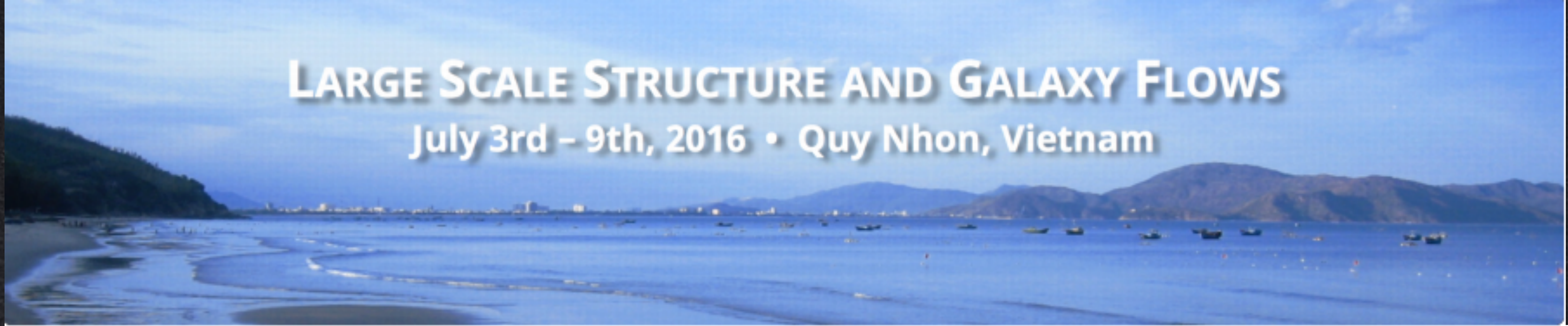
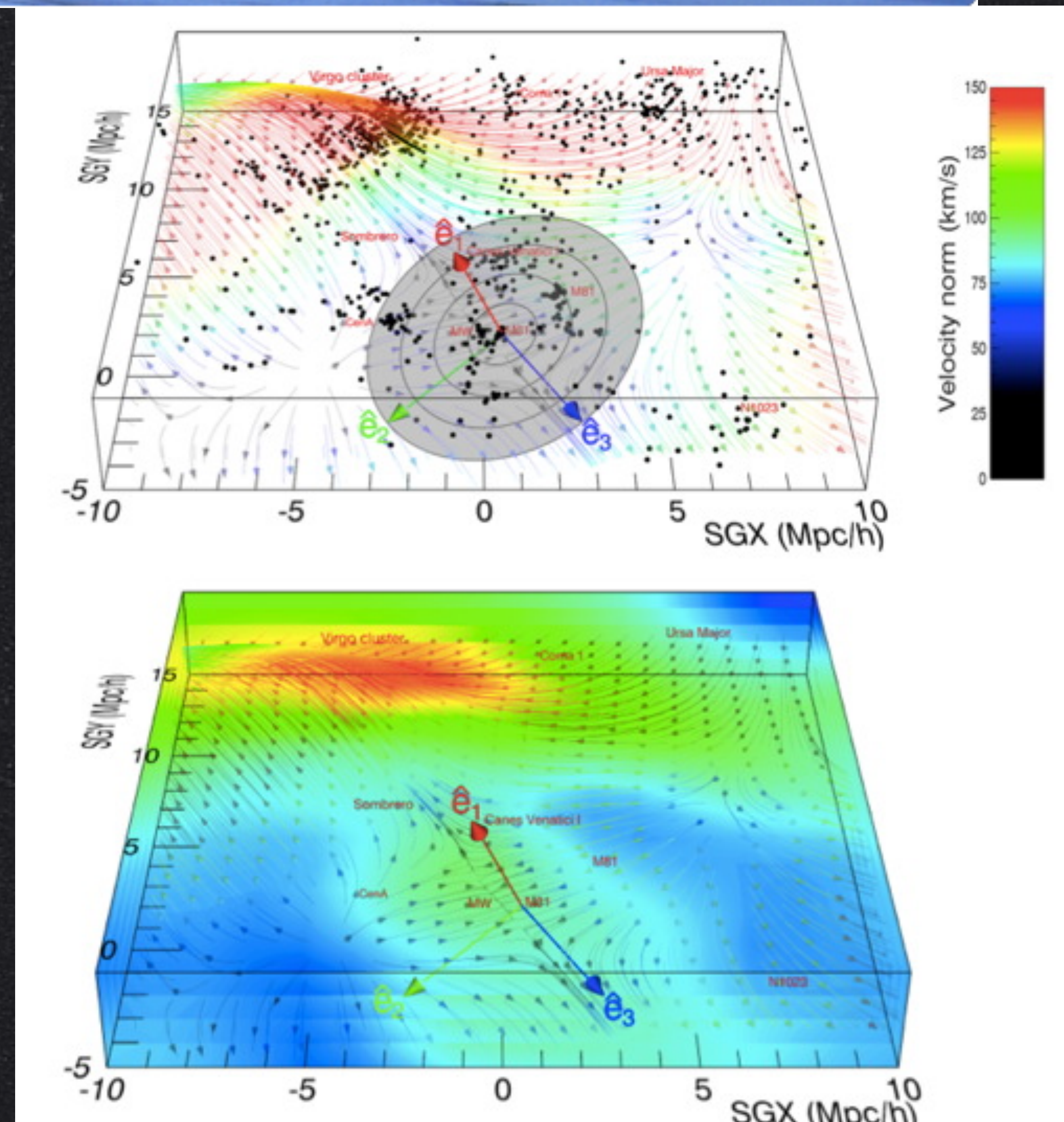


Convergence of Cosmic Flows: Problems and Opportunities

Velocity Correlation Function



Hume A. Feldman

Physics & Astronomy

University of Kansas

Likelihood Methods for Peculiar Velocities

A catalog of peculiar velocities galaxies, labeled by an index n

Positions r_n

Line-of-sight peculiar velocity estimators S_n

Uncertainties σ_n

Assume that observational errors are Gaussian distributed.

Estimators

Determination of the line-of-site peculiar (local) motion requires the measurement of the galaxy's distance

$$v = cz - H_o r$$

At large distances, we can include the effects of cosmic acceleration

$$z_{\rm mod} = z[1 + 0.5(1 - q_o)z - (1/6)(1 - q_o - 3q_o^2 + 1)z^2]$$

Since redshift is not an additive quantity

See also Davis & Scrimgeour 2014; Springob et al. 2014

$$(1 + z_{\rm mod}) = (1 + H_o r/c)(1 + v/c)$$

The peculiar velocity is

$$v = \frac{cz_{\rm mod} - H_o r}{1 + H_o r/c} \approx \frac{cz_{\rm mod} - H_o r}{1 + z_{\rm mod}}$$

Estimators

Determination of the line-of-site peculiar (local) motion requires the measurement of the galaxy's distance

Distance estimators give the distance moduli (μ), that is, log distances with Gaussian distributed errors.

Distance errors are skewed, not Gaussian and with non-zero average.

$$\langle r_e \rangle \neq r \qquad \Rightarrow \qquad \langle v_e \rangle \neq v$$

These undesirable features can lead to biases and invalidate our statistical assumptions about the errors in peculiar velocities.

Estimators

Proposal: estimate peculiar velocities using

$$v_e = cz \log(cz/H_o r_e)$$

Watkins & Feldman, 2015, MNRAS 450, 1868

log distance $\Rightarrow$ Gaussian distributed errors.

The uncertainty in the peculiar velocity

$$\delta v_e = cz\delta l_e$$

Uncertainty in the log distance

and

$$\langle \log(r_e) \rangle = \langle \log r \rangle$$

$$\Rightarrow \qquad \langle v_e \rangle \approx v$$

as long as the true $v \ll cz$

Estimators

A more accurate estimator at large redshift

$$v_e = \frac{cz_{mod}}{(1+z_{mod})} \log(cz_{mod}/H_o r_e)$$

with uncertainty

$$\delta v_e = cz_{mod} \delta l_e / (1 + z_{mod})$$

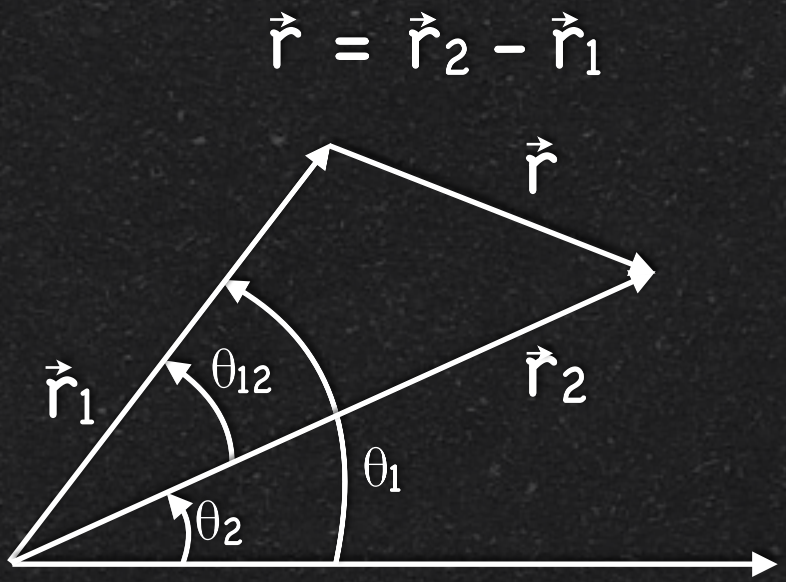
We assume that actual velocity (v) is small compared to the redshift, not the estimated velocity (v_e).

$v_e = \text{few} \times 10^3 \, \text{km/s}$ whereas $v = \text{few} \times 10^2 \, \text{km/s}$

Should hold quite well for galaxies at distances $\gtrsim 20 \, \text{Mpc}$

Wang, Rooney, Feldman & Watkins

Velocity Correlations



$$\Psi_{ij}(\vec{r}) = \langle v_i(\vec{r}_1) v_j(\vec{r}_2) \rangle$$ Correlation Tensor

$$\vec{u}_1 = \hat{r}_1 u_1 = \hat{r}_1(\hat{r}_1 \cdot \vec{v}_1)$$

The observable quantity

line-of-site peculiar velocities

$$\cos\theta_1 = \hat{r}_1 \cdot \hat{r} \qquad \cos\theta_2 = \hat{r}_2 \cdot \hat{r} \qquad \cos\theta_{12} = \hat{r}_1 \cdot \hat{r}_2$$

The Line-of-site velocity correlations

$$\psi_1(r) \equiv \frac{\Sigma_{\text{pairs}(r)} u_1 u_2 \cos\theta_{12}}{\Sigma_{\text{pairs}(r)} \cos^2\theta_{12}},$$

The dot product of the radial peculiar velocities

$$\psi_2(r) \equiv \frac{\Sigma_{\text{pairs}(r)} u_1 u_2 \cos\theta_1 \cos\theta_2}{\Sigma_{\text{pairs}(r)} \cos\theta_{12} \cos\theta_1 \cos\theta_2}.$$

The product of the components of the Line-of-site velocities along the vector separating the two galaxies

Górski, 1988, ApJL, 332, L7
Górski, Davis, Strauss, White & Yahil, 1989, ApJ 344: 1-19
Torment etal 1993, ApJ 411, 16-33
Brogani, etal, 2000, AJ, 119, 102

Velocity Correlations

The line-of-site velocity correlations are related to the peculiar velocity correlations

$$\langle u_{1i} u_{2j} \rangle = \hat{r}_{1i} \hat{r}_{2j} \langle v_m v_n \rangle \hat{r}_{1m} \hat{r}_{2n}$$

where

$$\langle v_m v_n \rangle = \left[\phi_{\|}(r) - \phi_{\perp}(r) \right] \hat{r}_m \hat{r}_n + \phi_{\perp}(r) \delta_{mn}$$

Monin and Yaglom, 1975
Górski, 1988, ApJL, 332, L7

We can express ψ_1 and ψ_2 in terms of $\phi_{\|}$ and $\phi_{\perp}$ assuming homogeneity and isotropy

$$\psi_1(r) = A(r)\phi_{\|} + [1 - A(r)]\,\phi_{\perp}$$
$$\psi_2(r) = B(r)\phi_{\|} + [1 - B(r)]\,\phi_{\perp}$$

Where A and B are the moments of the selection function

$$A(r) = \frac{\Sigma_{\mathrm{pairs}(r)} \cos\theta_1 \cos\theta_2 \cos\theta_{12}}{\Sigma_{\mathrm{pairs}(r)} \cos^2\theta_{12}}$$

$$B(r) = \frac{\Sigma_{\mathrm{pairs}(r)} \cos^2\theta_1 \cos^2\theta_2}{\Sigma_{\mathrm{pairs}(r)} \cos\theta_{12} \cos\theta_1 \cos\theta_2}$$

Velocity Correlations

Inverting to get the Line-of-site Radial and Transverse Velocity Correlations

$$\phi_{\|}(r) = \frac{[1 - B(r)]\,\psi_1(r) - [1 - A(r)]\,\psi_2(r)}{A(r) - B(r)}$$

$$\phi_{\perp}(r) = \frac{B(r)\psi_1(r) - A(r)\psi_2(r)}{B(r) - A(r)}$$

In general, use the distance estimator as r.

At large distances the redshift is a better estimator of the distance.

Use redshift measurements as the distances in the velocity correlations analysis.

