Anyonic Statistics Revealed by the Hong-Ou-Mandel Dip for Fractional Excitations

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The fractional quantum Hall effect (FQHE) is known to host anyons, quasiparticles whose statistics is intermediate between bosonic and fermionic. We show here that Hong-Ou-Mandel (HOM) interferences between excitations created by narrow voltage pulses on the edge states of a FQHE system at low temperature show a direct signature of anyonic statistics. The width of the HOM dip is universally fixed by the thermal time scale, independently of the intrinsic width of the excited fractional wave packets. This universal width can be related to the anyonic braiding of the incoming excitations with thermal fluctuations created at the quantum point contact. We show that this effect could be realistically observed with periodic trains of narrow voltage pulses using current experimental techniques.

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The fractional quantum Hall effect (FQHE) is an important example of a many-body system where electronic correlations have an essential impact [1]. When a fraction ν of the states of the lowest Landau level is occupied, the system reaches a state which cannot be understood without electronic interactions. The well-known Laughlin wave function describes the highly correlated ground state of the FQHE when $\nu = 1/(2n+1)$ for n integer. The fundamental excitations of the FQHE are anyons: quasiparticles which bear a fractional charge, and obey fractional statistics [2,3]. In a given Laughlin state, when two anyons are exchanged, the system acquires a phase $\exp(i\pi\nu)$, to be contrasted with the ± 1 of bosonic or fermionic statistics. More complex fractions exist, potentially hosting non-Abelian anyons, relevant for quantum computing applications [4].

A fractional charge $e/3$ was experimentally observed for the Laughlin state with $\nu = 1/3$ more than twenty years ago, by measuring the shot noise across a quantum point contact (QPC) in the tunneling regime, where individual fractional quasiparticles can tunnel between opposite edge states [5–8]. Fractional statistics, however, has proved more difficult to observe. Only very recently, two different experiments have been able to clearly show specific signatures directly associated with the fractional statistics of anyonic quasiparticles [9,10].

Electronic transport in FQHE occurs only through chiral edge modes, traveling along the boundary. These can be used as 1d electron beams enabling us to realize transport experiments inspired from quantum optics, such as the Hong-Ou-Mandel (HOM) interference experiment, where two identical photons are sent with a controlled time delay on a beam splitter [11]. The electronic counterpart was performed a few years ago in the integer QHE, where current correlations were shown to give precious information on the electronic wave packets and the many-body electronic state [12–15]. Recently, two-particle time-domain interferences were obtained in the FQHE, demonstrating that quasiparticles keep their coherence allowing for time-domain interference [16].

In this work, we show that using narrow periodic pulses of voltage, periodically exciting fractional charges, and measuring the HOM noise at the output of a QPC, one obtains a signal which is directly related to the anyonic statistics. To this aim, we first explain the unique properties of the time-dependent tunneling current at a QPC when a single fractional quasiparticle is incident, which are associated with braiding of the fractional quasiparticle with the thermal anyonic excitations occurring at the QPC. Our quantitative predictions, obtained with perturbative calculations performed using the nonequilibrium Keldysh Green's function formalism, could be checked with current experimental techniques, providing a relatively easy path for the study of fractional statistics.

We consider a FQH bar, with Laughlin filling factor $\nu = 1/(2n+1)$ for n integer, and describe the edge states in terms of the bosonic Hamiltonian $H_0 = (v_F/4\pi) \int dx \sum_{\mu=R,L} (\partial_x \phi_\mu)^2$, where $\phi_{R/L}$ are the bosonic fields describing the right- or left-moving edge states propagating with velocity v_F [17]. A bosonization identity, $\psi_{R/L}(x) = U_{R/L}/(2\pi a) e^{\pm i k_F x} e^{-i\sqrt{\nu}\phi_{R/L}(x)}$, relates the quasiparticle (QP) operator to the bosonic field, with a a small cutoff parameter a and $U_{R/L}$ a Klein factor. The presence of a QPC (at $x = 0$), in the weak backscattering regime, allows the tunneling of individual QP of charge $e^* = \nu e$ between the two edges. This is described by the tunneling Hamiltonian $H_T = \Gamma \psi_R^\dagger(0) \psi_L(0) + \text{H.c.}$ See Fig. 1 for a sketch of the setup.

