Supplemental material for
"Anyonic statistics revealed by the Hong-Ou-Mandel dip for fractional excitations"

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GREEN FUNCTIONS AND THEIR PROPERTIES

The quasiparticle Green function is defined as

$$\mathcal{G}_{R/L}\left(x,x';t^{\eta},t'^{\eta'}\right)=\left\langle T_K\psi^{\dagger}_{R/L}\left(x,t^{\eta}\right)\psi_{R/L}\left(x',t'^{\eta'}\right)\right\rangle. \tag{S1}$$

Using the properties of time ordering, and the linear dispersion along the edge, this can be recast under the simplified form

$$\mathcal{G}_{R/L}\left(x,x';t^{\eta},t'^{\eta'}\right)=\mathcal{G}_{R/L}\left(\sigma^{\eta\eta'}_{tt'}\left(t-t'\mp\frac{x-x'}{v_F}\right)\right), \tag{S2}$$

where $\sigma^{\eta\eta'}_{tt'}=\text{sign}(t-t')(\eta+\eta')/2+(\eta'-\eta)/2$ and

$$\mathcal{G}_{R/L}(t)=\left\langle\psi^{\dagger}_{R/L}(0,t)\psi_{R/L}(0,0)\right\rangle. \tag{S3}$$

Invoking the bosonization identity, this is further reduced as

$$\mathcal{G}_{R/L}(t)=\frac{1}{2\pi a}\left\langle e^{i\sqrt{\nu}\phi^{\dagger}_{R/L}(0,t)}e^{-i\sqrt{\nu}\phi_{R/L}(0,0)}\right\rangle=\frac{1}{2\pi a}e^{\nu G_{R/L}(t)}, \tag{S4}$$

where we introduced the bosonic Green function $G_{R/L}(t)=\left\langle\phi^{\dagger}_{R/L}(0,t)\phi_{R/L}(0,0)\right\rangle$.

From the free Hamiltonian H_0, one can readily extract the corresponding Green function for the bosonic modes as

$$G_{R/L}(t)=-\log\left[\frac{\sinh\left(i\frac{\pi a}{\beta v_F}-\frac{\pi t}{\beta}\right)}{\sinh\left(i\frac{\pi a}{\beta v_F}\right)}\right], \tag{S5}$$

so that the quasiparticle Green function ultimately reads

$$\mathcal{G}_{R/L}(t)=\frac{1}{2\pi a}\left[\frac{\sinh\left(i\frac{\pi a}{\beta v_F}\right)}{\sinh\left(i\frac{\pi a}{\beta v_F}-\frac{\pi t}{\beta}\right)}\right]^{\nu} \tag{S6}$$

One can easily show that this Green function is identical for right- and left-movers, so that we can safely drop the R/L subscript from this point onward.

As anyons obey fractional statistics, they show nontrivial exchange properties which ensure that, at equal time, one has

$$\psi^{\dagger}_R(0,t)\psi_R(x,t)=e^{-i\pi\nu\text{Sign}(x)}\psi_R(x,t)\psi^{\dagger}_R(0,t) \tag{S7}$$

Making use of the linear dispersion along the edge, this is rewritten as

$$\psi^{\dagger}_R(0,t)\psi_R\left(0,t-\frac{x}{v_F}\right)=e^{-i\pi\nu\text{Sign}(x)}\psi_R\left(0,t-\frac{x}{v_F}\right)\psi^{\dagger}_R(0,t) \tag{S8}$$

Since this is valid for any set of parameters (x,t), one can choose $x=v_Ft$, without loss of generality. Taking then the quantum average, this yields

$$\begin{aligned}\left\langle\psi^{\dagger}_R(0,t)\psi_R(0,0)\right\rangle&=e^{-i\pi\nu\text{Sign}(t)}\left\langle\psi_R(0,0)\psi^{\dagger}_R(0,t)\right\rangle\\ \mathcal{G}(t)&=e^{-i\pi\nu\text{Sign}(t)}\mathcal{G}(-t)\end{aligned} \tag{S9}$$

It follows that the value of the ratio $\mathcal{G}(t)/\mathcal{G}(-t)$ can be viewed as a direct consequence of the exchange statistics of anyons.

