Resolution Effects on the Distribution of Current through a Resonant Level

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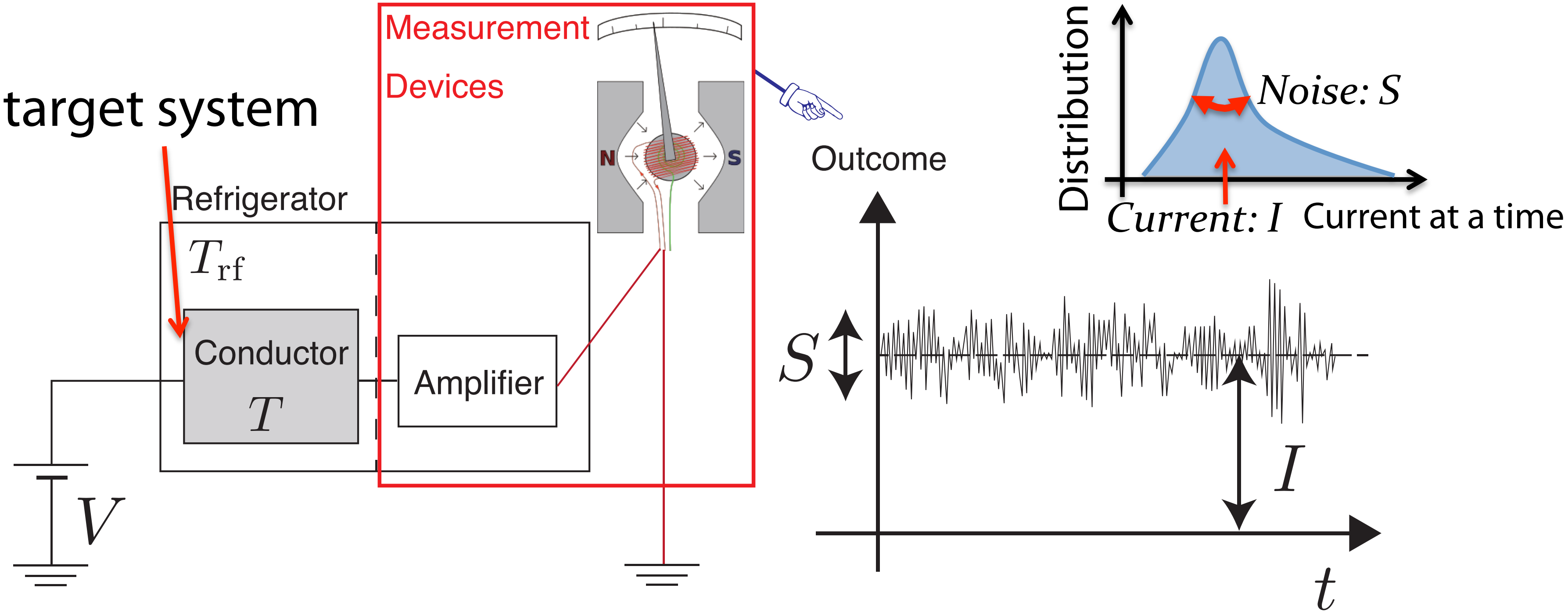
arXiv:1307.7535

Current Distribution and Resolution



Noise Measurement Intrinsic information about the target system

Shot Noise *Low-energy excitation/interaction*

Johnson Noise *Electric Temperature*

How the resolution affects the current and noise?

Johnson Noise

Johnson-Nyquist (J-N) relation

$$S_I = 2k_BTG$$
$$S_V = 2k_BTR$$

Left: Current/Voltage Noise in equilibrium

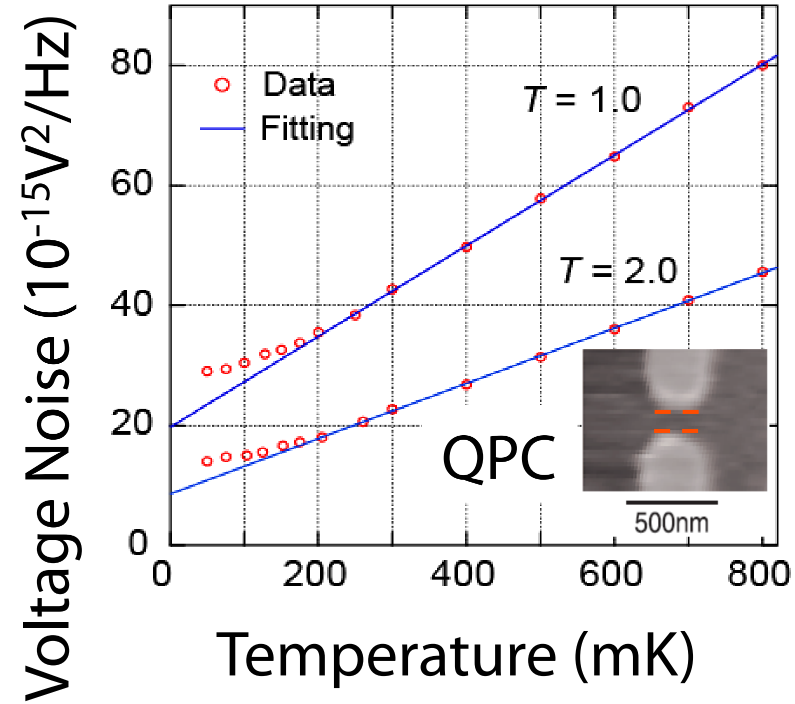
Right: Temperature x Conductance/Resistance

$$G = R^{-1} = \lim_{V\to 0} I/V$$

John B. Johnson, *Nature* **119**, 50 (1927); *Phys. Rev.* **32**, 97 (1928).

