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Vulnerability of weighted networks

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Abstract. In real networks complex topological features are often associated with a diversity of interactions as measured by the weights of the links. Moreover, spatial constraints may also play an important role, resulting in a complex interplay between topology, weight, and geography. In order to study the vulnerability of such networks to intentional attacks, these attributes must therefore be considered along with the topological quantities. In order to tackle this issue, we consider the case of the worldwide airport network, which is a weighted heterogeneous network whose evolution and structure are influenced by traffic and geographical constraints. We first characterize relevant topological and weighted centrality measures and then use these quantities as selection criteria for the removal of vertices. We consider different attack strategies and different measures of the damage achieved in the network. The analysis of weighted properties shows that centrality driven attacks are capable of shattering the network's communication or transport properties even at a very low level of damage in the connectivity pattern. The inclusion of weight and traffic therefore provides evidence for the extreme vulnerability of complex networks to any targeted strategy and the need for them to be considered as key features in the finding and development of defensive strategies.

Keywords: network dynamics, random graphs, networks, socio-economic networks, new applications of statistical mechanics

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Contents

1. Introduction

Network representation applies to large communication infrastructures (the Internet, e-mail networks, the World-Wide-Web), transportation networks (railroads, airline routes), biological systems (gene and/or protein interaction networks), and to a variety of social interaction structures [1]–[4]. Very interestingly, many real networks share a certain number of topological properties. For example, most networks are small worlds [5]: the average topological distance between nodes increases very slowly (logarithmically or even slower) with the number of nodes. Additionally, 'hubs' (nodes with very large degree k compared to the mean of the degree distribution $P(k)$) are often encountered. More precisely, the degree distributions exhibit in many cases heavy tails often well approximated for a significant range of values of degree k by a power-law behaviour ($P(k) \sim k^{-\gamma}$) [1, 2] from which the name scale-free networks originated. Real networks are however not only specified by their topology, but also by the dynamical properties of processes taking place on them, such as the flow of information or the traffic among the constituent units of the system. In order to account for these features, the edges are endowed with weights: for example, the air transportation system can be represented by a weighted network in which the vertices are commercial airports and the edges are non-stop passenger flights. In this context, a natural definition of link weights arises, such as the capacities (in terms of number of passengers) of the corresponding flights. Data on real weighted networks (communication and infrastructure networks, scientific collaboration networks, metabolic networks, etc) have been recently studied, giving particular attention to the relation between weight properties and topological quantities [6]–[8]. These findings have also generated several studies concerning modelling approaches in which the mutual influence of weights and topology plays an explicit role in determining the network's properties [9]–[13].

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One of the most striking effects of the complex topological features of networks concerns their vulnerability to attacks and random failures. Compared to 'regular' d-dimensional lattices and random graphs with a bounded degree distribution, heavy tailed networks can tolerate very high levels of random failure [14, 15]. On the other hand, malicious attacks on the hubs can swiftly break the entire network into small components, providing a clear identification of the elements which need the highest level of protection against such attacks [16, 17]. In this context it is therefore important to study how the introduction of traffic and geographical properties may alter or confirm the above findings. In particular we are interested in two main questions:

(i) which measures are best suited to assess the damage suffered by weighted networks and to characterize the most effective attack (protection) strategies;
(ii) how traffic and spatial constraints influence the system's robustness.

In this paper, our attention is therefore focused on weighted networks with geographical embedding and we analyse the structural vulnerability with respect to various centrality driven attack strategies. In particular, we propose a series of topological and weight-dependent centrality measures that can be used to identify the most important vertices of a weighted network. The traffic integrity of the whole network depends on the protection of these central nodes and we apply these considerations to a typical case study, namely the worldwide airport network. We find that weighted networks are even more vulnerable than expected in that the traffic integrity is destroyed when the topological integrity of the network is still extremely high. In addition all attack strategies, both local and non-local, perform with almost the same efficacy. The present findings may help in providing a quantitative assessment of the most vulnerable elements of the network and the development of adaptive reactions aimed at contrasting targeted attacks.

