Interactions and charge fractionalization in an electronic Hong-Ou-Mandel interferometer

Claire Wahl, J. Rech, T. Jonckheere and T. Martin

IXth Rencontres du Vietnam, Quy-Nhon, August 8, 2013

CPT, Marseille

C. Wahl et al., arXiv:cond-mat/1307.5257

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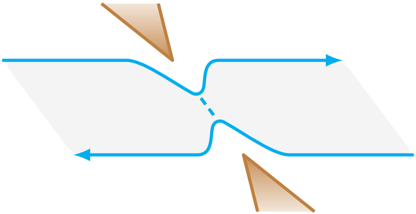
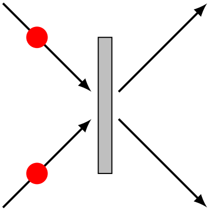
Electronic Quantum Optics

Electrons

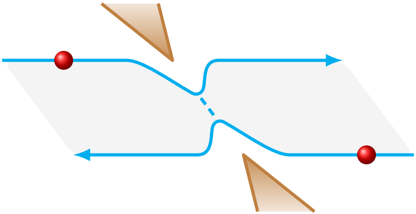


Photons



Electronic Quantum Optics

Electrons



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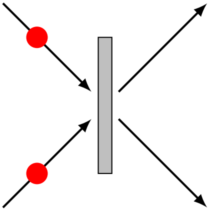
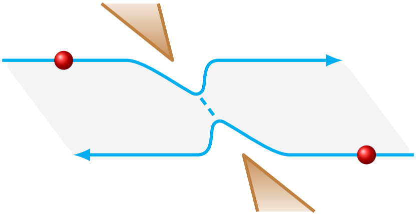


Electronic Quantum Optics

Electrons



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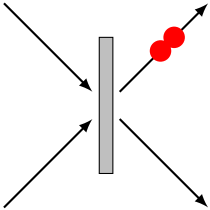
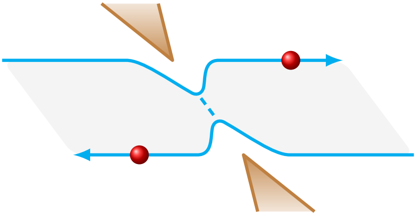


- bunching

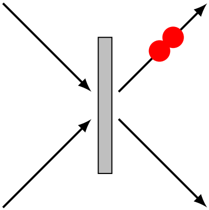
Electronic Quantum Optics

Electrons



- anti-bunching

Photons



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Electronic Quantum Optics

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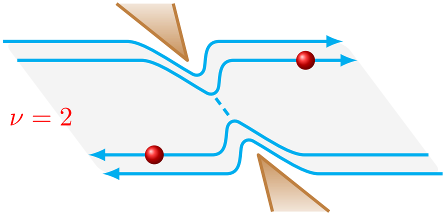

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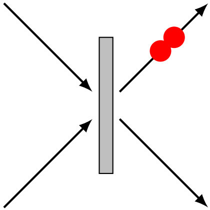

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Electronic Quantum Optics

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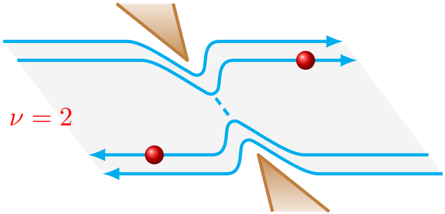
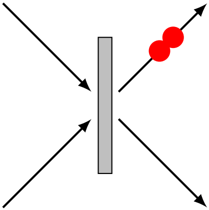


- anti-bunching
- effect of interactions ?

Photons



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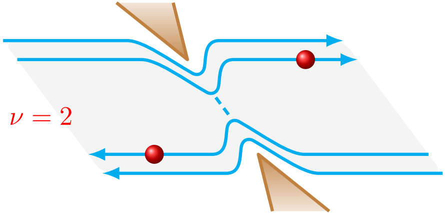
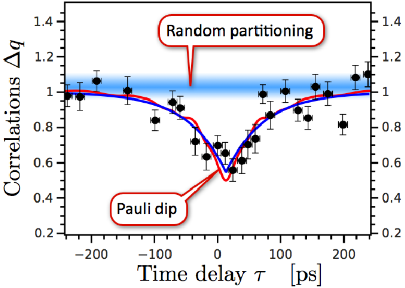
Electronic Quantum Optics

Electrons



- anti-bunching
- effect of interactions ?