Velocity Correlations

Working equations

The Line-of-site Velocity Correlations

$$\psi_1(r) \equiv \frac{\sum_{\mathrm{pairs}(r)} u_1 u_2 \cos\theta_{12}}{\sum_{\mathrm{pairs}(r)} \cos^2\theta_{12}} ,$$

$$\psi_2(r) \equiv \frac{\sum_{\mathrm{pairs}(r)} u_1 u_2 \cos\theta_1 \cos\theta_2}{\sum_{\mathrm{pairs}(r)} \cos\theta_{12} \cos\theta_1 \cos\theta_2} .$$

The Radial and Transverse Velocity Correlations

$$\phi_\parallel(r) = \frac{[1 - B(r)]\,\psi_1(r) - [1 - A(r)]\,\psi_2(r)}{A(r) - B(r)}$$

$$\phi_\perp(r) = \frac{B(r)\psi_1(r) - A(r)\psi_2(r)}{B(r) - A(r)}$$

The Moments of the Selection Functions

$$A(r) = \frac{\sum_{\mathrm{pairs}(r)} \cos\theta_1 \cos\theta_2 \cos\theta_{12}}{\sum_{\mathrm{pairs}(r)} \cos^2\theta_{12}}$$

$$B(r) = \frac{\sum_{\mathrm{pairs}(r)} \cos^2\theta_1 \cos^2\theta_2}{\sum_{\mathrm{pairs}(r)} \cos\theta_{12} \cos\theta_1 \cos\theta_2}$$

Velocity Correlations

In linear theory the radial and transverse correlation functions of the three-dimensional peculiar velocity field are a convolution of the power spectrum of density fluctuations with the window functions

$$\phi_{\parallel}(r) = \frac{f(\Omega_m)^2 H_0^2}{2\pi^2} \int dk\, P(k) \left[j_0(kr) - 2\frac{j_1(kr)}{kr} \right]$$

$$\phi_{\perp}(r) = \frac{f(\Omega_m)^2 H_0^2}{2\pi^2} \int dk\, P(k) \frac{j_1(kr)}{kr}$$

Górski, 1988, ApJL, 332, L7
Brogani, etal, 2000, AJ, 119, 102

Where

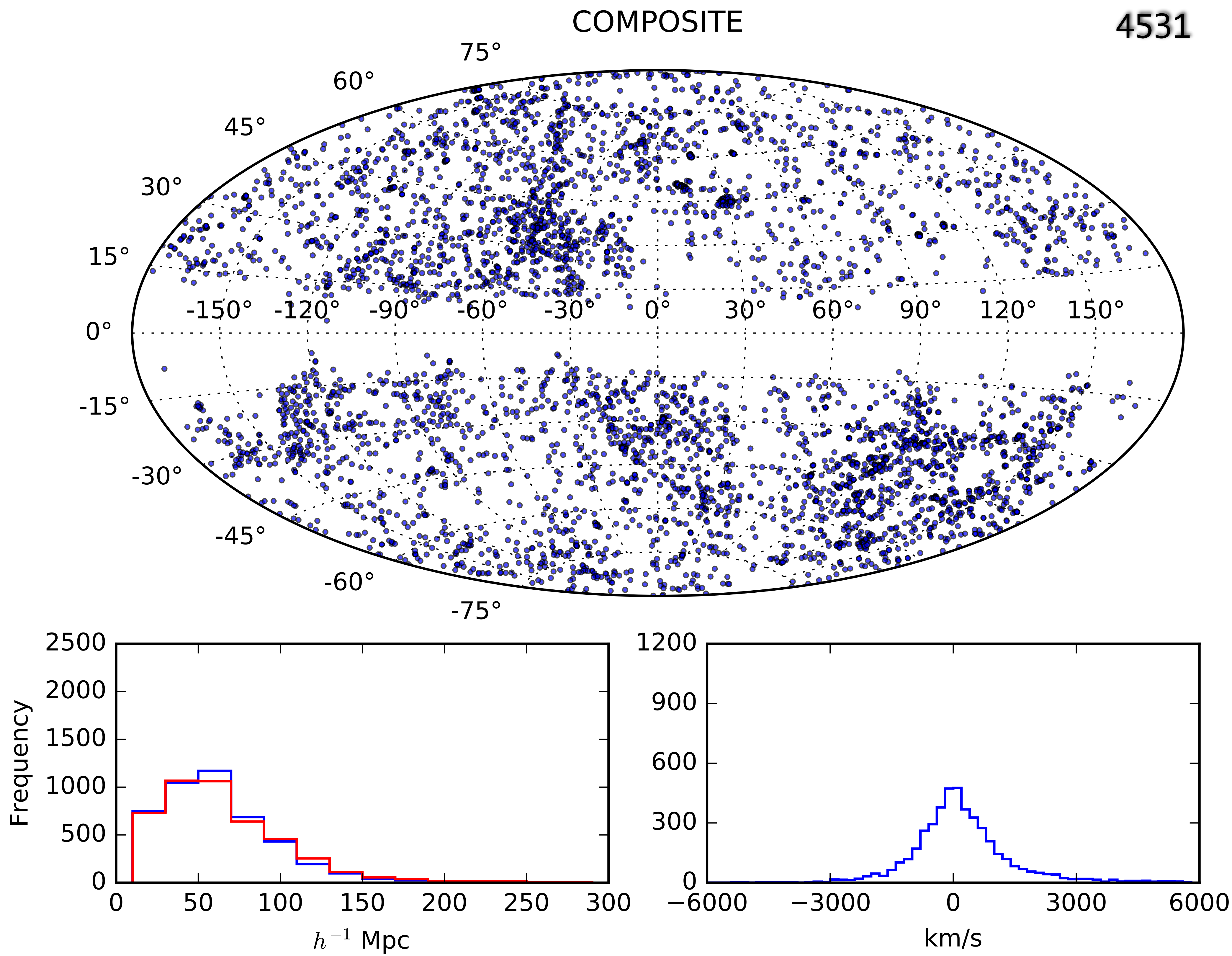
$j_i(x)$ is the i^{th} order spherical Bessel function

and

$$f(\Omega_m) \approx \Omega_m^{0.55}$$

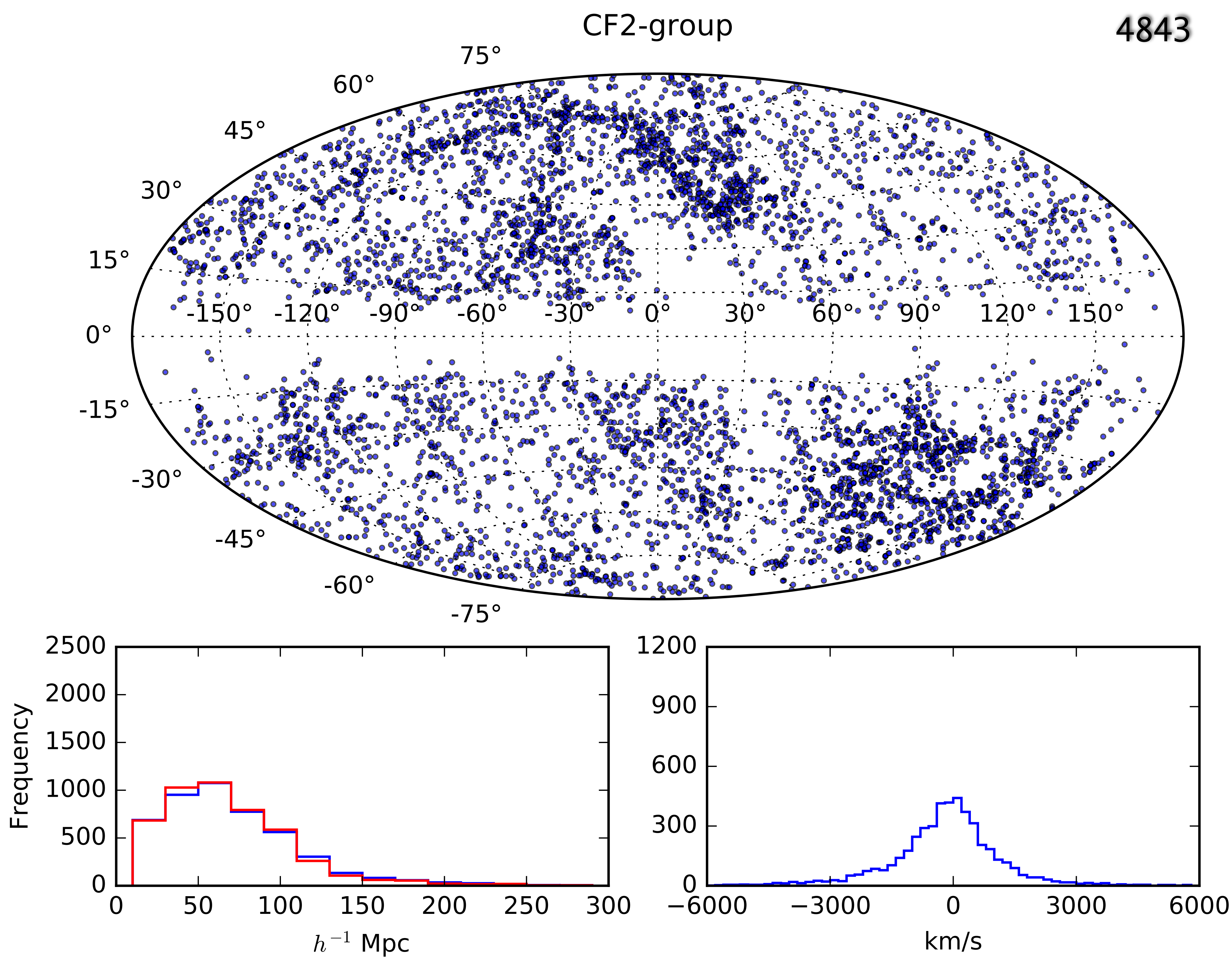
Linder, 2005, PRD, 72, 043529

Data



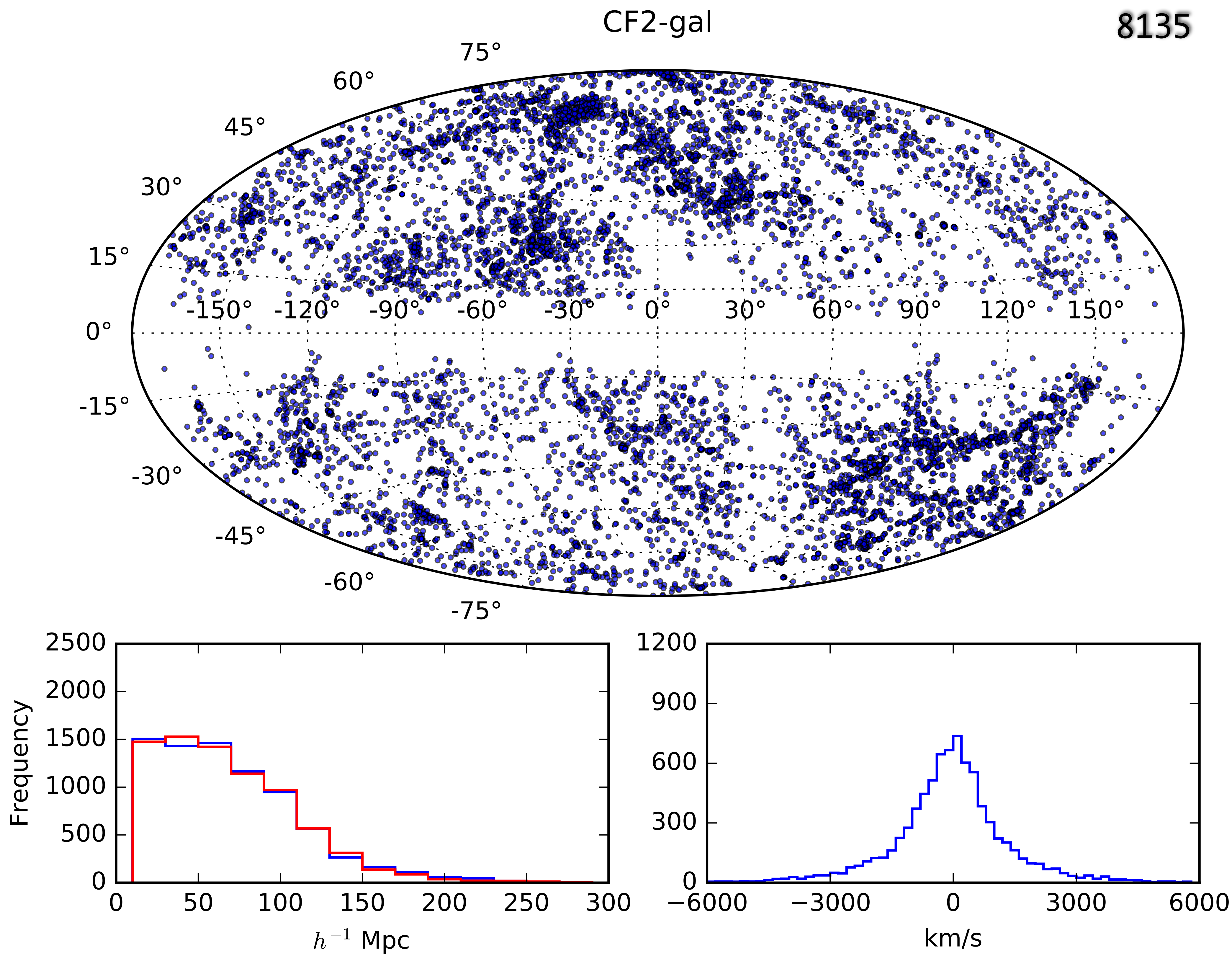
COMPOSITE
4531
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-150° -120° -90° -60° -30° 0° 30° 60° 90° 120° 150°
Frequency
2500
2000
1500
1000
500
0
0 50 100 150 200 250 300
h^{-1} Mpc
1200
900
600
300
0
−6000 −3000 0 3000 6000
km/s

