Variations on the Spin Hall effect

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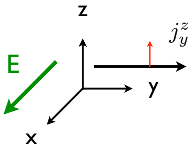
ROMA TRE
UNIVERSITÀ DEGLI STUDI

VietNam 2013
IXth Rencontres du Vietnam
August 4-10, 2013

Outline

1. SHE: intrinsic vs extrinsic mechanisms in a 2DEG
2. Onsager relations in the presence of SOC
3. Spin-thermoelectric effects: Nernst effect

The spin Hall effect (SHE) Dyakonov and Perel, JETP Lett. 13, 467 (1971); Phys. Lett. A 35, 459 (1971)



An applied electric field drives a transverse spin current

$$j_{yz} = \sigma^{sH} E_x$$

Symmetry argument Dyakonov PRL **99**, 126601 (2007)

- Charge current: odd under $\mathscr{T}$ and $\mathscr{P}$
- Spin current: odd under $\mathscr{P}$, even under $\mathscr{T}$
- For $\mathscr{P}$-invariant systems Onsager relations imply one parameter γ proportional to SOC

$$
\begin{aligned}
j_i &= \sigma E_i - \sigma^a \mathscr{E}_i^a - 4\, \varepsilon_{ija}\, \gamma\, j_{ja} \\
j_{ia} &= -\frac{\sigma}{4e^2} \mathscr{E}_i^a + \sigma^a E_i + \varepsilon_{iak}\, \gamma\, j_k, \quad \sigma^{sH} = \gamma\, \sigma
\end{aligned}
$$

$SU(2)$ language Mathur and Stone PRL **68**, 2964 (1992); Fröhlich and Studer RMP **65**, 733 (1993); Lyanda-Geller PRL **80**, 4273 (1998); Hatano et al. PRA **75**, 032107 (2007); Tokatly PRL **101**, 106601 (2008)

Potentials

$$
\begin{aligned}
\Psi &= \frac{1}{2} \Psi^a \sigma^a \\
\mathscr{A} &= \frac{1}{2} \mathscr{A}^a \sigma^a
\end{aligned}
$$

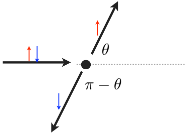
Fields

$$\mathscr{E}_i^a = -\partial_t \mathscr{A}_i^a - \nabla_{x_i} \Psi^a + \mathrm{i} [\Psi, \mathscr{A}_i]^a$$

$$\mathscr{B}_i^a = \frac{1}{2} \varepsilon_{ijk} \left(\nabla_{x_j} \mathscr{A}_k^a - \nabla_{x_k} \mathscr{A}_j^a + \mathrm{i} \left[\mathscr{A}_j, \mathscr{A}_k \right]^a \right)$$

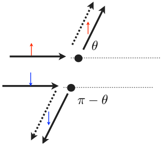
Biodiversity of the SHE (and iSHE) due to several mechanisms

Mott skew scattering (Smit 1958)



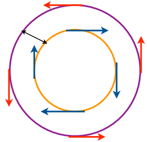
$$\mathscr{P} = 2\mathrm{Re}\left(\mathrm{i}A\frac{\lambda_0^2}{4}\right)\hat{\mathbf{p}}\times\hat{\mathbf{p}}'\cdot\boldsymbol{\sigma}$$

Side jump (Berger 1970)



$$\Delta\mathbf{x} = \frac{\lambda_0^2}{4}\boldsymbol{\sigma}\times\Delta\mathbf{p}.$$

The Rashba SOC



$$\alpha\hat{\mathbf{e}}_z\times\boldsymbol{\sigma}\cdot\mathbf{p}$$

and Dresselhaus . . .

Experiments interpretation (not exaustive list!)

