Bulk Flows in Theory and Practice

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Bulk Flow Theory

Peculiar Velocity Challenges:

- Can only measure radial component $s_i = \vec{v}_i \cdot \hat{r}_i$
- Individual velocities typically have *absurdly* large uncertainties
- Uncertainty in zero-point of distance relations or Hubble constant can lead to systematic error in velocities
- Velocity field only linear on large scales

Weighted Average to Reduce Uncertainty:

General Velocity Moments defined as

$$U_n \equiv \sum_i w_{n,i}\ s_i$$

Weights can be chosen to

- Minimize Uncertainty
- Estimate Standardized Flow (*i.e.* comparable between surveys)
- Match Physical Definition (*e.g.* integral of velocity over spherical volume)
- Probe $P(k)$ in particular way (*e.g.* focus on large, linear, scales)
- Minimize dependence on the value of H_o

or a combination of these.

Velocity Moment Example:

Bulk flow as integral of velocity field over spherical volume

$$\vec{U} = \int_{sphere} \vec{v} \, d^3x$$

For *ideal* survey (uniform, no errors)

$$w_{n,i} = \frac{\hat{r}_i \cdot \hat{r}_n}{r^2}$$

are the weights to estimate U_n (see Nusser 2014)

Bulk Flow for *Real* surveys with uncertainties

Weights should also have an uncertainty dependence.

Maximum Likelihood Method

- Minimizes uncertainty in Bulk Flow
- Bulk Flow estimate not comparable between surveys

Minimum Variance Method

- Estimate Standardized Bulk Flow (Comparable between surveys)
- Constraints easily incorporated
- Uncertainty cost not very high.

Statistics Aside

Distance Modulus $\mu \sim \log(r)$ has Gaussian errors, not distance r

Standard Estimator:

$$v_e = cz - H_o r_e$$

Biased and non-Gaussian errors

New, Improved Estimator:

$$v_e = cz \log(cz/H_o r_e)$$

Unbiased and Gaussian errors $\Rightarrow$ proper cancellation of errors

(See Watkins & Feldman 2015)

New Estimator

Unbiased:

$$\begin{aligned}\langle v_e \rangle &= -cz\left(\langle \log(H_o r_e) \rangle - \log(cz)\right) \\ &= -cz\left(\log(H_o r) - \log(cz)\right) \\ &= -cz\left(\log(cz - v) - \log(cz)\right) \\ &= -cz\left(\log(1 - v/cz)\right) \\ &\approx v\end{aligned}$$

assuming $v/cz \ll 1$ which it true for most galaxies and clusters.

Gaussian Distributed Errors since $\propto$ distance modulus μ

Go BIG

- Velocity perturbations enhanced on large scales relative to density perturbations, $P_v(k) = P(k)/k^2$, so peculiar velocities can tell us things that densities can't.
- We only know the theoretical *variance* of the bulk flow components. The only way to reduce this variance is to go to large scales. To have a possibility of getting a greater than 3σ result need to have $\sigma \sim 100$ km/sec per component.

Window Functions

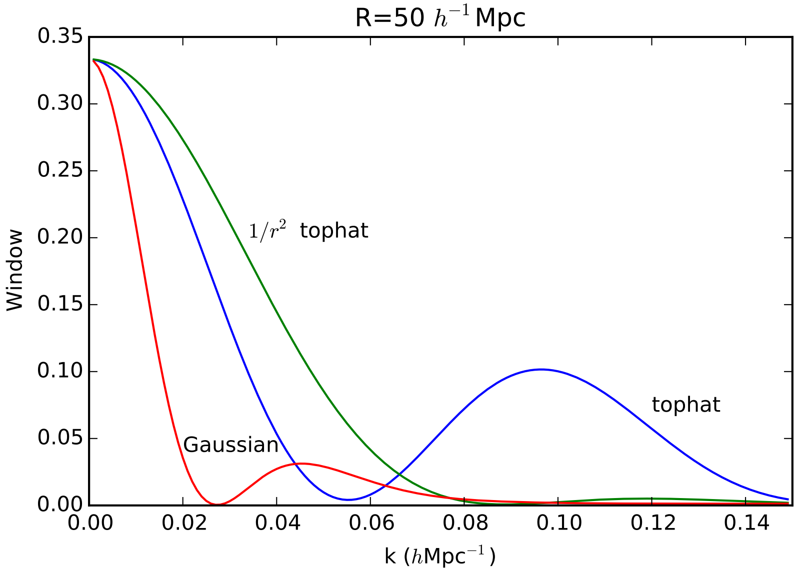
Advantage of Velocity Moments:

Easy to calculate angle-averaged Window Function $W(k)$ for U_n such that

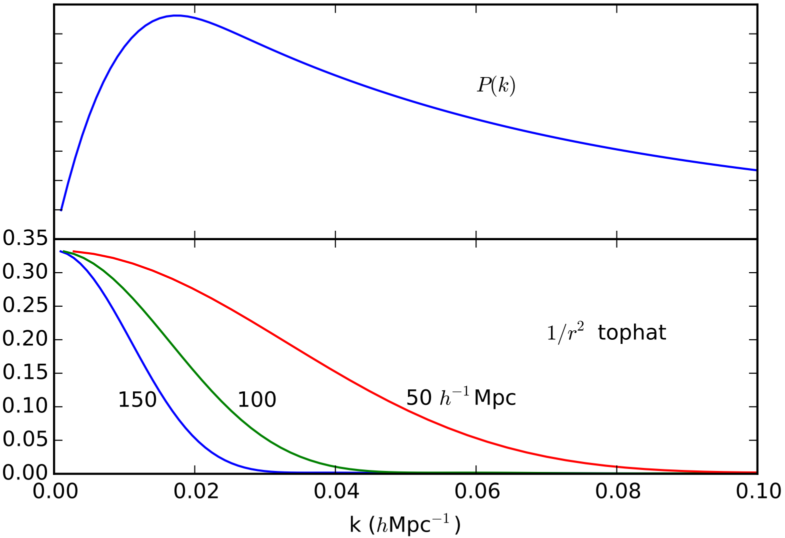
$$\langle U_n^2 \rangle = \int_0^\infty P(k)W(k)\ dk$$

- $W(k)$ tells us precisely what scales U_n probes.

Window Functions



Window Functions



Minimum Variance Method

$$U_n = \sum_i w_{n,i}\ s_i$$

Determine weights $w_{n,i}$ that minimize $\langle (U_n - u_n)^2 \rangle$, where u_n is velocity moment that would be measured by an ideal survey with desired radial distribution.

- Need to assume a velocity power spectrum.
- Uncertainty cost over Maximum Likelihood estimator generally small.
- Easy to incorporate *constraints* using Lagrange multipliers, *e.g.* normalization condition.

see Watkins, Feldman, & Hudson (2009) and Feldman, Watkins, & Hudson (2010)

Minimum Variance Method

MV method weights survey objects in order to probe space in same way as ideal survey.

- Objects in oversampled (undersampled) regions will be down(up)-weighted.
- Ensures that results won't be biased to reflect regions where there is more information, *e.g.* at small distances.

New Constraint:

Contribution to U_n from change in zero-point or Hubble constant $\propto \sum_i w_{n,i}\ cz_i$ (assuming new estimator).

Suggests constraint:

$$\sum_i w_{n,i}\ cz_i = 0$$

Creates moments U_n that are <u>independent</u> of choice of H_o or zero-point.

cz vs. r_{meas} as Distance Indicator:

How to determine positions of objects for MV method?

