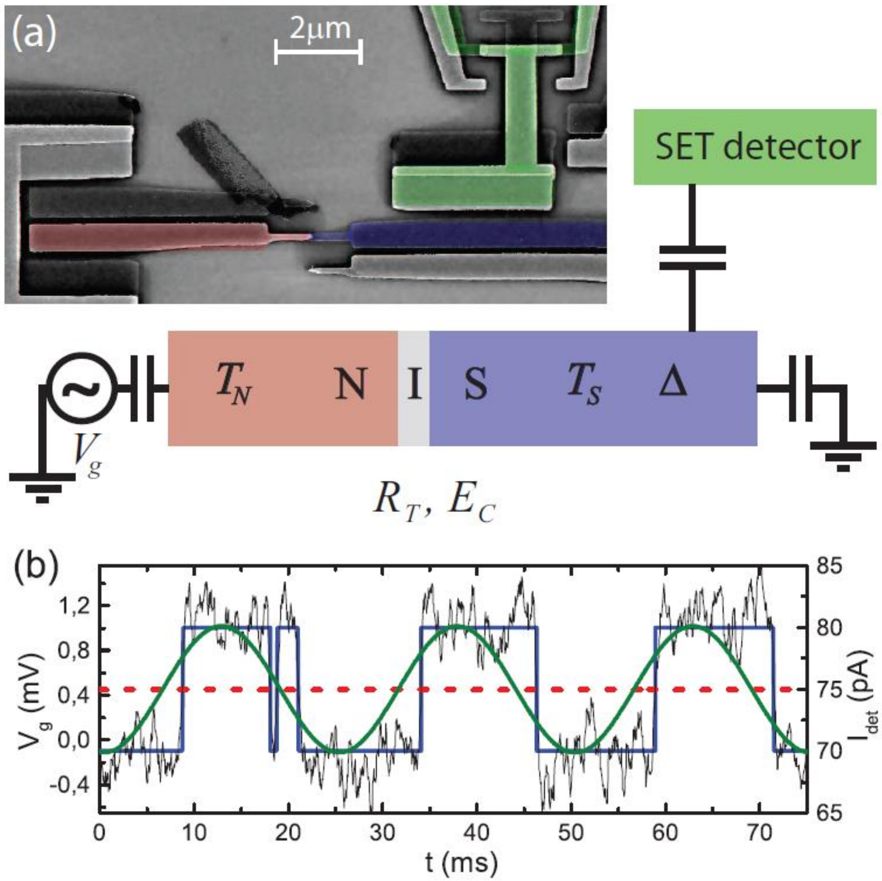
$$\langle e^{-\Delta S} \rangle = 1$$

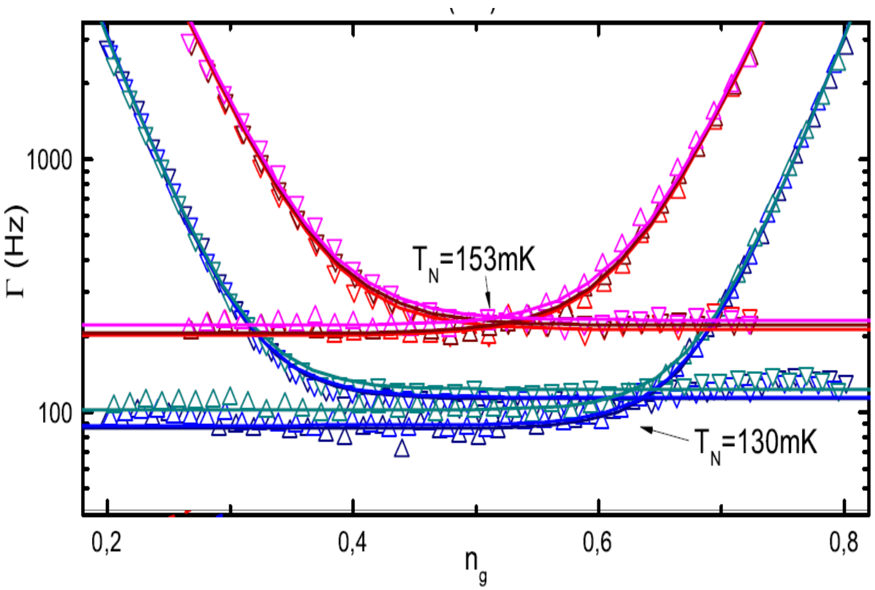


Experiments with un-equal temperatures

Coupling to two different baths



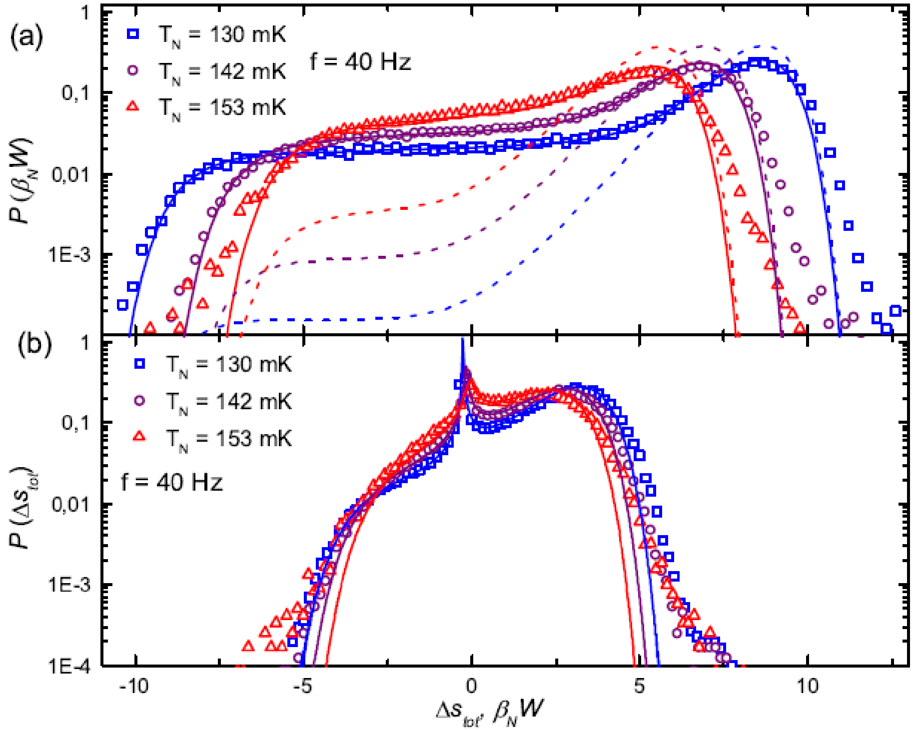




Typical values:

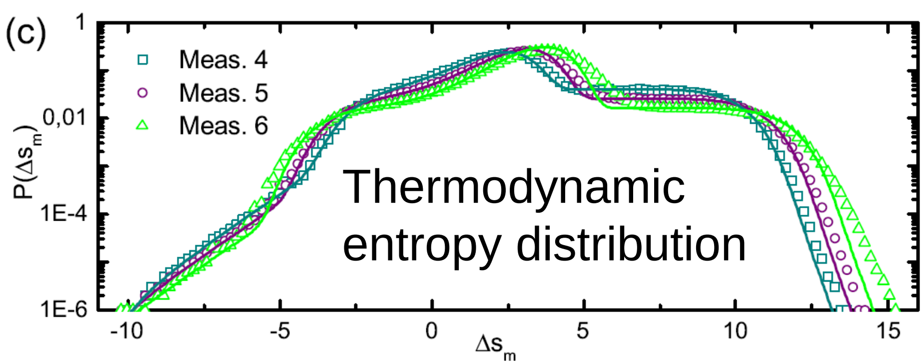
T_N=139 mK = T_{bath}, T_S = 180 mK

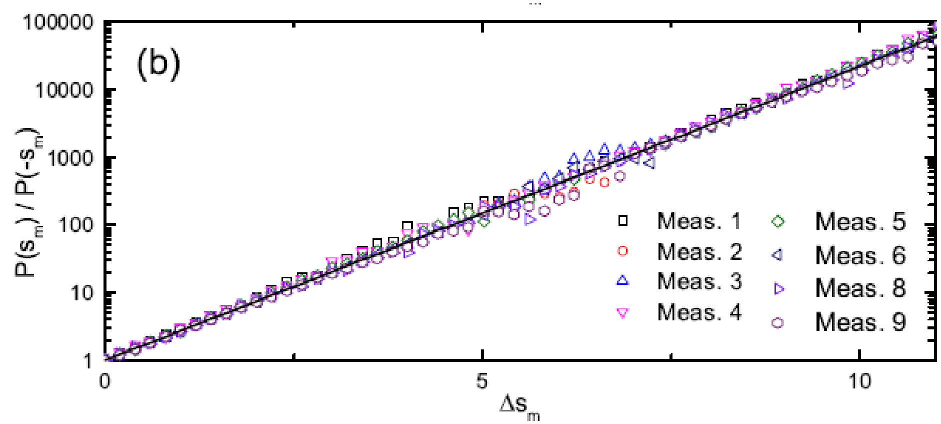
Experiments with un-equal temperatures



f (Hz)	T_N (mK)	n_0	$T_{S,0}$ (mK)	$T_{S,1}$ (mK)	$\langle e^{-\beta_N \bar{W}} \rangle$	$\langle e^{-\Delta s^{\rm st}_{\rm tot}} \rangle$	$\langle e^{-\Delta s^{\rm th}_{\rm tot}} \rangle$
20	130	0.526	174	177	93	1.085	1.063
40	130	0.516	174	177	129	1.064	1.053
80	130	0.507	176	178	180	1.074	1.083
20	142	0.513	179	181	20	1.064	1.030
40	142	0.509	179	181	30	1.054	1.047
80	142	0.505	180	181	45	1.096	1.100
120	141	0.504	181	182	68	1.241	1.324
40	153	0.502	184	184	11	1.095	1.058
80	153	0.503	184	185	15	1.140	1.139
120	153	0.502	185	186	20	1.301	1.370