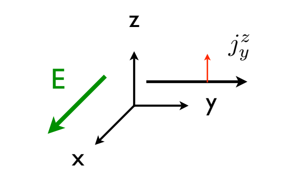
- GaAs film, extrinsic, $\gamma \sim 10^{-4}$ Kato et al. Sci. **306**, 1910 (2004)
- $In_xGa_{1-x}As$, extrinsic, $\gamma \sim 10^{-3}$ Garlid et al. PRL **105** 156602 (2010)
- 2DHG in GaAs, intrinsic, $\gamma \sim 10^{-2}$ Wunderlich et al. PRL **94**, 047204 (2005); 2DEG in ALGaAs extrinsic and intrinsic $\gamma \sim 10^{-3}$ Wunderlich et al. Nat. Phys. **5**, 675 (2009)
- Al, extrinsic, $\gamma \sim 10^{-4}$ Valenzuela and Tinkham, Nat. **442**, 176 (2006)
- Pt, Au, Mo, Pd, intrinsic, $\gamma \sim 10^{-3} - 10^{-2}$ Mosendz et al. PRB **82**, 214403 (2010)
- HgTe/(Hg,CdTe), intrinsic Brüne et al. Nat. Phys. **6**, 448 (2010)
- Ir-doped Cu, extrinsic, $\gamma \sim 10^{-1}$ Niimi et al. PRL **106**, 126601 (2011)
- β-Ta, intrinsic, $\gamma \sim 10^{-1}$ Liu et al. Sci. **336**, 555 (2012)

SHE in a Rashba 2DEG (Sinova et al. 2004)

Key arguments for the SHE

- Continuity-like equation for spin density Dimitrova 2005, Chalaev and Loss 2005

$$\partial_t s^y + \nabla \cdot \mathbf{j}^y = -2m\alpha j_y^z$$



$\Rightarrow$ No spin current in *static* and *uniform* conditions
$\Rightarrow$SHE still possible at *edges* and in *transient regime*

Mishchenko et al. PRL **93**, 226602 (2004); Raimondi et al. PRB **74**, 035340 (2006)

- $SU(2)$ vector potential language for SOC
  $\Rightarrow$ *diffusion* contribution from non-abelian covariant derivative
  $\Rightarrow$ *drift* contribution from Lorentz-like force due to $SU(2)$ *magnetic field*

$$j_y^z = -D(-\varepsilon^{zxy} 2m\alpha s^y) + \sigma^{sH} E_x$$

$\Rightarrow$ *diagrammatically* correspond to *vertex* and *bubble* terms

Raimondi and Schwab PRB **71**, 033311 (2005)

$\Rightarrow$ By product: *vanishing* of spin current implies in-plane spin polarization

Edelstein, Solid. State Commun. **73**, 233 (1990); Aronov and Lyanda-Geller, JETP Lett. **50**, 431 (1989)

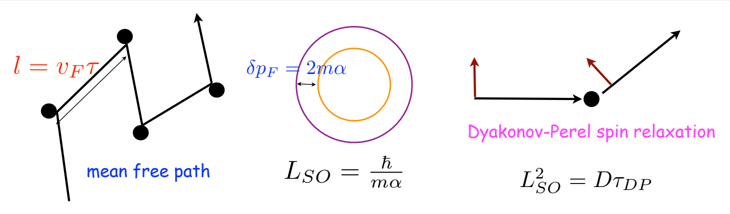
How to obtain SHE in the presence of Rashba SOC

Add extra source of spin relaxation

$$\partial_t s^y + \nabla \cdot \mathbf{j}^y = -2m\alpha j^z_y - \frac{1}{\tau_s} s^y$$

$$j^z_y = -D(-\varepsilon^{zxy} 2m\alpha s^y) + \sigma^{sH} E_x$$

Competition between τ_s and Dyakonov-Perel mechanism



$$\partial_t s^y = -\left(\frac{1}{\tau_{DP}} + \frac{1}{\tau_s}\right) s^y - 2m\alpha\sigma^{sH} E_x$$

$SU(2)$ formulation: Technical point

Combine $SU(2)$ with Keldysh GF: Gorini et al. PRB **82**, 195316 (2010)

- Introduce gauge invariant Green function In the standard $U(1)$ case

$$G(\mathbf{R}, t; \mathbf{p}, \varepsilon) \rightarrow G(\mathbf{R}, t; \mathbf{p} + e\mathbf{A}, \varepsilon)$$