COMPUTING THE TUNNELING CURRENT

Tunneling current when injecting a single quasiparticle

The tunneling current operator reads $I_T(t) = ie^*(\Gamma\psi_R^\dagger(0,t)\psi_L(0,t) - \text{H.c.})$. Here, we consider the situation where a single quasiparticle is incoming along the right edge, described by a prepared state of the form $|\varphi\rangle = \psi_R^\dagger(-x_0, -\mathcal{T})|0\rangle$.

To lowest order in Γ, the mean current is thus given by

$$\begin{aligned}\langle I_T(t)\rangle &= -\frac{i}{2}\int dt' \sum_{\eta,\eta'} \eta' \left\langle \varphi \left| T_K\ I_T\left(t^\eta\right) H_T\left(t'^{\eta'}\right)\right| \varphi\right\rangle \\ &= \frac{e^*}{2}\int dt' \sum_{\epsilon,\epsilon'}\sum_{\eta,\eta'} \epsilon\eta' \langle 0|T_K\ \psi_R(-x_0,-\mathcal{T}^-)\left(\Gamma\psi_R^\dagger(0,t^\eta)\psi_L(0,t^\eta)\right)^{(\epsilon)} \\ &\qquad\qquad \times \left(\Gamma\psi_R^\dagger(0,t'^{\eta'})\psi_L(0,t'^{\eta'})\right)^{(\epsilon')} \psi_R^\dagger(-x_0,-\mathcal{T}^+)|0\rangle \end{aligned} \tag{S10}$$

where $\epsilon = \pm$ is used to include the Hermitian conjugated terms, such that for $\epsilon = +$, one has for any operator O, $O^{(+)} = O$ while for $\epsilon = -$, one has $O^{(-)} = O^\dagger$.

Here, T_K ensures the time-ordering along the Keldysh contour, and $\eta, \eta' = \pm$ are Keldysh indices. Note that we consider the injection of QP to have happened in the distant past. The Kelsdysh indices added to the times $-\mathcal{T}$ have been chosen to ensure that the $\psi_R(-x_0, -\mathcal{T}^-)$ and $\psi_R^\dagger(-x_0, -\mathcal{T}^+)$ operators remain in the same position after time ordering, independently of the values of t and t', for $\mathcal{T}$ large enough. In particular, keeping in mind that $x_0 = v_F\mathcal{T}$ (corresponding to a quasiparticle reaching the QPC at $t = 0$), this allows us to simplify some of the resulting Green functions as

$$\mathcal{G}(-x_0, 0; -\mathcal{T}^-, t^\eta) = \mathcal{G}(-t) \tag{S11}$$

$$\mathcal{G}(0, -x_0; t^\eta, -\mathcal{T}^+) = \mathcal{G}(t), \tag{S12}$$

independently of η and t, provided that $t \ll \mathcal{T}$.

Using the bosonized form of the quasiparticle operators, we have

$$\langle I_T(t)\rangle = \Gamma^2\frac{e^*}{2}\int dt' \sum_{\epsilon}\sum_{\eta,\eta'} \epsilon\eta' \left[\mathcal{G}\left(\sigma_{tt'}^{\eta\eta'}\left(t-t'\right)\right)\right]^2 \left(\frac{\mathcal{G}(-t')\mathcal{G}(t)}{\mathcal{G}(t')\mathcal{G}(-t)}\right)^\epsilon \tag{S13}$$

Using the properties of the Green function derived in Eq. (S9), this then becomes

$$\langle I_T(t)\rangle = 2ie^*\Gamma^2\int_{-\infty}^t dt'\ \sin\left(2\pi\nu\int_{t'}^t d\tau\ \delta(\tau)\right) \times \left[\mathcal{G}(t-t')^2 - \mathcal{G}(t'-t)^2\right] \tag{S14}$$