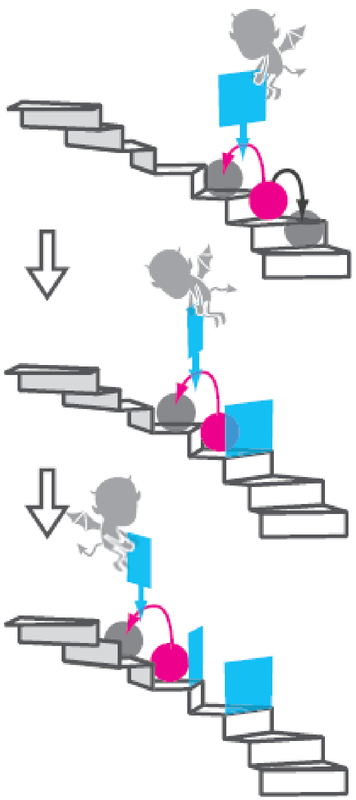
Work and (stochastic) entropy distributions





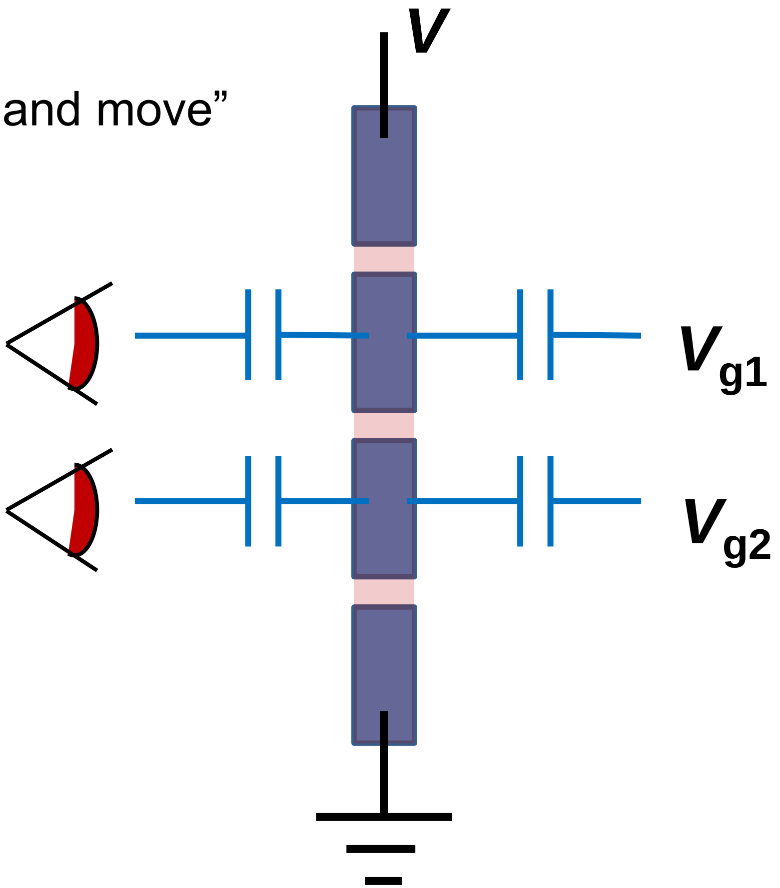
J.V. Koski et al., arXiv:1303.6405, Nature Physics (2013).