To better understand the importance of the anyonic statistics for tunneling at the QPC, let us first consider the somewhat simpler situation where a single QP of charge

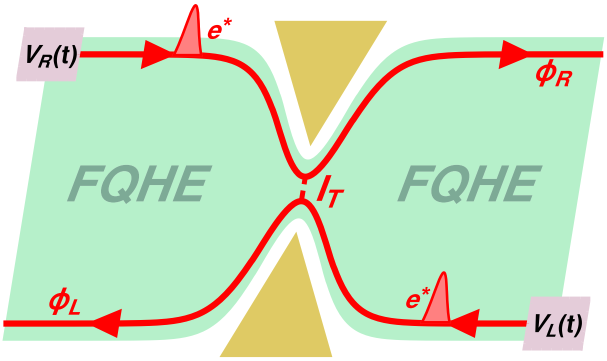

FIG. 1. The setup: a Hall bar in the Laughlin series, whose edge states are described by the bosonic fields ϕ_R and ϕ_L, with a QPC at position $x = 0$. The right- and left-moving edges are driven, respectively, by the time-dependent potential $V_R(t)$ and $V_L(t)$, resulting in a tunneling current I_T in between edge.

e^* is incoming on the R edge. To this aim, when computing physical quantities (current, etc.), we replace the ground state by a prepared state $|\varphi\rangle = \psi_R^\dagger(-x_0, -\mathcal{T})|0\rangle$, where a single QP has been added at an initial time $-\mathcal{T} < 0$. Without loss of generality, we choose $x_0 = v_F\mathcal{T}$, such that the QP reaches the QPC at $t = 0$. We now proceed with the perturbative calculation of the mean tunneling current $\langle I_T(t)\rangle$ at the QPC, using standard Keldysh Green's function formalism. The tunneling current operator is given by $I_T(t) = ie^*[\Gamma\psi_R^\dagger(0,t)\psi_L(0,t) - \mathrm{H.c.}]$. To lowest order in Γ, the mean current is given by [18]

$$\langle I_T(t)\rangle = -\frac{i}{2}\int dt' \sum_{\eta,\eta'} \eta' \langle\varphi|T_K I_T(t^\eta) H_T(t'^{\eta'})|\varphi\rangle, \quad (1)$$

where T_K is time ordering along the Keldysh contour, and $\eta, \eta' = \pm$ are Keldysh indices. Using the bosonized form of the quasiparticle operators, and keeping in mind that $x_0 = v_F\mathcal{T}$, we have

$$\langle I_T(t)\rangle = \Gamma^2 \frac{e^*}{2}\int dt' \sum_{\eta,\eta'} \eta' [\mathcal{G}(\sigma_{tt'}^{\eta\eta'}(t - t'))]^2 \times \left[\frac{\mathcal{G}(-t')\mathcal{G}(t)}{\mathcal{G}(t')\mathcal{G}(-t)} - \frac{\mathcal{G}(t')\mathcal{G}(-t)}{\mathcal{G}(-t')\mathcal{G}(t)}\right], \quad (2)$$

with

$$\mathcal{G}(t) = \frac{1}{2\pi a}\left[\frac{\sinh(i\pi a/(\beta v_F))}{\sinh(i\pi a/(\beta v_F) - \pi t/\beta)}\right]^\nu, \quad (3)$$

where $\mathcal{G}[\sigma_{tt'}^{\eta\eta'}(t - t')] = \mathcal{G}(0; t^\eta, t'^{\eta'}) = \langle 0|T_K\psi^\dagger(0, t^\eta) \psi(0, t'^{\eta'})|0\rangle$, with $\sigma_{tt'}^{\eta\eta'} = \mathrm{sign}(t - t')(\eta + \eta')/2 + (\eta' - \eta)/2$ accounting for the effect of time ordering along the Keldysh contour. $\mathcal{G}$ is the quasiparticle Green's function (identical for right and left movers), directly obtained from its bosonic counterpart, with β the inverse temperature. Note that the power ν leads to a slow decay of this Green's function at long times since $\nu < 1$, up to the thermal timescale $\tau_{\mathrm{Th}} = \hbar\beta$. In the limit of vanishing cutoff $a \to 0$, it is easy to check that $\mathcal{G}(t)/\mathcal{G}(-t) = \exp[-\mathrm{sign}(t) \times i\pi\nu]$. This directly arises from the nontrivial exchange properties of anyonic quasiparticles, exploiting their linear dispersion along the edge [18]. It follows that the last factor of Eq. (2) can be simplified as

$$\frac{\mathcal{G}(-t')\mathcal{G}(t)}{\mathcal{G}(t')\mathcal{G}(-t)} = \exp\left(-i\nu\int_{t'}^{t} d\tau\, 2\pi\delta(\tau)\right). \quad (4)$$

