Supplemental material for "Minimal Excitations in the Fractional Quantum Hall Regime"

J. Rech[1], D. Ferraro[1], T. Jonckheere[1], L. Vannucci[2,3], M. Sassetti[2,3], and T. Martin[1]
[1] *Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France*
[2] *Dipartimento di Fisica, Università di Genova, Via Dodecaneso 33, 16146, Genova, Italy*
[3] *CNR-SPIN, Via Dodecaneso 33, 16146, Genova, Italy*

EXTERNAL DRIVES AND CORRESPONDING FLOQUET COEFFICIENTS

The applied drive $V(t)$ is split into a DC and an AC part $V(t) = V_{dc} + V_{ac}(t)$, where by definition $V_{ac}(t)$ averages to zero over one drive period T.

The AC voltage is handled through the accumulated phase experienced by the quasiparticles $\varphi(t) = e^* \int_{-\infty}^{t} dt' V_{ac}(t')$ (with the fractional charge $e^* = \nu e$). We use the Fourier decomposition of $e^{-i\varphi(t)}$, defining the corresponding coefficients p_l as

$$p_l = \int_{-T/2}^{T/2} \frac{dt}{T} e^{il\Omega t} e^{-i\varphi(t)}. \tag{S1}$$

We focus on three types of drives:

$$\text{Cosine} \quad V(t) = V_{dc} \left[1 - \cos(\Omega t)\right], \tag{S2}$$

$$\text{Square} \quad V(t) = 2V_{dc} \sum_k \text{rect}\left(2\frac{t}{T} - 2k\right), \tag{S3}$$

$$\text{Lorentzian} \quad V(t) = \frac{V_{dc}}{\pi} \sum_k \frac{\eta}{\eta^2 + \left(\frac{t}{T} - k\right)^2}, \tag{S4}$$

where $\text{rect}(x) = 1$ for $|x| < 1/2$ ($= 0$ otherwise), is the rectangular function, and $\eta = W/T$ (W is the half-width at half-maximum of the Lorentzian pulse).

The corresponding Fourier coefficients of Eq. (S1) read, for non-integer $q = \frac{e^* V_{dc}}{\Omega}$:

$$\text{Cosine} \quad p_l = J_l(-q), \tag{S5}$$

$$\text{Square} \quad p_l = \frac{2}{\pi} \frac{q}{l^2 - q^2} \sin\left[\frac{\pi}{2}(l - q)\right], \tag{S6}$$

$$\text{Lorentzian} \quad p_l = q \sum_{s=0}^{+\infty} \frac{\Gamma(q + l + s)}{\Gamma(q + 1 - s)} \frac{(-1)^s e^{-2\pi\eta(2s+l)}}{(l + s)! s!}, \tag{S7}$$

CURRENT AND NOISE IN THE WEAK BACKSCATTERING REGIME

Fractional quantum Hall (FQH) edges at filling factor $\nu = 1/(2n + 1)$ are described in terms of a hydrodynamical model [1] through the Hamiltonian of the form

$$H = H_0 + H_V, \tag{S8}$$

$$H_0 = \frac{v_F}{4\pi} \sum_{\mu=R,L} \int dx \ (\partial_x \phi_\mu)^2, \tag{S9}$$

$$H_V = -\frac{e\sqrt{\nu}}{2\pi} \int dx \ V(x,t) \ \partial_x \phi_R \tag{S10}$$

where we apply a bias $V(x,t)$ which couples to the charge density of the right moving edge state. Here the bosonic fields satisfy $[\phi_{R,L}(x), \phi_{R,L}(y)] = \pm i\pi \text{Sgn}(x - y)$.

These bosonic fields propagate along the edge at velocity v_F and are directly related to the corresponding quasiparticle annihilation operators $\psi_\mu(x,t)$ ($\mu = R, L$) through the bosonization identity

$$\psi_{R/L}(x,t) = \frac{U_{R/L}}{\sqrt{2\pi a}} e^{\pm i k_F x} e^{-i\sqrt{\nu}\phi_{R/L}(x,t)}, \tag{S11}$$

where a is a short distance cutoff and U_μ are Klein factors.