Changing the integration variable to $\tau = -t'$, and using the expression of the Green function, we get:

$$\langle I_T(t)\rangle = \theta(t)2ie^*\frac{\Gamma^2}{(2\pi a)^2}\sin(2\pi\nu)\int_0^\infty d\tau\left[\left(\frac{\sinh(i\pi T\tau_0)}{\sinh(\pi T(i\tau_0 - t - \tau)}\right)^{2\nu} - \left(\frac{\sinh(i\pi T\tau_0)}{\sinh(\pi T(i\tau_0 + t + \tau)}\right)^{2\nu}\right] \tag{S15}$$

where $\tau_0 = a/v_F$, $T = 1/(k_B\beta)$ is the temperature, and we use $k_B = \hbar = 1$. Defining the reduced variables $\alpha = \pi T\tau_0$, $u = \pi T\tau$ and $z = \pi T t$, the first term in the integral can be written as

$$\int_0^\infty du \left(\frac{\sinh(i\alpha)}{\sinh(i\alpha - z - u)}\right)^{2\nu} = \int_0^\infty du\left(\frac{e^{i\alpha}-e^{-i\alpha}}{-e^{-i\alpha}e^z}\frac{1}{1-e^{2i\alpha}e^{-2z}e^{-2u}}\right)^{2\nu} e^{-2\nu\, u} \tag{S16}$$

$$= \left(e^{-z}\left(1-e^{2i\alpha}\right)\right)^{2\nu}\int_0^\infty du\left(1-e^{2i\alpha}e^{-2z}e^{-2u}\right)^{-2\nu} e^{-2\nu\, u} \tag{S17}$$

$$= \frac{1}{2}\left(e^{-z}\left(1-e^{2i\alpha}\right)\right)^{2\nu}\left(\frac{1}{\nu}\right)\ {}_2F_1\left(2\nu,\nu,\nu+1,e^{2i\alpha-2z}\right) \tag{S18}$$

where ${}_2F_1$ is the hypergeometric function. Using this result, the current can eventually be recast as

$$\begin{aligned}\langle I_T(t)\rangle &= \theta(t)\ 2e^*\left(\frac{\Gamma}{2\pi v_F\tau_0}\right)^2\frac{\sin(2\pi\nu)}{2\pi\nu T}\ e^{-2\nu\pi T t}\ \left(2\sin(\pi T\tau_0)\right)^{2\nu} \\ &\quad\times 2\mathrm{Im}\left[{}_2F_1\left(2\nu,\nu,\nu+1,e^{-2\nu\pi T t}e^{-2i\pi T\tau_0}\right)e^{i\pi\nu(1-2T\tau_0)}\right]\end{aligned} \tag{S19}$$

Taking then the leading order in the cutoff parameter τ_0 leads to

$$\langle I_T(t)\rangle = \theta(t)\; 2e^* \left(\frac{\Gamma}{2\pi v_F}\right)^2 \tau_0^{2\nu-2}\; \frac{2\sin(\pi\nu)\sin(2\pi\nu)}{\nu}\; e^{-2\nu\pi T t}(2\pi T)^{2\nu-1}\;\; {}_2F_1\left(2\nu,\nu,\nu+1,e^{-2\nu\pi T t}\right) \tag{S20}$$

We see that this is a function of $2\nu\pi T t = 2\nu\pi t/\beta$, which implies that the typical length scale for this function is $\sim \beta$. The behavior of the current in the two limits $t \ll \beta$ and $t \gg \beta$ is obtained by using the asymptotic behavior of the hypergeometric function:

$$2F_1\left(2\nu,\nu,\nu+1,e^{-2\nu\pi T t}\right) = \begin{cases} \nu\frac{\Gamma(\nu)^2}{\Gamma(2\nu)}\frac{\sin(\pi\nu)}{\sin(2\pi\nu)} - \frac{\nu}{1-2\nu}\left(1-e^{-2\nu\pi T t}\right) & t \ll \beta \\ 1+\frac{2\nu^2}{\nu+1}e^{-2\nu\pi T t} & t \gg \beta \end{cases}. \tag{S21}$$

Tunneling current when injecting a single electron

It is instructive to repeat the same kind of derivation, only this time considering the situation where a single electron is incoming along the right edge. The prepared state now takes the form $|\varphi\rangle = \Psi_R^\dagger(-x_0,-\mathcal{T})|0\rangle$, where the electron operator Ψ_R satisfies the bosonization identity $\Psi_R(x) = \frac{U_R}{2\pi a}e^{ik_F x}e^{-i\frac{1}{\sqrt{\nu}}\phi_R(x)}$.