The current can thus be written as

$$\langle I_T(t)\rangle = 2ie^*\Gamma^2\int_{-\infty}^{t} dt' \sin\left(2\pi\nu\int_{t'}^{t} d\tau\delta(\tau)\right) \times [\mathcal{G}(t - t')^2 - \mathcal{G}(t' - t)^2]. \quad (5)$$

One readily sees from Eq. (5) that the tunneling current has remarkable properties, which are unique to fractional charge tunneling in the FQHE [18]. It is of course zero for $t < 0$, i.e., before the arrival of the e^* QP. On the other hand, for $t > 0$, the t' integration is restricted to the negative portion of the real axis, and the current is simply proportional to $\sin(2\pi\nu)$. This, in turn leads to a nonzero current even for a time t taken long after the e^* QP has reached the QPC position, as a consequence of the slow decay in time of Green's function $\mathcal{G}(t - t')$.

The mean current thus remains finite for a long time interval, set by the thermal time scale τ_{Th}. We emphasize that this is in sharp contrast with the case of an incoming electron charge, since even for fractional edge states, the mean tunneling current is nonzero only at the specific time that the electron arrives at the QPC [18].

This nontrivial behavior of the tunneling current after the arrival of a single QP of charge e^* can be directly linked to the anyonic statistics of the fractional excitations. The phase $2\pi\nu$ occurring for $t' < 0 < t$ can be understood qualitatively from Eq. (1) by considering the time ordering of the right-moving edge operators (ψ_R, $\psi_R^\dagger$). From the expressions of $|\varphi\rangle$, I_T, and H_T, one readily sees that the average current in Eq. (1) involves a contribution of the form $T\psi_R(0)\psi_R^\dagger(t)\psi_R(t')\psi_R^\dagger(0)$, as the prepared state ensures that the QP reaches the QPC at time 0. For $t > 0$ and $t' < 0$, the time ordering thus requires us to bring both $\psi_R(0)$ and $\psi_R^\dagger(0)$ between the operators at t and t', yielding twice a phase $\pi\nu$. On the opposite, if t and t' have the same sign, one can easily see that the exchanges needed for the ordering now contribute with opposite phases, thus giving a zero net result. An equivalent point of view, developed in Refs. [19–21] is to see the expression of Eq. (1) as the interference between a process where a quasiparticle or quasihole (QP/QH) excitation is created at the location of the QPC at time t', before the passage of the e^* QP, and another where the QP/QH is created at time t, after the passage of the e^* QP. In both points of view, the tunneling current is nonzero because of the braiding of the incoming fractional QP with a thermal QP/QH excitation created at the QPC. This braiding results from the anyonic statistics, giving a nontrivial phase $\pi\nu$ when two quasiparticles are exchanged.

We now show that the same tunneling current, with the same signature of fractional statistics, can be obtained by applying a short voltage pulse that excites a fractional average charge. This is a highly nontrivial statement, as it is known that such a voltage pulse does not create the same many-body state as the one obtained by adding a single quasiparticle on top of the ground state [22].

The presence of an external time-dependent voltage bias leads to an extra term in the total Hamiltonian, of the form $H_V = -(2e\sqrt{\nu}/v_F)V(x,t)\partial_x\phi_R$. The voltage can be taken into account by using the following transformation: $\phi(x,t) = \phi^{(0)}(x,t) + e\sqrt{\nu}\int_{-\infty}^{t} dt' V(x',t')$, with $x' = x - v_F(t-t')$ and where $\phi^{(0)}(x,t)$ is the equilibrium bosonic field [22]. Assuming that the voltage is applied on a long contact, we can simplify $\int_{-\infty}^{t} dt' V[v_F(t'-t),t'] \simeq \int_{-\infty}^{t} dt' V(t')$. This leads to a time-dependent tunneling amplitude at the QPC $\Gamma(t) = \Gamma \exp[ie^*\int_{-\infty}^{t} dt' V(t')]$. Proceeding with the perturbative calculation of the tunneling current, one gets [18]

$$\langle I_T(t)\rangle = 2ie^*\Gamma^2 \int_{-\infty}^{t} dt' \sin\left(e^* \int_{t'}^{t} dt'' V(t'')\right) \times [\mathcal{G}(t-t')^2 - \mathcal{G}(t'-t)^2]. \tag{6}$$

