Electron interferometry in quantum Hall edge channels

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Quantum optics analogs with electrons, i.e. the controlled preparation, manipulation and measurement of single excitations in ballistic conductors

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• Mesoscopic capacitor [Fève et al., Science 316, 1169 (2007)]



Emits single electrons on-demand
 opens the way to all sorts of interference experiments!

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• Mesoscopic capacitor [Fève et al., Science 316, 1169 (2007)]



Emits single electrons on-demand
 opens the way to all sorts of interference experiments!

• Hong-Ou-Mandel setup in quantum optics





- pairs of identical photons sent on a beam-splitter
- dip in the coincidence rate
 bosonic statistics

Hong-Ou-Mandel interference experiment

- Setup
 - 2 single electron sources
 - counter-propagating channels coupled at QPC
 - measure output currents



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 $S_{RL}^{ ext{out}} = \int dt dt' \left[\langle I_R^{ ext{out}}(x,t) I_L^{ ext{out}}(x',t')
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Hong-Ou-Mandel interference experiment

- Setup
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 I_{I}^{out}

• Results at $\nu = 1$ for $|S_{RL}^{out}|$



- flat background contribution
 - → Hanbury-Brown and Twiss
- HOM/Fermi/Pauli dip down to 0
- signatures of injected object

 I_R^{out}

- Different possible types of interactions
 - between counter-propagating channels, near the QPC



(work in progress)

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 - between counter-propagating channels, near the QPC
 - between co-propagating channels at filling $\nu = 2$



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 - strong interactions within a channel: Fractional QHE
- Formalism and methods



	FQHE	u=2 (go listen to talk by C. Wahl)
 injection 	prepared state $ arphi angle$	prepared state $ arphi angle$
 propagation 	bosonized H	bosonized <i>H</i> + diagonalization
tunneling	perturbative expansion	scattering matrix

- Different possible types of interactions
 - between counter-propagating channels, near the QPC
 - between co-propagating channels at filling $\nu = 2$
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- Formalism and methods



(this talk)

- Hall bar in the Laughlin series $\nu = \frac{1}{2m+1}$ \rightarrow edge quasi-particles with fractional statistics
- Bosonization:

$$\psi_{R,L}(x) = \frac{U_{R,L}}{\sqrt{2\pi a}} \exp\left(-i\sqrt{\nu}\phi_{R,L}(x)\right)$$

• Hall bar in the Laughlin series $\nu = \frac{1}{2m+1}$ \rightarrow edge quasi-particles with fractional statistics • Bosonization: Klein factor chiral bosonic field $\psi_{R,L}(x) = \frac{U_{R,L}}{\sqrt{2\pi a}} \exp(-i\sqrt{\nu}\phi_{R,L}(x))$

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Hamiltonian



$$H = H_0 + H_T$$

$$H_0 = \frac{\hbar v_F}{4\pi} \int dx \left[(\partial_x \phi_R)^2 + (\partial_x \phi_L)^2 \right]$$

$$H_T = \sum_{\epsilon = \pm} \left[\Gamma \psi_R^{\dagger}(0) \psi_L(0) \right]^{\epsilon}$$

Hall bar in the Laughlin series ν = 1/(2m+1)
 → edge quasi-particles with fractional statistics
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• Hamiltonian



Problem!

Tunneling Hamiltonian is not quadratic → perturbative expansion in Γ

Jérôme Rech (CPT)

• Experimental single electron source in the IQHE



→ well described through Floquet scattering theory

• Experimental single electron source in the IQHE



- discrete levels
- tunable dot transmission via V_g
- control via V_{exc}

ightarrow well described through Floquet scattering theory

- Simplified model of injection: prepared state
 - injection in the past at $t=-T_0$: $|arphi
 angle=~\mathcal{O}^\dagger\left(-T_0
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ground-state

preparation operator

• Experimental single electron source in the IQHE



→ well described through Floquet scattering theory

- Simplified model of injection: prepared state
 - injection in the past at $t = -T_0$: $|\varphi\rangle = O^{\dagger}(-T_0) |0\rangle$
 - preparation operator $\mathcal{O}^{\dagger} = \mathcal{O}_{R}^{\dagger} \mathcal{O}_{L}^{\dagger}$ with preparation operator

$$\mathcal{O}_{R,L}^{\dagger} = \int dk \varphi_{R,L}(k) \left[\psi_{R,L}^{\dagger}(k;t=-T_0) \right]^{n_{R,L}}$$