Data

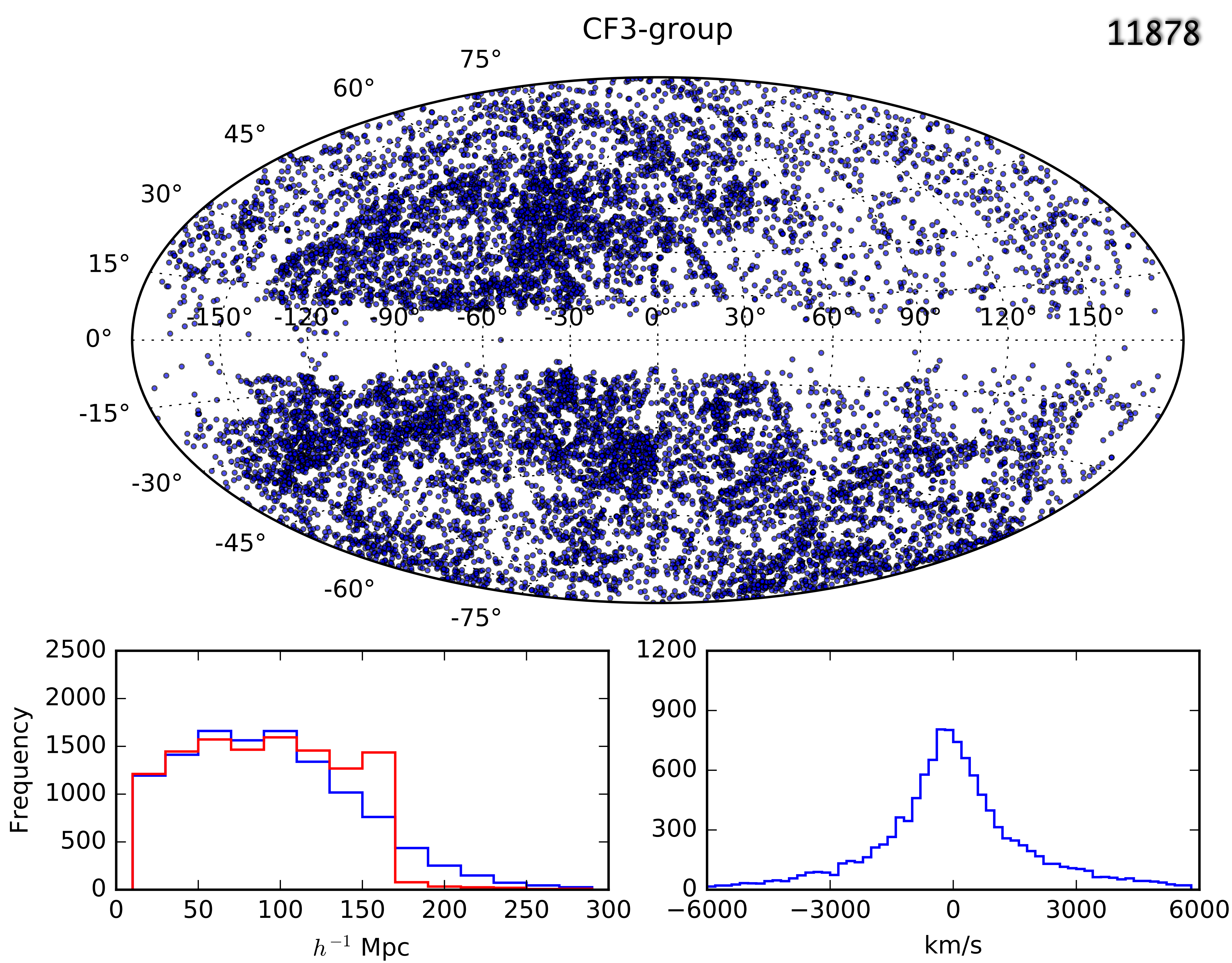


CF2-group
4843
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-150° -120° -90° -60° -30° 0° 30° 60° 90° 120° 150°
Frequency
2500
2000
1500
1000
500
0
0 50 100 150 200 250 300
h^{-1} Mpc
1200
900
600
300
0
−6000 −3000 0 3000 6000
km/s

Data

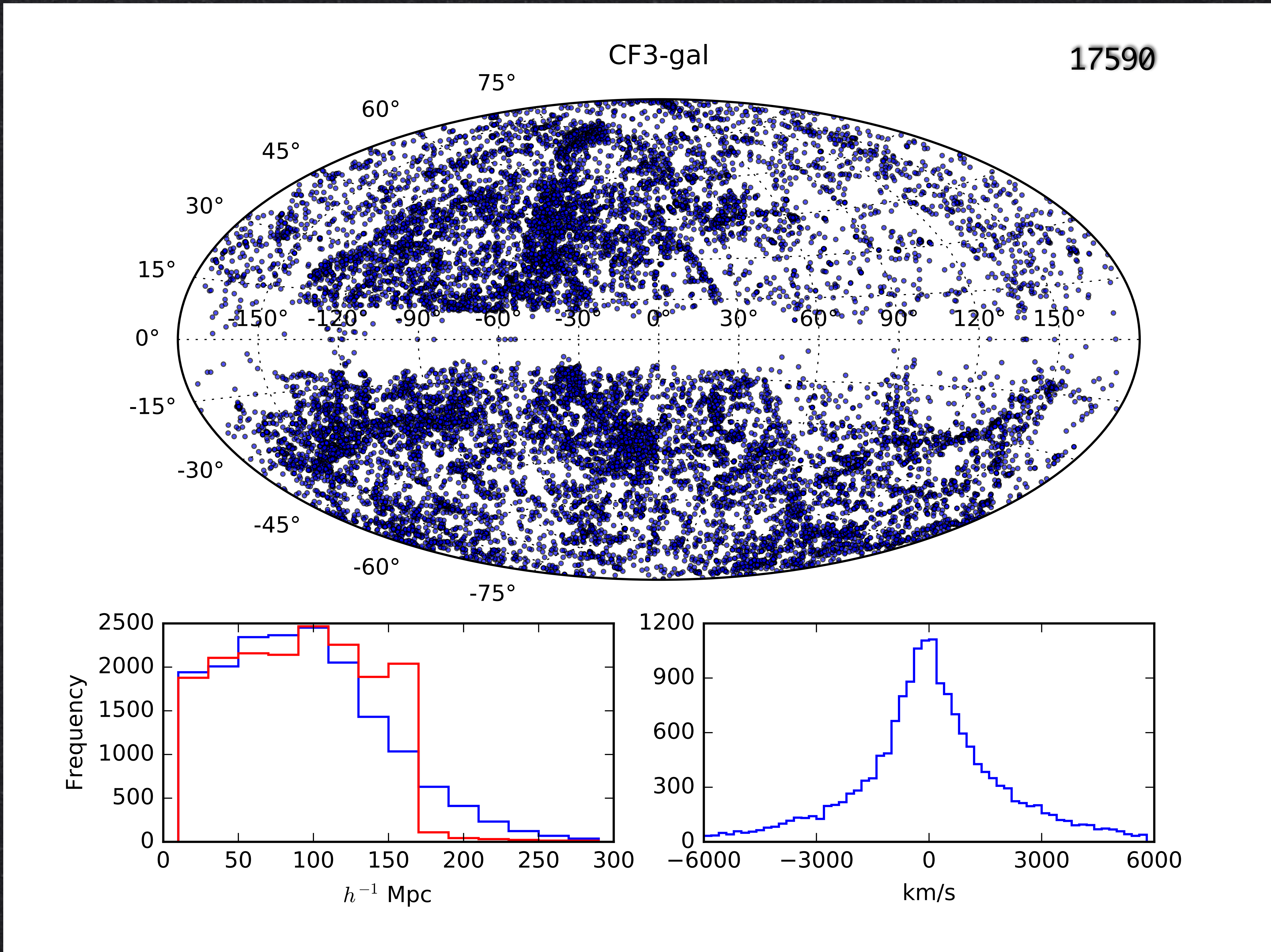

CF2-gal
8135
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-150°
-120°
-90°
-60°
-30°
0°
30°
60°
90°
120°
150°
Frequency
h^{-1} Mpc
km/s

Data



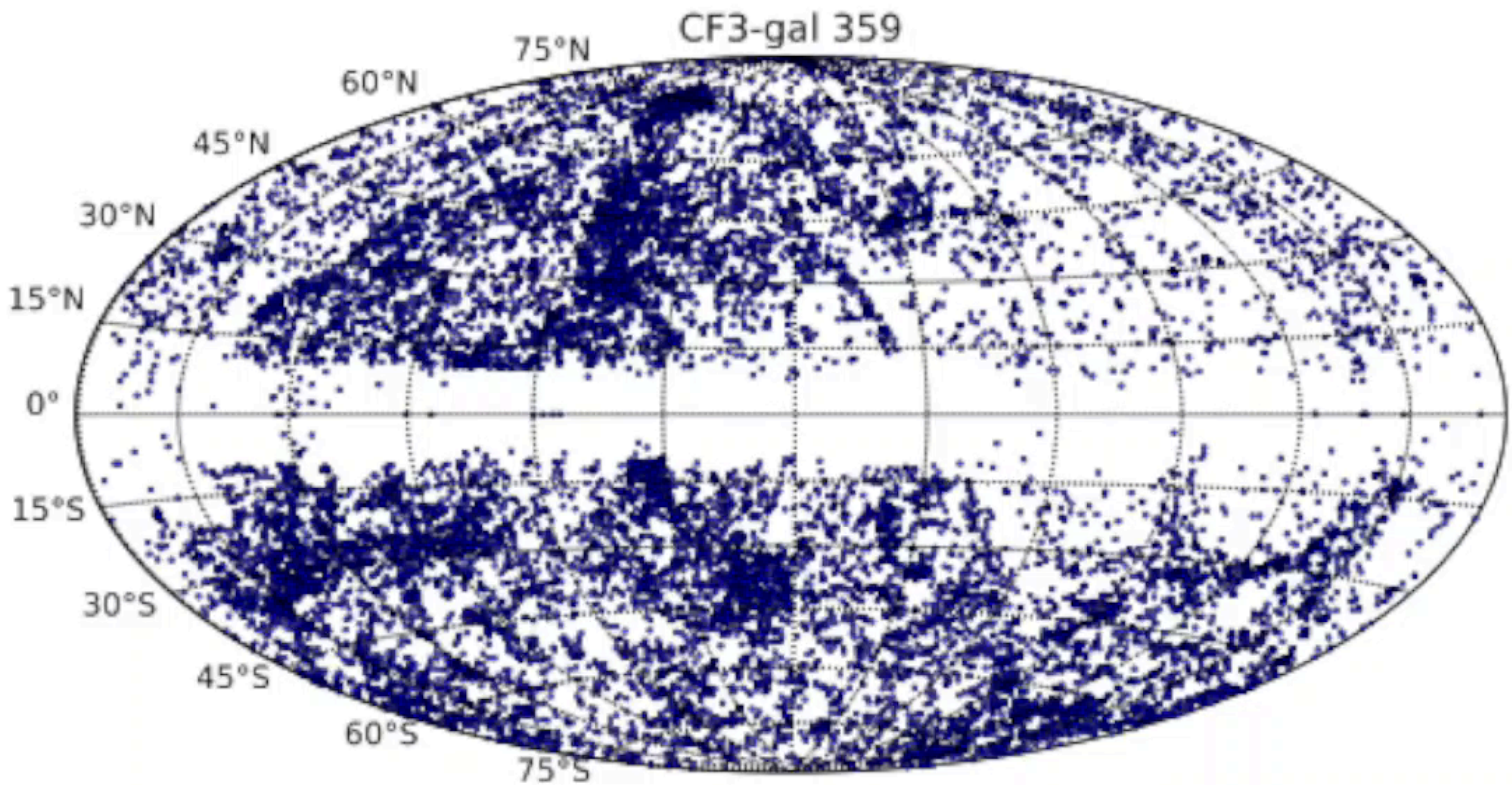
CF3-group
11878
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-150° -120° -90° -60° -30° 0° 30° 60° 90° 120° 150°
Frequency
2500
2000
1500
1000
500
0
0 50 100 150 200 250 300
h^{-1} Mpc
1200
900
600
300
0
−6000 −3000 0 3000 6000
km/s

Data


CF3-gal
17590
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-150° -120° -90° -60° -30° 0° 30° 60° 90° 120° 150°
Frequency
2500
2000
1500
1000
500
0
0 50 100 150 200 250 300
h^{-1} Mpc
1200
900
600
300
0
−6000 −3000 0 3000 6000
km/s

Data

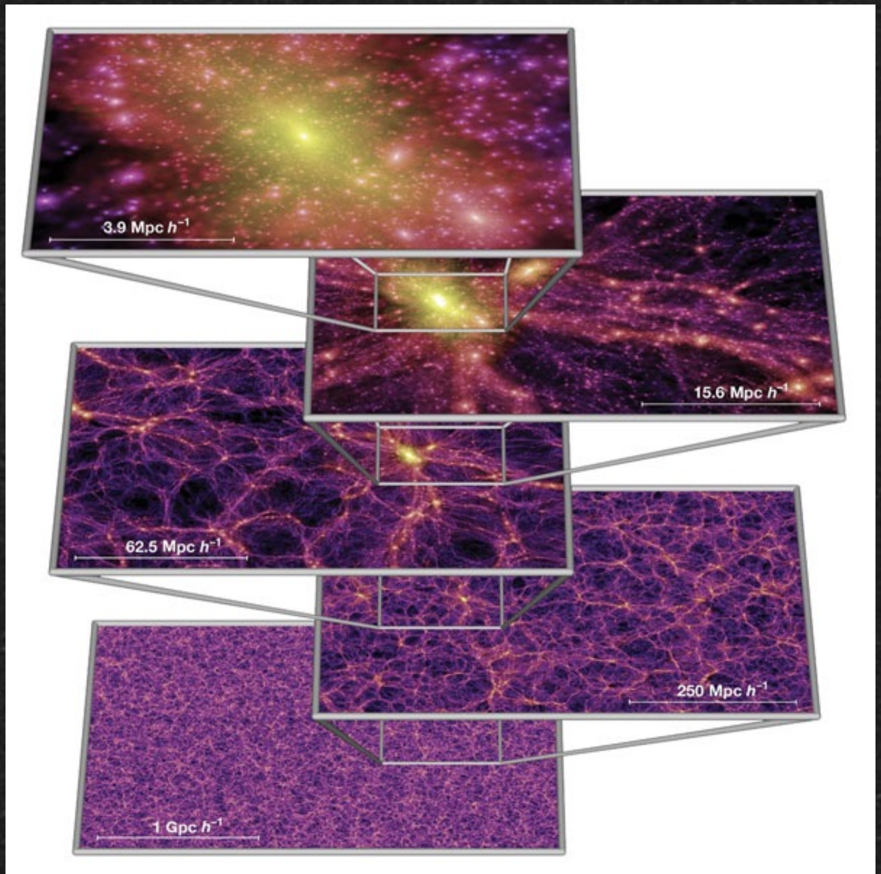
17590



Velocity Correlations

Use the Millennium Simulation, a dark matter only simulation using GADGET-2 simulation code

