Abstract

The hope to describe matter and its interactions in an unified theoretical framework is as old as Physics itself. Still, in 2005, World Year of Physics, we do not have this framework yet. We have reached a quasi-unified description of fundamental physics which relies on two pillars: quantum mechanics and general relativity. But a complete description of the quantum properties of the gravitational field, or, which is the same, the quantum properties of spacetime, is still missing. The problem is of course complex; but it is a major priority research direction of the international scientific community and the impact of a succesful solution would be major.

The last years have witnessed increasing interest for this problem, namely the problem of 'Quantum Gravity', and different ways to attack it have been developed. Loop Quantum Gravity (LQG) is a promising one and is receiveing increasing interest. LQG is based on the conservative viewpoint that it may be possible to perform a canonical non-perturbative quantization of general relativity using original techniques. In particular, space-time is no longer the fixed background where physics happens, but rather a dynamical quantity naturally treated in the quantization process in the same way in which quantum fields are. This illustrates a key feature of LQG: its 'background independence'. The price is an abstract description of states of quantum gravity, far from the classical geometrical image of space-time.

It is the goal of this project to recover conventional physics from results of LQG. To be more precise, we will concentrate on the following two major issues:
1. Can we show with precision that the low energy limit of LQG general relativity?
2. Can LQG include usual quantum field theory in its framework?
LQG has to give a clear and precise positive answer to these questions to be a serious candidate for a quantum theory of gravitation. We will attack these issues using techniques of Spin Foam models (SF) which have been introduced as a way to define dynamics in LQG and to find the physical states of the theory.

To answer the first question, we will use the recent results by Rovelli and his collaborators concerning the definition of the notion of graviton in the context of LQG and SF. The extraction of the perturbative notion of graviton from LQG, which is a non perturbative background independent formalism, is a promising result. Our aim is to show that this notion is related to the more conventional one, i.e. the one emerging from standard low-energy perturbative quantum gravity corresponding to the quantization of a spin-2 massless field. The link between LQG and classical gravity should the, become clear. We aim at computing graviton-graviton scattering amplitudes general n points correlation functions.

To adrress the second problem, we will use recent results, obtained by Freidel-Livine in the framework of SF models and by Noui-Perez in the canonical loop framework, which indicate the possibility of constructing a quantum theory of a scalar field coupled to three dimensional quantum gravity. We will extend the techniques employed, to the construction of quantum field theory of higher spins (Dirac or Yang-Mills) coupled to three dimensional quantum gravity. Finally, using Group Field Theory techniques, we will generalize this coupling to the four dimensional case.

The program we propose is ambitious but realistic. The results that we hope to obtain could represent a step ahead towards the understanding of the quantum properties of gravity.

CONTEXT and OBJECTIVES

Loop Quantum Gravity: an original direction of research

The idea of quantizing the gravitational interaction is almost as old as the classical theory of general relativity itself. During the past century this idea has found a number of obstacles, indicating that gravitation appear to escape all the methods that had proved so effective in understanding the physics of the other fundamental interactions.

The situation changed in the mid-eighties with Ashtekar's discovery of a new formulation for gravity. The new formalism presents general relativity as a gauge theory and has allowed a rapid development of a non-perturbative study of quantum gravity, in which the fixed background geometry, essential to standard quantum field theories, is not needed anymore. Such background independence provides LQG with an original perspective. It has quickly led to concrete answers to questions that were previously difficult to address: in particular, the construction of background independent quantum states, realized as abstract Wilson loops or spin networks; and the calculation of the spectrum of geometrical observables, to name a few examples. In light of these results, a better understanding of the foundations and potential outcomes of this approach have become an important challenge.

LQG was initially developed mostly under the impulse of Ashtekar, Rovelli and Smolin. The 90's have marked the emergence of a new dynamism and many interested young researchers have entered the field. A number of important results were then established: a microscopic description of black hole thermodynamics and horizons and a derivation of the Hawking formula, a theory of loop quantum cosmology, the development of a phenomenology of quantum gravity among others... A covariant version of the theory has been identified in the spinfoam formalism, especially under the impulse of Baez, Barrett, Freidel and Rovelli.

In twenty years, LQG has developed into a major candidate for a potential theory of quantum gravitation. The process has recently been acknowledged by the publication of two reference books, respectively by Rovelli and Thiemann.