For the $SU(2)$ case define a *locally* covariant Green function

$$G(\mathbf{R}, t; \mathbf{p}, \varepsilon) \rightarrow G(\mathbf{R}, t; \mathbf{p}, \varepsilon) - \frac{1}{2}\left\{\mathscr{A} \cdot \nabla_{\mathbf{p}}, G(\mathbf{R}, t; \mathbf{p}, \varepsilon)\right\}$$

- Define distribution function in terms of the covariant Green function

$$f(\mathbf{R}, \mathbf{p}, t) = \frac{1}{2}\left[1 + \int \frac{\mathrm{d}\varepsilon}{2\pi \mathrm{i}} G(\mathbf{R}, t; \mathbf{p}, \varepsilon)\right]$$

Key observation

Separation of terms: intrinsic $SU(2)$-fields in the *hydrodynamic derivative*, while extrinsic in the collision integral

Interplay of intrinsic and extrinsic SHE in the 2DEG

$$\left(\partial_t + \tilde{\nabla}_{\mathbf{x}} \cdot \left[\frac{\mathbf{p}}{m} f_{\mathbf{p}} + \frac{1}{2\tau}\frac{\lambda_0^2}{4}\left\{\mathbf{p}\times\sigma, f_{\mathbf{p}}\right\}\right] + \frac{1}{2}\left\{\mathscr{F}\cdot\nabla_{\mathbf{p}}, .\right\}\right) f_{\mathbf{p}} = I_0\left[f_{\mathbf{p}}\right] + \sum_{l=1,4} I_l\left[f_{\mathbf{p}}\right]$$

side-jump I_1

$$-\frac{1}{2}\sum_{\mathbf{p}'} W_{\mathbf{p}\mathbf{p}'}\frac{\lambda_0^2}{4}\left\{-e\mathbf{E}\times\sigma\cdot(\mathbf{p}'-\mathbf{p}), \frac{\partial f_{\mathbf{p}'}}{\partial \varepsilon_{\mathbf{p}'}}\right\}$$

skew-scattering I_2

$$-\sum_{\mathbf{p}'} W_{\mathbf{p}\mathbf{p}'}(\pi N_0 u)\frac{\lambda_0^2}{4}\left\{\mathbf{p}'\times\mathbf{p}\cdot\sigma, f_{\mathbf{p}'}\right\}$$

Elliott-Yafet I_3

$$I_3\left[f_{\mathbf{p}}\right] = -\sum_{\mathbf{p}'} W_{\mathbf{p}\mathbf{p}'}\frac{\lambda_0^4}{16}\left(\mathbf{p}'\times\mathbf{p}\right)_z^2\left(f_{\mathbf{p}} - \sigma^z f_{\mathbf{p}'}\sigma^z\right)$$

Extra term I_4

$$\frac{\lambda_0^2}{8}\sum_{\mathbf{p}'} W_{\mathbf{p}\mathbf{p}'}\varepsilon_{abc}\left\{(\tilde{\nabla}_{\mathbf{x}})_a\sigma_b, p_c f_{\mathbf{p}} - p'_c f_{\mathbf{p}'}\right\}$$

Spin-continuity equation (almost) unchanged and extra extrinsic drift terms

$$\partial_t s^y + \frac{1}{\tau_s}s^y = -2m\alpha j_y^z, \quad j_y^z = 2m\alpha D s^y + \left(\sigma_R^{sH} + \sigma_{sj}^{sH} + \sigma_{ss}^{sH}\right)E_x$$

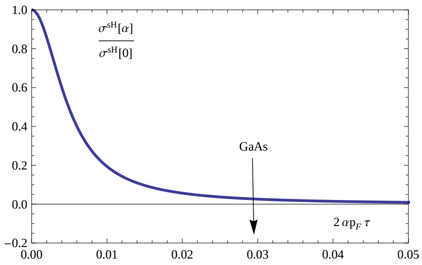
Spin Hall conductivity due to both intrinsic and extrinsic mechanisms

The uniform steady-state

$$\sigma^{sH} = \frac{\sigma_R^{sH} + \sigma_{sj}^{sH} + \sigma_{ss}^{sH}}{1 + \tau_s/\tau_{DP}}$$

