Galactic perturbations of the cosmic flow

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credit: Raphael Errani

Rencountres du Vietnam, Quy Nhon
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The timing argument

1- Relative motion between **mass-less** particles "A" and "G" in Universe with no radiation

$$\frac{\ddot{r}}{r} = -\frac{4\pi G}{3}\rho + H_0^2\Omega_\Lambda$$

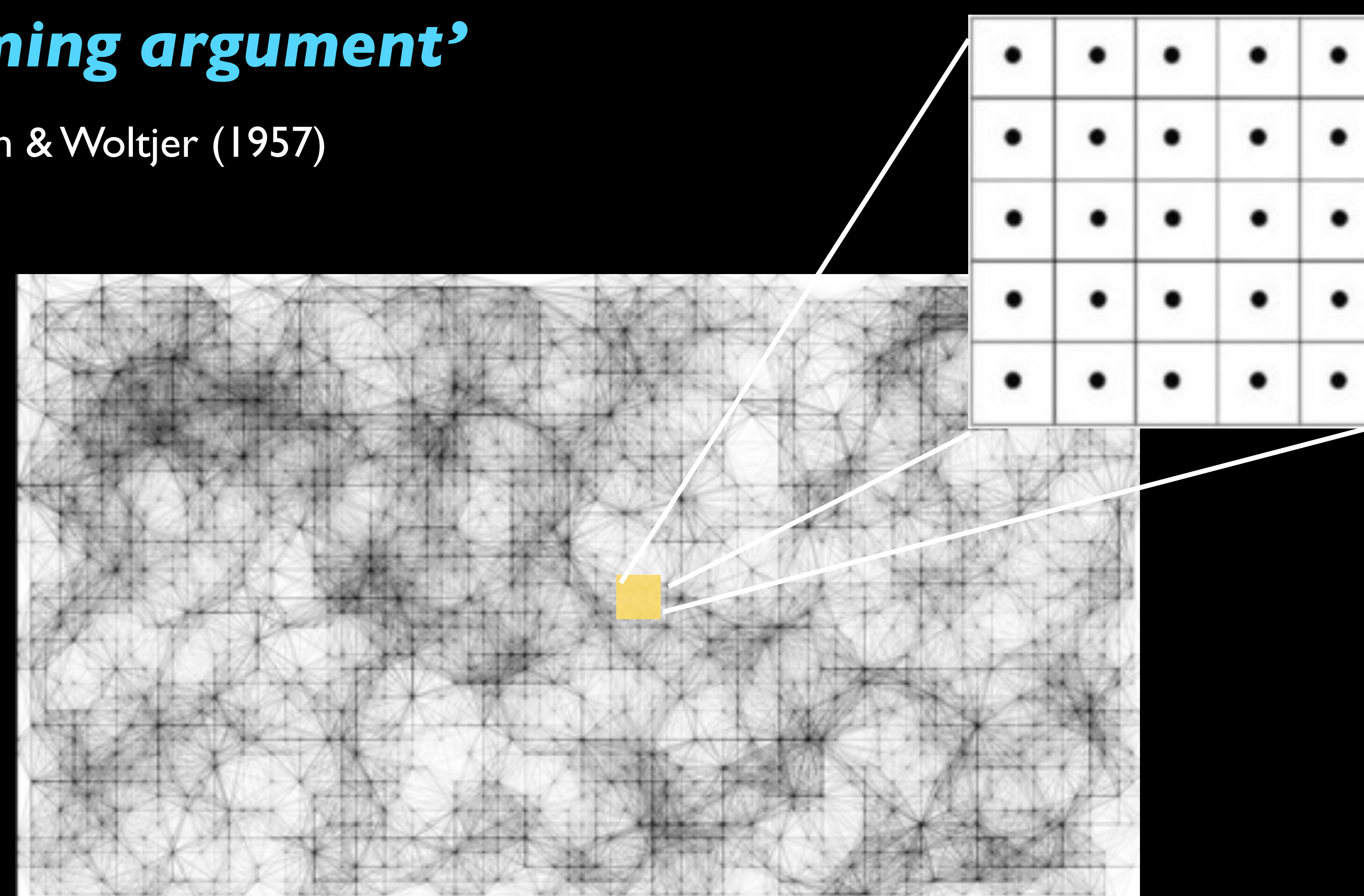
Friedmann eqs.

2- (Local) **perturbations of Hubble flow** by mass $M = M_A + M_G$

$$\left.\begin{array}{l} \ddot{r} = -\dfrac{GM}{r^2} + H_0^2\Omega_\Lambda r \\ \\ r(t = t_{\rm now}), v(t = t_{\rm now}) \end{array}\right\}$$

'Timing argument'

Kahn & Woltjer (1957)



Orbits

$$\ddot{r} = -\frac{GM}{r^2} + H_0^2 \Omega_\Lambda r.$$

relative motion between 2 massive particles in an expanding Universe

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The general solution is a Keplerian **radial** orbit

$$r = a(1 - \cos 2\eta); \qquad (1)$$

where

$$a = GM/(-2E)$$

is the semi-major axis of the orbit, **E** is the orbital energy, and **η** is an angle typically referred to as the eccentric anomaly, which can be calculated numerically from the following equation

$$2\eta - \sin 2\eta = (GM/a^3)^{1/2} t. \qquad (2)$$

THE DYNAMICAL AGE OF THE LOCAL GROUP OF GALAXIES

By D. Lynden-Bell
Institute of Astronomy, The Observatories, Cambridge

The distance to those Local Group members whose expansion has just been stopped by the gravity of the Local Group yields Mt^2 where M is the mass of the Group and t the time since expansion began. The distance and radial velocity of M31 yield a relationship between Mt^2 and t. Thus M and t may be deduced. Sandage–Tammann distances to Local Group members yield $t = 1{\cdot}6 \times 10^{10}$ years and $M = 3{\cdot}6 \times 10^{12}$ $M_{\odot}$. Accurate distances to outlying members of the Local Group could refine this method and make a lasting contribution to cosmogony and cosmology.