Focusing first on the case where no QPC is present, one can derive the following equations of motion for the bosonic fields

$$(\partial_t + v_F \partial_x)\,\phi_R(x,t) = \frac{e}{\hbar}\sqrt{\nu}V(x,t) \tag{S12}$$

$$(\partial_t + v_F \partial_x)\,\phi_L(x,t) = 0 \tag{S13}$$

It follows that the effect of the external voltage bias can be accounted for by a rescaling of the right-moving bosonic field $\phi_R(x,t) = \phi_R^{(0)}(x,t) + e\sqrt{\nu}\int_{-\infty}^{t} dt' V(x',t')$ (with $\phi_R^{(0)}$ the solution in absence of time dependent voltage) or alternatively by a phase shift of the quasiparticle operator of the form

$$\psi_R(x,t) \longrightarrow \psi_R(x,t)\; e^{-i\nu e \int_{-\infty}^{t} dt' V(x',t')} \tag{S14}$$

where $x' = x - v_F(t - t')$.

Accounting for this phase shift, the tunneling Hamiltonian which describes the scattering of single quasiparticles at the QPC ($x = 0$) in the weak backscattering regime is given by $H_T = \Gamma_0 \exp\left(i\nu e \int_{-\infty}^{t} dt' V(v_F(t'-t), t')\right)\psi_R^\dagger(0)\psi_L(0) +$ H. c., with the bare tunneling constant Γ_0.

Considering for simplicity the experimentally motivated situation of a long contact, located at a distance d from the QPC, one can write the bias voltage as $V(x,t) = V(t)\theta(-x-d)$ where $V(t)$ is a periodic time-dependent voltage. The tunneling Hamiltonian can then be simplified as

$$H_T = \Gamma_0 \exp\left(i\nu e \int_{-\infty}^{t-\frac{d}{v_F}} dt' V(t')\right)\psi_R^\dagger(0)\psi_L(0) + H.c. \tag{S15}$$

Note that the time delay d/v_F can safely be discarded as it corresponds to a trivial constant shift in time of the external drive.

The backscattering current is readily obtained from H_T, after defining $\Gamma(t) = \Gamma_0 \exp\left(ie^* \int_{-\infty}^{t} dt' V(t')\right)$ and $e^* = \nu e$:

$$I_B(t) = ie^* \left[\Gamma(t)\psi_R^\dagger(0,t)\psi_L(0,t) - \text{H.c.}\right]. \tag{S16}$$

Expanding to order Γ_0^2 and taking the average over one period, the backscattering current becomes

$$\overline{\langle I_B(t)\rangle} = -\frac{2ie^*}{T}\left(\frac{\Gamma_0}{2\pi a}\right)^2 \int_{-\infty}^{+\infty} d\tau e^{2\nu\mathcal{G}(-\tau)} \int_0^T d\bar{t} \sin\left[e^* \int_{\bar{t}-\frac{\tau}{2}}^{\bar{t}+\frac{\tau}{2}} dt'' V(t'')\right], \tag{S17}$$

with $\mathcal{G}(t-t')$, the connected bosonic Green's function $\mathcal{G}(t-t') = \langle \phi_\mu(0,t)\phi_\mu(0,t')\rangle_c$, which reads at zero temperature $\mathcal{G}(t-t') = -\log\left[1 + i\frac{v_F(t-t')}{a}\right]$.

The unsymmetrized shot noise is written in terms of $I_B(t)$ [2] as $S(t,t') = \langle I_B(t)I_B(t')\rangle - \langle I_B(t)\rangle\langle I_B(t')\rangle$. To second order Γ_0^2, the zero-frequency time-averaged shot noise becomes:

$$\begin{aligned}\mathcal{S} &= 2\int d\tau \int_0^T \frac{d\bar{t}}{T} S\left(\bar{t} + \frac{\tau}{2}; \bar{t} - \frac{\tau}{2}\right) \\ &= \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau e^{2\nu\mathcal{G}(-\tau)} \int_0^T \frac{d\bar{t}}{T} \cos\left[e^* \int_{\bar{t}-\frac{\tau}{2}}^{\bar{t}+\frac{\tau}{2}} dt'' V(t'')\right].\end{aligned} \tag{S18}$$

The excess shot noise at zero temperature then takes the form

$$\begin{aligned}\Delta\mathcal{S} &= \mathcal{S} - 2e^*\overline{\langle I_B(t)\rangle} \\ &= \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int d\tau \int_0^T \frac{d\bar{t}}{T} \exp\left[2\nu\mathcal{G}(-\tau) + ie^* \int_{\bar{t}-\frac{\tau}{2}}^{\bar{t}+\frac{\tau}{2}} dt'' V(t'')\right].\end{aligned} \tag{S19}$$