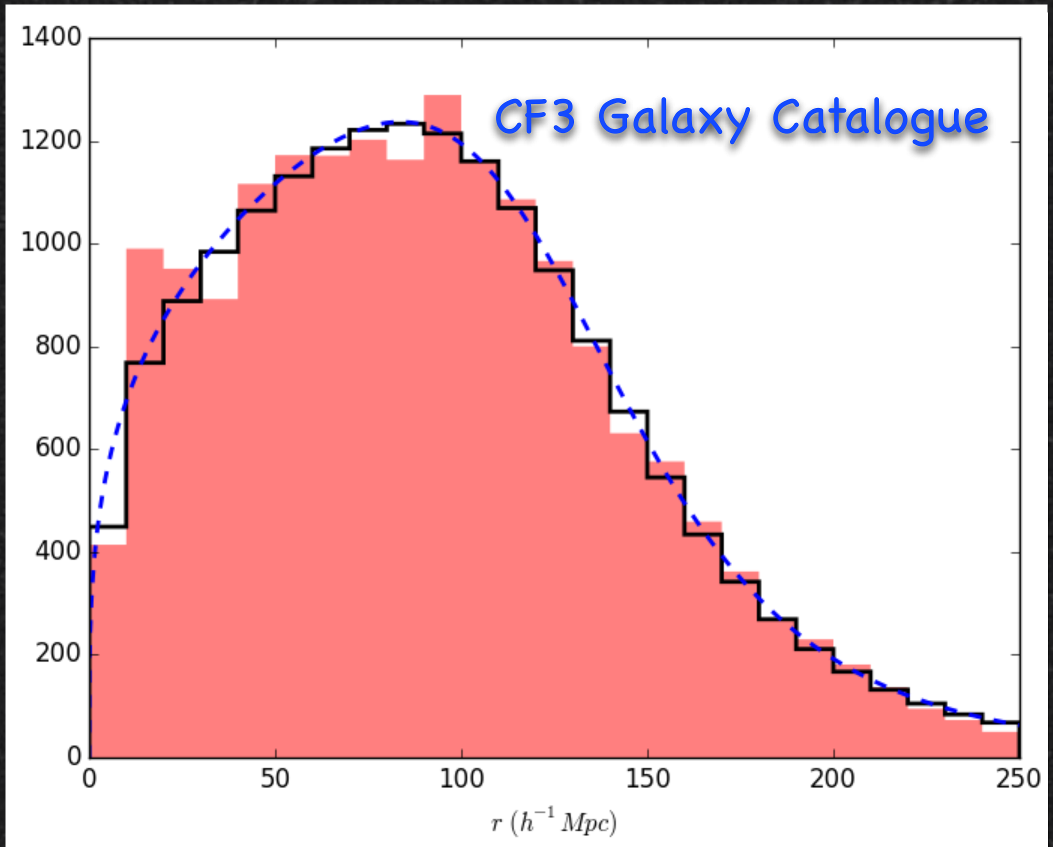
Matter density, Ω_m	0.25
Cosmological constant density, Ω_Λ	0.75
Baryon density, Ω_b	0.045
Hubble parameter, h $(100kms^{-1}Mpc^{-1})$	0.73
Amplitude of matter density fluctuations, σ_8	0.9
Primordial scalar spectral index, n_s	1.0
Box size (h^{-1}Mpc)	500
Number of particles	2160^3
Particle mass, m_p $(10^8 h^{-1} M_\odot)$	8.61
Softening, f_c $(h^{-1}kpc)$	5



Springel etal, 2005, Nature, 435, 629
De Lucia G., Blaizot J., 2007, MNRAS, 375, 2
http://wwwmpa.mpa-garching.mpg.de/millennium/http://wwwmpa.mpa-garching.mpg.de/millennium/
https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/

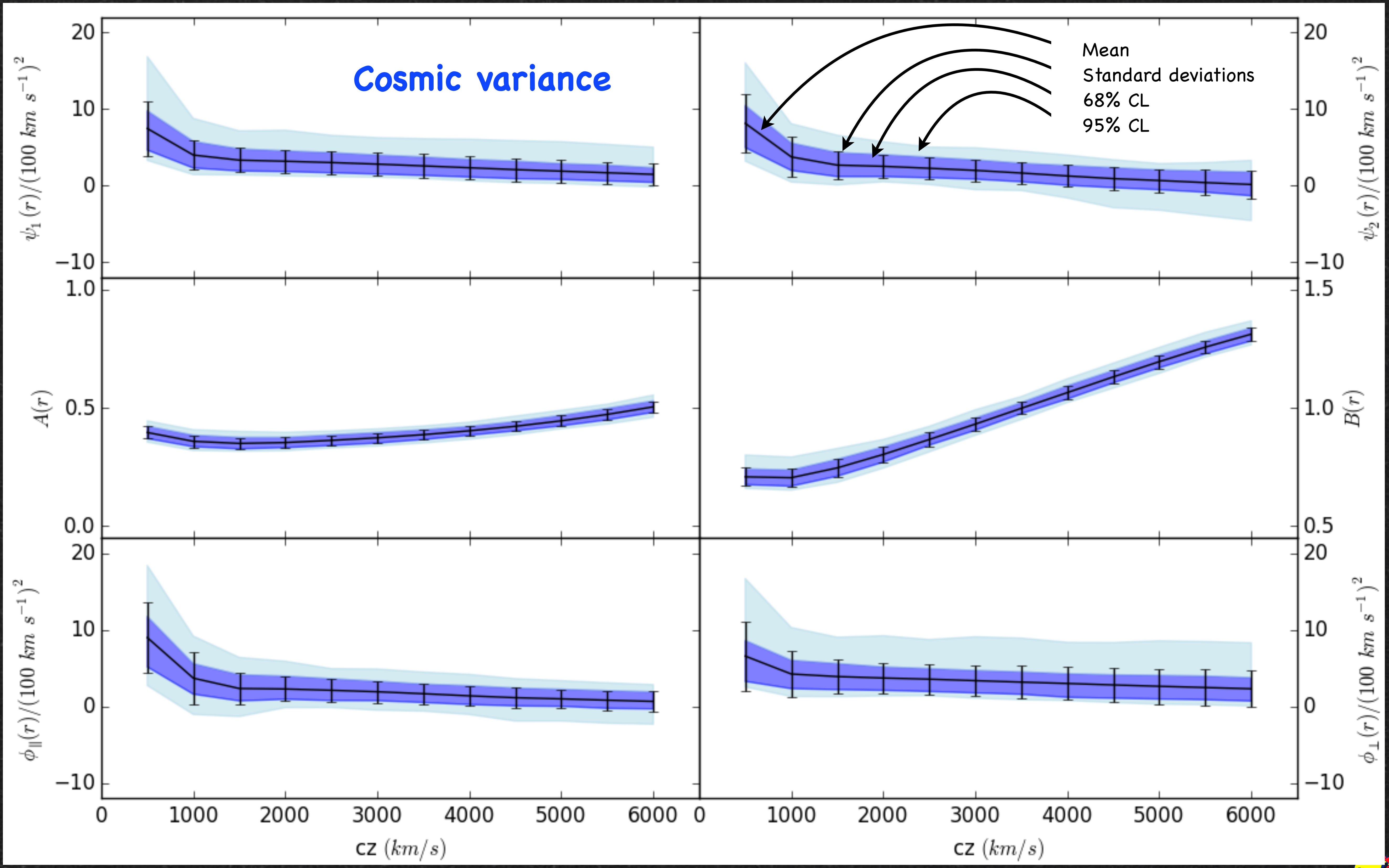
We select the galaxies for each mock survey by ensuring a best fit to the radial selection function fitted with

$$f(r) = \mathscr{A}\left(\frac{r}{r_0}\right)^{n_1}\left[1+\left(\frac{r}{r_0}\right)^{n_1+n_2}\right]^{-1}$$



Velocity Correlations

We chose 100 independent mock surveys and calculated the correlation functions with exact positions from the simulations



Velocity Correlations

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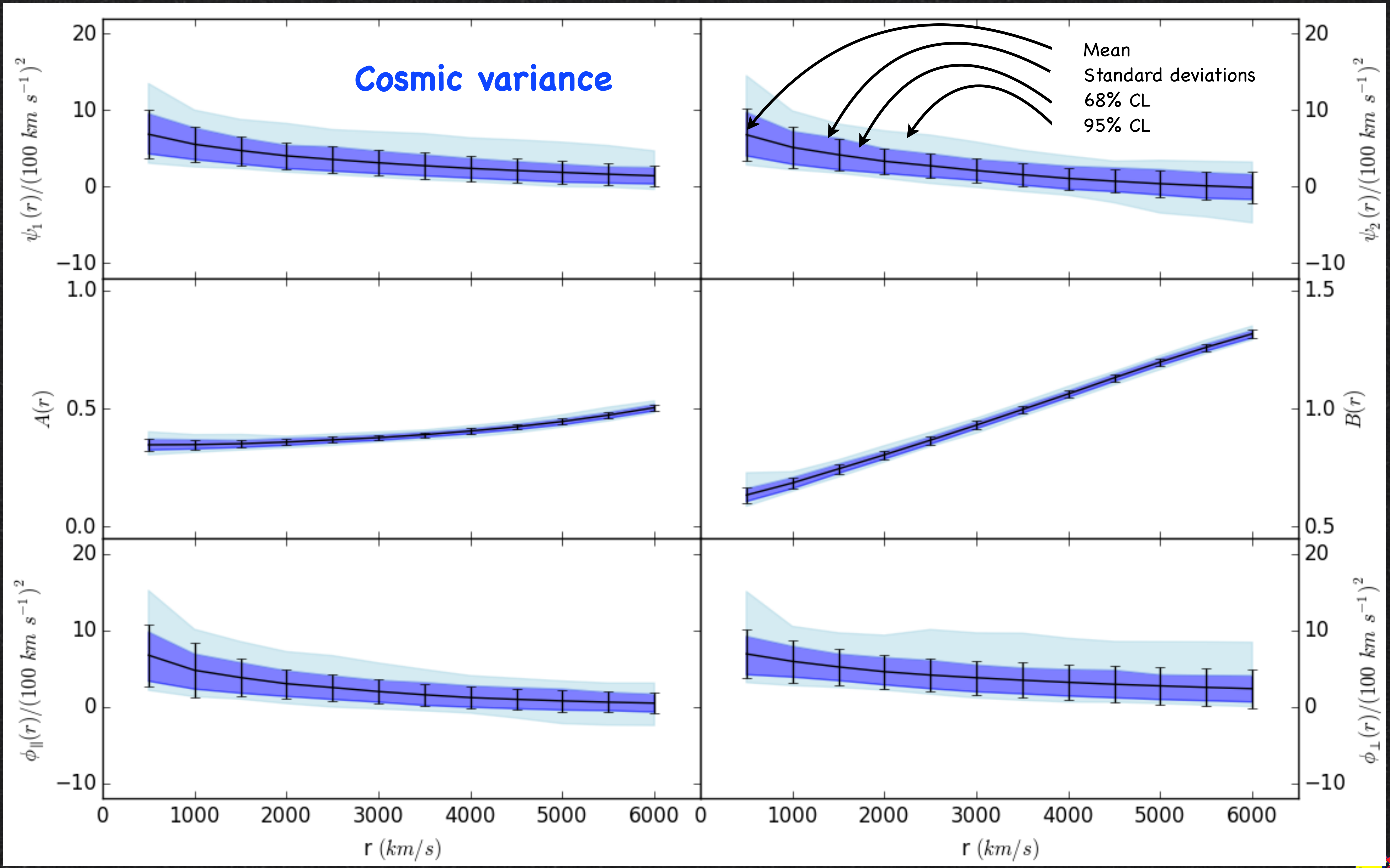
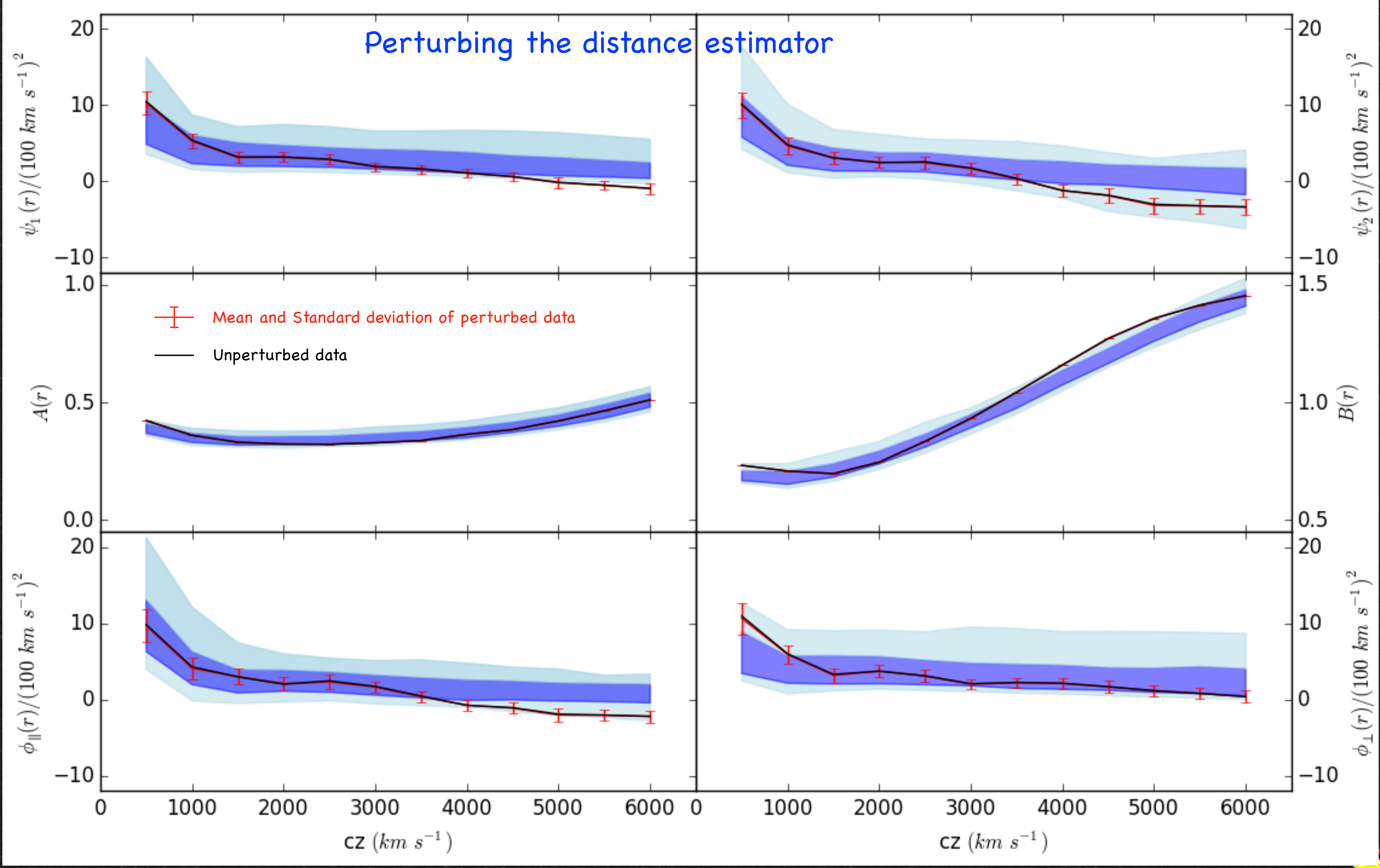


Velocity Correlations

The other main source of uncertainty in the correlation functions comes from the the uncertainty of the peculiar velocity measurements



