Coherent Quantum Phase-Slip in Josephson junction chains

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Zukunftskolleg
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Universität Konstanz

SEVENTH FRAMEWORK PROGRAMME

MARIE CURIE ACTIONS

“Nanophysics: from fundamentals to applications”, IXth Rencontres du Vietnam, Quy-Nhon, August 2013

Outline

1. Coherent Quantum Phase Slips (CQPS)

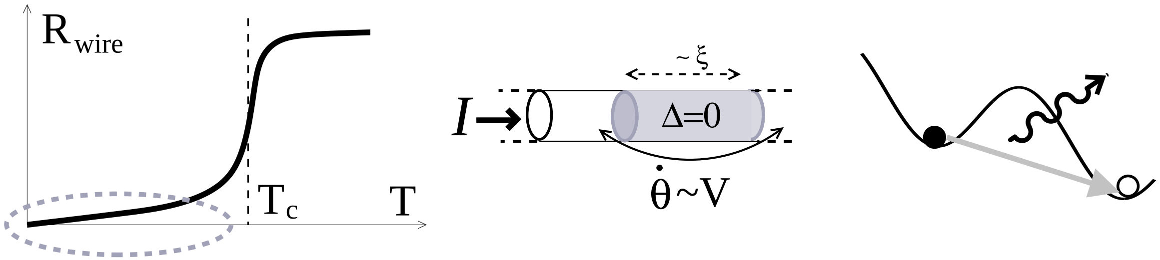
2. Josephson junction chains (theory & experiments)

3. Non-adiabatic dynamics

1. CQPS

Quantum Phase-Slip (QPS)

Incoherent QPS in superconducting nanowires

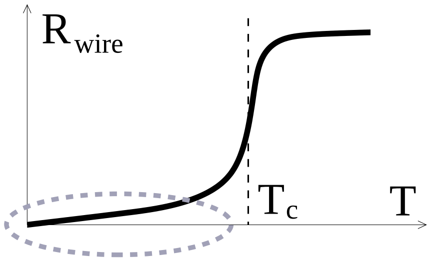


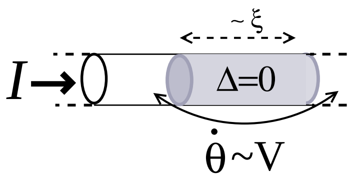
Arutyunov et al., Phys. Rep. 2008 (review)

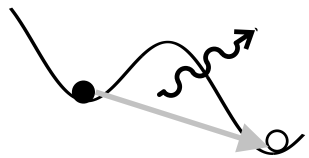
Giordano PRL 1988
Zaikin et al., PRL 1997
Bezryadin et al., Nature 2000
Lau et al., PRL 2001
Li et al., PRL 2011
Lehtinen et al., PRB 2012

Quantum Phase-Slip (QPS)

Incoherent QPS in superconducting nanowires







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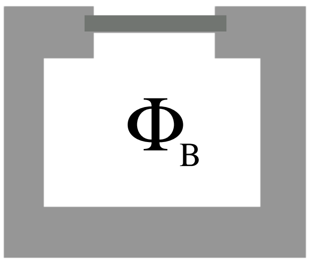
Coherent QPS superconducting nanowires

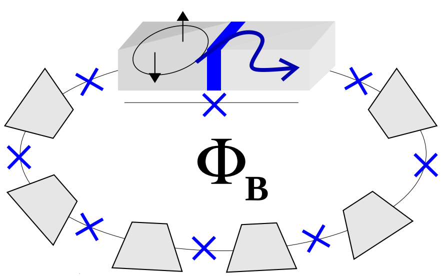


superconducting loop

theory
Mooij and Harmans, NJP 2005
Mooij and Nazarov, Nat. Physics 2006
Hriscu and Nazarov, PRL 2011
Vanević and Nazarov, PRL 2012

experiments
Astafiev et al., Nature 2012
Lehtinen et al., PRL 2012
Hongisto and Zorin, PRL 2012
Webster et al., PRB 2013

Coherent QPS in Josephson junction chains



theory
Orlando, Mooij et al., PRB 1999
Matveev,Larkin,Glazman, PRL 2002
Guichard and Hekking, PRB 2010
Rastelli et al., PRB 2013
Süsstrunk et al., arXiv:1304.4487

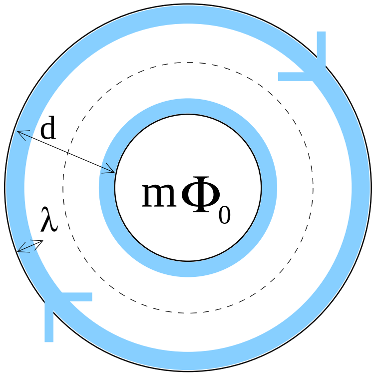
experiments
Mooij et al., Science 1999
Pop et al., Nat. Phys. 2010
Manucharyan et al., Science 2009
arXiv:0910.3039, PRB 2012
A. Ergül et al., arXiv:1305.7157

Persistent current eigenstates

a) Magnetic flux quantization

$$m\Phi_0 = L_g \; I \; + \Phi_{ex}$$

geometrical inductance



Persistent current eigenstates

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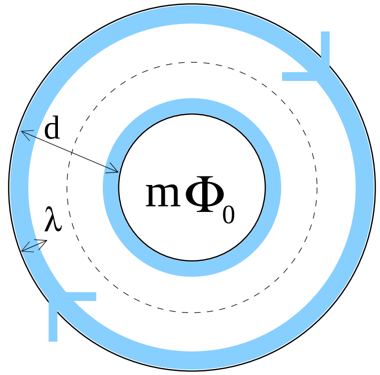
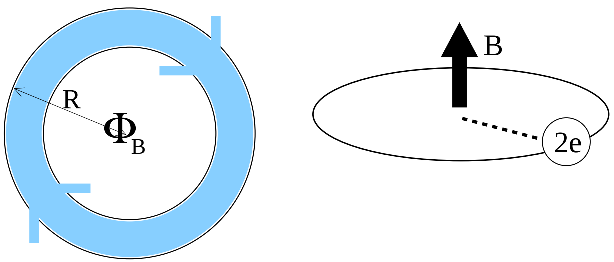


b) Fluxoid quantization: $\oint ds\,(d\varphi/ds) = 2\pi m$

$$\Phi_B = L_g I + \Phi_{ex} \simeq \Phi_{ex}$$

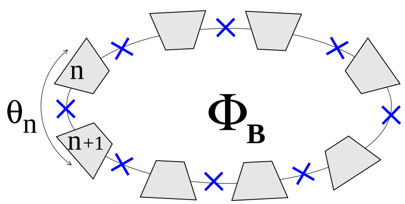
- Superconducting nano-rings $(L_k \gg L_g)$



$$I_m = (\Phi_0/L_k)\left(m - \frac{\Phi_B}{\Phi_0}\right)$$

kinetic inductance

- Josephson junction rings $(L_J \gg L_k \gg L_g)$

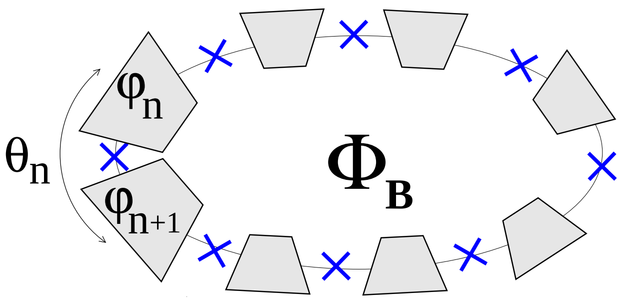


$$I_m = (\Phi_0/NL_J)\left(m - \frac{\Phi_B}{\Phi_0}\right)$$

Josephson inductance

Current eigenstates in JJ rings

- Homogeneous ring



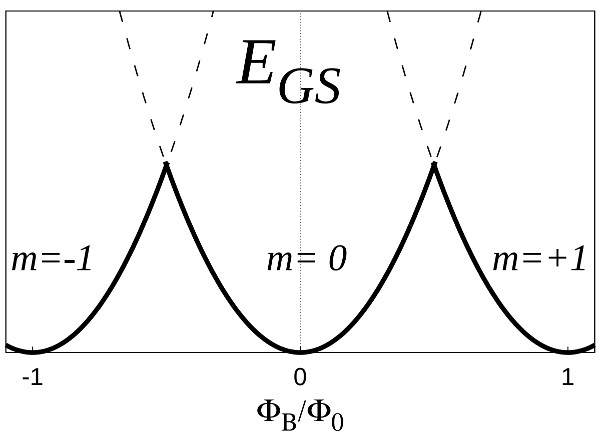
$$H_{JJ} = -E_J \sum_n \cos\left(\theta_n - 2\pi \frac{\Phi_B}{N\Phi_0}\right)$$

