Horizon Entropy and LQG

Carlo Rovelli

- Major steps ahead in LQG, recently
  (see in particular the talks of Pranzetti, Perez and Bianchi)

- Here: introduction to the problem,
  some general considerations,
  pointing out of questions

I: Unruh temperature [Unruh 1976. "Notes on black-hole evaporation"]

(Theoretical) fact: a thermometer moving with acceleration a in the vacuum state of a QFT on Minkowski space detects a temperature

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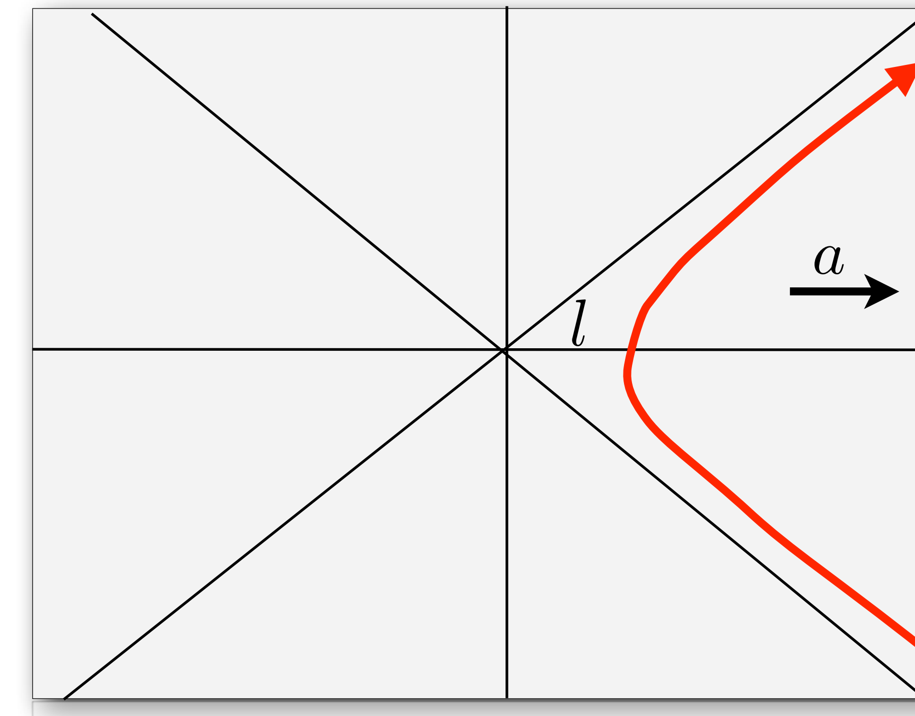
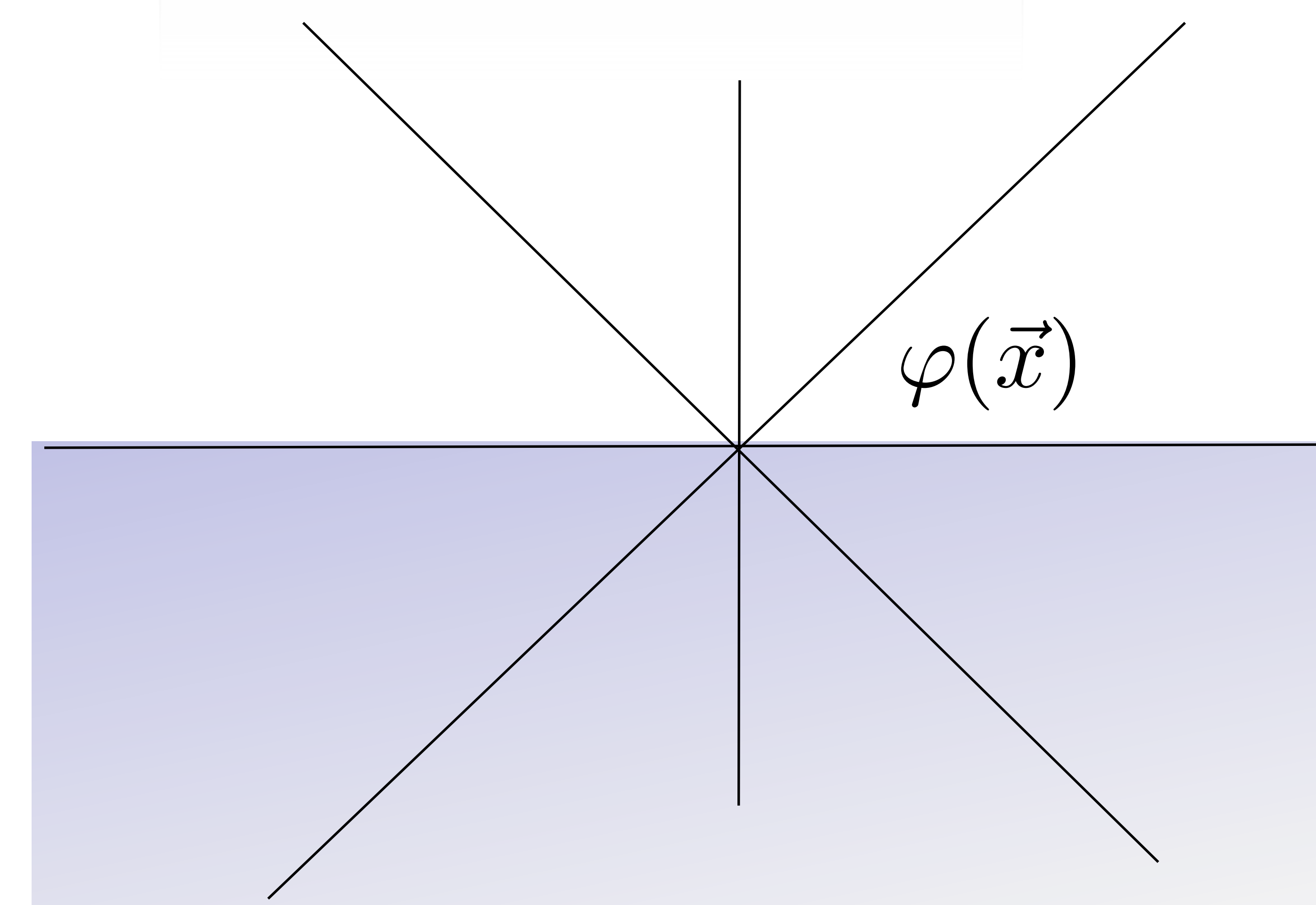


- No gravity.
- Many derivations:

$$|0\rangle = \lim_{t\to\infty} e^{-tH}|\psi\rangle$$

$$\langle\varphi|0\rangle = \int_{\partial\phi=\varphi} d\phi \; e^{-S_E}$$



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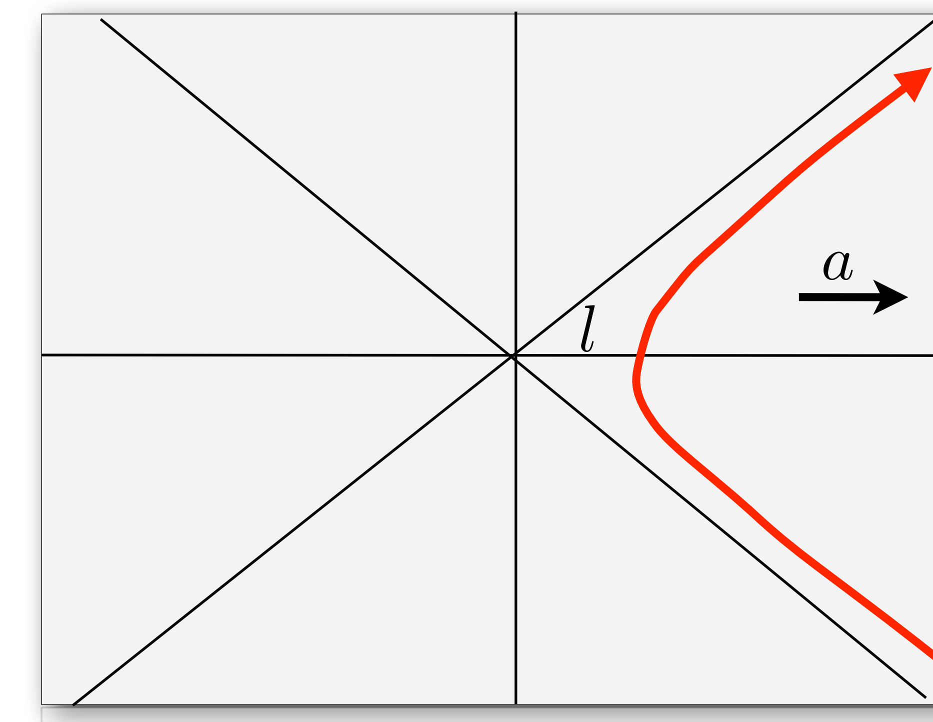

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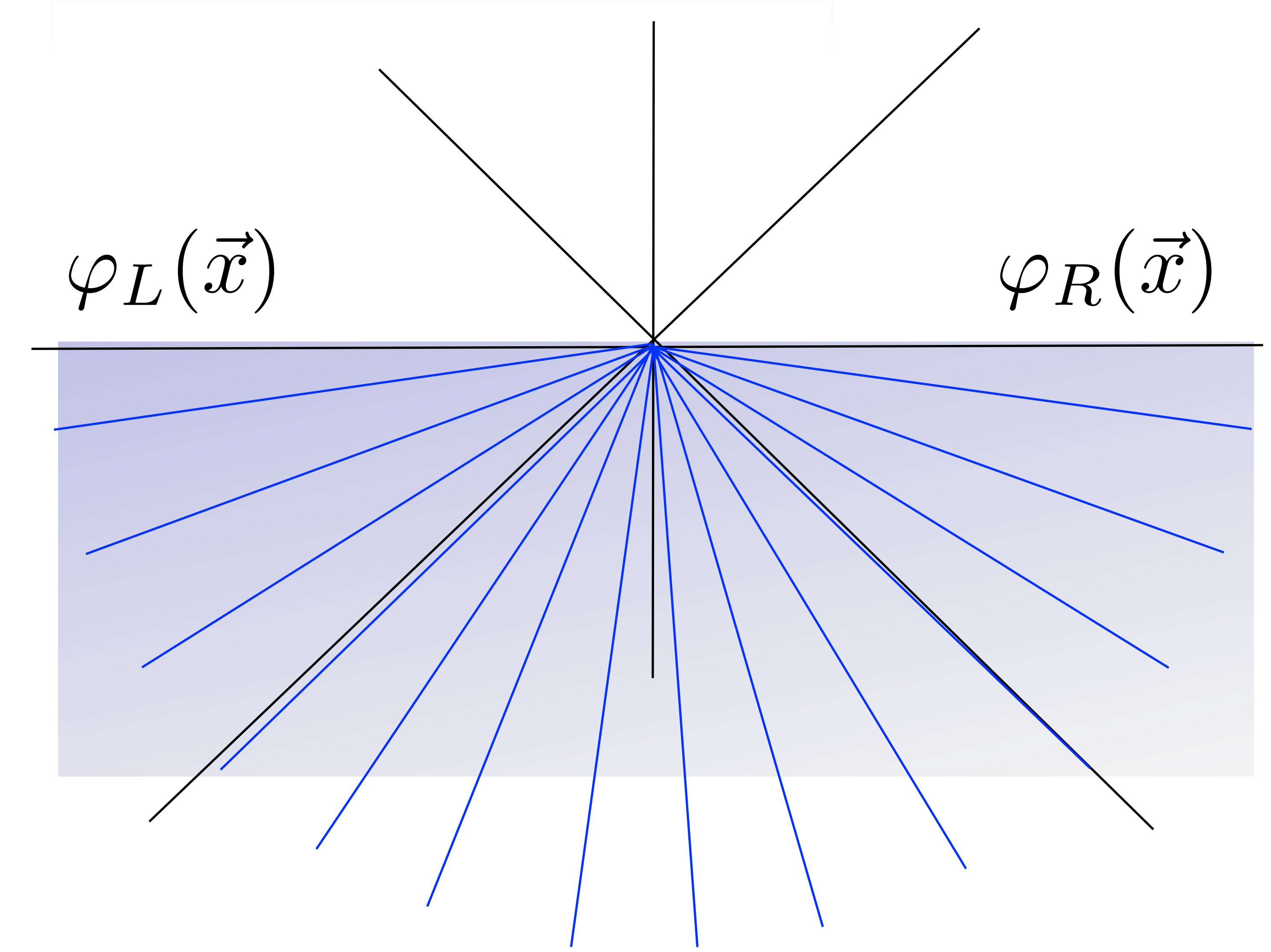
$$\langle\varphi_L,\varphi_R|0\rangle = \int_{\partial\phi=\varphi} d\phi\ e^{-S_E}$$

$$= \langle\varphi_L|e^{-\pi K}|\varphi_R\rangle$$

$$\rho(\varphi_R,\varphi_R') = \langle\varphi_R|e^{-2\pi K}|\varphi_R'\rangle$$

$$\rho = e^{-2\pi K}$$



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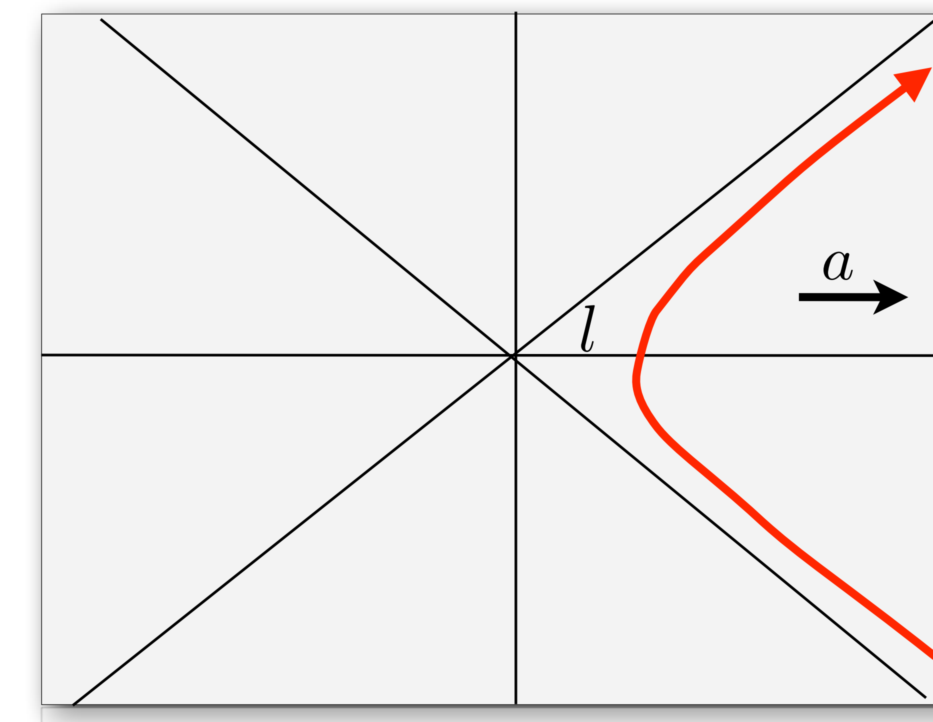