One can thus readily recover the result of Eq. (5), provided that one chooses a voltage pulse $V(t) = (2\pi/e)\delta(t)$, which excites a mean charge $e^* = \nu e$. The tunneling current $\langle I_T(t)\rangle$ is the same for a single QP of charge e^* arriving on the QPC, or when applying a very short voltage pulse $V(t)$ exciting a mean charge e^*.

This picture is further generalized by considering a voltage $V(t)$ composed of several short pulses of charge e^*. There, the phase of the sine term counts the number of fractional charges e^* that have passed through the QPC, each of them contributing a phase $2\pi\nu$. This then has important consequences for the tunneling current. For example, at filling factor $\nu = 1/3$, when two short fractionally charged pulses arrive at the QPC with a time delay much smaller than the thermal scale, the main contribution to the current in Eq. (6) comes with a factor $\sin(4\pi/3) < 0$, making it negative. Figure 2 shows the current for an ensemble of short pulses (each with a charge e^*, and a width $\delta t \ll \beta$). The dashed lines show the arrival times of the pulses at the QPC. We see that the current decreases slowly in absolute value after each pulse reaches the QPC, reflecting the slow decrease of the QP Green's function $\mathcal{G}$.

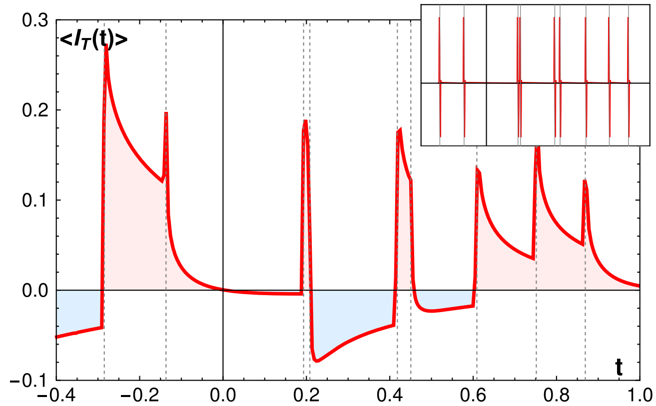

FIG. 2. Mean current $\langle I_T(t)\rangle$ (in units of e/β) as a function of t (in units of β) corresponding to Eq. (6) with $\nu = 1/3$, for a random ensemble of short pulses of width $\beta/100$, each carrying a charge $e/3$. The arrival times at the QPC are shown as dashed vertical lines (pulses for $t < -0.4\beta$ are not shown). Inset: same figure for pulses carrying a charge e.

More interestingly, the value of the current depends on the history of the pulses applied at earlier times. In particular, as argued above, the current can be negative when two pulses arrive at closely separated times (e.g., for t between 0.2 and 0.4β). The inset of Fig. 2 shows the equivalent picture when similar pulses, but carrying a charge e rather than e^*, are incident on the QPC. There, the current is nonzero only when the pulse reaches the QPC, with no effect from earlier pulses. Note that the same expression for the tunneling current, Eq. (6), allows one to describe a random stream of pulses, recovering known results for the collision between two Poissonian streams of charges e* [23].

While the use of voltage pulses is routinely performed, the measurement of time-dependent currents still constitutes an experimental challenge in quantum Hall junctions. We now propose a simpler alternative, within grasp of modern experiments, in order to reveal the effect of anyonic statistics. This relies on the measurement of the HOM noise, i.e., the current correlations resulting from two individual voltage pulses of fractional charge colliding at the QPC with a controllable time delay.