• Exponential wave-packets

$$\varphi_{R,L}(x) = \sqrt{\frac{2\Gamma}{v_F}} e^{\pm (i\epsilon + \Gamma)x/v_F}$$

Versatile: any paquet or number of qp

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ground-state

Problem!

Average not taken over the ground-state but over some prepared state $|\varphi\rangle$

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- Interaction representation and Keldysh contour
 - Usual case
- $C_{RL}(t,t') = \langle 0 | I_R(t) I_L(t') | 0 \rangle$ = $\langle 0 | S(-\infty,t) \tilde{I}_R(t) S(t,t') \tilde{I}_L(t') S(t',-\infty) | 0 \rangle$

Problem!

Average not taken over the ground-state but over some prepared state |arphi
angle

- Interaction representation and Keldysh contour
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$$C_{RL}(t,t') = \langle 0 | I_R(t) I_L(t') | 0 \rangle$$

= $\langle 0 | T_K \tilde{I}_R(t^-) \tilde{I}_L(t'^+) \tilde{S}_K | 0 \rangle$
 $T_K \exp\left(\frac{-i}{\hbar} \int_K dt_1 \tilde{H}_T(t)\right)$



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 $-\underbrace{\overset{t'}{\underbrace{}}_{t}\cdots \eta = +}_{t}\cdots \eta = -$

Present case

$$C_{RL}(t,t') = \langle \varphi | I_R(t) I_L(t') | \varphi \rangle$$

= $\langle 0 | S(-\infty, -T_0) \tilde{\mathcal{O}}(-T_0) S(-T_0, t) \tilde{I}_R(t) S(t, t') \rangle$
 $\times \tilde{I}_L(t') S(t', -T_0) \tilde{\mathcal{O}}^{\dagger}(-T_0) S(-T_0, -\infty) | 0 \rangle$

Problem!

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• Present case

$$C_{RL}(t, t') = \langle \varphi | I_R(t) I_L(t') | \varphi \rangle$$

= $\langle 0 | T_{K_4} \tilde{\mathcal{O}}(-T_0^4) \tilde{I}_R(t^3)$
 $\times \tilde{I}_L(t'^2) \tilde{\mathcal{O}}^{\dagger}(-T_0^1) \tilde{S}_{K_4} | 0 \rangle$
 $T_{K_4} \exp\left(\frac{-i}{\hbar} \int_{K_4} dt_1 \tilde{H}_T(t_1)\right)$



A few more technical details

• Second order perturbation in tunneling \tilde{H}_T

$$\begin{split} \langle I_{R}(x^{+},t) I_{L}(x^{-},t') \rangle &\to \frac{-1}{2\hbar^{2}} \int dt_{1} dt_{2} \sum_{\eta_{1}\eta_{2}} (-1)^{\eta_{1}+\eta_{2}} \sum_{\epsilon_{1},\epsilon_{2}} \Gamma^{\epsilon_{1}} \Gamma^{\epsilon_{2}} \\ &\times \langle T_{K_{4}} \mathcal{O}_{R} \left(-T_{0}^{4}\right) I_{R} \left(x^{+},t^{3}\right) \mathcal{O}_{R}^{\dagger} \left(-T_{0}^{1}\right) \psi_{R}^{\dagger} \left(0,t_{1}^{\eta_{1}}\right)^{\epsilon_{1}} \psi_{R}^{\dagger} \left(0,t_{2}^{\eta_{2}}\right)^{\epsilon_{2}} \rangle_{R} \\ &\times \langle T_{K_{4}} \mathcal{O}_{L} \left(-T_{0}^{4}\right) I_{L} \left(x^{-},t'^{2}\right) \mathcal{O}_{L}^{\dagger} \left(-T_{0}^{1}\right) \psi_{L} \left(0,t_{1}^{\eta_{1}}\right)^{\epsilon_{1}} \psi_{L} \left(0,t_{2}^{\eta_{2}}\right)^{\epsilon_{2}} \rangle_{L} \end{split}$$

