Supplemental material for "Negative delta-T noise in the Fractional Quantum Hall effect"

J. Rech, T. Jonckheere, B. Grémaud, and T. Martin
Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France

NON-INTERACTING FERMIONS: A SCATTERING THEORY CALCULATION

We consider here a two-terminal device where two fermionic non-interacting spinless reservoirs are brought together, thereby defining a junction where we assume a single conduction channel for simplicity. As is common in such devices, the tunnel barrier between the reservoirs is described by a transmission coefficient $\mathcal{T}(E)$ which we approximate by assuming it is energy-independent, $\mathcal{T}(E) \simeq \mathcal{T}$. The reservoirs, labeled $i = 1$ and 2 respectively, are characterized by their chemical potential μ_i and temperature T_i, leading to a description in terms of the Fermi distribution $f_i(E) = \left[1 + e^{\frac{E-\mu_i}{T_i}}\right]^{-1}$.

Using scattering theory, the current through the junction is readily obtained from

$$\langle \hat{I} \rangle = \frac{e}{2\pi} \int dE \, \mathcal{T} \left[f_1(E) - f_2(E) \right] \tag{1}$$

In the absence of a potential bias, the chemical potential of the two reservoirs are equal, $\mu_1 = \mu_2 = \mu$, and the resulting energy integral for the current identically vanishes, independently of the temperature on both sides of the junction.

Fluctuations away from this average value are characterized by the noise, i.e. the current-current correlations, which we consider here at zero-frequency. Within the scattering theory formalism, one obtains the standard expression for the zero-frequency noise

$$\begin{aligned} S &= 2 \int dt \left[\langle \hat{I}(t) \hat{I}(0) \rangle - \langle \hat{I}(t) \rangle \langle \hat{I}(0) \rangle \right] \\ &= \frac{e^2}{\pi} \int dE \left\{ \mathcal{T} \left[f_1(E) \left(1 - f_1(E)\right) + f_2(E) \left(1 - f_2(E)\right) \right] + \mathcal{T} \left(1 - \mathcal{T}\right) \left(f_1(E) - f_2(E) \right)^2 \right\} \end{aligned} \tag{2}$$

The first term corresponds to thermal-like noise, while the last one is a non-equilibrium contribution.

We now focus on the specific situation where no bias is applied, $\mu_1 = \mu_2 = \mu$, but the temperature of the two reservoirs are different and parametrized by

$$T_1 = \bar{T} - \frac{\Delta T}{2} \tag{3}$$

$$T_2 = \bar{T} + \frac{\Delta T}{2} \tag{4}$$

Noticing that $f_i(E) \left(1 - f_i(E)\right) = -T_i \frac{df_i(E)}{dE}$, the first contribution is readily obtained and reduces to

$$\frac{e^2}{\pi} \mathcal{T} \int dE \left[f_1(E) \left(1 - f_1(E)\right) + f_2(E) \left(1 - f_2(E)\right) \right] = \frac{e^2}{\pi} \mathcal{T} \left[T_1 + T_2 \right] = 2 \frac{e^2}{\pi} \mathcal{T} \bar{T} \tag{5}$$

The remaining, non-equilibrium, contribution does not reduce to a simple analytic form. Instead, assuming a small temperature difference compared with the average temperature of the reservoirs, we rely on an expansion in powers of the parameter $\frac{\Delta T}{2\bar{T}}$. Indeed, one has

$$\left[f_1(E) - f_2(E) \right]^2 = \left(\frac{\Delta T}{2\bar{T}} \right)^2 \left(2\bar{T} \left. \frac{\partial f(E)}{\partial T} \right|_{T=\bar{T}} \right)^2 + \frac{4}{3} \left(\frac{\Delta T}{2\bar{T}} \right)^4 \bar{T} \left. \frac{\partial f(E)}{\partial T} \right|_{T=\bar{T}} \bar{T}^3 \left. \frac{\partial^3 f(E)}{\partial T^3} \right|_{T=\bar{T}} + O\left[\left(\frac{\Delta T}{2\bar{T}} \right)^6 \right] \tag{6}$$

where $f(E) = \left[1 + e^{\frac{E-\mu}{T}}\right]^{-1}$.

The resulting integrals can be carried out as

$$\int dE \left(2\bar{T} \left. \frac{\partial f(E)}{\partial T} \right|_{T=\bar{T}} \right)^2 = \bar{T} \int du \, \frac{4u^2 e^{2u}}{(1+e^u)^4} = 2\bar{T}\frac{\pi^2-6}{9} \tag{7}$$

and

$$\int dE \; \bar{T} \left. \frac{\partial f(E)}{\partial T} \right|_{T=\bar{T}} \bar{T}^3 \left. \frac{\partial^3 f(E)}{\partial T^3} \right|_{T=\bar{T}} = \bar{T} \int du \; u^2 e^{2u} \frac{6+6u+u^2-4e^u\left(u^2-3\right)+e^{2u}\left(6-6u+u^2\right)}{(1+e^u)^6} = -\bar{T}\frac{7\pi^4-75\pi^2+90}{450} \tag{8}$$

where we introduced the reduced variable $u = \frac{E-\mu}{\bar{T}}$.