Let us first consider two narrow pulses of charge e^*, incoming on the two inputs of the QPC. The tunneling current noise is defined as

$$S(t,t') = \langle T_K \delta I_T(t^-)\delta I_T(t'^+)\rangle, \tag{7}$$

with $\delta I_T(t) = I_T(t) - \langle I_T(t)\rangle$, and $\pm$ are Keldysh indices. The HOM noise is the zero-frequency tunneling noise, when two pulses are incident on the QPC with a given time delay δt. It serves as a measure of the interference between the colliding excitations at the QPC. It can be written as [18]

$$S_{\mathrm{HOM}}(\delta t) = \frac{1}{2S_{\mathrm{HBT}}}\int_{-\infty}^{\infty} dt dt' \mathcal{G}(t'-t)^2 \times \{\cos[2\pi\nu f_{\delta t}(t,t')] - 1\}, \quad (8)$$

where $f_{\delta t}(t,t')$ is 1 if only one of the times t or t' is in the interval $[-\delta t/2, \delta t/2]$, and 0 otherwise, and normalization is given by twice the value of the Hanbury Brown–Twiss (HBT) noise S_{HBT} [24].

A very good approximation of Eq. (8) (exact for $\delta t/\beta \to 0$), is given by

$$S_{\mathrm{HOM}}(\delta t) \overset{\delta t \ll \tau_{\mathrm{Th}}}{\to} 1 - \exp\left(-2\pi\nu\frac{|\delta t|}{\beta}\right). \quad (9)$$

This result shows a behavior typical of a HOM dip for long and short time delays. For very large $|\delta t|$, it saturates to 1 as the two incident charges e^* reach the QPC at very distant times without interfering, therefore reproducing twice the amount of the HBT noise. For $\delta t = 0$, the HOM dip drops all the way to 0, as a result of perfect interference between the two identical incoming charges. This can be understood as a fractional charge injected from the left and another one injected from the right braiding with opposite phases with the thermal excitations of the QPC. At $\delta t = 0$ these phases cancel exactly. The most important result, however, is the behavior at intermediate δt: Eq. (9) shows that the width of the HOM dip is $\sim\beta$, set by the thermal timescale τ_{Th}, independently of the width of the incoming pulses. This is in sharp contrast with the conventional HOM dip, for example, between electronic wave packets in the integer QHE [14,25], where the dip width is directly proportional to that of the incoming wave packet. This striking result can be understood from our discussion of the tunneling current above. Indeed, we showed that, as a consequence of anyonic statistics and the braiding with thermal excitations, a single charge e^* reaching the QPC creates a nonzero current up to times $\sim\beta$ after the tunneling event occurred.

Two charges incident on both inputs of the QPC thus interfere up to times set by the thermal timescale, which explains the width of the HOM dip. The observation of such an HOM dip can thus provide a direct proof of the anyonic statistics of the incoming fractional charges. We now show how a realistic periodic voltage bias with frequency ω, sending pulses of charge qe (with noninteger q), can be used to observe the HOM dip of width $\sim\beta$. For illustrative purposes, we consider a periodic voltage $V(t)$ consisting of Lorentzian pulses, also known as levitons [26–29], but the results are independent of the actual shape of the voltage potential, as long as the pulse width is small compared to β. We use the Floquet formalism, where the essential ingredients are the coefficients p_l, which correspond to the Fourier coefficients of the phase $\phi(t) = e^*\int_{-\infty}^{t} dt' V_{\mathrm{ac}}(t')$ created by the ac part of the time-dependent voltage $V(t)$. The dc part of the voltage leads to a mean charge qe injected per period, with $q = e^* V_{\mathrm{dc}}/\omega$.

We consider that the voltages $V_R(t)$ and $V_L(t)$, applied on the right and left edge, respectively, differ by a time shift δt only, so that

$$V_L(t) = V_R(t-\delta t) = \frac{V_{\mathrm{dc}}}{\pi}\sum_k \frac{\eta}{\eta^2 + (t/T_0 - k)^2}, \quad (10)$$

where $T_0 = 2\pi/\omega$ is the period of the drive, and η is the finesse.

Figure 3 shows the normalized HOM noise for a periodic Lorentzian drive, at $\nu = 1/3$, with realistic values for the experimental parameters (frequency $\omega = 1 \times 2\pi$ GHz, and finesse $\eta = 0.01$) [30]. The black dotted line shows the shape of the narrow Lorentzian pulse over one period. The full curves show the HOM dip as a function of the time-shift δt. In panel (a), the average charge per pulse is fixed to $qe = e/3$, and the temperature T is varied from 250 down to 25 mK. At $T = 25$ mK, the hierarchy of the different timescales is thus: pulse width ($\sim$20 ps) $\ll$ thermal

