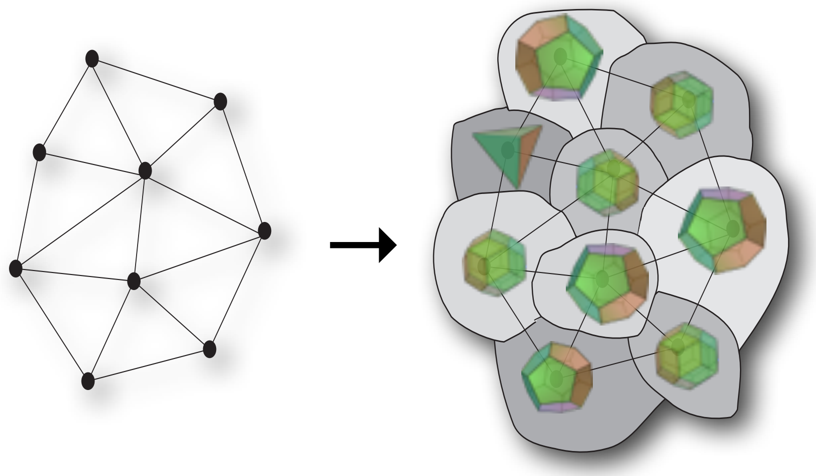
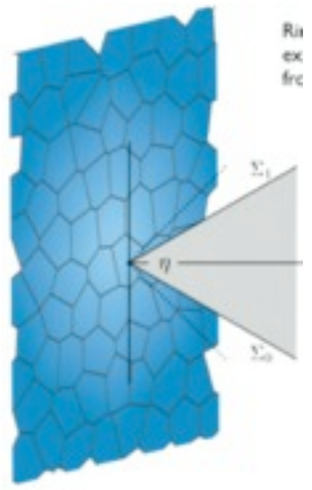
Covariant Loop Quantum Gravity



State of the art

Recent developments

Open problems



Carlo Rovelli

The problem

existence,
not uniqueness

no uncontrollable
infinities

Is there a consistent quantum theory

whose classical limit is general relativity,

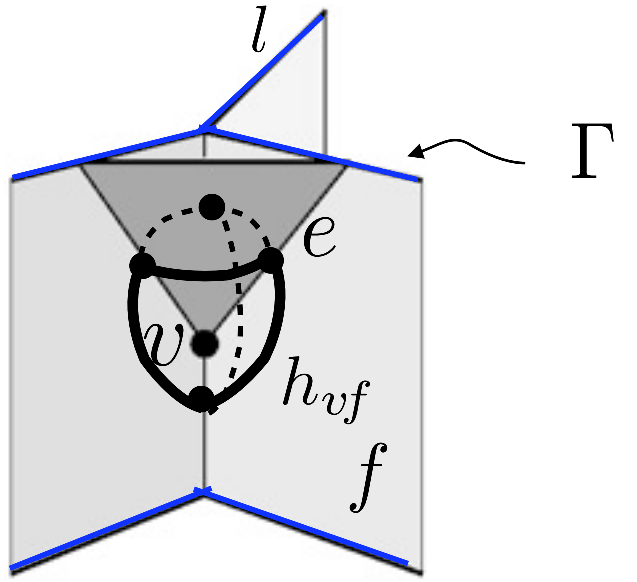
in 4 lorentzian dimensions, with its standard matter couplings?

unification
not addressed
(cfr QCD)

Covariant loop gravity dynamics

$\Lambda = 0$

Vertex amplitude
$$A(h_f) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f + 1)\ Tr_{j_f}[h_f Y_\gamma^\dagger g_e g_{e'}^{-1} Y_\gamma]$$



2-complex $\mathcal{C}$
(vertices, edges, faces)

Simplicity map
$$\begin{array}{rcl} Y_\gamma\ :\ \mathcal{H}_j & \to & \mathcal{H}_{j,\gamma j} \\ |j;m\rangle & \mapsto & |j,\gamma(j+1);j,m\rangle \end{array}$$

Transition amplitudes
$$W_{\mathcal{C}}(h_l) = N_{\mathcal{C}} \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$
$$h_f = \prod_v h_{vf}$$

$\Lambda > 0$

- SL(2,C) → SL(2,C)_q
- A ~ SL(2,C) Chern-Simons expectation value of the boundary graph of the vertex
- A = Vassiliev-Kontsevich invariant of the vertex graph.

Covariant loop gravity dynamics: other version

Explicit Y

$$W_{\mathcal{C}}(h_l) = \int_{(SL2C)^{2(E-L)-V}} dg'_{ve} \int_{(SU2)^{\mathcal{V}-L}} dh_{ef} \sum_{j_f}$$

$$N^{-1}_{\{j_f\}} \prod_f d_{j_f} \; \chi^{\gamma j_f, j_f}\Big(\prod_{e\in\partial f} g_{ef}^{\epsilon_{lf}}\Big) \prod_{e\in\partial f} \chi^{j_f}(h_{ef})$$

Coherent states

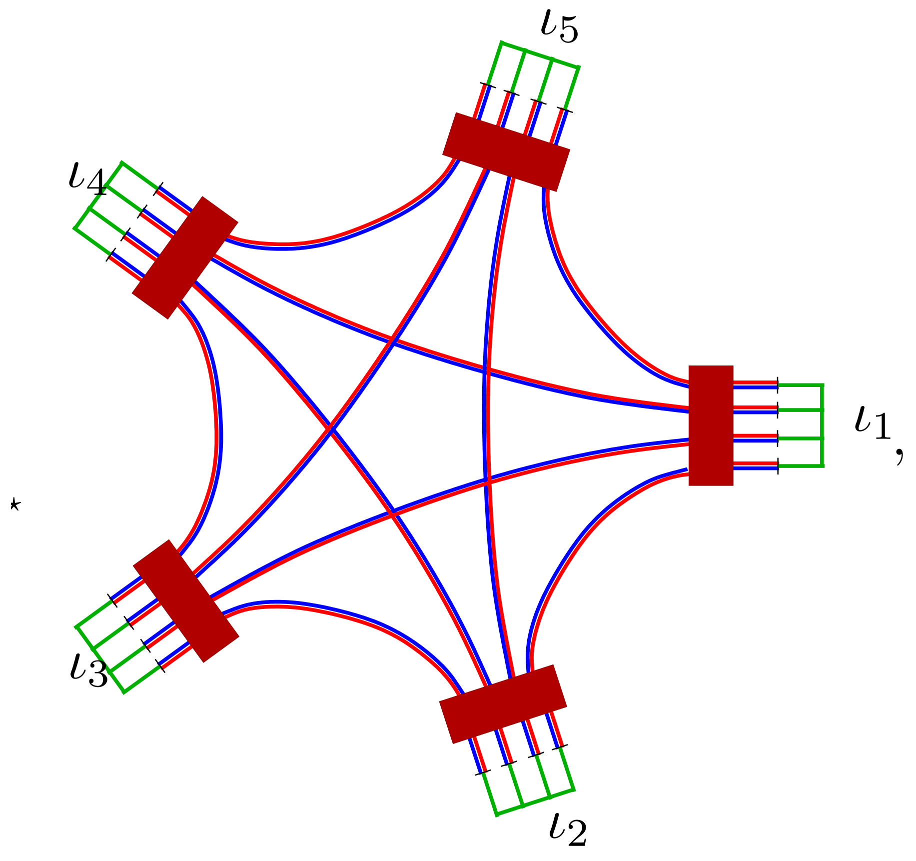
$$W_{\mathcal{C}}(h_j) = \sum_{j_f} \int d\tilde{g}_{ve} \int dn_{ef} \prod_f d_{j_f} \; \prod_v \langle j_f, -\vec{n}_{ef} | g_e g_{e'}^{-1} | j_f, \vec{n}_{e'f} \rangle_{\;\gamma(j_f+1), j_f}$$

Spinor integrals

n-j symbols

Alejandro Perez notation:

$$W_{\mathcal{C}}(h_l) = \sum_{j_f v_e} \prod_f (2j_f + 1) \; \prod_v$$



...