Following a similar derivation to the one above, one obtains instead of Eq. (S13), the following expression for the tunneling current

$$\langle I_T(t)\rangle = \Gamma^2\frac{e^*}{2}\int dt' \sum_{\eta,\eta'} \epsilon\eta' \left[\mathcal{G}\left(\sigma_{tt'}^{\eta\eta'}(t-t')\right)\right]^2 \left[\left(\frac{\mathcal{G}(-t')\mathcal{G}(t)}{\mathcal{G}(t')\mathcal{G}(-t)}\right)^{1/\nu} - \left(\frac{\mathcal{G}(t')\mathcal{G}(-t)}{\mathcal{G}(-t')\mathcal{G}(t)}\right)^{1/\nu}\right] \tag{S22}$$

From the properties of the quasiparticle Green function, Eq. (S9), one readily sees that for $t \neq 0$

$$\left(\frac{\mathcal{G}(-t')\mathcal{G}(t)}{\mathcal{G}(t')\mathcal{G}(-t)}\right)^{1/\nu} = \exp\left(-i\int_{t'}^{t} d\tau\; 2\pi\delta(\tau)\right) = 1, \tag{S23}$$

so that the tunneling current vanishes at all times $t \neq 0$ and is nonzero only at the specific time that the electron reaches the QPC.

Tunneling current in the presence of a time-dependent voltage

In the presence of a voltage bias, the tunneling part of the Hamiltonian can be written as

$$H_T(t) = \Gamma \exp\left[ie^*\int_{-\infty}^{t} dt'\; V(t')\right]\psi_R^\dagger(0,t)\psi_L(0,t) + \text{H.c.} \tag{S24}$$

where it now contains the effect of the applied votlage $V(t)$.

The tunneling current operator now reads

$$I_T(t) = ie^*\left(\Gamma\exp\left[ie^*\int_{-\infty}^{t} dt'\; V(t')\right]\psi_R^\dagger(0,t)\psi_L(0,t) - \text{H.c.}\right). \tag{S25}$$

Taking the quantum average, the mean tunneling current is given in full generality by

$$\begin{aligned}\langle I_T(t)\rangle = \frac{ie^*}{2}\sum_\eta\sum_\epsilon \epsilon\Big\langle T_K\left(\Gamma\exp\left[ie^*\int_{-\infty}^{t}dt'\; V(t')\right]\psi_R^\dagger(0,t^\eta)\psi_L(0,t^\eta)\right)^{(\epsilon)} \\ \times\exp\left[-i\sum_{\eta'}\eta'\int_{-\infty}^{\infty}dt'\; H_T(t'^{\eta'})\right]\Big\rangle\end{aligned} \tag{S26}$$

where the sum on $\epsilon = \pm$ is used to represent the Hermitian conjugate, and $\eta,\eta' = \pm$ are Keldysh indices.

Performing a perturbative expansion in the tunneling amplitude Γ, this gives up to second order

$$\langle I_T(t)\rangle = \frac{e^*}{2}\Gamma^2 \sum_{\eta,\eta'}\sum_{\epsilon} \epsilon\eta' \int_{-\infty}^{\infty} dt' \, \exp\left[i\,\epsilon\, e^* \int_{-\infty}^{t} dt' \, V(t')\right] \left\langle T_K \psi_R^\dagger(0,t^\eta)\psi_R(0,t'^{\eta'})\right\rangle \left\langle T_K \psi_L(0,t^\eta)\psi_L^\dagger(0,t'^{\eta'})\right\rangle \tag{S27}$$

Using the expression for the quasiparticle Green function, and performing explicitly the sum on the Keldysh indices η and η', one eventually gets

$$\langle I_T(t)\rangle = 2ie^*\Gamma^2 \int_{-\infty}^{t} dt' \, \sin\left(e^* \int_{t'}^{t} dt'' V(t'')\right) \left[\mathcal{G}(t-t')^2 - \mathcal{G}(t'-t)^2\right]. \tag{S28}$$

where the Keldysh summations end up restricting the t' integral from $-\infty$ to t.