Perturbed Hubble flow

Define 2 orbital frequencies (Lynden-Bell 1981):

$$\Omega = \left(\frac{GM}{r^3}\right)^{1/2}$$
$$\omega = \frac{v}{r}.$$

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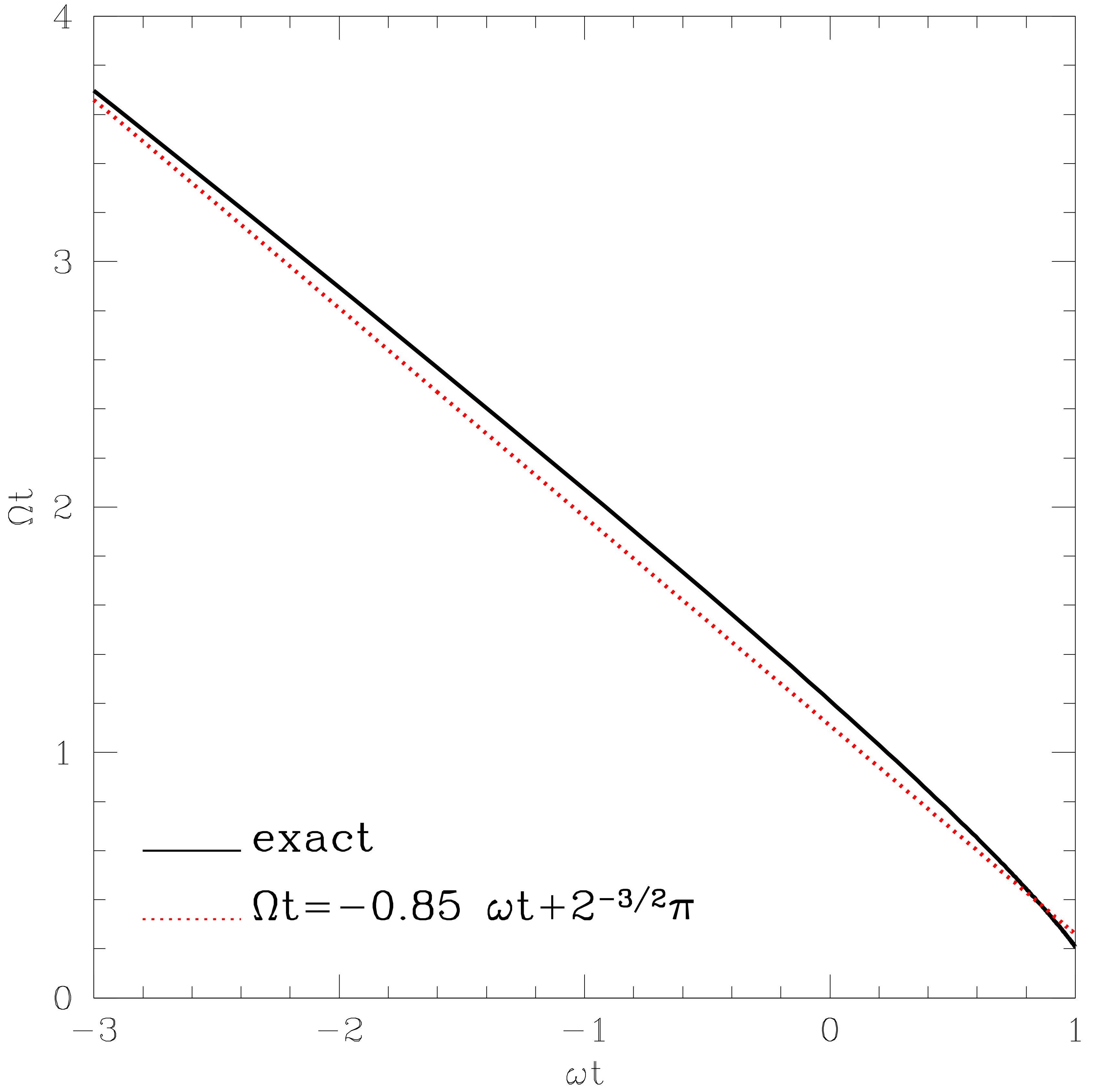
Multiply **ω** by the time **t** and insert **Ω t**

$$\omega t = \frac{\eta - \sin\eta\cos\eta}{\sin^3\eta}\cos\eta.$$



4
3
2
1
0
Ωt
−3
−2
−1
0
1
ωt
exact
Ωt=−0.85 ωt+2^{−3/2}π

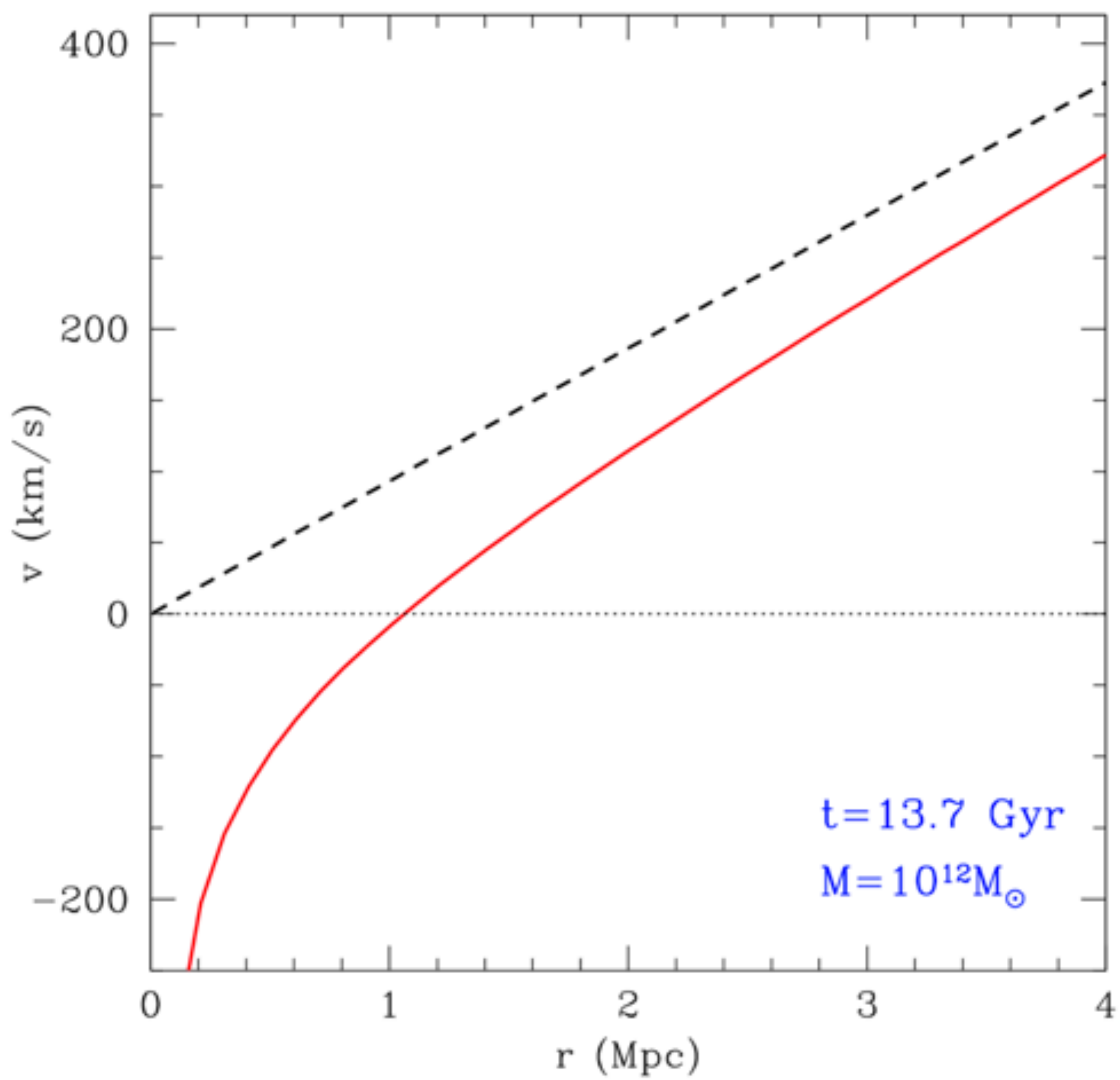
Perturbed Hubble Flow

$$\Omega t = -0.85\omega t + 2^{-3/2}\pi$$

$$\left(\frac{GM}{r^3}\right)^{1/2} t = -0.85\frac{v}{r}t + 2^{-3/2}\pi$$

solving for v=v(r)

$$v \simeq 1.2\frac{r}{t} - 1.1\left(\frac{GM}{r}\right)^{1/2}.$$ (red curve)