- when $\alpha \to 0$, $\sigma^{sH} \to \sigma_{ss+sj}^{sH}$ since $\tau_{DP} \to \infty$
- τ_s/τ_{DP} controlling parameter

Raimondi and Schwab, EPL **87**, 37008 (2009); Raimondi et al. Ann. Phys. **524**, 153 (2012)



Comparison with exps

GaAs estimates Sih et al., Nature Phys. **1**, 31 (2005)

$n_s = 10^{12}\mathrm{cm}^{-2}$, $\mu = 10^3\mathrm{cm}^2\mathrm{Vs}$, $N_0 v_o = -\frac{1}{2}$,
$\lambda_0 = 4.7 \times 10^{-8}\mathrm{cm}$, $\alpha = 10^{-12}\mathrm{eVm}$,
$\sigma\gamma_{sj} = -1.3 \times 10^{-7}\Omega^{-1}$,
$\sigma\gamma_{skew} = 4.3 \times 10^{-7}\Omega^{-1}$,
$\sigma\gamma_{intr} = -8.2 \times 10^{-9}\Omega^{-1}$
$\tau_s = 8.4 \times 10^3\mathrm{ps} \gg \tau_{DP} = 90\mathrm{ps}$

Remarks and interpretations

- Even weak Rashba SO ($2\alpha p_F \tau \ll 1$) has a strong effect
- By having α smaller one may reach the interesting region $\tau_s \approx \tau_{DP}$
- The exp does not provide γ, which could provide a consistency check for the estimation of α

Spin current definition and Onsager relations

Let me ask two questions:

- What is the correct definition of the spin current?
  Proposal to replace the conventional (and most natural) definition (which is not generally conserved because of e.g. spin-orbit interaction)

$$J_i^a = \frac{1}{2}\{v_i, s^a\}$$

  with a conserved one Shi et al. PRL **96**, 076604 (2006)

$$\mathscr{J}_i^a = J_i^a + P_i^a, \quad \partial_i P_i^a = -T^a$$

- Are Onsager relations satisfied for SHE and ISHE?
  Observation that a spin current (which may induce a ISHE) may be driven by different external spin-dependent ($SU(2)$) potentials Wang et al. PRB **85**, 165201 (2012)

...and answer that the two issues are intimately related Gorini et al. PRL **109**, 246604 (2012)

- Both definitions are legitimate (provided being careful)
- Connection between spin current definition and external driving field

How to see this

- Consider both the *spin-vector gauge*

$$H_V = H_0 + J_i^z \mathscr{A}_i^z, \quad \mathscr{A}_i^z = -t\mathscr{E}_i^z$$

- and *spin-scalar gauge*

$$H_S = H_0 + \frac{\tau^z}{2}\Psi^z, \quad \Psi^z = -r_i \mathscr{E}_i^z,$$

- Perform a gauge transformation from the scalar to the vector one

$$U = \exp\left(\mathrm{i}\frac{1}{2}\tau^z \chi\right)$$

In general:

$$H_S' = H_0 + \mathscr{J}_i^z \mathscr{A}_i^a, \mathscr{J}_i^z = -\frac{\mathrm{i}}{\hbar}\left[\frac{\tau^z}{2} r_i, H_S\right]$$

Note: only if H_S commutes with τ^z the current reduces to the conventional one

Rashba 2DEG

Using the conventional current

$$\sigma^{sH}(\omega) = -\frac{\gamma\sigma}{e}\frac{-i\omega+\tau_s^{-1}}{-i\omega+\tau_{DP}^{-1}+\tau_s^{-1}}$$

Using the conserved current

$$\tilde{\sigma}^{sH}(\omega) = -\frac{\gamma\sigma}{e}\frac{-i\omega}{-i\omega+2\tau_{DP}^{-1}}\frac{-i\omega-\tau_{DP}^{-1}+\tau_s^{-1}}{-i\omega+\tau_{DP}^{-1}+\tau_s^{-1}}.$$