Electronic Maxwell's demon



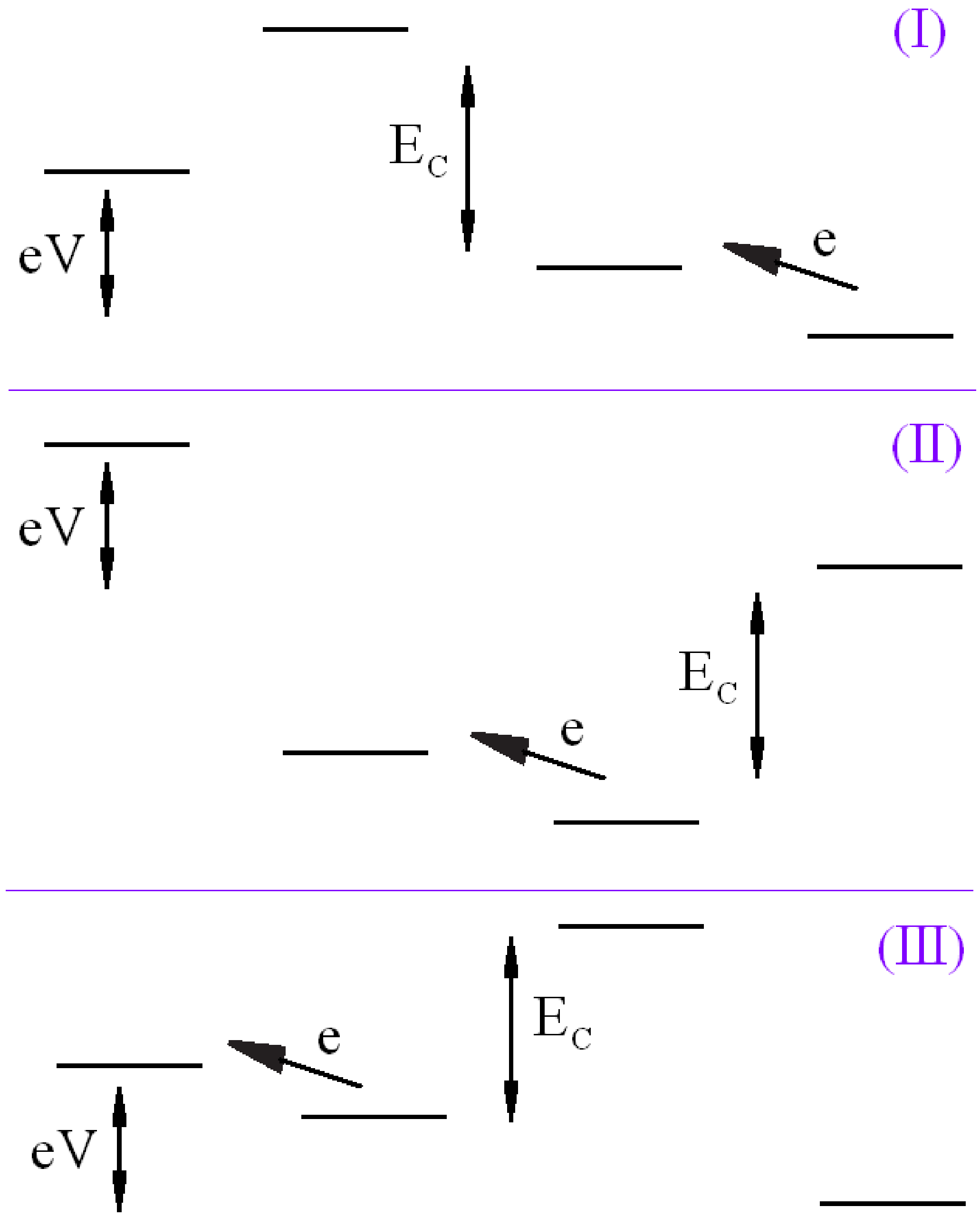
S. Toyabe et al., Nature Physics 2010

"watch and move"



D. Averin, M. Mottonen, and J. P., PRB 84, 245448 (2011)

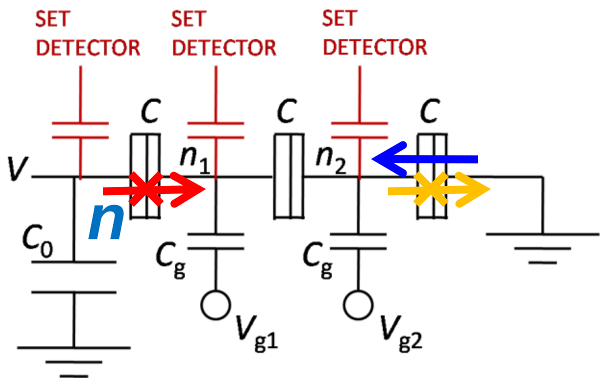
G. Schaller et al., PRB 84, 085418 (2011)
P. Strassberg et al., PRL 110, 040601 (2013).
J. Bergli et al., arXiv:1306.2742

