PhD proposal
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Data-driven development of new tools for higher-order networks

Network representations have proved invaluable to describe a large variety of natural and artificial systems: the elements of the system are represented by nodes, and their interactions by links. By design, such representations are however limited to pairwise interactions, and an increasing body of work [1,2] has shown that it can be necessary to go beyond this representation, as interactions between a larger number of components are crucial in many systems, from biological to social contexts (e.g., group discussions in a population).
Taking into group interactions (higher-order interactions, HOIs) leads to new interesting concepts and unveils new physical phenomena [1,2]. However, the resulting higher-order networks are also more complex mathematical objects, namely hypergraphs, where each hyperedge can enclose an arbitrary number of nodes, and their characterization is more complex. Tools and methods adequate to deal with hypergraphs still largely need to be developed. Open questions regard for instance the ability to characterise most connected or important elements and hyperedges within the system, to follow the temporal evolution of HOIs, to find their characteristic timescales and to compare the HOI structures of different systems. Higher-order networks are also the support of dynamical processes and, even if recent advances have tackled their modeling and study, many open questions remain in this context.

The PhD will consist in developing new approaches and tools to deal with higher order networks, in a data-driven fashion based on a number of available data sets such as co-authorship data, group email data, group online forum discussions, brain networks and particularly real-world interactions. In particular:

- we will define tools to characterize relevant and central structures in higher-order networks, adapting and generalizing concepts from network theory such as core-periphery structures or rich-clubs;
- we will consider methods to define a backbone of a higher order network, and possibly methods to reconstruct synthetic data similar to the original one, using the backbone as starting point [3];
- we will develop comparison methods between higher order networks, adapting methods built for networks [4,5,6] and moving to methods specific to higher order such as a comparison between hypercores, a type of structure we have recently defined and analysed [7];
- we will apply these methods to successive snapshots of temporal higher order networks: This will make it possible to extract states and timescales of such systems [8];
- we will expand the analysis of the cohesive structures of static hypergraphs to investigate their temporal stability, defining in particular temporal hypercores and temporally evolving core-periphery or rich club hierarchical organisations, distinguishing e.g. between a hierarchy whose shape evolves or a fixed shape in which elements can move between core and periphery over time.

In all cases, in the spirit of keeping description of systems as concise as possible, and to introduce additional levels of complexity only when needed, we will systematically compare the understanding obtained from using higher-order structures and the one obtained when the HOIs are not taken into account but simply projected on a network representation, and study how the definition of the HOIs from binary data affect these structures. For instance, when dealing with

temporal data, we will combine the tools built using network-based similarity (obtained when the HOIs are ignored and projected on pairwise interactions) and HOI-sensitive similarity measures to detect states and point changes due only to HOIs.
We will validate the methods developed on both on synthetic data (static or temporal) with tunable static and temporal properties, and on empirical data describing systems of different types (co-presence, collaborations, email exchanges, etc).

Finally, we will consider models of dynamical processes involving higher-order structures [9] and study the role of the various structures we have defined in these processes. We will examine e.g. the impact of seeding spreading process in central structures, or, for opinion forming models, of having stubborn individuals forming higher-order stable structures. We will possibly envision adaptive higher-order models in which the higher-order structure evolves as a reaction to the dynamical process unfolding.

The results obtained during the PhD will contribute to shape the currently very active direction of the interdisciplinary science of network towards taking into account higher order interactions in systems of different nature.

Secondment: The PhD will involve a secondment period at the Centai Institute in Turin, Italy, in collaboration with Giovanni Petri.

References
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