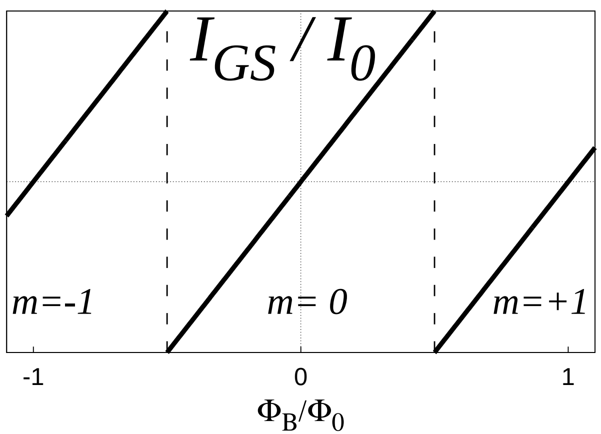
constraint

$$\varphi_{N+1} = \varphi_0 + 2\pi m \longrightarrow \sum_n \theta_n = 2\pi m$$

- Current states $|m\rangle$: minima of H_{JJ}, $\theta_n^{(m)} = 2\pi m/N$
- Ground state

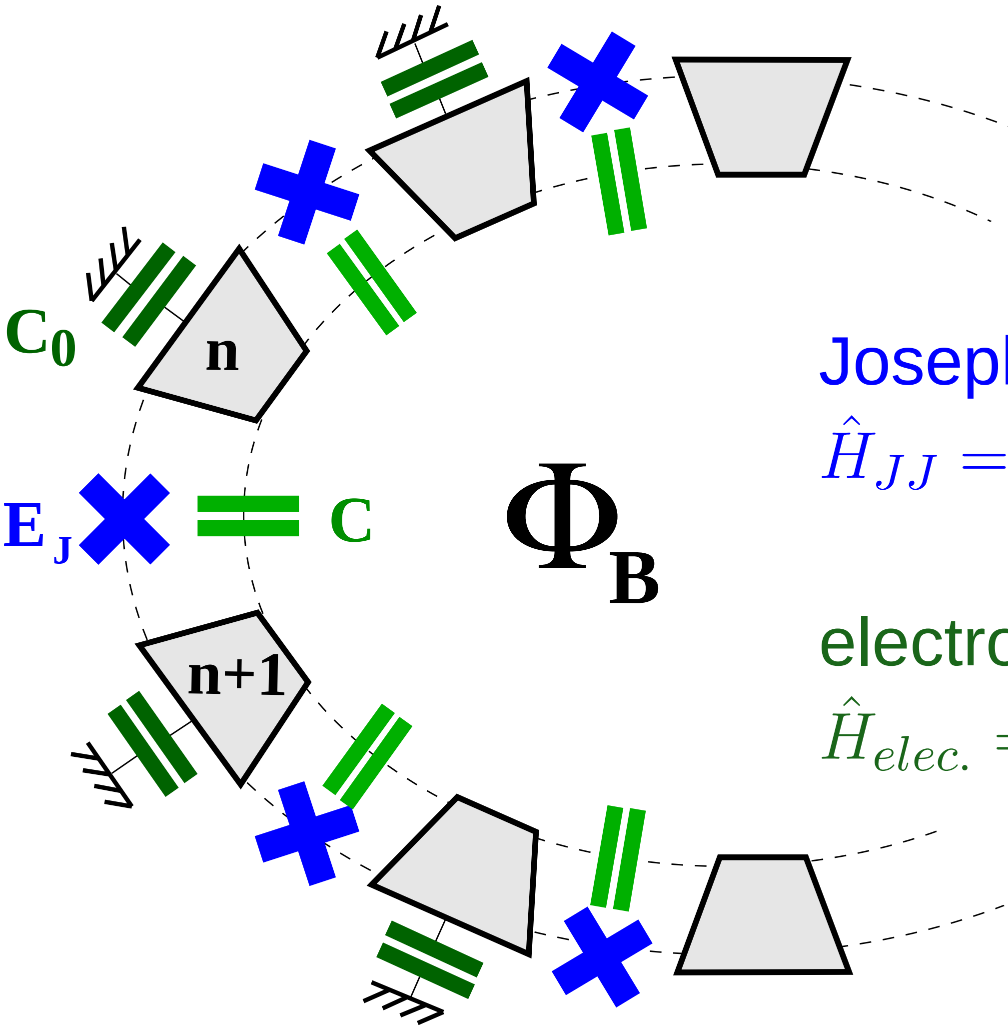
$$E_{GS} = \frac{E_J}{2N} \min_m \left(2\pi m - 2\pi \frac{\Phi_B}{\Phi_0}\right)^2 \qquad I_{GS} = \partial E_{GS}/\partial \Phi_B$$





2. Josephson junction chains (theory & experiments)

Microscopic model for a JJ ring



$$\hat{H} = \hat{H}_{JJ} + \hat{H}_{elec.}$$

Josephson coupling

$$\hat{H}_{JJ} = -E_J \sum_n \cos\left(\hat{\varphi}_n - \hat{\varphi}_{n-1} - \frac{2\pi\Phi_B}{N\Phi_0}\right)$$

electrostatic interaction

$$\hat{H}_{elec.} = \frac{1}{2}\sum_{n,n'} \bar{\bar{C}}^{-1}_{nn'}\hat{Q}_n\hat{Q}_m$$

Quantum Phase Model $\left[\hat{\varphi}_n, \hat{Q}_n\right] = 2e\, i$

CPQS in a JJ ring

- Regime $E_J \gg E_C = \frac{e^2}{2C}, E_0 = \frac{e^2}{2C_0}$

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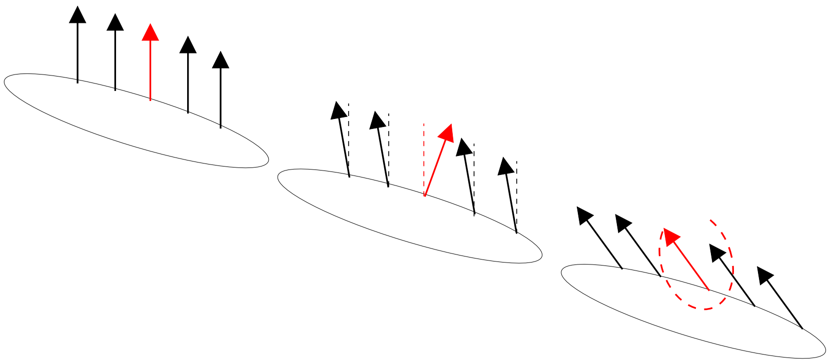
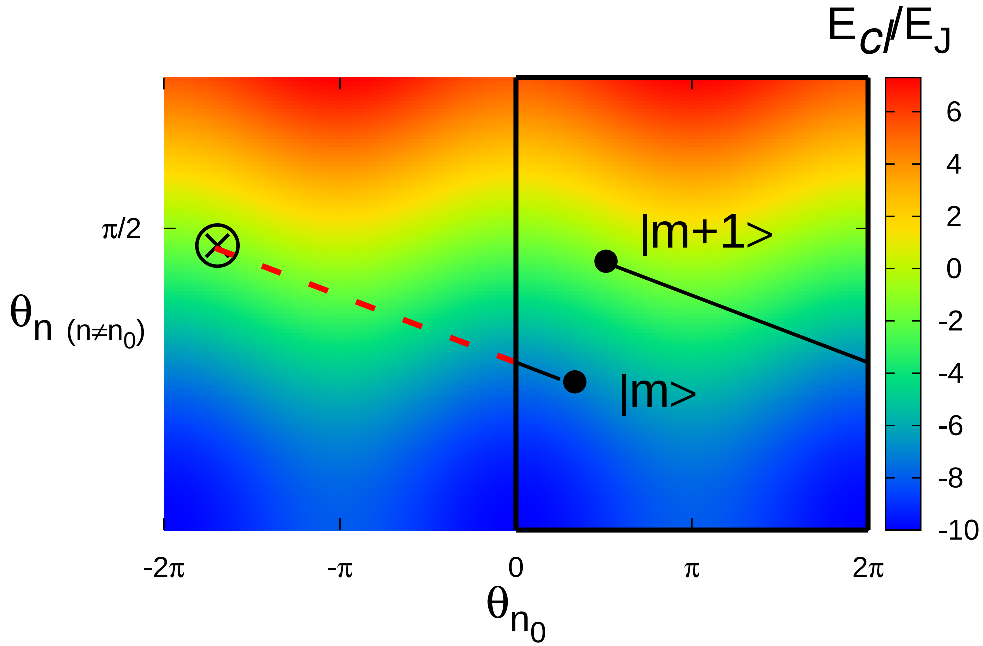


$\sum_n \theta_n = 0$ (mod$[2\pi]$)



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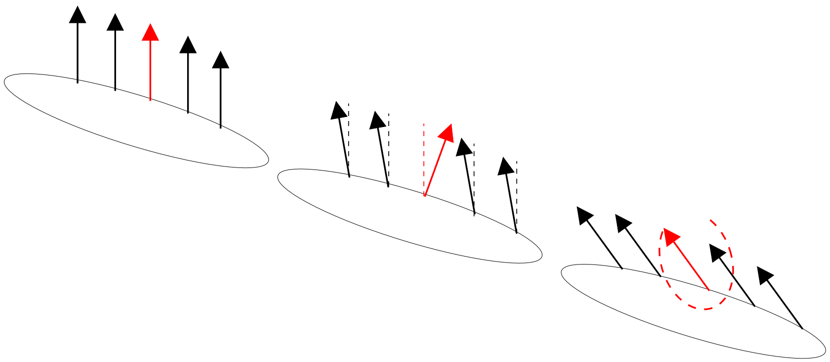