Experimental results



- low contrast

E. Bocquillon et al., Science 339, 2013

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Bosonization framework

- Finite temperature: $\Theta = 0.1\text{K}$

J. von Delft and H. Schoeller, Annalen Phys., 1998

Bosonization framework

- Finite temperature: $\Theta = 0.1\mathrm{K}$
- 2 channels per edge :

$$H_{\text{kin}} = \frac{\hbar}{\pi} \sum_{r=R,L} \int dx \; v_1 \left(\partial_x \phi_{1,r}\right)^2 + v_2 \left(\partial_x \phi_{2,r}\right)^2 \tag{1}$$

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Bosonization framework

- Finite temperature: $\Theta = 0.1\text{K}$
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$$H_{\text{kin}} = \frac{\hbar}{\pi} \sum_{r=R,L} \int dx \, v_1 \left(\partial_x \phi_{1,r}\right)^2 + v_2 \left(\partial_x \phi_{2,r}\right)^2 \tag{1}$$

- coupled via Coulomb interaction :

$$H_{\text{int}} = 2\frac{\hbar}{\pi} u \sum_{r=R,L} \int dx (\partial_x \phi_{1,r})(\partial_x \phi_{2,r}) \tag{2}$$

J. von Delft and H. Schoeller, Annalen Phys., 1998

Mixing angle θ

$$H_{\rm kin} + H_{\rm int} = \frac{\hbar}{\pi} \sum_{r=R,L} v_+ (\partial_x \phi_{+,r})^2 + v_- (\partial_x \phi_{-,r})^2 \qquad (3)$$

Mixing angle θ

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- Eigenmodes $\phi_{+,r}$ and $\phi_{-,r}$

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No coupling: $\theta = 0$

$\phi_{+,r} = \phi_{1,r}$
$\phi_{-,r} = \phi_{2,r}$
$v_+ = v_1$
$v_- = v_2$

Mixing angle θ

$$H_{\rm kin} + H_{\rm int} = \frac{\hbar}{\pi} \sum_{r=R,L} v_+ (\partial_x \phi_{+,r})^2 + v_- (\partial_x \phi_{-,r})^2 \qquad (3)$$

- Eigenmodes $\phi_{+,r}$ and $\phi_{-,r}$
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No coupling: $\theta = 0$

$\phi_{+,r} = \phi_{1,r}$
$\phi_{-,r} = \phi_{2,r}$
$v_+ = v_1$
$v_- = v_2$

Strong coupling: $\theta = \pi/4$

$\phi_{+,r} = \sqrt{2}/2(\phi_{1,r} + \phi_{2,r})$
$\phi_{-,r} = \sqrt{2}/2(\phi_{1,r} - \phi_{2,r})$
$v_1 = v_2 = v_F = 1$
$v_\pm = 1 \pm u$

Injecting a single electron

$$|\phi_L\rangle = \int dx \varphi_L(x) \psi^\dagger_{s,L}(x+L,0)|0\rangle \tag{4}$$

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Exponential packet

$$\varphi_L(x) = \sqrt{2\Gamma} e^{-i\epsilon_0 x} e^{-\Gamma x} \theta(x)$$

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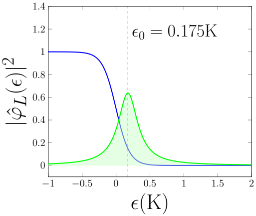

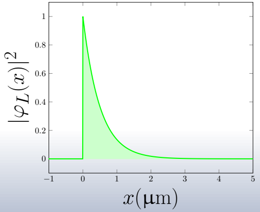

Exponential packet

$$\varphi_L(x) = \sqrt{2\Gamma} e^{-i\epsilon_0 x} e^{-\Gamma x} \theta(x)$$

wide in energy space

$\gamma = \epsilon_0/\Gamma = 1$

$\epsilon_0 = 175$mK

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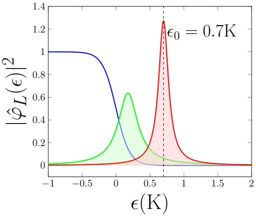