2. Network data set

In the following we use the worldwide air transportation network (WAN), built from the International Air Transportation Association database (www.iata.org). This database contains the direct flight schedules and available seat data from the vast majority of the world's airlines for the year 2002. The network obtained from the IATA database contains $N = 3880$ interconnected airports (vertices) and 18 810 direct flight connections (edges). This corresponds to an average degree of $\langle k \rangle = 9.7$, while the maximal one is $k_{\max} = 318$ showing a strong heterogeneity of the degrees. This is confirmed by the fact that the degree distribution can be described by the functional form $P(k) \sim k^{-\gamma} f(k/k_{\rm c})$, where $\gamma \simeq 2.0$ and $f(k/k_{\rm c})$ is an exponential cut-off which finds its origin in physical constraints on the maximum number of connections that can be handled by a single airport [18, 19]. The WAN is a small world: the average shortest path length, measured as the average number of edges separating any two nodes in the network, is $\langle \ell \rangle = 4.4$. The data contained in the IATA database allow one to go beyond the simple topological representation of the airport connections by obtaining a weighted graph [20] that includes the traffic w_{ij} and actual length d_{ij} of each link, specifying respectively the number of available seats in flights between cities i and j during the year 2002 and the Euclidean distance d_{ij} specifying the route length between cities i and j [6, 19, 21]. The weights are symmetric ($w_{ij} = w_{ji}$) for the vast majority of edges, so we work with a symmetric undirected graph.

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In addition to the very large degree fluctuations, both the weights and the strength are broadly distributed [6, 8] adding another level of complexity in this network.

3. Measures of centrality

A key issue in the characterization of networks is the identification of the most central nodes in the system. Centrality is however a concept that can be quantified by various measures. The degree is a first intuitive and local quantity that gives an idea of the importance of a node. Its natural generalization to a weighted graph is given by the strength of vertices defined for a node i as [22, 6]

$$s_i = \sum_{j \in \mathcal{V}(i)} w_{ij}, \tag{1}$$

where the sum runs over the set $\mathcal{V}(i)$ of neighbours of i. In the case of the air transportation network it quantifies the traffic of passengers handled by any given airport, with both a broad distribution and strong correlations with the degree, of the form $s(k) \sim k^{\beta_s}$ with $\beta_s \approx 1.5$ [6] (a random attribution of weights would lead to $s \sim k$ and thus $\beta_s = 1$).

Since space is also an important parameter in this network, other interesting quantities are the *distance strength* D_i and *outreach* O_i of i:

$$D_i = \sum_{j \in \mathcal{V}(i)} d_{ij}, \qquad O_i = \sum_{j \in \mathcal{V}(i)} w_{ij} d_{ij}, \tag{2}$$

where d_{ij} is the *Euclidean* distance between i and j. These quantities describe the cumulated distances of all the connections from the airport considered and the total distance travelled by passengers from this airport, respectively. They both display broad distributions and grow with the degree as $D(k) \sim k^{\beta_D}$ with $\beta_D \approx 1.5$ [21], and $O(k) \sim k^{\beta_O}$, with $\beta_O \approx 1.8$, showing the existence of important correlations between distances, topology, and traffic.

Such local measures however do not take into account non-local effects, such as the existence of crucial nodes which may have small degree or strength but act as bridges between different parts of the network. In this context, a quantity widely used for investigating node centrality is the so-called betweenness centrality (BC) [23], which counts the fraction of shortest paths between pairs of nodes that passes through a given node. More precisely, if σ_{hj} is the total number of shortest paths from h to j and $\sigma_{hj}(i)$ is the number of these shortest paths that pass through the vertex i, the betweenness of the vertex i is defined as $b_i = \sum_{h,j} \sigma_{hj}(i)/\sigma_{hj}$, where the sum is over all the pairs with $j \neq h$. Key nodes are thus part of more shortest paths within the network than less important nodes.

