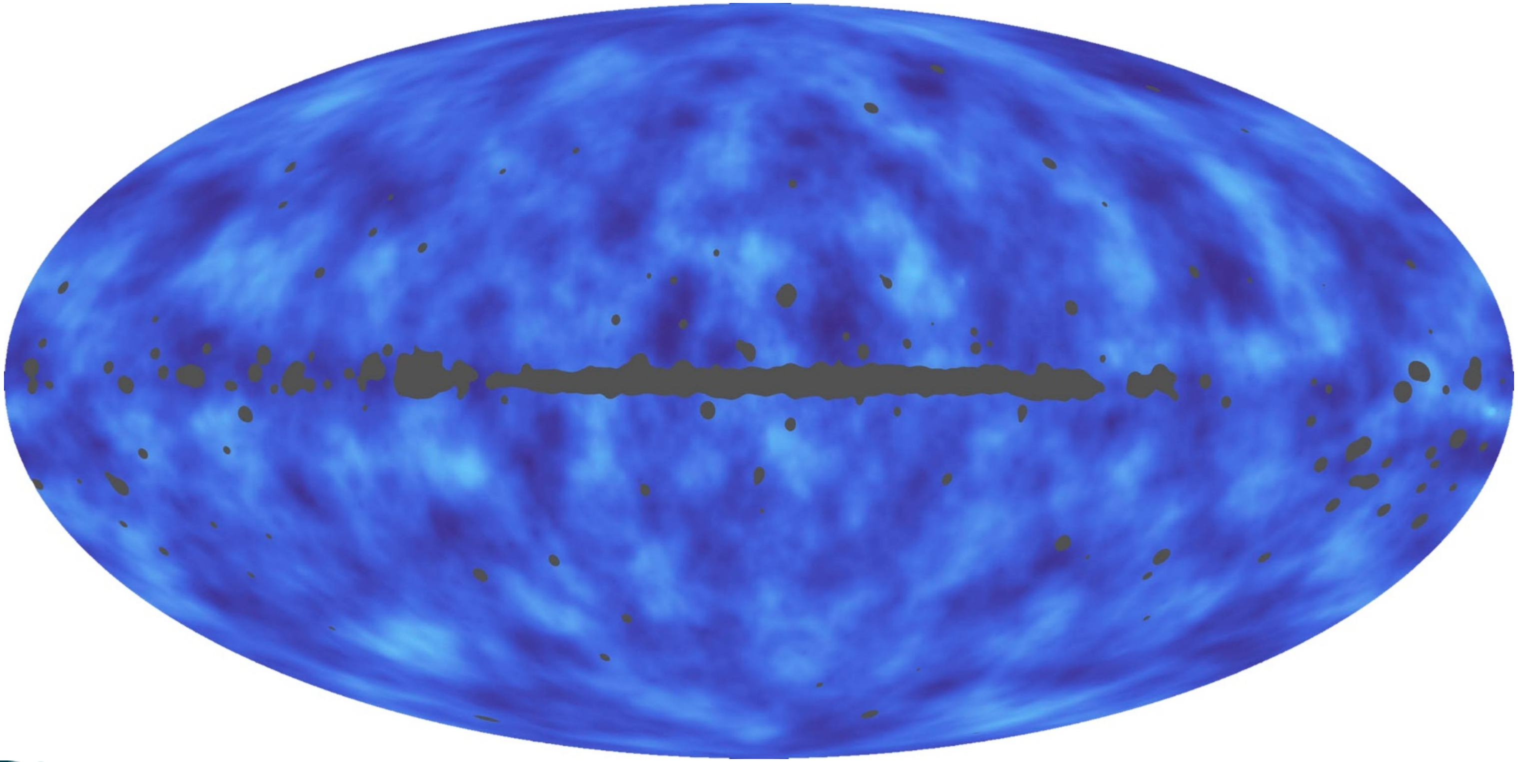
CMB Lensing

Antony Lewis

http://cosmologist.info/



UNIVERSITY OF SUSSEX

Science & Technology Facilities Council

European Research Council
Established by the European Commission
Supporting top researchers from anywhere in the world

Physics Reports review: astro-ph/0601594

GRG review: astro-ph/0911.0612

Weak Gravitational Lensing of the CMB

Antony Lewis[1]

Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, UK.[2]

Anthony Challinor

Astrophysics Group, Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, U.K.