Harry Nyquist, *Phys. Rev.* **32**, 110 (1928).

Apparent Deviation from the expected value of Johnson noise at very low T.



M. Hashisaka, Y. Yamauchi, S. Nakamura, S. Kasai, K. Kobayashi, and T. Ono, *J. Phys.: Conf. Ser.* **109**, 012013 (2008).

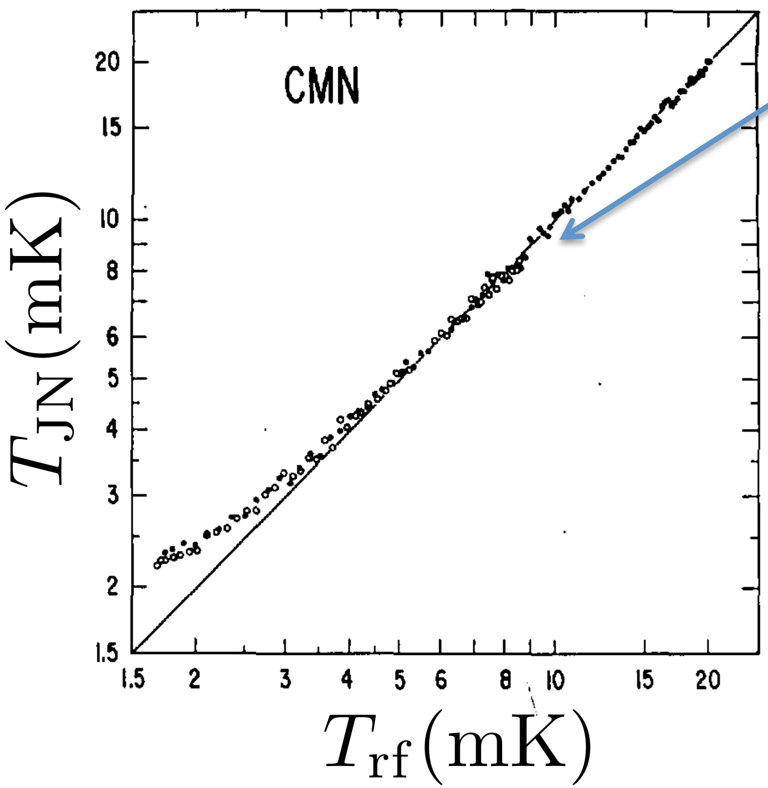
Fluctuatioin-Dissipation Theorem

Deviations appear below 200 mK

Deviations at Early 1970's

R. A. Webb, R. P. Giffard, and J. C. Wheatley, *J. Low. Temp. Phys.* **13**, 383 (1973).

cerium magnesium nitrate



1. J-N relation
2. $T_{\mathrm{rf}} = T$

$\longrightarrow T_{\mathrm{JN}} = T_{\mathrm{rf}}$

Deviations appear below 5.5 mK

One of these conditions is not satisfied!

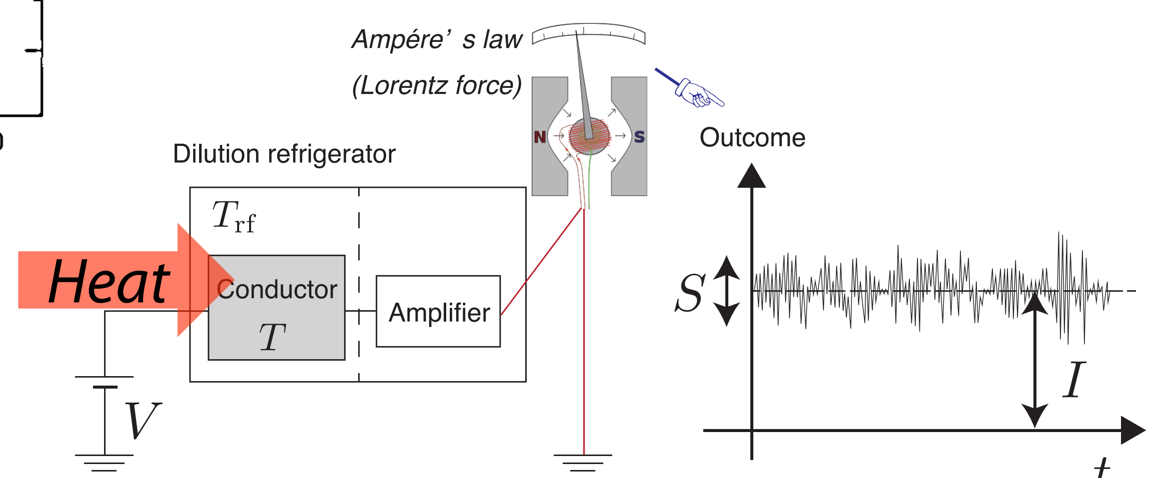
A possible explanation is a Heat Leak.

Second condition is not satisfied.

Simple! But it has not been directly confirmed.

Noise temperature

$$T_{\mathrm{JN}} \equiv S_I/2k_BG$$



Motivation

Johnson-Nyquist relation

intrinsic intrinsic

$$S_0 = 2k_B T G_0$$

$S_0 \not\parallel ?$ S measured

$G_0 \not\parallel ?$ G measured

"Intrinsic Noise" and "Intrinsic Current" of the target system.

But, we obtain the information about the current and noise with measurement devices which may affect outputs.

Can we obtain the intrinsic values by ~~ideal~~ actual measurement devices, in principle?