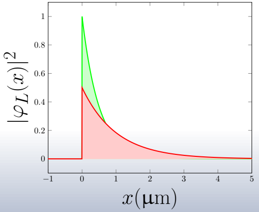

Exponential packet

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wide in energy space

$\gamma = \epsilon_0/\Gamma = 1$

$\epsilon_0 = 175$mK

energy-resolved

$\gamma = 8$

$\epsilon_0 = 0.7$K

Charge fractionalization

$$q_{s,L}(x,t) = e/\pi\partial_x\phi_{s,L}(x,t) \tag{5}$$

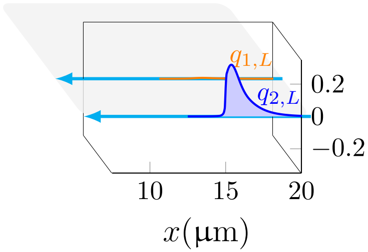
E. Berg et al., PRL 102, 2009 P. Degiovanni et al., PRB 81, 2010

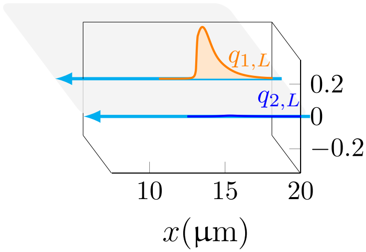
Charge fractionalization

$$q_{s,L}(x,t) = e/\pi \partial_x \phi_{s,L}(x,t) \quad (5)$$

Outer injection

Inner injection





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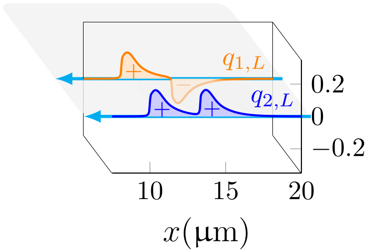
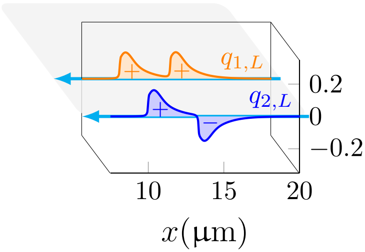


Inner injection



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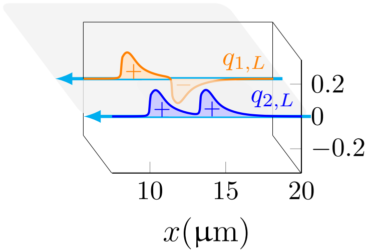

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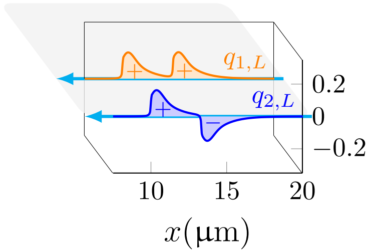

- fast charged mode: $\oplus$ $\oplus$ $\oplus$ $\oplus$

E. Berg et al., PRL 102, 2009 P. Degiovanni et al., PRB 81, 2010

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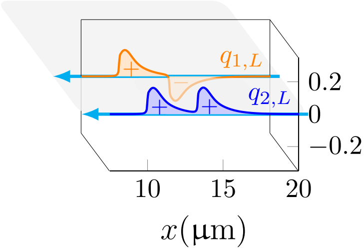
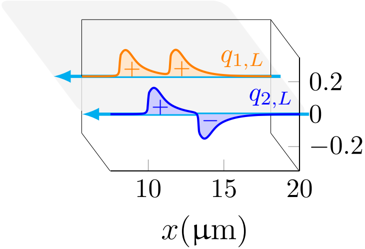


Inner injection



- fast charged mode: $\oplus$ $\oplus$ $\oplus$ $\oplus$
- slow neutral mode: $\oplus$ $\ominus$ $\ominus$ $\oplus$