Abstract

Weak gravitational lensing has several important effects on the Cosmic Microwave Background (CMB): it changes the CMB power spectra, induces non-Gaussianities, and generates a B-mode polarization signal that is an important source of confusion for the signal from primordial gravitational waves. The lensing signal can also be used to help constrain cosmological parameters and lensing mass distributions. We review the origin and calculation of these effects. Topics include: lensing in General Relativity, the lensing potential, lensed temperature and polarization power spectra, implications for constraining inflation, non-Gaussian structure, reconstruction of the lensing potential, delensing, sky curvature corrections, simulations, cosmological parameter estimation, cluster mass reconstruction, and moving lenses/dipole lensing.

Key words: Cosmic Microwave Background; Gravitational Lensing
PACS: 98.80.Es, 98.70.Vc, 98.62.Sb, 8.80.Hw

Weak lensing of the CMB

Duncan Hanson · Anthony Challinor · Antony Lewis

Received: date / Accepted: date

Abstract The cosmic microwave background (CMB) represents a unique source for the study of gravitational lensing. It is extended across the entire sky, partially polarized, located at the extreme distance of $z = 1100$, and is thought to have the simple, underlying statistics of a Gaussian random field. Here we review the weak lensing of the CMB, highlighting the aspects which differentiate it from the weak lensing of other sources, such as galaxies. We discuss the statistics of the lensing deflection field which remaps the CMB, and the corresponding effect on the power spectra. We then focus on methods for reconstructing the lensing deflections, describing efficient quadratic maximum-likelihood estimators and delensing. We end by reviewing recent detections and observational prospects.

Lensing warm up quiz: true or false?

1) Overdensities focus light rays, so the CMB looks hotter where there are overdensities along the line of sight

2) Even in linear theory lensing is mostly at low redshift because density perturbations grow with time

3) The lensing potential is nearly Gaussian because there are many lenses along the line of sight and perturbations nearly linear

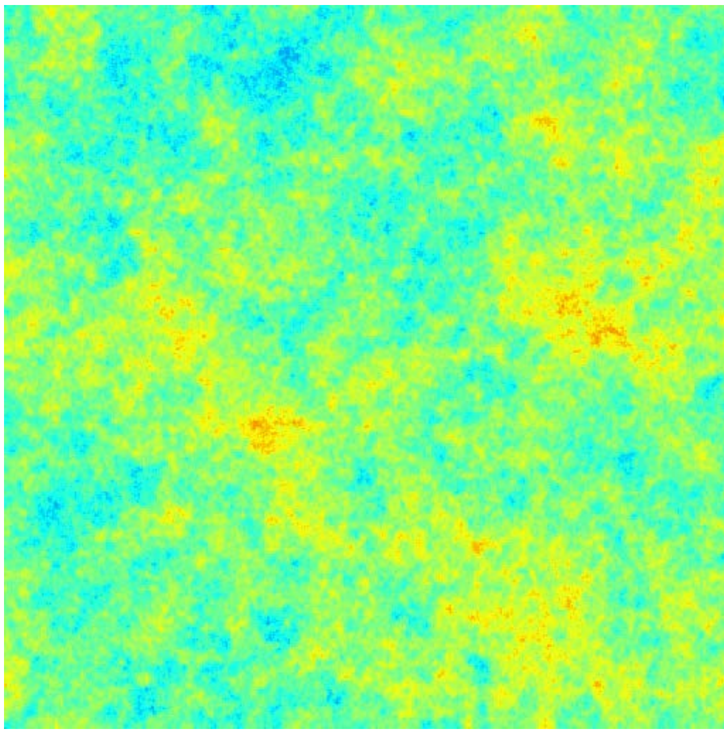
4) Lensing rotates polarization, partly turning E modes into B modes