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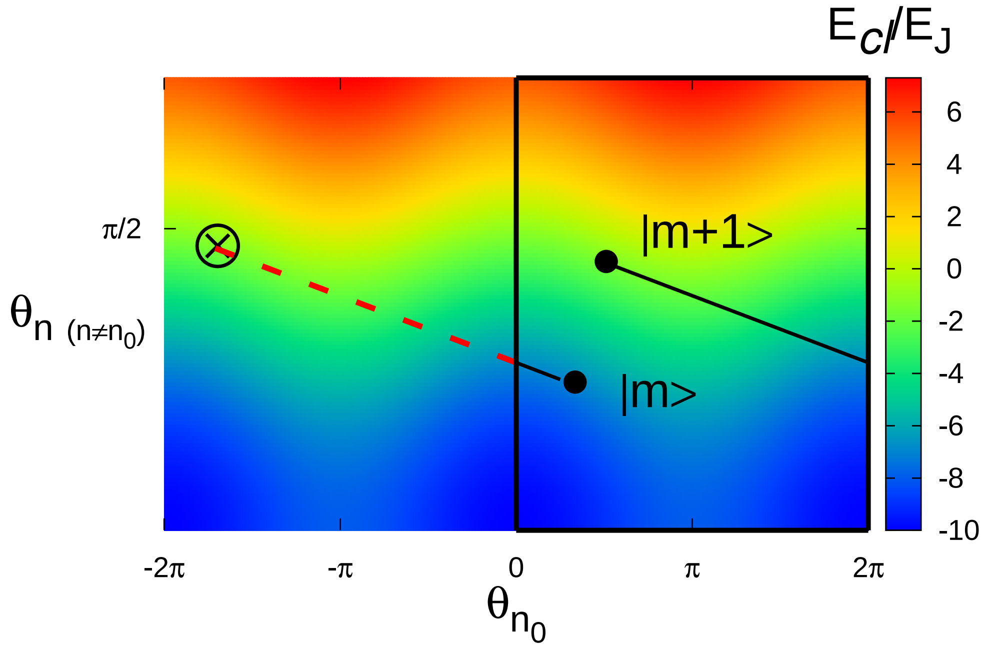
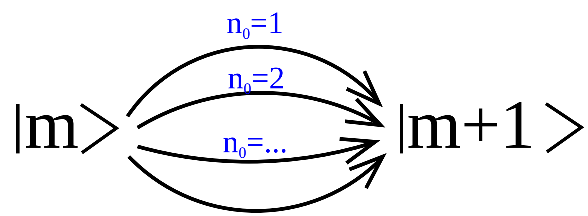


- Multidimensional space

υ = tunneling amplitude for θ_{n_0}



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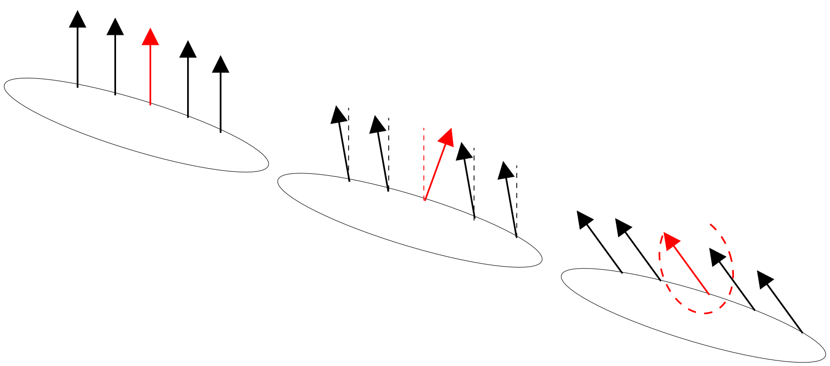
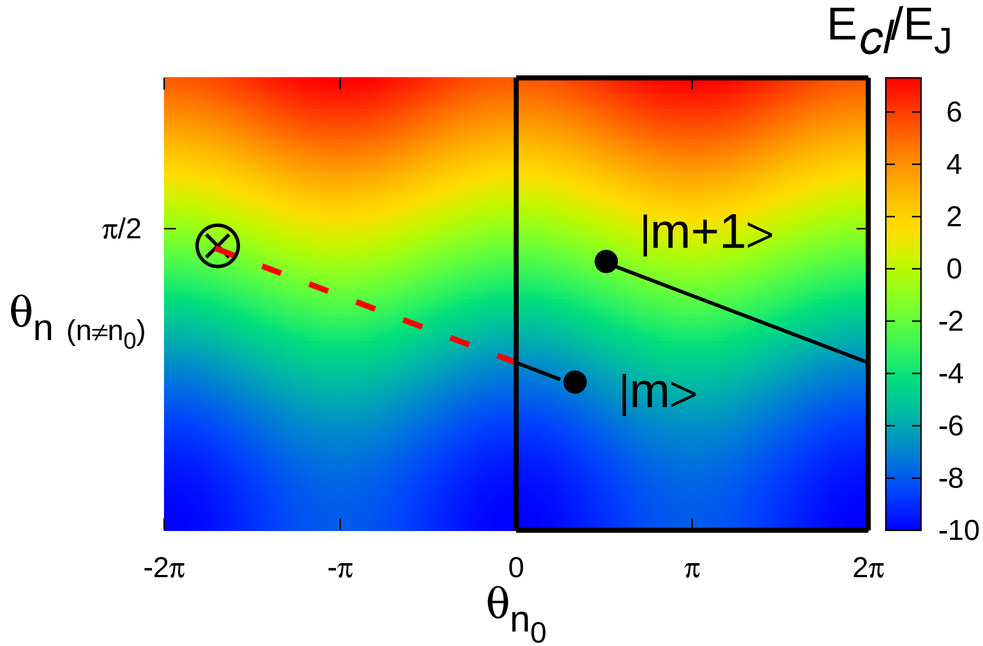
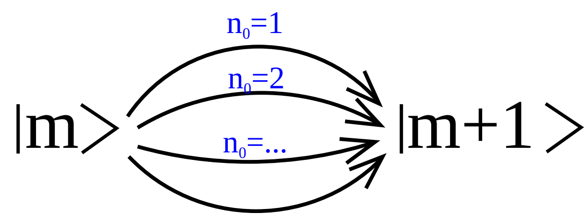


$\sum_n \theta_n = 0 \quad (\text{mod}[2\pi])$



- Multidimensional space

$\upsilon =$ tunneling amplitude for θ_{n_0}



- $\upsilon_{ring} = N\upsilon$ amplitude for $|m\rangle \leftrightarrow |m+1\rangle$

CPQS in a JJ ring

- Regime $E_J \gg E_C = \frac{e^2}{2C}, E_0 = \frac{e^2}{2C_0}$
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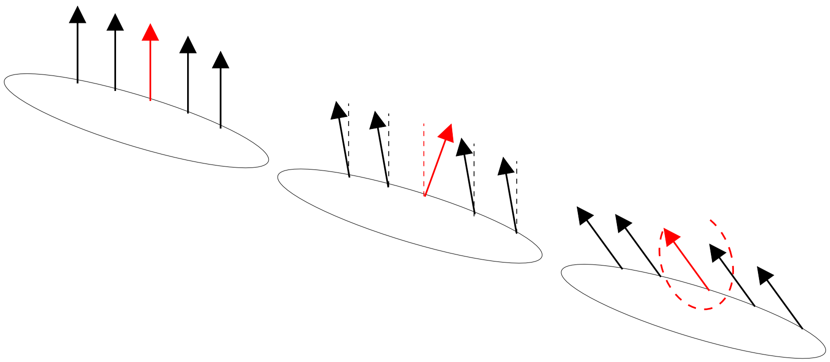
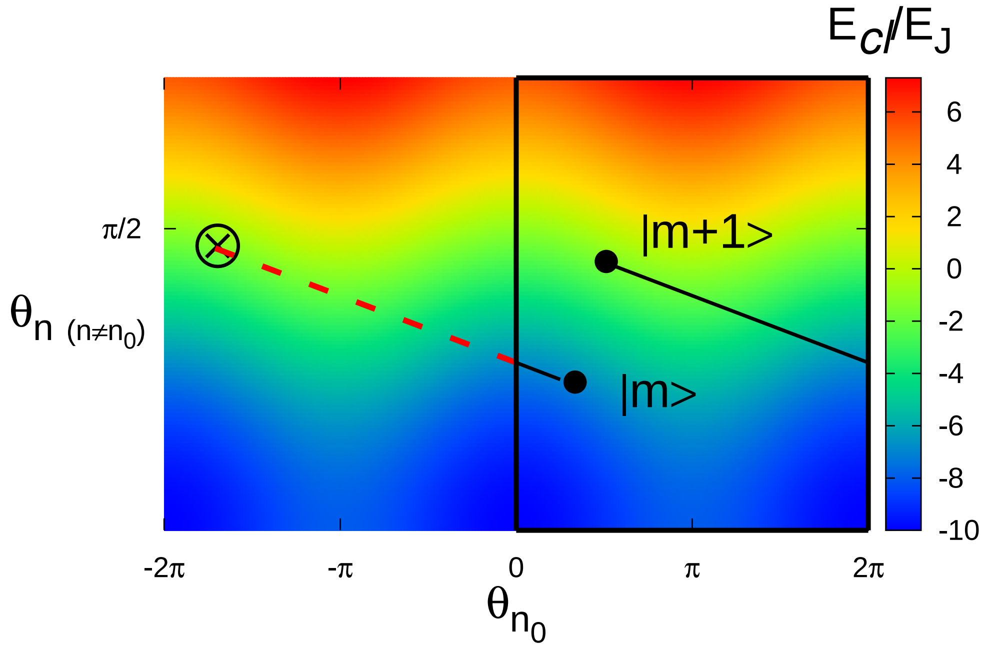
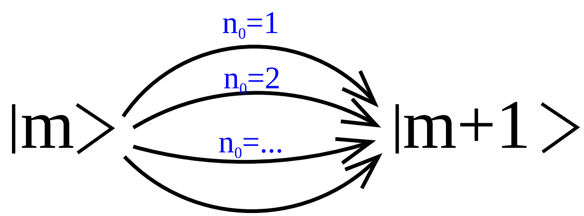


