Minimal Excitations in the Fractional Quantum Hall Regime

J. Rech,1 D. Ferraro,1 T. Jonckheere,1 L. Vannucci,2,3 M. Sassetti,2,3 and T. Martin1
1Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France
2Dipartimento di Fisica, Università di Genova, Via Dodecaneso 33, 16146 Genova, Italy
3CNR-SPIN, Via Dodecaneso 33, 16146 Genova, Italy

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We study the minimal excitations of fractional quantum Hall edges, extending the notion of levitons to interacting systems. Using both perturbative and exact calculations, we show that they arise in response to a Lorentzian potential with quantized flux. They carry an integer charge, thus involving several Laughlin quasiparticles, and leave a Poissonian signature in a Hanbury Brown–Twiss partition noise measurement at low transparency. This makes them readily accessible experimentally, ultimately offering the opportunity to study real-time transport of Abelian and non-Abelian excitations.

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Because of its potential application to quantum information processing, time-dependent quantum transport in open coherent nanostructures attracts prodigious attention. Recent years have seen the emergence of several attempts to manipulate elementary charges in quantum conductors [1–4]. This opened the way to the field of electron quantum optics [5] characterized by the preparation, manipulation, and measurement of single-particle excitations in ballistic conductors.

In this context, levitons—the time-resolved minimal excitation states of a Fermi sea—were recently created and detected in two-dimensional electron gas [4,6], 20 years after being theoretically proposed [7–9]. These many-body states are characterized by a single particle excited above Fermi level, devoid of accompanying particle-hole pairs [10]. The generation of levitons via voltage pulses does not require delicate circuitry and has thus been put forward as a solid candidate for quantum bit applications, in particular, the realization of electron flying qubits [11,12].

Interaction and quantum fluctuations strongly affect low-dimensional systems leading to dramatic effects like spin-charge separation and fractionalization [13–15]. These remarkable features were investigated by looking at both time-resolved current [16–18] and noise measurements [19–23]. While the emergence of many-body physics and the inclusion of interactions [24–27] was recently addressed in the framework of electron quantum optics, a conceptual gap still remains when it comes to generating minimal excitations. This is particularly true when the ground state is a strongly correlated state, as are the edge channels of a fractional quantum Hall (FQH) system [28], a situation which has remained largely unexplored so far for time-dependent drives [29]. The building blocks of such chiral conductors are no longer electrons but instead anyons, which have a fractional charge and statistics [30]. For Laughlin filling factors [31], these anyons are Abelian quasiparticles, but more exotic situations involving non-Abelian anyons [32] are predicted. Our understanding of these nontrivial objects would benefit from being able to excite only a few anyons at a time [33], allowing us to study their transport and exchange properties, and to combine them through interferometric setups. This calls for the characterization of minimal excitations in the FQH regime.

In this Letter, we study levitons in the edge channels of the fractional quantum Hall regime by analyzing the partition noise at the output of a quantum point contact (QPC). Our results rely on a dual approach combining perturbative and exact calculations of the noise in a Hanbury Brown–Twiss [34,35] configuration. We also provide results in the time domain, investigating leviton collisions with Hong-Ou-Mandel (HOM) [4,36] interferometry.

Consider a FQH bar (see Fig. 1) with Laughlin filling factor \(\nu = 1/(2n + 1)\) \((n \in \mathbb{N})\), described in terms of a hydrodynamical model [37] by the Hamiltonian \((\hbar = 1)\),

\[
H = \frac{v_F}{4\pi} \int dx \left( \sum_{\mu=R,L} (\partial_x \phi_\mu)^2 - \frac{2e\sqrt{\nu}}{v_F} V(x,t) \partial_x \phi_R \right),
\]

where the bosonic fields \(\phi_{R,L}\) propagate along the edge with velocity \(v_F\) and are related to the quasiparticle annihilation operator as \(\psi_{R,L}(x) = (U_{R,L}/\sqrt{2\pi a}) e^{\pm ik_F x} e^{-i/v_F \phi_{R,L}(x)}\) (with

![FIG. 1. Main setup. A quantum Hall bar equipped with a QPC connecting the chiral edge states of the fractional quantum Hall effect (FQHE). The left-moving incoming edge is grounded at contact 2 while the right-moving one is biased at contact 1 with a time-dependent potential \(V(t)\).](image-url)
\( a \) a cutoff parameter and \( U \) a Klein factor, and \( V(x, t) \) is an external potential applied to the upper edge at contact 1.