- σ Drude conductivity
- $\gamma = \gamma_{intr}+\gamma_{ss}+\gamma_{sj}$ charge-spin coupling parameter

$$\gamma_{intr} = -m\alpha^2\tau, \quad \gamma_{sj} = \frac{\lambda_0^2 m}{4\tau}$$

$$\gamma_{ss} = -\frac{(\lambda_0 p_F)^2}{16}(2\pi N_0 v)$$

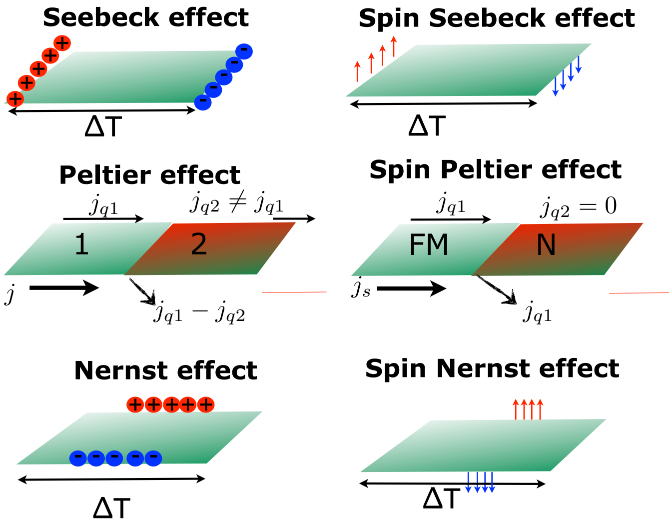
- τ_{DP}^{-1} Dyakonov-Perel relaxation
- τ_s^{-1} Elliott-Yafet relaxation

Rashba 2DHG Hughes et al. PRB **74**, 193316 (2006)

$$\sigma^{sH} = -\frac{\gamma\sigma}{e}, \quad \tilde{\sigma}(\omega) = -\frac{\gamma\sigma}{e}\frac{-i\omega}{-i\omega+\tau_{DP}^{-1}}, \quad \gamma = 6\tilde{\alpha}^3 k_F^4 m\tau$$

Experiment should tell which current is coupled to an external probe

Spin Caloritronics: Spin Nernst Effect



K. Uchida et al., Nat. Lett. **455**, 778 (2008); Nat. Mater. **9**, 894 (2010). C. M. Jaworski et al., Nat. Mat. **9**, 898 (2010). Review: Adachi et al. Rep. Prog. Phys. **76**, 036501 (2013).

A. Slachter et al., Phys. Rev. B **84**, 174408 (2011); J. Flipse et al., Nature Nanotechnology **7**, 166 (2012). B. Scharf et al., Phys. Rev. B **85**, 085208 (2012).

Ballistic limit for Rashba model: Z. Ma, Solid State Communications, **150**, 510 (2010

Present work: inclusion of disorder and interplay of several SO mechanisms

Phenomenological considerations and main result

Charge and heat currents

$$j_x = L_{11} E_x + L_{12}(-\nabla_x T)$$

- Electrical conductivity $L_{11} = \sigma$ and electric thermopower $S \equiv L_{12}/L_{11}$
- $L_{12} = e\mathscr{L} T \sigma'$ with Lorenz number $\mathscr{L} = \pi^2 k_B^2 / 3e^2$ and $\sigma' = \partial_\mu \sigma$, μ

Spin and heat currents in a simple situation

Assume inversion symmetry for a homogeneous and non-ferromagnetic material in the absence of magnetic fields M. I. Dyakonov, Phys. Rev. Lett. **99**, 126601 (2007):

$$j_y^z = -\gamma j_x \quad j_y^z = L_{11}^s E_x + L_{12}^s(-\nabla_x T), \quad S_s \equiv L_{12}^s / L_{11}^s \Rightarrow S_s = S$$

Spin and charge thermopowers coincide but what about when Rashba SOC is present?

- $S_s = S R_s$ with R_s depending on the various competing SO mechanisms
- $R_{so} \sim 3$ can be achieved, in principle, in GaAs samples with Rashba SO and extrinsic mechanisms $\Rightarrow$ heat-to-spin conversion could be more efficient than heat-to-charge

Detail: spin equivalent of the Mott formula

Same range of validity of the Wiedemann-Franz law based on single-particle description of transport, Fermi statistics of carriers, elastic scattering (G. V. Chester and Thellung, Proc. Phys. Soc. London **77**, 1005 (1961; M. Jonson and G. D. Mahan, Phys. Rev. B **21**, 4223 (1980); J. S. Langer, Phys. Rev. B **128**, 110 (1962).)