In weighted networks, unequal link capacities make some specific paths more favourable than others in connecting two nodes of the network. It thus seems natural to generalize the notion of betweenness centrality through a *weighted betweenness centrality* in which shortest paths are replaced with their weighted versions. A straightforward way to generalize the hop distance (number of traversed edges) in a weighted graph consists in assigning to each edge (i, j) a length ℓ_{ij} that is a function of the characteristics of the link (i, j). For example for the WAN, ℓ_{ij} should involve quantities such as the weight w_{ij} or the Euclidean distance d_{ij} between airports i and j. It is quite natural to assume that the effective distance between two linked nodes is a decreasing function of the weight of the link: the larger the flow (traffic) on a path, the more frequent and the faster will be the exchange of physical quantities (e.g. information, people, goods, energy). In other words, we consider that the 'separation' between nodes i and j decreases as w_{ij} increases. While a first possibility would be to define the length of an edge as the inverse of the weight, $\ell_{i,j} = 1/w_{ij}$, we propose to also take into account the geographical embedding of the network, through the following definition:

$$\ell_{ij} = \frac{d_{ij}}{w_{ij}}. \qquad (3)$$

It is indeed reasonable to consider two nodes of the networks as further apart if their geographical distance is larger; however, a large amount of traffic allows one to decrease the 'effective' distance by providing more frequent travel possibilities.

For any two nodes h and j, the weighted shortest path between h and j is the one for which the total sum of the lengths of the edges forming the path from h to j is minimum, independently of the number of traversed edges. We denote by σ^{w}_{hj} the total number of weighted shortest paths from h to j and by $\sigma^{\mathrm{w}}_{hj}(i)$ the number of them that pass through the vertex i; the weighted betweenness centrality (WBC) of the vertex i is then defined as

$$b^{\mathrm{w}}_i = \sum_{h,j} \frac{\sigma^{\mathrm{w}}_{hj}(i)}{\sigma^{\mathrm{w}}_{hj}}, \qquad (4)$$

where the sum is over all the pairs with $j \neq h$.[4] The weighted betweenness represents a trade-off between the finding of 'bridges' that connect different parts of a network, and taking into account the fact that some links carry more traffic than others. We note that the definition (4) is very general and can be used with any definition of the effective length of an edge ℓ_{ij}.

Centrality measure correlations

The probability distributions of the various definitions of centrality are all characterized by heavy tailed distributions. In addition a significant level of correlation is observed: vertices that have a large degree have also typically large strength and betweenness. When a detailed analysis of the different rankings is done, however, we observe that they do not coincide exactly. For example, in the case of the WAN the most connected airports do not necessarily have the largest betweenness centrality [18, 19, 21]. Large fluctuations between centrality measures also appear when inspecting the list of the airports ranked by using different definitions of centrality including weighted ones: strikingly, each definition provides a different ranking. In addition, some airports which are very central according to a given definition become peripheral according to another criterion. For example, Anchorage has a large betweenness centrality but ranks only 138th and 147th in terms of degree and strength, respectively. Similarly, Phoenix or Detroit have large strength but low ranks (>40) in terms of degree and betweenness.

[4] As already noted by Brandes, the algorithm proposed in [26] can be easily extended to weighted graphs, using in addition Dijkstra's algorithm [27] which provides a way to compute weighted shortest paths in at most $\mathcal{O}(EN)$ where E is the number of edges.

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Table 1. Similarity between the various rankings as measured by Kendall's τ. For random rankings of N values, the typical τ is of order 10^{-2}.

	k	D	s	O	BC	WBC
Degree k	1	0.7	0.58	0.584	0.63	0.39
Distance strength D	0.7	1	0.56	0.68	0.48	0.23
Strength s	0.58	0.56	1	0.83	0.404	0.24
Outreach O	0.584	0.68	0.83	1	0.404	0.21
Betweenness BC	0.63	0.48	0.404	0.404	1	0.566
Weighted BC	0.39	0.23	0.24	0.21	0.566	1

While previous analyses have focused on the quantitative correlations between the various centrality measures, here we focus on ranking differences according to the various centrality measures. A quantitative analysis of the correlations between two rankings of n objects can be done using rank correlations such as Kendall's τ [24]:

$$\tau = \frac{n_{\mathrm{c}} - n_{\mathrm{d}}}{n(n-1)/2} \qquad (5)$$

where n_{c} is the number of pairs whose order does not change in the two different lists and n_{d} is the number of pairs whose order was inverted. This quantity is normalized between -1 and 1: $\tau = 1$ corresponds to identical ranking while $\tau = 0$ is the average for two uncorrelated rankings and $\tau = -1$ is a perfect anticorrelation.