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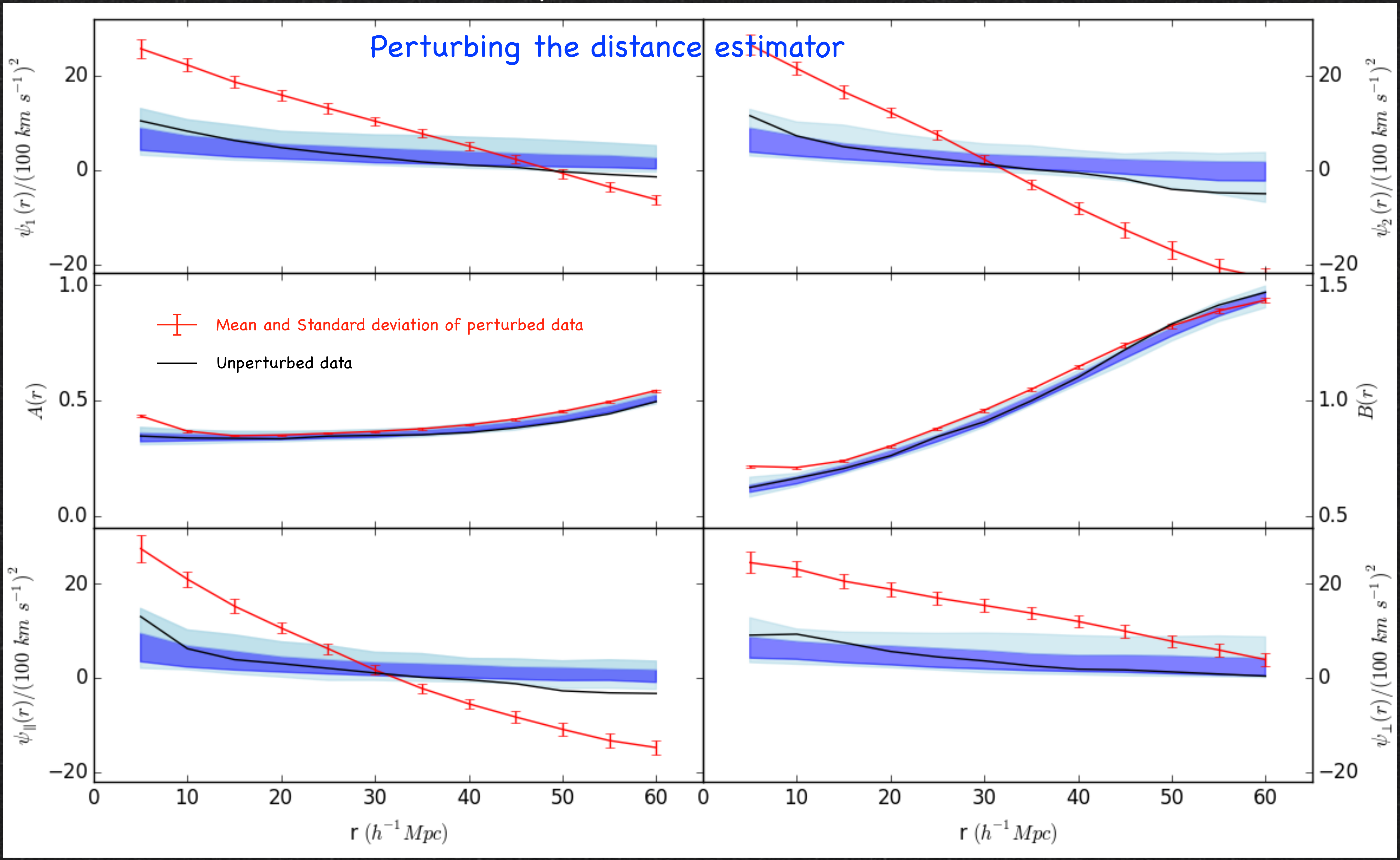

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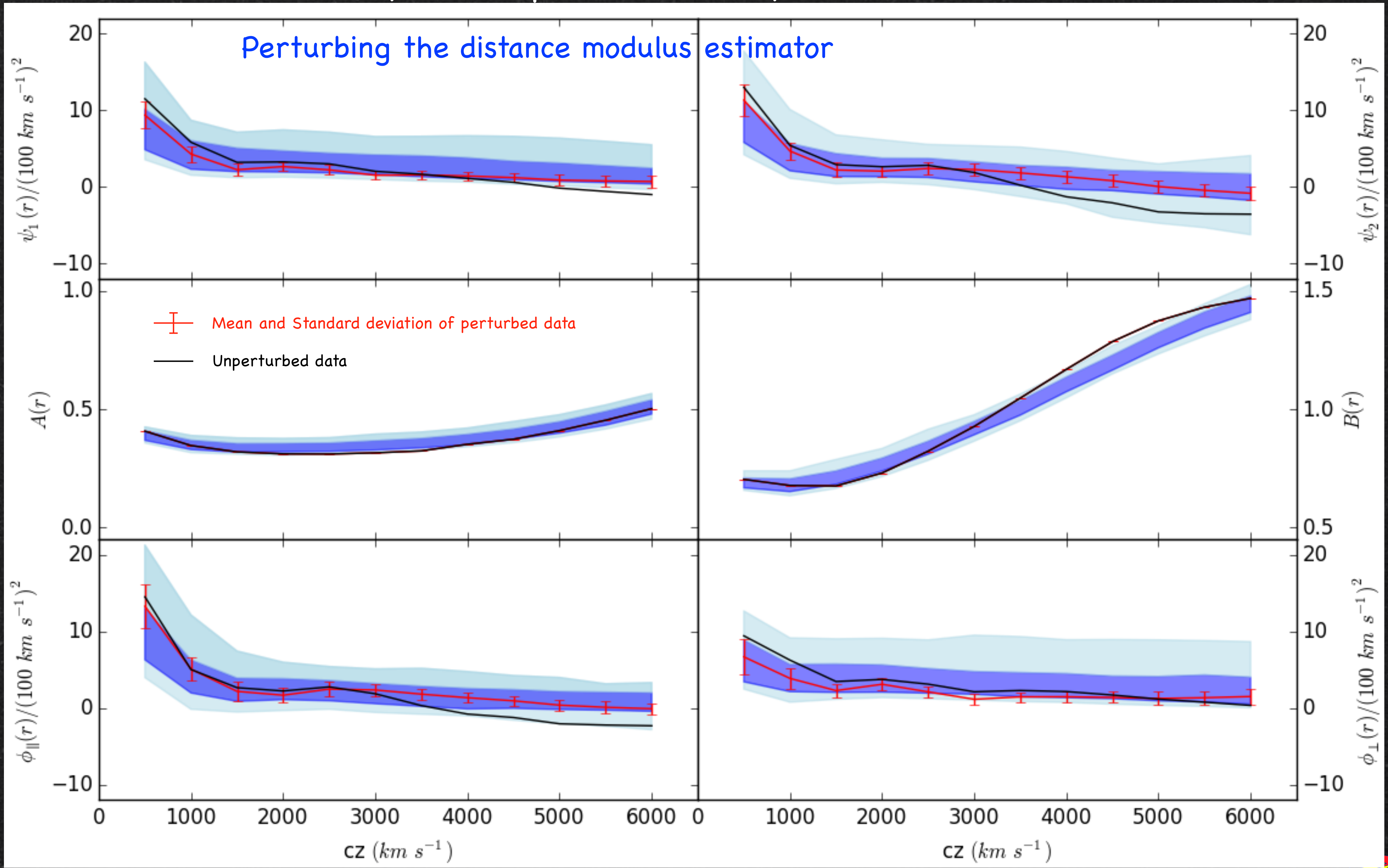

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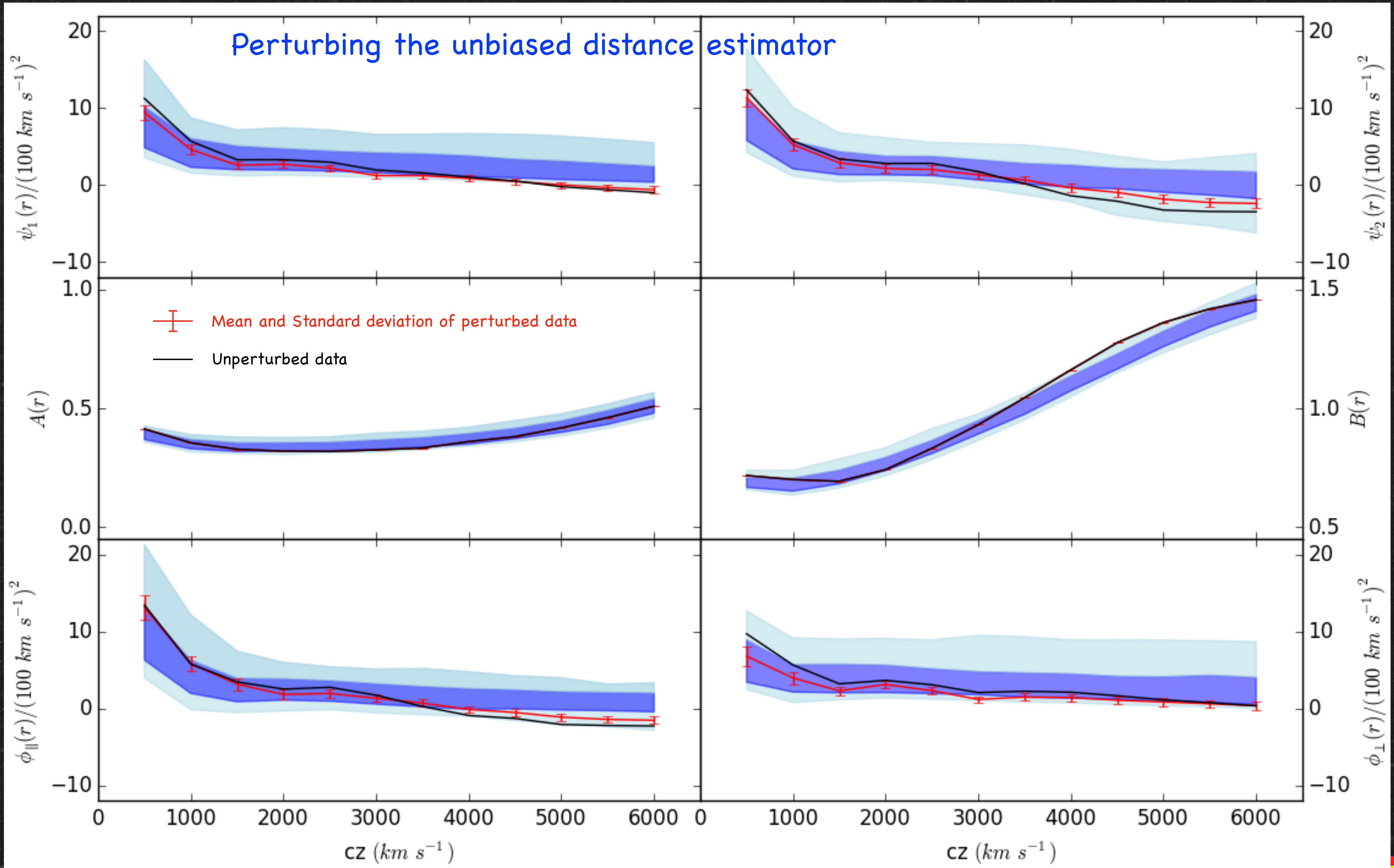
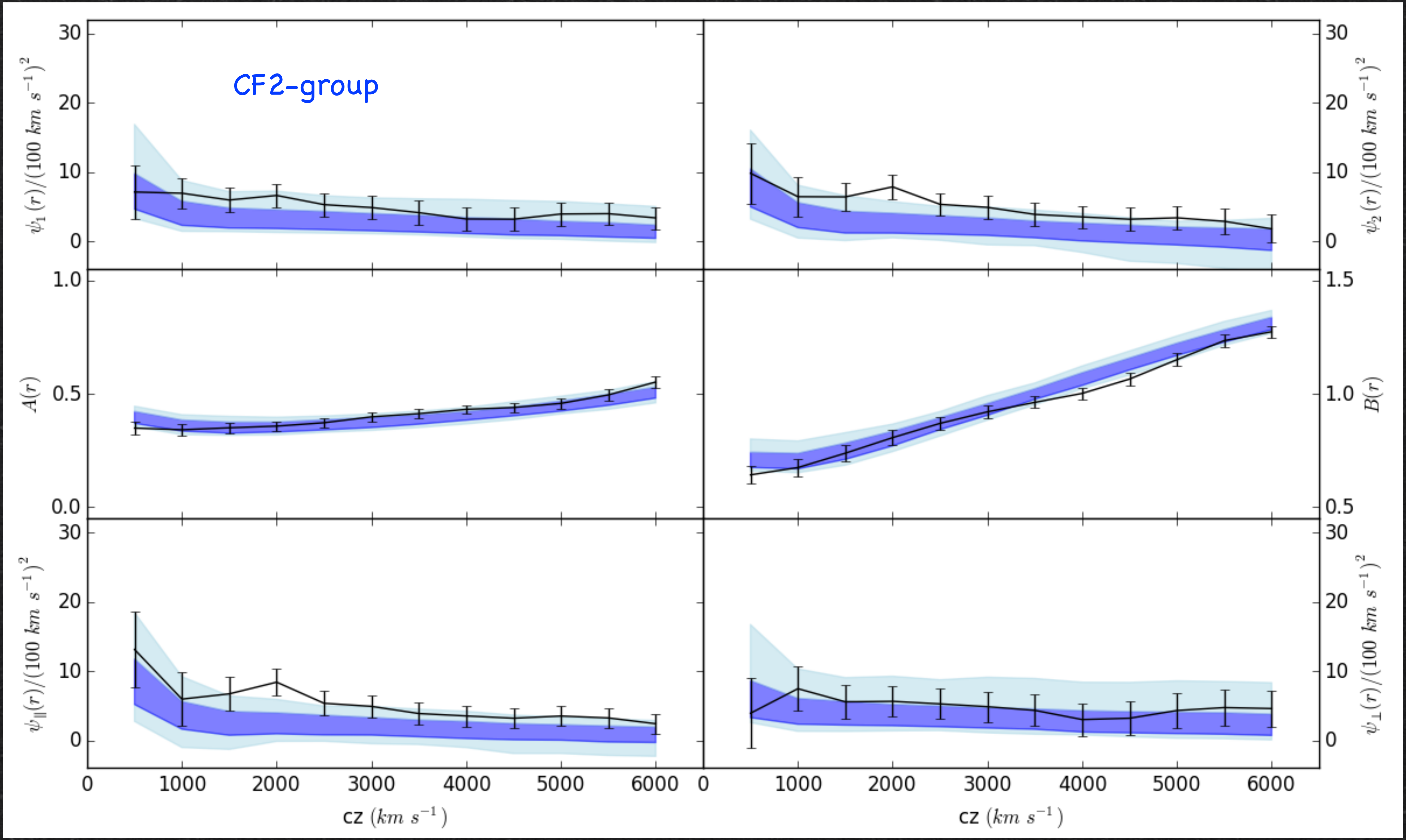