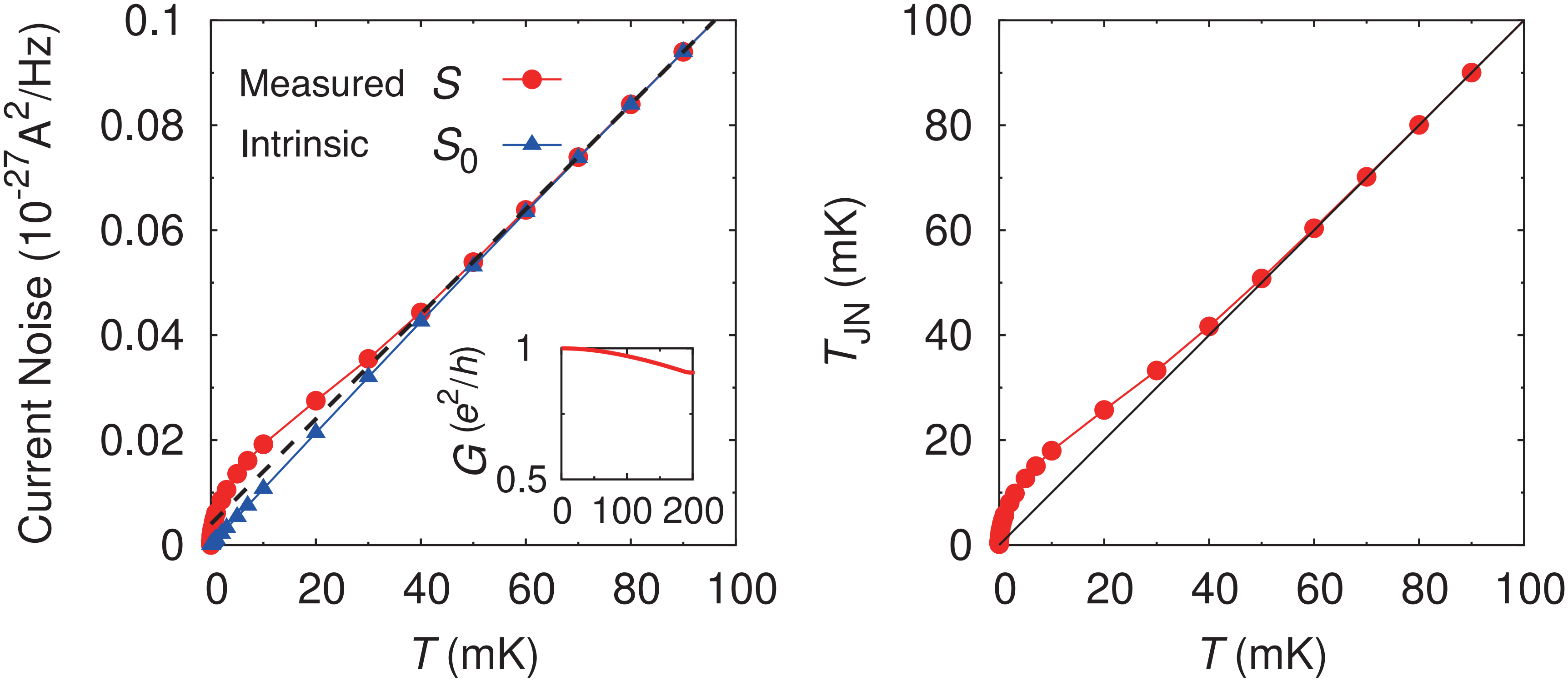
This problem is not clear!

"Resolution" has a close relation to the fact that we always have a smallest detectable change in every measurements.

Our Main Results

Limited Resolution does not affects the measured Current but, increases the measured Noise! *do not always satisfy the J-N relation*

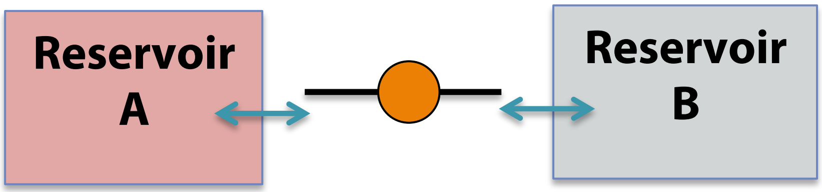
Resonant level model (Single channel QPC with perfect transmission)



Limited resolution becomes a possible explanation of deviations

Resonant Level Model

Spinless Electrons



$$\hat{\mathcal{H}}(t) = \hat{H}_0 + \hat{V}(t)$$

$$\hat{H}_0 = \hat{H}_\mathrm{A} + \hat{H}_\mathrm{B} + \hat{H}_\mathrm{S}$$

$$\hat{H}_\mathrm{A} = \sum_{x\in\mathrm{A}} \varepsilon_x^\mathrm{A} \hat{c}_x^\dagger \hat{c}_x$$

$$\hat{H}_\mathrm{B} = \sum_{x\in\mathrm{B}} \varepsilon_x^\mathrm{B} \hat{c}_x^\dagger \hat{c}_x$$

$$\hat{H}_\mathrm{S} = \varepsilon_0 \hat{d}^\dagger \hat{d}$$

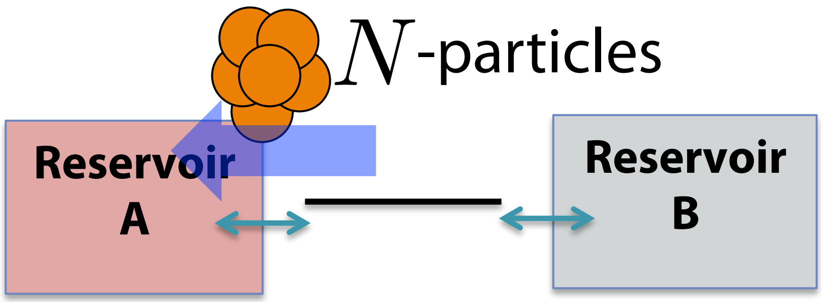
$$\hat{V}(t) = \hat{V}_\mathrm{A}(t) + \hat{V}_\mathrm{B}(t)$$

$$\hat{V}_X(t) = \sum_{x\in\mathrm{X}} (t_\mathrm{X}\theta(t)\hat{d}^\dagger \hat{c}_x + \mathrm{H.c.}) \qquad \text{for } \mathrm{X} = \mathrm{A}, \mathrm{B},$$

