Dynamic response of a mesoscopic capacitor in the presence of strong electron interactions

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We consider a one-dimensional mesoscopic capacitor in the presence of strong electron interactions and compute its admittance in order to probe the universal nature of the relaxation resistance. We use a combination of perturbation theory, renormalization-group arguments, and quantum Monte Carlo calculation to treat the whole parameter range of dot-lead coupling. The relaxation resistance is universal even in the presence of strong Coulomb blockade when the interactions in the wire are sufficiently weak. We predict and observe a quantum phase transition to an incoherent regime for a Luttinger parameter $K < 1/2$. Results could be tested using a quantum dot coupled to an edge state in the fractional quantum Hall effect.

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The dynamical response of mesoscopic conductors constitutes a mostly unexplored area of coherent quantum transport, which has recently led to groundbreaking experiments.1 The mesoscopic capacitor2 is one of its elementary building blocks: a quantum dot influenced by an ac gate voltage, which is put in contact with an electron reservoir. It has been studied so far at the single electron level with possible mean-field generalizations.3 Both the capacitance $C_{\mu}$ and the relaxation resistance $R_{\mu}$ obtained from the low-frequency expansion of the admittance $G(\omega) = -i\omega C_{\mu} + \omega^2 C_{\mu} R_{\mu}$ are fundamentally affected by the quantum coherence of the device. At zero temperature, a single spin-polarized channel yields a relaxation resistance $R_{q} = \hbar/(2e^2)$, which is independent of the dot-reservoir connection. Reference 1 has confirmed this result for a quantum dot with weak charging energy.

However, quantum dots with reduced size exhibit strong Coulomb blockade, and there is also a clear need to analyze whether electron-electron interactions in the leads are relevant. Here, taking rigorously these aspects into account, we prove that there is quantum phase transition from a coherent to an incoherent regime, where a relaxation resistance cannot be defined. For weak interactions, the universal behavior is recovered even in the presence of strong Coulomb blockade.

We consider a quantum dot (Fig. 1) connected to a reservoir modeled by a Luttinger-liquid lead, which allows to account exactly for Coulomb-blockade effects. We discuss separately the absence (Luttinger parameter $K=1$) or the presence ($K<1$) of interaction in the adjacent lead. This setup and the underlying physics is similar to that studied in Ref. 4, where attention was solely focused on the static occupation of a resonant level. Here we show that below $K = 1/2$, the Kosterlitz-Thouless (KT)-type phase transition driven by the dot-lead tunneling strength triggers a transition of dynamical transport from a coherent to an incoherent regime, hence provoking a deviation from the universal $R_{q} = \hbar/(2e^2)$. We use a combination of analytical [perturbation theory, renormalization group (RG)] and numerical (quantum Monte Carlo) approaches to monitor the capacitance and the relaxation resistance over the whole range of dot-lead connection. The present results can be applied to carbon-nanotube quantum wires as well as dots defined in the fractional quantum Hall effect.

The starting point is the Hamiltonian for a nonchiral, semi-infinite Luttinger liquid4 where the dot region corresponds to the interval $[0,L]$, 

$$H = \int_{0}^{L} dx \left[ \frac{v_F}{K^2} \left( \frac{\partial \phi}{\partial x} \right)^2 + v_F \left( \frac{\partial \theta}{\partial x} \right)^2 \right] - V \cos[2\phi(x=0)] + \frac{1}{\pi e^2} E_c \left[ \phi(x=0) - \left( \frac{\pi e V}{|\phi|} + k_e L \right) \right]^{2}. \tag{1}$$