5) The CMB lensing power spectrum peaks at $L \sim 60$, so temperature lensing reconstruction is sensitive to large-scale galactic foregrounds

CMB temperature

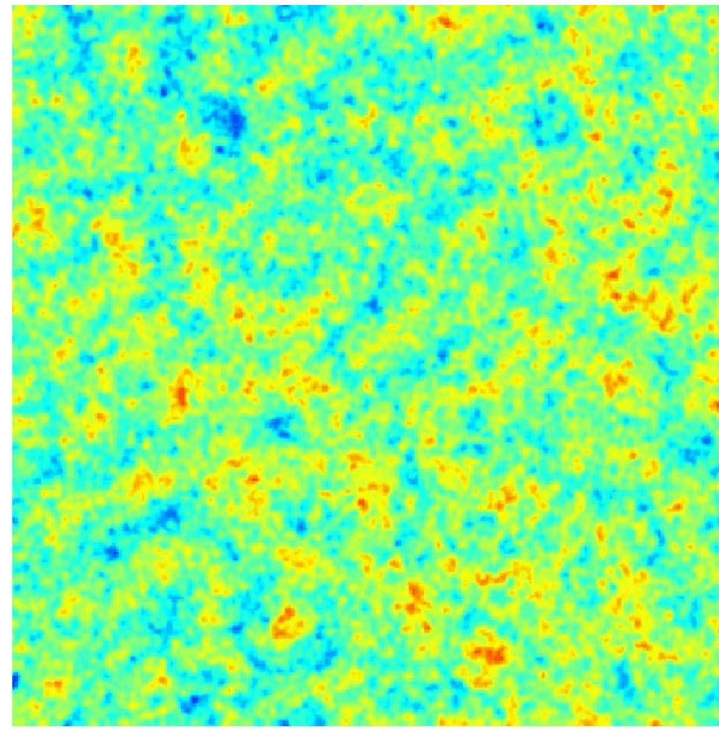
0th order uniform temperature + 1st order perturbations:

Perturbations: End of inflation

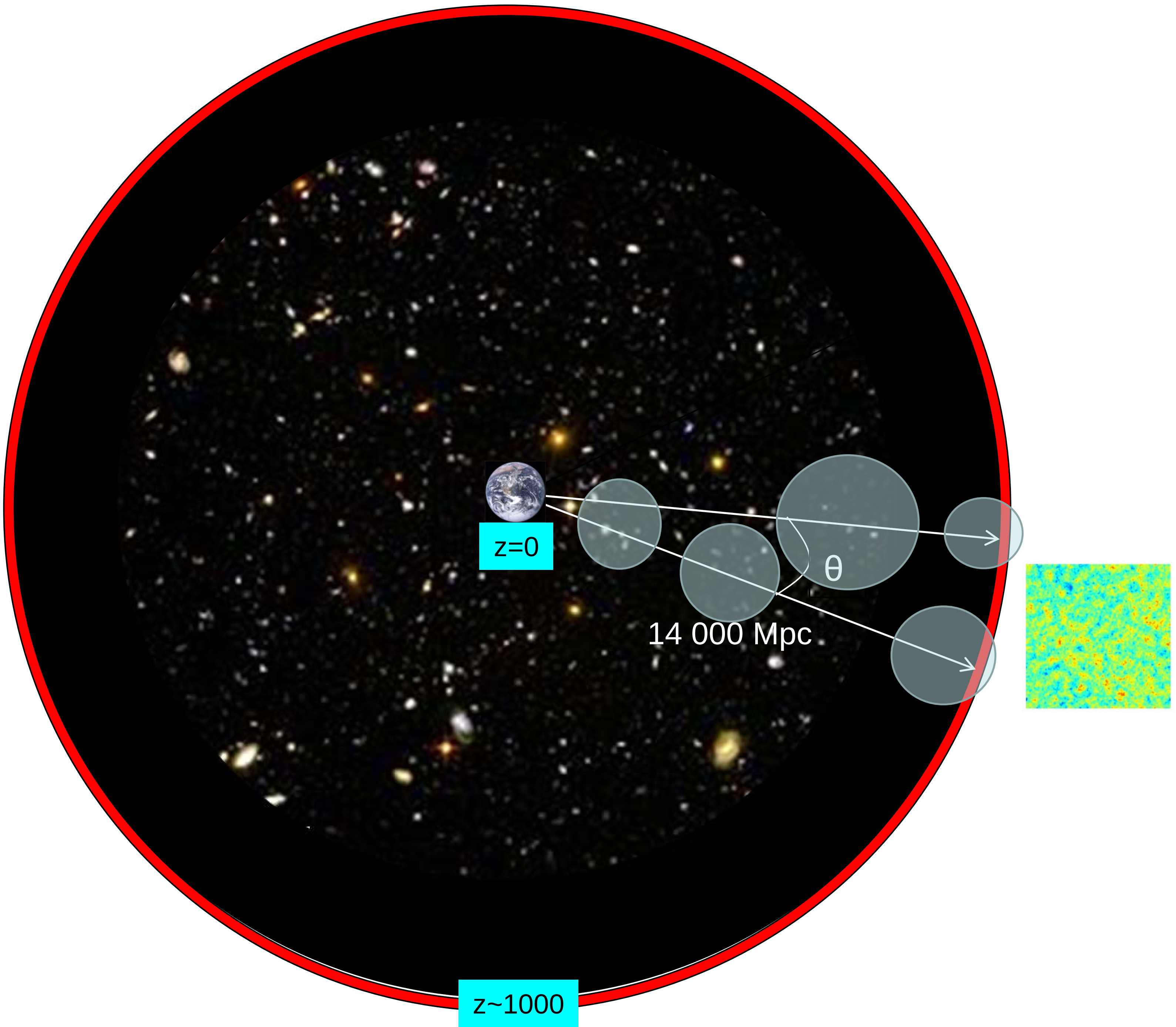


gravity+
pressure+
diffusion

Perturbations: Last scattering surface



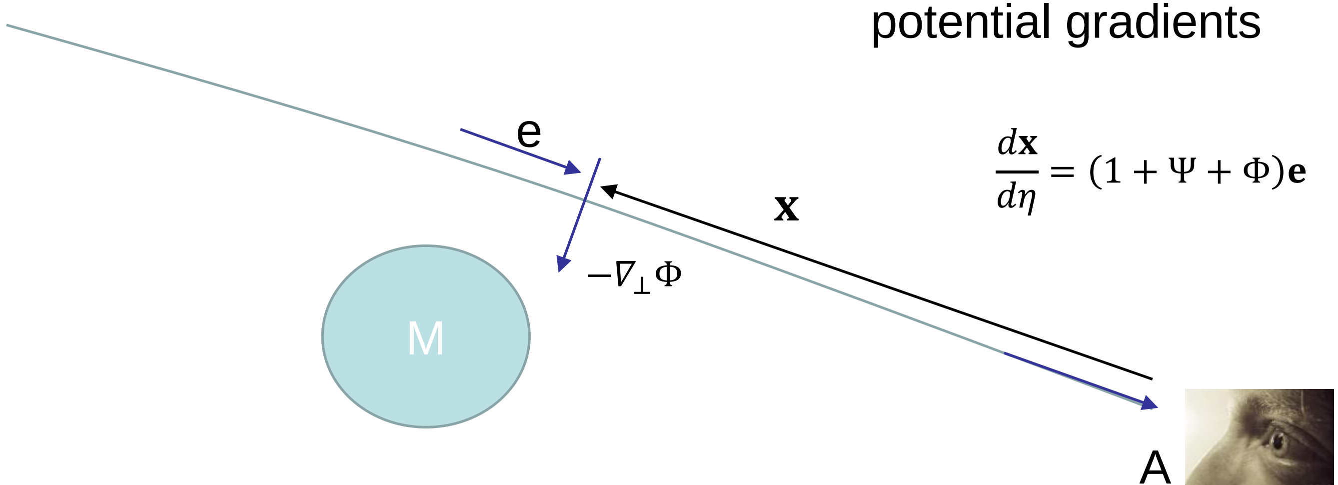
Expect nearly Gaussian
and isotropic linear perturbations


z=0
θ
14 000 Mpc
z~1000

Spatial components of the geodesic equation?