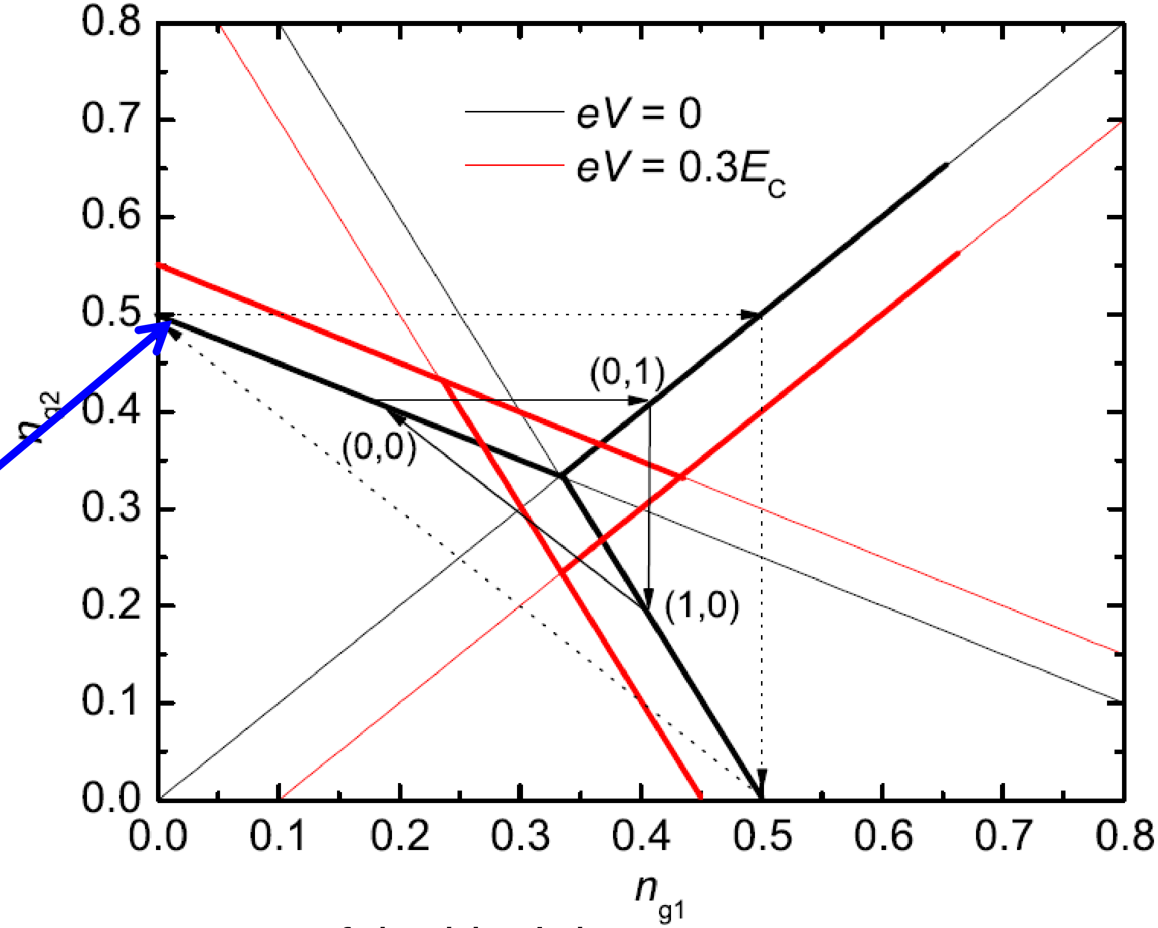


(I)
E_C
eV
e
(II)
eV
E_C
e
(III)
E_C
e
eV

Demon strategy

Adiabatic "informationless" pumping: $W = eV$ per cycle.
Ideal demon: $W = 0$





Energy costs for the transitions:

$$\Delta U_{1,+} = E_C/2 - eV/3$$

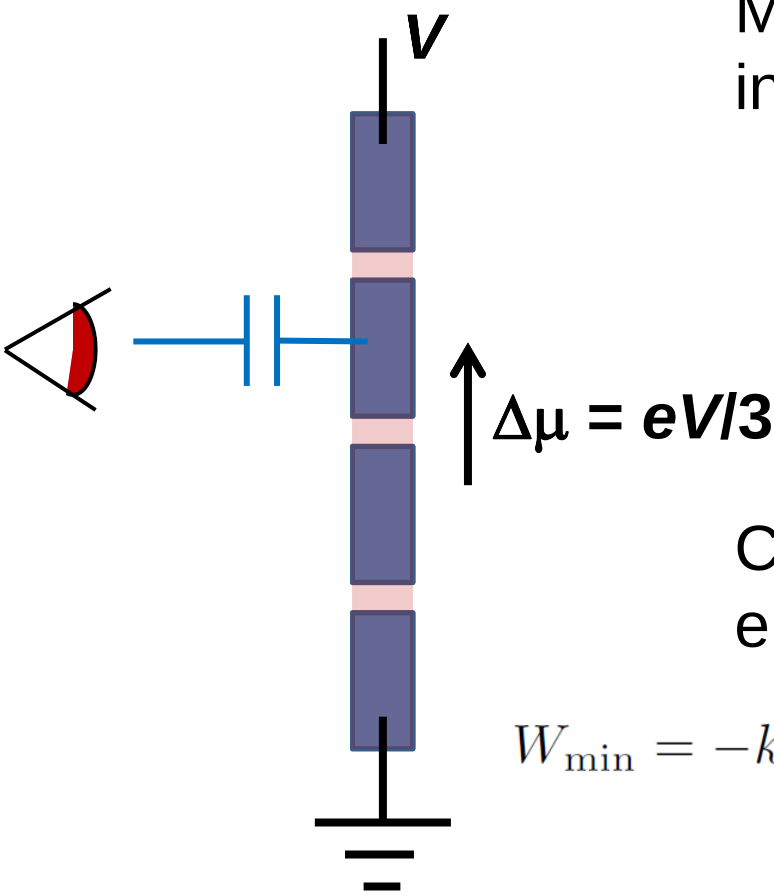
$$\Delta U_{3,-} = eV/3$$

Rate of return (0,1)->(0,0) determined by the energy "cost" $-eV/3$. If $\Gamma(-eV/3) << \tau^{-1}$, the demon is "successful". Here τ^{-1} is the bandwidth of the detector.