Splitting the voltage into its DC and AC part, and using the Fourier coefficients p_l introduced in Eq. (S1), one has

$$\Delta\mathcal{S} = \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \sum_l |p_l|^2 \int d\tau e^{i(l+q)\Omega\tau + 2\nu\mathcal{G}(-\tau)}$$
$$= \frac{2}{T}\left(\frac{e^*\Gamma_0}{v_F}\right)^2 \frac{1}{\Gamma(2\nu)}\left(\frac{\Omega}{\Lambda}\right)^{2\nu-2} \sum_l P_l \left|l+q\right|^{2\nu-1}\left[1-\mathrm{Sgn}\left(l+q\right)\right], \tag{S20}$$

where the zero-temperature expression of $\mathcal{G}(-\tau)$ has been used, which allows to perform the integration. As in the main text, we also introduced the notations $q = \frac{e^* V_{dc}}{\Omega}$ for the charge per pulse, $\Lambda = v_F/a$ for the high-energy cutoff of the chiral Luttinger liquid theory and $P_l = |p_l|^2$ for the probability for a quasiparticle to absorb ($l > 0$) or emit ($l < 0$) l photons [3].

At finite temperature, $\Delta\mathcal{S}$ needs to be slightly amended,

$$\Delta\mathcal{S} = \mathcal{S} - 2e^*\overline{\langle I_B(t)\rangle}\coth\left(\frac{q}{2\theta}\right)$$
$$= -\frac{2}{T}\left(\frac{e^*\Gamma_0}{v_F}\right)^2 \frac{2\theta}{\Gamma(2\nu)}\left(\frac{2\pi\Omega\theta}{\Lambda}\right)^{2\nu-2} \sum_l P_l \left|\Gamma\left(\nu + i\frac{l+q}{2\pi\theta}\right)\right|^2 \frac{\sinh\left(\frac{l}{2\theta}\right)}{\sinh\left(\frac{q}{2\theta}\right)}, \tag{S21}$$

in order to get rid of the thermal noise ($\Delta\mathcal{S} \to 0$ for large temperature). There, the reduced temperature is $\theta = \frac{k_B\Theta}{\Omega}$ (Θ the electron temperature) and the finite-temperature expression of $\mathcal{G}\left(-\tau\right)$ has been employed.

EXCITATION NUMBER AND SIGNATURES OF MINIMAL EXCITATIONS

The number of excitations created in a one-dimensional system of free fermions by the applied time-dependent drive $V(t)$ is given by the number of electrons and holes

$$N_e = \sum_k n_F(-k)\langle\psi_k^\dagger\psi_k\rangle, \tag{S22}$$
$$N_h = \sum_k n_F(k)\langle\psi_k\psi_k^\dagger\rangle, \tag{S23}$$

where $n_F(k)$ is the Fermi distribution and ψ_k is a fermionic annihilation operator in momentum space. Using the bosonized description for $\nu = 1$ [see Eq. (S11)], the number of electrons and holes becomes

$$N_{e/h} = v_F^2 \int \frac{dtdt'}{(2\pi a)^2} \exp\left[2\mathcal{G}(t'-t) \mp ie\int_{t'}^{t} d\tau V(\tau)\right]. \tag{S24}$$

Minimal excitations correpond to a drive which excites a single electron, while no particle-hole pairs are generated ($N_h = 0$). Generalizing this to a chiral Luttinger liquid (a FQH edge state) [1], this excitation should correspond to a vanishingly small value of the quantity

$$\mathcal{N} = v_F^2 \int \frac{dtdt'}{(2\pi a)^2} \exp\left[2\nu\mathcal{G}(t'-t) + ie^*\int_{t'}^{t} d\tau V(\tau)\right]. \tag{S25}$$

The excess noise [defined in Eq. (S19)] is thus identified as the most suitable quantity to study minimal excitations. At $\theta = 0$, one recovers in $\mathcal{N}$ precisely the excess noise, up to a prefactor which depends on the tunneling amplitude [see Eq. (S19)] [4]. At $\theta \neq 0$, thermal excitations contribute substantially to $\mathcal{N}$, motivating us to include a correction [the coth factor in Eq. (S21)] which gets rid of this spurious contribution in the high temperature limit.

