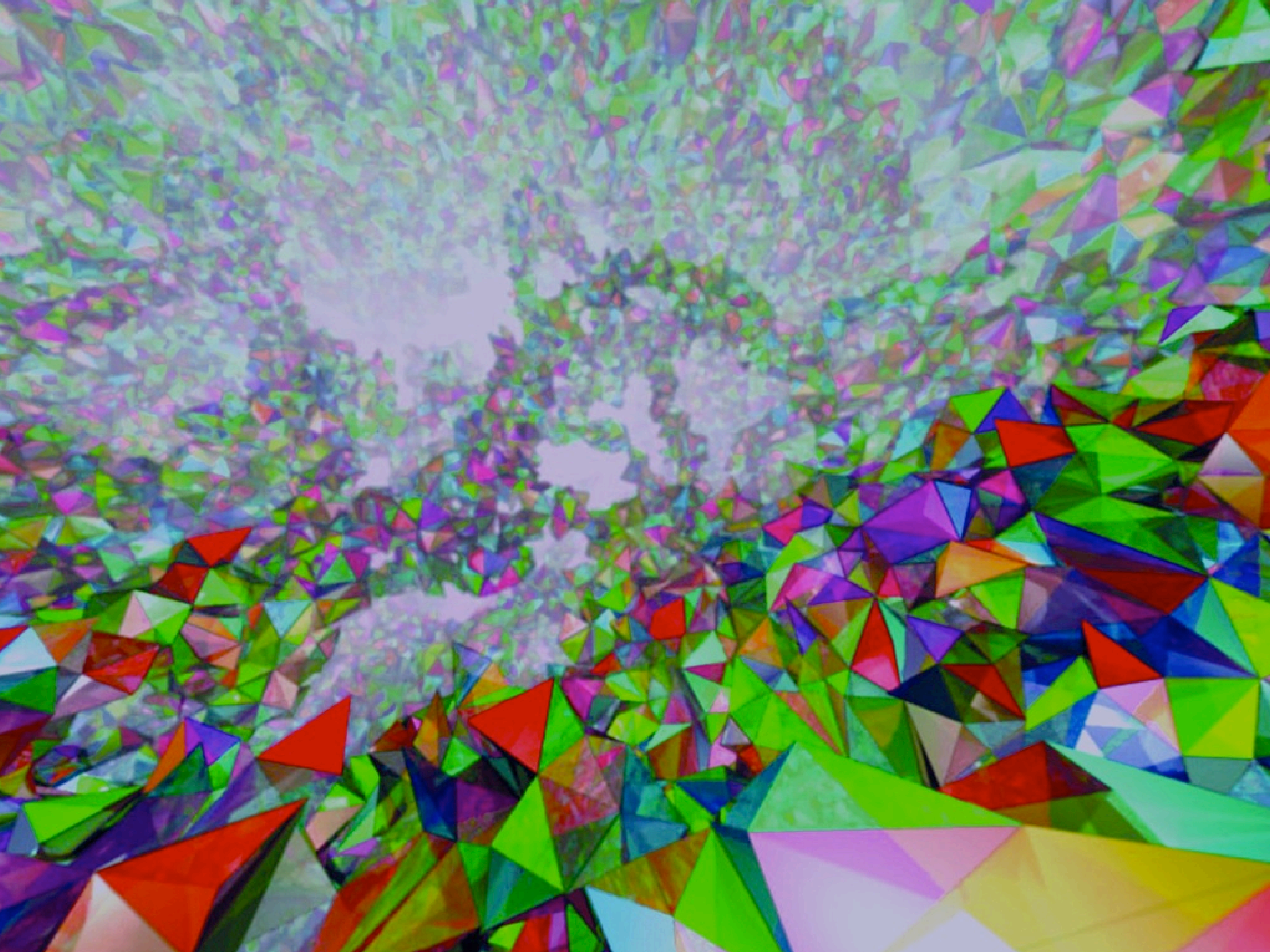


loop quantum gravity

Carlo Rovelli

January 2006, ENS Lyon

Quantum Gravity

- The problem is still open

- Several approaches studied: Dynamical Triangulations, Cosets, Twistors, Non-commutative geometry ...

- Two well-developed, but incomplete, theories: String Theory and Loop Quantum Gravity

- Traditional problems (big bang, black holes...) are now being addressed concretely

- Suggestions for experimental verifications. But no empirical support so far for any approach.

Loop quantum gravity:

- Developed from 1987
- ~ 100 people, ~ 30 groups (3 groups in France: ENS Lyon *Laurent Freidel, Etera Livine*; CPT Marseille *Alejandro Perez, CR*; Tour *Karim Noui*)
- Several books. Ex: CR, "Quantum Gravity" (Cambridge UP 2004)

➡ quantum properties of the gravitational field

➡ quantum spacetime

➡ background independent quantum field theory

General relativity is *two* different theories:

General relativity is *two* different theories:

- A *specific* theory of the gravitational field $$S = \frac{1}{16\pi G}\int \sqrt{g}R$$

General relativity is *two* different theories:

- A *specific* theory of the gravitational field $$S = \frac{1}{16\pi G}\int \sqrt{g}R$$

- A *general* modification of our understanding of spacetime

Spacetime $\longleftrightarrow$ **Gravitational field**

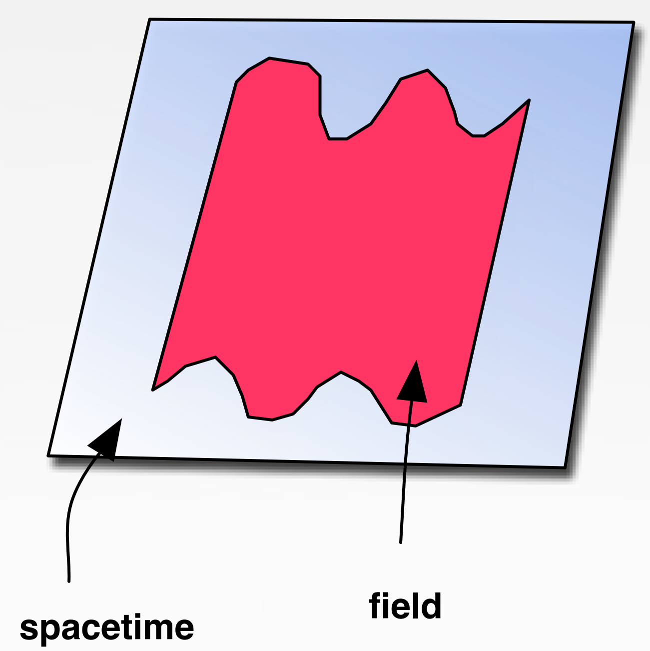
General relativity is *two* different theories:

- A *specific* theory of the gravitational field $S = 1/16\pi G \int \sqrt{g}\, R$

- A *general* modification of our understanding of spacetime

Spacetime ⟷ **Gravitational field**

Pre GR:



Background independence:

- Fields do not live on a background spacetime,
  They determine spacetime themselves.

- The problem of quantum gravity is to combine this with QFT.
  That is, to understand:

background independent quantum field theory

- Background independence is implemented by *gauge* invariance under "*active diffeomorphisms*": a smooth deformation of all the fields,

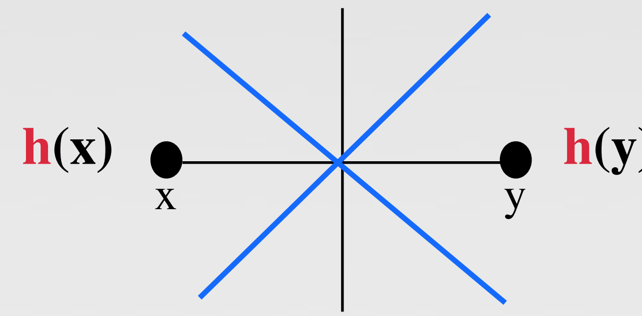
$$\begin{array}{lll} \phi_* : g & \mapsto & \phi_* g, \\ \phi_* : \psi_{matter} & \mapsto & \phi_* \psi_{matter}, \\ \phi_* : A & \mapsto & \phi_* A \end{array}$$

determined by an arbitrary smooth map $\phi : M \longrightarrow M$ of the spacetime coordinate manifold to itself.

- According to GR, no structure which is not invariant under this action exists in nature.

Roads to quantum gravity:

i) "Bring back a background spacetime"



$$g_{ab}(x) = \eta_{ab} + h_{ab}(x)$$

metric = gravitational field

metric

gravitational field

-> **string theory** (background independence to be recovered later on)

ii) Genuinely understand QFT without spacetime background

-> **loop quantum gravity**

$$g_{ab}(x) \rightarrow \hat{g}_{ab}(x)$$

– **" Take general relativity seriously "**

– **Most of usual tools of QFT must be abandoned** **(n-point functions, Poincare' invariance, vacuum, energy, momentum states, ...)**

– **Is the source of the short-scale divergences the unphysical assumption that there is a background ?**

Loop quantum gravity is the result of the combination of two traditional lines of thinking in theoretical physics:

i) Old ideas on **background independent quantum gravity**
(*Wheeler 64, DeWitt 63, Misner 69, Hawking 84, ...*)

ii) The idea that **flux lines (Wilson loops, holonomies)** are the natural variables in gauge theory
(*Wilson 74, Polyakov 79, Mandelstam 76, Migdal 84, ..., Faraday 1855*)

Both directions find stumbling blocks. But, remarkably, **the two provide the solution of the each other's difficulties**

i) background independent quantum gravity

canonical approach (*Wheeler 64, DeWitt 63, Isham 83 ...*)

- states: $\Psi(\mathbf{q})$ where **q**: 3d metric of a t=0 surface
- 3d diff: $\Psi(\mathbf{q}) = \Psi(\mathbf{q}')$ if there is a diff: $\mathbf{q} \rightarrow \mathbf{q}'$
- dynamics: Wheler-DeWitt eq $\mathbf{H}\Psi(\mathbf{q}) = 0$.

difficulties:

- Which space of states $\Psi(\mathbf{q})$? Which scalar product $< \Psi, \Phi >$?
- Operator **H** badly defined and divergent.
- No calculation possible.

covariant approach (*Misner 57, Hawking 79, Hartle 83...*)

- $$Z = \int Dg \; e^{iS[g]}$$

difficulties:

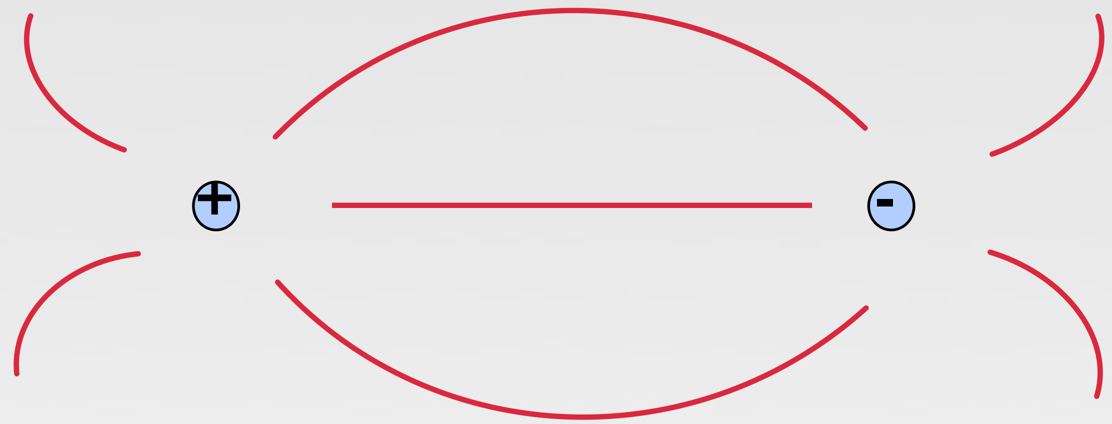
- Integral very badly defined
- Perturbative calculations bring short scale divergences back

ii) Loops: Gauge fields are "made up of lines"