Table 1 gives the values of τ for all the possible pairs of centrality rankings. For $N = 3880$, two random rankings yield a typical value of $\pm 10^{-2}$, so even the smallest observed $\tau = 0.21$ is a sign of a strong correlation. (All the values in this table were already attained for a sublist of only the first n most central nodes, with $n \approx 500$.) Remarkably, even a highly non-local quantity such as the BC is strongly correlated with the simplest local, non-weighted measure given by the degree. The weighted betweenness is the least correlated with the other measures (except with the betweenness), because ℓ_{ij} involves ratios of weights and distances.

Another important issue concerns how the centrality ranking relates to the geographical information available for infrastructure networks such as the WAN. Figure 1 displays the geographical distribution of the world's fifteen most central airports ranked according to different centrality measures. This figure highlights the properties and biases of the various measures: on one hand, topological measures miss the economical dimension of the worldwide airport while weighted measures reflect traffic and economical realities. Betweenness based measures on the other hand pinpoint the most important nodes in each geographical zone. In particular, the weighted betweenness appears as a balanced measure which combines traffic importance with topological centrality, leading to a more uniform geographical distribution of the most important nodes.

4. Vulnerability of weighted networks

4.1. Damage characterization

The example of the WAN enables us to raise several questions concerning the vulnerability of weighted networks. The analysis of complex network robustness has indeed been

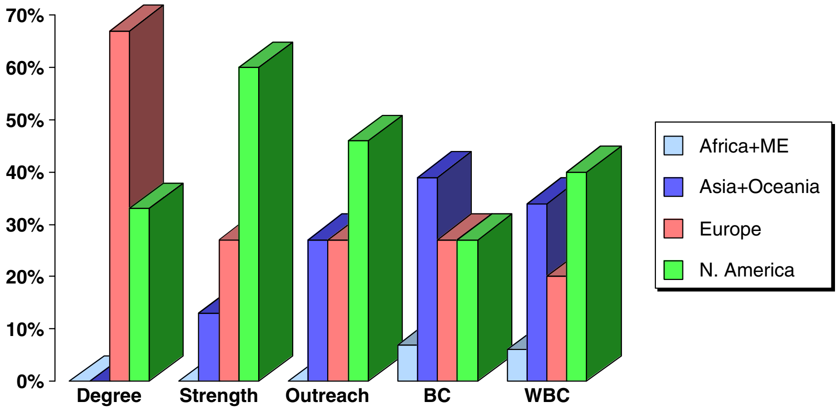

Figure 1. Geographical distribution of the world's 15 most central airports ranked according to different centrality measures. Topological measures miss the economical dimension of the worldwide airport. In contrast, the traffic aspect shows a clear dominance of North America. Non-local measures pinpoint important nodes in each geographical zone.

largely investigated in the case of unweighted networks [16, 14, 15, 25]. In particular, the topological integrity of the network N_g/N_0 has been studied, where N_g is the size of the largest component after a fraction g of vertices have been removed and N_0 is the size of the original (connected) network. When $N_g \simeq \mathcal{O}(1)$, the entire network has been destroyed[5].

Damage is generally studied for increasingly larger fractions g of removed nodes in the network, where the latter are chosen following different strategies. Heterogeneous networks with a scale-free degree distribution are robust against situations in which the damage affects nodes randomly. On the other hand, the targeted destruction of nodes following their degree rank is extremely effective, leading to the total fragmentation of the network at very low values of g [14]–[16]. Moreover, the removal of the nodes with largest betweenness typically leads to an even faster destruction of the network [25].