Redshift as distance indicator

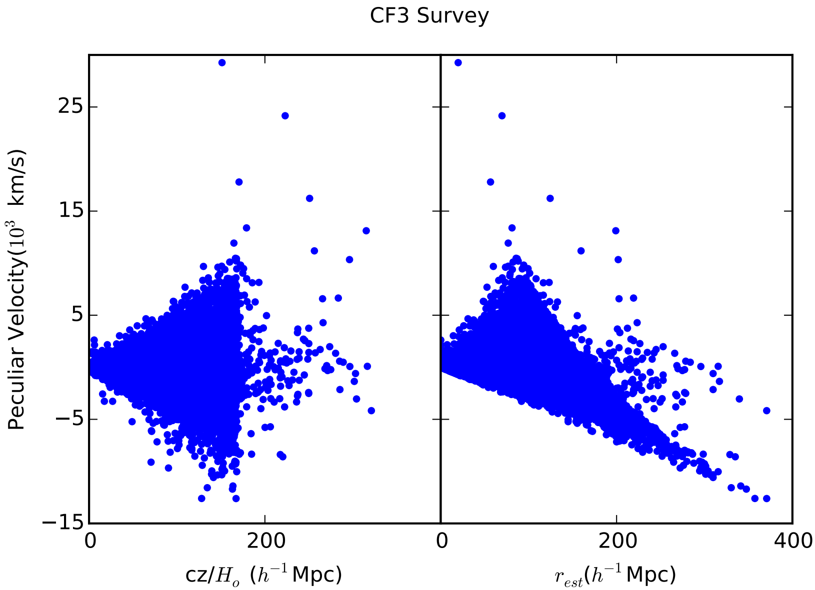
$$r_{cz} = cz/H_o = r - v/H_o$$

Distance as distance indicator

$$r_{meas} = r + \delta r$$

- For reasonably distant objects $v/H_o \ll \delta r$
- Reduces Malmquist biases

cz vs. r_{meas} as Distance Indicator:

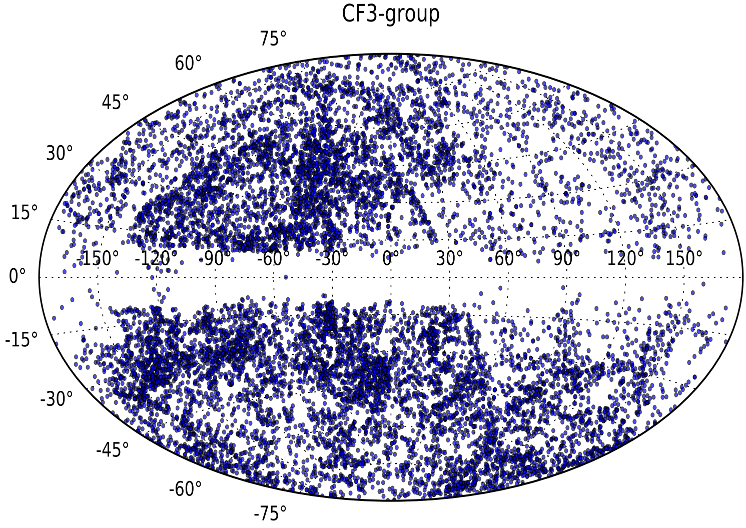


New Results: Measuring Bulk Flows using *CosmicFlows-3*

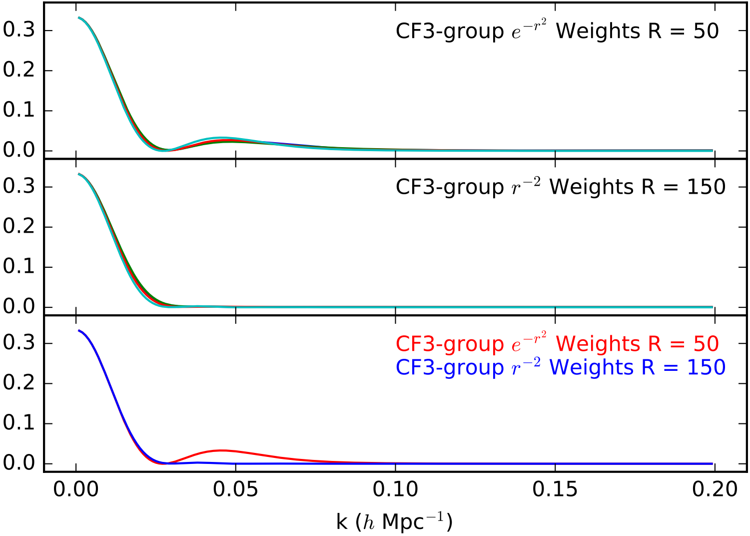
The Data: CosmicFlows-3 (Tully et al. (2016))

- Used 11,878 individual galaxies and groups of galaxies
  - Addition of 6dFGS, etc. $\Rightarrow$ Distribution not isotropic.
- Used cz/H_o as distance estimate
- Used new velocity estimator $v = cz \log(cz/H_o)$
- Used new constraint to make independent of choice of H_o
- Used both Gaussian Ball and $1/r^2$ tophat ideal surveys

