Early Warnings Indicators of financial crises via Auto Regressive Moving Average models

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Abstract

We address the problem of defining early warning indicators of financial crises. To this purpose, we fit the relevant time series through a class of linear models, known as Auto-Regressive Moving-Average (ARMA(p, q)) models. By running such a fit on intervals of the time series that can be considered stationary, we first determine

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the *typical* ARMA($\overline{p}, \overline{q}$). Such a model exists over windows of about 60 days and turns out to be an AR(1). Then, we define a distance Υ from such typical model in the space of the likelihood functions and compute it on short intervals of stocks indexes. Such a distance is expected to increase when the stock market deviates from its *normal* state for the modifications of the volatility which happen commonly before a crisis. We observe that Υ computed for the Dow Jones, Standard and Poor's and EURO STOXX 50 indexes provides an effective early warning indicator which allows for detection of the crisis events that showed precursors.

Keywords: Stochastic modeling, Tipping points detection, Extreme events

1 Introduction

In the late '70s, a succession of currency crises generated interest in Early warning indicators [22, 23]. Over the year, the indicators spread more generally to financial and economic crisis, generating methodological debates [17, 15]. Traditional statistical approach to this issue are based on specific properties of ideal statistical systems near critical transition: critical slowing down, modifications of the auto-correlation function or of the fluctuations [13], increase of variance and skewness [18], diverging susceptibility [21, 4, 20], diverging correlation length (see the book [25] for a comprehensive review). However, in many cases, these approaches fail to detect the financial crisis. First of all, such methods are *attractor* based, i.e. they assume that the system can be well described by relating the observation at the time t with the observation at the time $t + \tau$ by an empirical deterministic law describing a stationary state of the system (the so called attractor). This approach fails in describing financial data because, such processes involve a family of time scales rather than a single scale τ, [9, 3]. A second origin for the failure of traditional early warning indicators is due to the presence of human feed-backs on the system i.e. the constant attempt to keep economy in a state fit to make profits. Such feed-backs create some delays between the first early warning signals and the time at which the crisis is observed. In addition, traditional early warning indicators may be inapplicable in datasets containing a small number of observations (see *e.g.* [14]), which is usually the case in financial time series. This suggests that indicators based on single statistical properties are not well suited for financial analysis and that crisis detection must involve indicators based on global properties of the whole stochastic process. Here, we build a class of indicators based on the auto-regressive moving-average processes of order p, q ARMA(p, q), widely used to model and forecast the behavior of financial time series. We remark that the goal of this paper will not be to find the best model to describe stock indexes and make predictions: this would require at least the estimation of fractionally integrated (ARFIMA(p, d, q)) or conditionally heteroskedastic (GARCH(p)) models, among others. We will rather assess a *typical* ARMA(p, q) model able to capture the general features of the analyzed stock market and define the early warning indicators as deviations from such a model in a suitable likelihood space. In the first part of the paper, we recall some basics on ARMA(p, q) modeling and define corresponding early-warning indicators. We then check that these indicators are able to detect the transition in theoretical financial models. We conclude the paper by presenting and discussing the results of the analysis for some stock indexes.

2 The method

Let us consider a series X_t of an observable with unknown underlying dynamics. We further assume that for a time scale τ of interest, the time series $X_{t_1}, X_{t_2}, ..., X_{t_\tau}$ represents a stationary phenomenon. Since X_t is stationary, we may then model it by an ARMA(p, q) process such that for all t:

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \tag{1}$$

with $\varepsilon_t \sim WN(0, \sigma^2)$ - where WN stands for white noise - and the polynomials $\phi(z) = 1 - \phi_1 z_{t-1} - \cdots - \phi_p z_{t-p}$ and $\theta(z) = 1 - \theta_1 z_{t-1} - \cdots - \theta_q z_{t-q}$, with $z \in \mathbb{C}$, have no common factors. Notice that, hereinafter, the noise term ε_t will be assumed to be a white noise, which is a very general condition [7]. For a general stationary financial time series, this model is not unique. However there are several standard procedures for selecting the model which fits at best the data. The one we exploit in this paper is the Box-Jenkis procedure [5]. We choose the lowest p and q such that the residuals of ARMA(p, q) fit are uncorrelated: to this purpose, we perform a Ljung-Box test for the absence of serial correlation (see, for example, [7]). This fixes p and q, and thus our statistical model. There are other model selection procedures based on information criteria (Bayesan or Akaike information criteria). We tested, that they all give clear indications for discriminating the model and that they provide qualitatively the same results of the Box-Jenkis procedure. Intuitively, p and q are related to memory lag of the process, while the coefficients ϕ_i and θ_i represent the persistence: the higher their sum (in absolute value), the slower the system is forgetting its past history.

Our definition of early warning indicators requires first the identification of the basic ARMA($\overline{p}$,$\overline{q}$) process (with $\overline{p}$ and $\overline{q}$ fixed) which is best suited to describe a Stock index, for a time interval such that it can be considered stationary. This basic process plays the role of an *attractor* in the sense that it contains the information related to the dynamical properties of the system. With respect to the common attractors used in dynamical systems theory, dynamical indicators as the Lyapunov exponents are replaced by the coefficients ϕ_i and θ_j and the analogous of the attractor dimension is the number of terms p and q to be considered. We will comment on these analogies and on the possibility of choosing a reliable ARMA($\overline{p}$,$\overline{q}$) for stock market indexes in the next section. Now we turn to the ARMA based definition of the early warning indicators.

ARMA based Early warning indicators We consider a given Stock index, that is assumed to faithfully reflect the financial or economic conjuncture. In the absence of crisis, such index can be considered as stationary over a given time interval τ and can be fitted by a reference ARMA model. When crisis approach, the volatility of the index increases, and the best ARMA model describing the market will deviate from the basic one. The strongest the crisis, the larger the deviations will be. This suggest to introduce an early warning indicator as a suitable distance in the ARMA space from the reference model. For this, we introduce the Bayesian information criterion (BIC), measuring the relative quality of a statistical model, as:

$$BIC = -2 \ln \hat{L}(n, \hat{\sigma}^2, p, q) + k[\ln(n) + \ln(2\pi)], \tag{2}$$

where $\hat{L}(n, \hat{\sigma}^2, p, q)$ is the likelihood function for the investigated model and in our case $k = p + q$ and n the length of the sample. The variance $\hat{\sigma}^2$ is computed from the sample and is a series-specific quantity.

We can define a normalized distance between the referece ARMA($\overline{p}$, $\overline{q}$) and any other ARMA(p, q) model as the normalized difference between the $BIC(n, \hat{\sigma}^2, p+1, q)$ and the ARMA($\overline{p}$, $\overline{q}$) $BIC(n, \hat{\sigma}^2, \overline{p}, \overline{q})$:

$$\Upsilon = 1 - \exp\left\{|BIC(p+1, q) - BIC(\overline{p}, \overline{q})|\right\}/n. \tag{3}$$

with $0 \leq \Upsilon \leq 1$: it goes to zero if the dataset is well described by an ARMA($\overline{p}$, $\overline{q}$) model and tends to one in the opposite case.

It has been already observed that such indicators perform well in different physical systems, providing more information than the usual ones, based on the critical slow down due to the increase of correlations in the systems at the transition. These analyses have been reported in [12] where indicators similar to Υ have been used to model different physical systems: Ising and Langevin models, climate and turbulence.

3 Assessment of the reference model

We now perform the analysis on different stock indexes: the Dow Jones, the Standard & Poor's and the EURO STOXX 50. We consider databases of such indexes from January 1st 1990 to June 30th 2014 containing daily observations. The considered EURO STOXX 50 series is slightly longer (January 1st 1986 to June 30th 2014).