$\sum_n \theta_n = 0 \pmod{[2\pi]}$



- Multidimensional space

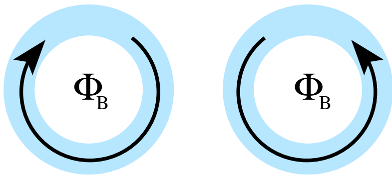
v = tunneling amplitude for θ_{n_0}

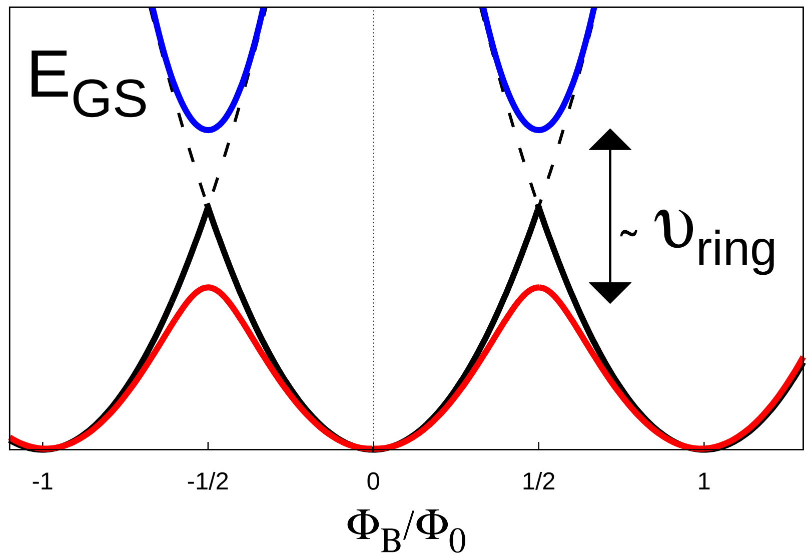


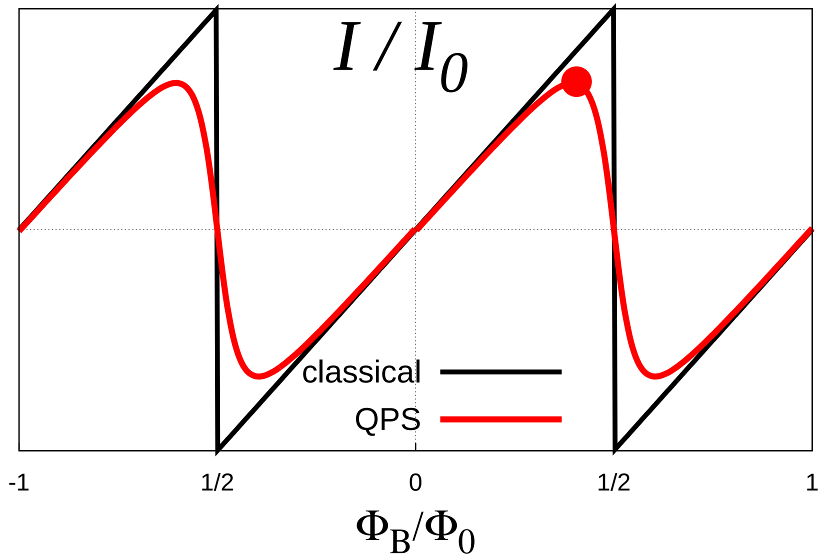
- $v_{ring} = Nv$ amplitude for $|m\rangle \leftrightarrow |m+1\rangle$

Phase-current relation

- Perturbative regime: $E_m, E_{m+1} \gg \upsilon_{ring}$







- General

$$\hat{H} = \sum_m E_m |m\rangle\langle m| - \upsilon_{ring} \sum_m (|m+1\rangle\langle m| + h.c.)$$

$$E_{GS}(\Phi_B) \longrightarrow I(\Phi_B) \longrightarrow \quad \bullet I_{max}$$

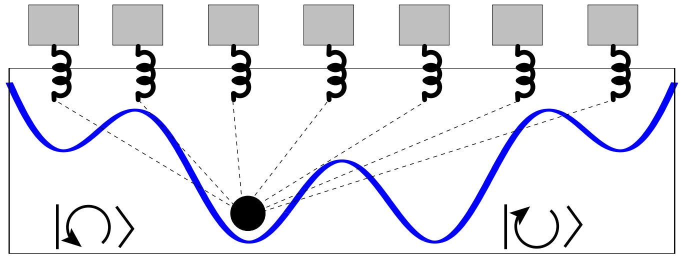
[Matveev,Larkin,Glazman, PRL 2002; Orlando, Mooij et al., PRB 1999]

Single tunneling amplitude υ

- One particle coupled to an external bath

θ_{n_0} : center pf QPS

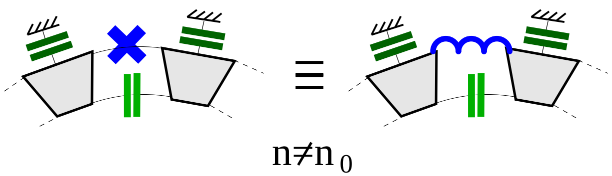
$\{\theta_{n\neq n_0}\}$: N-1 harmonic oscillators



- Regime $E_J \gg E_C, E_0$, instanton technique (Path Integral)

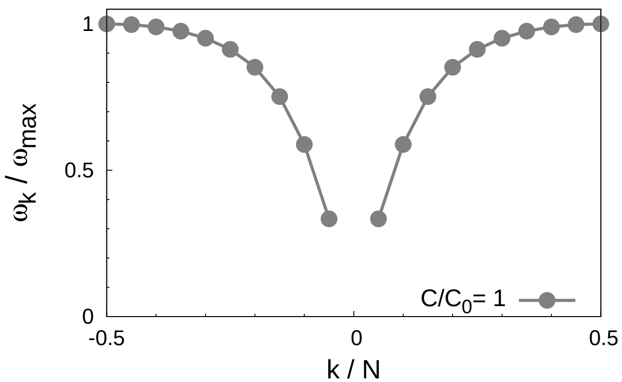
- Finite size effects $\upsilon = \upsilon(N)$