The recent development of an active research group in France

LQG has started on the other side of the Atlantic and has long been been missing from the French research laboratories. In France, it has only become an active and dynamical research subject since the arrival of Carlo Rovelli at the CPT in Marseille and Laurent Freidel at the ENS Lyon. The last years have seen the first Ph.D. thesis on this topic in France. The quality, originality and promise of these works have been recognized and rewarded with the hiring of four young researchers in the past two years: two maîtres-de-conférence (K. Noui and A. Perez) and two CNRS researchers (S. Alexandrov and E. Livine).

The present project, LQG-2006, reflects this dynamism. It aims to create a reference research group in France with an ambitious project involving both young and senior researchers. The cohesion of this group has already produced numerous successful collaborations. The project relies on a large national network involving four main nodes: the ENS in Lyon, the CPT in Marseille, the LMPT in Tours, the LPTA in Montpellier. These groups are among the most active ones outside Paris in theoretical physics.

In addition, there exist strong active collaborations between the project's team and various groups in the world: the ICPG of Penn State University (USA), the Perimeter Institute for Theoretical Physics (Canada), the La Sapienza university (Italy), the UNAM (Mexico), the Albert Einstein Institute for gravitational physics (Germany), Nottingham University (UK), etc... These links will play an important role for achieving the objectives of the LQG-2006 project.

Goals

Since it is not yet possible to test a quantum theory of gravity against real experiments, a candidate theory must at least be fully consistent, and pass a number of theoretical tests. For instance, it should provide a microscopic description of black hole thermodynamics and it should predict the actual nature of space-time at the Planck scale, but also admit a classical regime where general relativity and quantum field theory are valid. LQG gives us convincing answers to the first two criteria. However, a fully convincing way to address the latter requirement is not yet known.

This project has the ambitious goal to derive the low energy physics from the microscopic realm of LQG. The main tool will be Spin Foam (SF) formalism, which appear to offer an effective perspective for deriving a spacetime picture of the theory. This formalism is essentially a covariant quantization of gravity based on combinatorial structures that can be interpreted as evolution histories of the background independent LQG quantum gravitational states, namely histories of spin networks.

Surprisingly, the formalism has turned out to be essentially equivalent to the conventional Feynman diagrams expansion of certain quantum field theories defined on a group manifold. This equivalence has already been exploited as a powerful tool in various context, and we intend to utilize it to extract physical information on the low energy regime of LQG.

We intend to focus on the two following aspects of the problem:

1. **The low-energy limit of LQG: where is the graviton?**

   How can we relate the background independent formalism of LQG with conventional perturbative quantum field theoretical notions? That is, how does the graviton, the quantum of the linearized gravitational field over Minkowski space, appear from the abstract spin foam structures? How to concretely compute the graviton-graviton transition amplitudes from the background independent formalism?

2. **The coupling of quantum fields to LQG via the Spin Foams**

   What are the effects of quantum gravity on the dynamics of matter's quantum fields? How to describe the coupling of the usual matter fields to gravity in the framework of spinfoams: scalar and Dirac fields, Yang-Mills theory? What do the resulting effective field theories look like?

These questions are connected to well known open problems which remains unanswered to this day, in spite of a number of substantial advances:

-How to build coherent states for quantum gravity and thus how to recover the classical solutions to general relativity in a low energy limit in this context?

-What is the precise physical interpretation of a spin foam model? This question is crucial to establishing a precise link between the canonical approach (LQG) and the covariant formalism (SF) for quantum gravity. This is a question that has been recently solved by Perez and Noui in 3 dimensions, but so far remains elusive in 4 dimensions. Do spin foam models truly provide a solution to the LQG canonical Hamiltonian constraint?

Our strategy to tackle these issues will be on the one hand to study them within the toy model provided by three-dimensional gravity. On the other hand, to address them directly in the framework of gravity in four space-time dimensions. To this aim, we intend to develop the boundary amplitude formalism that has already appeared to be effective for the reconstruction of some graviton amplitudes.

We also plan to keep a constant communication open with parallel research directions aiming at providing a quantum description of gravity, in order to look for possible convergencies and exchange ideas and techniques. In particular, string theory, non-commutative geometry and dynamical triangulations. Within the project's team, Rovelli has collaborated and is in strict touch with the non-commutative geometry community and Alexandrov has also a specific competence and a lively research activity in string theory.