We first determine the time interval on which the series can be considered stationary. We run the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test for a unit root on the time series with increasing τ for $\tau > 10$ days. The test is successful for $10 < \tau < 90$ days but, as expected, results depend on how close to a crisis the window is taken. In order to preserve enough statistical information we chose a time window $\tau = 60$ days. We tested that results presented are in fact stable for $40 < \tau < 80$ days. We then compute for each index the reference ARMA($\overline{p}$, $\overline{q}$) model using a statistical strategy : for each window $\{X_t, ...X_{t+\tau}\}$ for $t = 1, 2, ..., T - \tau$, being T the total length of the series, we fit the best ARMA(p, q) describing the series in this time lag. We then count the frequency of all ARMA(p,q) that have the same total order $p + q$ and compute histograms of $p + q$. This is shown in Fig. 1 for each of the three indexes. From the figure, we see that there is a peak of probability around $\overline{p} + \overline{q} = 1$. Since p and q are integers, and since we exclude pure moving average (MA(1)) fits, this means that the most probable model has $\overline{p} = 1$, $\overline{q} = 0$. So, for each index, we choose the reference model as an AR(1).

Once the reference model is established, we can now compute the Υ indicator over all the time series, and see how it perform over a database of financial crises and by comparison with other financial indicators linked to the volatility of the markets. We thus consider the database [1] which reports the crises between 1940 and 1999, and the database [19] for the

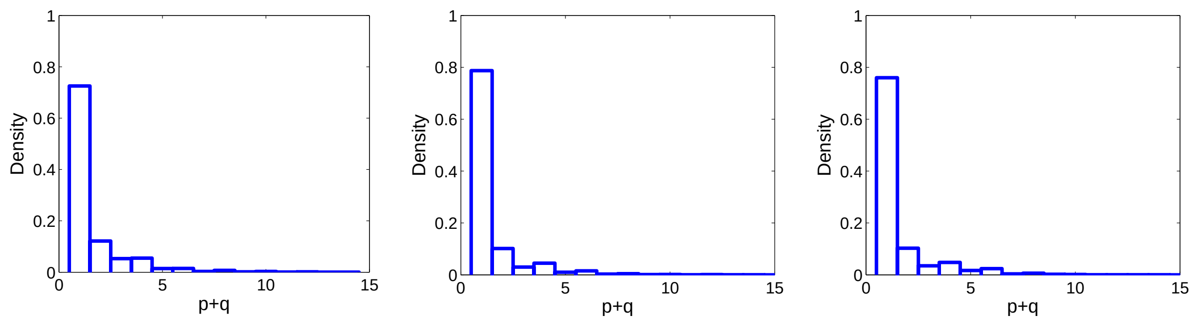

Figure 1: Assessment of the best ARMA($\overline{p}$,$\overline{q}$) model. The histograms represent the number of times (normalized to 1) that a certain $p+q$ ARMA order appear as best fit for sub-series of the originals in running windows of $\tau = 60$ days. Left panel: analysis for the Dow Jones index. Central panel: analysis for the Standard & Poor's index. Right panel: analysis for the EURO STOXX 50 index.

most recent ones. Naturally, we do not expect to get early warning indicators for crises connected to exogenous shocks, such as natural disasters (hurricanes, floods, earthquakes) or terrorism (Oklahoma bombs or September 11th 2001 events). The early warning, if any, should appear before the corresponding crisis.

4 Analysis

For the daily series of the Dow Jones, the Standard & Poor's and the EURO STOXX 50 indexes, we compute the quantity Υ defined in Eq. (3) using a running time windows of $\tau = 60$ days. At day $t + \tau$ we associate the Υ index computed using the observations $\{X_t, ..X_{t+\tau}\}$. We then smooth Υ data by using the moving average method with span of 5 days. We consider only values of $\Upsilon > 0.3$.