Working out the equation of motion for the field \( \phi_F \),

\[
\frac{\partial \phi_F(x, t)}{\partial t} + v_F \frac{\partial \phi_F(x, t)}{\partial x} = e\sqrt{\frac{2}{m^*}} V(x, t),
\]

one can relate it to the unbiased case using the transformation

\[
\phi_F(x, t) = \phi_F^0(x, t) + e\sqrt{\frac{2}{m^*}} \int_0^t dt' V(x', t'),
\]

with \( x' = x - v_F(t - t') \), and \( \phi_F^0(x, t) \) is the free chiral field. \( \phi_F^0(x, t) = \phi_F(x - vt, 0) \). Focusing first on the regime of weak backscattering (WB), the tunneling Hamiltonian describing the scattering between counterpropagating edges at the QPC can be written, in terms of the transformed fields, Eq. (2), as

\[
H_T = \Gamma(t) \psi_R^0(0) \psi_L(0) + \text{H.c.},
\]

where we introduced \( \Gamma(t) = \Gamma_0 \exp[i e^\omega \int_0^\infty dt' V(t')] \) [38], with the bare tunneling constant \( \Gamma_0 \), the fractional charge \( e^\omega = \nu e \), and assuming a voltage \( V(t) \) applied over a long contact, in accordance with the experimental setup [4], allowing us to simplify

\[
\int_0^\infty dt' V(t', t),
\]

The time-dependent voltage consists of an ac and a dc part \( V(t) = V_{dc} + V_{ac}(t) \), where by definition \( V_{dc} \) averages to zero over one period \( T = 2\pi/\Omega \). The dc part indicates the amount of charge propagating along the edge due to the drive. The total excited charge \( Q \) over one period is then

\[
Q = \int_0^T dt \langle I(t) \rangle = \nu e^2 \int_0^T dt V(t) = q e^\omega,
\]

where the fractional conductance quantum is \( G_0 = \nu e^2/2\pi \) and the number of electrons per pulse is \( q = (e^\omega V_{dc}/\Omega) \). The ac voltage generates the accumulated phase experienced by the quasiparticles \( \varphi(t) = e^\omega \int_0^t dt' V_{ac}(t') \), characterized by the Fourier components \( p_l \) of \( e^{-il\delta t} \).

In a 1D Fermi liquid, the number of electron-hole excitations resulting from an applied time-dependent voltage bias is connected to the current noise created by the pulse scattering on a QPC [7,9,39], which acts as a beam splitter, as in a Hanbury Brown–Twiss setup [34,35]. For FQH edge states, however, scattering at the QPC is strongly nonlinear as it is affected by interactions. Special care is thus needed for the treatment of the point contact, and the definition of the excess noise giving access to the number of excitations.

The quantity of interest is the photoassisted shot noise (PASN), i.e., the zero-frequency current noise measured from contact 3, and defined as

\[
\mathcal{S} = 2 \int dt \int_0^T dt' \left\langle \delta I_3(t + \frac{T}{2}) \delta I_3(t - \frac{T}{2}) \right\rangle,
\]

where \( \delta I_3(t) = I_3(t) - \langle I_3(t) \rangle \) and the output current \( I_3(t) \) reduces, since contact 2 is grounded, to the backscattered current \( I_B(t) \), readily obtained from the tunnel Hamiltonian:

\[
I_B(t) = i e^\omega \left[ \Gamma(t) \psi_R^\dagger(0, t) \psi_L(0, t) - \text{H.c.} \right].
\]

When conditions for minimal excitations are achieved in the perturbative regime, excitations should be transmitted independently, leading to Poissonian noise. It is thus natural to characterize minimal excitations as those giving a vanishing excess noise at zero temperature:

\[
\Delta S = S - 2 e^\omega \langle I_B(t) \rangle,
\]

where \( \langle I_B(t) \rangle \) is the backscattered current averaged over one period.