Putting all the contributions back together, one is left with

$$S \approx 2\frac{e^2}{\pi} \left\{ \mathcal{T}\bar{T} + \mathcal{T}\left(1-\mathcal{T}\right)\bar{T} \left[\frac{\pi^2-6}{9} \left(\frac{\Delta T}{2\bar{T}} \right)^2 - \frac{7\pi^4-75\pi^2+90}{675} \left(\frac{\Delta T}{2\bar{T}} \right)^4 \right] \right\} \tag{9}$$

BACKSCATTERED CURRENT AND NOISE

The system considered here is a Hall bar, in the fractional quantum Hall regime, restricting ourselves to filling factors in the Laughlin sequence, i.e. $\nu = 1/(2n+1)$. The Hall bar is equipped with a quantum point contact (QPC), placed at position $x = 0$. In the weak backscattering regime, quasiparticles are allowed to tunnel from one edge to the other through the bulk at the position of the QPC, leading to a tunneling Hamiltonian of the form

$$H_{\rm WB} = \Gamma_0 e^{ie^* V t} \psi_R^\dagger(0)\psi_L(0) + \text{H.c.} \tag{10}$$

where we introduced the effective charge $e^* = \nu e$ and used the Peierls substitution to make the voltage appear explicitly in the tunneling Hamiltonian rather than in the contacts.

The backscattered current is readily obtained from this tunneling Hamiltonian and reads

$$I_B(t) = -e\dot{N}_R = ie\left[N_R, H_{\rm WB}\right] = ie^* \left[\Gamma_0 e^{ie^*Vt}\psi_R^\dagger(0,t)\psi_L(0,t) - \Gamma_0 e^{-ie^*Vt}\psi_L^\dagger(0,t)\psi_R(0,t) \right] \tag{11}$$

The expectation value of the backscattered current can be conveniently expressed in the Keldysh formalism as

$$\mathcal{I}_B = \langle I_B(t) \rangle = \frac{1}{2} \sum_{\eta=+,-} \left\langle T_K I_B\left(t^{(\eta)}\right) e^{-i\int_c dt' H_T(t')} \right\rangle \tag{12}$$

which is further expanded up to second order in the tunneling parameter as (first order perturbation in H_T)

$$\begin{aligned} \mathcal{I}_B = \frac{e^*}{2}\Gamma_0^2 \int dt' \sum_{\eta,\eta'} \eta' \Big\{ & e^{ie^*V(t-t')} \left\langle T_K \psi_R^\dagger\left(0,t^{(\eta)}\right)\psi_R\left(0,t'^{(\eta')}\right)\right\rangle \left\langle T_K \psi_L\left(0,t^{(\eta)}\right)\psi_L^\dagger\left(0,t'^{(\eta')}\right)\right\rangle \\ & - e^{-ie^*V(t-t')} \left\langle T_K \psi_L^\dagger\left(0,t^{(\eta)}\right)\psi_L\left(0,t'^{(\eta')}\right)\right\rangle \left\langle T_K \psi_R\left(0,t^{(\eta)}\right)\psi_R^\dagger\left(0,t'^{(\eta')}\right)\right\rangle \Big\} \\ = -2i\frac{e^*\Gamma_0^2}{(2\pi a)^2} & \int_{-\infty}^{\infty} d\tau \sin\left(e^* V\tau\right) e^{\nu\mathcal{G}_R(-\tau)} e^{\nu\mathcal{G}_L(-\tau)} \end{aligned} \tag{13}$$

where we used that

$$\left\langle T_K \psi_\mu^\dagger(0,t^{(\eta)})\psi_\mu(0,t'^{(\eta')})\right\rangle = \left\langle T_K \psi_\mu(0,t^{(\eta)})\psi_\mu^\dagger(0,t'^{(\eta')})\right\rangle = \frac{1}{2\pi a}\exp\left[\nu\mathcal{G}_\mu\left(\sigma_{\eta\eta'}(t-t')\right)\right] \tag{14}$$

and introduced $\sigma_{\eta\eta'}(t-t') = \eta'\left[(1-\delta_{\eta\eta'})(t-t') + \delta_{\eta\eta'}|t-t'|\right]$.

The bosonic Green's function is given at finite temperature by

$$\mathcal{G}_\mu(\tau) = -\log\left[\frac{\sinh\left(\frac{\pi}{\beta_\mu}(i\tau_0 - \tau)\right)}{\sinh\left(i\frac{\pi}{\beta_\mu}\tau_0\right)}\right] \tag{15}$$

where $\beta_\mu = 1/T_\mu$ is the inverse temperature of the considered lead (recall that $k_B = 1$) and $\tau_0 = a/v_F$ is a short-time (or high-energy) cutoff.