For the Dow Jones index results are shown in Figure 2. The upper panel shows the series of Dow Jones from January 1st 1990 to June 30th 2014, the central panel the daily changes of the Dow Jones index and the lower panel Υ early warning indicator (blue), thresholded as described before. The red stars indicate crisis of economical origins as derived from the databases [1], whereas yellow stars refer to crisis of non-economical origin. Early warnings provided by Υ are empirically associated to the respective crisis by red lines. Question marks represent early warnings not associated to effective economical crises. The first thing to remark is that more than one warning is associated to the same crisis and that the interval between the early warning and the crisis is not constant. If we compare these results with the ones arising from physical systems and discussed in [12], the warnings for financial crises seem to appear *too early* with respect to what observed in controllable physical systems. The main difference of markets with respect to natural systems is that the former feel and react to the effect of the crisis by delaying its emergence until the time an official institution points to an economical problem. This usually happens when the crisis itself is unavoidable. For the Dow Jones time series, the matching between crisis and early warnings seem satisfactory although some warnings are of difficult interpretation.

A slightly different scenario appears when the Standard & Poor's stock index analysis is considered, as reported in Fig. 3. For this index some crises are well anticipated by an increase on Υ, some others instead are not captured. In order to explain the difference between the Dow Jones and the Standard & Poor's behavior we have to recall the way they are constructed: The major difference between them is that the Dow Jones includes a price-weighted average of 30 stocks whereas the Standard & Poor's is a market value-weighted index of 500 stocks. We can speculate that for indexes computed on a larger number of companies, crisis early warnings may be averaged out. In fact, if a relevant number of them will not suffer the effects of the crisis, no warning will be provided. Another possibility is that not all these companies have access to latest speculations on the market and therefore most of them cannot react in advance producing no early warnings at all.

The latter analysis concerns the EURO STOXX 50 index, extensevily analysed in [6], and it is reported in Fig. 4. As for the Dow Jones, the number of stocks considered is quite limited and crisis are well highlighted. We can remark that the average delay between early warning and crisis is shorter for the EURO STOXX than for the American indexes. This might be due to the fact that the European market usually follows the warnings and the speculations happening on the American side.