$$\frac{d\mathbf{e}}{d\eta} = -\nabla_\perp(\Phi + \Psi)$$

Where **e** is the spatial photon propagation direction
- deflection due to transverse potential gradients



$$\frac{d\mathbf{x}}{d\eta} = (1 + \Psi + \Phi)\mathbf{e} \qquad \frac{d\eta}{d\lambda} = \frac{E}{a}(1 - \Psi)$$

Full solution for $0^{th}+1^{st}$ order photon path

$$\mathbf{x}(\hat{\mathbf{n}};\eta) = \underbrace{-\mathbf{e}_A(\eta_A - \eta)}_{\text{FRW background solution}} + \underbrace{\mathbf{e}_A \int_{\eta_A}^{\eta} (\Phi + \Psi)\, \mathrm{d}\eta'}_{\text{Time delay}} - \underbrace{\int_{\eta_A}^{\eta} (\eta - \eta')\nabla_\perp(\Phi + \Psi)\mathrm{d}\eta'}_{\text{Lensing}}$$

Zeroth-order CMB

- CMB uniform blackbody at ~2.7 K
(+dipole due to local motion)

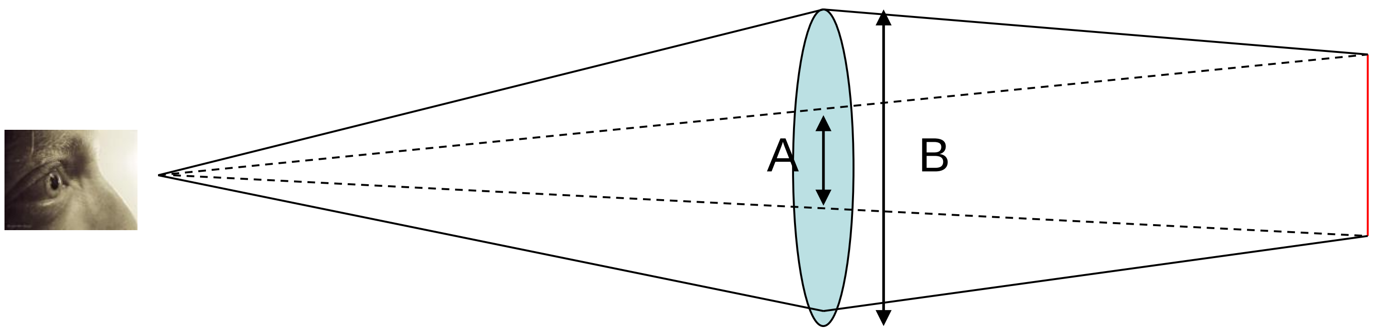
1st order effects

- Linear perturbations at last scattering, zeroth-order light propagation; zeroth-order last scattering, first order redshifting during propagation (ISW)
- usual unlensed CMB anisotropy calculation

- First order time delay, uniform CMB
- last scattering displaced, but temperature at recombination the same
- no observable effect

1^{st} order effects contd.

- First order CMB lensing: zeroth-order last scattering (uniform CMB ~ 2.7K), first order transverse displacement in light propagation



$$\frac{\text{Number of photons before lensing}}{\text{Number of photons after lensing}} = \frac{A^2}{B^2} = \frac{\text{Solid angle before lensing}}{\text{Solid angle after lensing}}$$

<u>Conservation of surface brightness</u>: number of photons per solid angle unchanged

uniform CMB lenses to uniform CMB – so no observable effect

2^{nd} order effects

- Second order perturbations at last scattering, zeroth order light propagation
  -tiny $\sim(10^{-5})^2$ corrections to linear unlensed CMB result

- First order last scattering ($\sim 10^{-5}$ anisotropies), first order transverse light displacement
  - this is what we call CMB lensing

