Master Physique Fondamentale et Applications

Internship 2026/2027

Year: M2

Laboratory: Centre de Physique Théorique (CPT)
Research Team: Nonlinear dynamics
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Project Title: Variational chaos indicators for hybrid dynamical systems

Keywords: Chaos, nonlinear dynamics, numerical simulations

Description: Chaotic dynamics, characterized by sensitivity to initial conditions, are ubiquitous in nature and arise across a wide range of scales and physical systems, from the Solar System to fluid dynamics, plasma physics, or the motion of artificial satellites orbiting the Earth. The existence of chaos has strong consequences for the predictability of future states of the dynamical system. Despite being deterministic, small uncertainties in the initial conditions can grow rapidly, limiting our ability to make reliable predictions in the long-term. This fundamental property has motivated the development of several computational methods for detecting and characterizing chaotic dynamics over the past several decades.

The objectives of this internship are twofold:

1. Introduction to chaos theory and variational indicators. The first objective is to become familiar with the theory of dynamical systems and chaos, as well as with numerical tools developed to detect and characterize chaotic dynamics in both discrete and continuous-time systems. Particular emphasis will be placed on a class of methods known as variational indicators, which rely on the evolution of deviation vectors. Specific examples include the Lyapunov exponents, or faster to converge indicators, such as the Fast Lyapunov Indicator (FLI), and the Smaller Alignment Index (SALI). See [1] for a pedagogical review on existing methods.

2. Extension to hybrid dynamical systems. The second and main objective is to extend this framework to hybrid dynamical systems, i.e., systems with a dynamic evolving continuously in time but undergoing instantaneous discrete events. A broad and physically relevant class of such systems is provided by billiards (e.g., a particle evolving continuously within a bounded domain and undergoing instantaneous reflections at the boundary). To account for the discrete events, a correction term appears in the variational equations associated to hybrid dynamical systems, see [2]. To the best of our knowledge, fast variational indicators, and their performances and properties, have never been investigated for this class of dynamical system. This is precisely the goal of the internship.

An extension of the internship to a paid research internship is possible (through ANR funding). The developed framework would be applied to a physical problem involving underwater long-range acoustic propagation.

Throughout this project, the student will learn techniques of nonlinear dynamics and more particularly Hamiltonian chaos (dimension reduction, phase space visualization, Lyapunov exponents and characterization of chaos) in a concrete physical setting. It will follow and review a set of seminal papers already identified. The internship contains a substantial part of numerical simulations.

References:

[1] C. H. Skokos, G.A. Gottwald and J. Laskar, eds. C*haos detection and predictability*, Berlin: Springer (2016).
[2] Ch. Dellago and H. A. Posch, Lyapunov instability in a system of hard disks in equilibrium and nonequilibrium steady states, Physical Review E, 53 (1996)