E. Berg et al., PRL 102, 2009   P. Degiovanni et al., PRB 81, 2010

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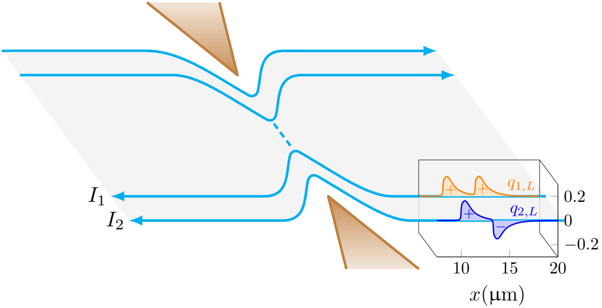
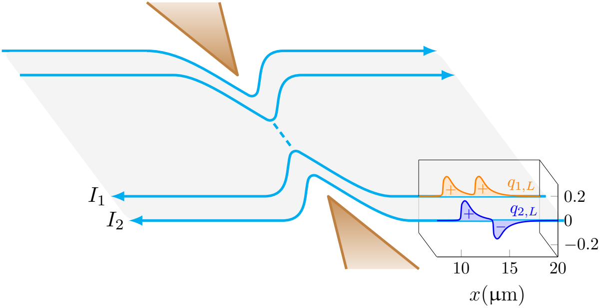
HBT setup



- Inner (outer) injections
- Backscattering: inner channels

HBT setup



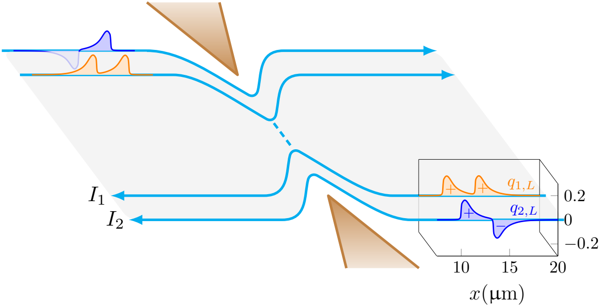
- Inner (outer) injections
- Backscattering: inner channels

$$S_{\text{HBT}} = \int dt dt' \langle I_1(t) I_1(t') \rangle - \langle I_1(t) \rangle \langle I_1(t') \rangle \tag{6}$$

averages performed on $|\phi_L\rangle$

H. Lee and L. Levitov, arXiv:cond-mat/9312013, 1993

HOM setup



- Inner (outer) injections
- Backscattering: inner channels
- Time delay δT

$$S_{\mathrm{HOM}} = \int dtdt' \langle I_1(t) I_1(t') \rangle - \langle I_1(t) \rangle \langle I_1(t') \rangle \tag{6}$$

averages performed on $|\phi_L\rangle \otimes |\phi_R\rangle$

G. Burkard and D. Loss, PRL 91, 2003
Ol'khovskaya et al., PRL 101, 2008

HBT Calculation

$$S_{\text{HBT}} = -\frac{2e^2\mathcal{R}\mathcal{T}}{(2\pi a)^3\mathcal{N}}\text{Re}\left\{\int dy_L\, dz_L\; \varphi_L(y_L)\varphi_L^*(z_L) g(0, z_L - y_L)\right.$$
$$\left.\int dt\, d\tau\, \text{Re}\left[g(\tau,0)^2\right]\left[\frac{h_s(t; y_L + L, z_L + L)}{h_s(t+\tau; y_L + L, z_L + L)} - 1\right]\right\} \quad (7)$$

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$$\mathcal{N} = \langle\phi_L|\phi_L\rangle$$

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$$g(t,x) = \left[\frac{\sinh\left(i\frac{\pi a}{\beta v_+}\right)}{\sinh\left(\frac{ia+v_+t-x}{\beta v_+/\pi}\right)}\frac{\sinh\left(i\frac{\pi a}{\beta v_-}\right)}{\sinh\left(\frac{ia+v_-t-x}{\beta v_-/\pi}\right)}\right]^{1/2} \qquad (8)$$

$$h_s(t;x,y) = \left[\frac{\sinh\left(\frac{ia-v_+t+x}{\beta v_+/\pi}\right)}{\sinh\left(\frac{ia+v_+t-y}{\beta v_+/\pi}\right)}\right]^{\frac{1}{2}}\left[\frac{\sinh\left(\frac{ia-v_-t+x}{\beta v_-/\pi}\right)}{\sinh\left(\frac{ia+v_-t-y}{\beta v_-/\pi}\right)}\right]^{\frac{3}{2}-s} \qquad (9)$$