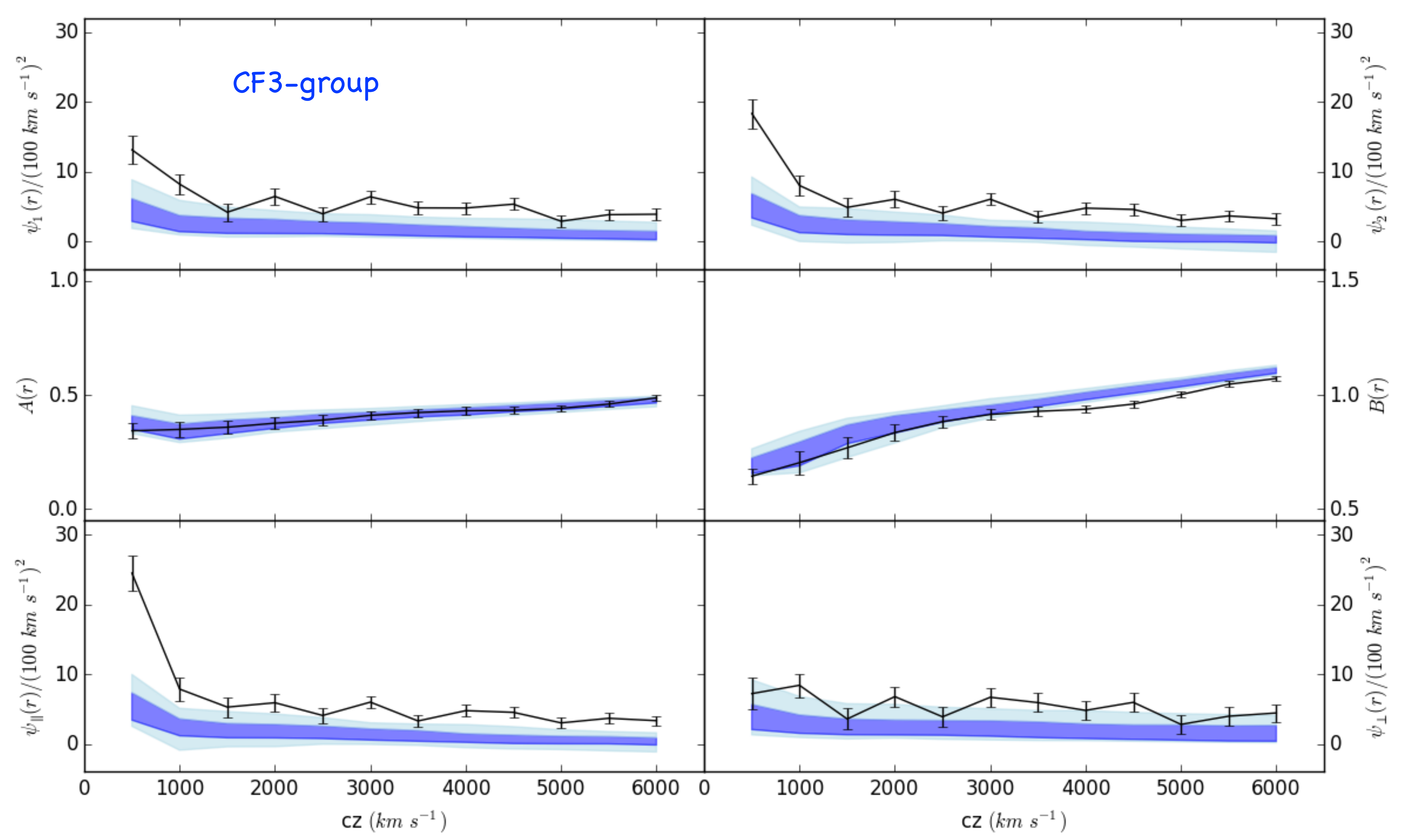
Velocity Correlations

Observational Data



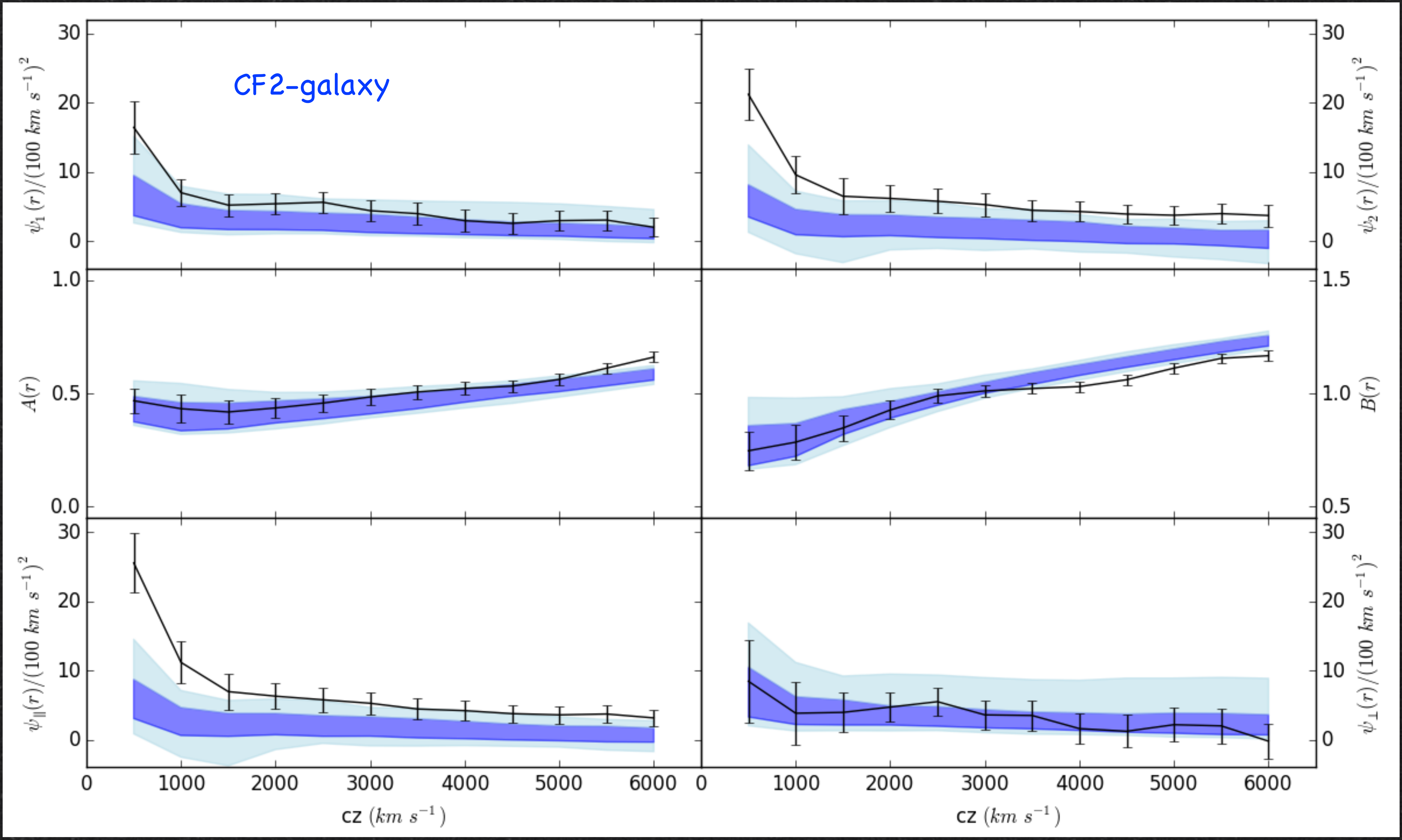
Velocity Correlations

Observational Data



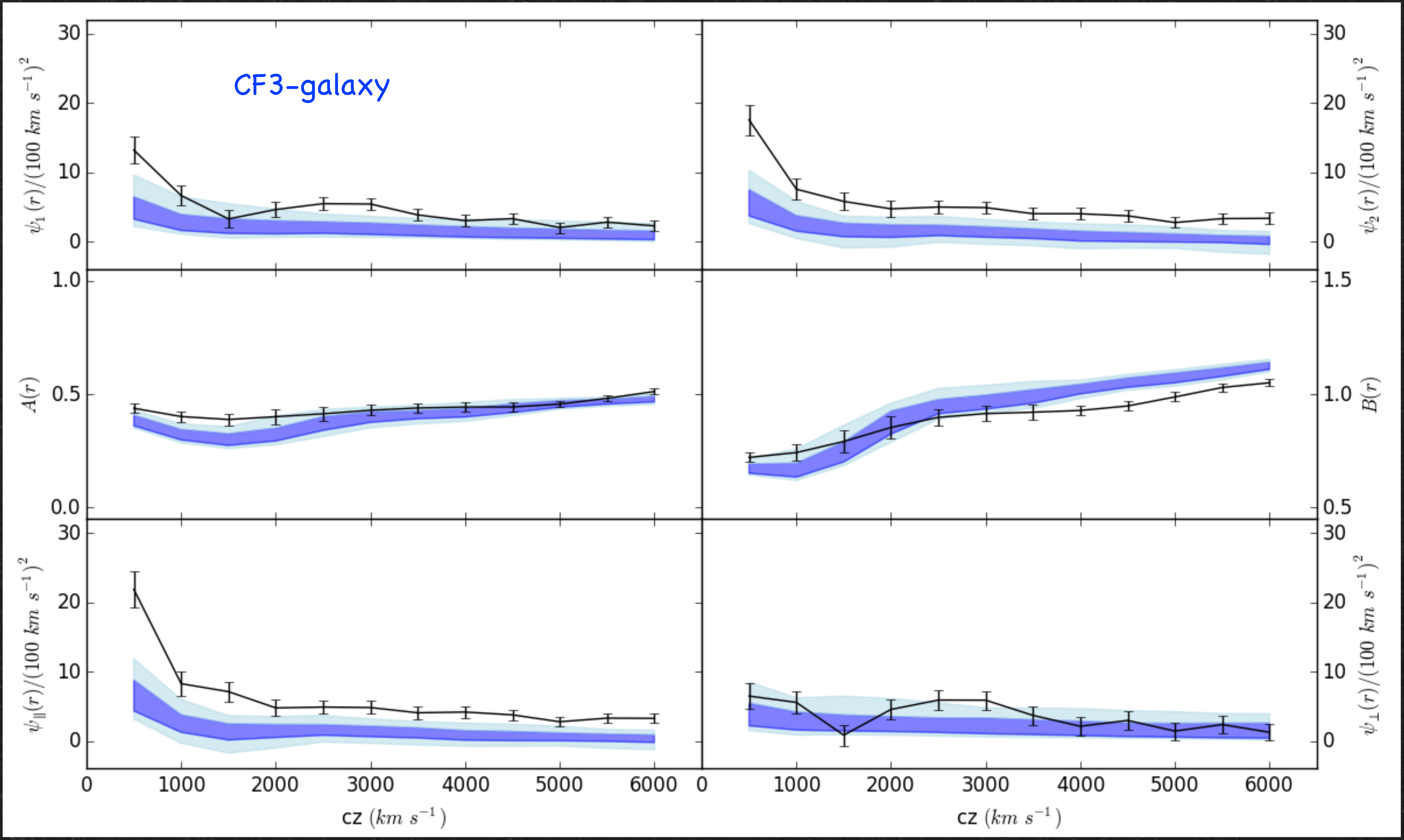
Velocity Correlations

Observational Data



Velocity Correlations

Observational Data

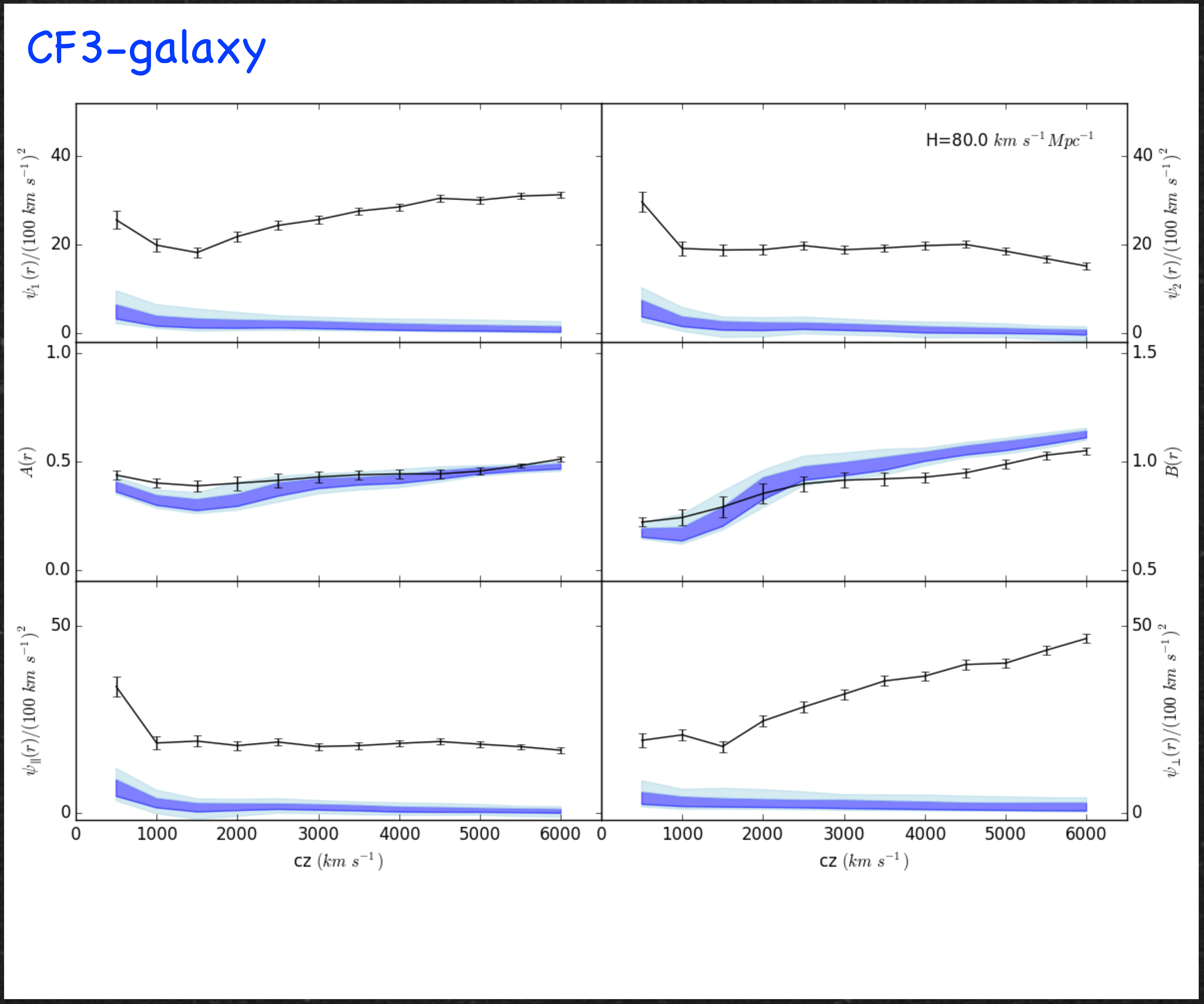


Velocity Correlations

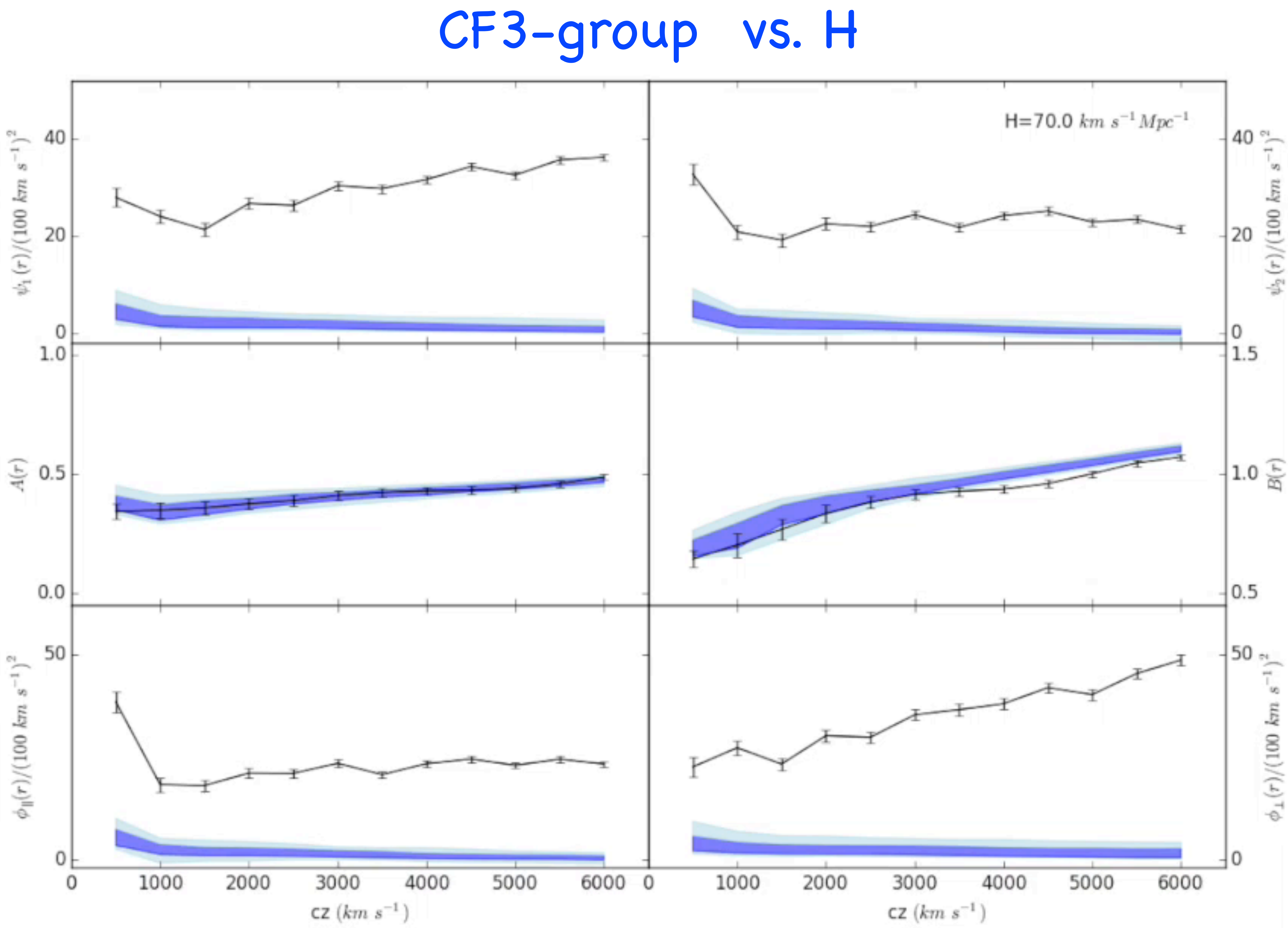
Use the correlation function to constrain H.

Create catalogues with different H.