The current noise can be written in terms of the backscattered current at the QPC as

$$S_B(t,t') = \langle I_B(t) I_B(t')\rangle - \langle I_B(t)\rangle \langle I_B(t')\rangle \tag{16}$$

It is then similarly obtained through perturbative expansion in the tunnel Hamiltonian writing, to second order in Γ_0

$$\begin{aligned}
S_B(t,t') &= \left\langle T_K \Delta I_B(t^{(+)}) \Delta I_B(t'^{(-)}) \exp\left(-i\sum_{\eta=\pm}\eta\int_{-\infty}^{+\infty} dt'' H_T(t''^{(\eta)})\right)\right\rangle \\
&= (e^*\Gamma_0)^2 \left[e^{ie^*V(t-t')}\left\langle T_K \psi_R^\dagger\left(0,t^{(+)}\right)\psi_R\left(0,t'^{(-)}\right)\right\rangle \left\langle T_K \psi_L\left(0,t^{(+)}\right)\psi_L^\dagger\left(0,t'^{(-)}\right)\right\rangle \right. \\
&\quad \left. + \, e^{-ie^*V(t-t')}\left\langle T_K \psi_L^\dagger\left(0,t^{(+)}\right)\psi_L\left(0,t'^{(-)}\right)\right\rangle \left\langle T_K \psi_R\left(0,t^{(+)}\right)\psi_R^\dagger\left(0,t'^{(-)}\right)\right\rangle\right] \\
&= 2\left(\frac{e^*\Gamma_0}{2\pi a}\right)^2 \cos\left[e^*V(t-t')\right]\exp\left[\nu\mathcal{G}_R\left(t'-t\right) + \nu\mathcal{G}_L\left(t'-t\right)\right]
\end{aligned} \tag{17}$$

From this, one readily sees that the noise only depends on the time difference, so that $S_B(t,t') = S_B(t-t')$. One can thus define the zero-frequency noise as

$$\begin{aligned}
\mathcal{S}_B &= 2\int d\tau\ S_B\left(\tau\right) \\
&= \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \cos\left(e^*V\tau\right)\exp\left[\nu\mathcal{G}_R\left(-\tau\right) + \nu\mathcal{G}_L\left(-\tau\right)\right] \\
&= \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \cos\left(e^*V\tau\right)\left[\frac{\sinh\left(i\pi T_R\tau_0\right)}{\sinh\left(\pi T_R(i\tau_0+\tau)\right)}\right]^\nu \left[\frac{\sinh\left(i\pi T_L\tau_0\right)}{\sinh\left(\pi T_L(i\tau_0+\tau)\right)}\right]^\nu
\end{aligned} \tag{18}$$

TEMPERATURE BIASED CASE: $T_R \neq T_L$ AND $V = 0$

Let us first consider the situation of zero bias voltage, and focus on the effect of a temperature difference between the two input ports of the QPC. In this particular situation, the expression for the backscattered current becomes trivial: as one can readily see from the general expression given in Eq. (13), the current vanishes in the absence of a voltage bias, no matter what the temperature difference is.

Things are different for the noise, as the temperature difference induces extra fluctuations of the current compared to the equilibrium case, which are then susceptible to be partitioned at the QPC. While a fully analytic expression is out of reach, we instead resort to a perturbative expansion in the temperature difference.

Introducing the notations

$$T_{R/L} = \bar{T} \pm \frac{\Delta T}{2} \tag{19}$$

and expanding the result of Eq. (18) to fourth order in ΔT, one has

$$\mathcal{S}_B = \mathcal{S}_0 + \left(\frac{\Delta T}{2\bar{T}}\right)^2 \mathcal{S}_2 + \left(\frac{\Delta T}{2\bar{T}}\right)^4 \mathcal{S}_4 + O\left[\left(\frac{\Delta T}{2\bar{T}}\right)^6\right] \tag{20}$$

Notice that, by symmetry of the parametrization in temperature, there are no linear terms in the temperature difference.

The various contributions can then be computed separately. The leading-order term corresponds to the thermal noise, it is given by

$$\mathcal{S}_0 = \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \left[\frac{\sinh\left(i\pi\bar{T}\tau_0\right)}{\sinh\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]^{2\nu} = \frac{e^2}{2\pi}\left(\frac{2\nu\Gamma_0}{v_F}\right)^2 \bar{T}\left(2\pi\bar{T}\tau_0\right)^{2\nu-2}\frac{\Gamma(\nu)^2}{\Gamma(2\nu)} \tag{21}$$

The leading correction reads

$$\begin{aligned}\mathcal{S}_2 &= \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \left[\frac{\sinh\left(i\pi\bar{T}\tau_0\right)}{\sinh\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]^{2\nu}\left[\frac{\nu\pi^2\bar{T}^2\tau_0^2}{\sinh^2\left(i\pi\bar{T}\tau_0\right)}+\frac{\nu\pi^2\bar{T}^2(i\tau_0+\tau)^2}{\sinh^2\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]\\ &= \frac{e^2}{2\pi}\left(\frac{2\nu\Gamma_0}{v_F}\right)^2\left(2\pi\bar{T}\tau_0\right)^{2\nu-2}\nu\bar{T}\frac{\Gamma(\nu)^2}{\Gamma(2\nu)}\left\{\frac{\nu}{2\nu+1}\left[\frac{\pi^2}{2}-\psi'(\nu+1)\right]-1\right\}\end{aligned} \tag{22}$$