HOM Calculation

$$S_{\text{HOM}}(\delta T) = -\frac{2e^2\mathcal{R}\mathcal{T}}{(2\pi a)^4\mathcal{N}}\text{Re}\left\{\int dy_L dz_L\; \varphi_L(y_L)\varphi_L^*(z_L) g(0, z_L - y_L)\right.$$
$$\times \int dy_R dz_R\; \varphi_R(y_R)\varphi_R^*(z_R) g(0, y_R - z_R) \int d\tau\, \text{Re}\left[g(\tau, 0)^2\right]$$
$$\left.\times \int dt \left[\frac{h_s(t; y_L + L, z_L + L)}{h_s(t + \tau; y_L + L, z_L + L)} \frac{h_s(t + \tau - \delta T; L - y_R, L - z_R)}{h_s(t - \delta T; L - y_R, L - z_R)} - 1\right]\right\} \tag{10}$$

HOM Calculation

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$$\left.\times\int dt\left[\frac{h_s(t;y_L+L,z_L+L)}{h_s(t+\tau;y_L+L,z_L+L)}\frac{h_s(t+\tau-\delta T;L-y_R,L-z_R)}{h_s(t-\delta T;L-y_R,L-z_R)}-1\right]\right\} \quad (10)$$

$$\mathcal{N} = \langle\phi_L|\phi_L\rangle\langle\phi_R|\phi_R\rangle$$

HOM Calculation

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$$\left.\times \int dt \left[\frac{h_s(t; y_L + L, z_L + L)}{h_s(t+\tau; y_L + L, z_L + L)} \frac{h_s(t+\tau-\delta T; L - y_R, L - z_R)}{h_s(t - \delta T; L - y_R, L - z_R)} - 1\right]\right\} \tag{10}$$

Numerics: quasi Monte Carlo algorithm

T. Hahn, Comput. Phys. Commun., 2005

Interferometry

Inner injections

Outer injections

Interferometry

Inner injections

Outer injections

- $\delta T = 0$

Interferometry

Inner injections

Outer injections

- $\delta T = 0$
  HOM dip

Interferometry

Inner injections

Outer injections

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Interferometry

Inner injections

Outer injections

- $\delta T = 0$
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HOM dip

- $\delta T = +2Lu/(1-u^2)$

Interferometry

Inner injections

Outer injections

- $\delta T = 0$
  HOM dip
- $\delta T = +2Lu/(1 - u^2)$
  S_{HBT} + asymmetric dip

HOM dip

T. Jonckheere et al., PRB 86, 2012

Interferometry

Inner injections

- $\delta T = 0$
  HOM dip
- $\delta T = +2Lu/(1-u^2)$
  S_{HBT} + asymmetric dip

Outer injections

HOM dip

S_{HBT} + asymmetric peak

T. Jonckheere et al., PRB 86, 2012

Interferometry

Inner injections

Outer injections

- $\delta T = 0$
  Inner: HOM dip
  Outer: HOM dip

- $\delta T = +2Lu/(1-u^2)$
  Inner: S_{HBT} + asymmetric dip
  Outer: S_{HBT} + asymmetric peak

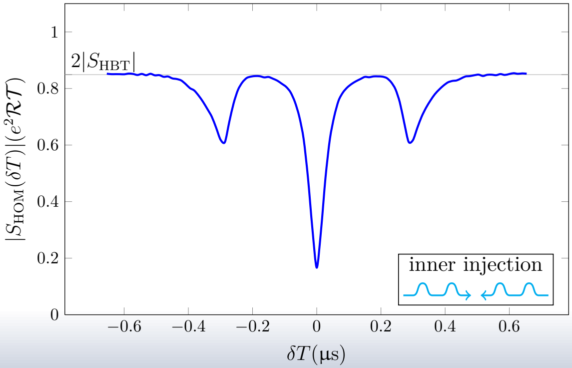
- $\delta T = -2Lu/(1-u^2)$
  Inner: S_{HBT} + asymmetric dip
  Outer: S_{HBT} + asymmetric peak