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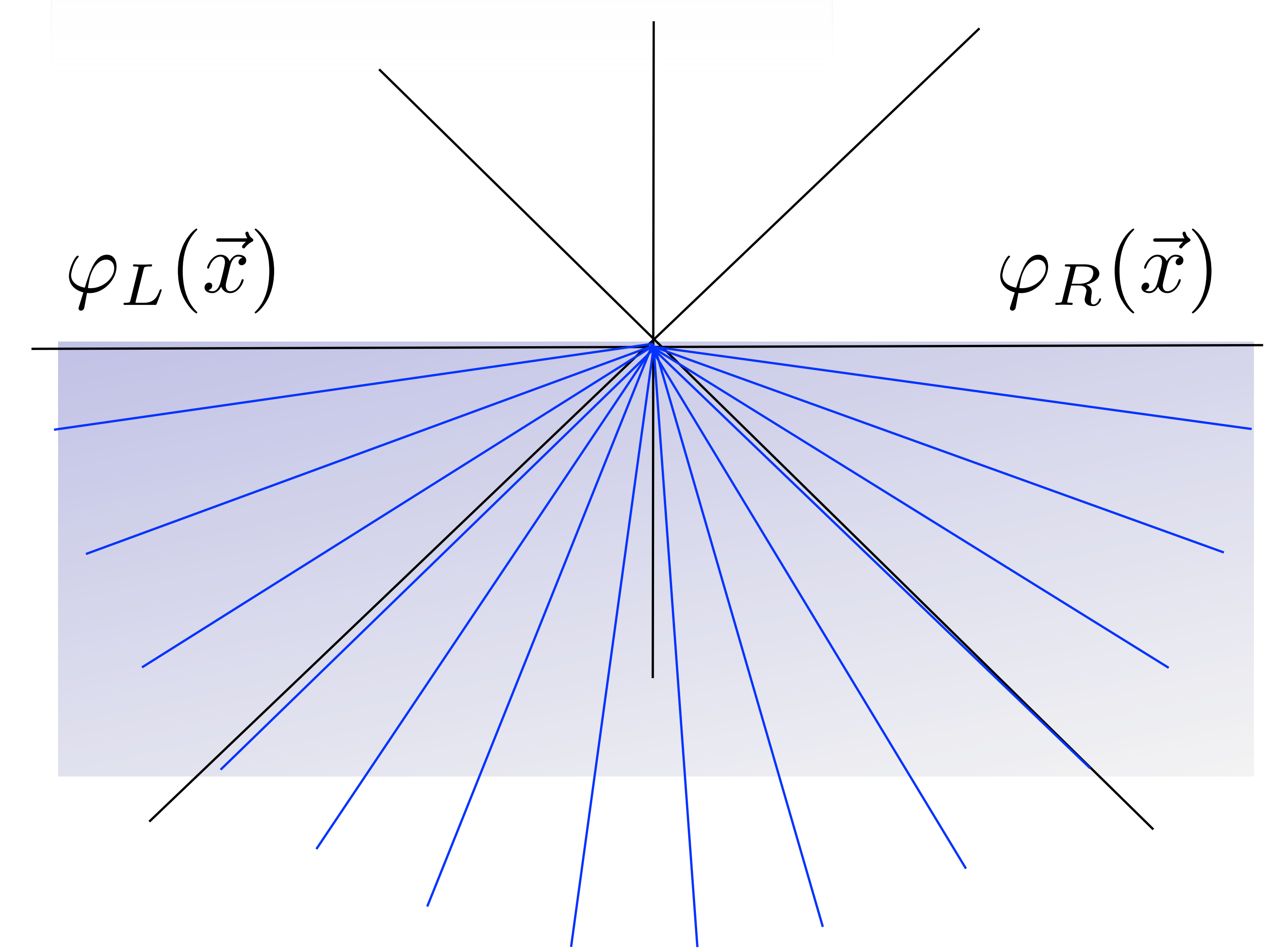
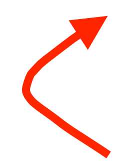
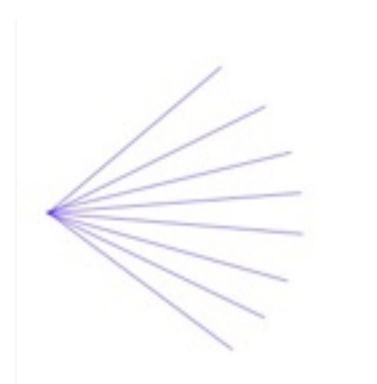
$$\rho = e^{-2\pi K} \quad = e^{-\frac{2\pi}{\hbar a}(\hbar a K)}$$

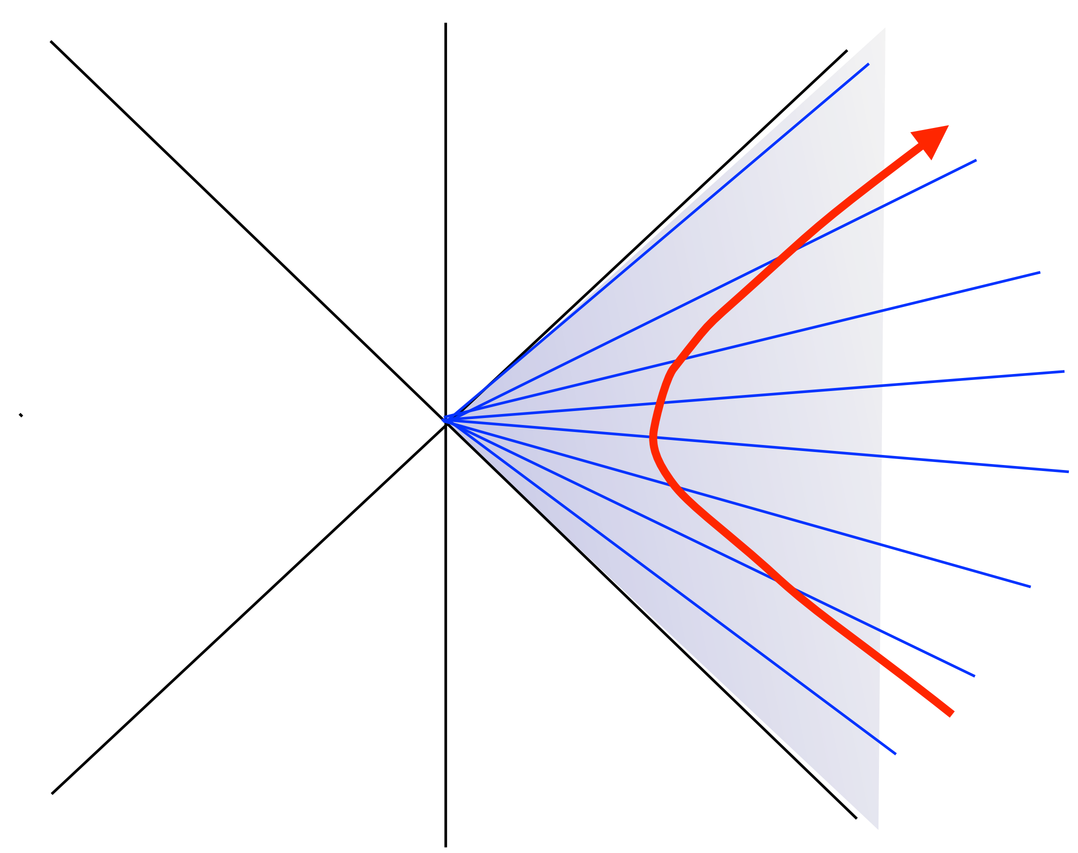


KMS [Kubo 1957, Martin & Schwinger 59]

local dimension-full flow and temperature  $\rho = e^{-\frac{2\pi}{\hbar a}(\hbar a K)}$ $e^{i\hbar a K}$ $T_U = \frac{1}{\beta} = \frac{\hbar a}{2\pi}$

global dimensionless flow and temperature  $\rho = e^{-2\pi K}$ e^{iK} $T_0 = \frac{1}{\beta_0} = \frac{1}{2\pi}$