The first part is the kinetic part, followed by the backscattering term at $x=0$ (strength $V$), and finally the contribution from the charging energy with $E_c = e^2/(2C)$ ($C$ is the geometrical capacitance). The canonically conjugated fields $\phi$ and $\theta$ satisfy the commutation relation $[\phi(x),\theta(x')] = (i\pi/2)\text{sgn}(x-x')$. The time-dependent gate voltage oscillates around $V_g$. Using the Matsubara imaginary time path-integral formulation, the quadratic degrees of freedom away from $x=0$ can be integrated out. The kinetic part of the effective action then reads $S_{\text{kin}} = (\pi K \beta)^{-1} \sum_{\omega_n} \left|\omega_n\right|^2 \left(1-e^{-\beta\varepsilon_{\omega_n}/|\Delta|}\right)^2$, where $\phi(\omega_n)$ is the Fourier transform of $\phi(x)$ (now specified at $x=0$), and $\Delta = \pi e V / L$ is the level spacing. The same action can be derived alternatively starting from a single chiral Luttinger-liquid “loop,” hence the relevance for the FQHE regime.6 Within linear response in the oscillating gate voltage, the (imaginary frequency) admittance can be expressed as

FIG. 1. Left: schematic view of the mesoscopic capacitor: a one-dimensional quantum dot, capacitively coupled to a gate with time-dependent voltage $V_c + V_g(t)$. Right: schematic phase diagram in the degenerate case. $V_c$ denotes the critical backscattering strength for $K=1/3$.  

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\[ G(i\omega_n) = \frac{e^2}{h} \frac{|\omega_n|}{\pi} \int_0^\beta d\tau (\phi(\tau) \phi(0)) e^{i\omega_n \tau}. \]  

The dynamical conductance is obtained by analytic continuation \( G(\omega) = G(i\omega_n = \omega + i\delta) \) while the capacitance reads

\[ C_\mu = \frac{e^2}{\beta^2} \left[ \langle \phi^2 \rangle - \langle \phi \rangle^2 \right] \left[ \phi = \frac{1}{\beta} \int_0^\beta d\tau \phi(\tau) \right]. \]

We start with a discussion of the weak barrier case, using perturbation theory in \( V/D \) (bandwidth \( D \)). The capacitance and relaxation resistance are derived as an expansion in orders of \( V \), \( C_\mu = C_{\mu}^{(0)} + C_{\mu}^{(1)} + C_{\mu}^{(2)} + \cdots \), and \( R_q = R_q^{(0)} + R_q^{(1)} + R_q^{(2)} + \cdots \). Introducing

\[ a_n = \frac{1}{\pi \sqrt{\beta}} \left( \frac{|\omega_n|}{1 - e^{-2\pi \kappa|\omega_n|/\Delta}} + \frac{E_C K}{\pi} \right), \]

one obtains to zeroth, first, and second order,

\[ G^{(0)}(i\omega_n) = \frac{e^2}{h} \frac{|\omega_n|}{\pi} \frac{1}{\beta a_n}, \]

\[ G^{(1)}(i\omega_n) = \frac{e^2}{h} \frac{|\omega_n|}{\pi} 2V \sqrt{F_0(0)} \frac{1}{\beta^2 a_n^2} \cos(2\pi N), \]

\[ G^{(2)}(i\omega_n) = \frac{e^2}{h} \frac{|\omega_n|}{\pi} 2V^2 F_0(0) \frac{1}{\beta^2 a_n^2} I(\omega_n), \]

where we defined

\[ F_\pm(u) = \exp \left[ -\sum_n \frac{\pm 2}{\beta^2 a_n^2} \cos(\omega_n u) \right], \]

\[ I(\omega_n) = \int_0^{\beta^2} du [\cos(4\pi N) V_+(\omega_n, u) + V_-(\omega_n, u)], \]

\[ V_\pm(\omega_n, u) = 1 - F_\pm(u) [1 \pm \cos(\omega_n u)], \]

\[ N = k_T L \pi + \frac{|e| V_g}{2 E_T} \quad \text{with} \quad E_T = E_C + \frac{\Delta}{2K^2}. \]