In the case of weighted networks, the quantification of the damage should consider also the presence of weights. In this perspective, the largest traffic or strength still carried by a connected component of the network is probably an important indicator of the network's functionality. For this reason, we define new measures for the network's damage:

$$I_{\mathrm{s}}(g) = \frac{\mathcal{S}_g}{\mathcal{S}_0}, \qquad I_{\mathrm{O}}(g) = \frac{\mathcal{O}_g}{\mathcal{O}_0}, \qquad I_{\mathrm{D}}(g) = \frac{\mathcal{D}_g}{\mathcal{D}_0}, \tag{6}$$

where $\mathcal{S}_0 = \sum_i s_i$, $\mathcal{O}_0 = \sum_i O_i$ and $\mathcal{D}_0 = \sum_i D_i$ are the total strength, outreach, and distance strength in the undamaged network and $\mathcal{S}_g = \max_G \sum_{i\in G} s_i$, $\mathcal{O}_g = \max_G \sum_{i\in G} O_i$, and $\mathcal{D}_g = \max_G \sum_{i\in G} D_i$ correspond to the largest strength, outreach, or distance strength carried by any connected component G in the network, after the removal of a density g of nodes. These quantities measure the *integrity* of the network with respect to either strength, outreach, or distance strength, since they refer to the relative traffic or flow that is still handled in the largest operating component of the network.

[5] Since the topological integrity focuses only on the largest component and overlooks the connectivity of smaller components, one can also monitor the average inverse geodesic length, also called the efficiency [28].

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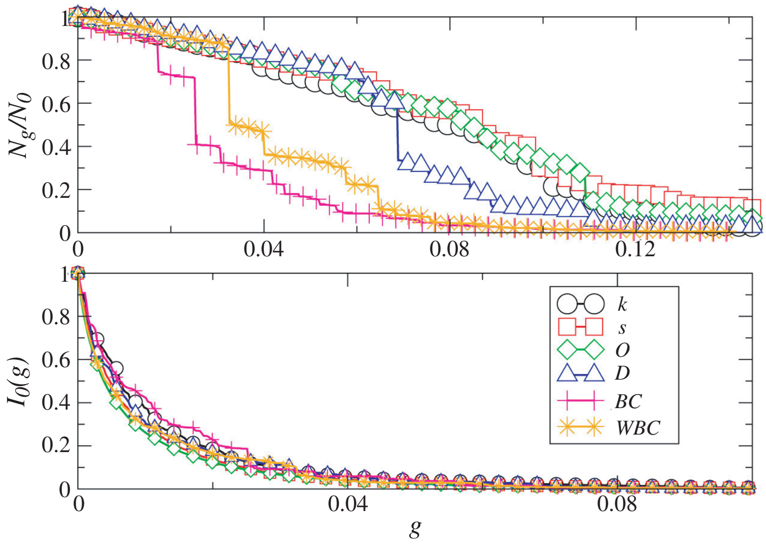

Figure 2. Effect of different attack strategies on the size of the connected giant component (top) and on the outreach (bottom).

4.2. Variable-ranking attack strategies

In order to evaluate the vulnerability of the air transportation network WAN, we study the behaviour of damage measures in the presence of a progressive random damage and of different attack strategies. Similarly to the simple topological case, weighted networks are inherently resilient to random damages. Even at a large density g of removed nodes, N_g/N_0 and all integrity measures decrease mildly and do not seem to have a sharp threshold above which the network is virtually destroyed. This is in agreement with the theoretical prediction for the absence of a percolation threshold in highly heterogeneous graphs [14, 15]. Very different is the scenario corresponding to the removal of the most central nodes in the network. In this case, however, we can follow various strategies based on the different definitions for the centrality ranking of the most crucial nodes: nodes can indeed be eliminated according to their rank in terms of degree, strength, outreach, distance strength, topological betweenness, and weighted betweenness. In addition, we consider attack strategies based on a recursive recalculation of the centrality measures on the network after each damage. This has been shown to be the most effective strategy [25], as each node removal leads to a change in the centrality properties of the other nodes. Such a procedure is somewhat akin to a cascading failure mechanism in which each failure triggers a redistribution on the network and changes the next most vulnerable node.