Density matrix at t=0

$$\hat{\rho}(0) = \frac{\mathrm{e}^{-\beta(\hat{H}_\mathrm{A} - \mu_\mathrm{A}\hat{N}_A)}}{\mathrm{Tr}\left[\mathrm{e}^{-\beta(\hat{H}_\mathrm{A} - \mu_\mathrm{A}\hat{N}_A)}\right]} \otimes \frac{\mathrm{e}^{-\beta(\hat{H}_\mathrm{B} - \mu_\mathrm{B}\hat{N}_B)}}{\mathrm{Tr}\left[\mathrm{e}^{-\beta(\hat{H}_\mathrm{B} - \mu_\mathrm{B}\hat{N}_B)}\right]} \otimes \hat{\rho}_\mathrm{S}(0)$$

Model of Current Measurement: Two-point measurement model



$\mathcal{T}$ measurement time

Δ particle measurement resolution

$$I = e\frac{dN}{d\mathcal{T}}$$

N is the particle number change of reservoir A during T.

The probability that $N = k\Delta$ is given by

k is a integer

$$\mathcal{P}(k;\mathcal{T},\Delta) = \mathrm{Tr}[\sum_l \hat{M}^{\dagger}_{k,l}(\mathcal{T},\Delta)\hat{M}_{k,l}(\mathcal{T},\Delta)\hat{\rho}(0)]$$

$$\hat{M}_{k,l}(\mathcal{T},\Delta) = \hat{P}_{l+k}(\Delta)\hat{U}(\mathcal{T},0)\hat{P}_l(\Delta)$$

Time evolution operator Projection Operator

Projection Operator

Projection Operator

of a particle number measurement in reservoir A

$$\hat{P}_k(\Delta) \equiv \int_{\chi_k - \frac{\Delta}{2}}^{\chi_k + \frac{\Delta}{2}} dx \delta(x - \hat{N}_A)$$

Outcome

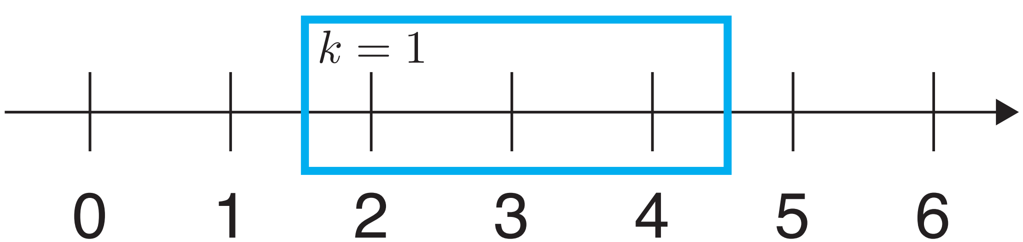
$$\chi_k \equiv \chi_0 + k\Delta$$

zero point deviation

Δ means the smallest detectable change in the particle number measurement.

$k = 1, \Delta = 3$

Outcome is 3.



Eigen values of $\hat{N}_A$

$\Delta = 1 \quad \chi_0 = 0$

M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. 81, 1665 (2009).

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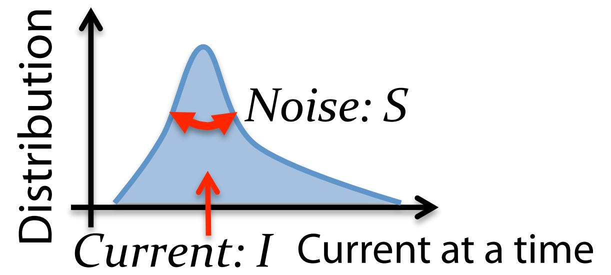
Cumulant Generating Function

$$\mathcal{P}(k;\mathcal{T},\Delta) = \mathrm{Tr}[\hat{M}_k^\dagger(\mathcal{T},\Delta)\hat{M}_k(\mathcal{T},\Delta)\hat{\rho}(0)]$$



$$\mathcal{C}(\lambda;\mathcal{T},\Delta) \equiv \partial_\mathcal{T} \ln\left[\sum_k \mathrm{e}^{ik\lambda}\mathcal{P}(k;\mathcal{T};\Delta)\right]$$

Cumulant Generating Function

$\mathcal{T}\Gamma \gg 1$

$\Gamma = 2\pi|t|^2\sigma_0$

$$\sim \partial_\mathcal{T} \ln\left[\sum_m \frac{\sin(\frac{\lambda+2\pi m}{2})}{\frac{\lambda+2\pi m}{2}} \exp[i\frac{\delta}{\Delta}2\pi m + \mathcal{T}\mathcal{C}_0(\frac{\lambda+2\pi m}{\Delta})]\right]$$

$\mathcal{C}_0(\lambda)$ is a Levitov-Lesovik type CGF obtained from Full Counting Statistics.