COMPUTING THE NOISE

General expression

The current noise is defined as:

$$S(t,t') = \left\langle T_K \delta I_T(t^-)\, \delta I_T(t'^+)\right\rangle \tag{S29}$$

with $\delta I_T(t) = I_T(t) - \langle I_T(t)\rangle$, and $\pm$ are Keldysh indices.

In the presence of a voltage bias applied to both edges, the tunneling part of the Hamiltonian can be written as

$$H_T(t) = \Gamma \exp\left[ie^* \int_{-\infty}^{t} dt' \, \left(V_R(t') - V_L(t')\right)\right] \psi_R^\dagger(0,t)\psi_L(0,t) + \text{H.c.} \tag{S30}$$

where we applied a standard gauge transformation in order to reabsorb the effect of the voltage drives into the tunneling amplitude. In this situation, the tunneling current operator reads

$$I_T(t) = ie^* \left(\Gamma \exp\left[ie^* \int_{-\infty}^{t} dt' \, \left(V_R(t') - V_L(t')\right)\right] \psi_R^\dagger(0,t)\psi_L(0,t) - \text{H.c.}\right). \tag{S31}$$

Substituting this back into Eq. (S29), one readily obtains, up to lowest order in the tunneling amplitude Γ

$$S(t,t') = 2\left(\frac{e^*\Gamma}{2\pi a}\right)^2 \cos\left(e^* \int_{t'}^{t} dt'' (V_R(t'') - V_L(t''))\right) \mathcal{G}(t-t')^2. \tag{S32}$$

In what follows, we focus on the Hanbury-Brown Twiss (HBT) and the Hong-Ou-Mandel (HOM) setups, corresponding respectively to applying a single voltage drive, or to applying both of them.

HOM noise for two narrow pulses of average charge e^*

We consider here the case of two infinitely short pulses so that both $V_R(t)$ and $V_L(t)$ are composed of a single delta function, with a time-shift δt between them. Focusing on pulses of average charge e^*, one can thus write

$$V_R(t) = \frac{2\pi}{e}\delta\left(t + \frac{\delta t}{2}\right) \qquad V_L(t) = \frac{2\pi}{e}\delta\left(t - \frac{\delta t}{2}\right). \tag{S33}$$

The cosine factor entering the expression for the noise in Eq. (S32) then simply reduces to either $\cos(2\pi\nu)$ or to 1, depending on the values of t and t', so that we write it as $\cos\left[2\pi\nu f_{\delta t}(t,t')\right]$. The newly defined function $f_{\delta t}(t,t')$ is 1 if one of the times t or t' is in the interval $[-\delta t/2, \delta t/2]$ while the other one is not, and reduces to 0 otherwise.

The HOM noise is defined as the zero-frequency noise due to the collision of these two excitations, as a function of the time-interval δt. Focusing on the zero-frequency contribution, and filtering out the equilibrium thermal noise (by subtracting the value in the absence of voltage drives), one has for the un-normalized HOM noise

$$\mathcal{S}_{HOM} = S(V_R,V_L) - S(0,0) = 2\left(\frac{e^*\Gamma}{2\pi a}\right)^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \left\{\cos\left[2\pi\nu f_{\delta t}(t,t')\right] - 1\right\} \mathcal{G}(t-t')^2 \tag{S34}$$

Similarly, one can work out the expression for the corresponding noise when only one of the drives is present. The resulting HBT noise reads

$$\mathcal{S}_{HBT} = S(V_R, 0) - S(0,0) = 2\left(\frac{e^*\Gamma}{2\pi a}\right)^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \left[\cos\left(2\pi\nu \frac{1-\text{sign}(t\times t')}{2}\right) - 1\right] \mathcal{G}(t-t')^2 \tag{S35}$$

The standard HOM noise ratio is then defined as the ratio of the un-normalized HOM noise to twice the HBT noise, so that

$$\begin{aligned} S_{HOM}(\delta t) &= \frac{\mathcal{S}_{HOM}}{2\mathcal{S}_{HBT}} \\ &= \frac{\int dt dt' \left\{\cos\left[2\pi\nu f_{\delta t}(t,t')\right] - 1\right\} e^{2\nu G(t'-t)}}{2\int dt dt' \left[\cos\left(2\pi\nu \frac{1-\text{sign}(t\times t')}{2}\right) - 1\right] e^{2\nu G(t'-t)}} \end{aligned} \tag{S36}$$