NON-PERTURBATIVE TREATMENT AT $\nu = 1/2$

The Hamiltonian is identical to that of Eqs. (S9)-(S15). Introducing new bosonic fields $\phi_\pm = \frac{\phi_R \mp \phi_L}{\sqrt{2}}$, it becomes:

$$H_0 = \frac{v_F}{4\pi}\sum_{r=\pm}\int dx \left(\partial_x\phi_r\right)^2, \tag{S26}$$
$$H_T = \Gamma(t)\frac{e^{i\phi_-(0)}}{2\pi a} + \Gamma^*(t)\frac{e^{-i\phi_-(0)}}{2\pi a}. \tag{S27}$$

This form of H_T (specific to $\nu = 1/2$) allows us to refermionize the $e^{i\phi_-}$ field, and to decouple ϕ_+, following Ref. [5]. A new fermionic field $\psi(x)$ and a Majorana fermion field f, which satisfy $\{f, f\} = 2$ and $\{f, \psi(x)\} = 0$, are introduced. These obey the equations of motion:

$$-i\partial_t\psi(x,t) = iv_F\partial_x\psi(x,t) + \frac{\Gamma(t)}{\sqrt{2\pi a}}f(t)\delta(x), \tag{S28}$$

$$-i\partial_t f(t) = 2\frac{1}{\sqrt{2\pi a}}\left[\Gamma^*(t)\psi(0,t) - \text{H.c.}\right]. \tag{S29}$$

Solving this set of equations near the position $x = 0$ of the quantum point contact (QPC), one can relate the fields ψ_b and ψ_a corresponding to the new fermionic field taken respectively before and after the QPC:

$$\psi_a(t) = \psi_b(t) - \gamma\Omega e^{i\varphi(t)+iq\Omega t}\int_{-\infty}^{t} dt' e^{-\gamma\Omega(t-t')}\left[e^{-i\varphi(t')-iq\Omega t'}\psi_b(t') - \text{H.c.}\right]. \tag{S30}$$

The backscattered current is the difference of the left-moving current after and before the QPC:

$$I_B(t) = \frac{ev_F}{2}\left[\psi_b^\dagger(t)\psi_b(t) - \psi_a^\dagger(t)\psi_a(t)\right]. \tag{S31}$$

After some algebra, the time-averaged backscattered current becomes:

$$\overline{\langle I_B\rangle} = -\frac{e}{T}\gamma\sum_l P_l \,\text{Im}\left[\Psi\left(\frac{1}{2} + \frac{\gamma - i(q+l)}{2\pi\theta}\right)\right], \tag{S32}$$

where $\Psi(z)$ is the digamma function, and the dimensionless tunneling parameter is $\gamma = \frac{|\Gamma_0|^2}{\pi a v_F \Omega}$.

Similarly, the zero-frequency time-averaged shot noise defined in Eq. (S18) takes the form:

$$\mathcal{S} = \frac{e^2}{T}4\gamma^2\sum_{klm}\frac{\text{Re}\left(p_k^* p_l p_{l+m}^* p_{k+m}\right)}{m^2+4\gamma^2}\text{Re}\left[\left(\frac{\frac{2\gamma^2}{m} - i\gamma}{\tanh\left(\frac{l-k}{2\theta}\right)} - \frac{m + i\gamma + \frac{2\gamma^2}{m}}{\tanh\left(\frac{k+l+m+2q}{2\theta}\right)}\right)\Psi\left(\frac{1}{2} + \frac{\gamma - i(k+q)}{2\pi\theta}\right)\right]. \tag{S33}$$

Finally, the excess shot noise is obtained using the known $\theta = 0$ DC results [5] for the backscattered current $\langle I_B\rangle_{dc}$ and corresponding zero-frequency noise $\mathcal{S}_{dc}$:

$$\langle I_B\rangle_{dc} = \frac{e}{2\pi}\xi\arctan\left(\frac{eV_{dc}}{2\xi}\right), \tag{S34}$$

$$\mathcal{S}_{dc} = \frac{e^2}{2\pi}\xi\left[\arctan\left(\frac{eV_{dc}}{2\xi}\right) - \frac{\frac{eV_{dc}}{2\xi}}{1+\left(\frac{eV_{dc}}{2\xi}\right)^2}\right], \tag{S35}$$

where we introduced $\xi = \frac{|\Gamma_0|^2}{\pi a v_F}$. This allows to express the DC noise, not as a function of the applied bias, but rather as a function of the charge $\mathcal{Q}_{\Delta t} = \Delta t\langle I_B\rangle_{dc}$ transferred through the QPC over a given time interval Δt

$$\mathcal{S}_{dc}(\mathcal{Q}_{\Delta t}) = e\frac{\mathcal{Q}_{\Delta t}}{\Delta t} - \frac{e^2\xi}{4\pi}\sin\left(\frac{4\pi}{\xi e}\frac{\mathcal{Q}_{\Delta t}}{\Delta t}\right). \tag{S36}$$