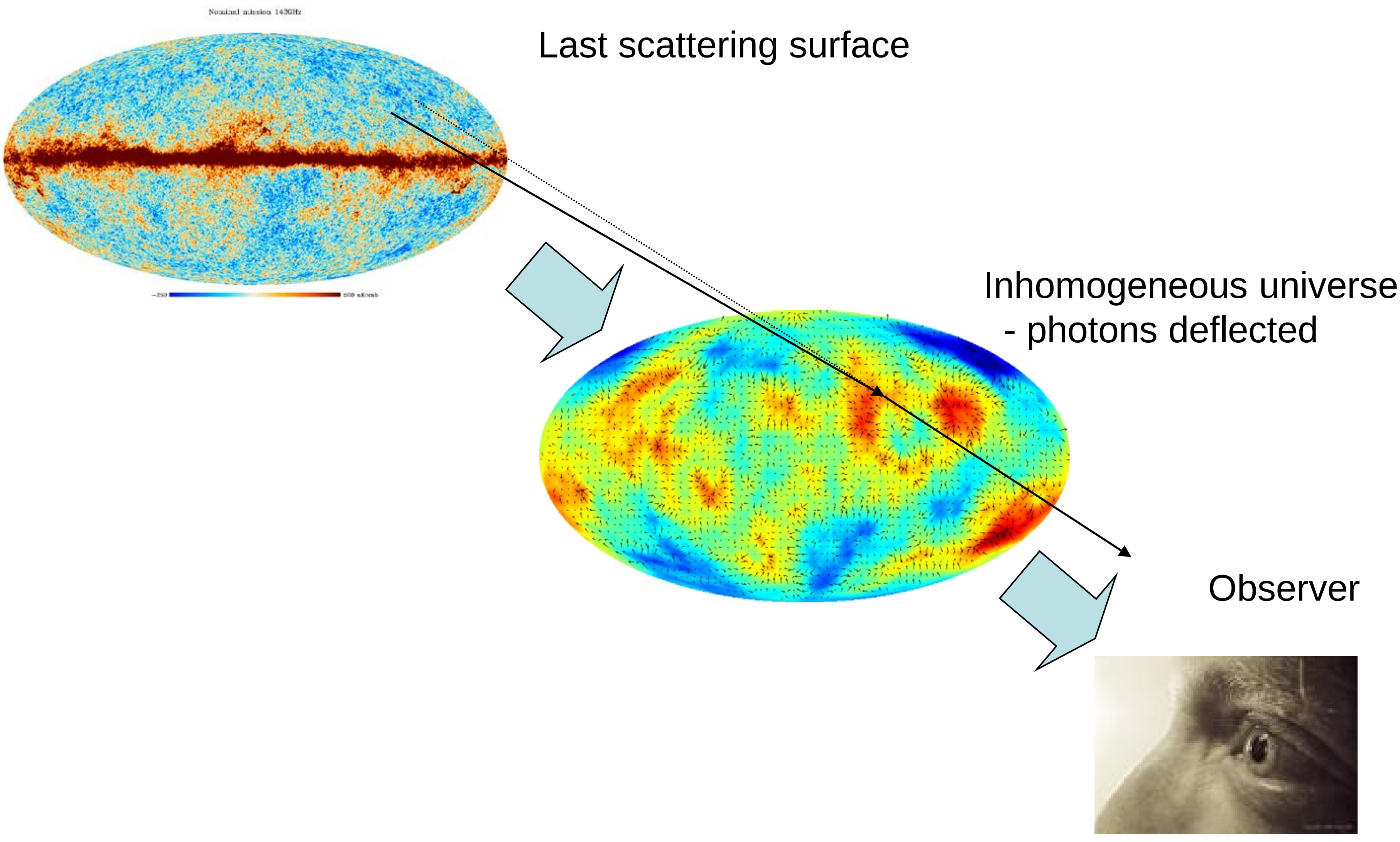
- First order last scattering, first order time delay
  - delay ~1MPc, small compared to thickness of last scattering
  - coherent over large scales: very small observable effect   Hu, Cooray: astro-ph/0008001

- First order last scattering, first order anisotropic expansion
  $\sim(10^{-5})^{2:}$ small but non-zero contribution to large-scale bispectrum
  [equivalent to mapping from physical to comoving $\mathbf{x}$ - the Maldacena consistency relation bispectrum on the CMB]