and the next order one is yet more involved, writing

$$\begin{aligned}\mathcal{S}_4 = \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \left[\frac{\sinh\left(i\pi\bar{T}\tau_0\right)}{\sinh\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]^{2\nu} &\left[\frac{\nu(\nu-1)}{2}\frac{\pi^4\bar{T}^4\tau_0^4}{\sinh^4\left(i\pi\bar{T}\tau_0\right)}+\frac{\nu}{3}\frac{\pi^4\bar{T}^4(i\tau_0+\tau)^4}{\sinh^2\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right.\\ &-\frac{\nu}{3}\frac{\pi^4\bar{T}^4\tau_0^4}{\sinh^2\left(i\pi\bar{T}\tau_0\right)}+\frac{\nu(\nu+1)}{2}\frac{\pi^4\bar{T}^4(i\tau_0+\tau)^4}{\sinh^4\left(\pi\bar{T}(i\tau_0+\tau)\right)}\\ &\left.+\nu^2\frac{\pi^2\bar{T}^2\tau_0^2}{\sinh^2\left(i\pi\bar{T}\tau_0\right)}\frac{\pi^2\bar{T}^2(i\tau_0+\tau)^2}{\sinh^2\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]\\ = \frac{e^2}{2\pi}\left(\frac{2\nu\Gamma_0}{v_F}\right)^2\bar{T}\left(2\pi\bar{T}\tau_0\right)^{2\nu-2}\frac{\Gamma(\nu)^2}{\Gamma(2\nu)}&\\ \times\left\{\nu\frac{\pi^4\nu^2(4+3\nu)-12\pi^2\nu\left(2\nu^2+3\nu-3\right)+12\left(4\nu^3+4\nu^2-5\nu-3\right)}{24\left(4\nu^2+8\nu+3\right)}\right.&\\ +\nu^2\frac{4\nu^2+6\nu-6-\pi^2\nu(4+3\nu)}{8\nu^2+16\nu+6}\psi'(\nu+1)+\nu^3\frac{4+3\nu}{2\left(4\nu^2+8\nu+3\right)}\left[\psi'(\nu+1)\right]^2&\\ \left.+\nu^3\frac{4+3\nu}{12\left(4\nu^2+8\nu+3\right)}\psi^{(3)}(\nu+1)\right\}&\end{aligned} \tag{23}$$

Bringing all these contributions together, one is left with the following expression for the noise

$$\mathcal{S}_B = \mathcal{S}_{\mathrm{WB}}^0\left[1+\left(\frac{\Delta T}{2\bar{T}}\right)^2\mathcal{C}_\nu^{(2)}+\left(\frac{\Delta T}{2\bar{T}}\right)^4\mathcal{C}_\nu^{(4)}\right] \tag{24}$$

where $\mathcal{S}_{\mathrm{WB}}^0=\mathcal{S}_0$ and the coefficients $\mathcal{C}_\nu^{(n)}$ take the form

$$\mathcal{C}_\nu^{(2)} = \nu\left\{\frac{\nu}{2\nu+1}\left[\frac{\pi^2}{2}-\psi'(\nu+1)\right]-1\right\} \tag{25}$$

$$\begin{aligned}\mathcal{C}_\nu^{(4)} = {}&\nu\frac{\pi^4\nu^2(4+3\nu)-12\pi^2\nu\left(2\nu^2+3\nu-3\right)+12\left(4\nu^3+4\nu^2-5\nu-3\right)}{24\left(4\nu^2+8\nu+3\right)}\\ &+\nu^2\frac{4\nu^2+6\nu-6-\pi^2\nu(4+3\nu)}{8\nu^2+16\nu+6}\psi'(\nu+1)+\nu^3\frac{4+3\nu}{2\left(4\nu^2+8\nu+3\right)}\left[\psi'(\nu+1)\right]^2\\ &+\nu^3\frac{4+3\nu}{12\left(4\nu^2+8\nu+3\right)}\psi^{(3)}(\nu+1)\end{aligned} \tag{26}$$

As a first step, one can check that in the special situation of filling factor $\nu=1$, one recovers the expected values for the prefactors above, namely

$$\mathcal{C}_1^{(2)} = \frac{\pi^2}{9}-\frac{2}{3}\simeq 0.42996 \tag{27}$$

$$\mathcal{C}_1^{(4)} = -\frac{7\pi^4}{675}+\frac{\pi^2}{9}-\frac{2}{15}\simeq -0.04688 \tag{28}$$

where we used that $\psi'(2) = \frac{\pi^2}{6} - 1$ and $\psi^{(3)}(2) = \frac{\pi^4}{15} - 6$.

