Master Physique Fondamentale et Applications

Faculté des sciences amU

# Internship 2026/2027

**Year**: M2

**Laboratory:** Centre de Physique Théorique (CPT)
**Research Team:** Nonlinear dynamics
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**Project Title:** Billiard maps in underwater acoustic

**Keywords:** Chaos, nonlinear dynamics, numerical simulations

**Description:** In the high-frequency limit, underwater acoustic waves admit an elegant Hamiltonian formulation. The resulting Hamiltonian is analogous to that of classical mechanics. The acoustic rays (or rays, for short) correspond to the trajectories governed by the canonical equations. This connection allows to investigate properties of rays using the geometrical and analytical tools of Hamiltonian theory. When the ocean sound-speed profile is allowed to vary with range as well as depth, the system becomes generally nonintegrable. In this regime, rays may exhibit chaotic behavior characterized by sensitive dependence on initial conditions – a phenomenon known as ray chaos, recognised by the community in the early 1990s. See [1-2] for an introduction to the topic.

The objectives of this internship are twofold:

**1. Implementation and numerical treatment of the model.** The first objective is to become familiar with the Hamiltonian formulation of underwater acoustic ray propagation. The student will develop its own symbolic and numerical tools to study the corresponding dynamics. This will involve to implement and validate numerical schemes to integrate numerically ray trajectories. This first stage will provide the foundations for the subsequent investigation of the system's dynamical properties.

**2. Billiard maps and phase-space exploration.** The second and main objective is to investigate the dynamics of rays through a collision-based Poincaré map, analogous to those used in classical billiard systems. The construction allows to reduce the continuous dynamical system to a discrete dynamical system of lower dimension. Instead of sampling trajectories at fixed range intervals (Poincaré range map, see [1,2]), the collision map is based on successive encounters with the boundaries of the waveguide, providing a complementary description of the dynamics. The student will construct and analyse the corresponding map and explore the structure of its phase space for different values of the model parameters (such as the strength and spatial period of the sound-speed perturbation). The goal is to establish a zoology of dynamical regimes, identifying and characterising the main structures that emerge, such as regular regions, chaotic seas, and the possible presence of mixed phase space and sticky trajectories. If time permits, the study will be extended to waveguides with non-flat boundaries

(non-flat sea surface or sea bottom), in order to explore how more complex geometries modify the phase-space structure and the resulting dynamics.

Throughout this project, the student will learn techniques of nonlinear dynamics and more particularly Hamiltonian chaos (phase space visualization and characterization of chaos) in a concrete physical setting. It will follow a set of seminal papers already identified. The internship contains a substantial part of numerical simulations.

**References:**

[1] I. P. Smirnov, A. L. Virovlyansky, and G. M. Zaslavsky, Ray chaos, travel time modulation, and sensitivity to the initial conditions, The Journal of the Acoustical Society of America 117, 1595 (2005)

[2] I. P. Smirnov, A. L. Virovlyansky, and G. M. Zaslavsky, Theory and applications of ray chaos to underwater acoustics, Physical Review E, Volume 64, 036221