Covariant loop gravity dynamics: the Polish version (See Lewandowski talk)

Partition function:

$$W_{\mathcal{C}} = \sum_{j_f} \mu(j_f)\ Tr_{\mathcal{C}}\left[\prod_e P_e\right]$$

$$\mu(j_f) = \prod_f (2j_f + 1)$$

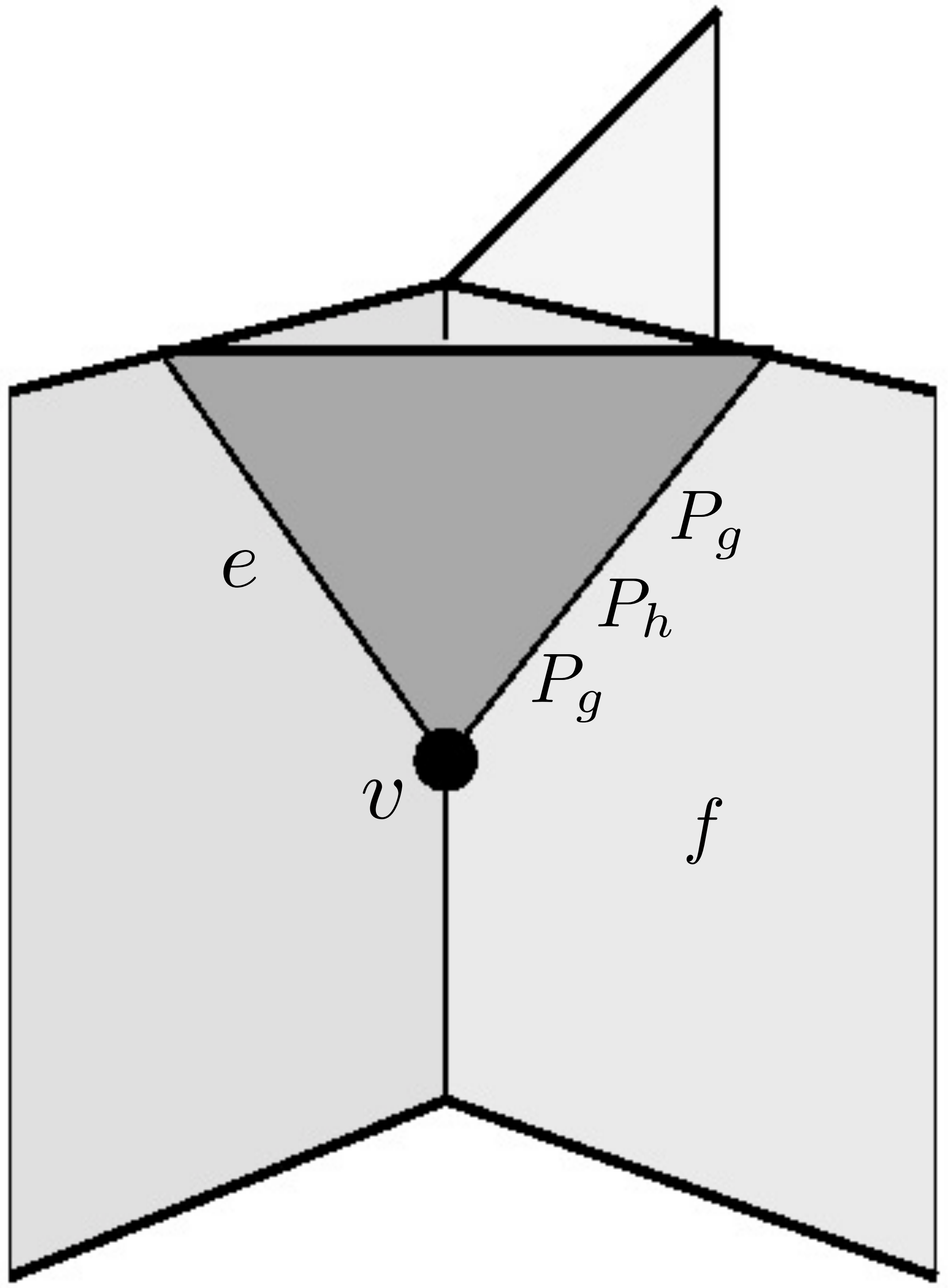
on each face $H_f = (k, \nu) = (j_f, \gamma(j_f + 1))$

on each edge $H_e = \otimes_{f \in e} H_f$

P operators: $P_e = P_g P_h P_g$

Projector on SL(2,C) invariant part

Projector on minimal weight



The four major results

1. The boundary states represent classical geometries.

   (Canonical LQG 1990', Penrose spin-geometry theorem 1971).

2. Boundary geometry operators have discrete spectra.

   (Canonical LQG main results, 1990').

3. The classical limit of the vertex amplitude converges (appropriately) to the Regge Hamilton function (with cosmological constant).

   (Barrett *et al*, Conrady-Freidel, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2012).

4. The amplitudes with positive cosmological constant are UV and IR finite: $W^q_{\mathcal{C}} < \infty$

   (Han, Fairbairn, Meusburger, 2011).

Other important recent results

1. Fermions and Yang-Mills fields can be coupled to LQG

   (Bianchi, Han, Magliaro, Perini, Wieland, CR).

2. Amplitudes are locally Lorentz invariance

   (Speziale, CR).

3. The amplitude leads immediately to the Bekenstein-Hawking entropy (no fixing of γ) $S = \frac{A}{4}$

   (Frodden-Gosh-Perez, Bianchi, 2011-2012).

There aren't "many models".

There is only **one** Lorentzian theory in 4d that works.

Several possible variants in the definition:

- Which class of two-complexes? (Bahr, Puchta)
- Pre-factors (see later)
- ... $N_{\mathcal{C}}$

General relativity

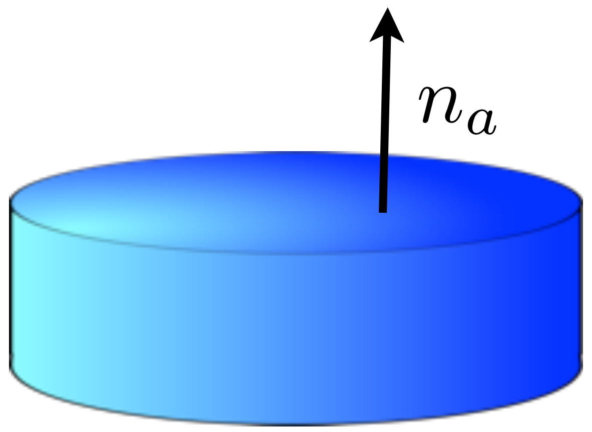
Einstein Hilbert action $S[g] = \int \sqrt{-\det g}\; R[g]$

Tetrads $g_{ab} \to e^i_a$ $\qquad g_{ab} = e^i_a\, e^i_b$ $\qquad e = e_a dx^a \in R^{(1,3)}$

Spin connection $SL(2,C)$ $\qquad \omega = \omega_a dx^a \in sl(2,C)$ $\qquad \omega(e): \quad de + \omega \wedge e = 0$

GR action $S[e,\omega] = \int e \wedge e \wedge F^*[\omega]$

GR Holst action $S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma} \int e \wedge e \wedge F[\omega]$



Canonical variables $\omega, \qquad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$

On the boundary $n_i = e^a_i n_a$ $\qquad n_i e^i = 0 \quad SL(2,C) \to SU(2)$

$B \to (K = nB, L = nB^*)$

$\vec{K} + \gamma \vec{L} = 0$ $\qquad$ “Linear simplicity constraint”

Main tool: SL(2,C) ***unitary*** irreducible representations (why so little used in physics?)

SU(2) unitary representations: $2j \in Z$ $\quad |j;m\rangle \in \mathcal{H}_j$ $\quad L^2 = j(j+1)$

SL(2,C) unitary representations: $2k \in N, \quad p \in R^+$ $\quad |(k,p);j,m\rangle \in \mathcal{H}_{k,p} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,p},$

SL(2,C) Casimir's: $K^2 - L^2 = p^2 - k^2 + 1$ $\quad \vec{K} \cdot \vec{L} = pk$

γ-simple representations: $p = \gamma(k+1)$

SU(2) → SL(2,C) map:

$$\begin{aligned} Y_\gamma : \quad \mathcal{H}_j \quad &\rightarrow \mathcal{H}_{j,\gamma(j+1)} \\ |j;m\rangle &\mapsto |(j,\gamma(j+1))\,;\; j,m\rangle \end{aligned}$$

Image of Y_γ :
minimal weight subspace
$j = k$

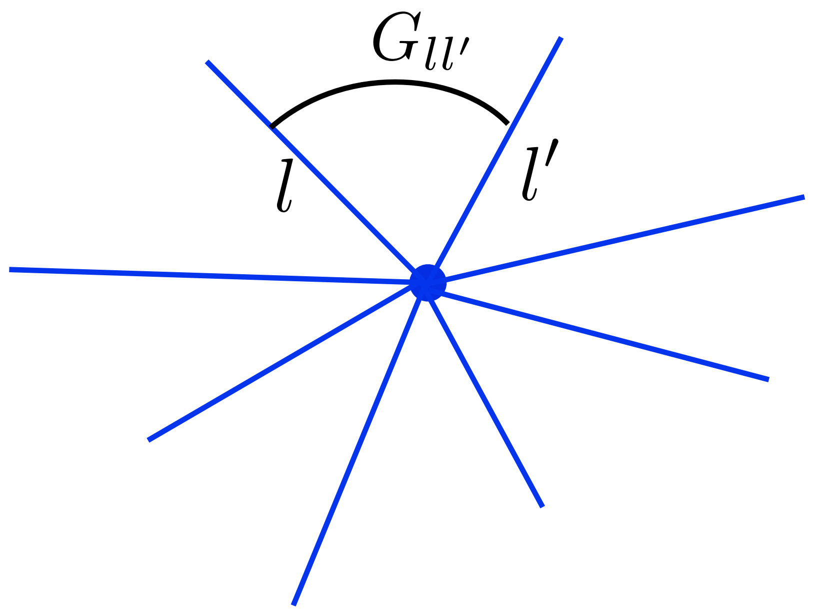
Main property: $\vec{K} + \gamma\vec{L} = 0$ weakly on the image of Y_γ