PROJET DESCRIPTION

One of the most interesting results of Loop Quantum Gravity (LQG) is the discrete spectrum of geometrical quantities. This result has been obtained via a spectral analysis of the self-adjoint operators associated to functions of the gravitational–field operator, functions that in general relativity are interpreted as metric properties of spacetime. The result follows therefore from the mixing of rather conventional and relatively well–established physical inputs from quantum field theory and general relativity. Its consequences, however, are numerous and far reaching. Not only it has found intriguing tentative applications in black–hole physic, cosmology and astrophysics; but, more importantly, it has also opened the way to a completely background–independent formulation of quantum field theory (QFT), in which quantum states are not defined over a given manifold, but, rather, space itself appears as a collection of the quantum excitations of an underlying diff-invariant quantum field. Precisely as in classical general relativity, solutions of the theory do not live on spacetime: it is spacetime which is a solution of the theory. The translation of this perspective to the covariant picture has merged with a number of other lines of research to lead to the spinfoam (SF) formalism, where transition amplitudes are computed as sums over combinatorial objects (the spinfoams), loosely interpretable as discretized spacetimes.

These developments offer a fresh perspective on the possibility of thinking quantum spacetime at the Planck scale. They also offer a tool for explicit computations, and, in perspective, the hope of a complete and coherent background independent formulation of QFT and of a candidate quantum theory for the gravitational field.

Towards low energies

The conceptual novelty of the LQG formalism is substantial. Formally, LQG is simply a standard quantum theory, defined by a Hilbert space H, an algebra of quantum operators, and an equation coding quantum evolution; its naive classical limit is general relativity, possibly with its standard additional matter couplings. As the result of the implementation of diffeomorphism gauge invariance, however, states $|s\rangle$ in H are described by spin networks, which are abstract combinatorial objects. Classical spacetime is not a fundamental ingredient in this picture. On the one hand, this is not surprising: it is the analog of the fact that in the quantum description of the hydrogen atom in the $|E, l, m\rangle$ basis, we can compute transition amplitudes between basis states (say when interacting with a field) without reference to the trajectory of the electron: spacetime is, like the trajectory of the electron, a quantity that looses its meaning in this formulation. This reflects the most characteristic and interesting aspect of LQG, namely its background independence.

On the other hand, however, the disappearance of classical spacetime makes it difficult to connect LQG with full classical general relativity, or to compute standard low energy scattering amplitude from the full background independent formalism, or to recover conventional flat–space QFT in an appropriate limit. This is a serious problem for the theory for two reasons. First, computing low–energy physics starting from the full theory is difficult, hence it is difficult to explore the theory's phenomenology and the possibility of comparing it with observation. Second, and more importantly, for LQG to be credible as a candidate quantum theory of gravity, its low energy limit must be precisely shown to lead to conventional general relativity and to conventional QFT, which are of course extremely well empirically supported. Aim of the present project is to address this problem directly.

In other words, the current LQG description of spacetime remains rather distant from the standard low energy picture of spacetime and QFT on a background, and our objective is to fill this gap. We want thus to address the following two questions.

- How to recover in full classical general relativity from LQG transition amplitudes? How to recover, in a suitable approximation, the conventional quantum field of the linearized gravitational field around flat space? How to recover the notion of graviton, and to compute its scattering amplitude?

- How to recover the conventional QFT of conventional matter fields, from full LQG?

We are convinced that LQG must answer these questions, if it pretend to be a credible candidate for a quantum theory of gravity.

Posed in these manner, these questions illustrate the direction we intend to take, but are clearly ambitious and rather wide. A number of very recent developments, however, open the way, we think, to possible explicit solutions of these problems, and make this project, we are convinced, realistic.