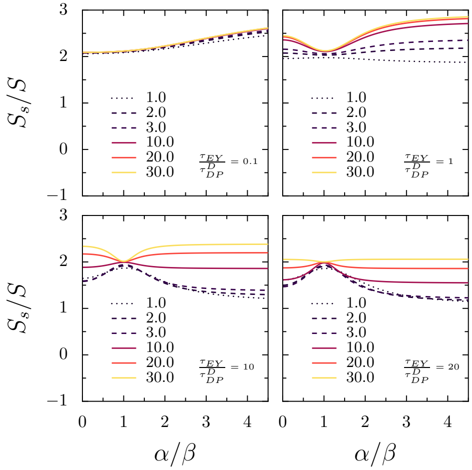
The spin thermopower

$$S_s = -e\mathscr{L}T\sigma'_{sH}/\sigma_{sH} = -e\mathscr{L}T\frac{\sigma'}{\sigma}\left[1+\frac{\gamma_{\mathrm{ss}}}{\gamma}-\frac{\zeta}{1+\zeta}\left(1-\frac{2\tau_{\mathrm{s}}}{\tau_{\mathrm{EY}}}\right)\right].$$

- $\gamma = \gamma_{\mathrm{intr}} + \gamma_{\mathrm{sj}} + \gamma_{\mathrm{ss}}$ Combine the various spin-charge current couplings
- $\frac{1}{\tau_{DP}} = (2m)^2(\alpha^2+\beta^2)D$ Combine D-P relaxation from Rashba and Dresselhaus SO interaction
- $\frac{1}{\tau_s} = \frac{1}{\tau_{EY}} + \frac{4}{3\tau_{sf}}$ Combine spin relaxation from SO from impurities and from magnetic impurities
- $\zeta = \frac{\tau_{\mathrm{s}}}{\tau_{DP}} - 4\frac{\tau_{\mathrm{s}}^2/(\tau_{DP}^R\tau_{DP}^D)}{\tau_{\mathrm{s}}/\tau_{DP}+1}$

The spin thermopower S_s of a disordered 2D-electron gas Borge et al. PRB **87**, 085309 (2013)

Each panel shows the ratio S_s/S as a function of the ratio α/β (Rashba / Dresselhaus) for a given Elliott-Yafet scattering strength, strong to weak from top left to bottom right



Magnetic scattering is strongest for the dotted curve, $\tau_{sf}/\tau_{DP}^D = 1$, and strong (weak) for the dashed (solid) curves, $\tau_{sf}/\tau_{DP}^D = 2, 3 (10, 20, 30)$. Panel 3 corresponds to standard GaAs

- mobility $\mu = 10^4 \mathrm{cm}^2/\mathrm{Vs}$
- density $n = 10^{12} \mathrm{cm}^{-2}$
- effective extrinsic wavelength $\lambda_0 = 4.7 \times 10^{-8} \mathrm{cm}$
- Dresselhaus coupling constant $\hbar\beta = 10^{-12} \mathrm{eVm}$

Then $\gamma_{ss} \gg \gamma_{intr}, \gamma_{sj},\ \tau_{EY} \gg \tau_{DP}^D$.

Conclusions and acknowledgements

- Non trivial interplay in the SHE between Rashba and impurity driven SOCs
- Spin current definition associated to the gauge choice for spin potentials
- Onsager relations hold irrespective of the choice
- Spin thermopower enhanced with respect to charge thermopower

Collaborators

- Juan Borge
- Cosimo Gorini
- Andrei Shelankov
- Peter Schwab
- Giovanni Vignale

Papers

- EPL 87, 37008 (2009)
- PRB **82**, 195316 (2010)
- Ann. Phys. **524**, 153 (2012)
- PRL **109**, 246604 (2012)
- PRB **87**, 085309 (2013)

Thank you for your attention!