Boost generator Rotation generator

Boundary geometry : standard LQG kinematics

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$

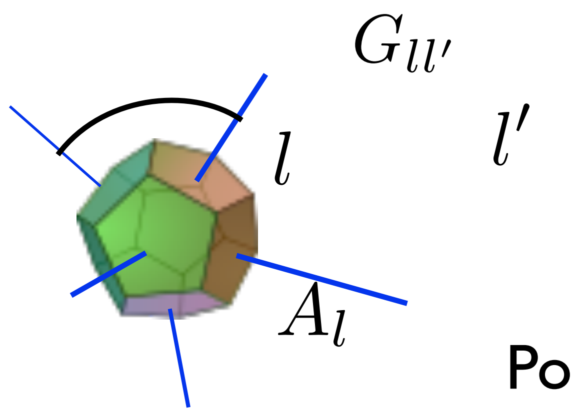
Operator: $\vec{L}_l = \{L^i_l\}, i = 1, 2, 3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$ $\sum_{l\in n} \vec{L}_l = 0$



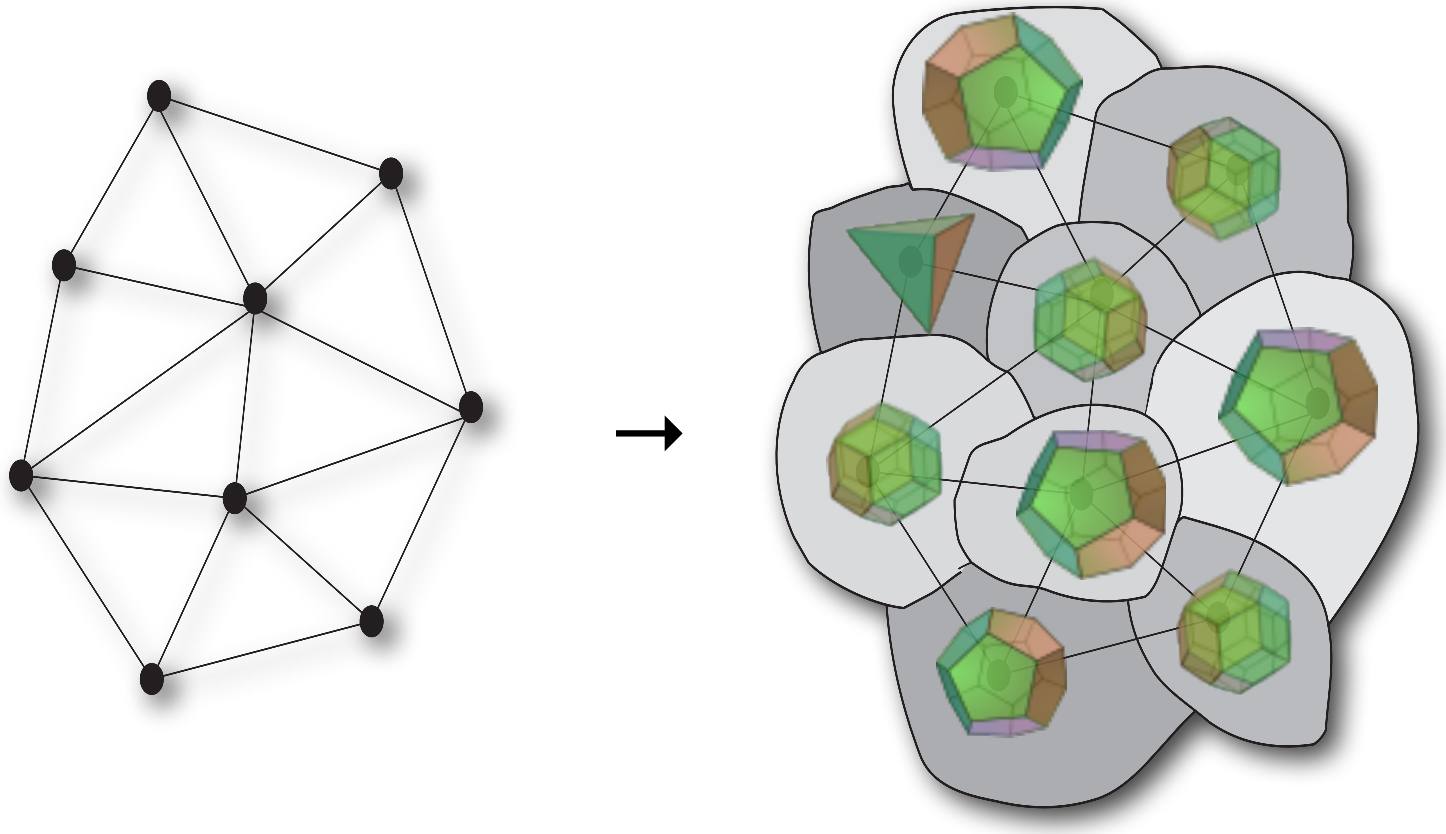
The gauge invariant operator: $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ satisfies $\sum_{l\in n} G_{ll'} = 0$

Is precisely the Penrose metric operator on the graph

It satisfies 1971 Penrose spin-geometry theorem, and 1897 Minkowski theorem: semiclassical states have a geometrical interpretation as polyhedra.



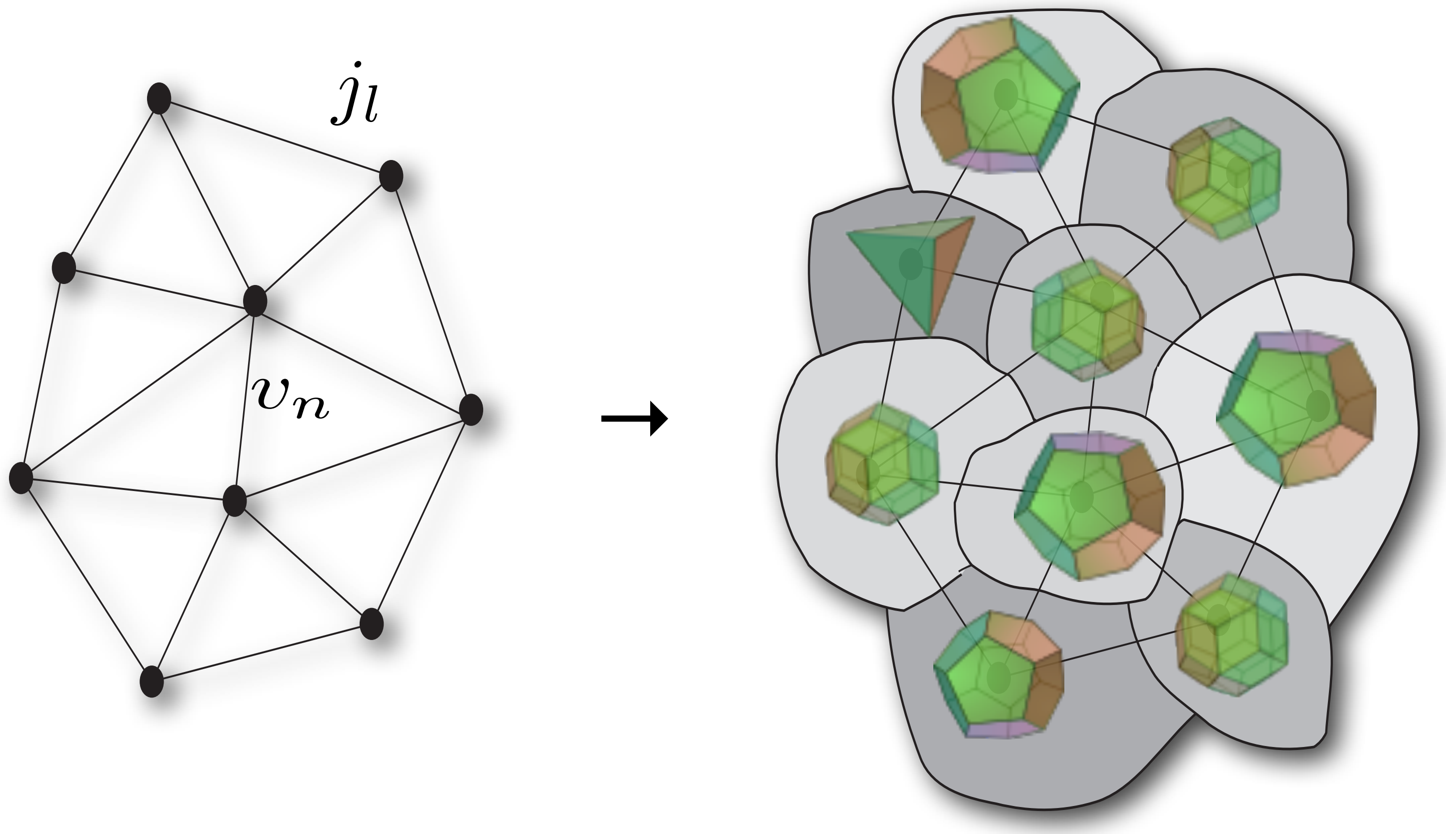
Polyhedron



Boundary geometry

area $$A_l^2 = G_{ll}$$ volume $$V_n^2 = \frac{2}{9}\, \vec{L}_{l_1} \cdot (\vec{L}_{l_2} \times \vec{L}_{l_3})$$