Since we are using cz as the distance, only v_p changes

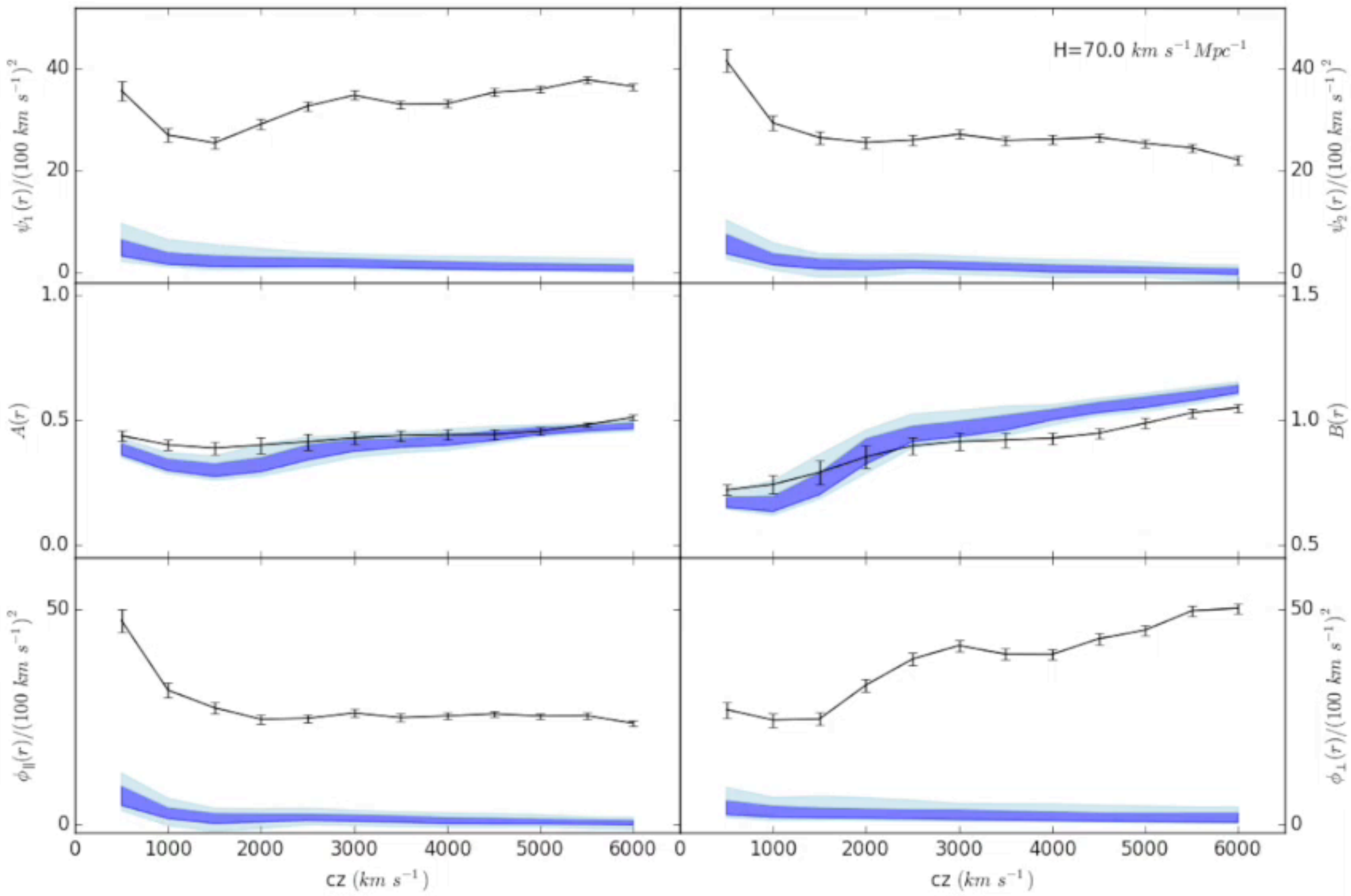


Velocity Correlations

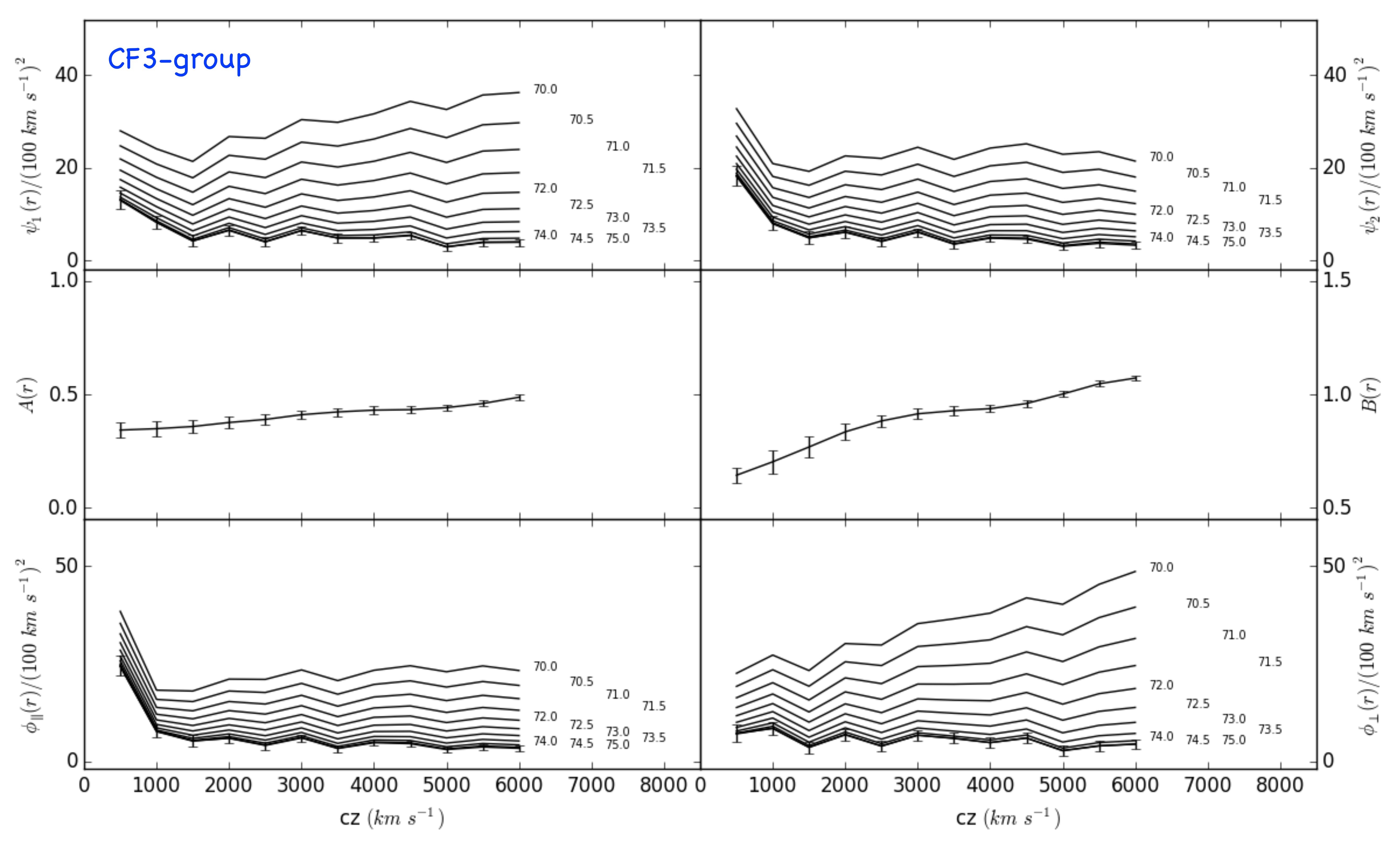


Velocity Correlations

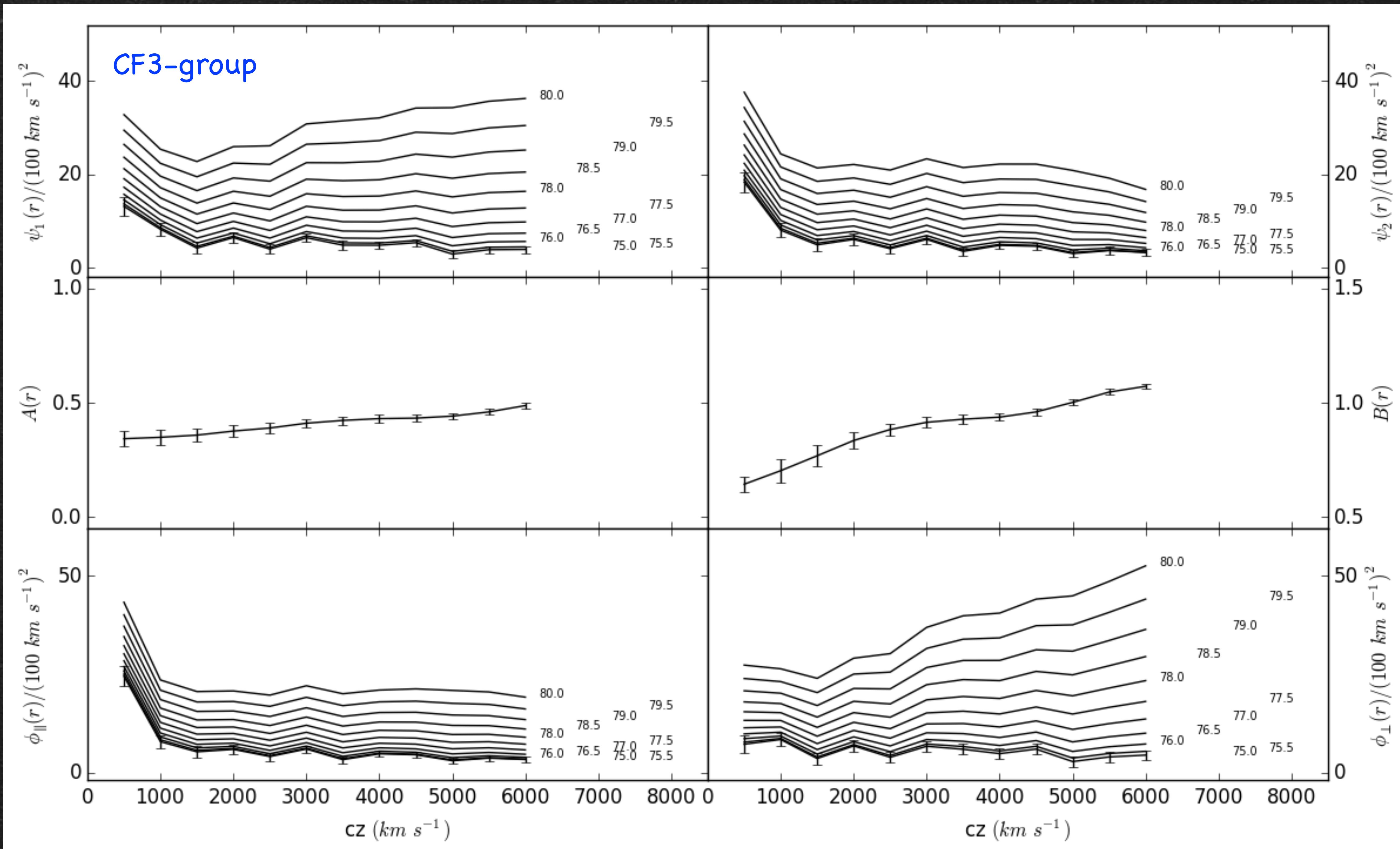
CF3-galaxy vs. H



Velocity Correlations



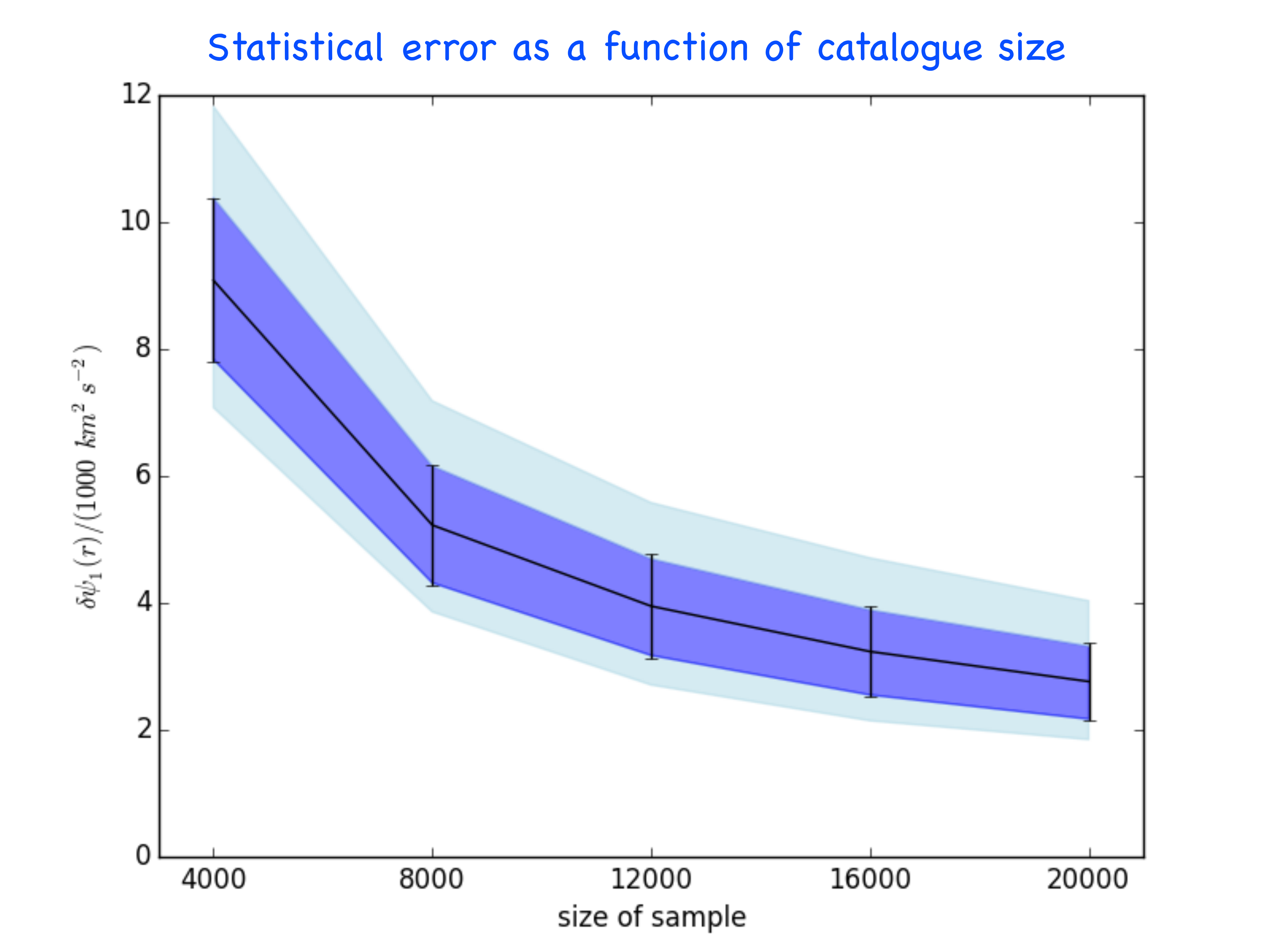
Velocity Correlations



Velocity Correlations

Errors as a function of sample size

The statistical errors get smaller with sample size

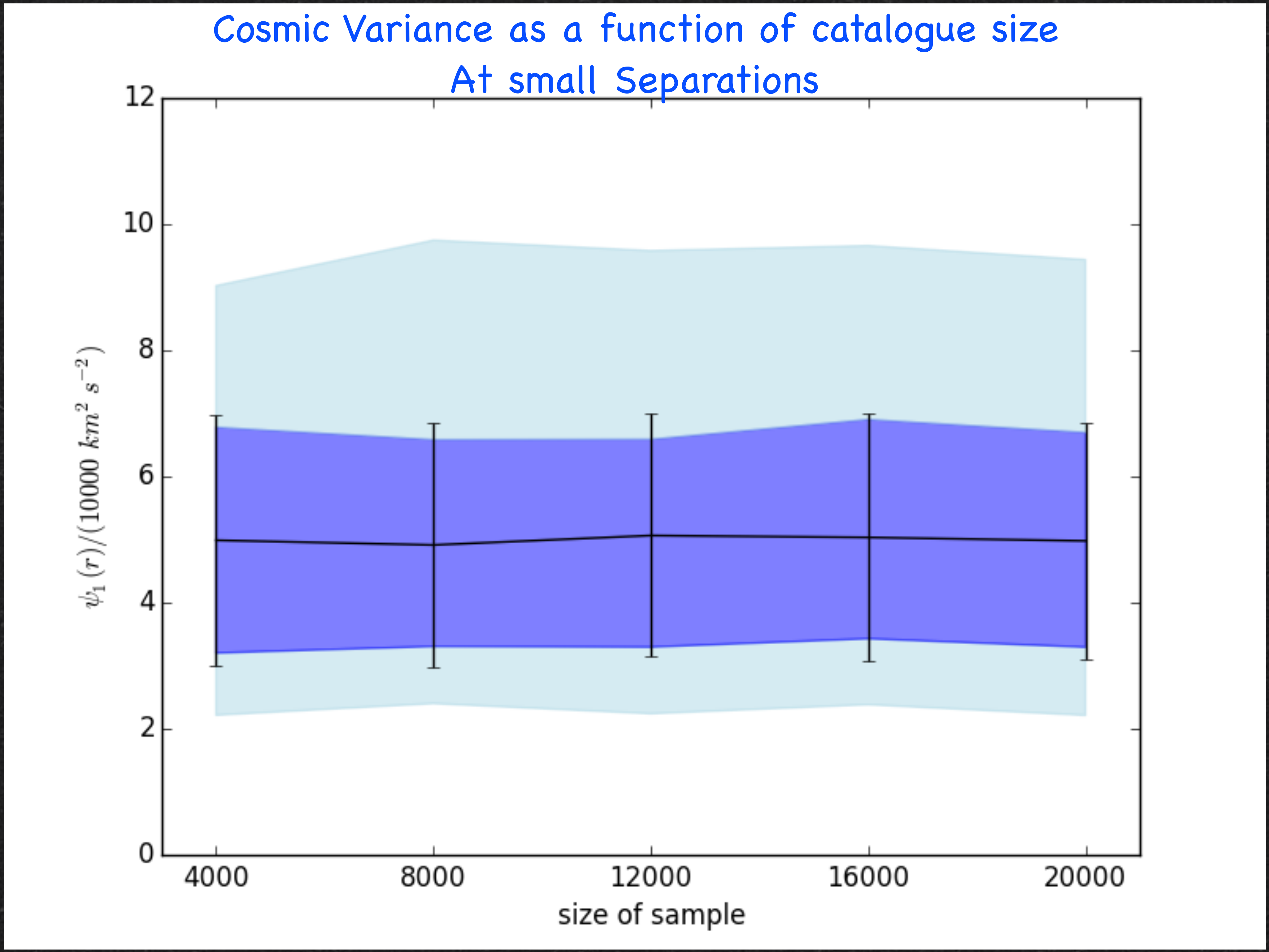


Velocity Correlations

Errors as a function of sample size

The statistical errors get smaller with sample size

Cosmic Variance stays the same



Velocity Correlations