Perturbed Hubble Flow

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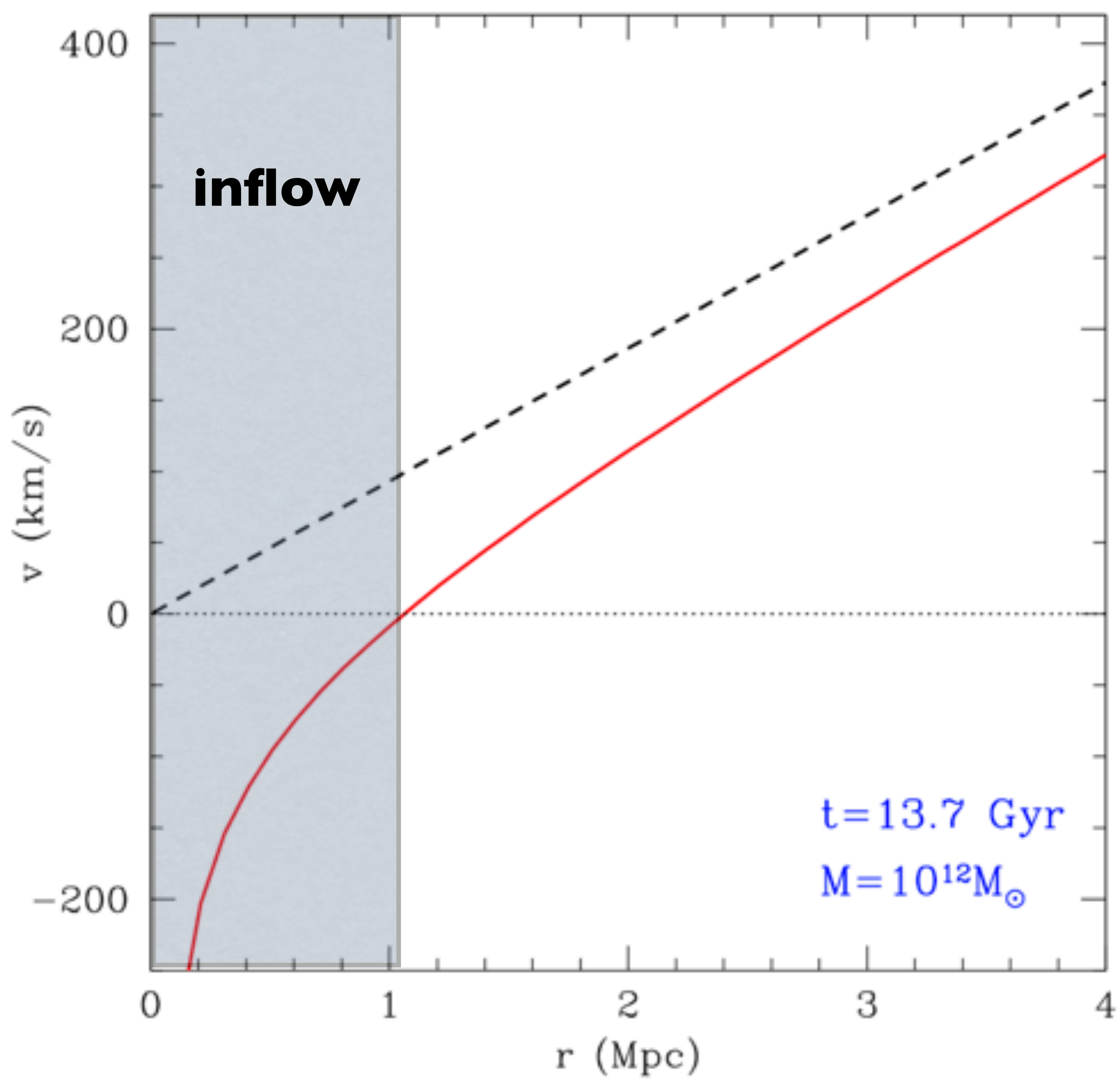
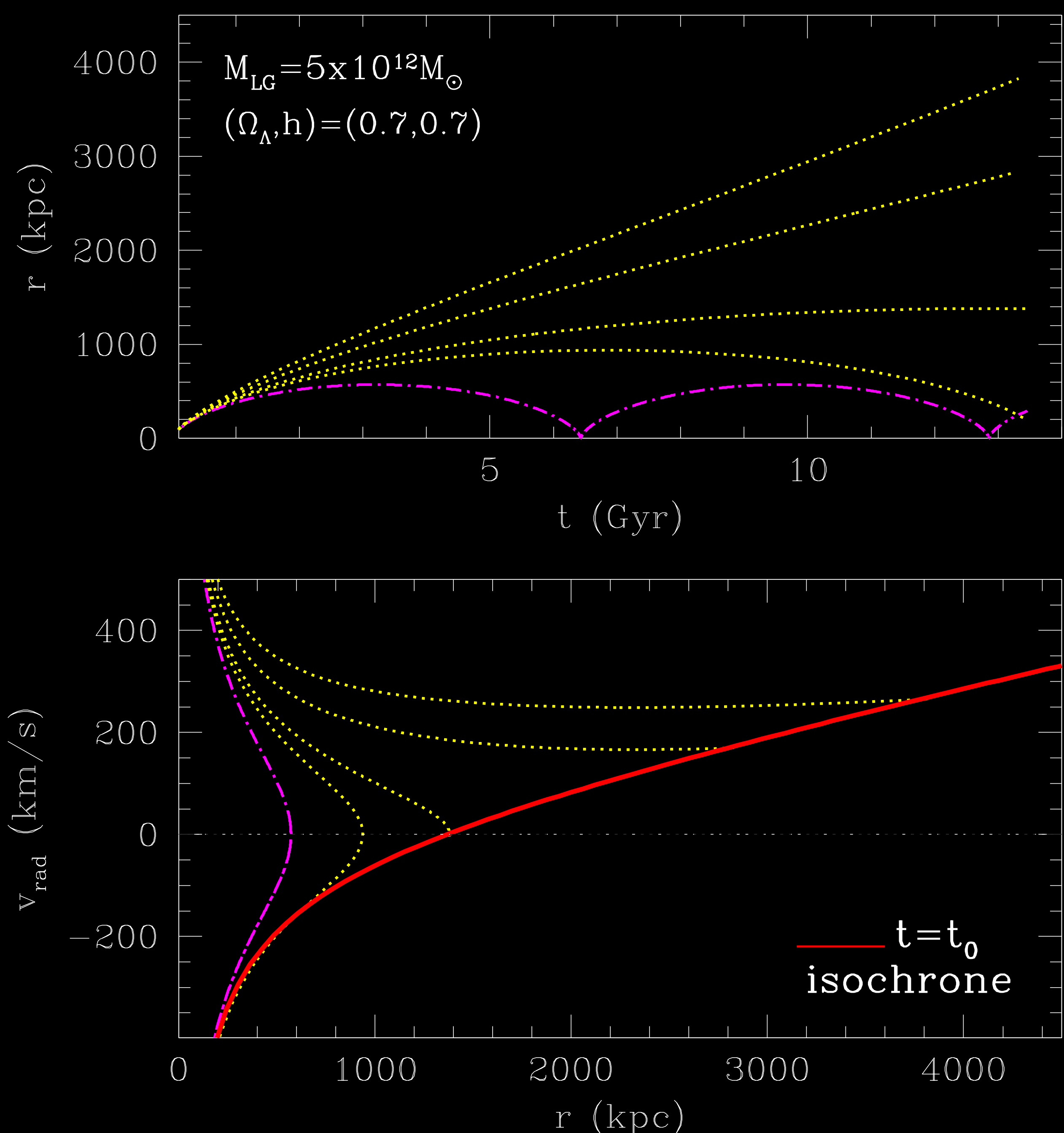
The expansion of the Universe is momentarily halted (**v=0**) at $\mathbf{r_0=(GM\ t^2)^{1/3}}$ (*turn-around radius*).