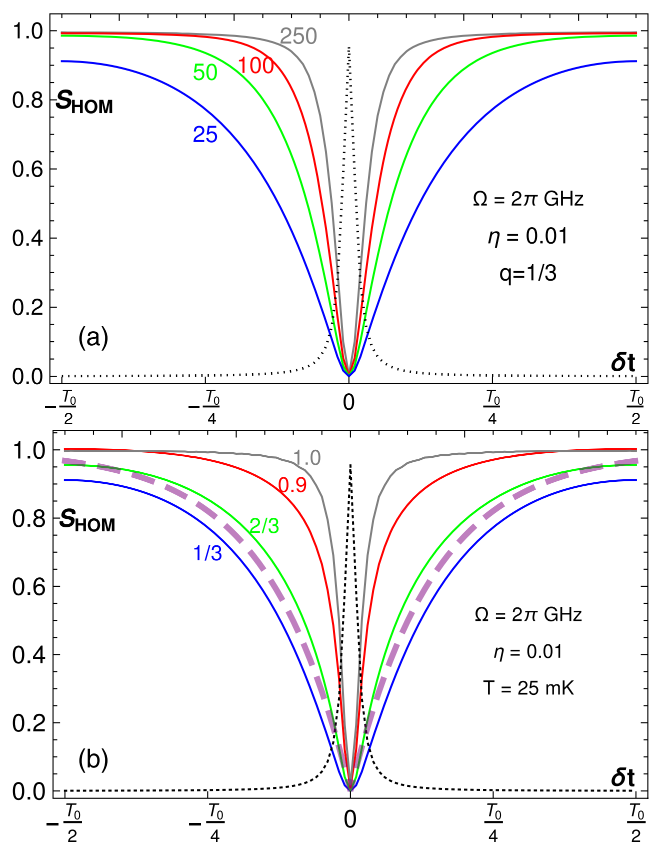

FIG. 3. HOM noise as a function of δt for a filling factor $\nu = 1/3$, for $V(t)$ made of voltage pulses with Lorentzian shape of finesse $\eta = 0.01$, with $\omega = 2\pi/T_0 = 2\pi$ GHz. (a) pulses of charge $e/3$, and T in mK indicated near each curve. (b) $T = 25$ mK, and the charge of each pulse (in units of e) shown near each curve. The thick dashed line shows the theoretical prediction of Eq. (9) for two infinitely narrow pulses at $T = 25$ mK. The dotted curve shows the shape of $V(t)$ over one period.

timescale (∼300 ps) < period (∼1000 ps). One can readily see that, while the width of the HOM dip is close to that of the Lorentzian pulse at $T = 250$ mK, it significantly increases as the temperature is lowered, ultimately being much larger at $T = 25$ mK. We consider, in panel (b), a fixed temperature $T = 25$ mK, and an injected charge per period which varies from $qe = e$ down to $qe = e/3$. There, the width of the HOM dip is similar to that of the incoming pulse for $q = 1$ (corresponding to the injection of a full electron per period on each edge), before increasing substantially as q is lowered, recovering a wide HOM dip for $q = 1/3$. The thick dashed line corresponds to the analytical prediction of Eq. (9) for $T = 25$ mK. This shows a very good agreement with the full numerical result obtained for $q = 1/3$, with only a small underestimation of the width of the dip associated with the assumption of infinitely sharp pulses.

In conclusion, we have shown that the anyonic statistics of quasiparticles in the FQHE has direct consequences on the HOM interference of excitations created by narrow voltage pulses. Contrarily to the usual picture, where the width of the HOM dip is trivially related to the temporal extension of the incoming excitations, here it is fixed by the thermal scale, which dominates at low temperature. We have shown how this can be explained by the anyonic braiding of the incoming quasiparticles with thermal excitations naturally occurring at the QPC. Reducing temperature increases the thermal time, which enhances the timescale on which braiding is effective, and thus leads to a wider HOM dip. Our proposal could be realized with current experimental techniques, and could lead to an original and relatively simple way to observe directly the consequences of anyonic statistics in the FQHE. A natural extension of this work would be to consider more exotic fractions like $\nu = 2/5$ or $\nu = 2/3$ [31,32], or even $\nu = 5/2$, characterized by non-Abelian statistics [4,33–35].

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for the experimental parameters, which allows us to take a smaller value for the finesse, as higher harmonics of the base frequency ω are more easily accessed. The experiment of Ref. [28] used pulses with width as small as 30 ps, which is similar to the width of 20 ps that we consider in Fig. 3.

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