Quite surprisingly, using some specific values of the digamma function and its derivatives, one can also show that for a (fictitious) filling factor of $\nu = 1/2$, one has for the coefficients of the expansion

$$\mathcal{C}^{(2)}_{1/2} = \frac{1}{2}\left\{\frac{1}{4}\left[\frac{\pi^2}{2} - \psi'\left(\frac{3}{2}+1\right)\right] - 1\right\} = 0 \tag{29}$$

$$\mathcal{C}^{(4)}_{1/2} = \frac{11\pi^4 + 48\pi^2 - 384}{3072} - \frac{8+11\pi^2}{256}\psi'\left(\frac{1}{2}+1\right) + \frac{11}{256}\left[\psi'\left(\frac{1}{2}+1\right)\right]^2 + \frac{11}{1536}\psi^{(3)}\left(\frac{1}{2}+1\right) = 0 \tag{30}$$

where we used that $\psi'\left(\frac{3}{2}\right) = \frac{\pi^2}{2} - 4$ and $\psi^{(3)}\left(\frac{3}{2}\right) = \pi^4 - 96$. These results raise the question of the fate of the ΔT noise in the special situation of filling factor $\nu = 1/2$. Having a closer look at the full expression for the noise, we have for this special filling factor

$$\mathcal{S}_B = \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \sqrt{\frac{\sinh\left(i\pi T_R \tau_0\right)}{\sinh\left(\pi T_R(i\tau_0+\tau)\right)}}\sqrt{\frac{\sinh\left(i\pi T_L \tau_0\right)}{\sinh\left(\pi T_L(i\tau_0+\tau)\right)}} \tag{31}$$

As it turns out, in the limit of vanishingly small cutoff $a \to 0$, this result is independent of ΔT, so that the delta-T noise exactly vanishes at this order in the tunneling parameter Γ_0.

STRONG BACKSCATTERING REGIME

All the results presented so far have been obtained in the regime of weak backscattering. It is, however, interesting to investigate the fate of the ΔT noise in the opposite regime of strong backscattering, where the tunneling Hamiltonian now reads

$$H_{\mathrm{SB}} = \Gamma e^{ieVt}\Psi^\dagger_R(0)\Psi_L(0) + \mathrm{H.c.} \tag{32}$$

where $\Psi_{R/L}$ is the operator associated with the annihilation of a full electron.

This leads to substantial modifications in the expressions for the average current and zero-frequency noise, which now become

$$\mathcal{I}_B = -2i\frac{e\Gamma}{(2\pi a)^2}\int d\tau\ \sin\left(eV\tau\right)\exp\left[\frac{\mathcal{G}_R\left(-\tau\right) + \mathcal{G}_L\left(-\tau\right)}{\nu}\right] \tag{33}$$

$$\mathcal{S}_B = \left(\frac{e\Gamma}{\pi a}\right)^2\int d\tau\ \cos\left(eV\tau\right)\exp\left[\frac{\mathcal{G}_R\left(-\tau\right) + \mathcal{G}_L\left(-\tau\right)}{\nu}\right] \tag{34}$$

These expressions could be readily obtained from the ones derived in the weak backscattering regime upon performing a duality transformation, i.e. substituting $e^* \to e$, $\Gamma_0 \to \Gamma$ and $\nu \to \frac{1}{\nu}$.

It follows that one can similarly extend our results for the ΔT noise to this regime of strong backscattering, leading to

$$\mathcal{S}_B = \mathcal{S}^0_{\mathrm{SB}}\left[1 + \left(\frac{\Delta T}{2\bar{T}}\right)^2 \mathcal{C}^{(2)}_{1/\nu} + \left(\frac{\Delta T}{2\bar{T}}\right)^4 \mathcal{C}^{(4)}_{1/\nu}\right] \tag{35}$$

where one has

$$\mathcal{S}^0_{\mathrm{SB}} = \frac{e^2}{2\pi}\left(\frac{2\Gamma}{v_F}\right)^2 \bar{T}\left(2\pi\bar{T}\tau_0\right)^{\frac{2}{\nu}-2}\frac{\Gamma\left(\frac{1}{\nu}\right)^2}{\Gamma\left(\frac{2}{\nu}\right)} \tag{36}$$

and the coefficients $\mathcal{C}^{(n)}_{1/\nu}$ take the form

$$\mathcal{C}^{(2)}_{1/\nu} = \frac{1}{\nu}\left\{\frac{1}{2+\nu}\left[\frac{\pi^2}{2} - \psi'\left(\frac{1}{\nu}+1\right)\right] - 1\right\} \tag{37}$$

$$\begin{aligned}\mathcal{C}^{(4)}_{1/\nu} = {} & \frac{1}{\nu^2}\frac{\pi^4\left(4\nu+3\right) - 12\pi^2\left(2+3\nu-3\nu^2\right) + 12\left(4+4\nu-5\nu^2-3\nu^3\right)}{24\left(4+8\nu+3\nu^2\right)} \\ & + \frac{1}{\nu^2}\frac{4+6\nu-6-\pi^2\left(4\nu+3\right)}{8+16\nu+6\nu^2}\psi'\left(\frac{1}{\nu}+1\right) + \frac{1}{\nu^2}\frac{4\nu+3}{2\left(4+8\nu+3\nu^2\right)}\left[\psi'\left(\frac{1}{\nu}+1\right)\right]^2 \\ & + \frac{1}{\nu^2}\frac{4\nu+3}{12\left(4+8\nu+3\nu^2\right)}\psi^{(3)}\left(\frac{1}{\nu}+1\right)\end{aligned} \tag{38}$$

Note that, one readily sees from these results that the case $\nu = 1/2$ is no longer special, suggesting that the vanishing of the ΔT noise for this specific value of the filling factor has to do with the weak backscattering regime and does not extend beyond it.