NOTE: $\mathbf{r_0}$ grows with time even when **M**=const.

$$\ddot{r} = -\frac{GM}{r^2}$$

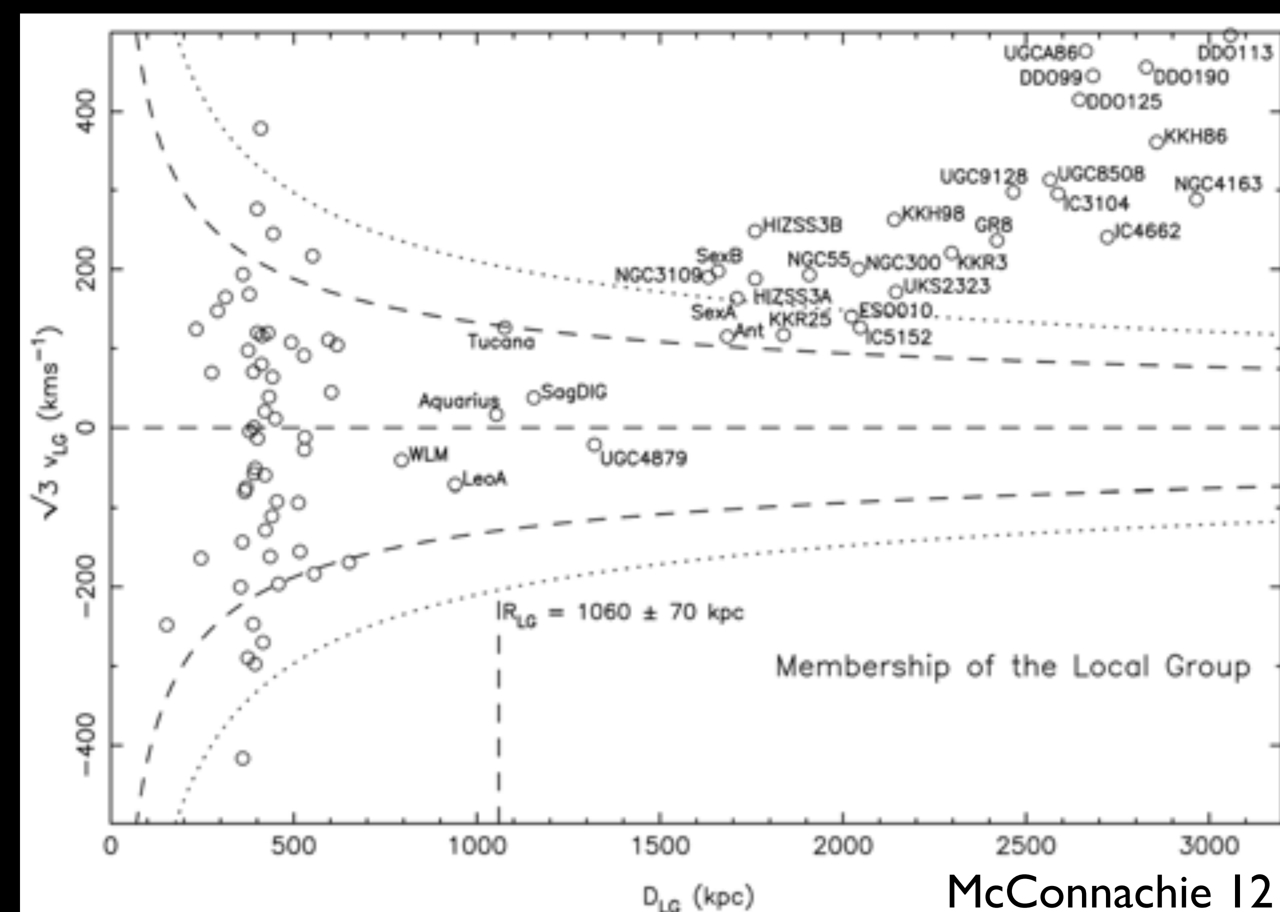
Galaxies around the Local Group



$$v = 1.2\frac{r}{t} - 1.1\left(\frac{GM}{r}\right)^{1/2}$$

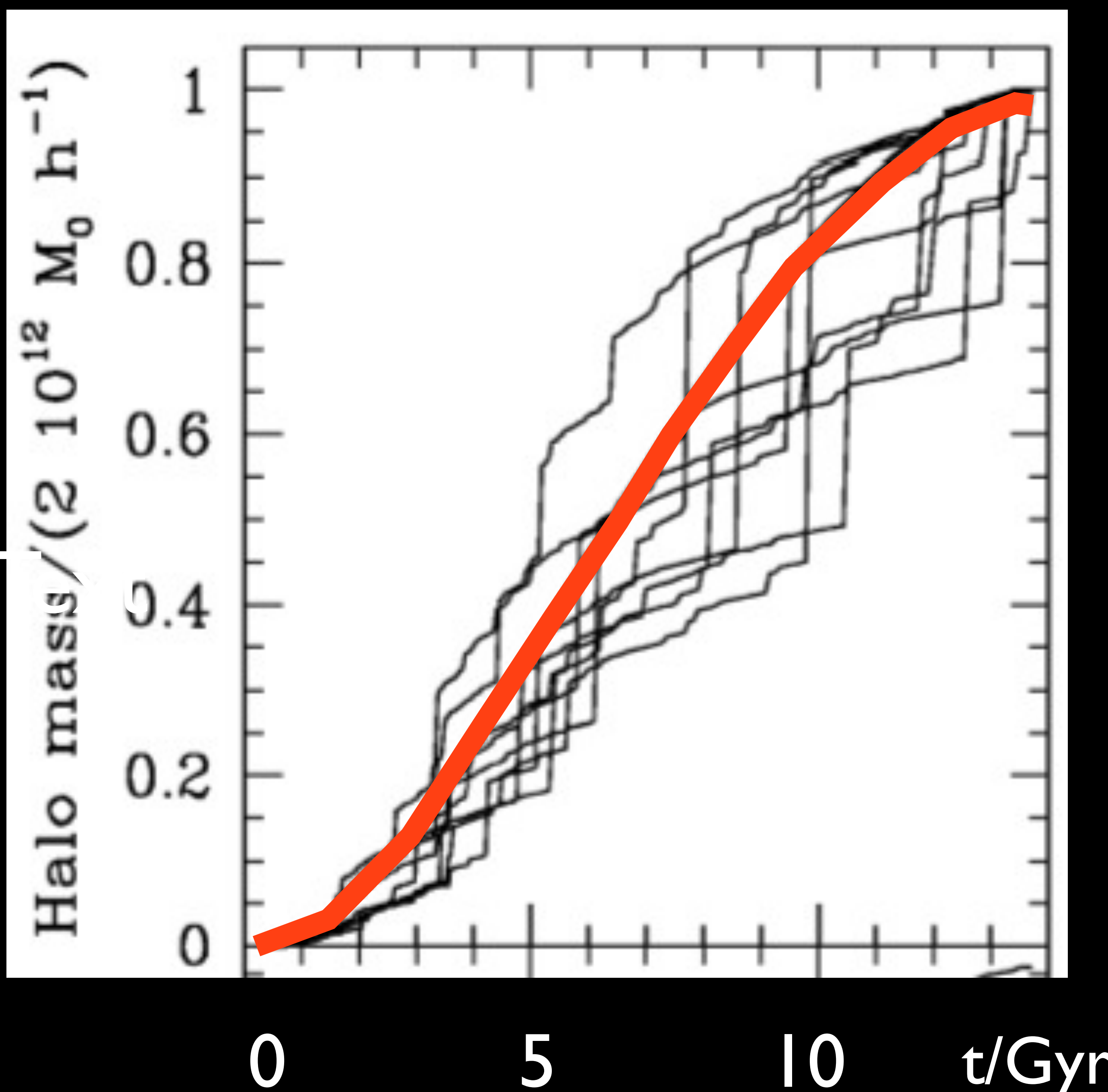
Isochrones can be used to model the local Hubble flow

Lynden-Bell (1981); Sandage (1986);
Peñarrubia et al (2014, 2016)