T. Jonckheere et al., PRB 86, 2012

Current correlations

Wide packets in energy

$\epsilon_0 = 175\text{mK} \quad \gamma = 1$



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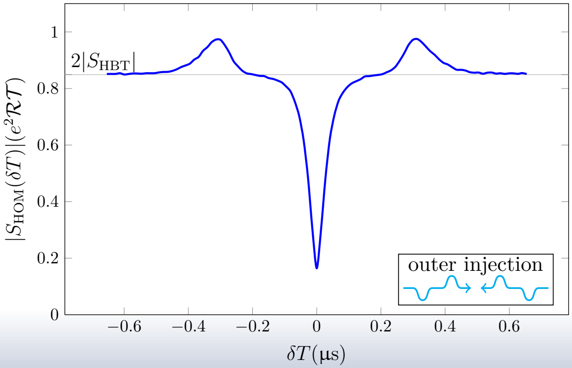
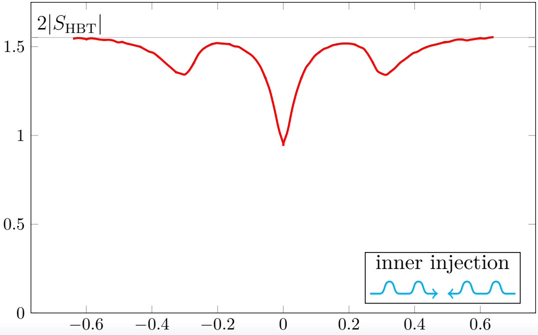


Current correlations

Energy resolved packets

$\epsilon_0 = 0.7\text{K} \quad \gamma = 8$



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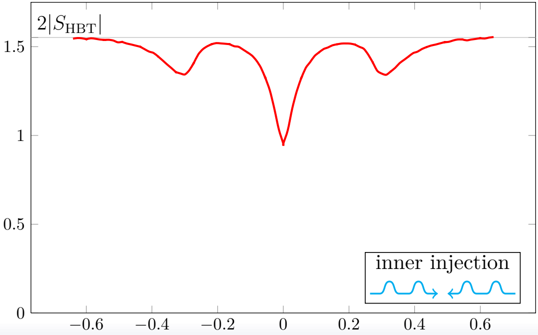
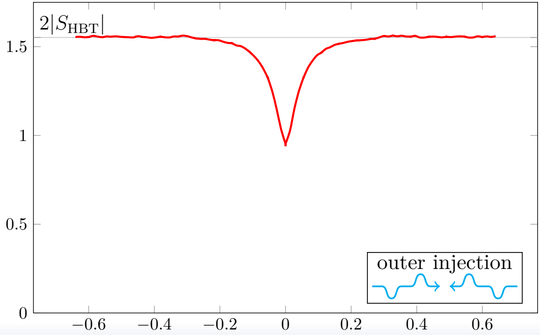


- dramatic loss of contrast (agreement with experimental results) !!

Current correlations

Energy resolved packets

$\epsilon_0 = 0.7\text{K} \quad \gamma = 8$



- dramatic loss of contrast (agreement with experimental results) !!

- no peaks

Conclusion

- **Interactions account for the observed loss of contrast!**

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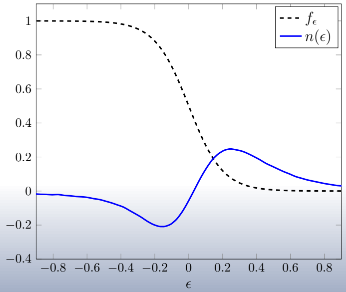
- Fast and slow excitations interference → **lateral dips/peaks**

Lateral peaks

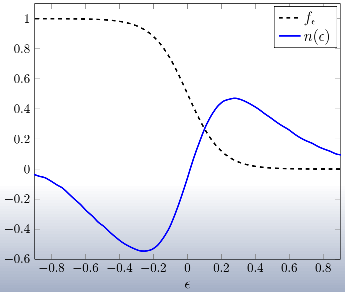
$\nu = 1$: electron-hole interferometry

$$\frac{S_{\mathrm{HOM}}(\delta T)}{2S_{\mathrm{HBT}}} = 1 + \left| \frac{\int_0^\infty dk\, \phi_e(k)\phi_h^*(k)\, e^{-ik\delta t} f_k(1-f_k)}{\int_0^\infty dk\, |\phi_e(k)|^2 (1-f_k)^2} \right|^2 \tag{11}$$

Wide in energy



Energy-resolved



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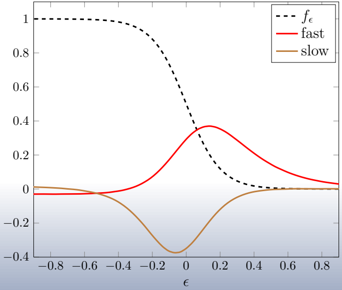
Wide in energy



Energy-resolved