- Harmonic modes $\upsilon = \upsilon(\{\omega_k\})$



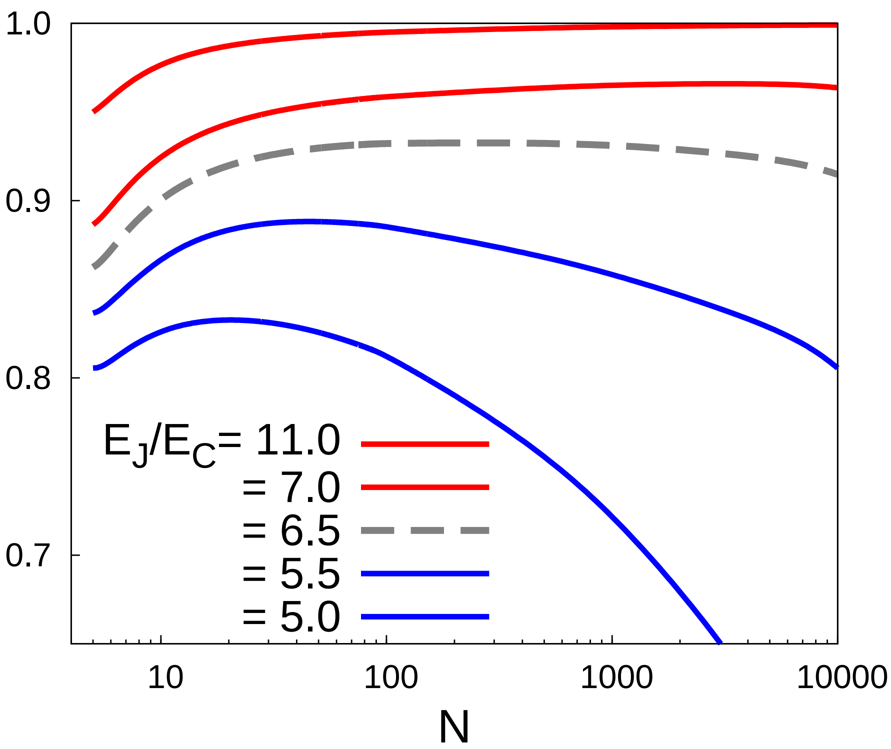
$\longrightarrow$ LC-line

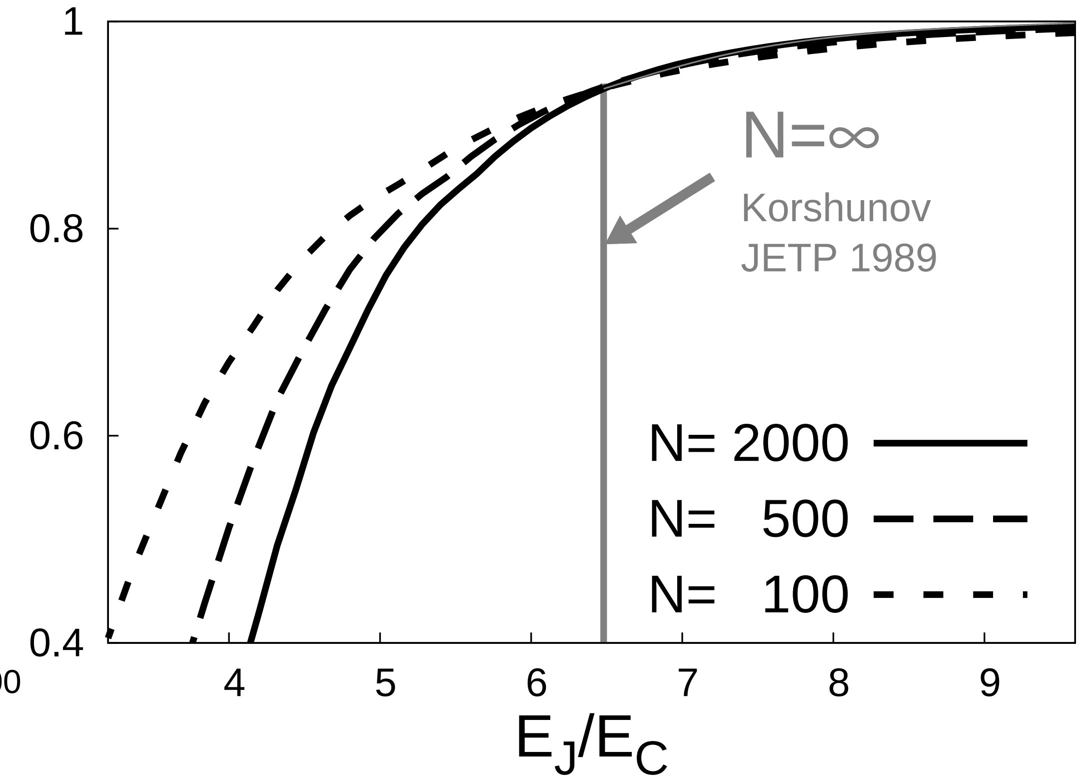
electrodynamic modes



Theoretical results: I_{max}/I_0

Example $(C = 2C_0)$

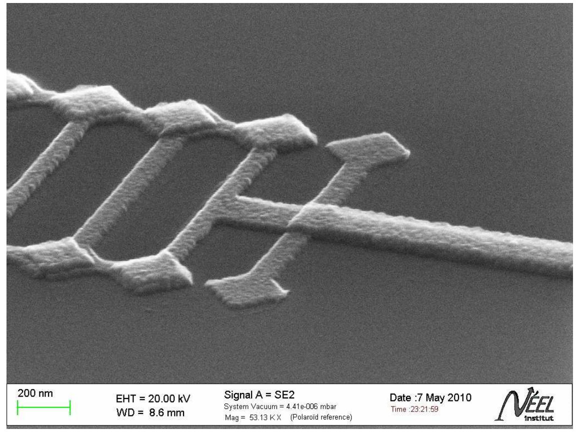




- Finite size effects: non-monotonic behavior
- Superconductor/Insulator:
  Quantum phase transition for $N \to \infty$

[Rastelli et al., PRB 2013]

Experiments: setup

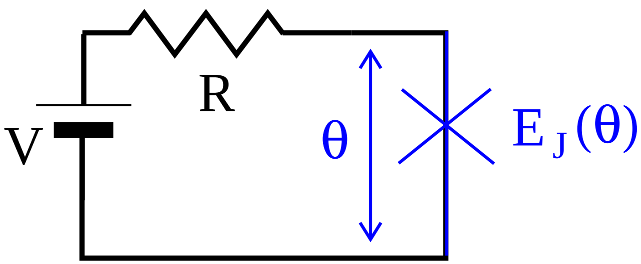


- Samples
  $N = 10, 20, 30, \ldots, 110$

- Voltage-bias configuration

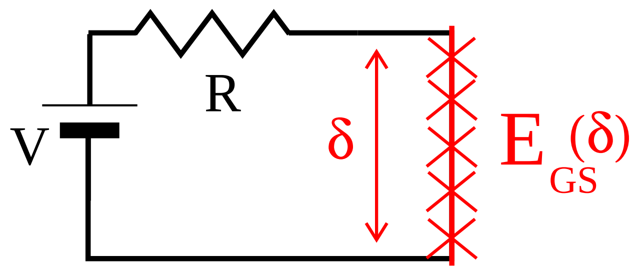
single JJ

$$\frac{1}{R}\left(V - \frac{\hbar}{2e}\frac{d\theta}{dt}\right) = I_J \sin(\theta)$$

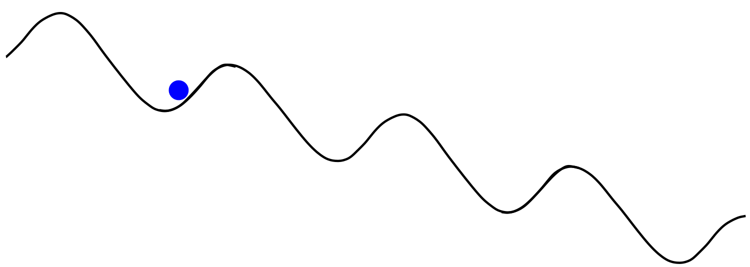


JJ chains

$$\frac{1}{R}\left(V - \frac{\hbar}{2e}\frac{d\delta}{dt}\right) = I(\delta)$$



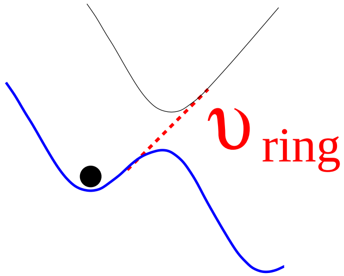
- Mechanical analog



Experiments: caveats

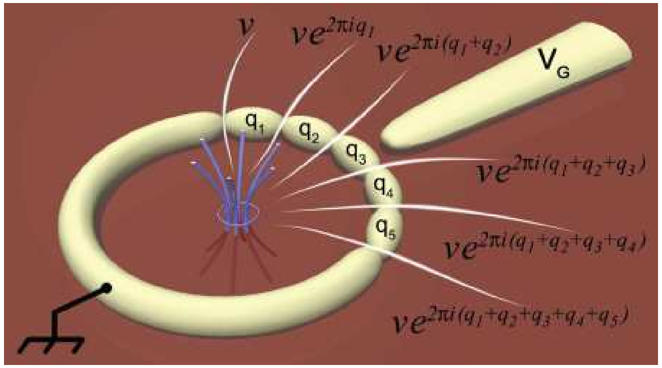
- Zener-transitions

$$P_c = \frac{1}{2} = e^{-2\pi \frac{\upsilon^2_{ring}}{E_J(2eV_c)}} \longrightarrow V_c \quad \text{threshold}$$



- Charged impurities

$$\upsilon_{ring} = \upsilon \sum_n e^{i\frac{2\pi}{2e}q_n} = \upsilon N_{eff}$$

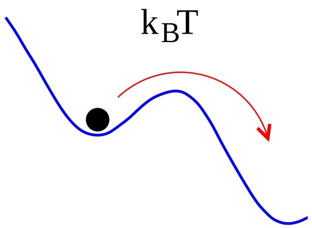


(Pop,Douçot,Ioffe,et al., PRB 2012)