Power of the ideal demon:

$$P = (eV/3)\Gamma(eV/3)$$

Information to energy in MD



Measurements repeated at time intervals τ

$$p_1(\tau) = p_1^{\mathrm{eq}}(1 - e^{-\Gamma_\Sigma \tau})$$

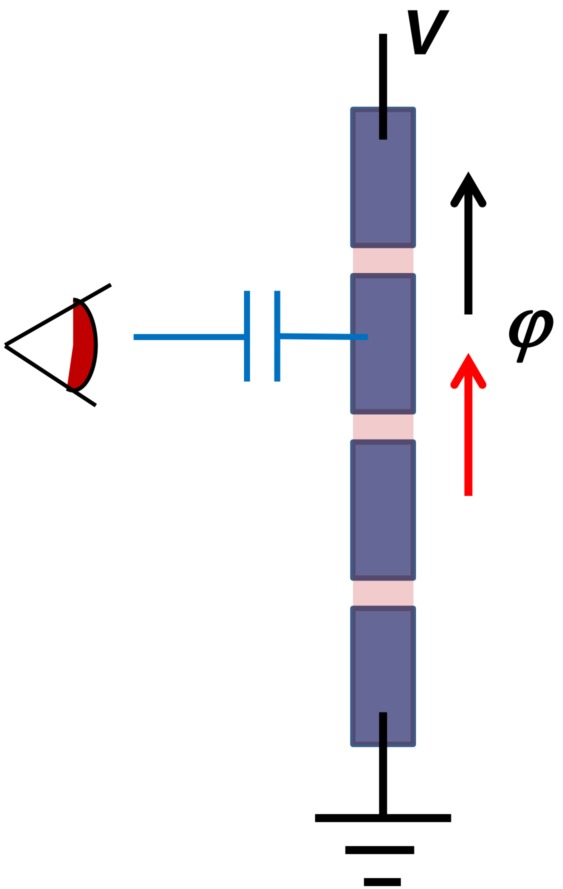
$$\langle \Delta U \rangle = p_1(\tau)\Delta\mu$$

Copying the measurement result costs energy (Landauer, Bennett, Sekimoto)

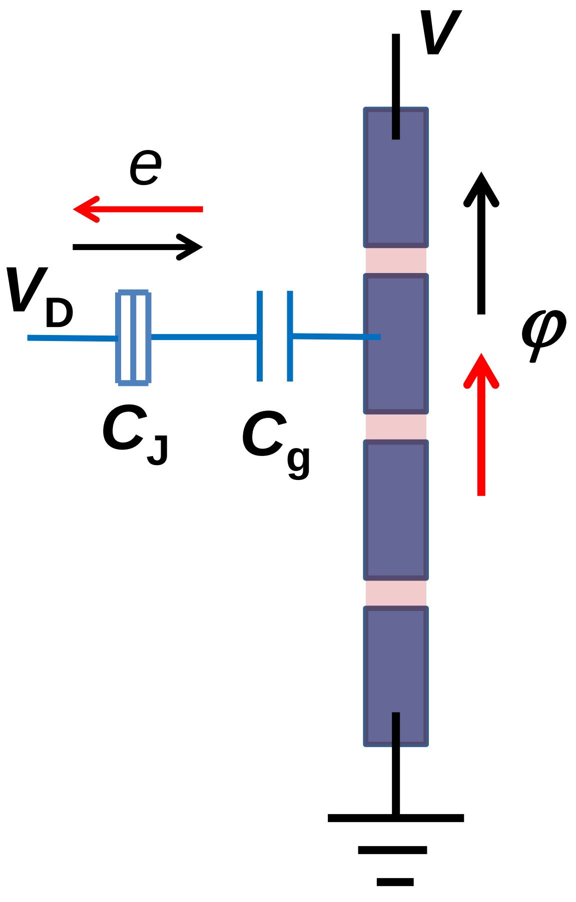
$$W_{\min} = -k_B T[(1 - p_1(\tau))\ln(1 - p_1(\tau)) + p_1(\tau)\ln(p_1(\tau))]$$

$$\langle \Delta U \rangle / W_{\min} = \frac{1}{1 + \frac{k_B T}{\Delta\mu}(1 + e^{\Delta\mu/k_B T})\ln(1 + e^{-\Delta\mu/k_B T})} \leq 1$$