Substituting the actual value of $f_{\delta t}(t,t')$, this can be further rewritten as

$$\begin{aligned} S_{HOM}(\delta t) &= \frac{\int_0^{|\delta t|} dt \int_0^{\infty} dt' \left[e^{2\nu G(t+t')} + e^{2\nu G(-t-t')}\right]}{\int_0^{\infty} dt \int_0^{\infty} dt' \left[e^{2\nu G(t+t')} + e^{2\nu G(-t-t')}\right]} \\ &= \frac{\text{Re}\left[\int_0^{|\delta t|} dt \int_0^{\infty} dt' e^{2\nu G(t+t')}\right]}{\text{Re}\left[\int_0^{\infty} dt \int_0^{\infty} dt' e^{2\nu G(t+t')}\right]} \\ &= 1 - \frac{\text{Re}\left[\mathcal{I}(\delta)\right]}{\text{Re}\left[\mathcal{I}(0)\right]} \end{aligned} \tag{S37}$$

where we introduced

$$\mathcal{I}(\delta) = \int_0^{\infty} dz\ z \left(\frac{\sinh(i\alpha)}{\sinh(i\alpha - z - \delta)}\right)^{2\nu} \tag{S38}$$

with the reduced variable $\delta = \pi\,|\delta t|\,/\beta$, and the infinitesimal $\alpha = \pi\tau_0/\beta$.

This integral can be worked out as

$$\mathcal{I}(\delta) = -\frac{1}{4}\left(1 - e^{2i\alpha}\right)^{2\nu} e^{-2\nu\delta} \partial_\gamma \left[\frac{1}{\nu+\gamma}\, {}_2F_1\left(2\nu, \nu+\gamma; \nu+\gamma+1; e^{2i\alpha} e^{-2\delta}\right)\right]_{\gamma=0} \tag{S39}$$

where one clearly sees that for $\delta \ll 1$, the exponential prefactor dominates, so that

$$\mathcal{I}(\delta) \underset{\delta \ll 1}{\simeq} e^{-2\nu\delta} \mathcal{I}(0) \tag{S40}$$

It follows that, in the regime where $|\delta t|/\beta \to 0$, one has

$$S_{HOM}(\delta t) \underset{|\delta t|/\beta \to 0}{\longrightarrow} 1 - e^{-2\pi\nu \frac{|\delta t|}{\beta}} \tag{S41}$$

HOM noise for two narrow pulses of average charge qe

The previous results can be easily extended to the case of pulses carrying a different charge. We now define

$$V_R(t) = \frac{2\pi q}{\nu e}\delta\left(t + \frac{\delta t}{2}\right) \qquad V_L(t) = \frac{2\pi q}{\nu e}\delta\left(t - \frac{\delta t}{2}\right). \tag{S42}$$

Following the lines of the previous calculation, one can similarly obtain an expression for the HOM noise ratio as

$$S_{HOM}(\delta t) = \frac{\int dt dt' \left\{\cos\left[2\pi q f_{\delta t}(t,t')\right] - 1\right\} e^{2\nu G(t'-t)}}{2\int dt dt' \left[\cos\left(2\pi q \frac{1-\text{sign}(t\times t')}{2}\right) - 1\right] e^{2\nu G(t'-t)}} \tag{S43}$$

Interestingly, while the resulting integrals are finite for different domains in time, they always contain a prefactor $\cos(2\pi q) - 1$. For $q \notin \mathbb{Z}$, this prefactor simplifies between numerator and denominator, leaving us with the same expression as Eq. (S37), independently of q. This, however, is specific to the very short pulse situation, as a finite extent leads to slightly different contributions for the numerator and denominator, which depend on q in a nontrivial way.