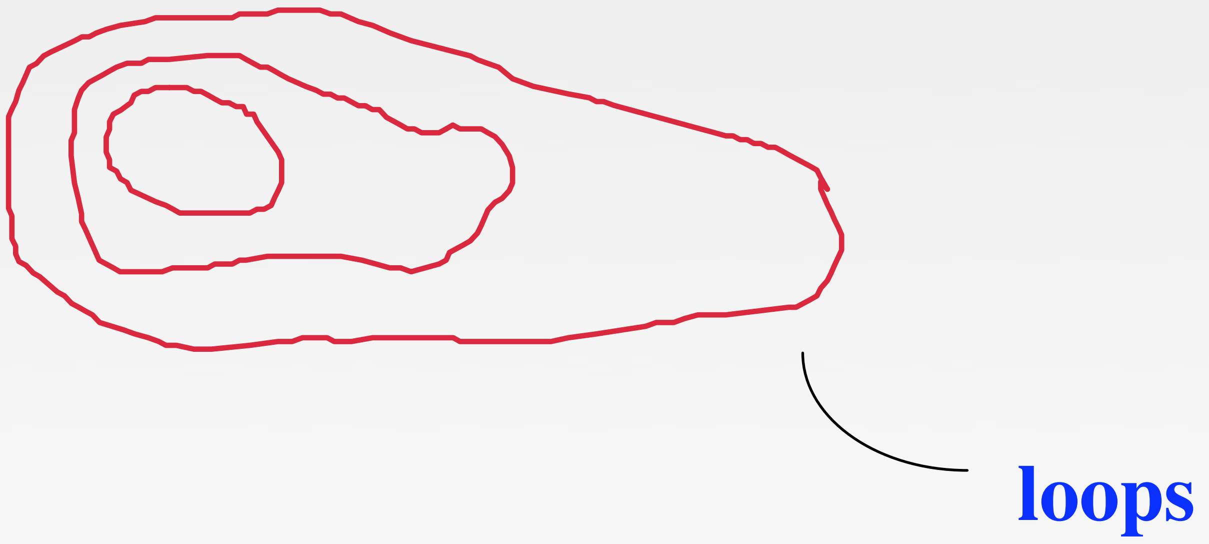
Faraday lines :

ii) Loops: Gauge fields are “made up of lines”

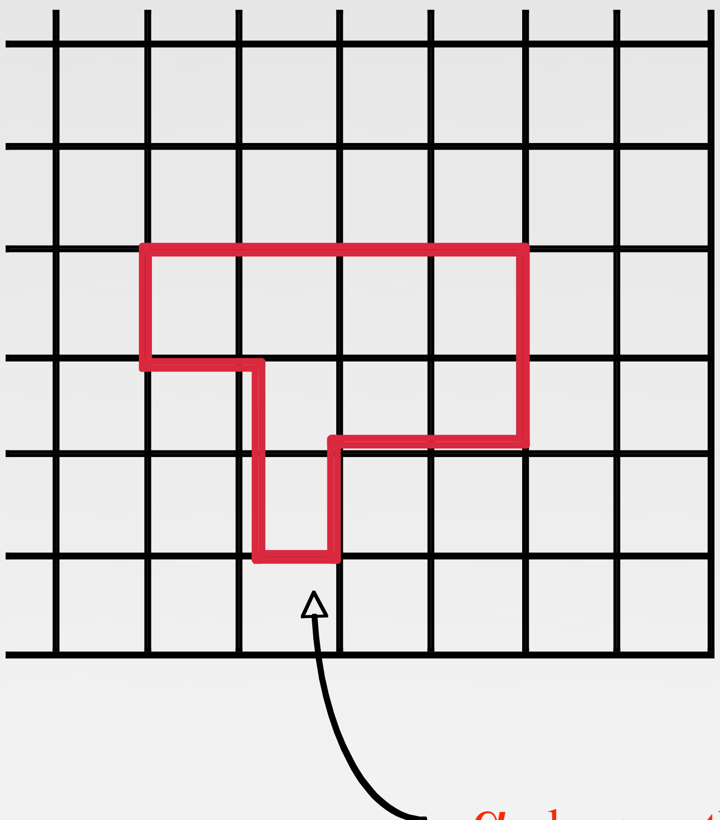
Faraday lines :



In vacuo:



Lattice Yang Mills quantum theory (spacial lattice) *(Wilson Kogut Susskind 1975)*



Variable: U_e in the gauge group **G**

State: $\Psi(U_e)$ in $L_2[G^{(\text{number of edges})}]$

Operators: Magnetic field $B_{\text{Plaquette}}$: $(U_1 U_2 U_3 U_4)$

Electric field: $\mathbf{E_e} = -i\hbar\, \partial/\partial U_e$

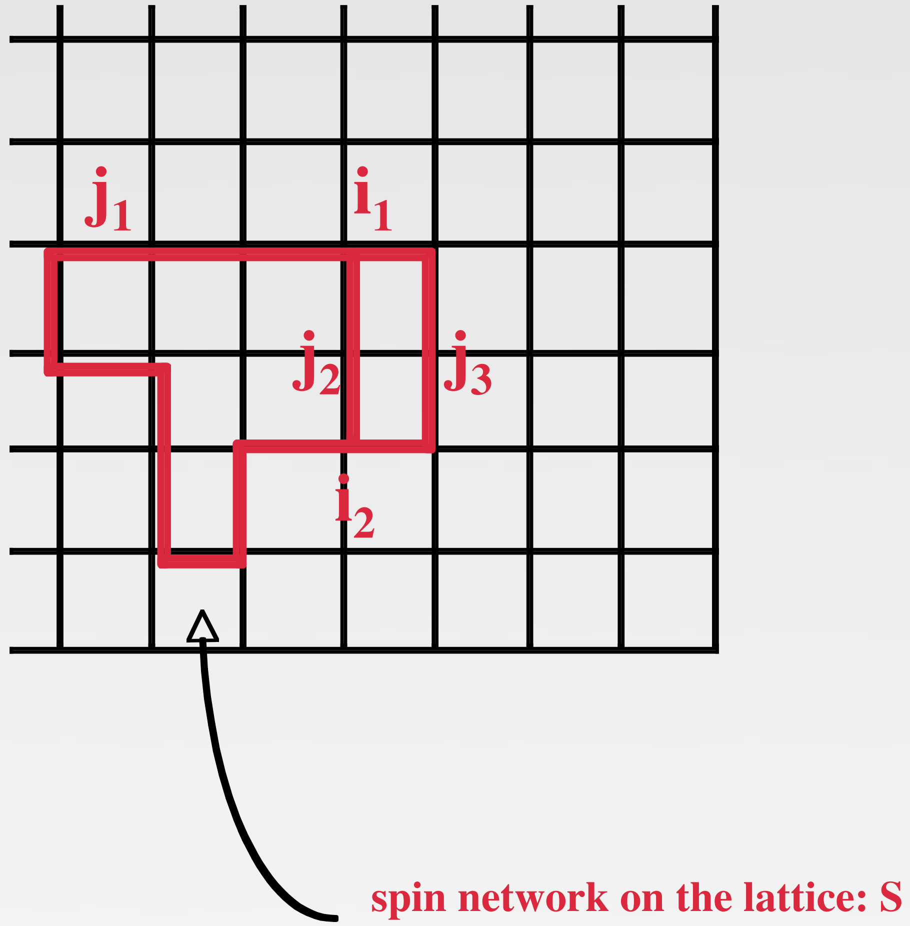
Dynamics: $H = B^2 + E^2$

Loop state: $\Psi_\alpha(U_e) = \mathrm{Tr}\,(U_l \dots U_e \dots U_N) = \langle U_e | \alpha \rangle$

Main property: $|\alpha\rangle$ is an eigenstate of E_l with eigenvalue having support on α .

$|\alpha\rangle$ is a quantum excitation of a single Faraday line.

Generalization: spin-networks and spin-network states

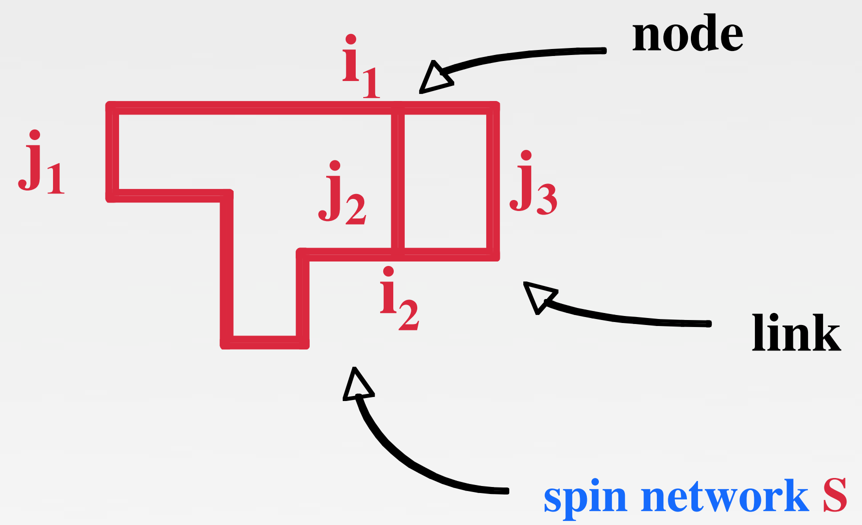


Spin network $S = (\Gamma\,,\ j_l\,,\ i_n\,)$

graph Γ

spins j_l on links

intertwiners i_n on nodes



Spin network state: $\Psi_S(U_e) = R^{(j1)}(\,U_l) \dots R^{(jL)}(\,U_L)\ i_1 \dots i_N = \ < U_e \,|\, S >$

The spin network states $|\,S>$ form an othonormal basis.