L. S. Levitov and G. B. Lesovik,JETP Lett. 58, 230 (1993); arXiv cond-mat/9401004 (1994).

$\delta \equiv N_A^0 - \chi_0 \bmod \Delta$ Degrees of Freedom of the initial density matrix within the resolution

Initial particle number of reservoir A

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Cumulant Generating Function

$\delta \equiv N_A^0 - \chi_0 \mod \Delta$ We cannot obtain the initial information practically

Random averaging (analogy of quenched random systems).

$$\langle \cdots \rangle_\delta \equiv \int_0^\Delta \frac{d\delta}{\Delta} \cdots$$

$$\mathcal{C}(\lambda; \mathcal{T}, \Delta) \equiv \partial_{\mathcal{T}} \langle \ln \Big[\sum_m \frac{\sin(\frac{\lambda+2\pi m}{2})}{\frac{\lambda+2\pi m}{2}} \exp[i \frac{\delta}{\Delta} 2\pi m + \mathcal{T} \mathcal{C}_0 (\frac{\lambda + 2\pi m}{\Delta})] \Big] \rangle_\delta$$

$\Delta = 1$ *We can distinguish electrons, one by one.*

$\mathcal{C}(\lambda; \mathcal{T}, 1) = \mathcal{C}_0(\lambda)$ Full Counting Statistics

Our quantum measurement scheme is an extension of FCS.

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Current and Noise

Measured current $I = I_0$ Intrinsic ← Full counting statistics independent of $\mathcal{T}\ \Delta$

Measured noise $S = S_0 + \langle \Delta S \rangle_\delta$

Intrinsic Excess noise

$$S_0|_{V=0} = 2k_B T G_0$$ J-N relation is satisfied!

$$\langle \Delta S \rangle_\delta \geq 0$$ Always positive

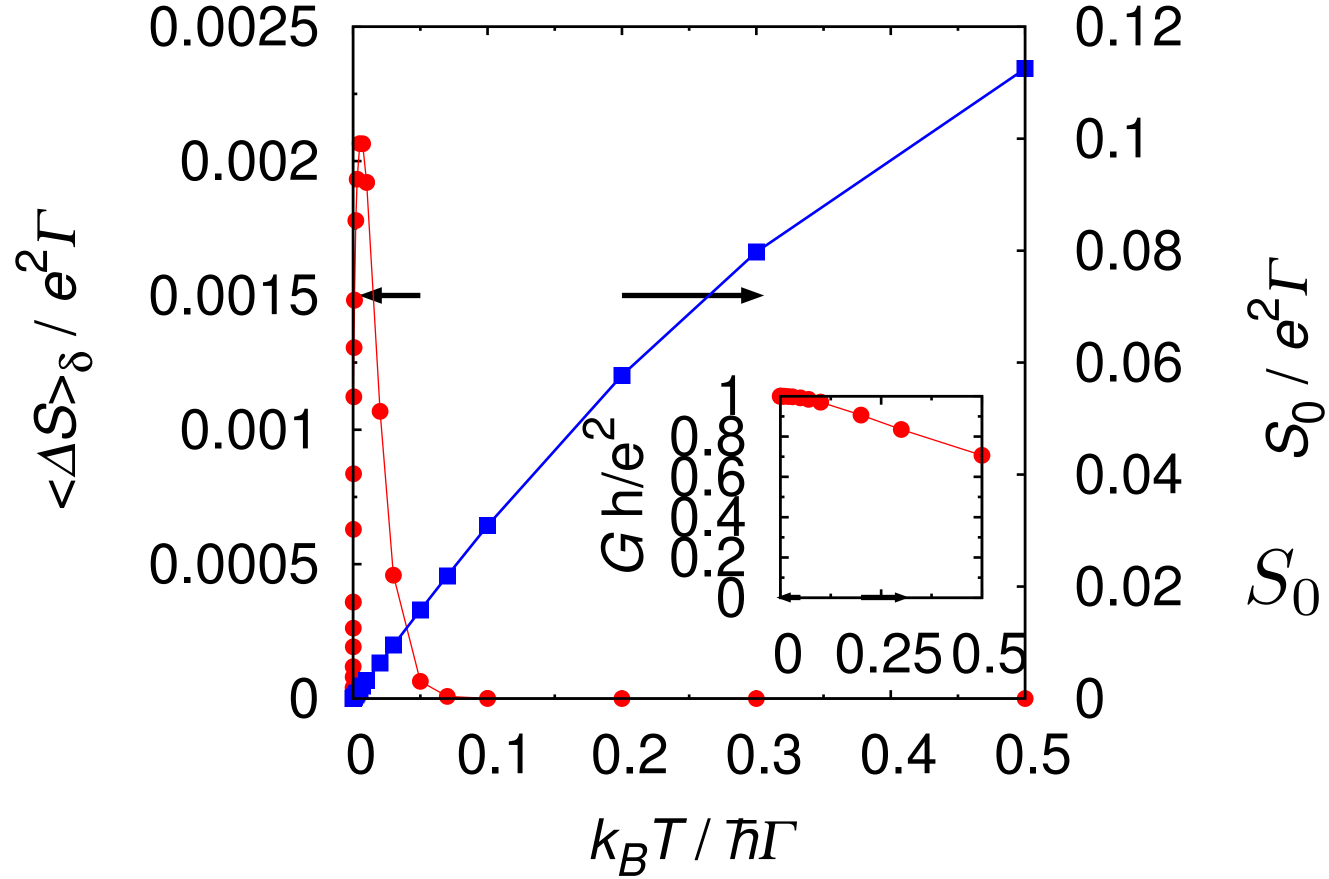
$$G_0 \equiv \lim_{V \to 0} \partial I_0 / \partial V$$

$$G \equiv \lim_{V \to 0} \partial I / \partial V = G_0$$

Johnson-Nyquist relation is not satisfied between S and G

$$\langle \Delta S \rangle_\delta \geq 0 \Rightarrow S|_{V=0} \geq 2k_B T G$$

Temperature Dependence



$\mathcal{T}\Gamma = 1000$

$\Delta = 10$

$\varepsilon_0 = 0$

$$S_0 = 2k_B T G$$

$$\langle \Delta S \rangle_\delta$$

- ✓ Strong enhancement at very low temperatures
- ✓ Cannot be calibrated by usual empirical methods!