Low energy limit of LQG

A simple way to code the physics of a conventional QFT on flat space is to define its n-point functions. The usual definition of n-point functions is notoriously useless for a generally–covariant theory, because diffeomorphisms invariance makes the dependence of a n-point function from its arguments completely trivial. This is a characteristic difficulty of the background independent context, and reflects the different physical meaning of the coordinates on flat spacetime and in a general relativistic setting. For instance, a conventional two-point function $W(x, y)$ is a function of the background distance between x and y; in a general relativistic setting, there is no background distance between two coordinate points. A way to circumvent this difficulty has been recently discussed by Rovelli, Testa, Oeckl and others, and is based on the boundary formalism (well known for instance in lattice gauge theory). The idea is to consider a different class of n-point functions, defined in terms of a closed 3-surface Σ that bounds a finite portion of spacetime, and which are explicitly dependent on the state of the field on Σ. In the flat space context, this is only a minor modification in the formalism. In a general relativistic context, on the other hand, this strategy circumvents the problem mentioned before, since the (mean) boundary value of the gravitational field on Σ provides precisely an appropriately covariant notion of distance between the arguments of the n-point functions, if these are chosen on the boundary.

This strategy has been used in conjunction with spinfoam methods by Modesto and Rovelli, to begin the computation of the 2-point function, starting from the fully background independent LQG formalism. The results appear to be promising. The correct low energy graviton propagator appears in a suitable approximations. Aim of these investigation has so far been mostly to develop a general tool for computing n-point functions in a background independent contexts.

Our objective is to develop this research direction in a number of directions. First, to solidify the preliminary calculations of Modesto and Rovelli, to clarify all assumptions made, and to extend them to higher orders and to others n-point functions. Second, to apply this calculation scheme to other versions of the theory's dynamics, in order to test the existing alternatives, and to compare them with the expected low energy behavior. Eventually, if this test is successful, we will consider using this tool for exploring the first deviations provided by LQF with respect to the (inconsistent) conventional perturbative expansion of quantum general relativity.

Matter in LQG

In three spacetime dimensions, a particle can be consistently interpreted simply as a conical singularity of the two-dimensional geometry of space. Chargeless bosonic matter can therefore be simply represented as a (singular) aspect of geometry. In a sense, matter degrees of freedom appear naturally in the gravitational theory. An analogous result in the physical case of four spacetime dimensions has long been searched and it is still under active investigation, but so far without clear success. It therefore appears, so far, that matter degrees of freedom must be explicitly added to the gravitational theory in order to describe ordinary matter. Is it possible to consistently couple all standard–model fields and their dynamics to LQG? The answer is formally yes, and the theory appears to maintain its ultraviolet finiteness properties also when matter is added. But, as for pure gravity, the relation between the Planck scale implicitly defined dynamics and the low energy theory is nontrivial. Again, in order for LQG to be able to claim to be a coherent theoretical framework, it must be able to accommodate standard matter and to clearly lead to conventional QFT on a background in an appropriate limit.

Recent work by Freidel and Livine has open novel perspectives in this regard. In three dimensions, Freidel and Livine have shown that it possible to rigorously derive the effective quantum field theory of a scalar self-gravitating field, starting from a SF model. Remarkably, the quantum properties of the gravitational field manifest itself in the effective QFT by a spacetime non-commutativity. More precisely, there is a limit of the full theory leading precisely to a QFT over a non-commutative spacetime. In the construction, a quantum group appear as a symmetry of an auxiliary formulation of the theory. These results have been reproduced by Noui and Perez in the Hamiltonian context.

In the context of the present project, we intend to generalize this construction to the Dirac and Yang-Mills fields and begin the investigation of the possibility of extending them to four spacetime dimensions.

Loop quantum gravity and Spinfoams formalism

The precise relation between the covariant spinfoam formulation and the canonical construction of the LQG Hilbert space has been clarified in 3 spacetime dimensions, but a number of details remain unclear in 4 spacetime dimensions.

The work of Alexandrov has opened a new perspective for clarifying these difficulties, but proposing a version of LQG in which 4d local Lorentz covariance remain explicit, unlikely conventional LQG, where a time gauge is used to fix it. Alexandrov formulation is more complicated and harder to study, but it opens the way to a full clarification of the relation between LQG and SF in 4d.

An essential aspect of the present project is to complete this clarification, which is essential for the overall consistence of the 4d theory.