Universe has a finite age!

But ... how accurate is to assume M=const ?



hierarchical growth via mergers

$$\langle M(z)\rangle \approx M_0 \exp[-z/z_c]$$

Wechsler et al. (2002)

What is the effect of M(t) on the mass derived from timing argument?

Time-dependent potentials

$$F(r,t) = -GM(t)r^n$$

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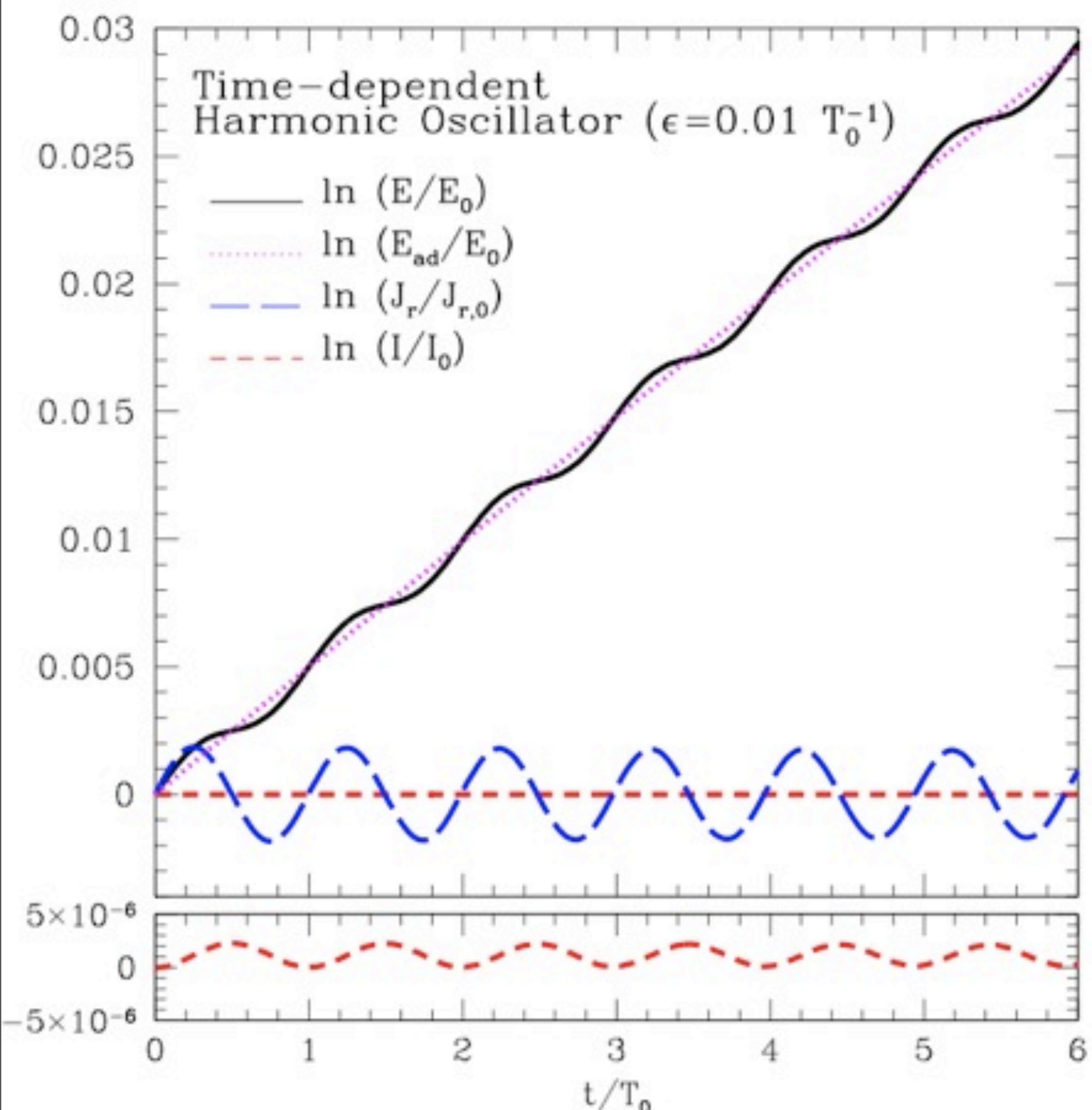
For power-law fields