Loop states in continuum Yang Mills theory (ex: Maxwell)

Maxwell potential $A(x)$

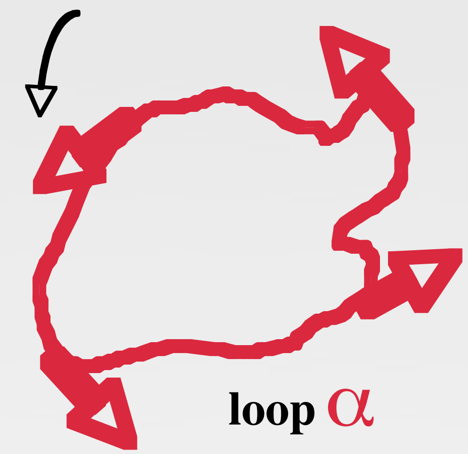
State: $\Psi(A)$

Operators: magnetic field $B = dA$

electric field: $E(x) = -i\hbar\delta/\delta A(x)$

Gauge : $\Psi(A) = \Psi(A')$ if $A' = A + d\lambda$

Dynamics: $H = B^2 + E^2$



Loops

Loop states: $\Psi_\alpha(A) = e^{i\int_\alpha A} = U_\alpha(A) = \langle A \,|\, \alpha \rangle$

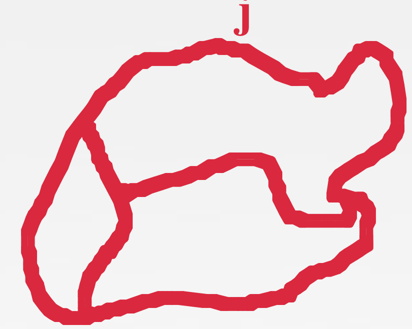
Eigenstates of $E(x)$ with eigenvalue

$$E_a(x) = \int ds \;\; d\alpha/ds \;\; \delta(\alpha(s), x)$$

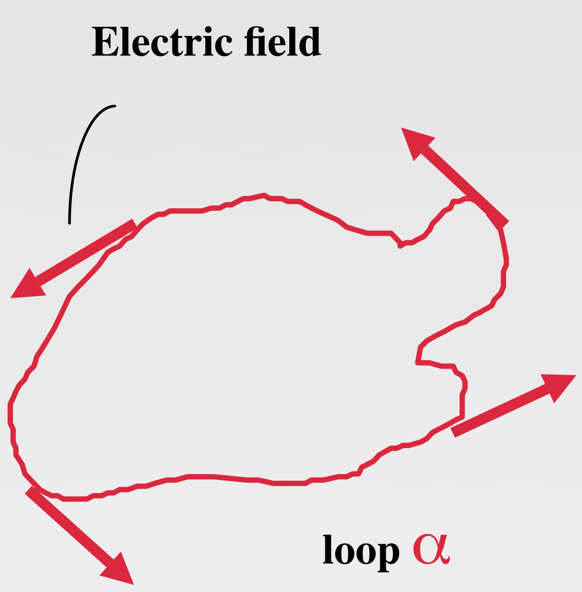
Notice: $\mathrm{div}\, E_a(x) = 0$!

Spin network states:

$$\Psi_S(A) = R^{(j_1)}(U_{l_1}(A))\ldots R^{(j_L)}(U_{l_L}(A))\; i_{l_1} \ldots i_{l_N} = \langle A \,|\, S \rangle$$



Loop states in Yang Mills theory

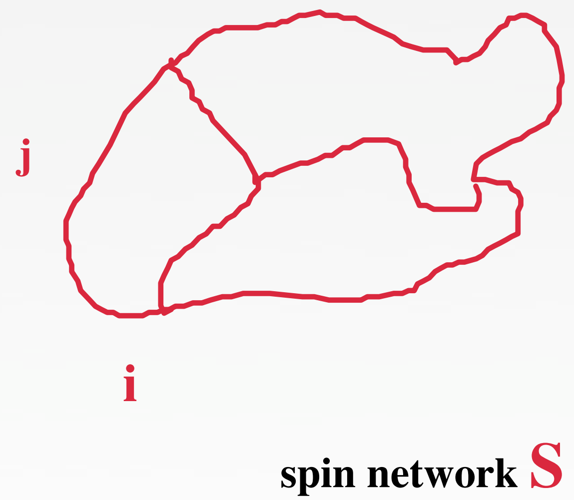


$|\alpha>$ **:** quantum state where

the electric field is concentrated

on a single Faraday line

Generalization: spin network



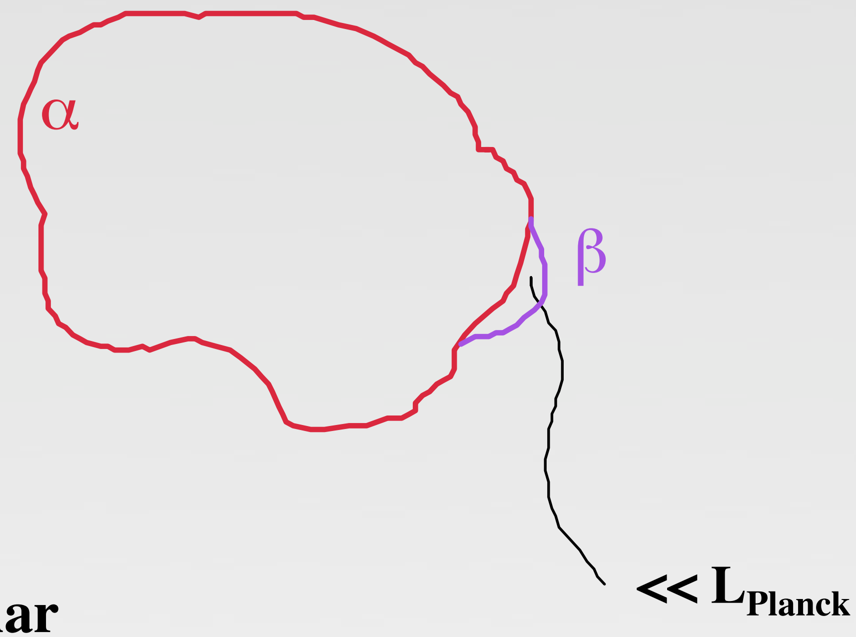
$|S>$ **:** quantum state where

the electric field is concentrated

on a network of Faraday lines, carrying

quantum numbers on edges and nodes

Difficulties :



- $< \alpha \,|\, \alpha > \; = \; \bullet$

  The loop states $|\, \alpha >$ in the continuum are **too singular**

- $< \alpha \,|\, \beta > \; = \; 0$ for β infinitely close to α

  States are **“too many”:**
  Hilbert space is “enormous” (nonseparable)

- **Reason: an *infinitesimal* displacement in space yields a *different* loop state**

But:

A loop formulation of gravity solves both sets of difficulties

Loop quantum gravity

i) connection formulation of GR (*Cartan 29, Weyl 29, Swinger 63, Sen 82, Ashtekar 86, Barbero 95*) **:**

$\mathbf{q} \rightarrow A, E$

E = triad field: $(\det q)\, q^{ab} = \mathrm{Tr}(E^a E^b)$

A = spin connection

ii) quantum theory:

States: $\Psi(A)$

Operators: A, $E(x) = -i \hbar G\, \delta / \delta A(x)$

Gauge: $\Psi(A) = \Psi(A')$ if A' is gauge *or diff* equivalent to A

Dynamics: $H = \mathrm{Tr}\, B\, EE$

Loop quantum gravity

i) connection formulation of GR (*Cartan 29, Weyl 29, Swinger 63, Sen 82, Ashtekar 86, Barbero 95*) **:**

$\mathbf{q} \;\rightarrow\; A,\; E$

E = triad field: $\quad(\det q)\, q^{ab} = \mathrm{Tr}(E^a E^b)$

A = spin connection

ii) quantum theory:

States: $\Psi(A)$

Operators: A, $\quad E(x) = -i\hbar G\, \delta / \delta A(x)$

Gauge: $\Psi(A) = \Psi(A')$ if A' is gauge *or diff* equivalent to A

Dynamics: $H = \mathrm{Tr}\, B\, EE$

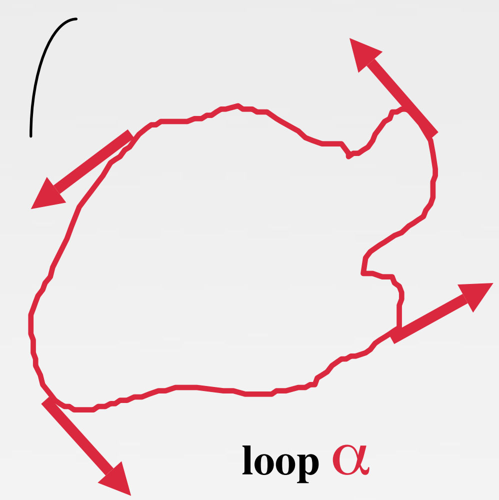
Loop states: $\Psi_\alpha(A) = \langle A \mid \alpha \rangle = \mathrm{Tr}\; e^{\int_\alpha A}$ (*Smolin CR 88*)

Spin network states: $\Psi_S(A) = \langle A \mid S \rangle$ (*Smolin CR 95*)

scalar product:

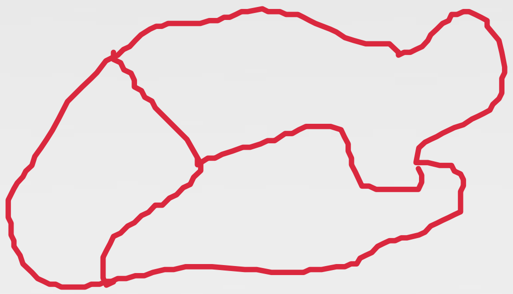
$\langle S' \mid S \rangle = 0$ if the graph is different

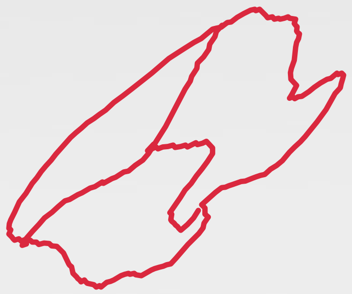
$\langle S' \mid S \rangle$ as in lattice gauge theory if same graph



Diffeomorphism invariance :

| **S** > and | **S’** > are gauge equivalent if **S** can be transformed into **S’** by **a diffeomorphism !**

S =


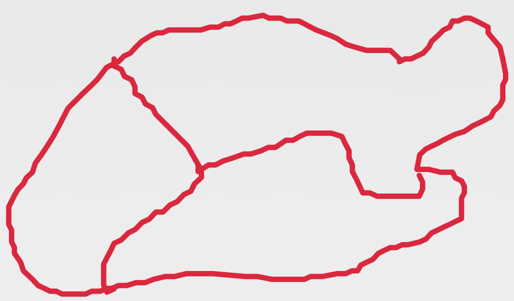
S’ =


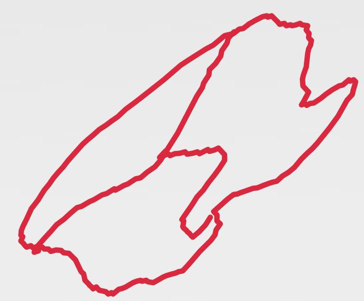
S’’ =


$$\langle S \,|\, \Psi \rangle = \langle S' \,|\, \Psi \rangle = \langle S'' \,|\, \Psi \rangle$$ **for any** Ψ.

Diffeomorphism invariance :

$|\,S\,\rangle$ and $|\,S'\,\rangle$ are gauge equivalent if **S** can be transformed into **S'** by **a diffeomorphism !**

S = 

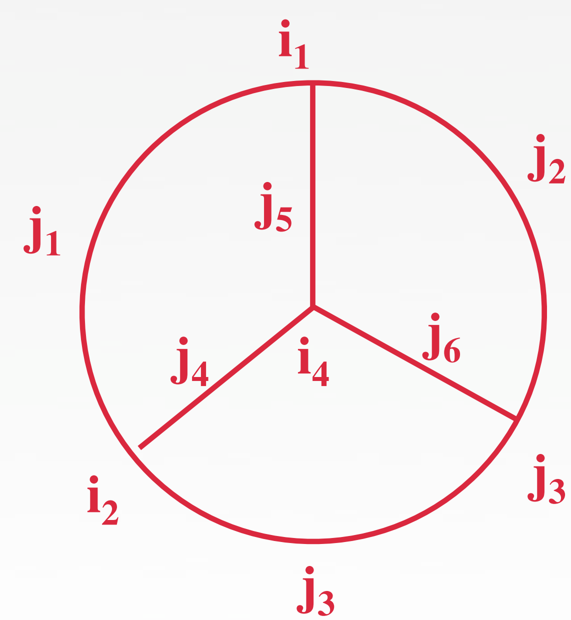
S' = 

S'' = 

$$\langle S\,|\,\Psi\rangle = \langle S'\,|\,\Psi\rangle = \langle S''\,|\,\Psi\rangle \qquad \text{for any } \Psi.$$

States are determined only by an *abstract* graph γ with j's and i's (*Smolin CR 1988*)

s-knot states $|\,s\,\rangle = |\gamma, j, i\,\rangle$, where

s = 

The kinematics of the theory is well defined !