VOLTAGE DEPENDENCE

We now extend the results of the previous sections, applying an external bias voltage in addition to the small temperature difference. Starting from the general expression of Eq. (18) in the weak backscattering regime, and expanding in powers of the temperature difference ΔT, we have, up to second order

$$\begin{aligned}\mathcal{S}_B = {} & \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \cos\left(e^* V\tau\right)\left[\frac{\sinh\left(i\pi\bar{T}\tau_0\right)}{\sinh\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]^{2\nu}\left\{1 + \left(\frac{\Delta T}{2\bar{T}}\right)^2\left[\frac{\nu\pi^2\bar{T}^2\tau_0^2}{\sinh^2\left(i\pi\bar{T}\tau_0\right)} + \frac{\nu\pi^2\bar{T}^2(i\tau_0+\tau)^2}{\sinh^2\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]\right. \\ & + \left(\frac{\Delta T}{2\bar{T}}\right)^4\left[\frac{\nu(\nu-1)}{2}\frac{\pi^4\bar{T}^4\tau_0^4}{\sinh^4\left(i\pi\bar{T}\tau_0\right)} + \frac{\nu}{3}\frac{\pi^4\bar{T}^4(i\tau_0+\tau)^4}{\sinh^2\left(\pi\bar{T}(i\tau_0+\tau)\right)} - \frac{\nu}{3}\frac{\pi^4\bar{T}^4\tau_0^4}{\sinh^2\left(i\pi\bar{T}\tau_0\right)}\right. \\ & \left.\left. + \frac{\nu(\nu+1)}{2}\frac{\pi^4\bar{T}^4(i\tau_0+\tau)^4}{\sinh^4\left(\pi\bar{T}(i\tau_0+\tau)\right)} + \nu^2\frac{\pi^2\bar{T}^2\tau_0^2}{\sinh^2\left(i\pi\bar{T}\tau_0\right)}\frac{\pi^2\bar{T}^2(i\tau_0+\tau)^2}{\sinh^2\left(\pi\bar{T}(i\tau_0+\tau)\right)}\right]\right\}\end{aligned} \tag{39}$$

This can be written in a similar form as before, only now involving voltage-dependent coefficients, as

$$\mathcal{S}_B = \mathcal{S}^0_{\rm WB}(V)\left\{1 + \left(\frac{\Delta T}{2\bar{T}}\right)^2\mathcal{C}^{(2)}_\nu(V) + O\left[\left(\frac{\Delta T}{2\bar{T}}\right)^4\right]\right\} \tag{40}$$

where the noise in the absence of a voltage difference reads

$$\mathcal{S}^0_{\rm WB}(V) = \frac{e^2}{2\pi}\left(\frac{2\nu\Gamma_0}{v_F}\right)^2\bar{T}\left(2\pi\bar{T}\tau_0\right)^{2\nu-2}\frac{\left|\Gamma\left(\nu+i\frac{e^*V}{2\pi\bar{T}}\right)\right|^2}{\Gamma\left(2\nu\right)}\cosh\left(\frac{e^*V}{2\bar{T}}\right) \tag{41}$$

and the coefficient $\mathcal{C}^{(2)}_\nu(V)$ is given by

$$\begin{aligned}\mathcal{C}^{(2)}_\nu(V) = -\nu + \frac{\nu^2 + \left(\frac{e^*V}{2\pi\bar{T}}\right)^2}{2\nu+1} & \left\{-2\pi{\rm Im}\ \psi\left(\nu+1+i\frac{e^*V}{2\pi\bar{T}}\right)\tanh\left(\frac{e^*V}{2\bar{T}}\right)\right. \\ & \left. + \frac{\pi^2}{2} + 2\left[{\rm Im}\ \psi\left(\nu+1+i\frac{e^*V}{2\pi\bar{T}}\right)\right]^2 - {\rm Re}\ \psi'\left(\nu+1+i\frac{e^*V}{2\pi\bar{T}}\right)\right\}\end{aligned} \tag{42}$$

which lead back to the expressions of Eqs. (24) and (25) in the limit of vanishingly small voltage bias.

In the special case $\nu = 1$, this can be worked out explicitly as

$$\mathcal{C}^{(2)}_1(V) = \pi\frac{\pi\frac{e^*V}{2\pi\bar{T}}\left[1+\left(\frac{e^*V}{2\pi\bar{T}}\right)^2\right] - \left[1+3\left(\frac{e^*V}{2\pi\bar{T}}\right)^2\right]\tanh\left(\frac{e^*V}{2\bar{T}}\right)}{3\frac{e^*V}{2\pi\bar{T}}\left[\sinh\left(\frac{e^*V}{2\bar{T}}\right)\right]^2} \tag{43}$$

where we used that ${\rm Im}\ \psi\left(2+iz\right) = \frac{\pi}{2\tanh(\pi z)} - \frac{1}{2z} - \frac{z}{1+z^2}$.