The excess noise at $\theta = 0$ associated with an arbitrary drive $V(t)$ is then defined as the difference between the photo-assisted shot noise (PASN) and the DC noise $\mathcal{S}_{dc}(\mathcal{Q}_T)$ obtained for the same charge $\mathcal{Q}_T = T\overline{\langle I_B\rangle}$ transferred during one period of the AC drive

$$\Delta\mathcal{S} = \mathcal{S} - 2e^*\overline{\langle I_B\rangle} + \frac{(e^*)^2}{T}2\gamma\sin\left(\frac{T}{\gamma e^*}\overline{\langle I_B\rangle}\right), \tag{S37}$$

where we reintroduced the effective charge $e^* = e/2$.

HONG-OU-MANDEL COLLISION OF LEVITONS

A periodic voltage bias is now applied to both right- and left-moving incoming arms of the QPC. We focus here on single leviton collisions with identical potential drives, up to a tunable time delay τ. The drives are periodic Lorentzians with a single electron charge per pulse ($q_R = q_L = 1$). Using a gauge transformation, this amounts to computing the noise in the case of a single total drive $V_{\rm Tot}(t) = V(t) - V(t-\tau)$ applied to the right incoming branch only.

In the context of electron quantum optics, a standard procedure is to compare this so-called Hong-Ou-Mandel (HOM) noise to the Hanbury-Brown and Twiss (HBT) case where single levitons scatter on the QPC without interfering, leading to the definition of the following normalized HOM noise [6, 7]

$$\Delta Q(\tau) = \frac{\mathcal{S}_{V(t)-V(t-\tau)} - \mathcal{S}_{\rm vac}}{\mathcal{S}_{V(t)} + \mathcal{S}_{V(t-\tau)} - 2\mathcal{S}_{\rm vac}}. \tag{S38}$$

Thermal fluctuations are eliminated by subtracting the vacuum contribution $\mathcal{S}_{\rm vac}$ to each instance of the noise.

Taking advantage of the gauge transformation, we use the expressions for the noise established earlier in the perturbative and the exact cases. This calls for a new set of Fourier coefficients $\tilde{p}_l$ associated with the total drive $V_{\rm Tot}(t)$:

$$\begin{aligned}\tilde{p}_l =& \sum_m p_m p^*_{m-l} e^{i(l-m)\Omega\tau} \\ =& \frac{2i\sin\left(\pi\frac{\tau}{T}\right) e^{i(l+1)\pi\tau/T}(1-z^2)}{1-z^2 e^{-2i\pi\tau/T}} \left[\left(\frac{1}{e^{2i\pi\tau/T}-1} + \frac{1}{1-z^2}\right)\delta_{l,0} - \left(z e^{-i\pi\tau/T}\right)^{|l|}\right],\end{aligned} \tag{S39}$$

where $z = e^{-2\pi\eta}$ and the Fourier coefficients p_l corresponding to a periodic Lorentzian drive $V(t)$ with $q = 1$ take the form

$$p_l = \theta_H\left(l + 1^-\right) z^l \left(1 - z^2\right) - \delta_{l,-1} z, \tag{S40}$$

with $\theta_H(x)$ the Heaviside step function.

In the WB case, the PASN is given by Eq. (S18), which gives at finite temperature

$$\mathcal{S}_{V_{\rm Tot}(t)} = S^0 \frac{(2\pi\theta)^{2\nu-1}}{\pi\Gamma(2\nu)} \sum_l |\tilde{p}_l(\tau)|^2 \left|\Gamma\left(\nu + i\frac{l}{2\pi\theta}\right)\right|^2 \cosh\left(\frac{l}{2\theta}\right), \tag{S41}$$

$$\mathcal{S}_{V(t)} = S^0 \frac{(2\pi\theta)^{2\nu-1}}{\pi\Gamma(2\nu)} \sum_l |p_l|^2 \left|\Gamma\left(\nu + i\frac{l+q}{2\pi\theta}\right)\right|^2 \cosh\left(\frac{l+q}{2\theta}\right), \tag{S42}$$

while the vacuum contribution reduces to

$$\mathcal{S}_{\rm vac} = S^0 \frac{(2\pi\theta)^{2\nu-1}}{\pi\Gamma(2\nu)} \left[\Gamma\left(\nu\right)\right]^2. \tag{S43}$$

Combining the results from Eqs. (S38) through (S43), we obtain for the normalized HOM noise

$$\Delta Q(\tau) = \frac{\sin^2\left(\frac{\pi\tau}{T}\right)}{\sin^2\left(\frac{\pi\tau}{T}\right) + \sinh^2\left(2\pi\eta\right)}. \tag{S44}$$

Remarkably, this result is independent of both temperature and filling factor. Indeed, thermal contributions factorize in the exact same way in the numerator and denominator, leading to a universal profile. This result also corresponds to that of Ref. [8] for $\nu = 1$.