In figure 2 we report the behaviour of N_g/N_0 and of the outreach integrity $I_{\mathrm{O}}(g)$ for all cases. As expected, all strategies lead to a rapid breakdown of the network with a very small fraction of removed nodes. More precisely, the robustness level of the network depends on the quantity under scrutiny. First, the size of the giant component decreases faster upon removal of nodes which are identified as central according to global (i.e. betweenness) properties, instead of local ones (i.e. degree, strength), showing that, in order to preserve the structural integrity of a network, it is necessary to protect not only

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the hubs but also strategic points such as bridges and bottleneck structures. Indeed, the betweenness, which is recomputed after each node removal, is the most effective quantity for pinpointing such nodes [25]. The weighted betweenness combines shortest paths and weights and leads to an intermediate result: some of the important topological bridges carry a small amount of traffic and are therefore part of more shortest paths than weighted shortest paths. These bridges have therefore a lower rank according to the weighted betweenness. The weighted betweenness is thus slightly less efficient for identifying bridges. Finally, we note that all locally defined quantities yield a slower decrease of N_g and that the removal of nodes with the largest distance strength is rather effective since it targets nodes which connect very distant parts of the network.

Interestingly, when the attention shifts on the behaviour of the integrity measures, one finds a different picture in which all the strategies achieve the same level of damage (the curves of $I_{\mathrm{s}}(g)$ and $I_{\mathrm{D}}(g)$ present shapes very close to the one of $I_{\mathrm{O}}(g)$). Most importantly, their decrease is even faster and more pronounced than for topological quantities: for N_g/N_0 still of the order of 80%, the integrity measures are typically smaller than 20%. This emphasizes how the purely topological measure of the size of the largest component does not convey all the information needed. In other words, the functionality of the network can be temporarily jeopardized in terms of traffic even if the physical structure is still globally well connected. This implies that weighted networks appear more fragile than would be thought from considering only topological properties. All targeted strategies are very effective in dramatically damaging the network, reaching the complete destruction at a very small threshold value of the fraction of removed nodes. In this picture, the maximum damage is achieved still by strategies based on non-local quantities such as the betweenness which lead to a very fast decrease of both topological and traffic related integrity measures. On the other hand, the results for the integrity show that the network may unfortunately be substantially harmed also by using strategies based on local quantities more accessible and easy to calculate.

4.3. Single-ranking attack strategies

The previous strategies based on a recursive recalculation of the centrality measures on the network are however computationally expensive and depend upon a global knowledge of the effect of each node removal. It is therefore interesting to quantify the effectiveness of such a strategy with respect to the more simple use of the ranking information obtained for the network in its integrity. In this case the nodes are removed according to their initial ranking calculated for the undamaged network. As shown in figure 3, successive removals of nodes according to their initial outreach or BC lead to a topological breakdown of the network which is maximized in the case of recalculated quantities [25]. This effect is very clear in the case of global measures of centrality such as the betweenness that may be altered noticeably by local rearrangements. When traffic integrity measures are studied, however, differences are negligible (figure 3, bottom curves): a very fast decrease of the integrity is observed for all strategies, based either on initial or recalculated quantities. The origin of the similarity between the two strategies can be traced back by studying how much the centrality ranking of the network vertices is scrambled during the damage process. In order to quantify the reshuffling of the ranking of the nodes according to various properties, we study the previously used rank correlation as measured