HOM noise in the Floquet formalism

The applied voltages on the right and left edges are now given by periodic Lorentzian pulses. They are identical except for a time-shift δt, so that

$$V_L(t) = V_R(t - \delta t) = \frac{V_{DC}}{\pi} \sum_k \frac{\eta}{\eta^2 + (t/T_0 - k)^2} \tag{S44}$$

In the Floquet formalism, the essential ingredients are the coefficients p_l, which are the Fourier components of the accumulated phase $\phi(t) = e^* \int_{-\infty}^{t} dt' V_{AC}(t')$ created by the AC part of the time-dependent voltage. In practice, it is convenient to introduce the time-dependent voltage $V_{\text{diff}}(t) = V_R(t) - V_L(t)$ which naturally appears in the expression of the noise.

Starting back from the general expression of Eq. (S32), and inserting the p_l coefficients associated with a generic drive $V(t)$ (this allows us to replace V with V_R, V_L or V_{diff}), one can write

$$\begin{aligned} S(t,t') &= 2\left(\frac{e^*\Gamma}{2\pi a}\right)^2 \cos\left[e^* \int_{t'}^{t} dt'' V(t'')\right] \mathcal{G}(t-t')^2 \\ &= \left(\frac{e^*\Gamma}{2\pi a}\right)^2 \sum_{l,m} p_l^* p_m \left(e^{ie^* V_{DC}(t-t')} e^{il\omega t} e^{-im\omega t'} + e^{-ie^* V_{DC}(t-t')} e^{-im\omega t} e^{il\omega t'}\right) \mathcal{G}(t-t')^2 \end{aligned} \tag{S45}$$

where $\omega = \frac{2\pi}{T_0}$ is the frequency of the drive.

In this Floquet formalism, the zero-frequency noise is now defined as

$$\mathcal{S} = \int d\tau \int_0^{T_0} \frac{d\bar{t}}{T_0} S\left(\bar{t} + \frac{\tau}{2}, \bar{t} - \frac{\tau}{2}\right) \tag{S46}$$

which becomes

$$\begin{aligned} \mathcal{S} &= \int d\tau \int_0^{T_0} \frac{d\bar{t}}{T_0} \left(\frac{e^*\Gamma}{2\pi a}\right)^2 \sum_{l,m} p_l^* p_m \left(e^{ie^* V_{DC}\tau} e^{il\omega\left(\bar{t}+\frac{\tau}{2}\right)} e^{-im\omega\left(\bar{t}-\frac{\tau}{2}\right)} + e^{-ie^* V_{DC}\tau} e^{-im\omega\left(\bar{t}+\frac{\tau}{2}\right)} e^{il\omega\left(\bar{t}-\frac{\tau}{2}\right)}\right) \mathcal{G}(\tau)^2 \\ &= 2\left(\frac{e^*\Gamma}{2\pi a}\right)^2 \sum_l |p_l|^2 \int d\tau \cos\left[(l+q)\omega\tau\right] \mathcal{G}(\tau)^2 \end{aligned} \tag{S47}$$

where we introduced the average charge $q = \frac{e^* V_{DC}}{\omega}$ injected by the drive over one period.

Introducing the coefficients $p_{\text{diff},l}$ for the voltage difference $V_{\text{diff}}(t)$, as well as the coefficients $p_{L,l}$ and $p_{R,l}$ corresponding to $V_L(t)$ and $V_R(t)$ applied individually, and noticing that $V_{R,DC} = V_{L,DC} = \frac{q\omega}{e^*}$, while $V_{\text{diff},DC} = 0$, one finally has for the HOM noise ratio

$$S_{HOM}(\delta t) = \frac{\mathcal{S}_{HOM}}{2\mathcal{S}_{HBT}} = \frac{\sum_l F(p_{\text{diff},l}, 0) - |\Gamma(\nu)|^2}{\sum_l \left[F(p_{L,l}, q) + F(p_{R,l}, q)\right] - 2\,|\Gamma(\nu)|^2} \tag{S48}$$

with

$$F(p_l, q) = |p_l|^2 \left|\Gamma\left(\nu + i\frac{l+q}{2\pi\theta}\right)\right|^2 \cosh\left(\frac{l+q}{2\theta}\right) \tag{S49}$$

and $\theta = k_B T/\hbar\omega$ is the reduced temperature. Note that this expression is very general and can describe any kind of periodic potentials, provided that one uses the correct corresponding expressions of the p_l coefficients.