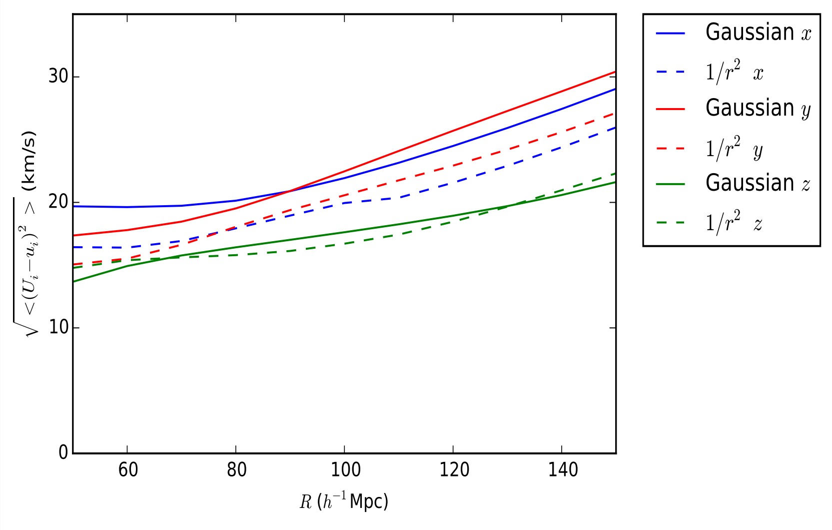
CosmicFlows3: Galactic Coordinates



How Big can we go?: Window Functions



Expected difference with Ideal Survey: $\sqrt{\langle (U_i - u_i)^2 \rangle}$

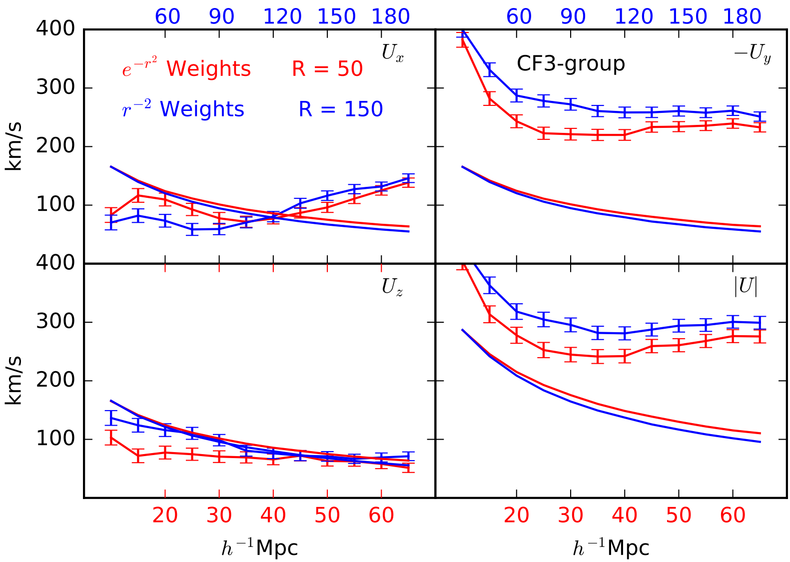


Better match to ideal case with $1/r^2$ tophat weighting

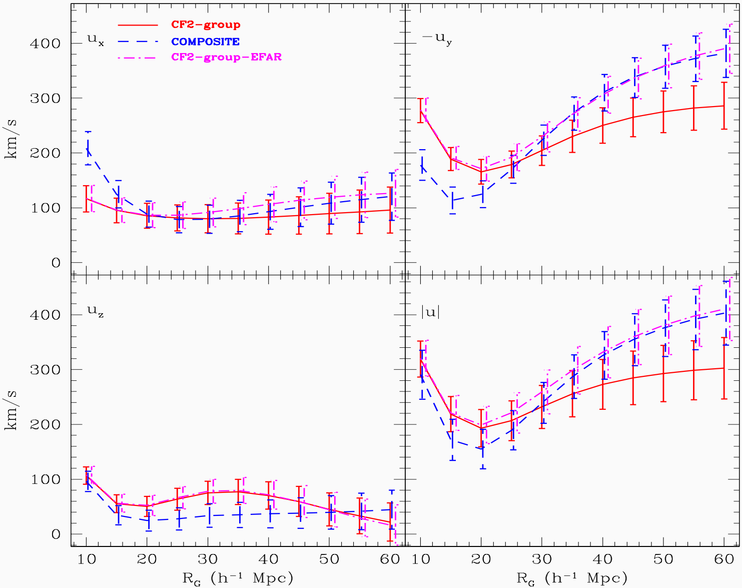
Advantages of $1/r^2$ tophat weighting over Gaussian

- More desirable window function.
- Smaller σ for Bulk Flow Moments
- Better match to realistic surveys.
- Matches intuitive physical definition of Bulk Flow

New Results from the CF3



Compare to Previous Results



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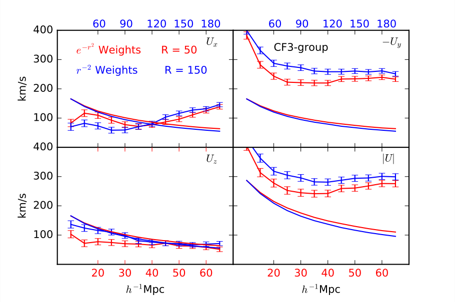
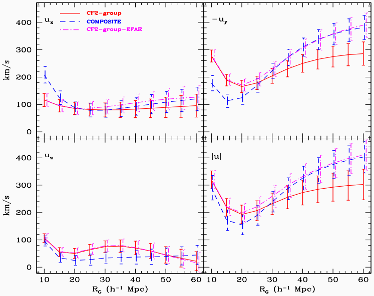
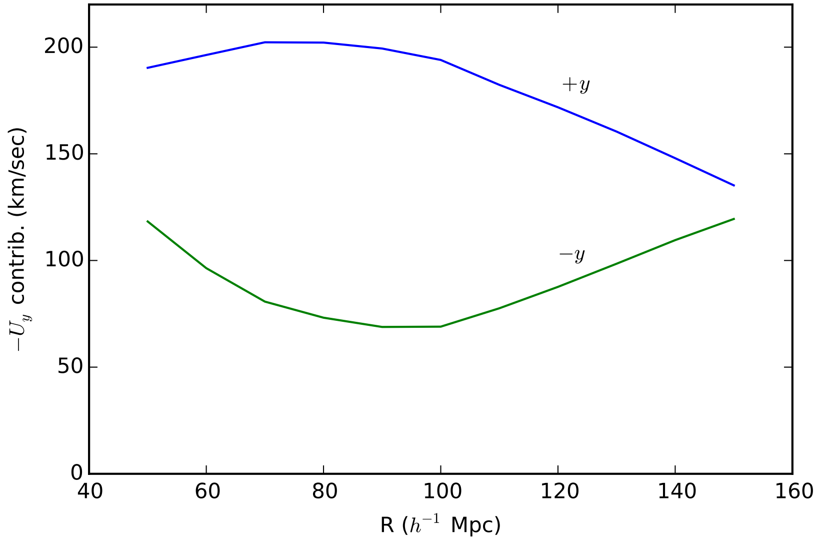