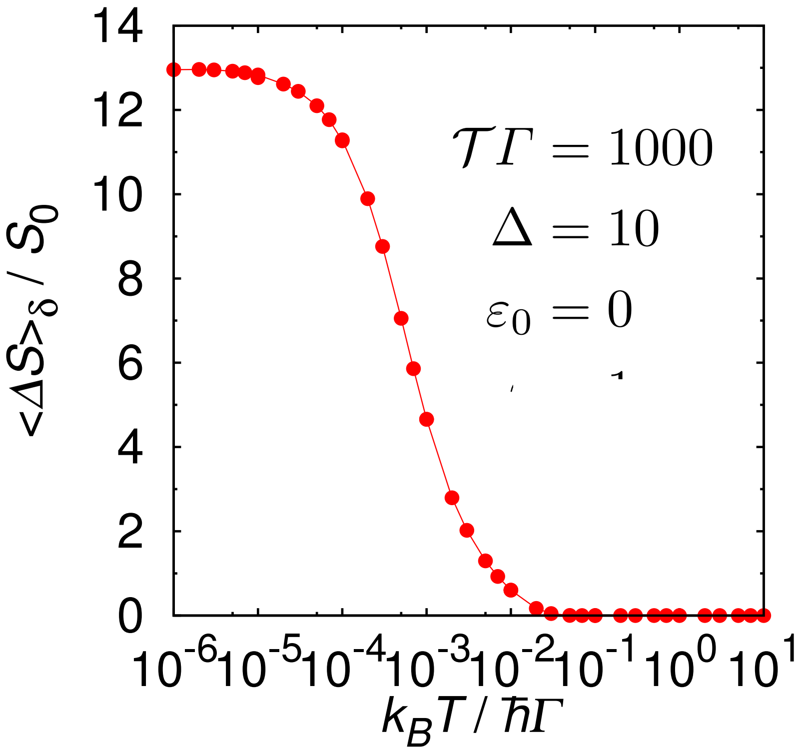
Deviation from J-N relation

Deviation from J-N relation between S and G

$$\frac{S}{2k_B TG} - 1 = \langle \Delta S \rangle_\delta / S_0$$

Measurable in experiment
Directly indicates the deviation

Ratio of Noises