(Separable) Hilbert space H is well-defined

Self-adjoint field operators are well-defined.

(Notice: this was neither true in metric quantum gravity nor in continuous loop YM theory)

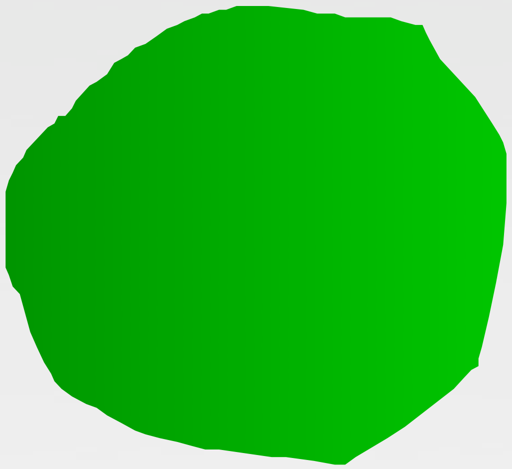
- What does it mean ?
- What do the s-knot states mean ?
- Which physics follows from this ?

Interpretation of the spin network states $| S >$

Interpretation of the spin network states $| S >$

Volume: $V(R)$: function of the gravitational field

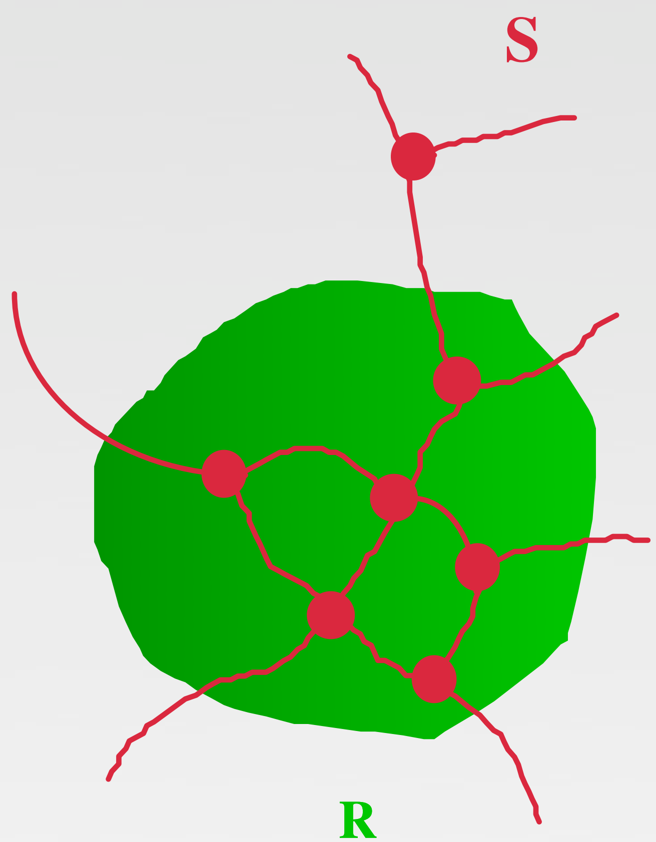
$$V(R) = \int_R \sqrt{g} = \int_R \sqrt{EEE} \quad \rightarrow \quad V(R) \text{ operator}$$



- $V(R)$ is a well-defined self-adjoint operator

- The volume receives a contribution for each node of $| S >$ inside R

Interpretation of the spin network states | S >



Volume: V(R): function of the gravitational field

$$V(R) = \int_R \sqrt{g} = \int_R \sqrt{EEE} \quad \rightarrow \quad V(R) \text{ operator}$$

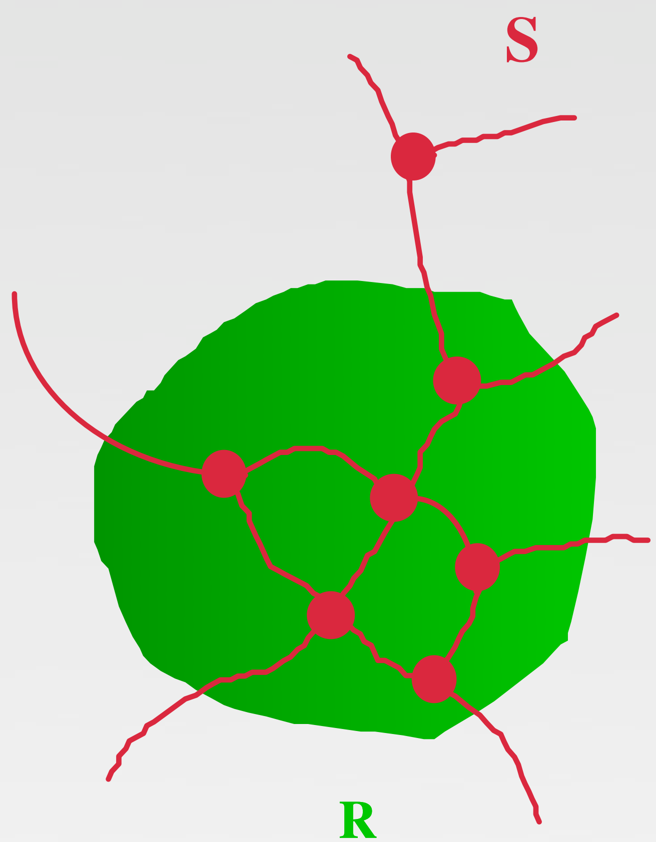
- V(R) is a well-defined self-adjoint operator

- The volume receives a contribution for each node of | S > inside R

Interpretation of the spin network states | S >

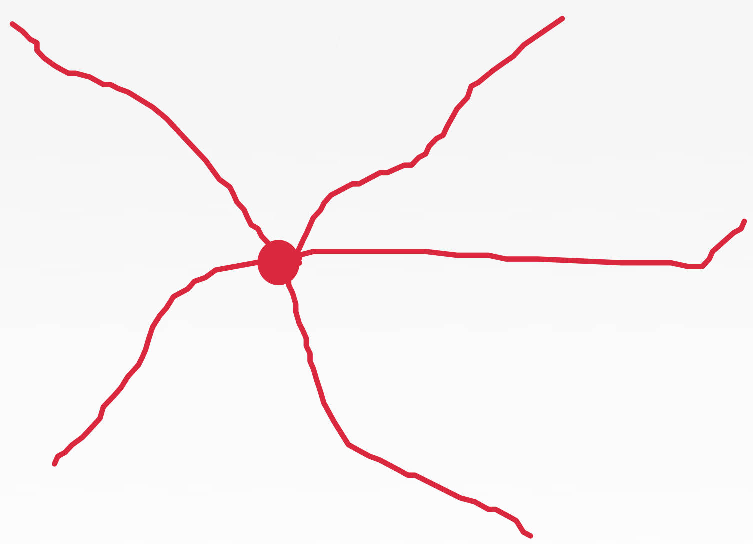
Volume: V(R): function of the gravitational field

$$V(R) = \int_R \sqrt{g} = \int_R \sqrt{EEE} \quad \rightarrow \quad V(R) \text{ operator}$$

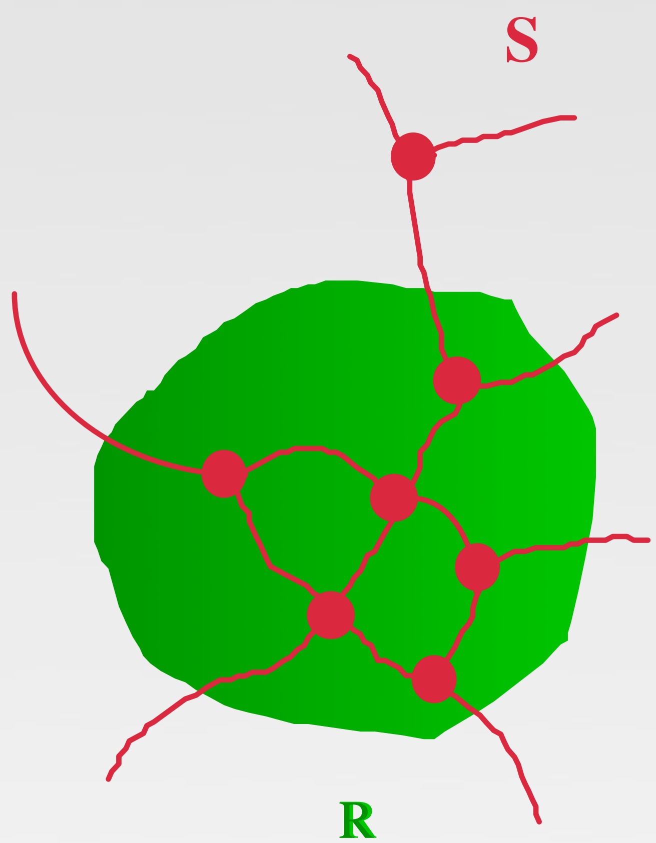


- V(R) is a well-defined self-adjoint operator

- The volume receives a contribution for each node of | S > inside R



Interpretation of the spin network states | S >

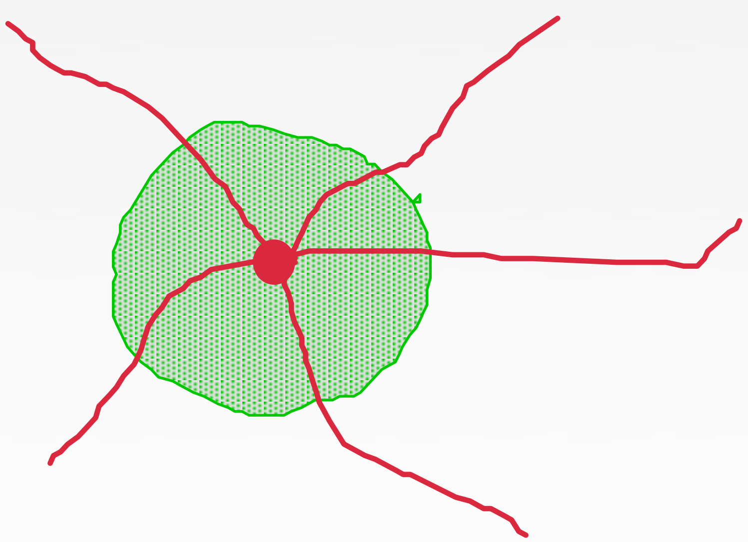


Volume: V(R): function of the gravitational field

$$V(R) = \int_R \sqrt{g} = \int_R \sqrt{EEE} \quad \rightarrow \quad V(R) \text{ operator}$$