APPLYING THE VOLTAGE BIAS TO A POINT-LIKE OR A LONG CONTACT

In the main text, we focus on the experimentally relevant case of a long contact [8] where electrons travel a long way through ohmic contacts before reaching the mesoscopic conductor, accumulating a phase shift along the way. In a previous work [4] however, the authors consider applying the voltage pulse through a point-like contact. Here we show that these two approaches are equivalent.

Indeed, one can see starting from the Hamiltonian (S8), and solving the corresponding set of equations of motion for the fields that the external bias can be accounted for by implementing a phase shift of the quasiparticle operator, which we recall here

$$\psi_R(x,t) \longrightarrow \psi_R(x,t)\ e^{-i\nu e\int_{-\infty}^{t} dt' V(x-v_F(t-t'),t')}. \tag{S45}$$

Several choices for the external drive $V(x,t)$ are thus acceptable, provided that the integral $\int_{-\infty}^{t} dt' V(x-v_F(t-t'),t')$ leads to the same result. Indeed, this phase shift is the only meaningful physical quantity, which gives us some freedom in the choice of $V(x,t)$. We further consider two different options.

In the present work, we apply the voltage to a long contact, so that $V_1(x,t) = \theta(-x-d)V(t)$. This leads to a phase shift of the form

$$\int_{-\infty}^{t} dt' V_1(x-v_F(t-t'),t') = \int_{-\infty}^{t} dt'\ \theta(-x+v_F(t-t')-d)V(t') = \int_{-\infty}^{t-\frac{x+d}{v_F}} dt' V(t'). \tag{S46}$$

Following now Ref. 4, one can recast the single particle Hamiltonian defined in their Eq. (4) into a form similar to the one presented here in Eq. (S10) provided that one defines the applied voltage drive as $V_2(x,t) = v_F\delta(x+d)\int_{-\infty}^{t} d\tau V(\tau)$. This, in turn, leads to the phase shift

$$\int_{-\infty}^{t} dt' V_2(x-v_F(t-t'),t') = \int_{-\infty}^{t} dt' v_F \delta(x-v_F(t-t')+d)\int_{-\infty}^{t'} d\tau V(\tau) = \int_{-\infty}^{t-\frac{x+d}{v_F}} d\tau V(\tau) \tag{S47}$$

One thus readily sees that at the level of the phase shift experienced by the quasiparticles as a result of the external drive, the protocol presented in Ref. 4 and the one presented in the text are completely equivalent.

[1] X. G. Wen, Adv. Phys. **44**, 405 (1995).
[2] A. Crépieux, P. Devillard, and T. Martin, Phys. Rev. B **69**, 205302 (2004).
[3] J. Dubois, T. Jullien, C. Grenier, P. Degiovanni, P. Roulleau, and D. C. Glattli, Phys. Rev. B **88**, 085301 (2013).
[4] J. Keeling, I. Klich, and L. S. Levitov, Phys. Rev. Lett. **97**, 116403 (2006).
[5] C. de C. Chamon, D. E. Freed, and X. G. Wen, Phys. Rev. B **53**, 4033 (1996).
[6] E. Bocquillon, V. Freulon, J.-M. Berroir, P. Degiovanni, B. Plaçais, A. Cavanna, Y. Jin, and G. Fève, Science **339**, 1054 (2013).
[7] E. Bocquillon, V. Freulon, F. D. Parmentier, J.-M. Berroir, B. Plaçais, C. Wahl, J. Rech, T. Jonckheere, T. Martin, C. Grenier, D. Ferraro, P. Degiovanni, and G. Fève, Ann. Phys. **526**, 1 (2014).
[8] J. Dubois, T. Jullien, F. Portier, P. Roche, A. Cavanna, Y. Jin, W. Wegscheider, P. Roulleau, and D. C. Glattli, Nature **502**, 659 (2013).