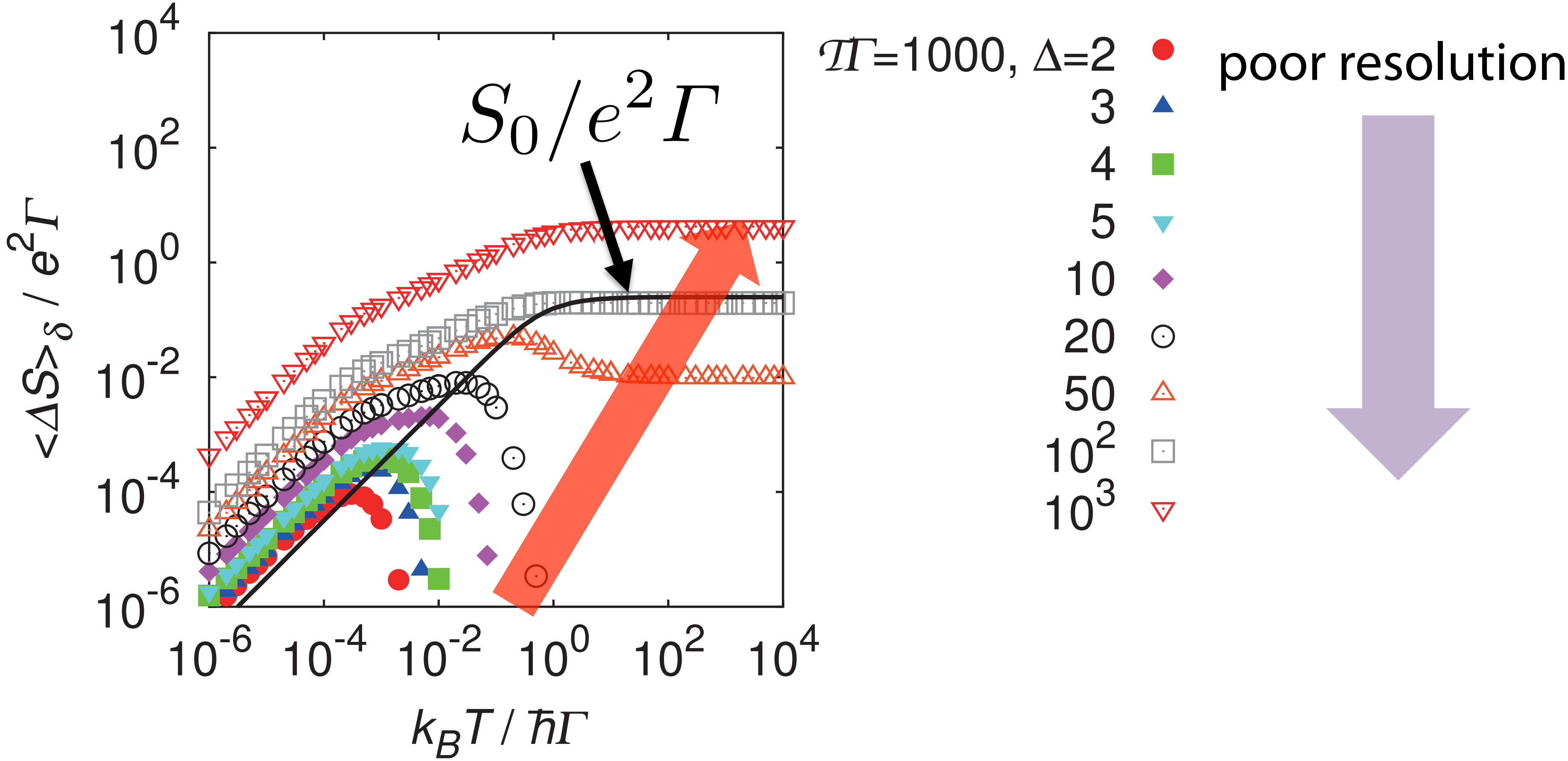


The deviation is enhanced and saturated at very low temperature

Saturated value of deviation

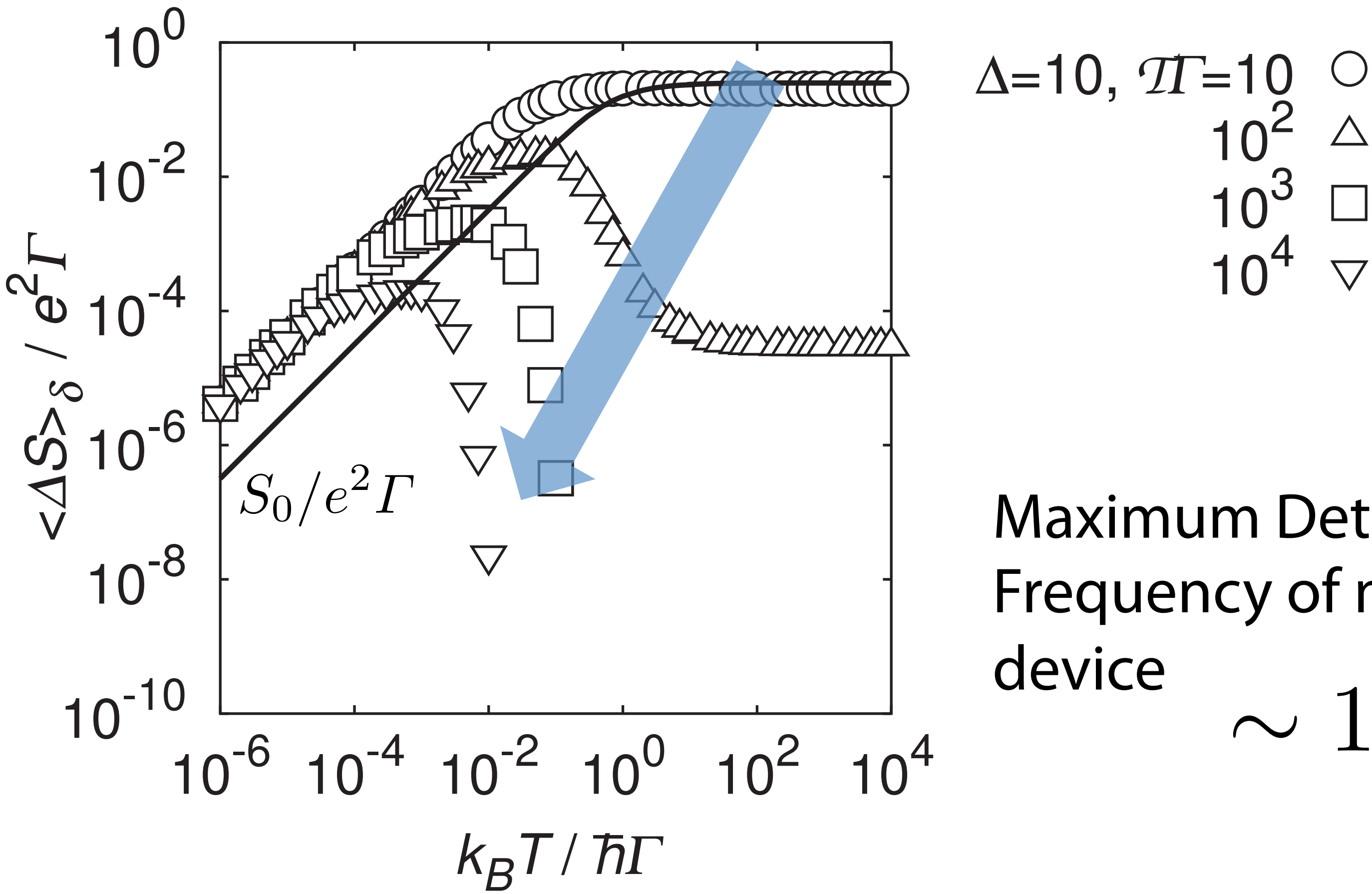
$$\sim \Delta$$

Delta Dependence



Excess noises are increased by increasing Delta.