- Thermal fluctuations

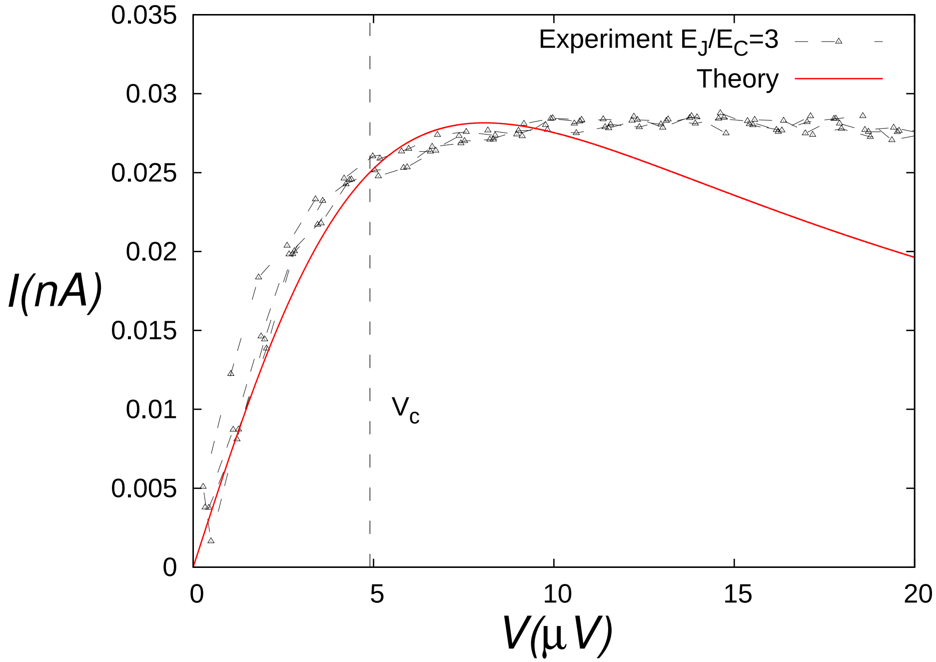
$(T_{exp} \sim 50 - 100mK)$

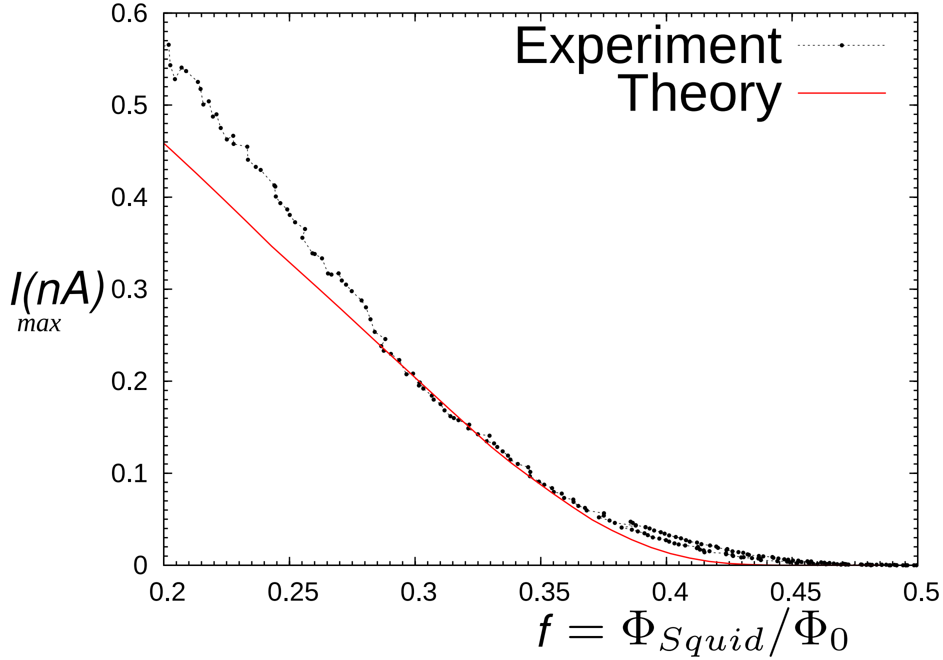
1 junction $E_J \simeq 1000mK$ JJ Chains $E_{GS} \sim 100mk$



Experiments: results

Example $N = 50$



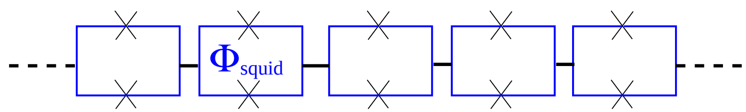


- Chains of SQUIDS:

$$E_J = E_J^{Squid} \cos\left(\pi \Phi_{Squid}/\Phi_0\right)$$

experimentally tunable

1 element $\equiv$ 1 SQUID



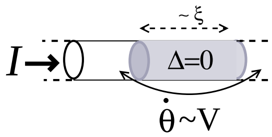
- Current-scale: 1 junction $I_J \simeq 50nA$ $\longrightarrow$ JJ chains $I_{max} \simeq 0.1nA$

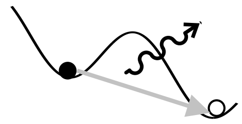
[Weissl, Rastelli et al., (unpublished)]

3. Non-adiabatic dynamics

Dynamics

- Incoherent QPSs



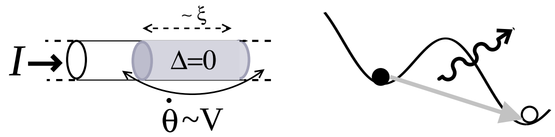


$V = \Phi_0 \Gamma \longrightarrow$ rate

(T=0K)

Dynamics

- Incoherent QPSs



$V = \Phi_0 \Gamma \longrightarrow$ rate

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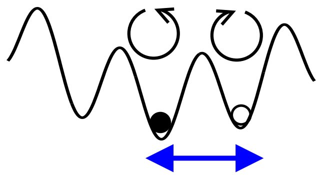
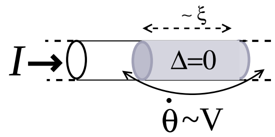
- Coherent QPSs

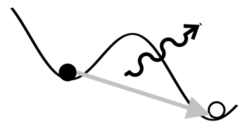


$v \longrightarrow$ quantum amplitude

Dynamics

- Incoherent QPSs





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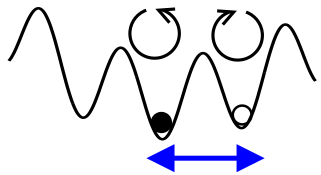
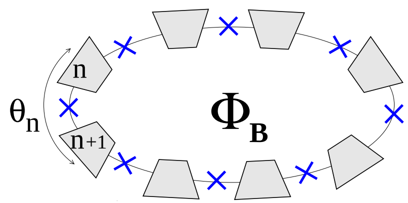


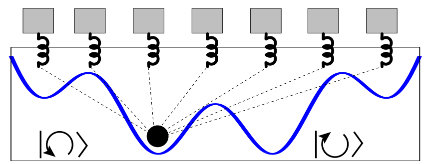
$v \longrightarrow$ quantum amplitude

- QPS in JJ rings

1 Josephson junction coupled

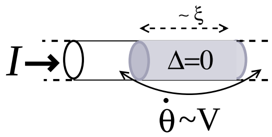
to the harmonic modes $\{\omega_k\}$

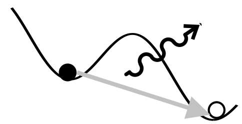




Dynamics

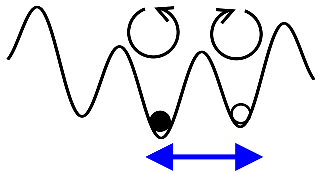
- Incoherent QPSs





$V = \Phi_0 \Gamma \longrightarrow$ rate (T=0K)

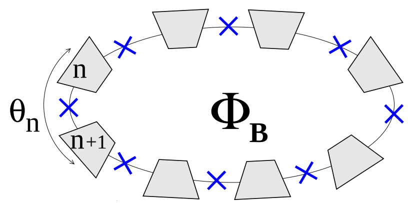
- Coherent QPSs

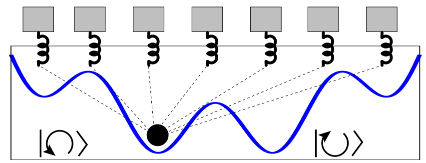


$v \longrightarrow$ quantum amplitude

- QPS in JJ rings

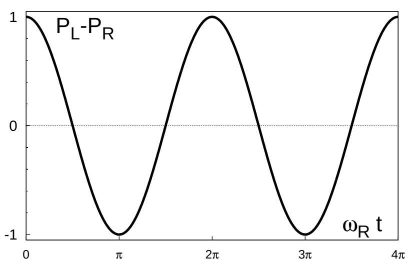
1 Josephson junction coupled to the harmonic modes $\{\omega_k\}$





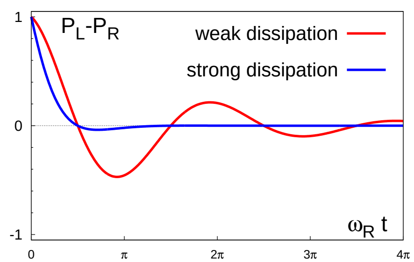
a) Adiabatic coherent dynamics $v \ll \omega_{min}$