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• Current in terms of chiral bosonic fields

$$I_{R,L}(x) = \pm \frac{e^* v_F}{2\pi \sqrt{\nu}} \partial_x \phi_{R,L}(x)$$

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Average of bosonic operators

$$\langle T_{K_4} \prod_{n=1}^{N} e^{\alpha_n \phi_{R,L}(x_n,t_n)} \rangle_{R,L} = \exp\left[\sum_{n=1}^{N} \sum_{m=n+1}^{N} \alpha_n \alpha_m \mathcal{G}\left(\sigma_{\eta_n \eta_m}^{t_n t_m} \left\{ t_n - t_m \mp \frac{x_n - x_m}{v_F} \right\} \right) \right]$$

with $\sigma_{\eta_n\eta_m}^{t_nt_m} = \delta_{\eta_n\eta_m}(-1)^{\eta_n} \mathsf{Sgn}(t_n - t_m) + (1 - \delta_{\eta_n\eta_m}) \mathsf{Sgn}(\eta_n - \eta_m)$

Final expression

• Current cross-correlations

$$S_{RL}(x^+, x^-; t, t') = \frac{\langle \varphi | I_R(x^+, t) I_L(x^-, t') | \varphi \rangle}{\langle \varphi | \varphi \rangle} - \frac{\langle \varphi | I_R(x^+, t) | \varphi \rangle}{\langle \varphi | \varphi \rangle} \frac{\langle \varphi | I_L(x^-, t') | \varphi \rangle}{\langle \varphi | \varphi \rangle}$$

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• Zero-frequency, finite temperature expression

Benchmarking against $\nu = 1$

• Vacuum contribution (no injection)

$$\mathcal{S}^{\mathsf{vac}}(\omega=0) = -\frac{(e^*\Gamma)^2 \Gamma(\nu)^2}{2\pi^2 a v_F \Gamma(2\nu)} \left(\frac{2\pi a}{\beta v_F}\right)^{2\nu-1}$$

Noise from Fermi sea at finite *T* [T. Martin, Les Houches LXXXI]

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• Hanbury-Brown and Twiss setup at $\nu = 1$: noise reduction!

$$S_{RL} = -\left(\frac{e\Gamma}{v_F}\right)^2 \int \frac{dk}{2\pi} (1 - 2f_k) \frac{|\varphi_R(k)|^2 (1 - f_k)^2}{\int \frac{dk}{2\pi} |\varphi_R(k)|^2 (1 - f_k)}$$

→ antibunching with thermal excitations [Bocquillon et al., PRL 108, 196803 (2012)]

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→ antibunching with thermal excitations [Bocquillon et al., PRL 108, 196803 (2012)]



- well-defined dip reflecting indistinguishability
- reaches 0 for synchronized injections
- in agreement with expected exponential [Ol'khovskaya et al., PRL 101, 166802 (2008)]

Injecting single electrons (u = 1/3)





- dip does not reach 0
- contrast smaller for resolved packets
- shape is not simply given by the wave-packet
- weak temperature dependence

Injecting single quasi-particles



- presence of a dip
- approaches 0 at $\delta T = 0$
- striking temperature dep.
- intriguing wave-packet (in)dependence

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- similar behavior $\forall \nu$
- surprisingly good fit

$$\frac{\mathcal{S}_{\mathrm{HOM}}}{2\mathcal{S}_{\mathrm{HBT}}} = 1 - \exp\left(-2\nu\frac{\pi |\delta\,T|}{\beta}\right)$$

Jérôme Rech (CPT)

• A general formalism for the computation of noise correlations for electron quantum optics setups in the quantum Hall regime

• When injecting electrons, the current cross-correlations show a dip, with less than optimal contrast

• When injecting single quasi-particles, an exponential dip appears, like nothing seen in the IQHE