The \( \Sigma_\omega \) in \( F_\pm \) is limited by \( D \). From Eqs. (5)–(7), the capacitance at low temperature becomes

\[ C_\mu^{(0)} = \frac{e^2 \pi}{h E_T}, \]

\[ C_\mu^{(1)} = -C_\mu^{(0)} \frac{\Omega^2}{E_T} 2V \sqrt{F_0(0)} \cos(2\pi N), \]

\[ C_\mu^{(2)} = C_\mu^{(0)} \frac{\Omega^2}{E_T} 2V^2 F_0(0) I(\omega_n \to 0) \cos(4\pi N). \]

It is clear from these expressions that the total capacitance \( C_\mu \) is a periodic function of \( N \) with period 1. Below we focus on the interval \( 0 \leq N < 1 \). The results for the relaxation resistance, at low temperature, are simple since the computation
right bottom panel of Fig. 2), $C_\mu$ exhibits an abrupt increase at a finite $V$, signaling a possible transition. One can see that $C_\mu$ grows as $\propto 1/T$ in the large barrier region.

To reveal the origin of the transition behavior, we next examine the strong barrier limit using an instanton method which was developed for the Kondo model.\textsuperscript{3} Near the degeneracy point $N=1/2$, the configuration of the bosonic field $\phi$ can be represented in the dilute instanton gas approximation, \begin{equation}
\phi(\tau) = \sum_{j=1}^{2n} s_j \Theta(\tau - \tau_j) + \frac{\pi}{2}(1-s),
\end{equation} where $s_j = s(-1)^{j-1}$ and $s \sim 1$ denotes the separation between the well minima ($\Theta$ is the step function). Inserting Eq. (16) in the full effective action, the partition function becomes \begin{equation}
Z = \sum_{n=0}^{\infty} \int_0^{\tau_n} \cdots \int_0^{\tau_2} \int_0^{\tau_1} \exp \left[ \frac{1}{2K} \sum_{j \neq k} s_j s_k \log \left( \frac{\tau_j - \tau_k}{\tau_c} \right) - u \sum_j s_j \tau_j \right],
\end{equation} where $t$ is the tunneling amplitude between the well minima and $\tau_c$ is the short-time cutoff. $u=(2N-1)E_T$ denotes the deviation from the degeneracy point. Note the similarity between this partition function and that which was proposed in the context of dissipative Josephson junctions.\textsuperscript{10} One can therefore identify the scaling equations,\textsuperscript{11} \begin{equation}
\frac{d\tau}{dt} = \left( 1 - \frac{s^2}{2K} \right) \tau, \quad \frac{ds^2}{dt} = -4s^2 \tau^2,
\end{equation} which are familiar in the context of a Kosterlitz-Thouless transition in the two-dimensional XY model. No further arguments are needed when one deviates from the degeneracy point since $t$ is small, starting from $u \neq 0$. Eq. (19) predicts that $u$ will further increase, leading the system further away from the degeneracy point. This means that $\phi$ will be trapped in an effective harmonic potential, and one thus recover the result of Eq. (5), which is therefore universal. For the charge degenerate case $N=1/2$, the transition corresponds to a Kondo-type transition associated with the charge pseudospin on the dot. Equation (18) determines the tendency of the dot-lead transmission as temperature is lowered; $(t,s^2)$ flows along one of the hyperbolic curves $B^2 - 4t^2 = \text{const}$, where $B = 1 - s^2/(2K)$. For $K > 1/2$, the tunneling strength always grows upon reducing the temperature, and the system reaches the Kondo fixed point where the dot is strongly coupled to the reservoir. An electron freely tunnels in and out of the dot irrespective of the initial tunneling strength. In particular, at $K=1$, this implies that the charge relaxation resistance is universal, i.e., $R_g = h/(2e^2)$, as a consequence of the unitary limit of the underlying Kondo model. On the other hand, for $K < 1/2$, there is the possibility that at a critical, sufficiently weak transmission $t$ ("large" $V$), the RG flow always drives the system into a weak-coupling configuration with specified charge. Then the charge fluctuation remains finite, i.e., $\langle \Delta^2 \rangle - \langle \phi^2 \rangle = (\pi s/2)^2$ so that the capacitance diverges as $\propto 1/T$ at low temperatures [see Eq. (3)]. This explains the transition observed for the capacitance in the strongly interacting case.