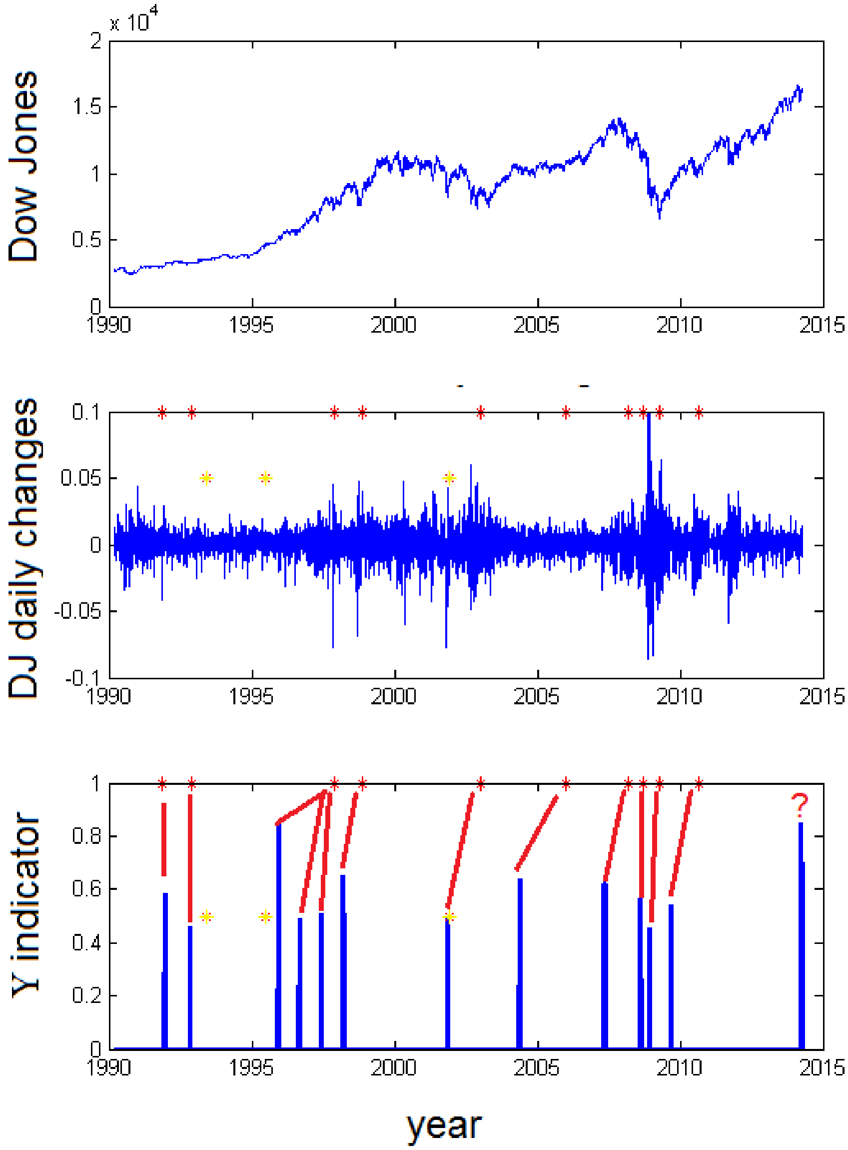

Figure 2: Early detection of crisis based on the Dow Jones stock index analysis. Upper panel: the series of Dow Jones from January 1st 1990 to June 30th 2014. Central panel: daily changes of the Dow Jones index, red stars indicate crisis of economical origin found in the databases [1, 19], yellow stars refer to crisis of non-economical origin. Lower panel: Υ early warning indicator (blue) associated to the respective crisis by red lines. Question marks represent early warnings not associated to effective economical crisis. See text for more descriptions.