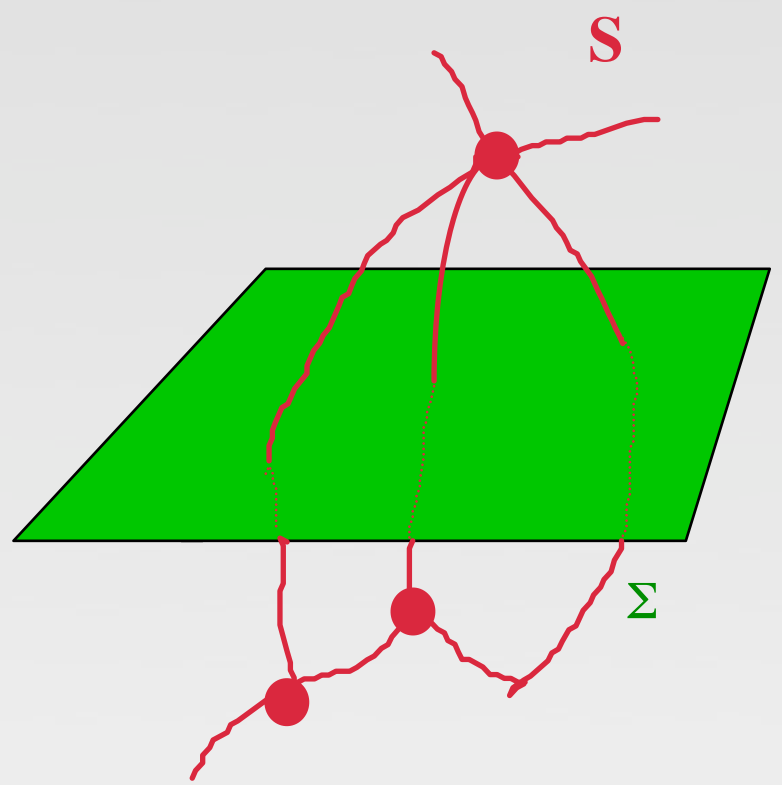
- V(R) is a well-defined self-adjoint operator

- The volume receives a contribution for each node of | S > inside R



Node = "Chunk of space"
with quantized volume

Area: $A(\Sigma)$



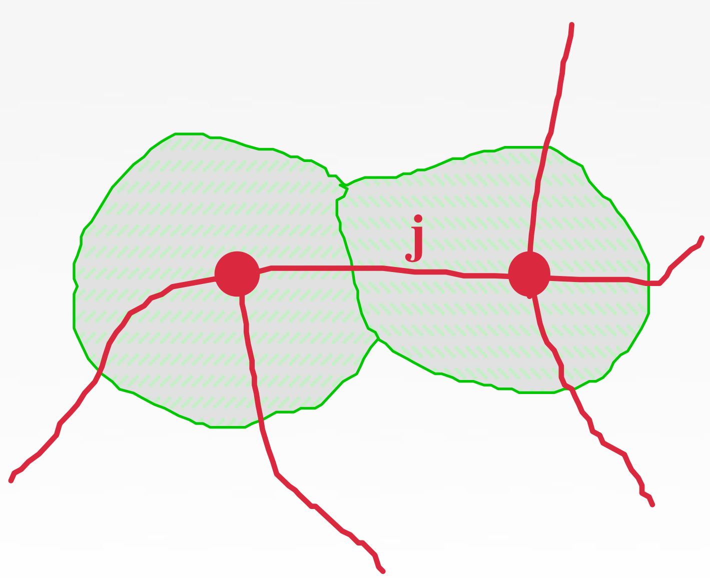
$A(\Sigma) = \int_\Sigma \sqrt{g} = \int_\Sigma \sqrt{EE}$ ➞ $A(\Sigma)$ operator

- Theorem: $A(\Sigma)$ well-defined selfadjoint operator in $\mathcal{H}$.
- Area gets a contribution for each link of $|S>$ that intersects S

Area eigenvalues: $$A = 8\pi\,\hbar G\,\gamma \sum_i \sqrt{j_i(j_i+1)}$$

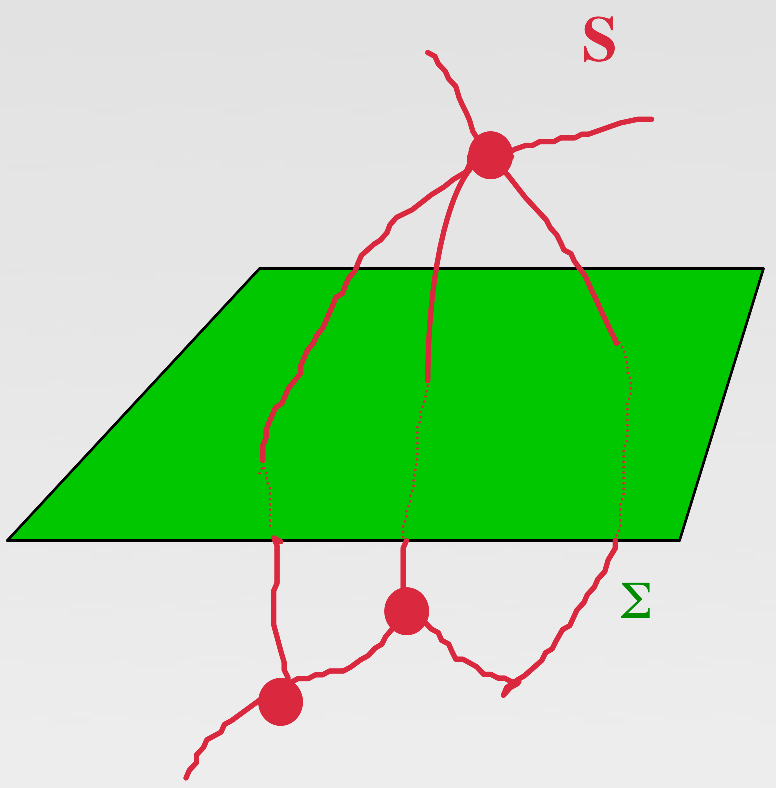
(*Smolin CR 95*)

(γ = Immirzi parameter)



Area: A(Σ)

$A(\Sigma) = \int_\Sigma \sqrt{g} = \int_\Sigma \sqrt{EE} \quad \rightarrow \quad A(\Sigma)$ operator

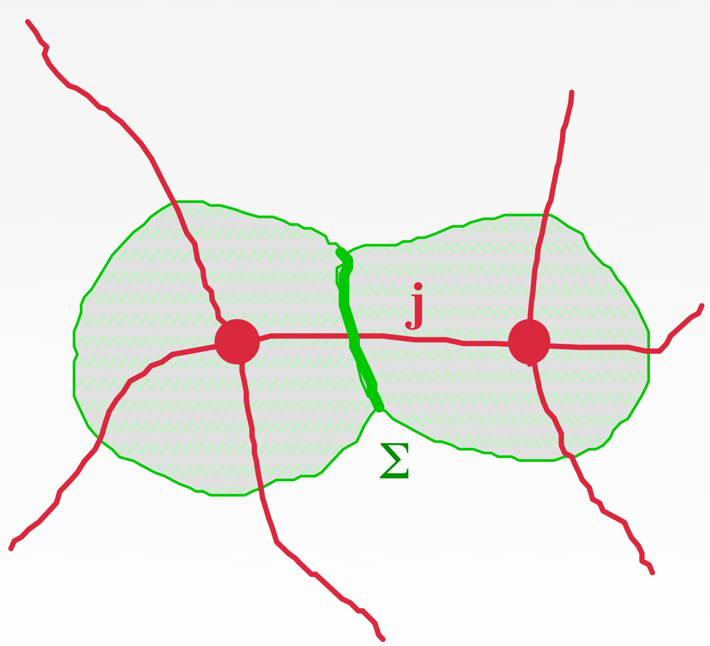


- Theorem: A(Σ) well-defined selfadjoint operator in H .
- Area gets a contribution for each link of | S > that intersects S

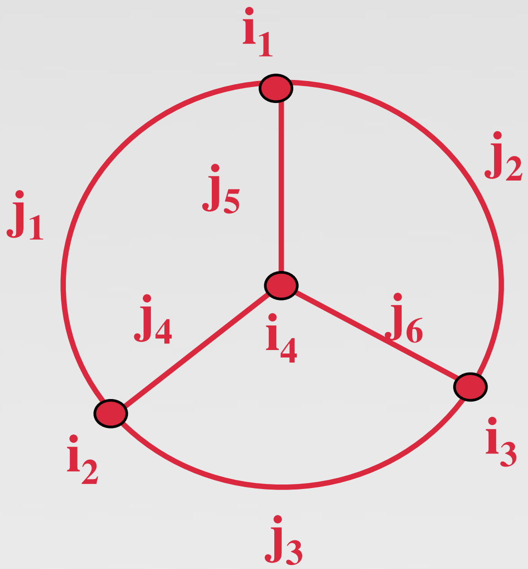
Area eigenvalues: $$A = 8\pi\,\hbar G\,\gamma \sum_i \sqrt{j_i(j_i+1)}$$

(*Smolin CR 95*)

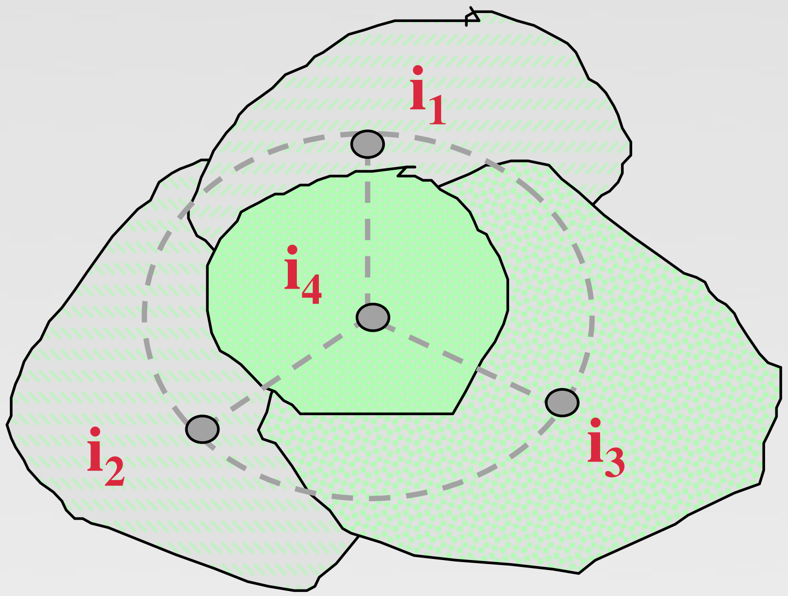
(γ = Immirzi parameter)



Link = "Quantum of surface"
with quantized area



=



$$| \, s > \; = \; | \, \gamma \, , \, j_l \, , \, i_n >$$

s-knot state

connectivity between the elementary
quantum chunks of space

quantum numbers
of area

quantum numbers
of volume

Spin network states represent discrete quantum excitations of spacetime

- Spin networks are not excitations in space: they are excitations of space.

  ➞ Background independent QFT

- **Discrete structure of space at the Planck scale** **-** in quantum sense

  Follows from:

  standard quantum theory (cfr granularity of oscillator’s energy)

  + standard general relativity (because space is a field)

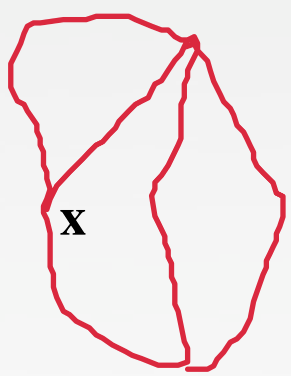
- **Area and volume are quantized:** eigenvalues known.

