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Probabilistic aspects of dynamical systems

It is well known that chaotic dynamical systems are sensitive dependent on initial conditions. Thus, it is not possible to analyze their long term pointwise behavior. However, statistical properties of such systems can be quite stable with respect to small changes of initial conditions. This has motivated the development of a major research direction via an ergodic theoretic approach that uses probabilistic, geometric and functional analytic techniques. In particular, for a dynamical system with some hyperbolic properties, ergodic theory aims to understand whether the long-term behavior of the system is governed by a dynamic equilibrium represented by a Sinai-Ruelle-Bowen (SRB) measure; it enjoys, for a suitable class of observables, a certain rate of correlation decay; it satisfies a Central Limit Theorem (CLT) and other important statistical limit laws.

This focus issue meant to shed some light on recent discoveries in this area of research. In particular, the aim of this issue is to reveal important trends of contemporary research in ergodic theory that aims to expand the understanding of statistical properties of dynamical systems reached in the last 50 years to

(a) (non)uniformly or partially hyperbolic dynamical systems
(b) systems whose natural invariant measure is infinite
(c) unveil new connections with probability theory and stochastic analysis

Indeed, there is a fruitful exchange between probability theory, stochastic analysis and ergodic theory of dynamical systems [3,4,8,9]. The latter often suggest new questions that require advanced probabilistic approaches and tools, and conversely probability theory offers techniques which allow to analyze and understand new fine statistical properties of a dynamical system. Among the probabilistic techniques that had a strong impact in studying dynamical systems is the ‘coupling’ approach introduced to dynamical systems by Young, often called a Young Tower, as a tool to reduce a nonuniformly hyperbolic system or a partially hyperbolic system to a uniformly hyperbolic one. Such a technique is used in [5] to study statistical properties of *almost* Anosov diffeomrphisms.

In parallel to the tower technique, there is a functional analytic approach to study statistical properties of dynamical systems. Although the functional analytic technique is so far mainly limited to uniformly hyperbolic systems, it has the advantage of providing information on the isolated spectrum of the associated transfer operator acting on a suitable Anisotropic Banach space [6]. Moreover, such techniques are amenable to perturbations even in situations where the associated transfer operator does not admit a spectral gap [1].

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However, unlike in the setting of [1,6], not all dynamical systems preserve a probabilistic SRB measure. In fact, many systems preserve a natural measure which is infinite (σ)-finite. Indeed the paper [5] presents important examples of such systems. Moreover, the contributions [2,7] study mixing properties and precise limit laws (in the appropriate sense) for systems that preserve an infinite measure. The papers [2,7] include applications to specific systems such as billiards and maps with indifferent fixed points.

Declaration of Competing Interest

None.

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