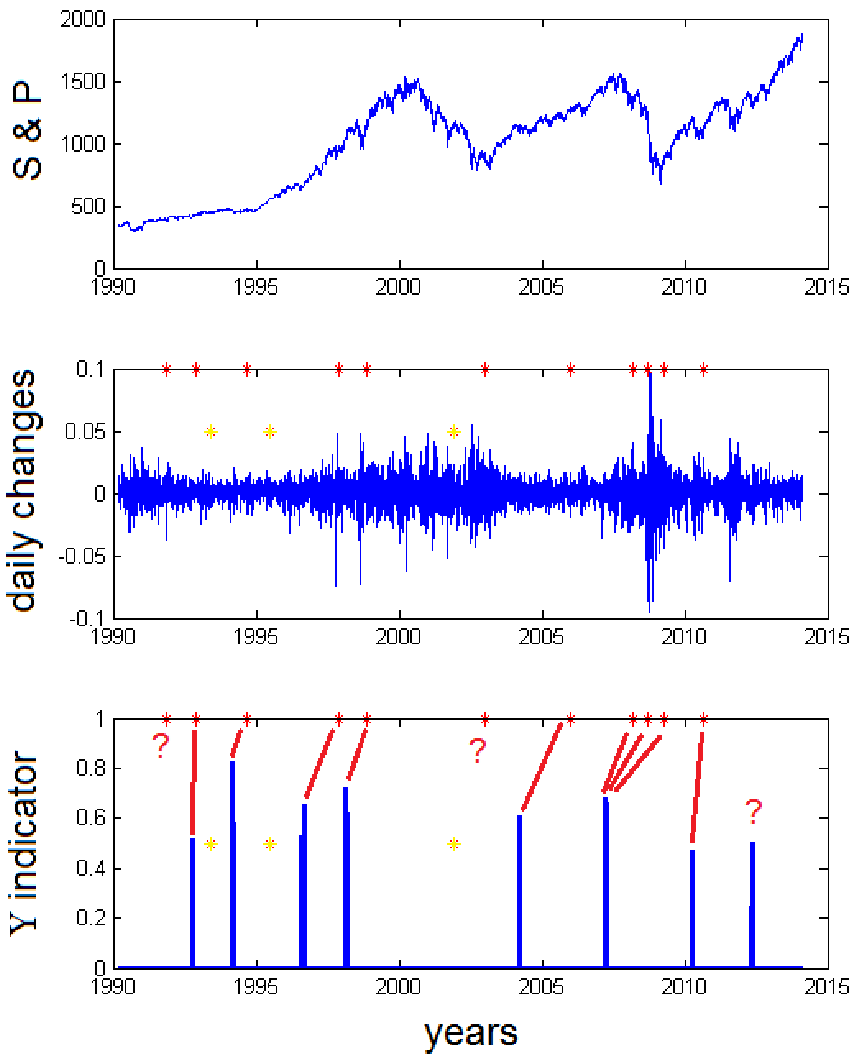

Figure 3: Early detection of crisis based on the Standard & Poor's stock index analysis. Upper panel: the series of Standard & Poor's from January 1st 1990 to June 30th 2014. Central panel: daily changes of the Standard & Poor's index, red stars indicate crisis of economical origin found in the databases [1, 19], yellow stars refer to crisis of non-economical origin. Lower panel: Υ early warning indicator (blue) associated to the respective crisis by red lines. Question marks represent early warnings not associated to effective economical crisis. See text for more descriptions.