  Quantitative physical predictions from the theory

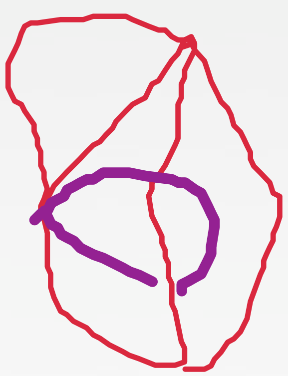
Dynamics

- **H Ψ = 0** (***Thiemann 98***)
- regularization ➡ in the limit in which the cut of removed
  H is <u>finite</u> on diff-invariant states
- **H = B EE,** regularization: $B = \lim \rightarrow_{\alpha\to 0} U_\alpha(A)$

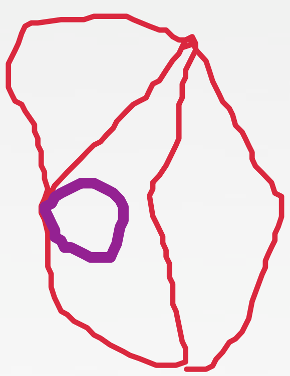
H_α (x)



~

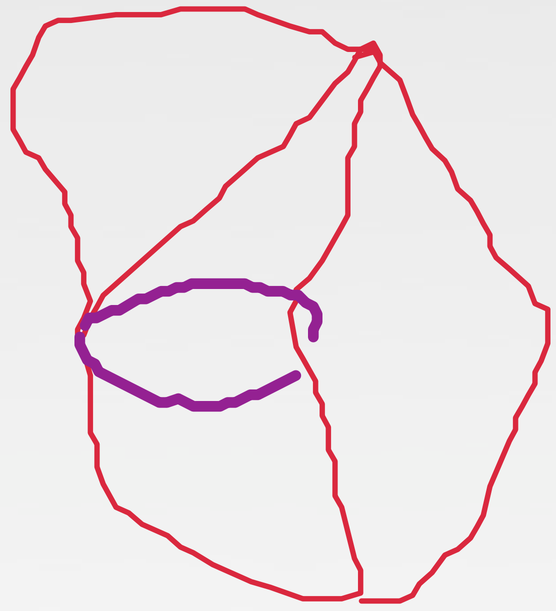


$\alpha \rightarrow 0$

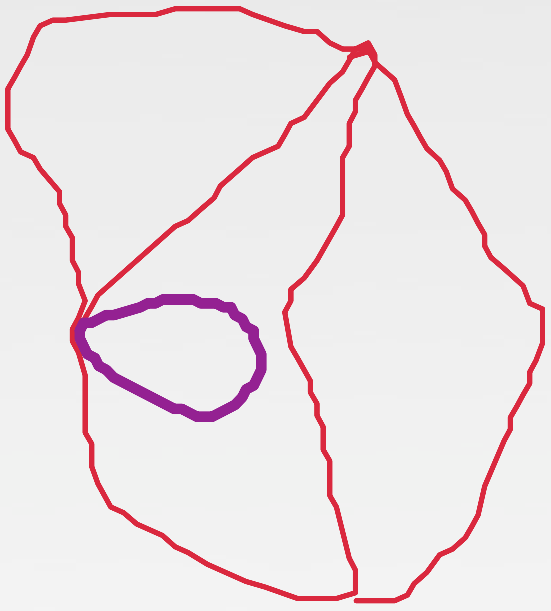


$\alpha \to 0$

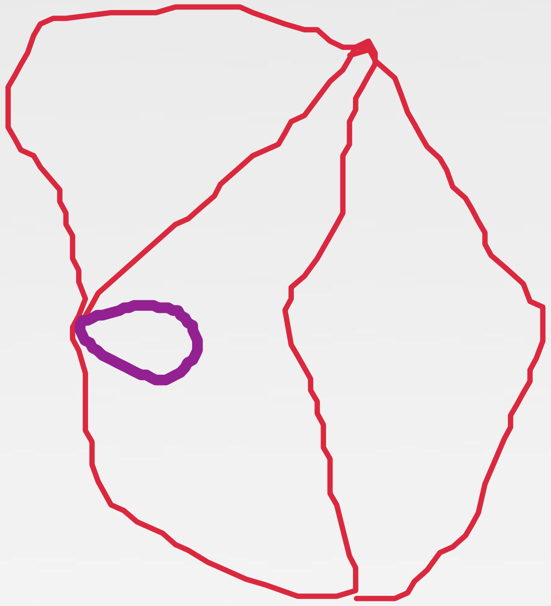
$\alpha \rightarrow 0$



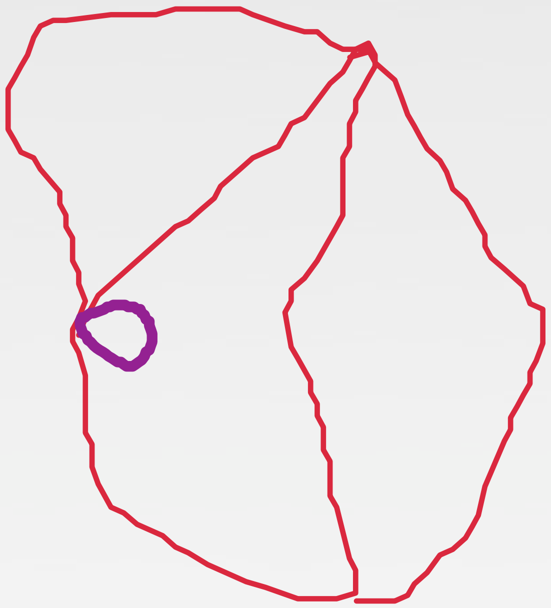
$\alpha \to 0$



$\alpha \to 0$



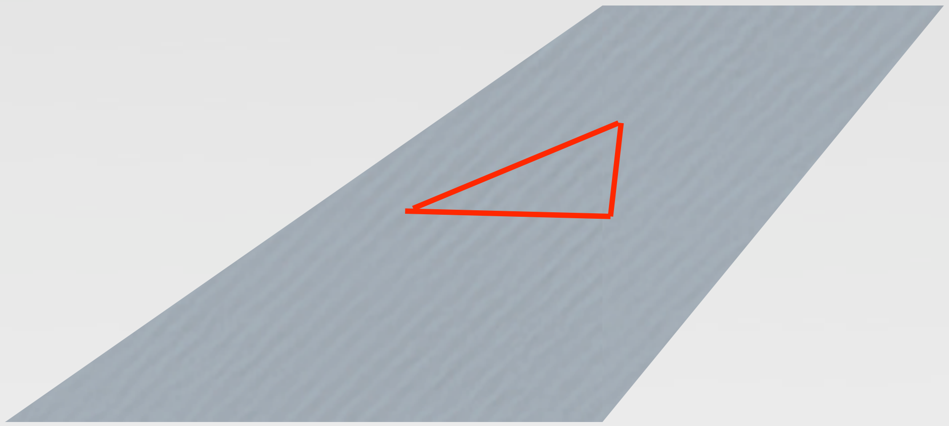
$\alpha \to 0$



The limit $\alpha \to 0$ is trivial because
there is no short distance structure at all in the theory !

- The theory is naturally ultraviolet finite

Strings: closed lines on space, forming matter



Loops: closed lines forming space, and matter

`quantum threads weaving a classical geometry' (*Ashtekar Smolin CR 89*)

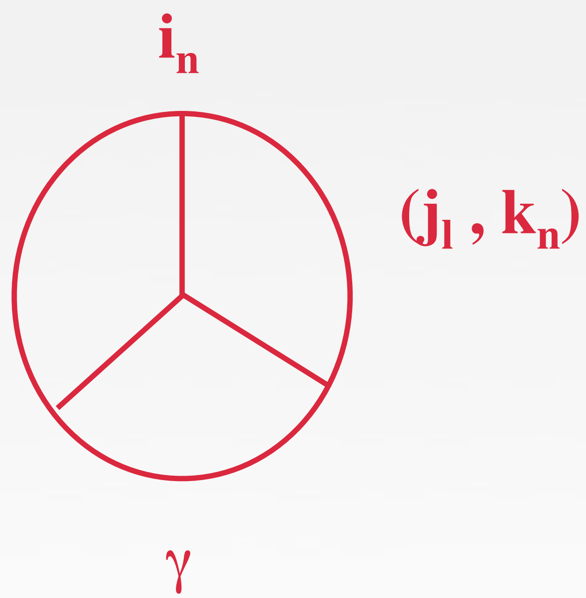
Developments

Developments

Matter:

- YM, fermions, scalars
- Same techniques: The gravitational field is *not* special
- UV finiteness remains (*Thieman 99*)
- YM and fermions on spin networks = on a Planck scale lattice !

  Notice: no lattice spacing to zero !



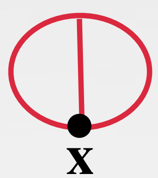
quantum numbers
of matter

$$|\, s > \;=\; |\, \gamma , j_l , i_n , k_l >$$

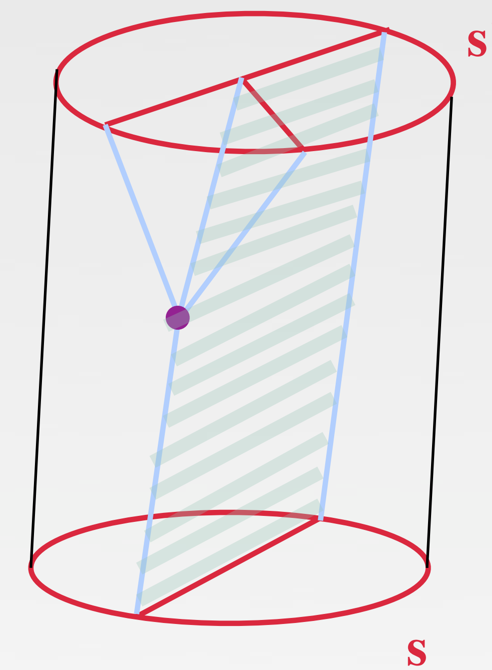
Developments

Covariant formulation (spinfoam formalism) :

- Compute transition amplitudes perturbatively as a sum over: evolutions of spin networks: **spinfoams**.

H(x) 

= C 

< s' | s >$_{Ph}$ = 

+ ...