- finite-size systems
- renormalized v_N

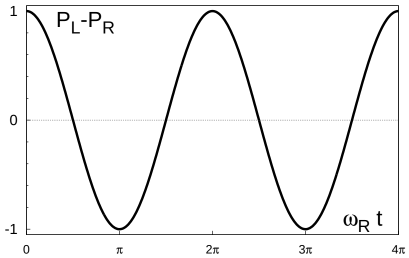


b) Incoherent dynamics in the thermodynamic limit $N{=}\infty$

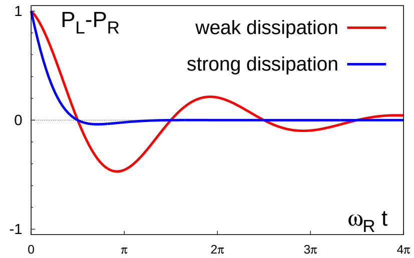
- Caldeira-Leggett dissipative model
- $\omega = vk$ ohmic friction, $Z_{line} = \sqrt{L_J/C_0}$



a) Adiabatic coherent dynamics $v \ll \omega_{min} \sim 1/N$

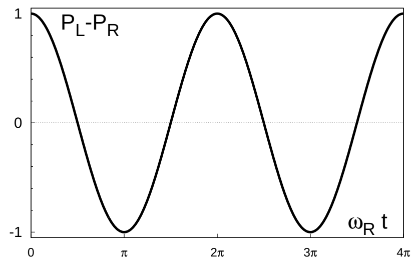


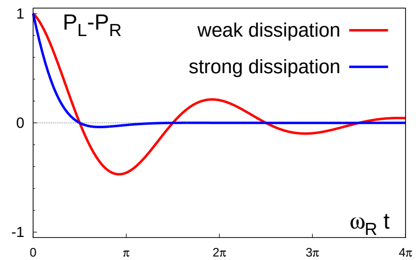
b) Incoherent dynamics in the thermodynamic limit $N{=}\infty$



a) Adiabatic coherent dynamics $\upsilon \ll \omega_{min} \sim 1/N$

b) Incoherent dynamics in the thermodynamic limit $N=\infty$



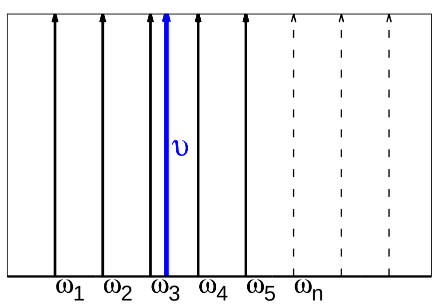


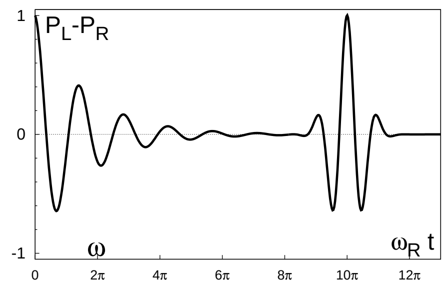
← intermediate regime →

$N^* < N < \infty$

c) Non-adiabatic dynamics

- $\omega_{min}(\sim 1/N) \ll \upsilon$
- N discrete: no dissipation

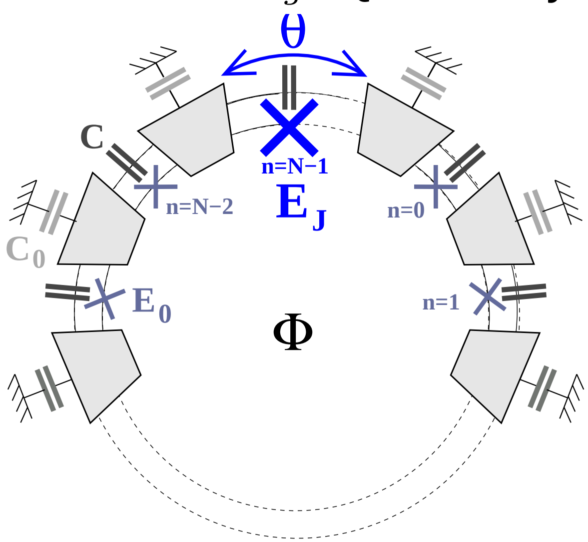


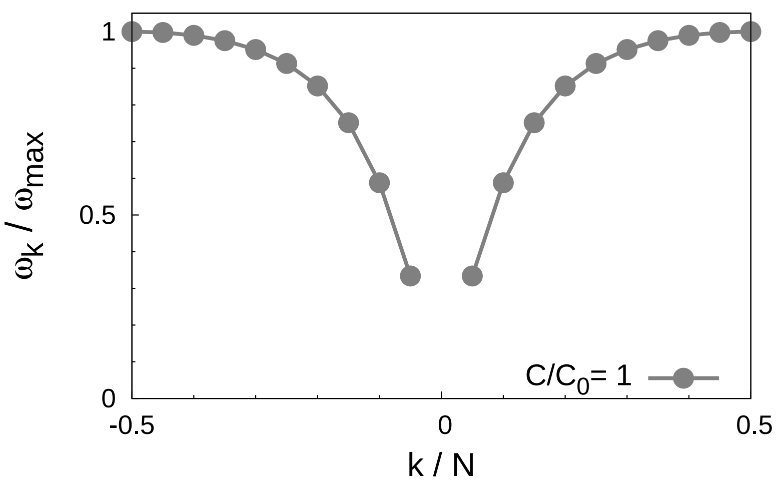


revivals

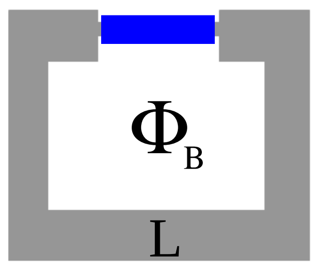
Systems

- Inhomogeneous ring with a weak element
  - $v \gg v_{ring}$: QPSs only in 1JJ
  - N ring: LC-line, electrodynamic modes $\{\omega_k\}$

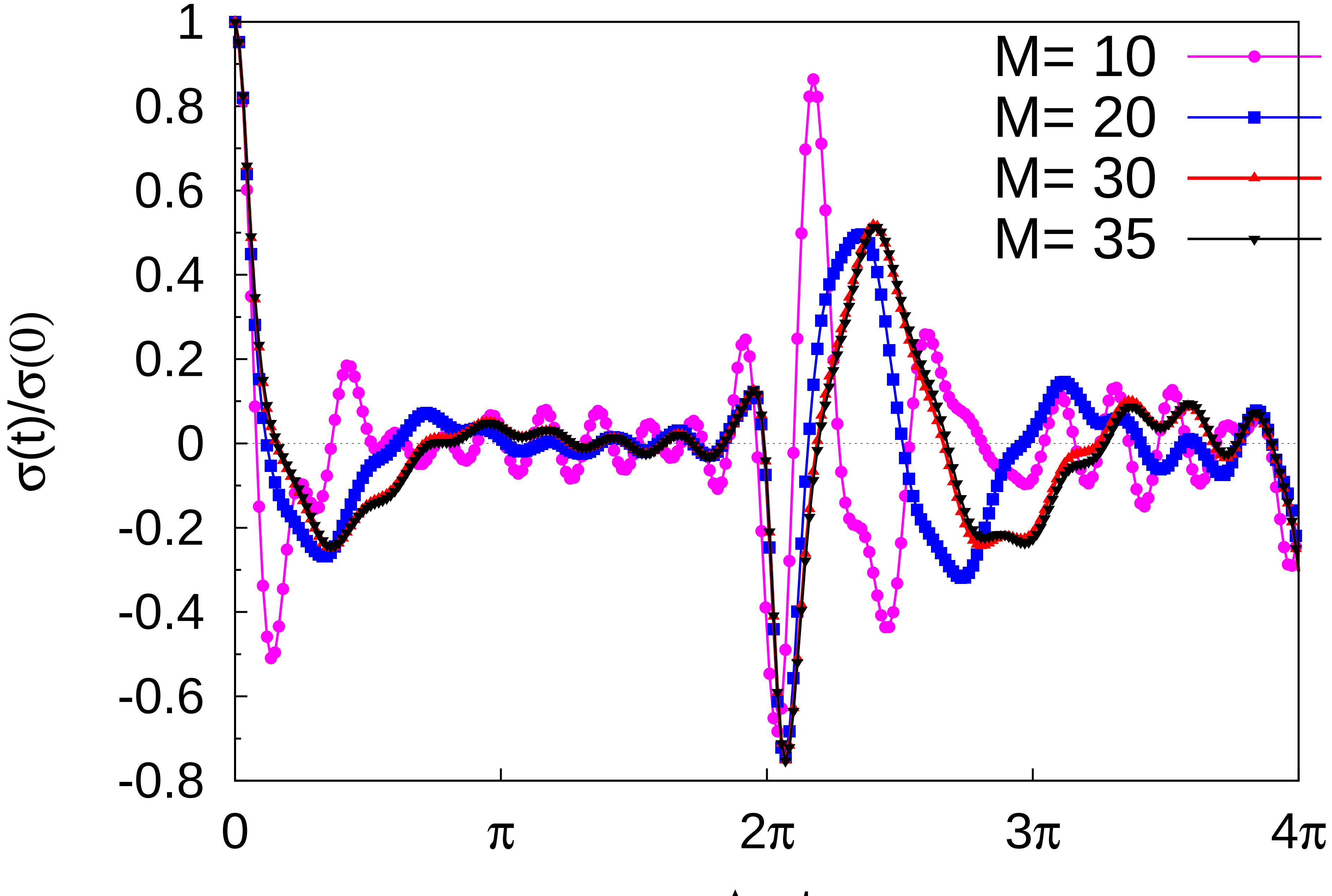




- Superconducting loop with a weak link
  - short coherent conductor
  - finite loop L: Mooij-Schön modes $\{\omega_k\}$



Results (example)



[Rastelli, Vanević, Belzig, unpublished]