- Area and volume (A_l, V_n) form a complete set of commuting observables and have discrete spectra
- → basis (spin network basis) $|\Gamma, j_l, v_n\rangle$



Geometry is quantized:

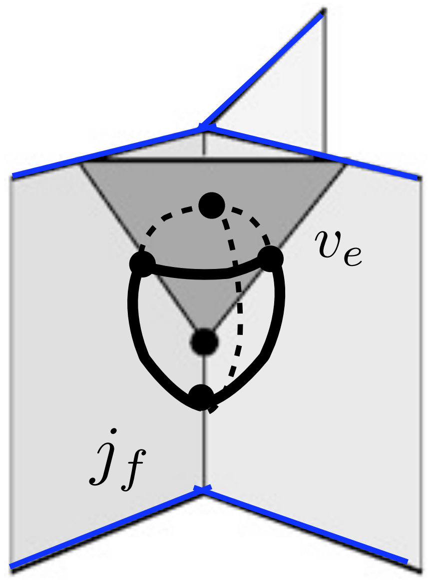
(i) eigenvalues are discrete
(ii) the operators do not commute

- Using this basis, the amplitude reads $$Z_{\mathcal{C}} = \sum_{j_f, v_e} N_{\{j_f\}} \prod_f (2j_f{+}1) \prod_v A_v(j_f, v_e)$$

→ Ultraviolet finiteness

Large distance limit

A “spinfoam”: a two-complex colored with spins on faces and intertwiners on edges.



$$Z_{\mathcal{C}} = \sum_{j_f, v_e} N_{\{j_f\}} \prod_f (2j_f+1) \prod_v A_v(j_f, v_e)$$

Theorem : For a 5-valent vertex
[Barrett, Pereira, Hellmann, Gomes, Dowdall, Fairbairn 2010]

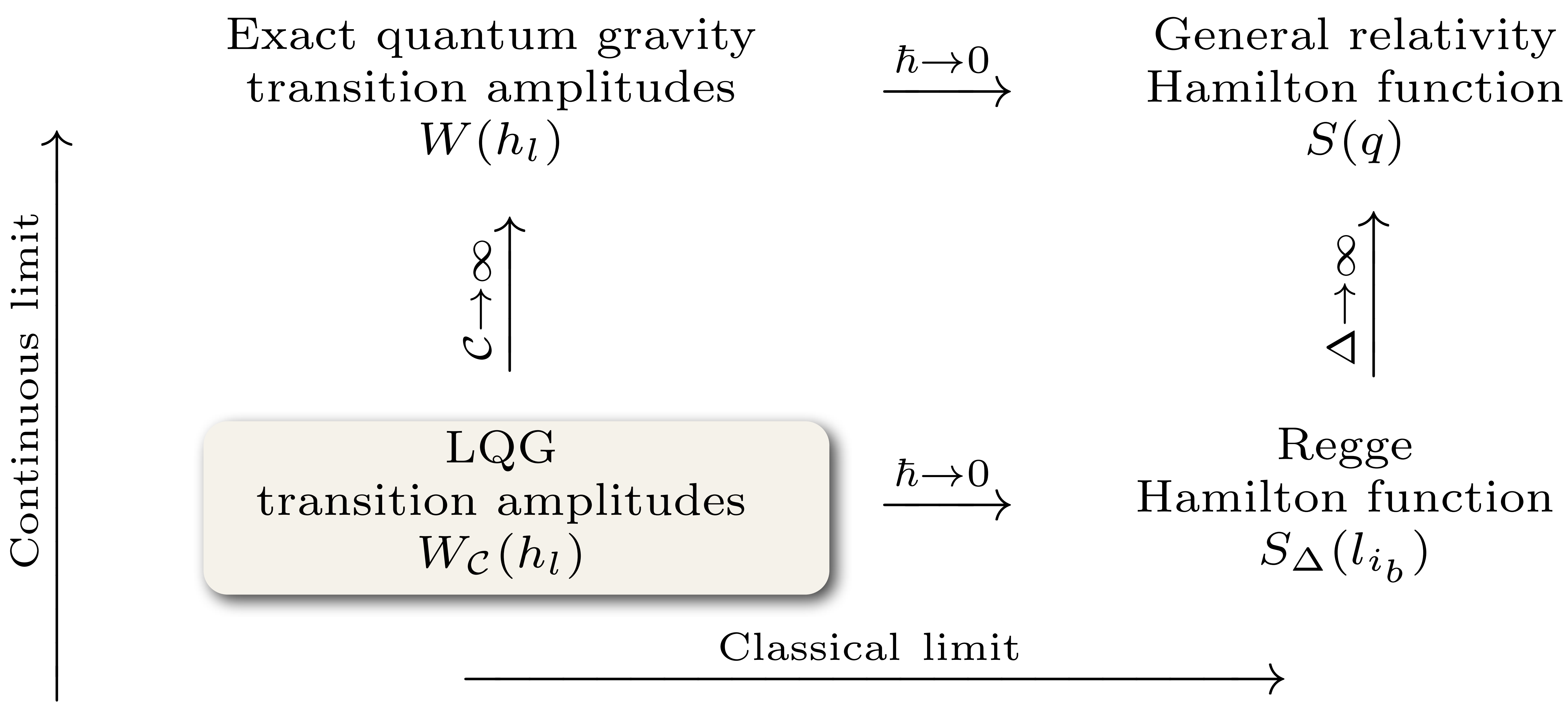
$$A(j_f, v_e) \underset{j \gg 1}{\sim} e^{iS_{\text{Regge}}} + e^{-iS_{\text{Regge}}}$$

$$W_{\mathcal{C}} \xrightarrow[j \gg 1]{} e^{iS_{\Delta}} \qquad Z_{\mathcal{C}} \xrightarrow[C \to \infty]{} \int Dg \; e^{iS[g]}$$

Theorem : For a 5-valent vertex
[Han 2012]

$$A^q(j_f, v_e) \underset{j \gg 1, q \sim 1}{\sim} e^{iS^{\Lambda}_{\text{Regge}}} + e^{-iS^{\Lambda}_{\text{Regge}}} \qquad q = e^{\Lambda \hbar G}$$

Structure of the theory



$$\begin{tikzcd}
\begin{array}{c}\text{Exact quantum gravity}\\ \text{transition amplitudes}\\ W(h_l)\end{array} \arrow[r, "\hbar\to 0"] & \begin{array}{c}\text{General relativity}\\ \text{Hamilton function}\\ S(q)\end{array} \\
\begin{array}{c}\text{LQG}\\ \text{transition amplitudes}\\ W_{\mathcal{C}}(h_l)\end{array} \arrow[r, "\hbar\to 0"] \arrow[u, "\mathcal{C}\to\infty"] & \begin{array}{c}\text{Regge}\\ \text{Hamilton function}\\ S_{\triangle}(l_{i_b})\end{array} \arrow[u, "\triangle\to\infty"]
\end{tikzcd}$$

Continuous limit

Classical limit

Regime of validity of the expansion: $L_{Planck} \ll L \ll \sqrt{\frac{1}{R}}$

Structure of the theory

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\end{tikzcd}$$

Continuous limit

Classical limit

Notice:

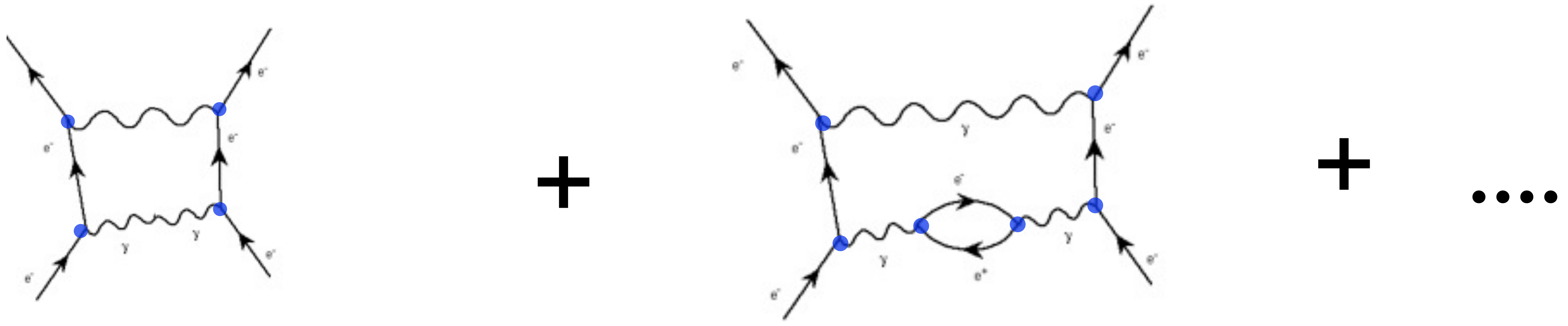
- No critical point
- No infinite renormalization
- Physical scale:
- Cfr: condensed matter away from critical points