- Ingredients:
  - flow: $e^{i\hbar a K t}$
  - state: $|0\rangle \quad \rightarrow \quad \rho$
  - algebra: $\mathcal{A}_R$



$f_{AB}(t) = \rho(A(t)B)$ KMS condition $f_{AB}(t) = f_{BA}(-t + i\beta)$

$$\tilde{f}_{AB}(\omega) = e^{-\beta\omega}\tilde{f}_{BA}(-\omega)$$

Interpreting KMS

KMS condition

$$\tilde{f}_{AB}(\omega) = e^{-\beta\omega}\tilde{f}_{BA}(-\omega) \qquad f_{AB}(t) = \rho(A(t)B)$$

Couple the field to a 2-level system

$$V = g(|0\rangle\langle 1| + |1\rangle\langle 0|)A$$

Up-transition amplitude

$$\begin{aligned} W_+(t) &= g\int_{-\infty}^{t} dt\ (\langle 1| \otimes \langle f|)\ \alpha_t(V)\ (|0\rangle + |i\rangle) \\ &= g\int_{-\infty}^{t} dt\ e^{it\Delta E}\langle f|\alpha_t(A)|i\rangle. \end{aligned}$$

Transition probability per unit time

$$p_+ = \frac{dP_+}{dt} = g^2\ \tilde{f}_{A^\dagger A}(\Delta E)$$

$$p_- = g^2\ f_{AA^\dagger}(-\Delta E)$$

$$\frac{p_+}{p_-} = e^{-\beta\Delta E} = e^{-\Delta S}$$

The KMS condition implies that a system moving with the flow has transition probabilities which are thermal (Boltzamanian)

Entropy ?

- The state is pure!
  There may be thermality and temperature with pure states !

- Where does the uncertainty that characterizes thermality comes from?

From quantum mechanics.

- One way of viewing this: the accelerating thermometer does not couple to $\varphi_l(\vec{x})$

  Therefore we can trace out these d.of f.: quantum entanglement produces entropy

Entropy is then the von Newman entanglement entropy

$$S_{VN} = -Tr[\rho \log \rho] = -Tr_R[Tr_L(|0\rangle\langle 0|) \log(Tr_L(|0\rangle\langle 0|))]$$

- However:
  If you put a mirror at the origin, the state on the right algebra is pure, but a coupled accelerated system <u>still</u> measures Unruh temperature. [Smerlak, CR 2011 "Unruh effect without trans-horizon entanglement"]

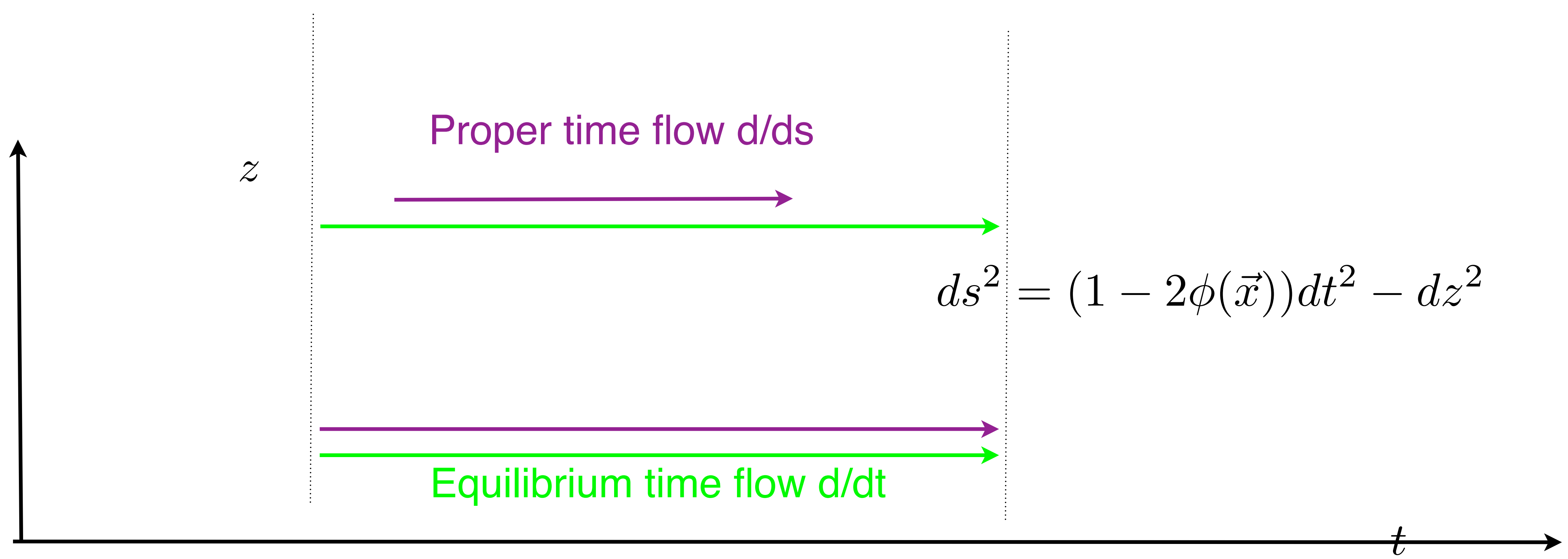
- In any case: There is nothing to "count".

II. Gravity

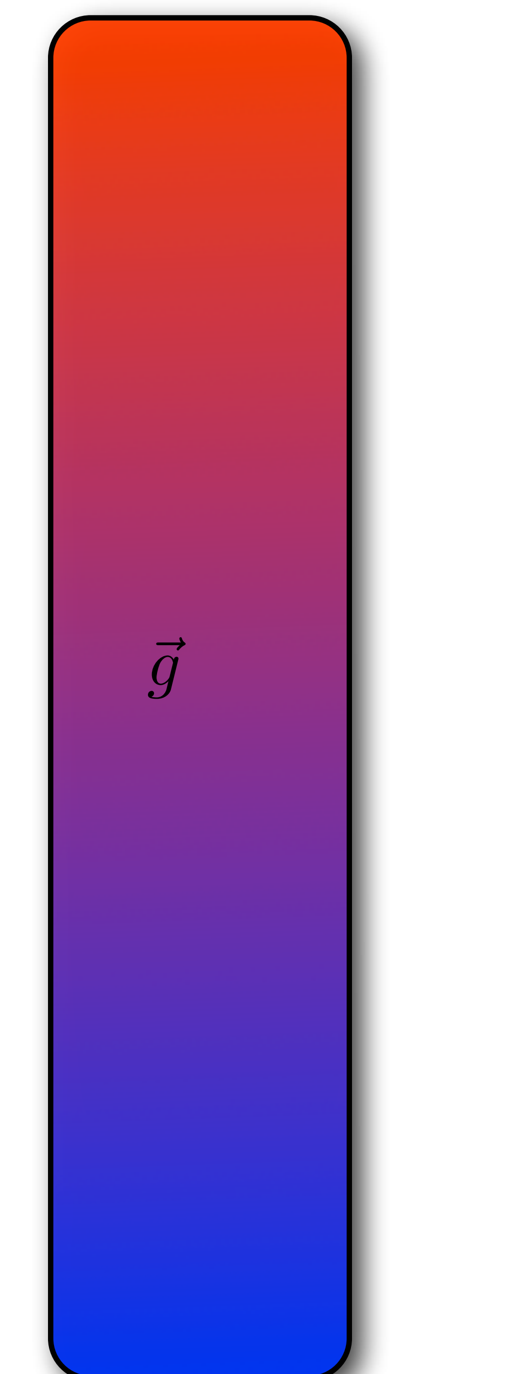
(Theoretical) fact: temperature is not constant in a (stationary) gravitational field
[Tolman 1930 “On the Weight of Heat and Thermal Equilibrium in General Relativity”. Ehrenfest 1930 “Temperature Equilibrium in a Static Gravitational Field,”]