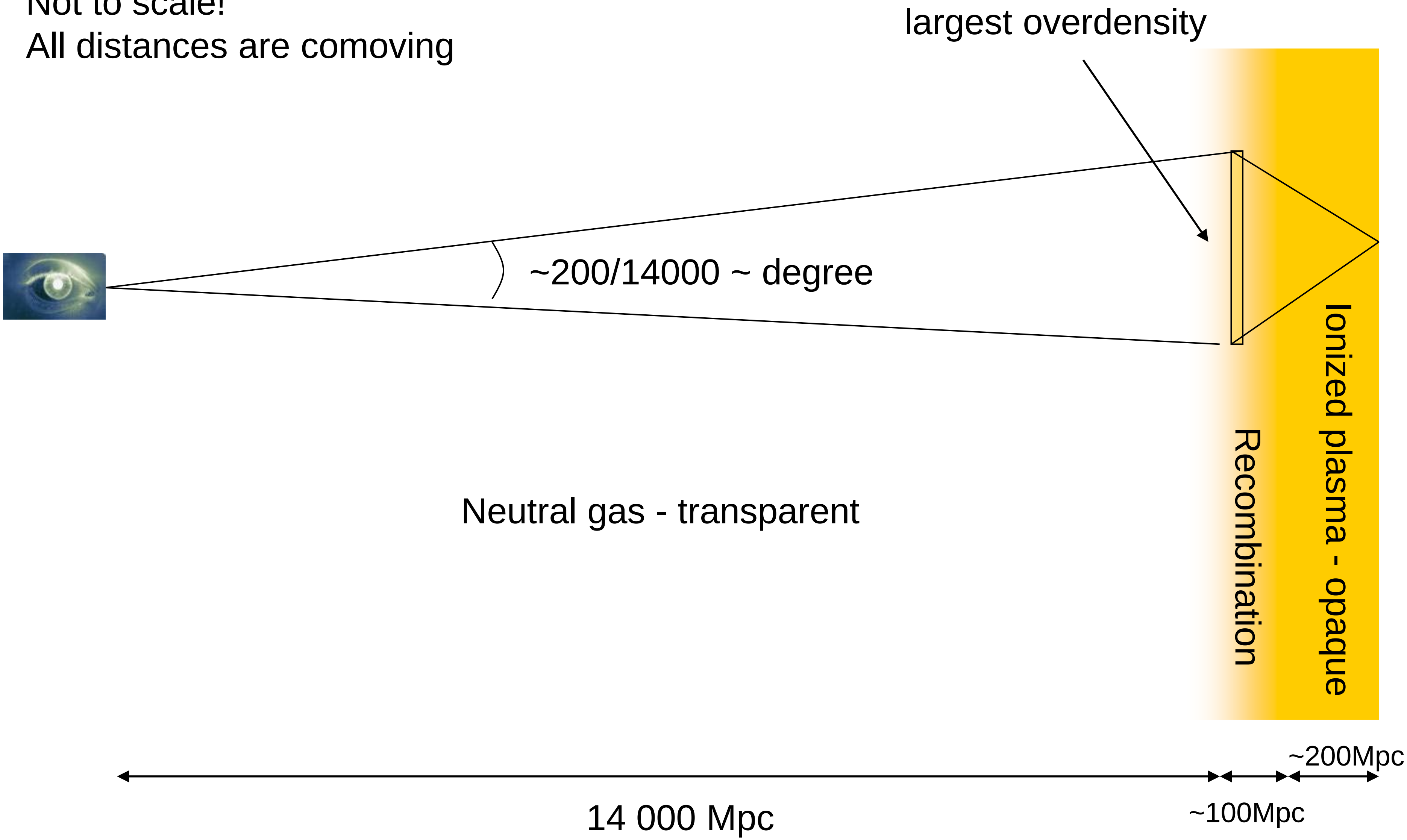
- First order last scattering, first order anisotropic redshifting
  $\sim(10^{-5})^2$: gives non-zero but very small contribution to large-scale bispectrum

- Others
  e.g. Rees-Sciama: second (+ higher) order redshifting
  SZ: second (+higher) order scattering, etc….