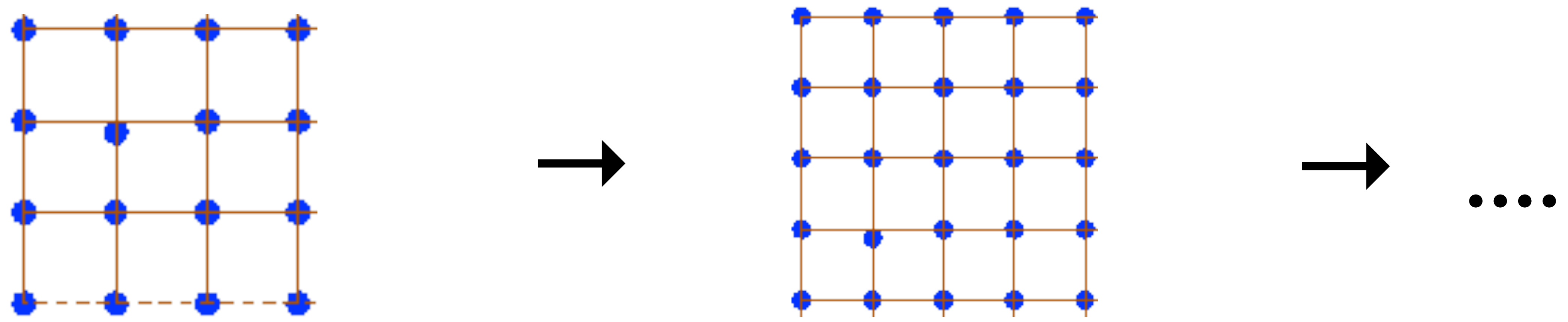
- QFT : critical phenomenon

- Quantum Gravity: non-critical phenomenon

Convergence between the QED and the QCD pictures

- *All* physical QFT are constructed via a truncation of the *d.o.f.* (cfr: QED: particles, QCD Lattice).
- All physical calculation are performed within a truncation.
- The limit in which all *d.o.f.* is then recovered is pretty different in QED qnd QCD:





What about Quantum Gravity?

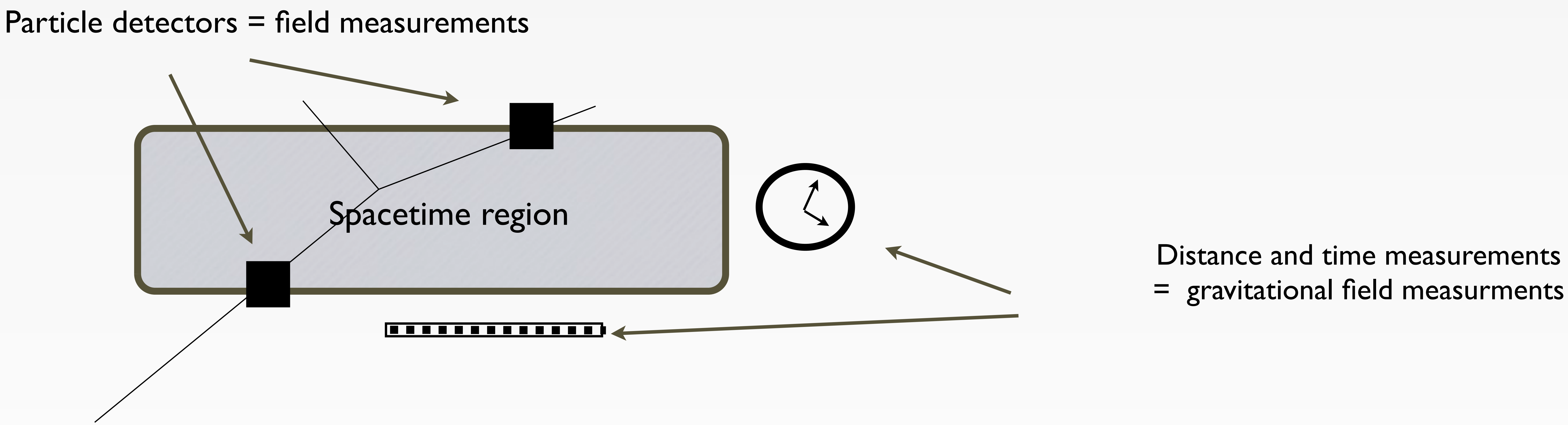
Lattice site = small region of space = excitations of the gravitational field = quanta of space = quanta of the field

Diff invariance !

Physics

- Scattering amplitudes
- Cosmology
- Black hole entropy

Boundary values of the gravitational field = geometry of box surface
= distance and time separation of measurements



Particle detectors = field measurements

Spacetime region

Distance and time measurements
= gravitational field measurments

In GR, distance and time measurements
are field measurements like the other ones:
they are part of the **boundary data** of the problem.

(i) Gravitational waves.

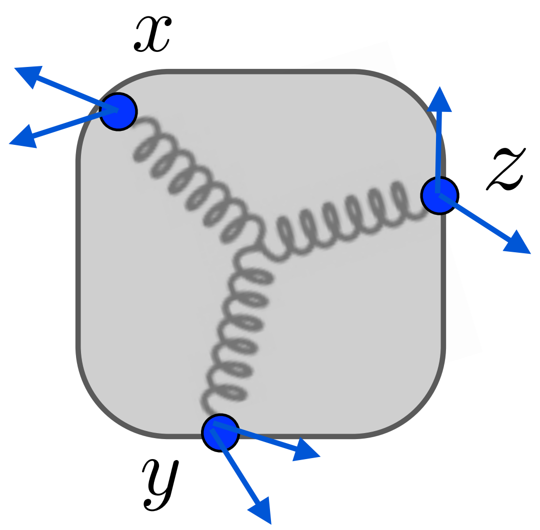
Result: The free graviton propagator is recovered

[Alesci, Bianchi Magliaro Perini 2009 , Ding 2011, Zhang 2011,]

(ii) Scattering.

Result: The Regge n-point function is recovered in the large j limit

[Zhang, CR 2011]



(iii) Loop quantum cosmology.

Result:

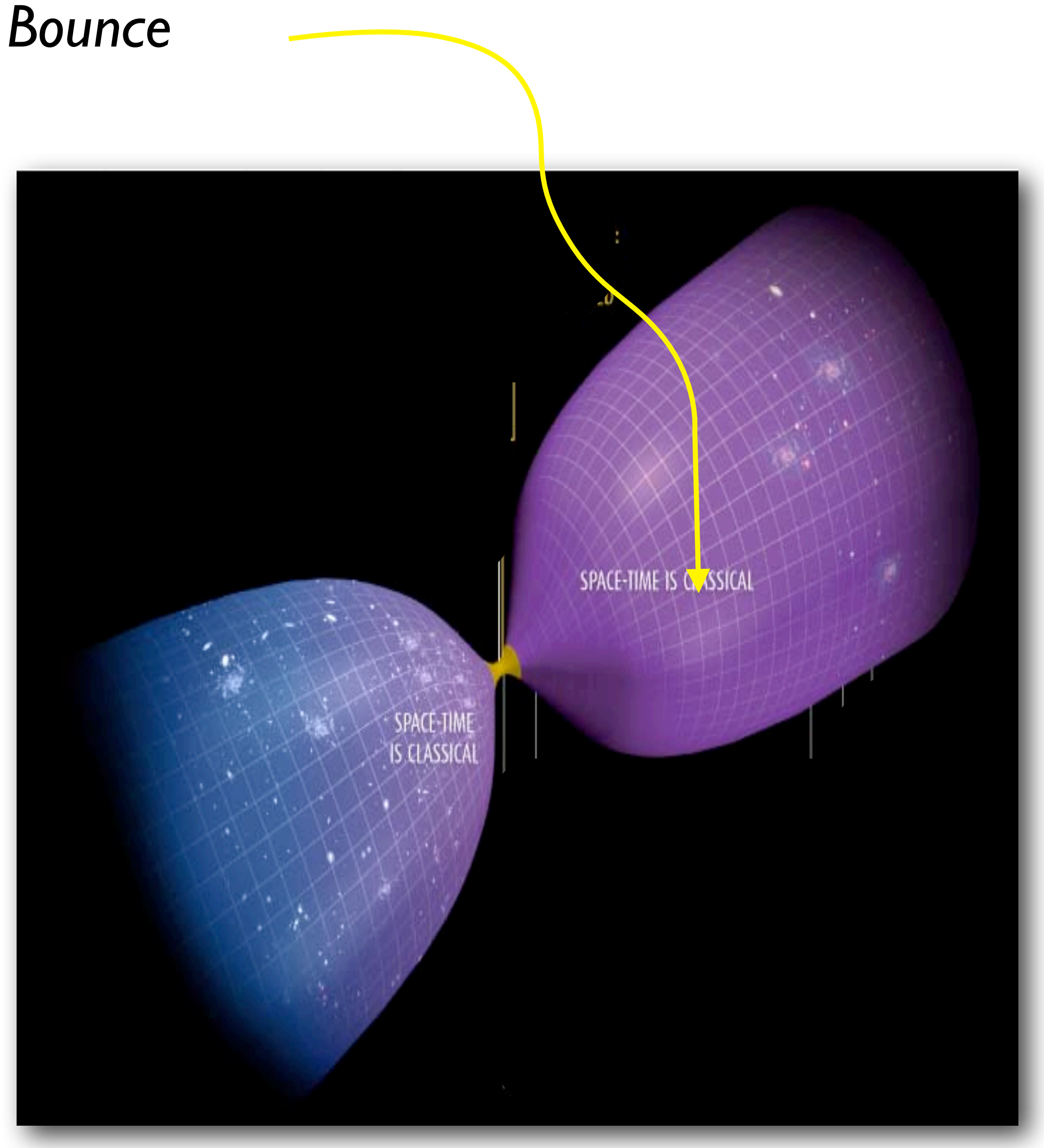
Friedman equation for the scale factor

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1-\frac{\rho}{\rho_c}\right) - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Mukhanov-Sasaki equations for perturbations [Cailleteau, Barrau, Grain, Mielczarek, Vidotto]

$$\ddot v - \left(1-2\frac{\rho}{\rho_c}\right)\Delta v - \frac{\ddot z}{z}v = 0$$