$$||\xi||\ T(\vec{x}) \quad = \quad constant$$

$$\sqrt{g_{00}(\vec{x})}\ T(\vec{x}) \quad = \quad constant$$



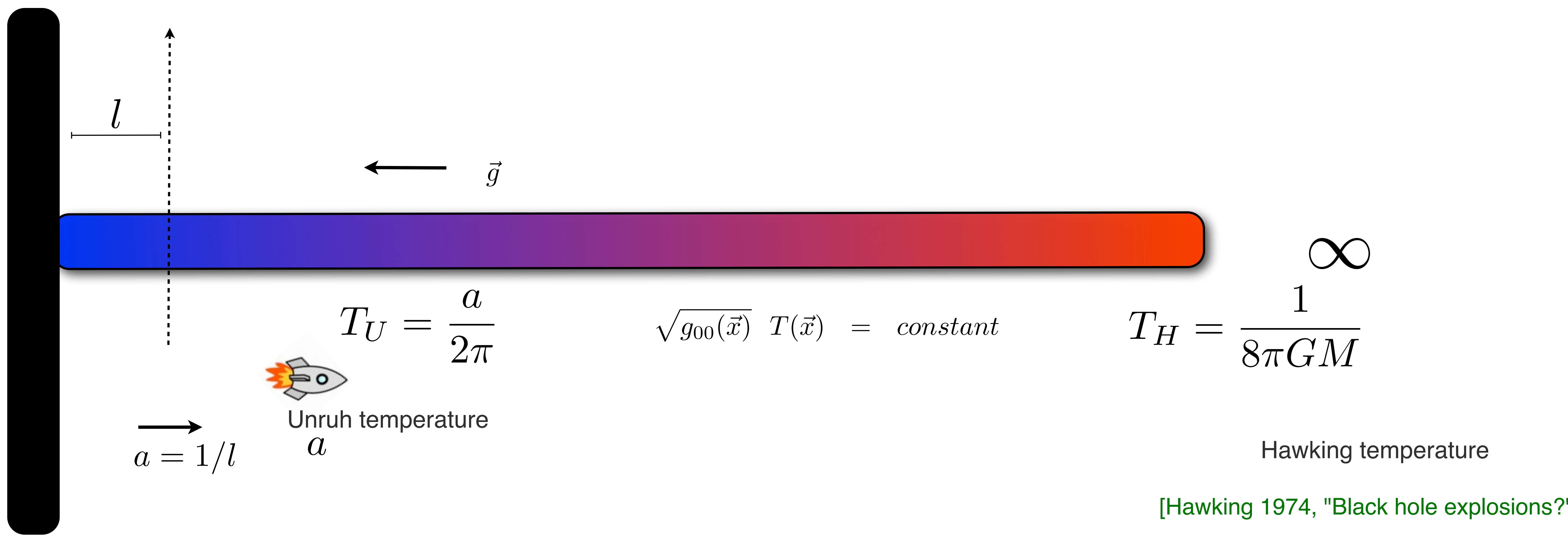
$$ds^2 = (1 - 2\phi(\vec{x}))dt^2 - dz^2$$



There are two relevant notions of time flow, which are distinct: proper time, and equilibrium flow time (or thermal time).

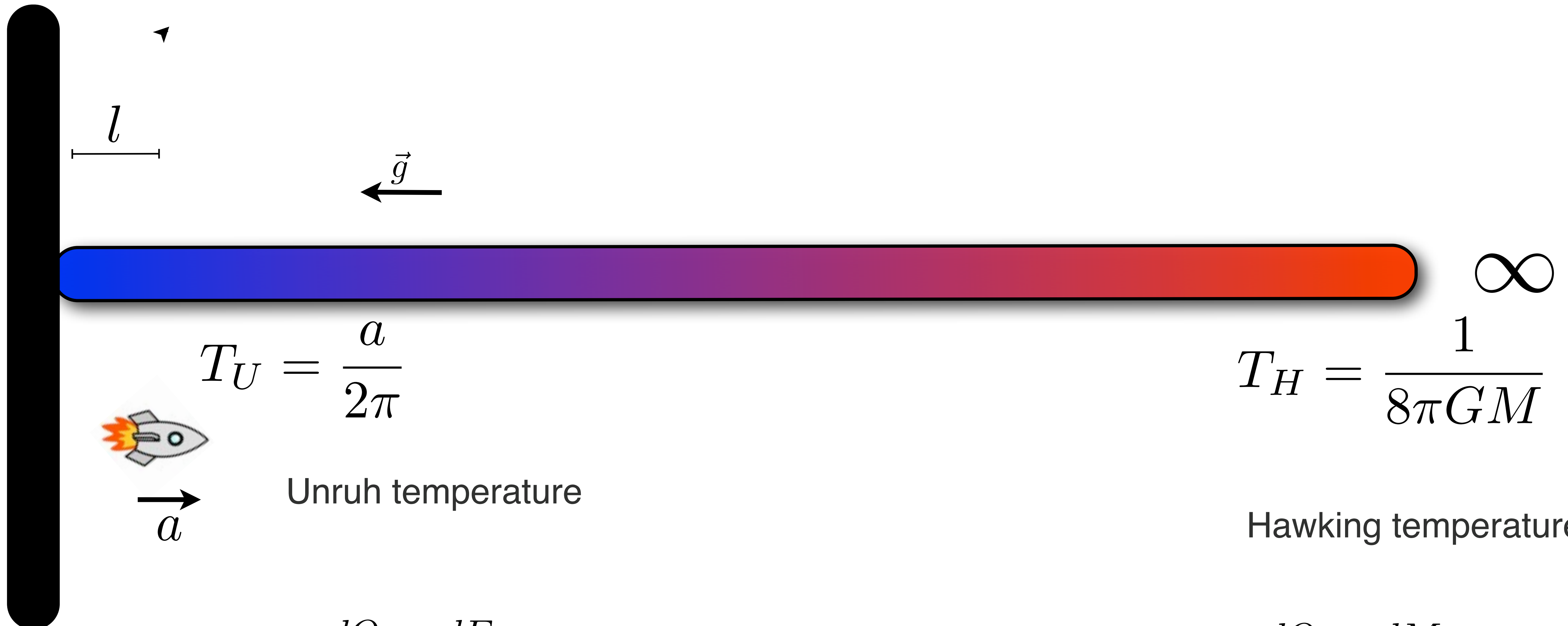
(Tolman effect can also be can be viewed as a manifestation of a key property of thermal time: temperature is locally the rate of flow of thermal time with respect to proper time [Smerlak, CR 2010 “Thermal time and the Tolman-Ehrenfest effect: temperature as the `speed of time’ ”]).