CROSSED CORRELATIONS AND BACKSCATTERED NOISE

In the setup considered here, and represented in Fig. 1 of the main text, the current operators I_μ at the position of the contacts x_μ ($\mu = 3, 4$) are given by

$$I_3(x_3, t) = \frac{e\sqrt{\nu}}{2\pi} v_F \partial_x \phi_R(x_3, t) + \frac{\nu e^2}{2\pi} V \tag{44}$$

$$I_4(x_4, t) = -\frac{e\sqrt{\nu}}{2\pi} v_F \partial_x \phi_L(x_4, t). \tag{45}$$

We now focus on the weak backscattering regime, where the tunneling Hamiltonian is given by Eq. (10). Expanding to second order in the tunneling parameter Γ_0, following a similar derivation to the one presented in the previous sections, one has for the average currents at the contacts

$$\langle I_3(x_3, t)\rangle = 2i\frac{\nu e\Gamma_0^2}{(2\pi a)^2}\int d\tau \sin(\nu eV\tau)\exp\left[\nu\mathcal{G}_R(-\tau) + \nu\mathcal{G}_L(-\tau)\right] + \frac{\nu e^2}{2\pi}V = -\mathcal{I}_B + \frac{\nu e^2}{2\pi}V \tag{46}$$

$$\langle I_4(x_4, t)\rangle = -2i\frac{\nu e\Gamma_0^2}{(2\pi a)^2}\int d\tau \sin(\nu eV\tau)\exp\left[\nu\mathcal{G}_R(-\tau) + \nu\mathcal{G}_L(-\tau)\right] = \mathcal{I}_B, \tag{47}$$

where we used the expression for the backscattered current obtained in Eq. (13).

The current crossed correlations measured at the contacts can similarly be defined as

$$S_{34}(t - t') = \langle I_3(x_3, t) I_4(x_4, t')\rangle - \langle I_3(x_3, t)\rangle \langle I_4(x_4, t')\rangle \tag{48}$$

Substituting the expression for the current operators and performing an expansion to second order in the tunneling Hamiltonian, one obtains after some algebra

$$\begin{aligned} S_{34}(t - t') = &-\left(\frac{e\nu v_F \Gamma_0}{4\pi^2 a}\right)^2 \sum_{\eta_1,\eta_2}\int d\tau \cos(\nu eV\tau)\exp\left[\mathcal{G}_R(\sigma_{\eta_1\eta_2}(\tau)) + \mathcal{G}_L(\sigma_{\eta_1\eta_2}(\tau))\right] \\ &\times \int d\bar{t}\frac{\pi T_R}{v_F}\left[\coth\left(\pi T_R\left(\bar{t} - \frac{\tau}{2} - i\eta_1\tau_0\right)\right) - \coth\left(\pi T_R\left(\bar{t} + \frac{\tau}{2} - i\eta_2\tau_0\right)\right)\right] \\ &\quad\times \frac{\pi T_L}{v_F}\left[\coth\left(\pi T_L\left(\bar{t} - \frac{\tau}{2} - t + t' - i\eta_1\tau_0\right)\right) - \coth\left(\pi T_L\left(\bar{t} + \frac{\tau}{2} - t + t' - i\eta_2\tau_0\right)\right)\right]. \end{aligned} \tag{49}$$

Following the same prescription as the one used for the backscattered current, one can define the zero-frequency crossed correlations of the current as

$$\begin{aligned} \mathcal{S}_{34} &= 2\int d\tau S_{34}(\tau) \\ &= \left(\frac{e\nu\Gamma_0}{\pi a}\right)^2 \Big\{-\int d\tau \cos(\nu eV\tau)\exp\left[\nu\mathcal{G}_R(\tau) + \nu\mathcal{G}_L(\tau)\right] \\ &\qquad + i\int d\tau \cos(\nu eV\tau)\left(T_R + T_L\right)\tau \exp\left[\nu\mathcal{G}_R(\tau) + \nu\mathcal{G}_L(\tau)\right]\Big\} \end{aligned} \tag{50}$$

Using the expressions for the backscattered current and noise, Eqs. (13) and (18), one can readily write

$$\mathcal{S}_{34} = 2\left(T_R + T_L\right)\frac{\partial\mathcal{I}_B}{\partial V} - \mathcal{S}_B \tag{51}$$

therefore providing a connection between tunneling conductance, crossed correlations and backscattered noise.

WEAKLY COUPLED NON-CHIRAL LUTTINGER LIQUIDS

In this section, we consider the slightly different case of two weakly coupled non-chiral Luttinger liquids. The full Hamiltonian of such a system is given by $H = H_1 + H_2 + H_T$ with

$$H_\mu = \frac{v}{2}\int dx\left[K\left(\partial_x\phi_\mu\right)^2 + \frac{1}{K}\left(\partial_x\theta_\mu\right)^2\right] \tag{52}$$

$$H_T = \Gamma\sum_{r,r'}\Psi^\dagger_{r,1}(0)\Psi_{r',2}(0) + \text{H.c.} \tag{53}$$

where v and K are the standard Luttinger parameters, and Γ is the tunneling constant. Note that we restrict ourselves to repulsive interactions, so that $K < 1$.

Following conventional notations, the bosonic modes ϕ_μ and θ_μ are related to the physical fermionic fields $\Psi_{r,\mu}$ via the bosonization identity

$$\Psi_{r,\mu}(x) = \frac{U_{r,\mu}}{\sqrt{2\pi a}} e^{irk_F x} e^{i\sqrt{\pi}[\phi(x)+r\theta(x)]} \tag{54}$$

where $U_{r,\mu}$ is a Klein factor, a is a short distance cutoff, and $r = \pm 1$ corresponds to right/left movers.