Bounce



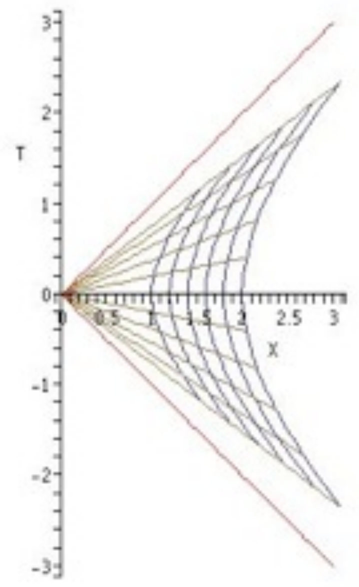
Best hope for empirical confirmation ?

$$\rho_c = \left(\frac{8\pi G}{3}\,\gamma^2 a_o\right)^{-1}$$

(iv) Spinfoam cosmology: (See Vidotto's talk)

(iv) Black hole entropy.

$$\boxed{S = \frac{A}{4\hbar G}}$$



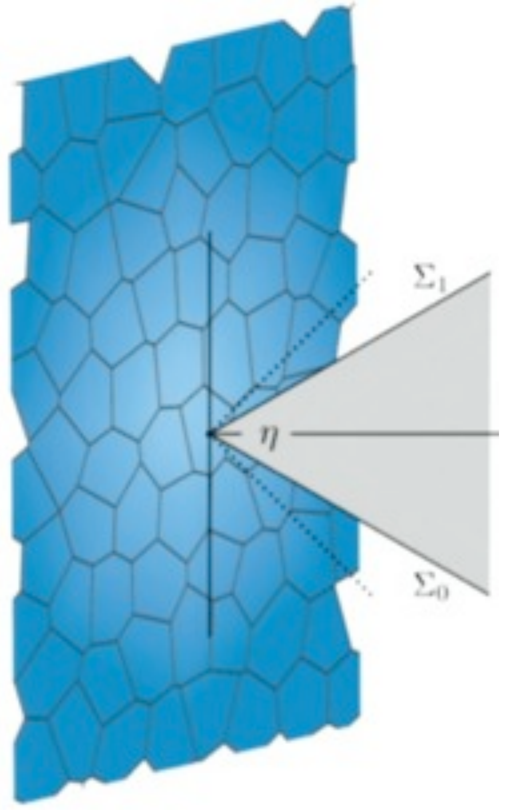
1. Local near-horizon geometry is Rindler geometry, where the stationary killing field is boost

3. Local equilibrium time evolution is the generator of boosts aK : on

4. Local energy is aA $E = \langle j|aK|j\rangle = \gamma j = \dfrac{Aa}{8\pi G}$ (Frodden, Gosh, Perez formula)

5. State $|\Psi\rangle = \otimes_f |j\rangle$ is thermal at temperature $T = \dfrac{a\hbar}{2\pi}$ [Bianchi, 2012]

6. Entropy $dS = \dfrac{dE}{T} = \dfrac{dAa}{8\pi G}\dfrac{2\pi}{a\hbar} = \dfrac{dA}{4\hbar G}$ [Bianchi, 2012]



Main open issue: Do radiative corrections destroy the viability of the expansion?

$$\begin{tikzcd}
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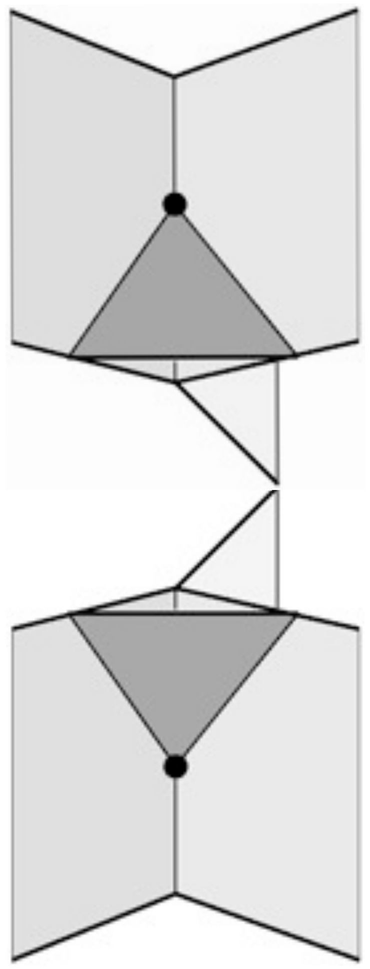
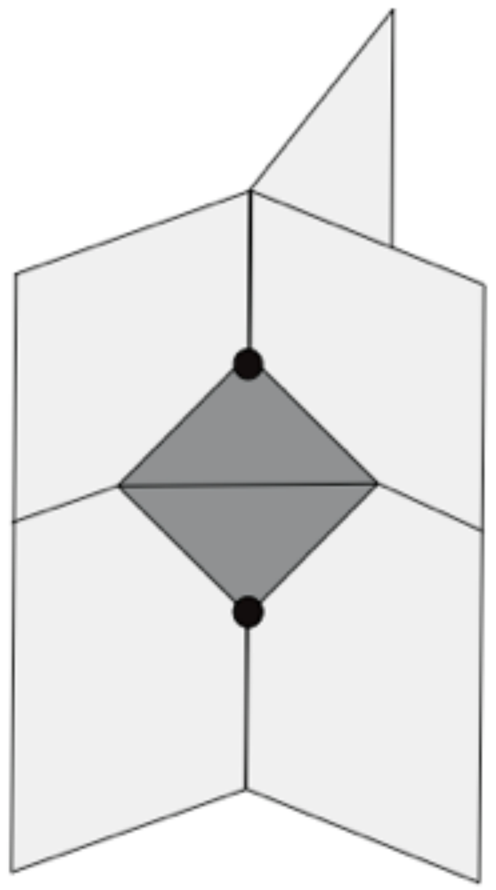
& {} \arrow[r, "\text{Classical limit}"] & {}
\end{tikzcd}$$

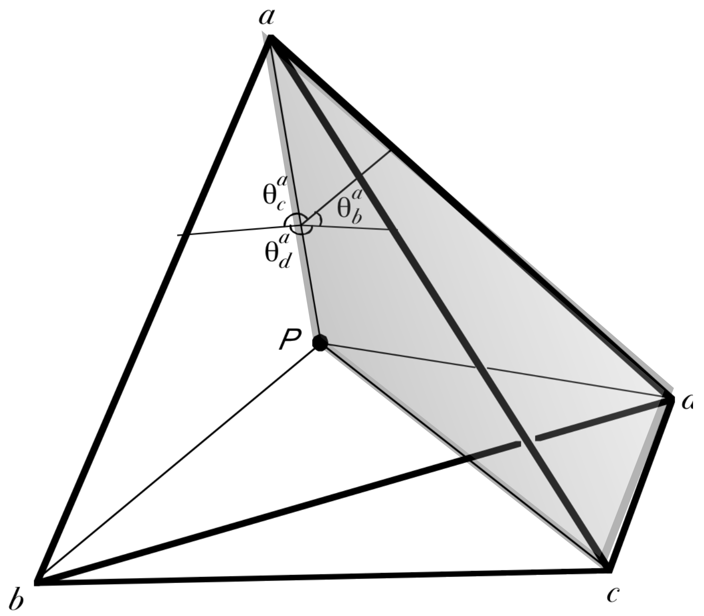
- Radiative corrections come from large spins and from large graphs with many spins.
- They are finite.
- Where is the expansion viable?