T Dependence

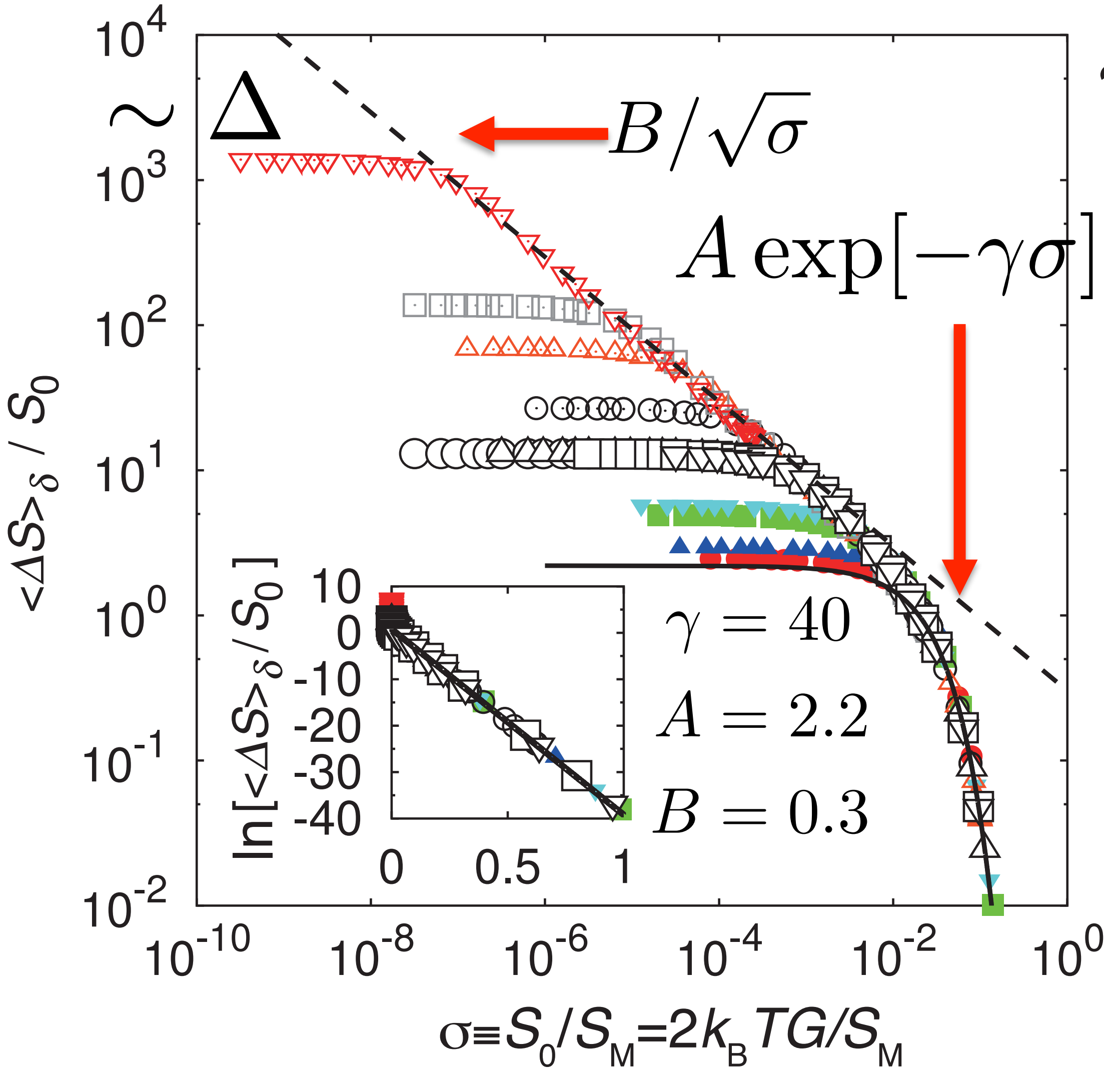


Long time measurement

Maximum Detectable Frequency of measurement device $\sim 1/\mathcal{T}$

Limitation of Frequency band of measurement device improves detectability of S_0

Single-parameter Scaling of Deviation



$$S_M \equiv (e\Delta)^2/\mathcal{T}$$

$$\sigma = 1$$

criterion whether the deviation practically appears or not.

Summary

Resolution effects on Current Statistics

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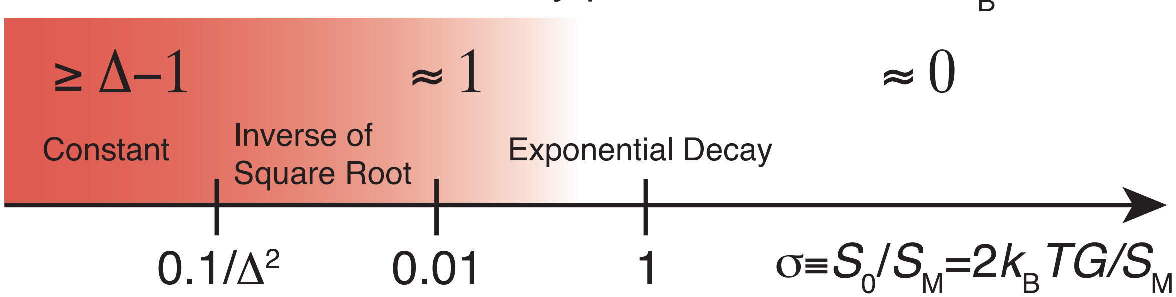
✓ Noise enhancement by limited resolution

✓ Deviation from Jonson-Nyquist relation

✓ Scaling behavior of deviation

Deviation From the Johnson-Nyquist Relation $S / 2k_B TG - 1$



✓ Consistent with experimental results