The backscattered current operator is readily obtained from the tunneling Hamiltonian and reads

$$\mathcal{I}_B = ie\Gamma \sum_{r,r'} \Psi^\dagger_{r,1}(0)\Psi_{r',2}(0) + \text{H.c.} \tag{55}$$

The backscattered noise is similarly defined as

$$S_B(t,t') = \langle I_B(t) I_B(t')\rangle - \langle I_B(t)\rangle \langle I_B(t')\rangle \tag{56}$$

and can be expanded to second order in the tunneling parameter Γ yielding

$$\begin{aligned} S_B(t,t') &= \left\langle T_K \Delta I_B(t^{(+)})\Delta I_B(t'^{(-)}) \exp\left(-i\sum_{\eta=\pm}\eta\int_{-\infty}^{+\infty} dt'' H_T(t''^{(\eta)})\right)\right\rangle \\ &= 2\left(\frac{e\Gamma}{2\pi a}\right)^2 \sum_{r,r'} \exp\left\{\pi\left[\mathcal{G}^{+-}_{\phi\phi,1}(t,t') + \mathcal{G}^{+-}_{\theta\theta,1}(t,t') + r\mathcal{G}^{+-}_{\phi\theta,1}(t,t') + r\mathcal{G}^{+-}_{\theta\phi,1}(t,t')\right]\right\} \\ &\qquad \times \exp\left\{\pi\left[\mathcal{G}^{+-}_{\phi\phi,2}(t,t') + \mathcal{G}^{+-}_{\theta\theta,2}(t,t') + r'\mathcal{G}^{+-}_{\phi\theta,2}(t,t') + r'\mathcal{G}^{+-}_{\theta\phi,2}(t,t')\right]\right\} \end{aligned} \tag{57}$$

where, for simplicity, we focused on the case of a temperature difference, setting the voltage bias $V = 0$.

The Keldysh Green's functions are then readily expressed in terms of the conventional bosonic propagators $\mathcal{G}^{+-}_{\alpha\beta,\mu}(t,t') = \mathcal{G}_{\alpha\beta,\mu}(t'-t)$, whose expressions are obtained from standard Luttinger liquid derivations and are given by

$$\mathcal{G}_{\phi\phi,\mu}(\tau) = -\frac{1}{2\pi K}\log\left[\frac{\sinh(\pi T_\mu(i\tau_0-\tau))}{\sinh(i\pi T_\mu \tau_0)}\right] \tag{58}$$

$$\mathcal{G}_{\theta\theta,\mu}(\tau) = -\frac{K}{2\pi}\log\left[\frac{\sinh(\pi T_\mu(i\tau_0-\tau))}{\sinh(i\pi T_\mu \tau_0)}\right] \tag{59}$$

$$\mathcal{G}_{\phi\theta,\mu}(\tau) = 0 \tag{60}$$

$$\mathcal{G}_{\theta\phi,\mu}(\tau) = 0 \tag{61}$$

where $\tau_0 = a/v_F$ is a short time cutoff.

The zero-frequency backscattered noise then reads

$$\begin{aligned} \mathcal{S}_B &= 2\int d\tau\ S_B(\tau) \\ &= 4\left(\frac{e\Gamma}{\pi a}\right)^2 \int d\tau \exp\left\{\pi\left[\mathcal{G}_{\phi\phi,1}(\tau) + \mathcal{G}_{\theta\theta,1}(\tau) + \mathcal{G}_{\phi\phi,2}(\tau) + \mathcal{G}_{\theta\theta,2}(\tau)\right]\right\} \\ &= 4\left(\frac{e\Gamma}{\pi a}\right)^2 \int d\tau \left[\frac{\sinh(i\pi T_1\tau_0)}{\sinh(\pi T_1(i\tau_0-\tau))}\right]^{\frac{1}{2}\left(K+\frac{1}{K}\right)} \left[\frac{\sinh(i\pi T_2\tau_0)}{\sinh(\pi T_2(i\tau_0-\tau))}\right]^{\frac{1}{2}\left(K+\frac{1}{K}\right)} \end{aligned} \tag{62}$$

Interestingly, the resulting expression is similar (up to a trivial numerical prefactor) to the one obtained in the chiral case, Eq. (18), provided that one introduces an effective filling factor related to the Luttinger parameter K, namely

$$\nu_{\text{eff}} = \frac{1}{2}\left(K + \frac{1}{K}\right) \tag{63}$$

This allows us to use some of the results obtained in the chiral case in order to extend them to the non-chiral one. Most importantly, since $K < 1$, this effective filling factor always satisfies $\nu_{\text{eff}} \geq 1$, thus leading to a *positive* contribution to the delta-T noise.

This derivation can easily be extended to include more degrees of freedom (spinfull Luttinger liquids, nanotubes) ultimately leading to similar expressions with an effective filling factor involving the multiple Luttinger liquid parameters, but still satisfying $\nu_{\text{eff}} \geq 1$.