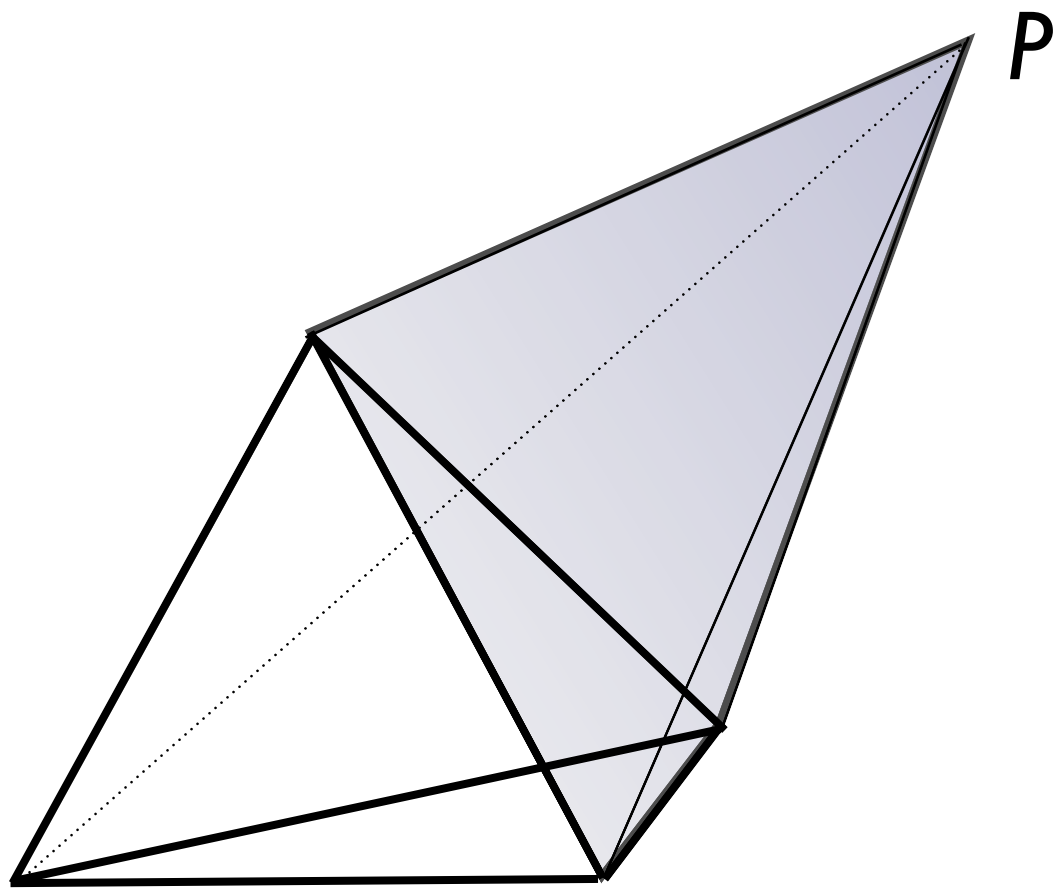




Main open issue: Nature of the large radiative corrections: “antispacetimes”

$$A(j_f, v_e) \underset{j\gg 1}{\sim} e^{iS_{\rm Regge}} + e^{-iS_{\rm Regge}}$$





$$A \underset{j\gg 1}{\sim} \left(e^{iS_{\rm Regge}} + e^{-iS_{\rm Regge}}\right)\left(e^{iS_{\rm Regge}} + e^{-iS_{\rm Regge}}\right)\left(e^{iS_{\rm Regge}} + e^{-iS_{\rm Regge}}\right)\left(e^{iS_{\rm Regge}} + e^{-iS_{\rm Regge}}\right)$$
$$\underset{j\gg 1}{\sim} A_{++++} + A_{+++-} + ..$$

↑

This is the term that diverges!
(Christodoulou, Lanvik, Riello, CR)

Divergences are due to
“antispacetimes”

Main open issue: Do radiative corrections destroy the viability of the expansion?

$$A(j_f, v_e) \underset{j\gg1}{\sim} e^{iS_{\text{Regge}}} + e^{-iS_{\text{Regge}}}$$

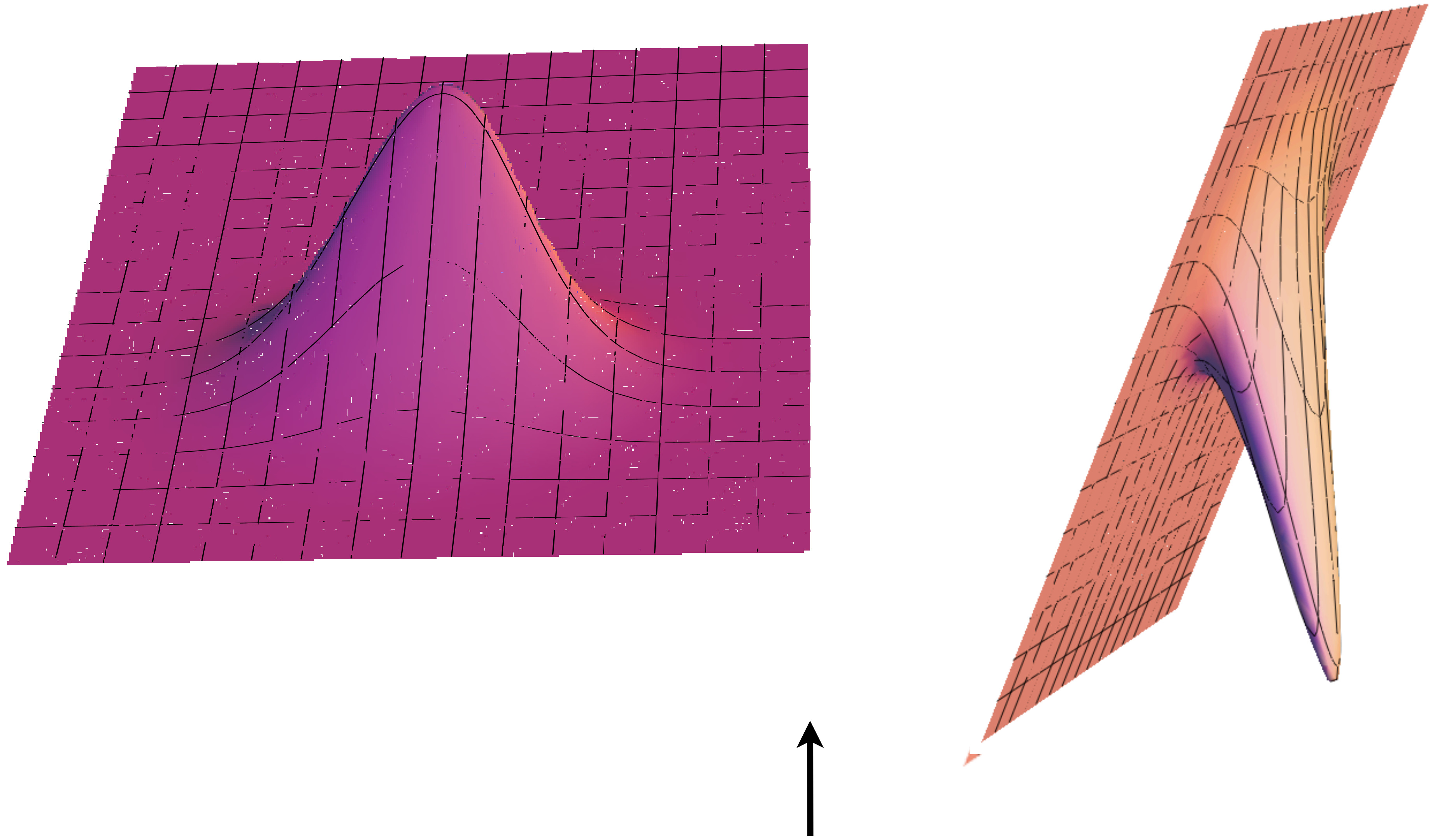
The physics of “antispacetimes”

(Christodoulou, Riello, CR)

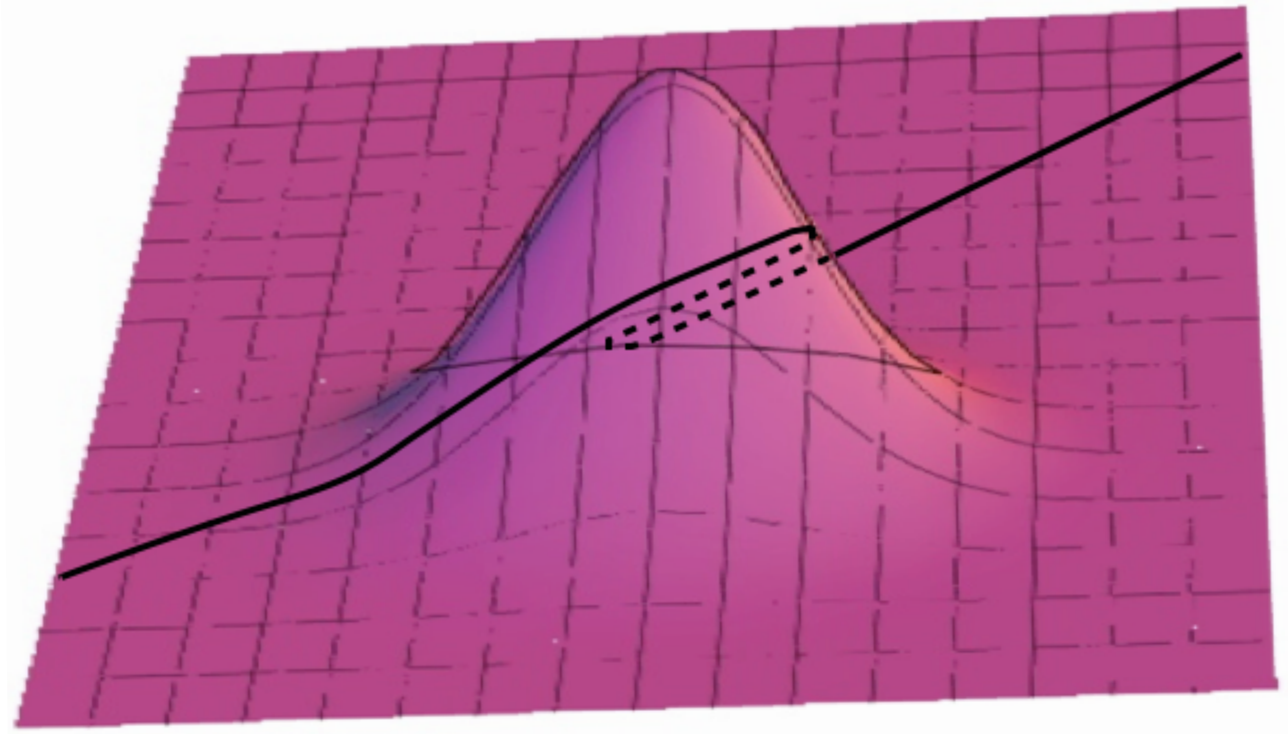
$$e^0 = df(t,\vec{x}), \qquad e^i = dx^i$$