Tolman effect on black holes



The Hawking temperature is a peeling of the Unruh temperature, due to tidal gravitational forces

Entropy ?



$$T_U = \frac{a}{2\pi}$$

Unruh temperature

$$T_H = \frac{1}{8\pi GM}$$

Hawking temperature

$$dS = \frac{dQ}{T} = \frac{dE}{T_U} = d\frac{Aa}{8\pi G}\frac{2\pi}{a} = d\frac{A}{4G}$$

$$dS = \frac{dQ}{T} = \frac{dM}{T_H} = (8\pi GM)dM = d(4\pi GM^2) = d\frac{A}{4G}$$

$$E_{FGP} = \frac{Aa}{8\pi G}$$

[Frodden, Gosh, Perez 2011
"A local first law for black hole thermodynamics."]

Local energy that fell into the horizon

Energy of what? Of the gravitational field.
Entropy of what? Of the gravitational field.

Interesting moral:
The relevant physics appears to be local.

III. Dynamical gravity

- What is the problem?

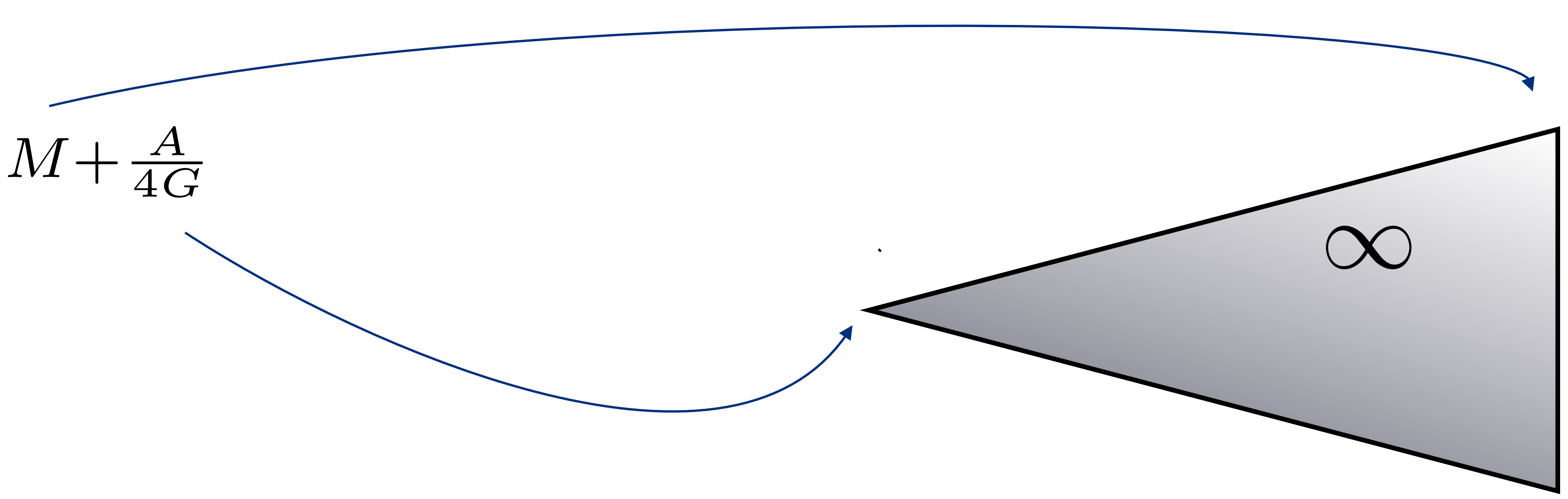
There are 3 independent indications that there is (thermodynamical) entropy in the gravitational field:

i) There is a measurable inverse temperature as integration factor for energy transfer. The energy goes into the gravitational field. Therefore there is thermodynamical entropy into the gravitational field.

ii) Total entropy $S = \frac{A}{4G} + S_{matter}$ appears to be increasing in all processes we can think about.

iii) A formal "statistical" calculation gives the same result from integration over geometries
[Gibbons Hawking 1977 "Cosmological Event Horizons, Thermodynamics, and Particle Creation"]

$$Z(\beta) = Tr[e^{-\beta H}] = \int_{\Omega_\beta} dg\ e^{-S_E} = e^{I(\beta)} = e^{-\beta M + \frac{A}{4G}}$$



$$Z(\beta) = e^{-\beta E + S}$$

- Therefore there is a statistical entropy associated to the gravitational field !

- The **classical** statistical mechanics of gravity is not yet developed!

III. Dynamical gravity

- What should a quantum theory do?

i) Give a general conceptual framework for understanding entropy on a background independent context. Local?

ii) Gives a rigorous version of the Gibbons-Hawking computation, in a well defined mathematical context, and without the need of a full scale semiclassical approximation.

[→ Bianchi]

III. Dynamical gravity

General covariant thermodynamics, and general covariant statistical mechanics ?

- Difficulty: time. General covariant systems are not standard Hamiltonian systems evolving in time.

- An attempt: [CR 1993: "Statistical mechanics of gravity and the thermodynamical origin of time"
  "The Statistical state of the Universe."]

- Hints:

(i) State → Boundary state

(ii) Flow determines equilibrium state

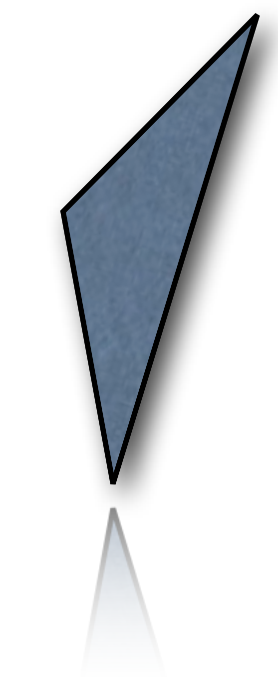
→ i. State → Tomita flow ("thermal time")

ii. Equilibrium = local proportionality between thermal time and proper time.

iii. Temperature = local proportionality factor between thermal time and proper time.

LQG

- Consider a single quantum of space: an element of area.



- According to LQG, this lives in a representation of SU(2).

$$\mathcal{H}_j \quad \ni \quad |j, m\rangle$$

- The dynamics of the theory is given by a map

$$Y: \quad \mathcal{H}_j \quad \rightarrow \quad \mathcal{H}_{p,k} \qquad\qquad p = \gamma(j+1), \quad k = j$$