Using the zero-temperature bosonic correlation function \( \langle \phi_F^R/L(t) \phi_F^R/L(0) \rangle_c = - \log (1 + i \Lambda \tau) \), this excess noise is computed perturbatively up to order \( \Gamma^2_0 \), yielding [38]

\[
\Delta S = \frac{2}{T} \left( \frac{e^\omega \Gamma_0}{v_F} \right)^2 \left( \Omega \right)^{2\nu - 2} \frac{1}{\Gamma(2\nu)} \times \sum_l P_l |l + q|^{2\nu - 1} [1 - \text{sgn}(l + q)],
\]

where \( \Lambda = v_F/\gamma \) is a high-energy cutoff and \( P_l = |p_l|^2 \) is the probability for a quasiparticle to absorb \( (l > 0) \) or emit \( (l < 0) \) \( \nu \) photons, which depends on the considered drive [38]. These probabilities \( P_l \) also depend on \( q \), as the ac and dc components of the voltage are not independent. Indeed, we are interested here in a periodic voltage \( V(t) \) consisting of a series of identical pulses, with \( V(t) \) close to 0 near the beginning and the end of each period. This implies that the ac amplitude is close to the dc one. Our formalism could also be used to perform a more general analysis by changing these contributions independently. In particular, fixing the dc voltage and changing the ac amplitude allows us to perform a spectroscopy of the probabilities themselves. Conversely, changing the dc voltage at fixed ac amplitudes, we can reconstruct the tunneling rate associated with each photoassisted process [10] in the same spirit as finite frequency noise calculations [40]. However, this broader phenomenology does not provide any additional information concerning the possibility of creating minimal excitation by applying periodic pulses.

In Fig. 2, we show the variation of the excess noise as a function of \( q \), for several external drives at \( \nu = 1/3 \) and various reduced temperatures \( \theta = k_B T/\Omega \) (\( \Theta \) is the electronic temperature). At \( \theta = 0 \), only the periodic Lorentzian drive leads to a vanishing excess noise, and only for integer values of \( q \). This confirms that as mentioned in earlier work [9], optimal pulses have a quantized flux and correspond to Lorentzians of area \( \int dt V = m 2\pi/\nu^2 \) (with \( m \) an integer number of fractional flux quanta). More intriguingly, however, this vanishing of \( \Delta S \) occurs for specific values of \( q \): while levitons in the FQH are also minimal excitations, they do not carry a fractional charge and instead correspond to an integer number of electrons. This shows that integer levitons are minimal excitation states even in the presence of strong electron-electron interactions, and that it is
ΔS is reminiscent of the orthogonality catastrophe[7,39,41].

disturbance of the ground state, and ultimately leads to the
producing less than a full electronic charge produces a strong
producing a seemingly diverging contribution. Indeed, excit-
tions of the perturbative treatment at
temperature results (see Fig.2) cure some inherent limi-
tions of the perturbative treatment at \( \theta = 0 \) (diverging
behavior close to integer \( q \)). The noiseless status of the
Lorentzian drive is confirmed, as \( \Delta S \approx 0 \) at low enough
temperature for some values of \( q \) (yet shifted compared to
the \( \theta = 0 \) ones).

Our perturbative analysis is valid when the differential
conductance is smaller than \( G_0 \). This condition can be
achieved on average (\( \Gamma_0 \) is then bounded from above), but it
is not fulfilled in general when the voltage drops near zero
because of known divergences at zero temperature. In order
to go beyond this WB picture, we now turn to an exact
nonperturbative approach for the special filling \( \nu = 1/2 \).
While this case does not correspond to an incompressible
quantum Hall state, it nevertheless provides important
insights concerning the behavior of physical values of \( \nu 
\) beyond the WB regime. The agreement between the two
methods in the regime where both are valid makes our
results trustworthy.