$$f(t,\vec{x}) = t - 2\alpha\tau\ e^{-\frac{\vec{x}^2}{\sigma}}\ (\arctan(t/\tau) + \pi/2)$$

$$\gamma^I e_I^\mu \left(\partial_\mu + \omega_\mu\right)\psi + m\psi = 0$$

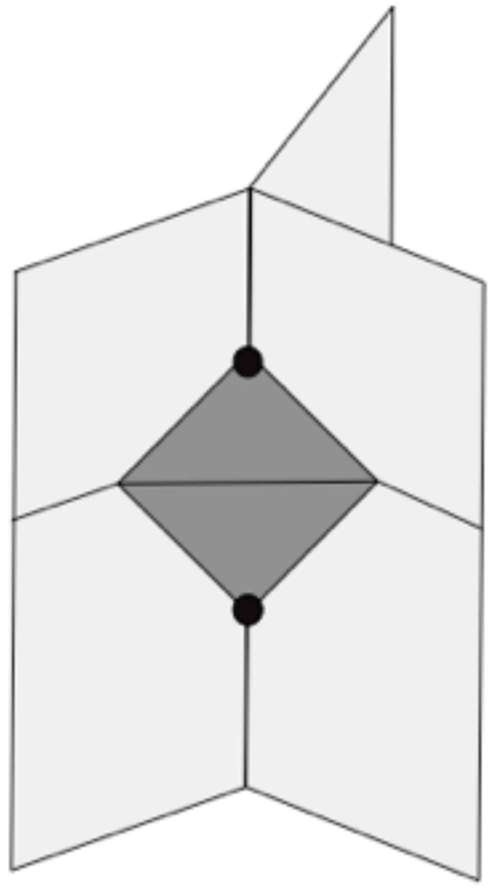


A spacetime “pocket”



Fermions detect negative Lapse regions

Main open issue: Do radiative corrections destroy the viability of the expansion?



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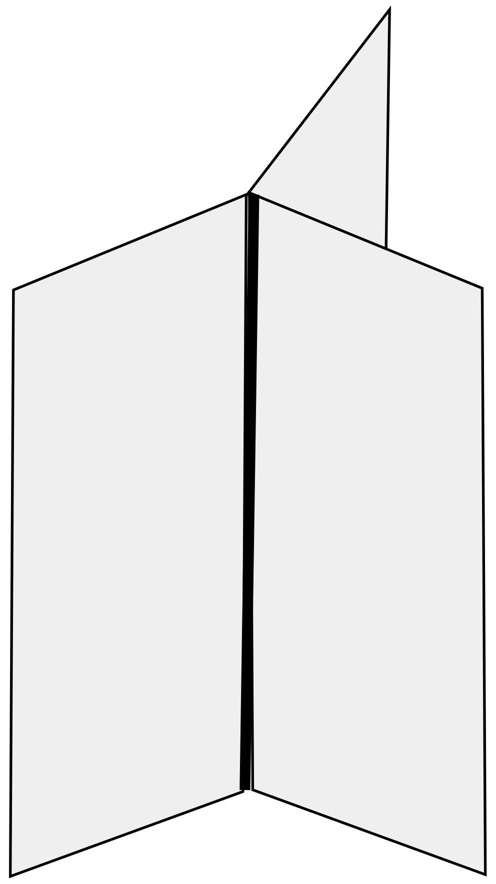
Large factor

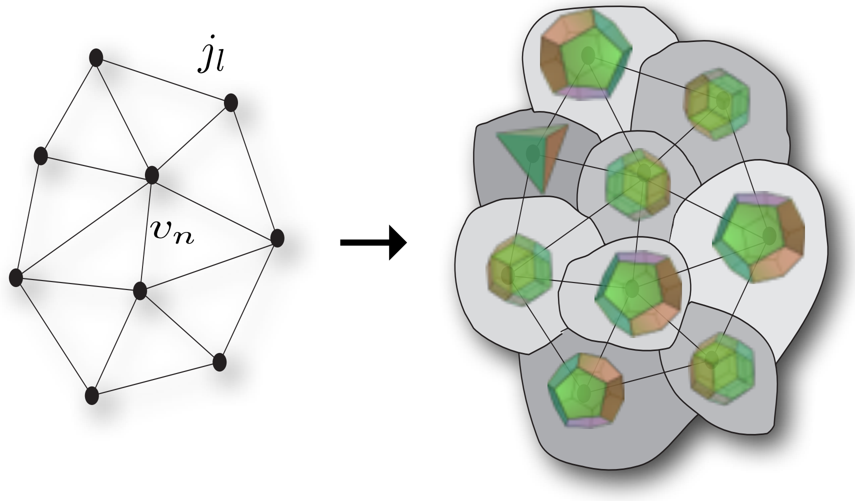


+ Small corrections

Reabsorbed here

$$W_{\mathcal{C}}(h_l) = N_{\mathcal{C}} \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

LQG: Summary



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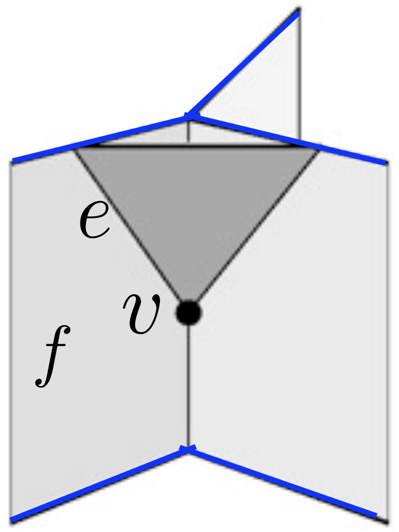
Continuous limit

Classical limit

$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

$$A(h_{ab}) = \sum_{j_{ab}} \int_{SL(2C)} dg_a \prod_{ab} tr_{j_{ab}}[h_{ab} Y_\gamma^\dagger g_a g_b^{-1} Y_\gamma]$$

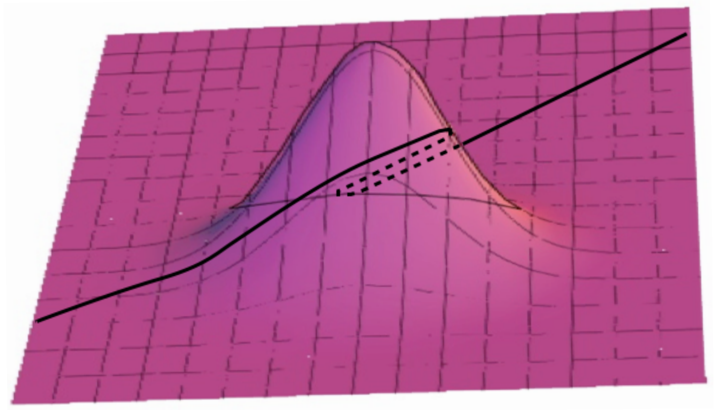
$$Y_\gamma : \quad \begin{array}{l} \mathcal{H}_j \quad \to \mathcal{H}_{j,\gamma j} \\ |j;m\rangle \mapsto |j,\gamma j; j, m\rangle \end{array}$$

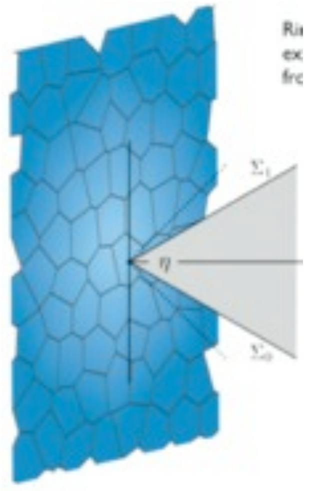


Physics: (i) Propagator, (ii) *n*-point functions,

(iii) Early cosmology → Bounce + perturbations,

(iv) Black-Hole thermodynamics.





Main open issue: Large radiative corrections,

(i) Related to "antispacetimes",

(ii) Can be absorbed in pre-factors?