- The main property of this map is that on its image

$$\vec{K} \quad \overset{Y}{=} \quad \gamma\, \vec{L}$$

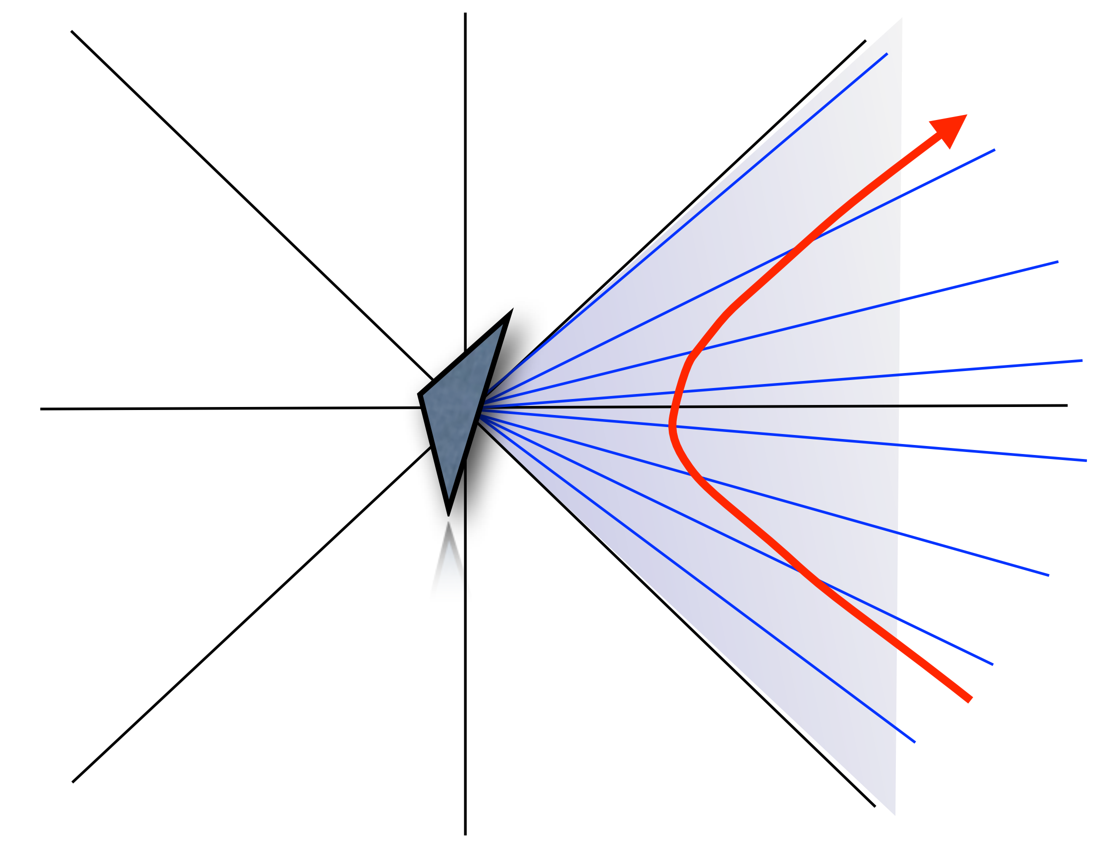
LQG → A/4 [Bianchi 2012: "Entropy of Non-Extremal Black Holes from Loop Gravity"]

- Consider a state: $|j,j\rangle$

- A flow $e^{i\hbar(aK)}$

- An algebra $\mathcal{A}: \quad \mathcal{H}_j \to \mathcal{H}_j$

Define $f(t) = \langle j,j|e^{iKt}|j,j\rangle$



Eugenio Bianchi main result: $f(t) = f(-t + 2\pi i)$

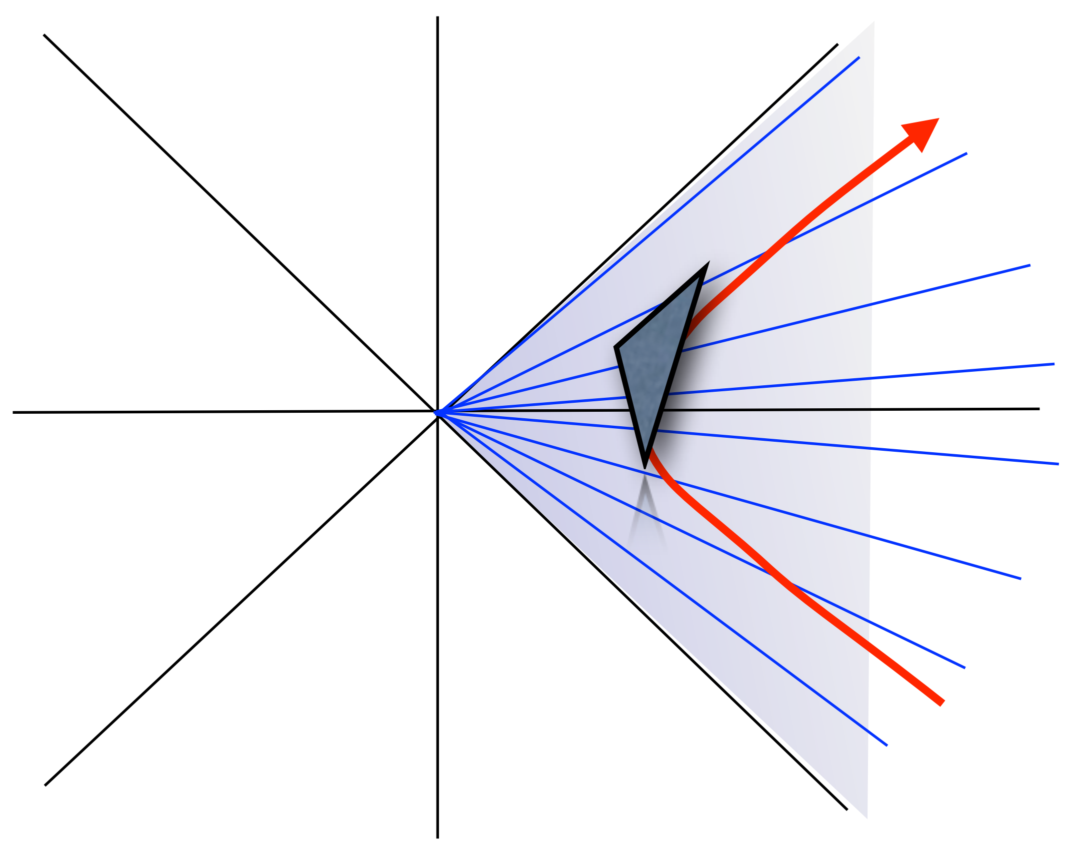
$$\tilde{f}(\omega) = e^{-\beta\omega}\tilde{f}(-\omega) \qquad T_0 = \frac{1}{2\pi}, \qquad T_U = \frac{a}{2\pi}$$

Eugenio Bianchi second result: $(\vec{K} = \gamma\vec{L})$

$$E = \langle j,j|aK|j,j\rangle = \langle j,j|a\gamma L|j,j\rangle = a\gamma j = a\frac{A}{8\pi G} \qquad A_{j,m} = 8\pi G\gamma\sqrt{j(j+1)}.$$

LQG

Purely from the quantum theory:



$$T_0 = \frac{1}{2\pi}, \qquad T_U = \frac{a}{2\pi}$$

$$E_{FGP} = \frac{aA}{8\pi G}$$

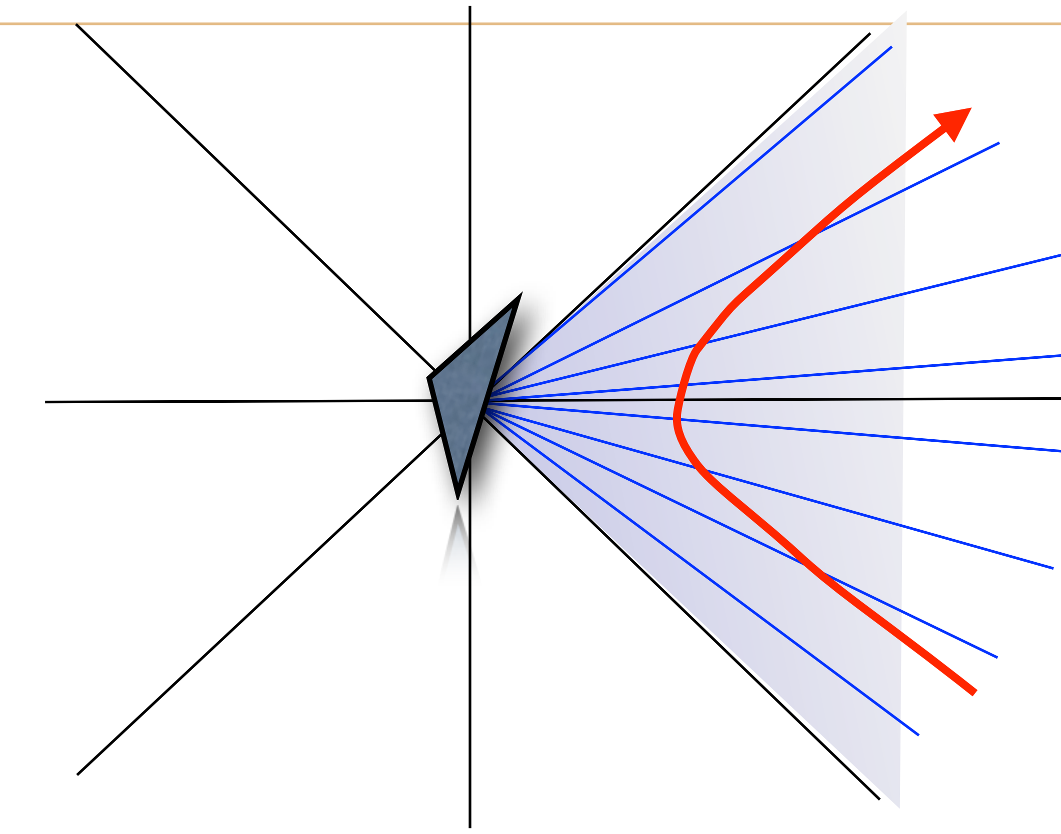
From which the thermodynamical entropy follows immediately, from $\frac{p_+}{p_-} = e^{-\beta \Delta E} = e^{-\Delta S}$

Therefore there exists a straightforward calculation:

$$LQG \quad \rightarrow \quad S = \frac{A}{4G}$$

where there is no background, and nothing ill-defined, valid for all black holes.

Observations



(i) The central equation of LQG acquires a novel geometrical interpretation:

$$\vec{K} \stackrel{Y}{=} \gamma \vec{L} = \frac{A}{8\pi G}$$

(ii): Direct derivation of classical theory from covariant LQG

[Smolin 2012 "General relativity as the equation of state of spin foam."],

(iii) Traditional counting in LQG is based on identifying Energy with Area.

Doing a microcanonical state counting, of states with given Area.

[Rovelli 1996, Krasnov 1998], [Ashtekar-Baez-Corichi-Krasnov 1998, Domagala-Lewandowski 2004] [Barbero *et al* 2008].

This is half-correct (because of the equation above) <u>and</u> half-wrong (because of the equation above).

(a) There is nothing to count. (b) The state whose Von Neumann entropy is relevant is

$$e^{-\beta K} \neq e^{-\beta \frac{A}{8\pi G}}$$

Relation with area counting

(iv) Canonical partition function with the Area. $A_{j,m} = 8\pi G\gamma\sqrt{j(j+1)}.$

$$Z(\beta) = \sum_{j,m} e^{-\beta E_{FGP}} = \sum_j (2j+1)e^{-\beta a\gamma\sqrt{j(j+1)}}$$

$$S = \frac{2\pi}{a}\frac{aA}{8\pi G} + \log Z$$
$$= \frac{A}{4G} + \log\sum_j (2j+1)e^{-2\pi\gamma\sqrt{j(j+1)}}$$

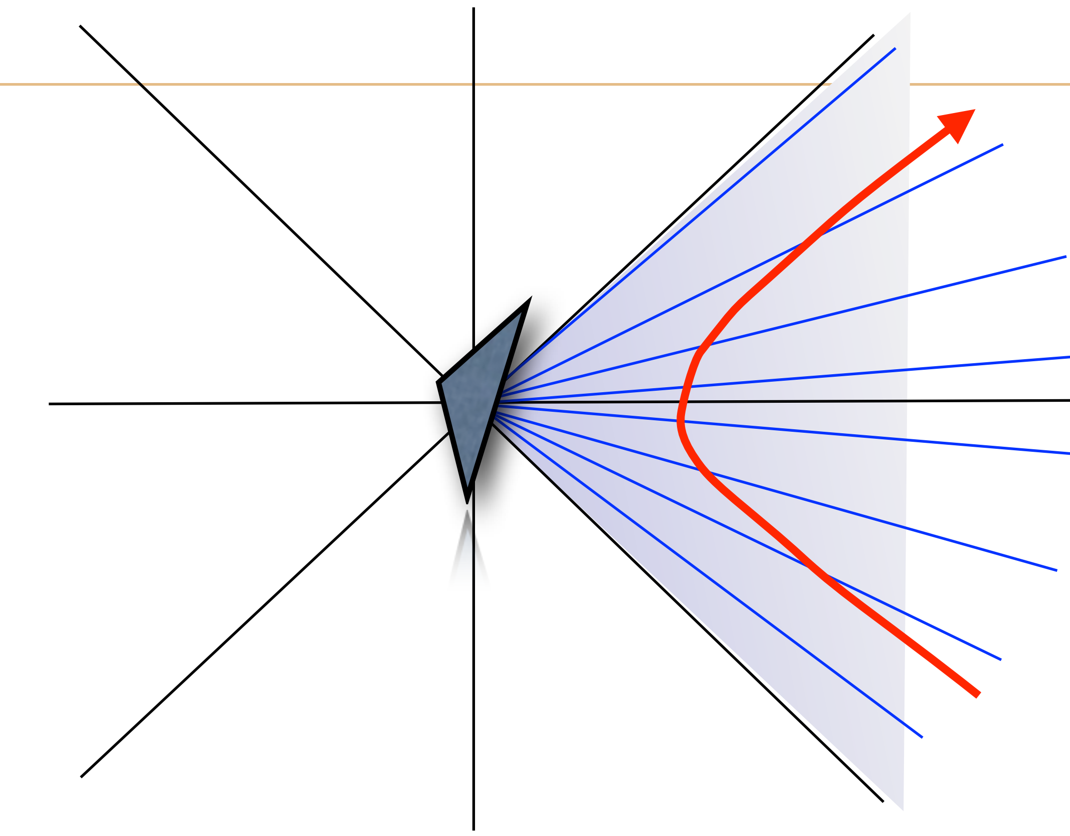
This gives the right result only if $\sum_j (2j+1)e^{-2\pi\gamma\sqrt{j(j+1)}} = 1$ namely if $\gamma = \gamma_0$ [Frodden, Gosh, Perez]

Therefore $\gamma = \gamma_0$ is the value that makes the Helmholtz free energy vanish.

Ghosh and Peres [2012 “Black hole entropy and isolated horizons thermodynamics.”] have suggested that the Helmholtz free-energy term can also be interpreted as an effect of varying the number of punctures, which can be controlled by means of an appropriate chemical potential. The exchange of heat that does not modify the area is then a modification in the number of punctures, at fixed area.

$$dE = dQ + \mu dN$$

What is missing ?



(i) A clean quantum general covariant statistical mechanics is missing.
What is this Entropy, exactly?

(ii) Is there a relation between $|j, j\rangle$ and $\rho = e^{-2\pi K}$?

(iii) What is exactly the KMS algebra, here?

(iv) What is the partition function? $Z(\beta_o, A) = e^{-\frac{A}{8\pi G}(\beta_0 - 2\pi)}.$

$$E_o = -\partial_{\beta_o} Z = \frac{A}{2\pi G}$$

$$S = T_o E_o + \log Z = \frac{A}{4G}$$

$$\beta_o = \frac{dS}{dA} = \frac{dA}{4G}\frac{2\pi G}{dA} = 2\pi$$

$\rightarrow$

$$Z(\beta, A) = 1$$

(v) The Helmoltz free energy of the local thermodynamics vanishes. Why?