Methodology: the spinfoam formalism and its relation with group field theory

The spinfoam (SF) formalism is the main tool that we intend to exploit, both in order to study the classical limit of the theory and in order to derive conventional matter QFT. A SF model is a theory that in principle allows us to associate a transition amplitude $W[s]$ to any state $|s\rangle$ of quantum gravity (seen as a state on a closed surface Σ enclosing a spacetime interaction region) or a transition amplitude $W[s_f, s_i]$ two any couple of states (seen as a states on an initial and a final time-slice). Different models, giving different amplitudes $W[s]$, have been constructed.

In general $W[s]$ is defined as a sum over spinfoams σ of a spinfoam amplitude $A[\sigma]$. A spinfoam is a two-complex, namely a collection of abstract faces joining in edges, in turn joining in nodes, colored with representation of the local internal gauge group of the theory, determined by the spacetime dimensions and the signature considered. This abstract structure is not unconventional: a conventional quantum Yang-Mills theory on a lattice can be easily re-expressed in this language, by "Fourier" transforming, using Weyl theorem, from the group elements associated to the plaquettes to the irreducible representations, and thus transforming the integral over the group elements at fixed boundary field, into a sum over assignments of representations to plaquettes. The novelty in the gravitational case is that the (connectivity of the) lattice itself is not necessarily fixed, but rather it can be one of the dynamical variables summed over. This reflects the background independence of the formalism.

This SF formalism appears to be a very natural framework for a covariant approach to background independent theories. Very remarkably, indeed, the very same formalism has been derived in a number of very different ways, starting from quite different points of view. For instance, it has been derived from canonical LQG by considering the possibility of computing transition amplitudes as histories of spin networks (Reisenberger and Rovelli); from discretization of various formulations of general relativity (Ponzano and Regge, Freidel and Krasnov), from the study of models of quantum properties of geometry (Barrett and Crane), from the generalization to higher dimensions of the matrix models that describe 2d quantum gravity (Ooguri, Crane), and others. This impressive convergence of research direction to a substantially unique formalism makes the SF formalism a powerful and intriguing tool for expressing background independent quantum dynamics.

From topological BF theory to GFT.

In 3d, quantum gravity is a BF topological theory, where the gauge group is $G = SO(3)$ (or $G = SO(2,1)$, according on the signature considered). The partition function of the theory $\mathcal{Z}(\mathcal{M})$ is a topological invariant of the 3d manifold $\mathcal{M}$. It can be defined by choosing a triangulation $\mathcal{T}$ of $\mathcal{M}$: irreducible representations of G are assigned to each face of the dual triangulation, and $\mathcal{Z}(\mathcal{M})$ is defined as sum over the representations of a statistical weight associated to vertices, faces and links. These are natural objects in the representation theory of G. This is precisely the 3d quantum gravity model defined (with completely different motivations) by Ponzano and Regge in the late 1960'. Convergence can be assured by going over to the case in which a cosmological constant is present, which amounts to a quantum deformation of the group. The properties of convergence, and the classical limit of this theory are well understood. The relation with LQG, pointed out by Rovelli in 1993, has recently been solidly established by Noui and Perez, who have shown that the theory can be derived from the LQG quantum canonical constraint, and gives the correct transition amplitude between LQG states defined on the boundary $\partial\mathcal{M}$.

In 4d, general relativity is not anymore a topological theory, but Plebanski has shown that it can be expressed as a 4d BF topological theory plus a constraint. The implementation of the Plebanski constraint in the sum model leads to a non–topological model first considered (with different motivations) by Barrett and Crane, which has raised much interest and has been intensely studied.

Since the 1990, it is known that the 3d topological invariants defined by the Ponzano–Regge theory can be obtained from a perturbation expansion of a quantum field theory over a group. (Group Field Theory or GFT). The action $S(\lambda)$ of such a theory contains a coupling constant λ and the partition function can be expanded in the form $\mathcal{Z}(\lambda) = \sum_\Gamma \lambda^{V_\Gamma} \frac{A(\Gamma)}{Sym(\Gamma)}$ where $A(\Gamma)$ is the amplitude of the Feynman graph Γ, and V_Γ is the number of vertices. De Pietri, Freidel, Krasnov and Rovelli have shown that the Barrett Crane model can be obtained in the same manner and Reisenberger and Rovelli have shown than *any* spinfoam model can be obtained in this manner. Freidel has shown that this technique allows the general construction of the physical Hilbert product of LQG in general. These results have provided quantum gravity with a novel powerful tool: transition amplitudes can be computed within the auxiliary GFT, using all the conventional powerful machinery of perturbative quantum field theory.