Single-electron box as detector



Single-electron box as detector



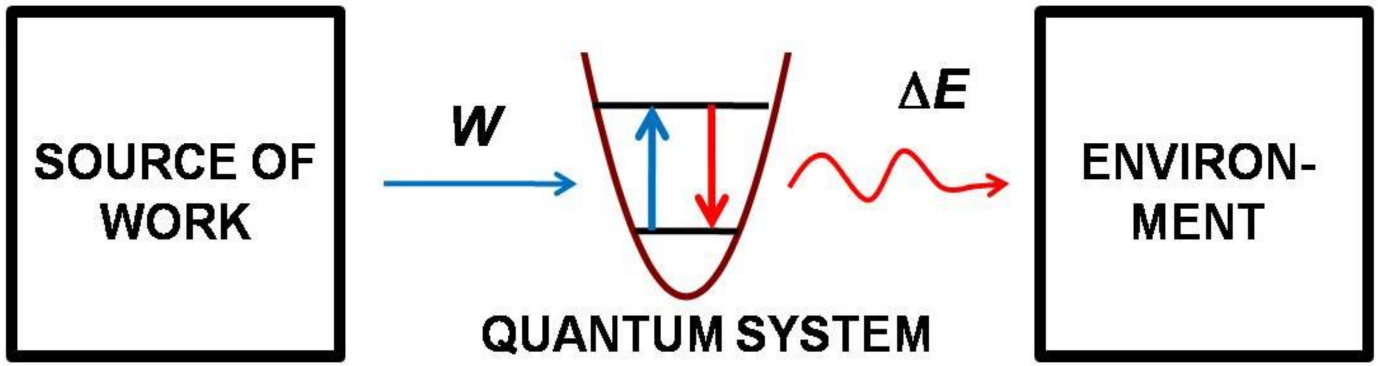
$$\delta E = e\frac{C_g}{C_\Sigma}(\varphi^{\mathrm{out}} - \varphi^{\mathrm{in}})$$

$$\delta E \geq eV/3$$

Distribution of dissipation in a quantum system: calorimetry

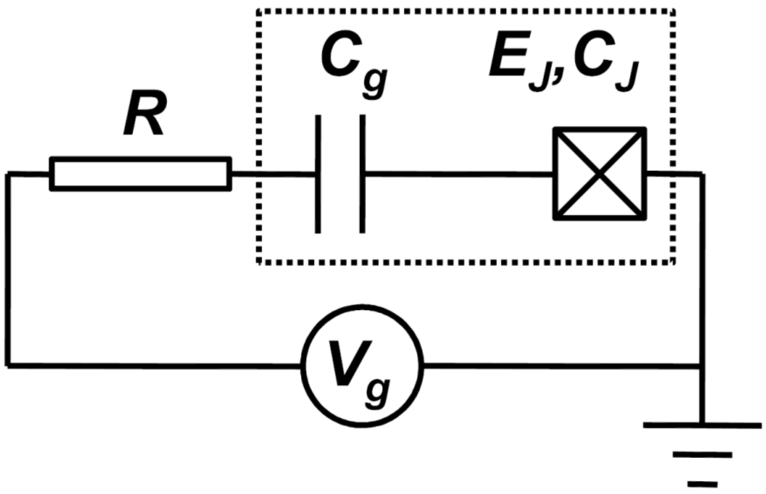
Work in a quantum system?

We propose to measure the photons exchanged between the system and environment



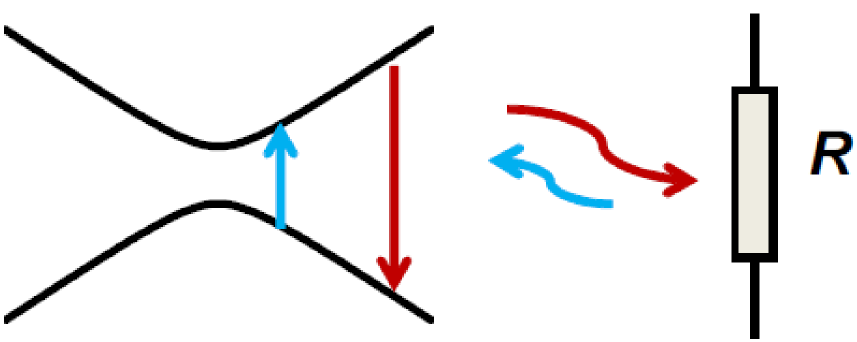
arXiv:1212.5808

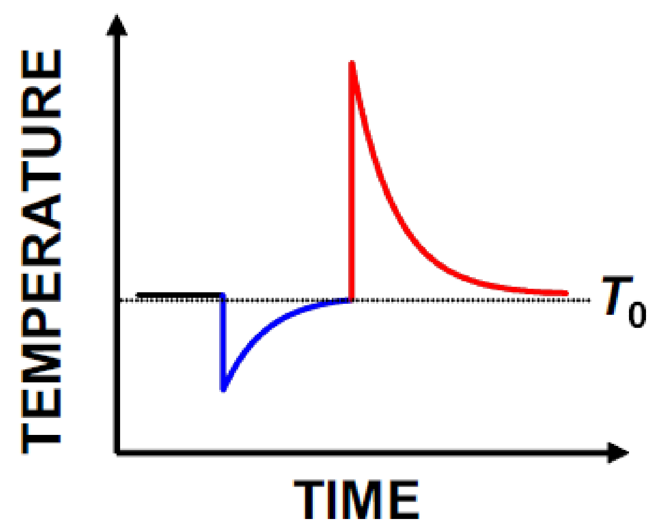
Measurement of work distribution of a two-level system (CPB)



Calorimetric measurement: Measure temperature of the resistor after relaxation.