$$\ddot{R} + r'^{n-1}GM(t)R^n = -r'^{n-1}GM_0R^{-3}.$$

$$R(t) \approx \left[\frac{M_0}{M(t)}\right]^{1/(3+n)}$$

Invariant space

Integrals of motion in **(r',τ)** become **dynamical invariants** in **(r,t)** Peñarrubia 2014, 2016



$$I = \left(\frac{d\mathbf{r}'}{d\tau}\right)^2 + \Phi'(\mathbf{r}') = \frac{1}{2}(R\mathbf{v} - \dot{R}\mathbf{r})^2 + \frac{1}{2}\ddot{R}Rr^2 + R^2\Phi(\mathbf{r},t)$$

$$E = \underbrace{\frac{I}{R^2}}_{\text{Adiabatic term } E_a} + \underbrace{(\mathbf{r}\cdot\dot{\mathbf{r}})\frac{\dot{R}}{R} - \frac{1}{2}r^2\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right)}_{\text{Deviation } \Delta};$$

r · v independent of angular momentum

Δ varies in phase with the particle orbit

Friday, 8 July 16

Isochrones in time-dependent potentials

Expressing the frequencies in the invariant coordinates $(\mathbf{r}', \tau)$

$$\Omega(t) \mapsto \frac{1}{R^2}\left[\frac{GM_0}{r'^3}\right]^{1/2} = \frac{\Omega'}{R^2(t)}$$

$$\omega(t) \mapsto \frac{1}{r'}\frac{dr'}{d\tau} = \frac{1}{R(t)r'}\left[2\left(\frac{E_0}{R^2(t)} + \frac{GM_0}{r'R^2(t)}\right)\right]^{1/2} = \frac{\omega'}{R^2(t)}$$

for $\frac{dM}{dt} \ll \frac{M_0}{t_0}$ (adiabatic)

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In a Keplerian potential **n=-2**

$$R(t) \approx \left[\frac{M_0}{M(t)}\right]^{1/(3+n)} = \frac{M_0}{M(t)}$$

The isochrones evolve as

$$\Omega' t + m\omega' t = nR^2(t) = n\left[\frac{M_0}{M(t)}\right]^2;$$

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At present **$M(t_0)=M_0$** and thus **$R(t_0)=1$**

$$\Omega' t_0 + m\omega' t_0 = n$$

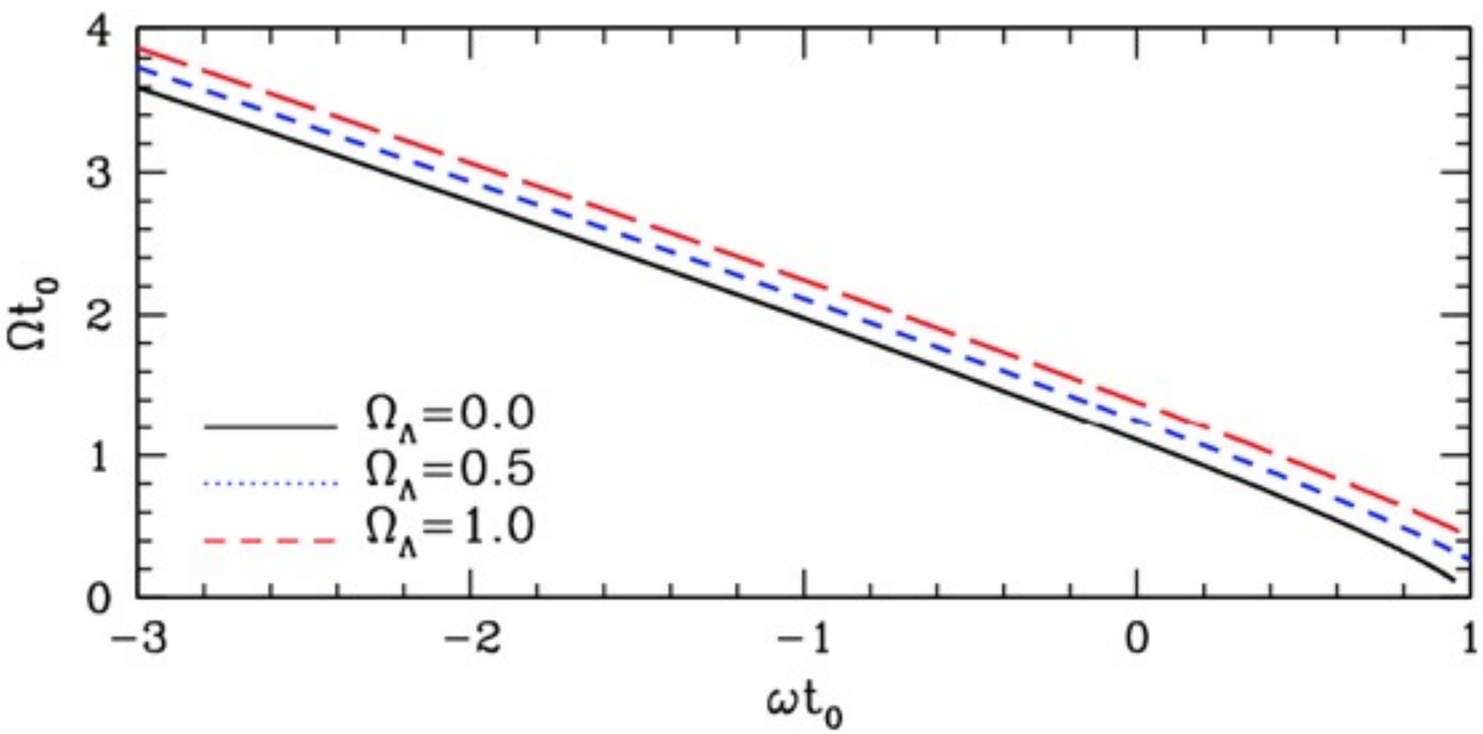
*Perturbed Hubble flow contains **NO** information on past **M(t)***