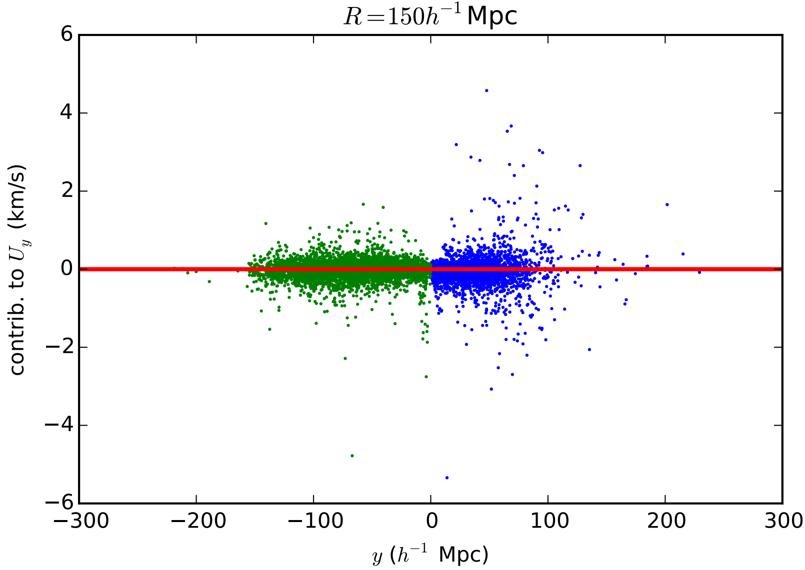
Key Results for CF3:

- Bulk Flow doesn't increase with distance. $\ddot\smile$
- Generally lower amplitude Large Scale Flow
- But...still doesn't fall off as fast as expected.