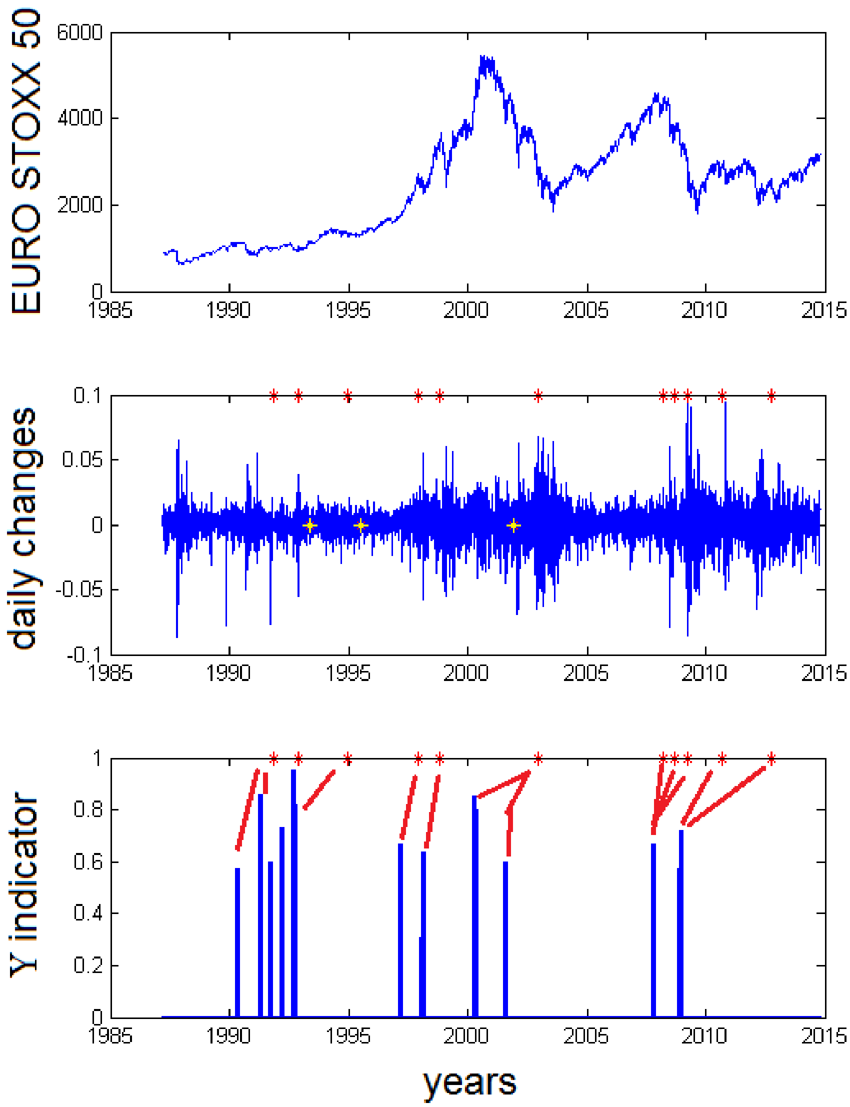

Figure 4: Early detection of crisis based on the EURO STOXX 50 index analysis. Upper panel: the series of EURO STOXX 50 from January 1st 1986 to June 30th 2014. Central panel: daily changes of the EURO STOXX 50 index, red stars indicate crisis of economical origin found in the databases [1, 19], yellow stars refer to crisis of non-economical origin. Lower panel: Υ early warning indicator (blue) associated to the respective crisis by red lines. Question marks represent early warnings not associated to effective economical crisis. See text for more descriptions.

5 Discussion

We have introduced an early warning indicator Υ for financial crises of economical origin based on the ARMA models. The indicator provides, for each day, a [0-1] distance with respect to a reference model ARMA($\overline{p}$, $\overline{q}$) which is able to fit the data on a certain window τ far from the crisis events.

The first result of the paper is that it is possible to statistically deduct such ARMA($\overline{p}$,$\overline{q}$) model which turns out to be the simple AR(1) often emerging in the description of natural phenomena driven by a Langevin equation, such as heavy particles in gas [2], polymers [24] and even turbulence [10]. We strongly remark that such model is not the best model to describe all the data of a specific index, but is the best one on a certain window τ chosen as parameter of the method. In some sense, the introduction of this model plays the role of the *attractor* of a physical system: attractor dimension is replaced by the total terms p and q needed to describe the series and Lyapunov exponents are linked to the magnitude of coefficients ϕ_i and θ_i. In physical systems crisis happen when the system departs from its attractor and explore new portions of the phase space. In terms of ARMA processes, crises happen when the model departs from the reference one and the series is fitted by other orders than $\overline{p}, \overline{q}$. It has been reported in [12] that this picture is true for physical systems ranging from toy models (Ising dynamics, Langevin problem) up to complex systems (turbulent flows).

For stock indexes, the indicator is useful for most of the crises reported in the databases, at least for the years we tested (1990 - 2014). The Υ indicator has different response functions according to the indexes to which it is applied. The larger the number of stocks used to construct the index, the lower is the early warning power of Υ. We have conjectured that such phenomenon is due to the fact that in indexes including more companies early effects of the crisis get averaged out either because some companies is not interested by the crisis, either because they do not have at their disposal any instruments to figure it out.

The instruments currently used to keep track of the markets volatility have been introduced by the CBOE (Chicago Board Options Exhange). Among such indexes, one of the most used is the VIX [16], which measures the expected volatility at 30 days for the

Standard & Poor's. This index has been created for the investors to have an information on the pure volatility of the market in the next month. In some sense, the VIX index should *feel* the modification of the markets and anticipate their change. However, as reported by CBOE [11, 8] VIX was not devised to predict stock prices, the direction of the market and highs or lows. Moreover, our Υ indicator is entirely based on the available data, rather on a forecast as the CBOE indexes. For a such a reason Υ will not be used as a substitute for the VIX index but rather as a completely different financial tool. As explained in [16], the VIX index is used to foresee on a month window the change of volatility in the market, not systemic changes in the behavior of the market. When a statistical comparison between the two indexes is done, the two indexes appear to be not correlated and they do not have the same statistical behavior: by construction the VIX index possess the same long memory and correlation structures of the S& P data series, whereas Υ consists of a series of peaks whose significance appears only when the original data series deviates from the AR(1) model. Υ therefore does not replace the VIX indexes: it is intended to forecast market shocks on the long term and not to foresee short terms volatility fluctuations.

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