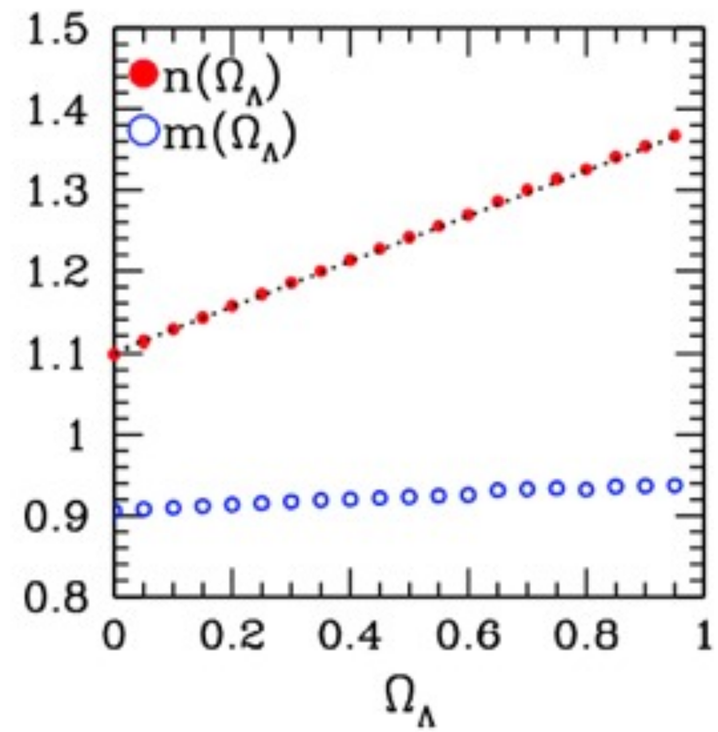
Isochrones with Dark Energy

The above relation can be generalized to $\Omega_\Lambda \neq 0$

$$\ddot{r} = -\frac{GM}{r^2} + H_0^2 \Omega_\Lambda r.$$

$$v \approx (n - \Omega t_0)\frac{r}{m t_0}$$



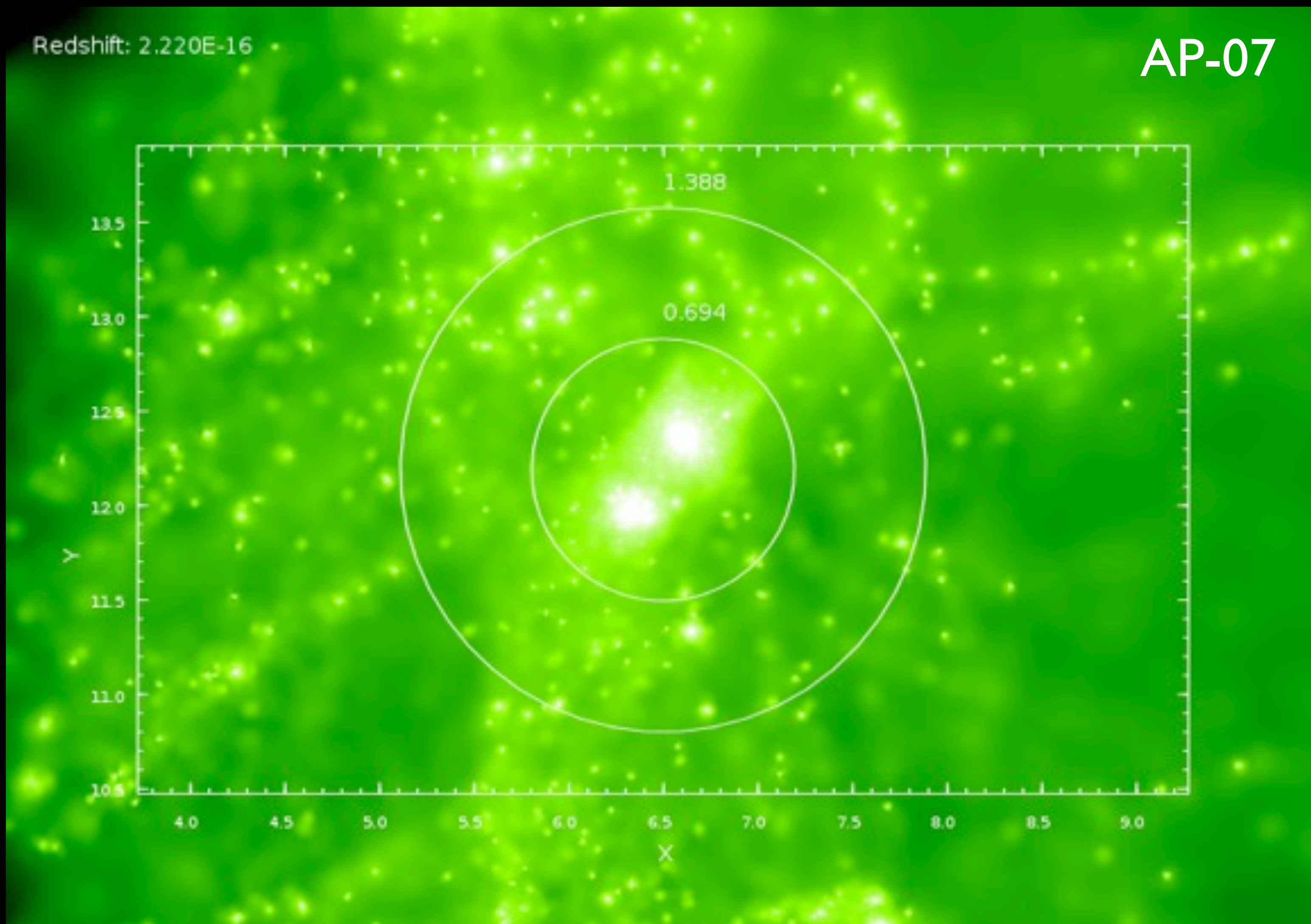


$$v \approx 1.2\frac{r}{t_0} - 1.1\left(\frac{GM}{r}\right)^{1/2} + 0.16\Omega_\Lambda \frac{r}{t_0}$$

LCDM isochrones

Dark energy steepens the Hubble flow by **$\delta_v = v - v(\Omega_\Lambda=0) \sim 0.16\ \Omega_\Lambda\ r/t_0$** independently of **M**. For galaxies within 3Mpc and t_0 =13.46 Gyr this implies **δ_v**< 24 \kms

Tests with cosmological N-body sims.



12 Local Group realizations
(Sawala+16; Fattahi+16)

50 random substructures between 1-3 Mpc

Fit total mass (M), mass ratio (q) and hyperparameter (σ)