$+y$ vs. $-y$ contributions



Contribution to U_y for each object $c_i = w_i s_i$



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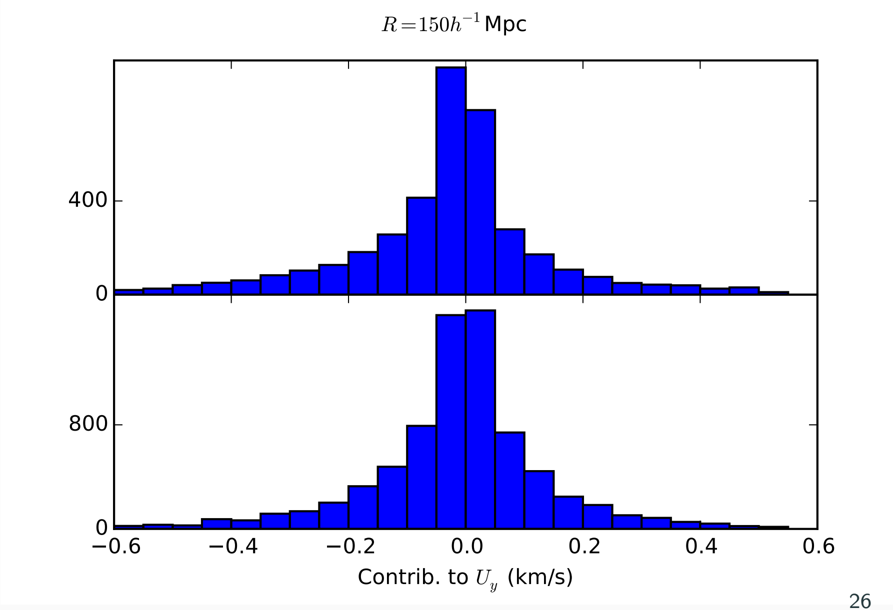
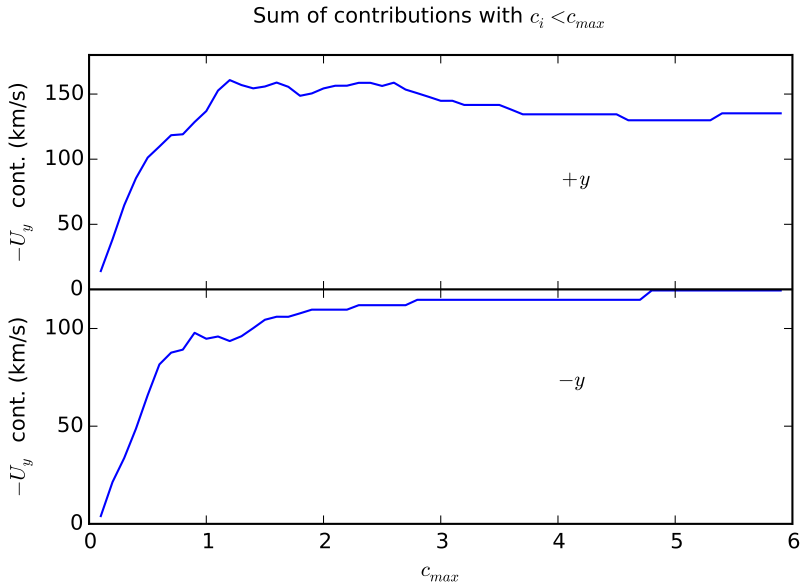
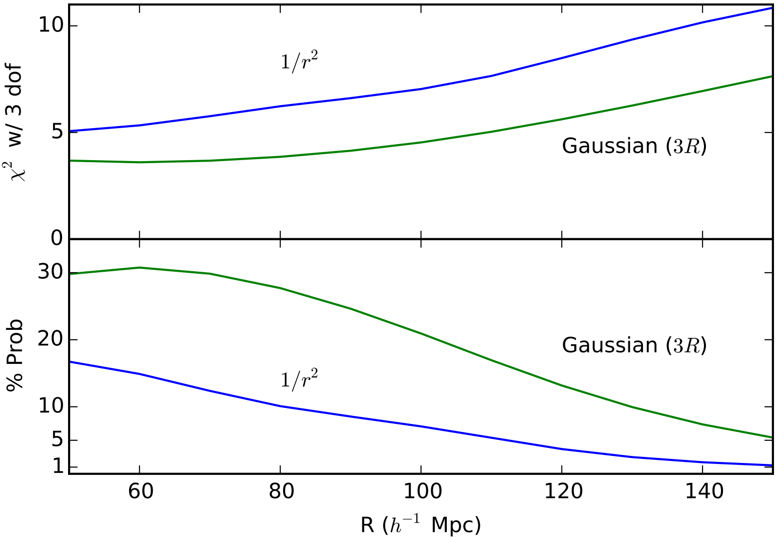


Sum of contributions to U_y with $c_i < c_{max}$



χ^2 with 3 degrees of freedom



Summary:

- CF3 catalog gives bulk flow w/ more “reasonable” R dependence.
- Smaller bulk flow on Large scales: ~ 300km/s for $R = 150h^{-1}$Mpc for $1/r^2$ weights
- Still “Tension” with Standard Cosmological Model: $\sim 1\%$ Prob. for $1/r^2$ weights w/ $R = 150h^{-1}$Mpc
- Need more distance measurements in $+y$ direction