- Feynman sum is over spinfoams: colored two-complexes (*Reisenberger CR 97, Baez 98*)
- Several models (*Barrett Crane 99, Oriti 01, Livine 03*)
- Finiteness theorems (*Perez 01*)
- Derivation from auxiliary field theory on a group (*DePietri Freidel Krasov CR 00*)
- Matter couplings (*Freidel Livine 04*)
- Clean relation canonical-covariant in 3d (*Perez Noui 02*)
- 3d - noncommutative QFT as “classical” limit (*Girelli Livine 04*)

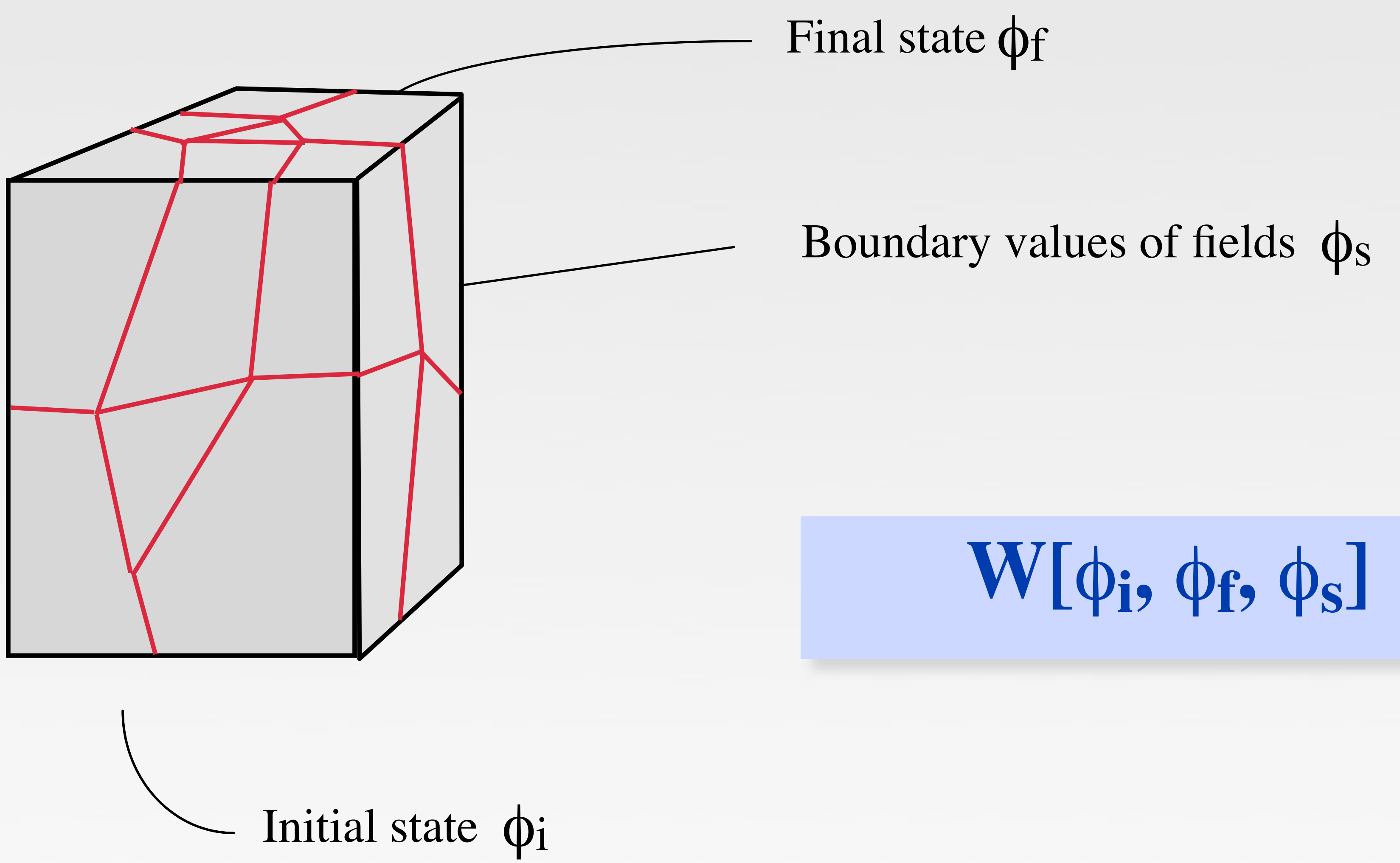
Developments

Mathematical:

- Diffeomorphism invariant measure (*Ashtekar Lewandowski 93*)
- C* algebraic techniques (*Ashtekar Isham 92*)
- Category (*Baez 99, Crane 97*)
- “Quantum geometry”
- Uniqueness of the representation (*Fleishhack 04, Lewandowski Okolow Sahlmann and Thiemann 05*)

Developments

Transition amplitudes: amplitude for boundary values of fields

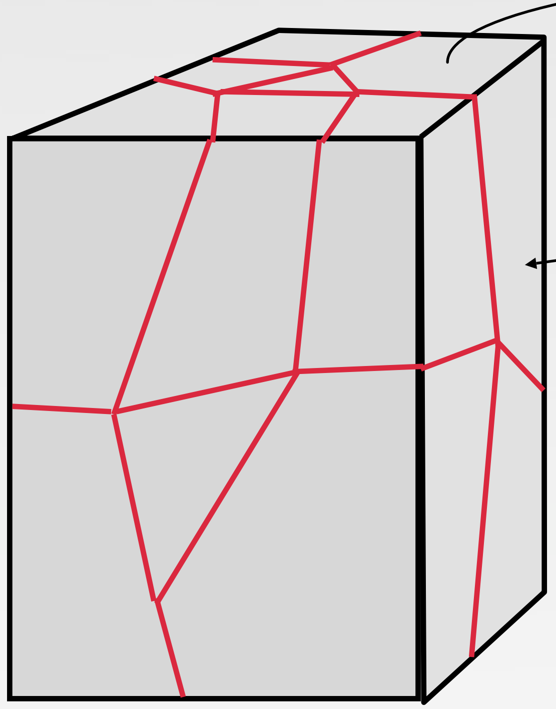


$$\mathbf{W[\phi_i, \phi_f, \phi_s]}$$

Developments

Transition amplitudes: amplitude for boundary values of fields

Final state ϕ_f



Boundary values of fields ϕ_s

<u>Including elapsed time !!</u>

Initial state ϕ_i

$$\mathbf{W[\phi_i, \phi_f, \phi_s]}$$

Elapsed time

Boundary values of the gravitational field = geometry of box surface

= distance and time separation of measurements

➡ n-point functions (as function of boundary geometry).

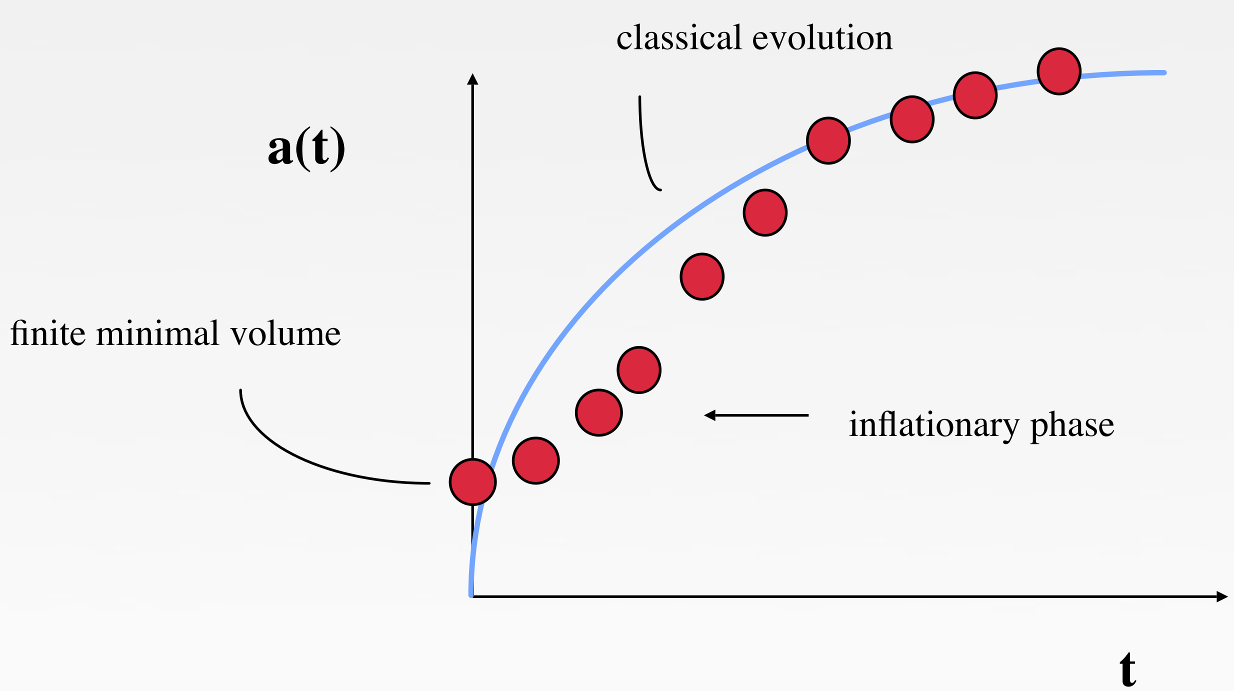
Graviton propagator *(Modesto Speziale CR 05)*

Applications

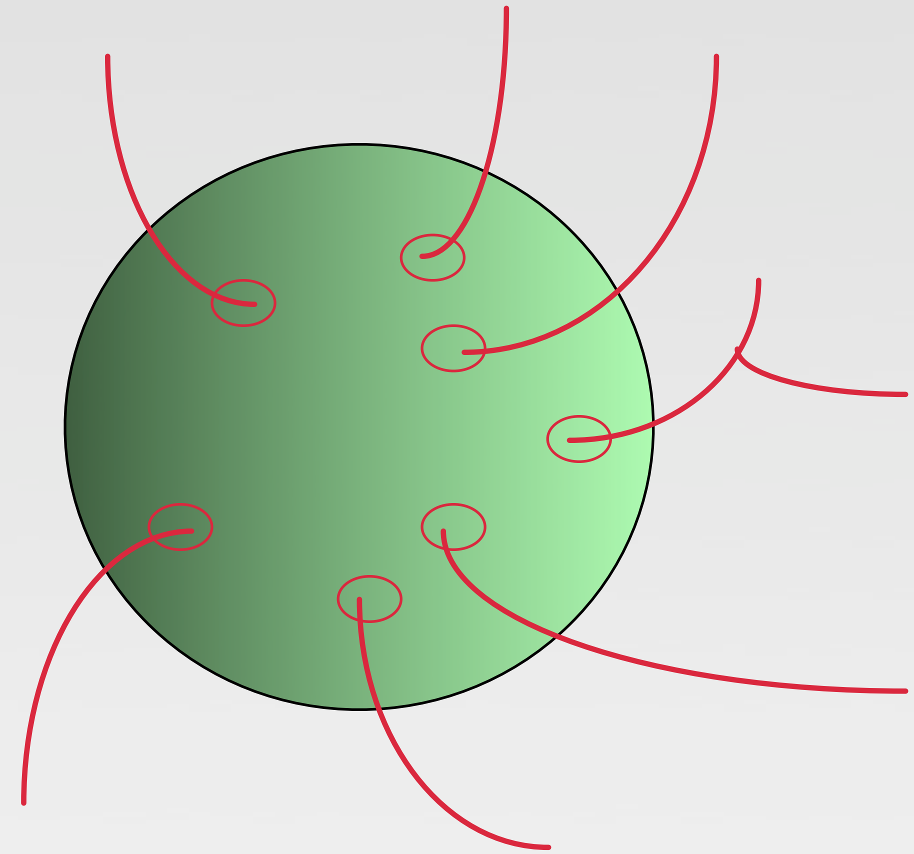
Applications

Loop cosmology:

- Discrete cosmological time
- Big Bang singularity removed (*Bojowald 01, Ashtekar 2004*)
- Inflationary behavior at small a(t) (*Bojowald 02*)
- Computable detectable effects on primordial power spectrum? (*Hofmann Winkler 05*)