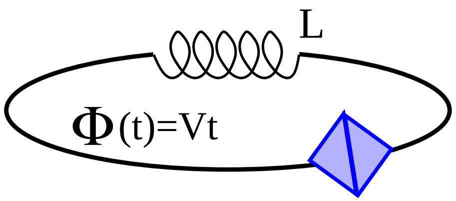
Thanks for your attention

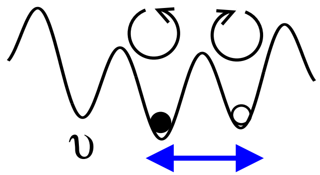
collaborators:

Grenoble		Konstanz
F. W. J. Hekking	(experiments) W. Guichard I. Pop (Yale) T. Weissl O. Feofanov (EPF Lausanne) I. Matei (CEA-Grenoble) O. Buisson	W. Belzig M. Vanević

QPS junction

- system





$$\delta(t) = 2\pi \frac{\Phi(t)}{\Phi_0}$$

$$\hat{H} = \left(\frac{\hbar}{2e}\right)^2 \frac{(\delta(t) - 2\pi\hat{m})^2}{2L} + \upsilon \sum_m \left(|m\rangle\langle m+1| + h.c.\right)$$

- conjugate (charge) operator

$$[\hat{m}, \hat{q}] = 2ei \qquad e^{i\frac{2\pi}{2e}\hat{q}} = |m\rangle\langle m+1| \qquad \frac{d\hat{q}}{dt} = \frac{\partial \hat{H}}{\partial \Phi} = \hat{I}$$

$$\hat{H} = -\left(\frac{\hbar^2}{2L}\right)\frac{\partial^2}{\partial \hat{q}^2} - 2\upsilon \cos\left(\frac{2\pi}{2e}\hat{q}\right) - V_{bias}\hat{q}$$

- duality with current-bias Josephson Junction

$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos(\hat{\theta}) - \frac{\hbar}{2e} I \hat{\theta}$$

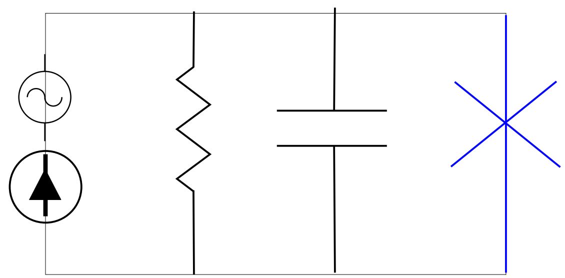
(Likharev & Zorin, J. Low Temp. Phys. 1985)

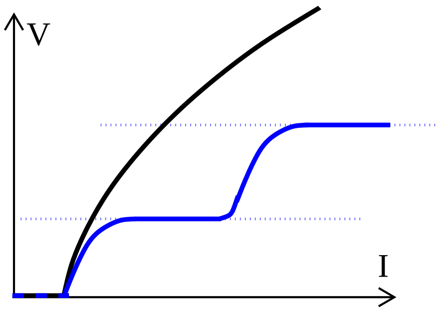
(Mooij & Nazarov, Nat. Phys. 2006)

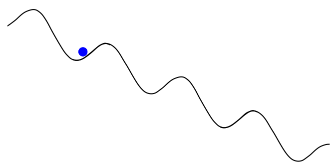
Quantum metrology

- classical dynamics for θ in JJ $\rightarrow$ Shapiro steps $V = n\,\frac{\hbar}{2e}\,\omega$

$$I + I_{mw}(\omega) = \frac{\hbar/2e}{R}\frac{d\theta}{dt} + C\frac{\hbar}{2e}\frac{d^2\theta}{dt^2} + I_J \sin(\theta)$$

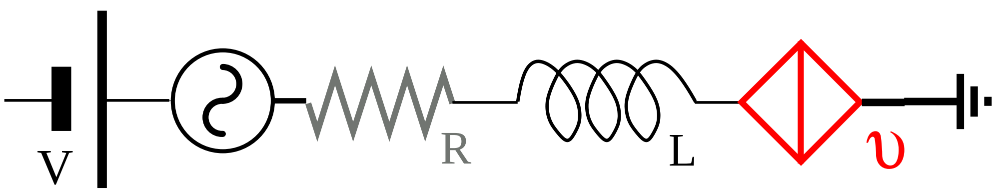


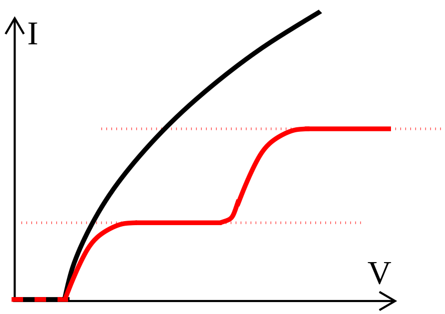




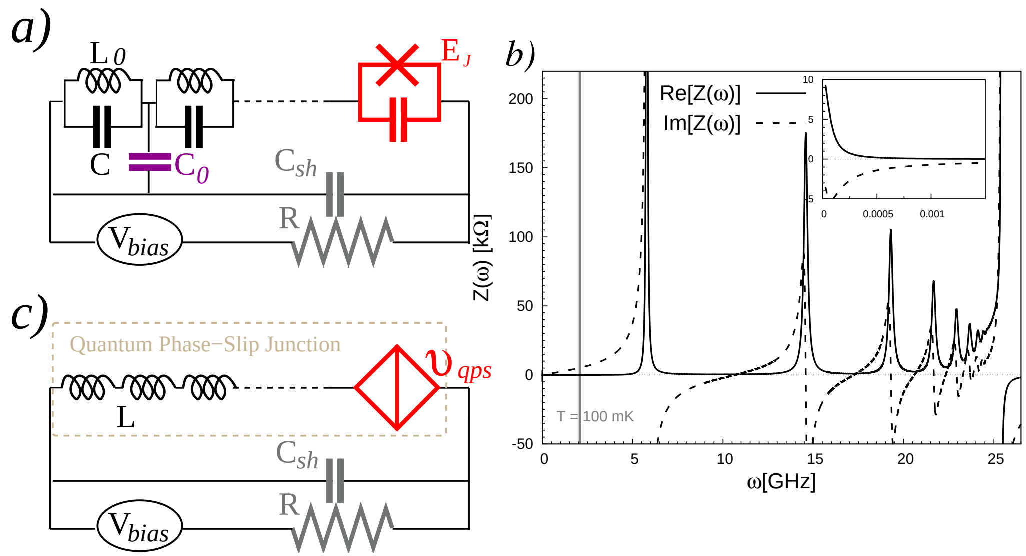
- classical dynamics for q in QPS-J $\rightarrow$ dual steps $I = n\,\frac{2e}{2\pi}\,\omega$

$$V + V_{mw}(\omega) = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + V_x \sin\left(\frac{2\pi}{2e}q\right)$$





Experiments QPS Junction

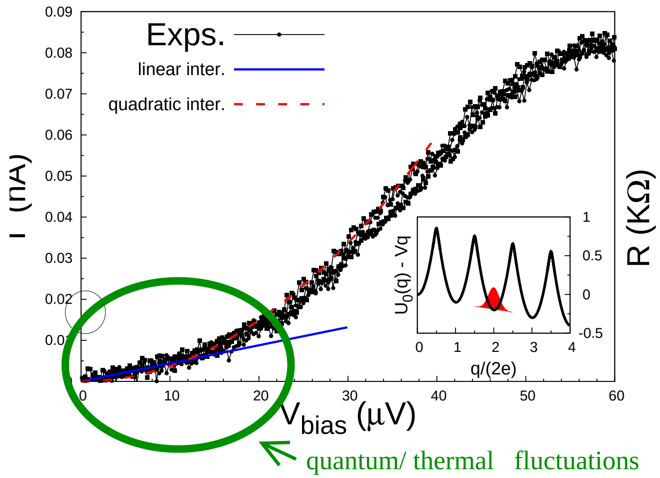


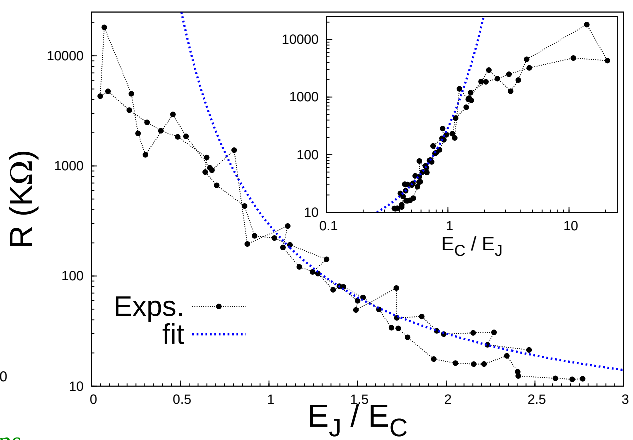
Inhomogeneous chains

$N = 18, 28, 38, 48, 88, 108$

$+1$ weak element $E_J \ll E_0$

Measurements (no microwaves irradiation)





activation

$$I = \left\langle \frac{d\hat{q}}{dt} \right\rangle = 2e\,(\Gamma_+ - \Gamma_-)$$

quantum/classical

structures under microwaves irradiation (but no steps...)

[Guichard, Rastelli et al., unpublished]