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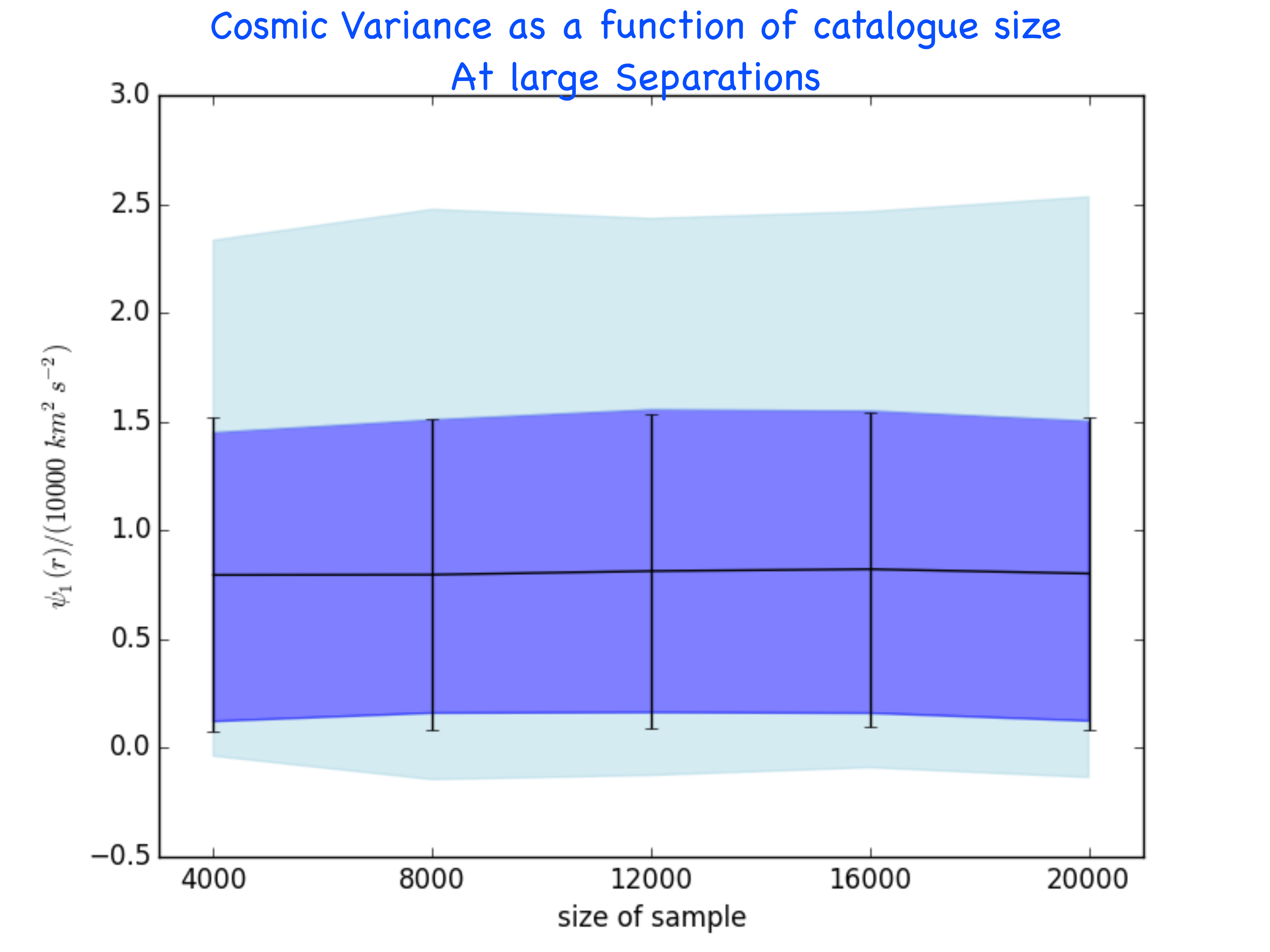

Velocity Correlations
Conclusions

Correlation function statistic is robust and stable across different samples.

Provides a constraint on $H = 75 \pm 2$ km s^{-1} Mpc^{-1}

Coherence length of ~1,500 ± 500 km s^{-1}

Amplitude 5–10 $(10^2 \text{ km s}^{-1})^2$

Statistical errors get smaller with sample size

Cosmic variance is independent of sample size

Agrees with ΛCDM expectations with Planck/WMAP parameters

$\Gamma = h\,\Omega = 0.3 \pm 0.15$

$\sigma_8 = 0.85 \pm 0.6$ preliminary results

$\eta_8 = \Omega^{0.55}\,\sigma_8 = 0.42 \pm 0.2$

Thank you