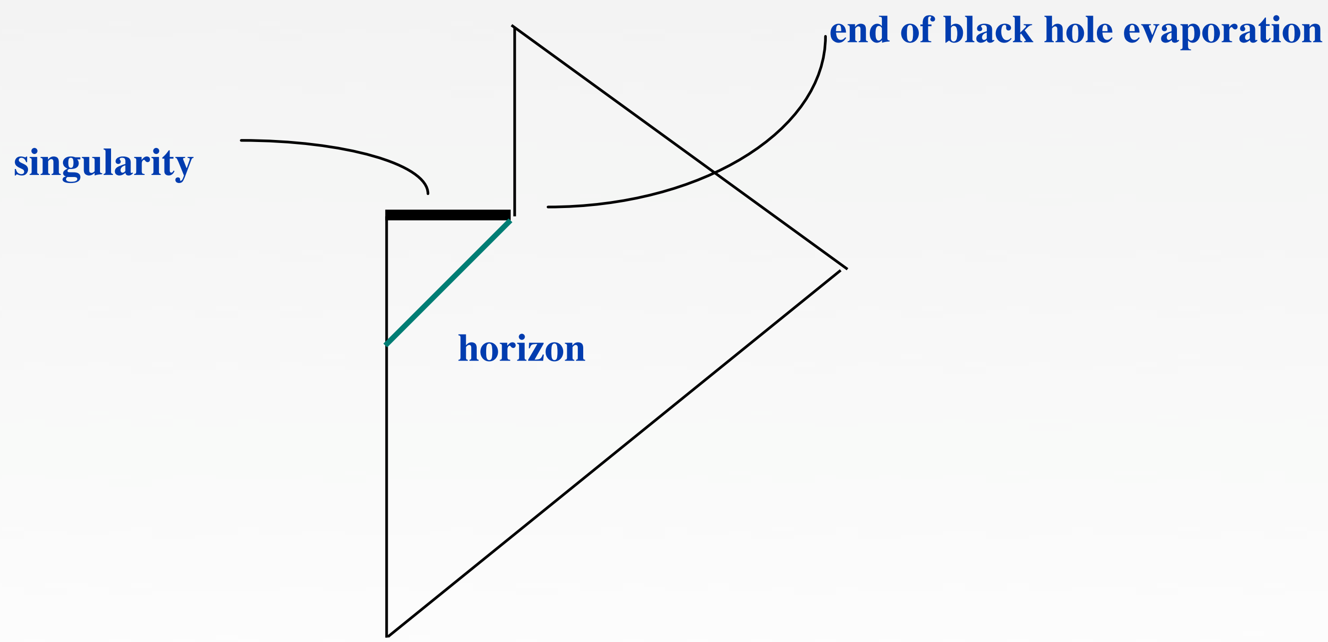


Applications

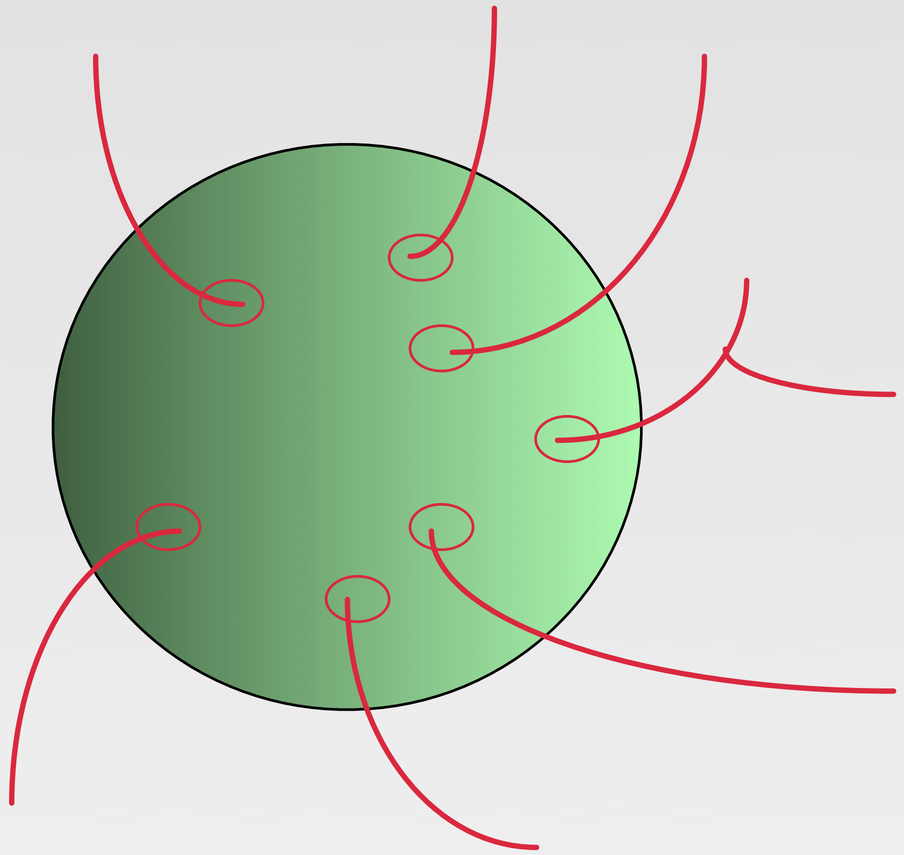


Black holes:

- Entropy finite, proportional to the area (*CR 96, Ashtekar Baez Corichi Krasnov 98*)
- Physical black holes (unlike strings)
- S = A/4   if a dimensionless free parameter (Immirzi parameter) is fixed
- R = 0 singularity under control (*Modesto 05*):

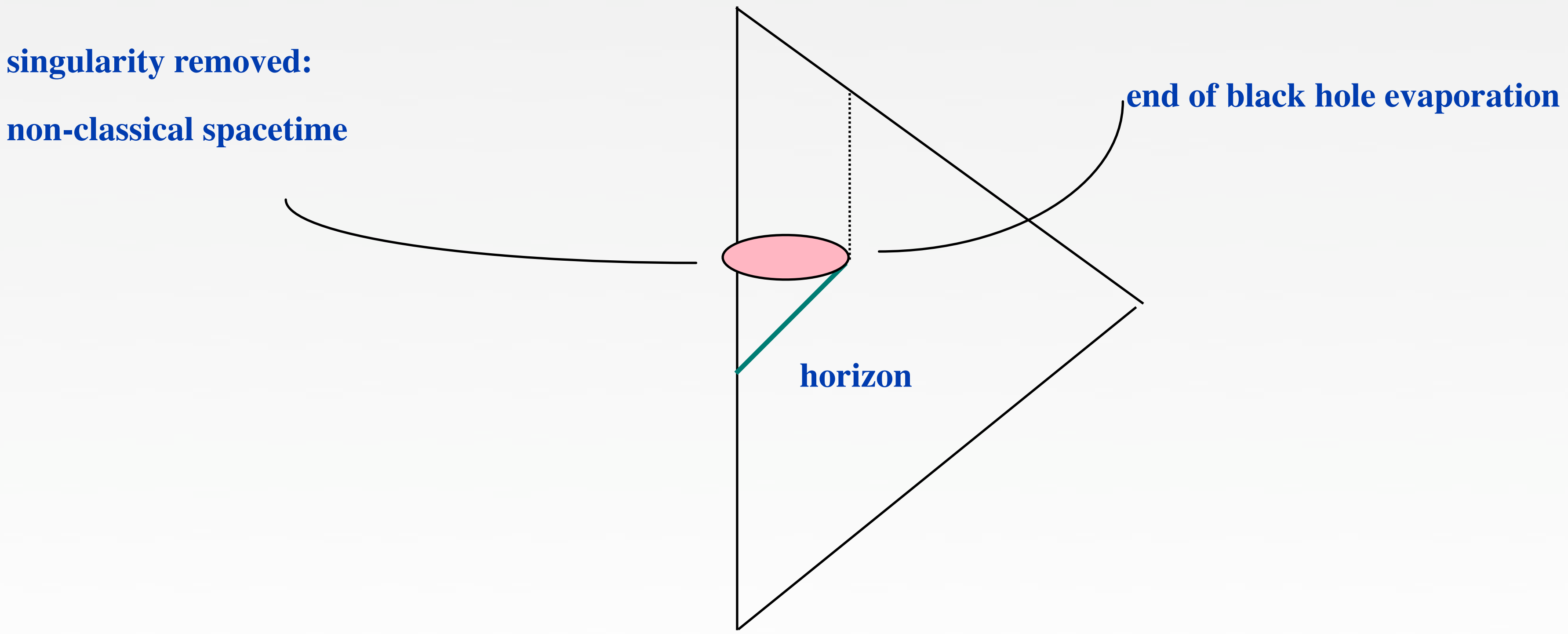


Applications



Black holes:

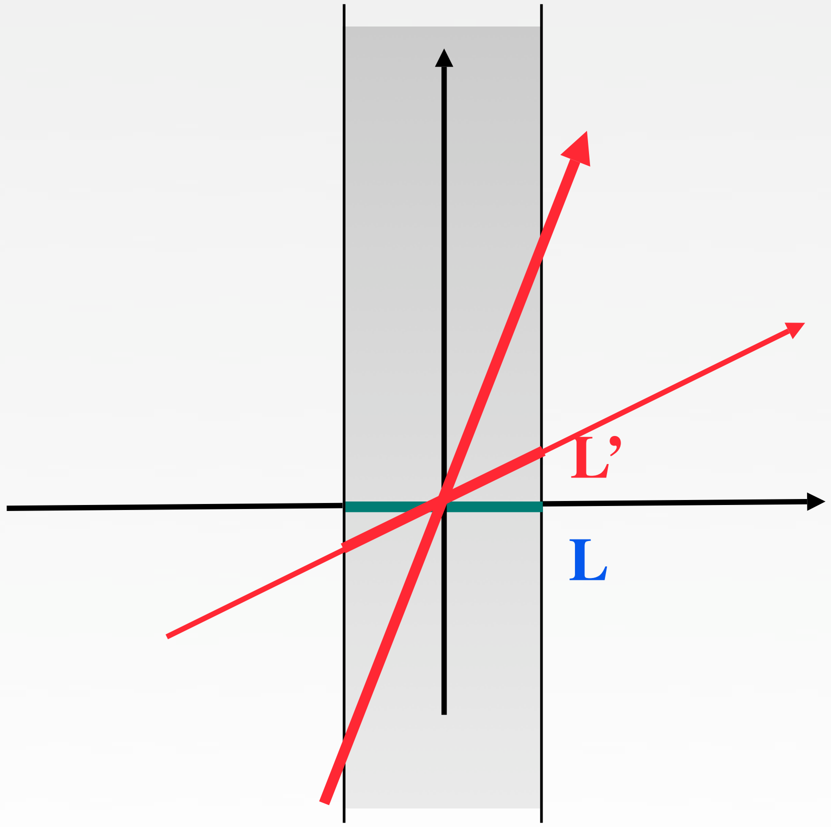
- Entropy finite, proportional to the area (*CR 96, Ashtekar Baez Corichi Kranov 98*)
- Physical black holes (unlike strings)
- S = A/4 if a dimensionless free parameter (Immirzi parameter) is fixed
- R = 0 singularity under control (*Modesto 05*):



Applications

Phenomenology:

- Observables effects of small scale discreteness? (Cosmic rays, threshold effects...)
- Breaking of Lorentz invariance? Not necessarily ! (*Speziale 03*)



$$[\, L \,,\, L' \,] \neq 0$$

Summary

- Loop quantum gravity is a well-defined background independent quantization of GR in 4d.
- Merges “in depth” QFT with GR.
- Predicts discrete structure of space at Planck scale.
- UV finite.
- Yields detailed quantitative physical predictions (spectra).
- Includes matter.
- Numerous tentative applications in cosmology, black hole physics, astrophysics.

Is the theory physically correct ?

We do not know

Only substantial experimental support can give us confidence in this (or any other) quantum theory of gravity

But

To a large extent, a well-defined, background-independent, UV-finite quantum theory of general relativity, (in 4 dimensions and without supersymmetry), exists, and deserves to be better studied.

Summary

- Loop quantum gravity is a well-defined background independent quantization of GR in 4d.
- Merges "in depth" QFT with GR.
- Predicts discrete structure of space at Planck scale.
- UV finite.
- Yields detailed quantitative physical predictions (spectra).
- Includes matter.
- Numerous tentative applications in cosmology, black hole physics, astrophysics.