It is using this powerful machinery that the graviton propagator mentioned above has been computed, and this is the main technical tool that we intend to use.

Program: expected results

A. Pure gravity: n-point functions

Following the lines indicated above, we expect to be able to obtain the following results:

1. *Notion of graviton.*

   Show clearly how the spin 2 excitations of linearized general relativity appear in LQG. Show that in an appropriate low–energy limit LQG describes a flat space with gravitons.

2. *Graviton-graviton transition amplitudes.*

   Complete the calculation of the graviton 2-point function at the lowest order. Study the second order. Study the 4-point function. Compare the 4-point function with the one defined by linearized general relativity.

   If successful, study the corrections, presumably of order length/Planck length, to the conventional perturbative predictions.

3. *Definition of coherent states.*

   Understand, in this context, the role of background geometry (here appearing as a boundary state of the interaction region). Compare the result with the coherent states studied in LQG by Thiemann, Ashtekar, and others.

B. Coupling to matter fields

We will start by studying the 3d theory, where the topological invariance of the purely gravitational theory greatly simplify the problem. In addition, the fact that particles can be represent simply as conical singularities of the metric provides a natural tool for studying the gravity–matter coupling.

1. *Scalar field theory.*

   Standard quantum field theoretical methods have never provided a convincing quantum field for a scalar field with gravitational self interactions. The original idea developed by Freidel and Livine in the SF context and by Noui and Perez in the LQG context is to couple point particles to a quantum spacetime, and then reconstruct the quantum field theory by reconstructing the Fock space from the particle states. Remarkably, this can be done, and a fully consistent theory can be defined. Freidel et Livine have shown that the resulting theory is in fact equivalent to a theory of a scalar field coupled to gravity defined over a non-commutative spacetime. The result has been once more recovered by Perez and Noui in the LAG context. This is a remarkable concrete realization of the old idea that non-commutativity of spacetime emerges as an effect of quantum gravitational effects at the Planck scale.

   We will study the explicit consequences of this result on the n-point functions of the scalar field and on eigenvalues of certain observables.

   We will generalize this construction, so far given only in the Euclidean context, to the Lorentzian context.

2. *Role of quantum groups.*

   The spacetime non-commutativity mentioned above is associated to the fact that the symmetry group of the theory is in fact a quantum group. It is $DSU(2)$, Drinfeld double of the classical group $SU(2)$. It appears rather clearly that quantum groups play a role in LQG, although the precise meaning of this appearance is not yet clear. In order to try to clarify it, we will begin by studying the emergence of $U_q(su(2))$ (with q real, or root of unity, depending on the lorentzian or riemannian signature) in the loop quantization of 3d gravity, in the presence of a cosmological constant. If, in addition, particles are coupled to the model, one may expect that the symmetry group be $D(U_q(su(2)))$, the Drinfeld double of $U_q(su(2))$. These investigations should help us to understand the role that quantum groups could play in the framework of the physically interesting 4d case.

3. *Higher spins: Dirac and Yang-Mills.*

   The coupling of quantum gravity to fields with spin 1/2 and spin 1 can be obtained as simple generalization of the model, and can be formulated in the canonical LQG as well as in the covariant SF formalisms.

   We will study the resulting theories in detail, using techniques similar to the one used for the scalar field. In particular, we want to derive the form that the effective Dirac and Yang-Mills equations will take on a quantum geometry. This may give, in particular, a rigorous derivation of the photon's dispersion relation (in 3d).

4. *GFT models.*

   Finally, we plan to reformulate these 3d models in the GFT language, namely as group field theories on a group, in order to have them in a form as close as possible to the formalism that is been used in 4d.

C. Precise relation between SF and LQG in 4d

Alexandrov's formulation of LQG, in which the time gauge for the internal group is not chosen, appears to be the natural tool for constructing a rigorous bridge between the SF and loop formalisms. We intend to use this tool and try to solve this technical issue in a definitive form. For this, the first step is to provide a precise description of the kinematical state space in Alexandrov's formulation and to compute the physical scalar product on this space. This, we hope, should give the complete SF formalism. Alexandrov and Livine have already begin addressing this problem.