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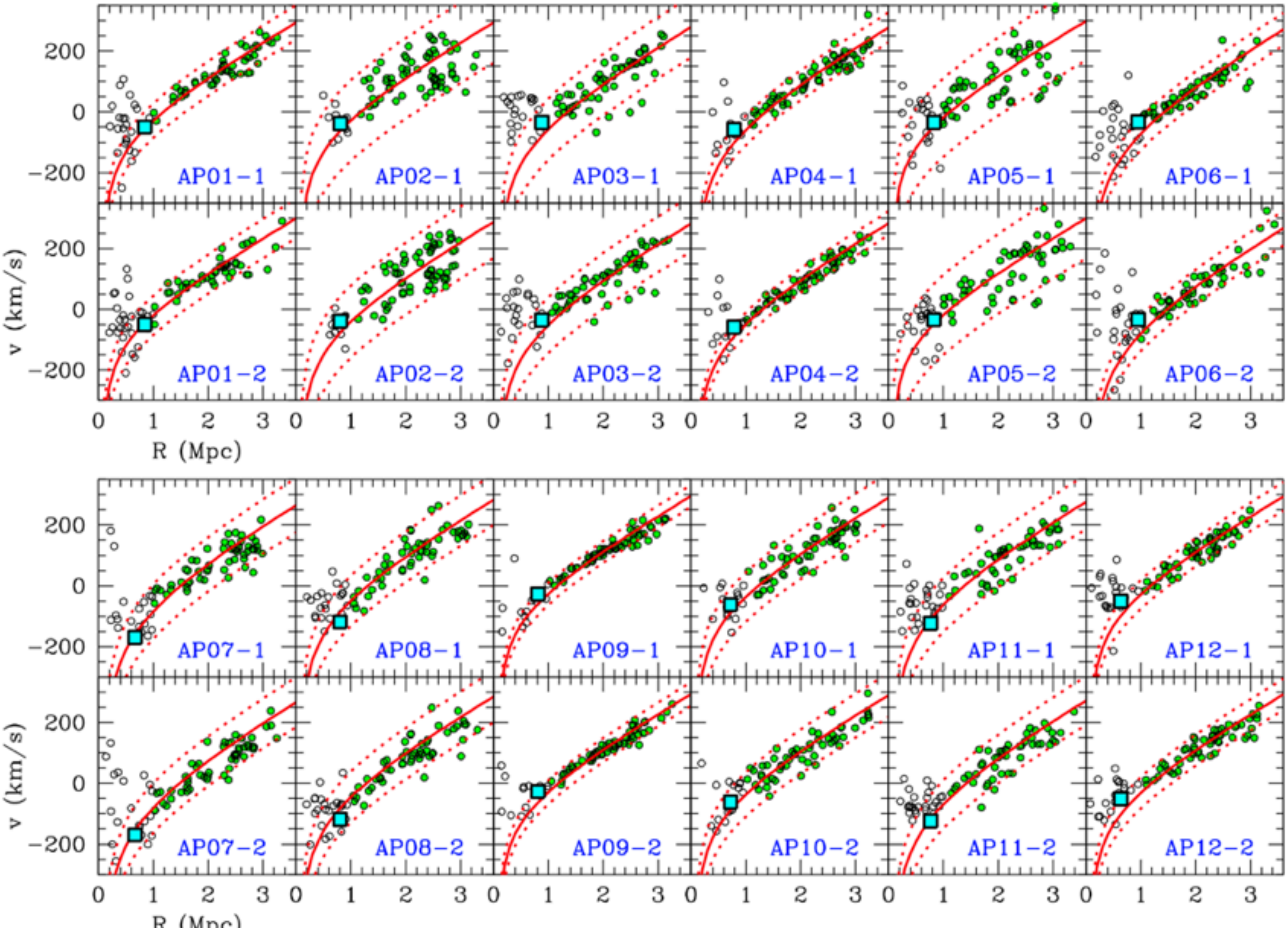


Summary

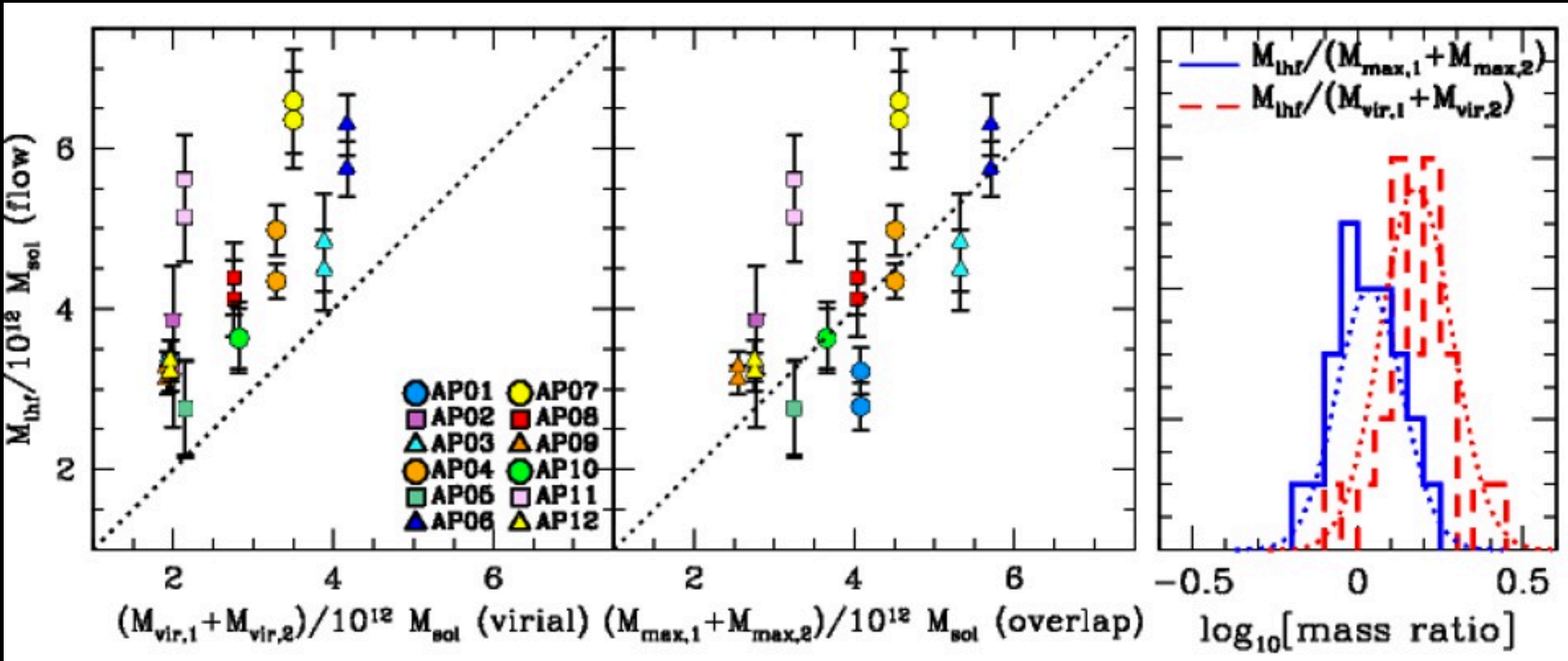
- **The timing argument describes the local expansion of the Universe in a quasi non-linear regime (no shell crossing)**

- **For a point-mass the perturbed Hubble flow in LCDM can be derived analytically**

- **The analytical isochrones provides a simple tool for modelling Local Group dynamics within a Bayesian framework** (Peñarrubia+14,16)

- **The iso-chrone $v=v(r,t_0)$ is an adiabatic invariant, i.e. independent of $M(t)$ insofar as $dM/dt << M/t_0$**

- **Future work: test analytical approach with cosmological N-body models of the Local Group (e.g. Apostle, CLUES?) as well as external galaxies (issue: projection effects)**



$M_{lhf}/10^{12}$ M_{sol} (flow)
$(M_{vir,1}+M_{vir,2})/10^{12}$ M_{sol} (virial)
$(M_{max,1}+M_{max,2})/10^{12}$ M_{sol} (overlap)
AP01
AP02
AP03
AP04
AP05
AP06
AP07
AP08
AP09
AP10
AP11
AP12
$M_{lhf}/(M_{max,1}+M_{max,2})$
$M_{lhf}/(M_{vir,1}+M_{vir,2})$
$\log_{10}$[mass ratio]