We thus extend the refermionization approach for filling
factor \( \nu = 1/2 \) [42,43] to a generic ac drive [38]. Starting
from the full Hamiltonian expressed in terms of bosonic
fields, one can now write the tunneling contribution
introducing a new fermionic entity, \( \psi(x,t) \propto
\exp[\phi(x,t)/\sqrt{\nu}] \). Solving the equation of motion for
\( \psi(x,t) \) near \( x = 0 \), one can define a relation between this
new field taken before (\( \psi_b \)) and after (\( \psi_a \)) the QPC,

\[
\psi_a(t) = \psi_b(t) - \nu \Omega e^{i\nu t + i\nu \Omega t} \int_{-\infty}^{t} dt' e^{-\nu \Omega (t-t')}
\times [e^{-i\nu (t-t') - i\nu \Omega t'} \psi_b(t') - \text{H.c.}],
\]

allowing us to treat the scattering at the QPC at all orders.
Expressing the current and noise in terms of \( \psi_b \) and \( \psi_a \),
and using the standard correlation function
\( \langle \psi_b^\dagger(t) \psi_b(t') \rangle = \int (d\omega/2\pi\nu_F) e^{i\omega(t-t') \gamma} f(\omega) \)
\( \) (with \( f \) the Fermi function), we derive an exact solution for both the backscattered current and PASN. As the dc noise at a QPC does not remain
Poissonian when its transmission increases, our definition of
\( \Delta S \) is further extended to treat the nonperturbative
regime. In the \( \nu = 1 \) case, where an exact solution exists,
it is standard to compare the PASN to its equivalent dc
counterpart [4,9] obtained with the same \( V_{dc} \), and \( V_{ac} = 0 \).
Here, in order to account for the nontrivial physics involved
at the QPC in the FQH, it makes sense to compare our
PASN to the dc noise which one obtains for the same
charge transferred at the QPC, over one period of the ac
drive. At zero temperature, \( \Delta S \) is redefined as

\[
\Delta S = S - 2e^\nu \langle I_B(t) \rangle + \frac{\sigma^4}{2} \sin^2 \left( \frac{T}{\gamma e^\nu \langle I_B \rangle} \right),
\]

where \( \sigma^4 = \nu e/e^2 \), and \( \gamma = (\Gamma_0^2/\pi v_F \Omega) \) is the
dimensionless tunneling parameter. This definition
coincides with the Poissonian one Eq. (6) at low \( \gamma \), in that it
vanishes for the same values of \( q \).

Results for \( \Delta S \) at \( \nu = 1/2 \) are presented in Fig. 3. At low
\( \gamma \), structures appear as a function of \( q \), which are very
similar to the perturbative calculation (Fig. 2). For the
Lorentzian drive only, the excess noise approaches zero
close to integer values of \( q \) in the tunneling regime \( \gamma \ll 1 \).
When increasing \( \gamma \) the position of these minima gets shifted
and the excess noise eventually becomes featureless,
independently of the ac drive. In the \( \gamma \to +\infty \) limit

\[\text{FIG. 2.} \ \text{Excess noise in units of } S_0 = (2/\eta)(e^\nu \Gamma_0/\nu_F)^2(\Omega/\Lambda)^{2\nu-2} \text{ as a function of the number of electrons per pulse } q, \text{ for different reduced temperatures } \theta \text{ and filling factor } \nu = 1/3, \text{ in the case of a square (bottom), a cosine (middle), and a periodic Lorentzian drive with half width at half maximum } \eta = W/T = 0.1 \text{ (top).} \]
FIG. 3. Rescaled excess noise \(\Delta S/\gamma\) in units of \((e^2/T)\) as a function of the number of electrons per pulse \(q\), for different values of the dimensionless tunneling parameter \(\gamma = (|\Gamma_0|^2/\pi e v_F \Omega)\). Results are obtained at zero temperature, with filling factor \(\nu = 1/2\), in the case of a square (bottom), a cosine (middle), and a periodic Lorentzian drive with \(\eta = 0.1\) (top).

(not shown), the Lorentzian drive shows signatures of Poissonian electron tunneling at the QPC occurring at \(q\) multiples of \(\nu\), consistent with the duality property of the FQH regime [42]. This Poissonian behavior, not observed for other drives, is also confirmed by the strong backscattering perturbative treatment. At finite temperature, our results are almost unaffected for \(\theta \lesssim \gamma\), while larger temperatures tend to smear any variations in \(q\).