”Typical parameters”:
$\Delta T_R \sim$ 10 mK over 0.01 - 1 ms time

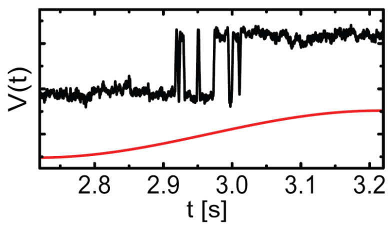


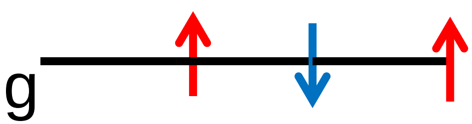


Quantum jump approach for dissipation

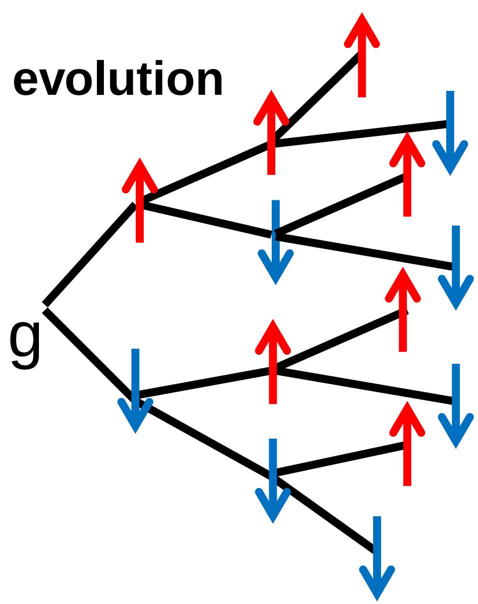
The jump method: Dalibard et al., PRL **68**, 580 (1992); Plenio and Knight, RMP **70**, 101 (1998). We apply the jump method to a driven qubit

Classical evolution

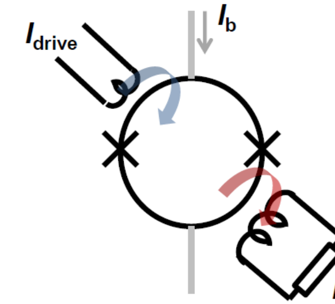


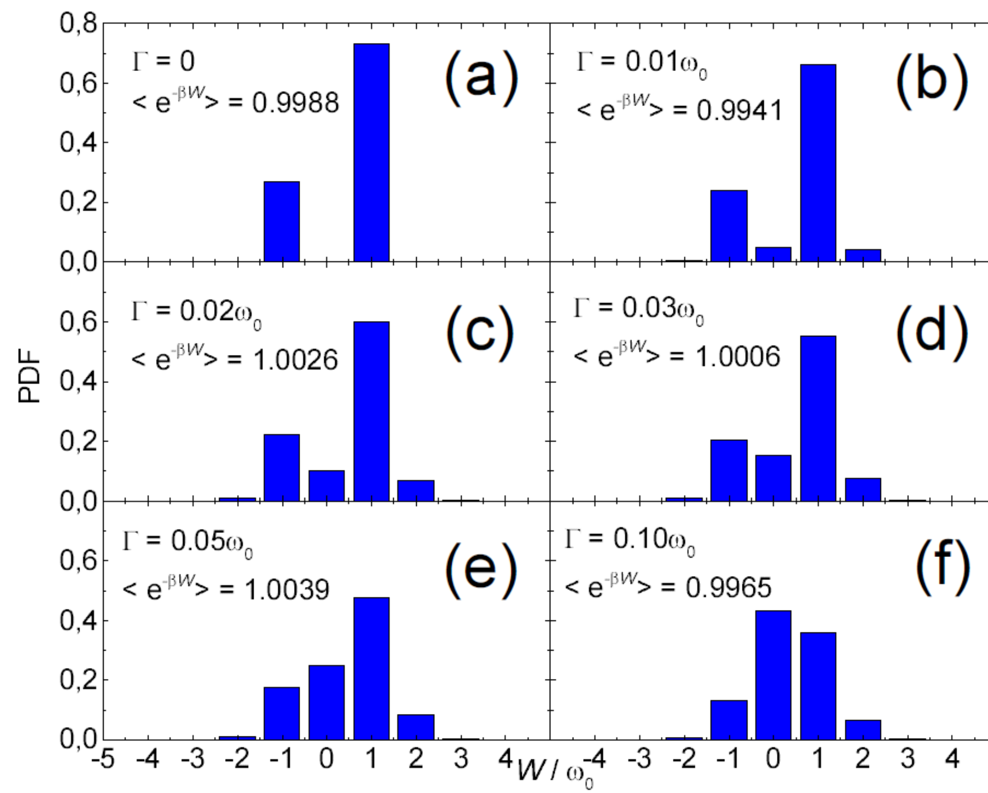


Quantum evolution



π pulse with dissipation





F. Hekking and J.P., arXiv:1305.5207

Summary

Distribution of dissipation measured and analyzed in a single-electron box via charge counting statistics. Possibility to realize Maxwell's demon

Work and dissipation in a quantum two-level system: a calorimetric measurement proposed for a superconducting qubit, quantum jump analysis briefly presented