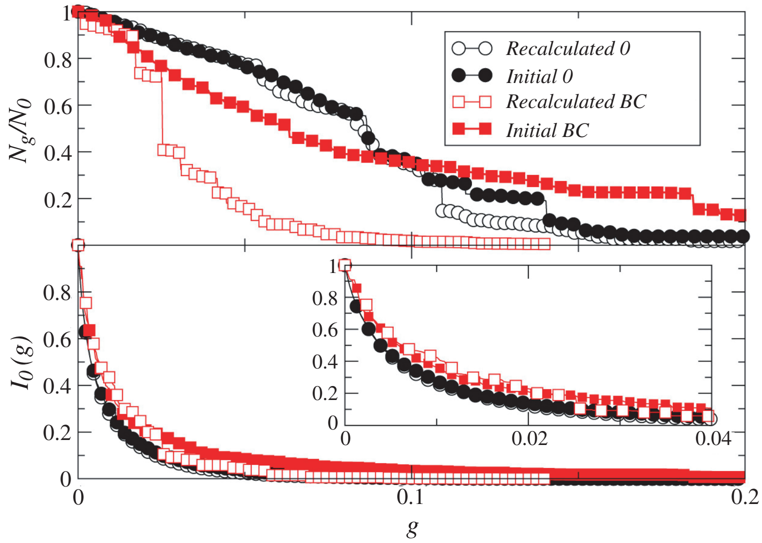

Figure 3. Removal of nodes according to the ranking calculated at the beginning of the process (empty symbols) or to recalculated rankings (full symbols). The decreases of N_g and $I_\mathrm{O}(g)$ are comparable for both cases. Inset: initial decrease of $I_\mathrm{O}(g)$ for very small values of g.

by Kendall's τ, computed between the rankings of the nodes according to a given property before and after each removal. In all cases, τ remains very close to 1, showing that the reshuffling caused by any individual removal remains extremely limited. Slightly smaller values are observed when we compare the rankings of the betweenness or of the weighted betweenness. This can be understood since such quantities are non-local and the betweennesses is more prone to vary when any node in the network is removed. This evidence brings both good and bad news concerning the protection of large scale infrastructures. On one hand, the planning of an effective targeted attack needs only to gather information on the initial state of the network. On the other hand, the identification of crucial nodes to protect is an easier task that is somewhat weakly dependent on the attack sequence.

4.4. Geographical heterogeneity

As shown in figure 1, various geographical zones contain different numbers of central airports. The immediate consequence is that the different strategies for node removal have different impacts in different geographical areas. Figure 4 highlights this point by showing the decrease of two integrity measures representative of topological and traffic integrity. These quantities were measured on subnetworks corresponding to the six following regions: Africa, Asia, Europe, Latin and North America, and Oceania. Figure 4 displays the case of a removal of nodes according to their strength (other removal strategies lead to similar data). While the curves of topological damage are rather intertwined, the decrease of the different integrity measures is much faster for North America, Asia, and Europe than Africa, Oceania, and Latin America; in particular the removal of the first nodes does not

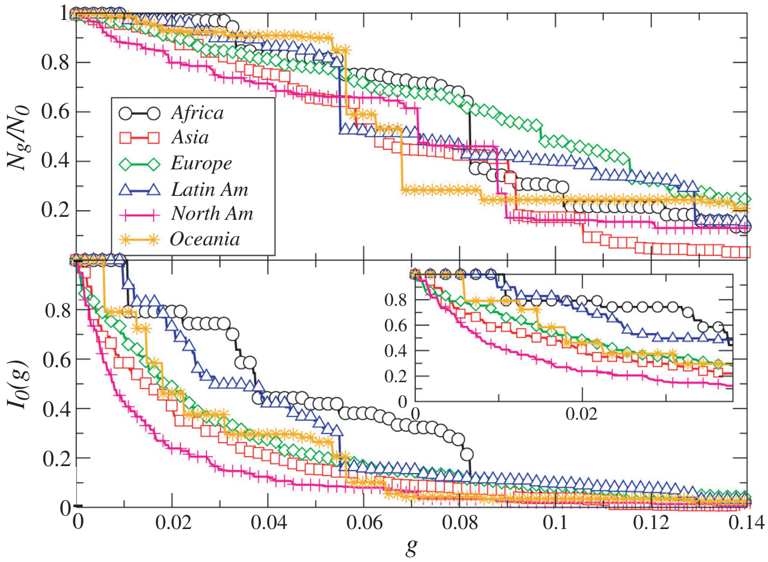

Figure 4. Geographical effect of the removal of nodes with largest strength. The integrity decreases strongly in regions such as North America, while a 'delay' is observed for the zones with smaller initial outreach or strength.

affect these last three zones at all. Such plots demonstrate two crucial points. First, various removal strategies damage differently the various geographical zones. Second, the amount of damage according to a given removal strategy strongly depends on the precise measure used to quantify the damage. More generally, these results lead to the idea that large weighted networks can be composed from different subgraphs with very different traffic structures and thus different responses to attacks.