We now describe the effect of the KT transition on the dynamical properties. If $\omega \ll 1/\tau_{RC}$ holds (with $RC$ time $\tau_{RC}$), the charge relaxation resistance can be defined in the low-frequency expansion $G(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_g + O(\omega^3)$. However, the validity of this expansion is not obvious since the KT transition may influence $\tau_{RC}$ itself. Instead, we investigate the low-frequency resistance using \begin{equation}
R(i\omega_n) = \frac{1}{G(i\omega_n)} - \frac{1}{\omega_n C_\mu},
\end{equation} where $G(i\omega_n)$ and $C_\mu$ are defined in Eqs. (2) and (3), respectively. The extrapolation $R(i\omega_n \to 0)$ gives the real part of the impedance in the low-frequency limit, hence $\tau_{RC} = R(0)\tau_i$. In Fig. 3, we plot $R(0)$ for $K=1/3$, and $1/5$ as a function of $V$ at temperature $T=0.04$. For $K=1$, $R(0)$ equals $h/(2e^2)$ irrespective of $V$, in agreement with the universal charge relaxation resistance.\textsuperscript{23} For $K=1/3$ and $1/5$, the universality is observed in the weak barrier region whereas $R(0)$ is abruptly enhanced with increasing $V$, reflecting the RG flow to the weak-coupling regime due to the KT transition. The temperature dependence of $R(0)$ for $K=1/3$ is shown in the inset of Fig. 3, which indicates that $R(0)$ diverges as $T \to 0$ in the strong barrier region.

The KT transition plays a crucial role in the relevance of the universal charge relaxation resistance. If $2r+B>0$, the system scales to the weak barrier limit, where $\tau_{RC}$ is independent of temperature. If $2r+B<0$, on the other hand, the scaling Eq. (18) predict $s^2 \to \text{const}$ and $t \propto T^{-B}$ so that $\tau_{RC}$ roughly scales as $\propto T^{-1}(2r+B+\text{const})$, which grows faster than the (thermal) coherence time $\tau_{coh} \propto 1/T$ as temperature is lowered. These observations suggest that if $2r+B>0$ coherent transport can be realized by lowering temperature to guarantee $\tau_{RC} < \tau_{coh}$ while if $2r+B<0$ electronic transport in the dot decoheres before charge relaxation is achieved. In the latter case, the quantum dot effectively acts as a reservoir and consequently the dynamical property of the system is
In conclusion, the study of the mesoscopic capacitor in the presence of strong electron-electron interaction shows that the relaxation resistance for a dot connected to Luttinger liquid is universal $R_q = \hbar/(2e^2K)$ as long as interactions are sufficiently weak. Below $K<1/2$, this resistance is governed by the strength of the dot-lead coupling: at the charge degeneracy point, there is a critical coupling strength, governed by a KT-type phase transition, below which the dot acts as an incoherent reservoir and the low-frequency resistance exceeds the universal value. In this incoherent regime, the charge relaxation resistance cannot be defined anymore due to the divergence of the $RC$ time.

Results could be probed experimentally using quantum dots connected to an edge state in the FQHE regime. Another experimental probe could use one dimensional quantum wires (nonchiral Luttinger liquids) with the limitation that the operating frequency would have to be larger than the inverse time of flight within the wire, in order to avoid renormalization effects due to eventual Fermi liquid leads connected to this wire.\textsuperscript{12}

\textit{Note added.} At this point, we would like to note that a preprint\textsuperscript{13} discussing the same setup has recently appeared. Similar results are obtained in the weak barrier regime, and the universality of the charge relaxation resistance is emphasized. However, it does not address the quantum phase transition which drastically affects the nature of the resistance.

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6. In the FQHE regime, $V$ denotes the strength of quasiparticle tunneling.


11. We assume that the local scatterer at $x=0$ does not renormalize significantly the bulk interaction parameter $K$.