Levitons can also be explored in the time domain through electronic Hong-Ou-Mandel interferometry [4,6]. By driving both incoming channels, one can study the collision of synchronized excitations onto a beam splitter, as two-particle interferences reduce the current noise at the output, leading to a Pauli-like dip. Figure 4 shows the normalized HOM noise \(\Delta Q\) [24] as a function of the time delay \(\tau\) between applied drives. While this does not constitute a diagnosis for minimal excitations, it reveals the special nature of levitons in the WB regime, as the normalized HOM noise is independent of temperature and filling factor [38,44], reducing at \(q = 1\) to the universal form:

\[
\Delta Q(\tau) = \frac{\sin^2(\frac{\gamma\tau}{2})}{\sin^2(\frac{\gamma\tau}{2}) + \sinh^2(2\pi\eta)}.
\]

The same universal behavior is also obtained for fractional \(q = \nu\) in the strong backscattering regime (tunneling of electrons at the QPC). Interestingly, although the HOM noise and the PASN are very different from their Fermi liquid counterparts, an identical expression for \(\Delta Q(\tau)\) was also obtained in this case [10] (where it is viewed as the overlap of leviton wave packets).

Finally, in addition to the excess noise, the time-averaged backscattered current \(\langle I_B(t)\rangle\) also bears peculiar features. In contrast to the Ohmic behavior observed in the Fermi liquid case, \(\langle I_B(t)\rangle\) shows large dips for integer values of \(q\) (see Fig. 5). These dips are present for all types of periodic drives, and cannot be used to detect minimal excitations. However, the spacing between these dips provides an alternative diagnosis (from dc shot noise [19,20]) to access the fractional charge \(q^*\) of Laughlin quasiparticles, as \(q^*\) is known from the drive frequency and the amplitude \(V_{dc}\) [45].

Real-time quasiparticle wave packet emission has thus been studied in a strongly correlated system, showing the existence of minimal excitations (levitons) in edge states of the FQH. These occur when applying a periodic Lorentzian drive with quantized flux, and can be detected as they produce Poissonian noise at the output of a Hanbury Brown–Twiss setup in the weak backscattering regime.

FIG. 4. Normalized HOM noise \(\Delta Q\) at \(q = 1\), as a function of the time delay \(\tau\) between pulses. Results are presented in the WB case at \(\nu = 1/3\) and \(\theta = 0.1\), for a square, a cosine, and periodic Lorentzian drive with \(\eta = 0.1\). Inset: HOM setup with applied drives on both incoming arms.

FIG. 5. Averaged backscattered current \(\langle I_B(t)\rangle\) as a function of the number of electrons per pulse \(q\), in the case of a square, cosine, and periodic Lorentzian drive with \(\eta = 0.1\) and \(\theta = 0.1\). Results are presented for \(\nu = 1/3\) in the perturbative regime in units of \(F^0 = \sqrt{\gamma}(|\Gamma_0|/e)^2(\Omega/\Lambda)^{2k-2}\) (top) and for the exact treatment at \(\nu = 1/2\) in units of \((e/T)\) (bottom).
Although FQH quasiparticles typically carry a fractional charge, the charge of these noiseless excitations generated through Lorentzian voltage pulse corresponds to an integer number of $e$. Furthermore, our findings are confirmed for arbitrary tunneling using an exact refermionization scheme. Remarkably enough, in spite of the strong interaction, two FQH leviton collisions bear a universal Hong-Ou-Mandel signature identical to their Fermi liquid analog. Possible extensions of this work could address more involved interferometry of minimal excitations as well as their generalization to non-Abelian states.

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Note added in proofs—Recently, it came to our attention that a simple argument can rule out the possibility of minimal excitations beyond the results presented here. Starting from Eq. (7), one readily sees that a minimal excitation $(\Delta \nu = 0)$ can only be realized if $P_\nu = 0$ for all $\nu \leq -q$, independently of the filling factor. At $\nu = 1$, it was shown [7–9] that minimal excitations were associated with quantized Lorentzian pulses, so that this type of drive is the only one satisfying the constraint of vanishing $P_\nu$. Since this condition is independent of $\nu$, it follows that also at fractional filling, minimal excitations can only be generated using Lorentzian drives with quantized charge $q \in \mathbb{N}$.