5. Conclusions

In summary, we have identified a set of different but complementary centrality measures for weighted networks. The various definitions of centrality are correlated but lead to different rankings since different aspects (weighted or topological, and local or global) are taken into account. The study of the vulnerability of weighted networks to various targeted attack strategies shows that complex networks are more fragile than expected from the analysis of topological quantities when the traffic characteristics are taken into account. In particular, the network's integrity in terms of traffic carried is vanishing significantly before the network is topologically fragmented. Moreover, we have compared attacks based on initial centrality ranking with those using quantities recalculated after each removal, since any modification of the network (e.g. a node removal) leads to a partial reshuffling of these rankings. Strikingly, and in contrast to the case for purely topological damage, the integrity of the network is harmed in very similar manners in the two cases. All these results warn about the extreme vulnerability of the traffic properties of weighted networks, and signals the need to pay particular attention to weights and traffic in the design of protection strategies.

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References

[1] Barabási A-L and Albert R, 2002 *Rev. Mod. Phys.* **74** 47
[2] Dorogovtsev S N and Mendes J F F, 2003 *Evolution of Networks: from Biological Nets to the Internet and WWW* (Oxford: Oxford University Press)
[3] Pastor-Satorras R and Vespignani A, 2004 *Evolution and Structure of the Internet: A Statistical Physics Approach* (Cambridge: Cambridge University Press)
[4] Amaral L A N, Scala A, Barthélemy M and Stanley H E, 2000 *Proc. Nat. Acad. Sci.* **97** 11149
[5] Watts D J and Strogatz S H, 1998 *Nature* **393** 440
[6] Barrat A, Barthélemy M, Pastor-Satorras R and Vespignani A, 2004 *Proc. Nat. Acad. Sci.* **101** 3747
[7] Almaas E, Kovács B, Viscek T, Oltvai Z N and Barabási A-L, 2004 *Nature* **427** 839
[8] Li W and Cai X, 2004 *Phys. Rev.* E **69** 046106
[9] Barrat A, Barthélemy M and Vespignani A, 2004 *Phys. Rev. Lett.* **92** 228701
[10] Barrat A, Barthélemy M and Vespignani A, 2004 *Phys. Rev.* E **70** 066149
[11] Bianconi G, 2005 *Europhys. Lett.* **71** 1029
[12] Dorogovtsev S N and Mendes J F F, 2004 *Preprint* cond-mat/0408343
[13] Wang W-X, Wang B-H, Hu B, Yan G and Ou Q, 2005 *Phys. Rev. Lett.* **94** 188702
[14] Cohen R, Erez K, ben-Avraham D and Havlin S, 2000 *Phys. Rev. Lett.* **85** 4626
[15] Callaway D S, Newman M E J, Strogatz S H and Watts D J, 2000 *Phys. Rev. Lett.* **85** 5468
[16] Albert R A, Jeong H and Barabási A-L, 2000 *Nature* **406** 378
[17] Pastor-Satorras R and Vespignani A, 2001 *Phys. Rev. Lett.* **86** 3200
[18] Guimerà R, Mossa S, Turtschi A and Amaral L A N, 2005 *Proc. Nat. Acad. Sci.* **102** 7794
[19] Guimerà R and Amaral L A N, 2004 *Eur. Phys. J.* B **38** 381
[20] Clark J and Holton D A, 1998 *A First Look at Graph Theory* second reprint (Singapore: World Scientific)
[21] Barrat A, Barthélemy M and Vespignani A, 2005 *J. Stat. Mech.* P05003
[22] Yook S H, Jeong H, Barabasi A-L and Tu Y, 2001 *Phys. Rev. Lett.* **86** 5835
[23] Freeman L, 1977 *Sociometry* **40** 35
[24] Press W H, Flannery B P, Teukolsky S A and Vetterling W T, 1992 *Numerical Recipes in Fortran* 2nd edn (Cambridge: Cambridge University Press)
[25] Holme P, Kim B J, Yoon C N and Han S K, 2002 *Phys. Rev.* E **65** 056109
[26] Brandes U, 2001 *J. Math. Sociol.* **25** 35
[27] Dijkstra E W, 1959 *Numer. Math.* **1** 269
[28] Latora V and Marchiori M, 2001 *Phys. Rev. Lett.* **87** 198701

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