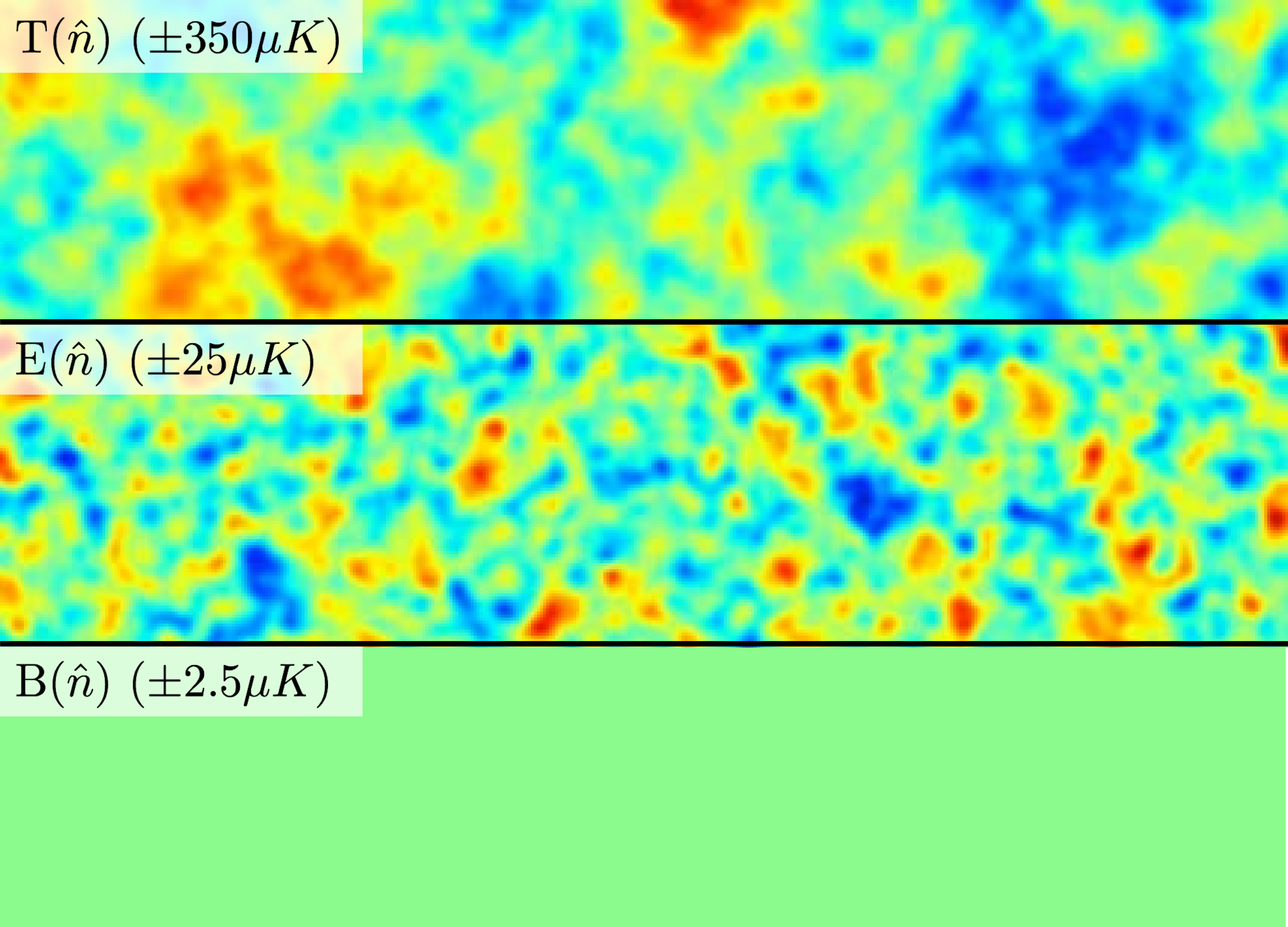
Weak lensing of the CMB perturbations



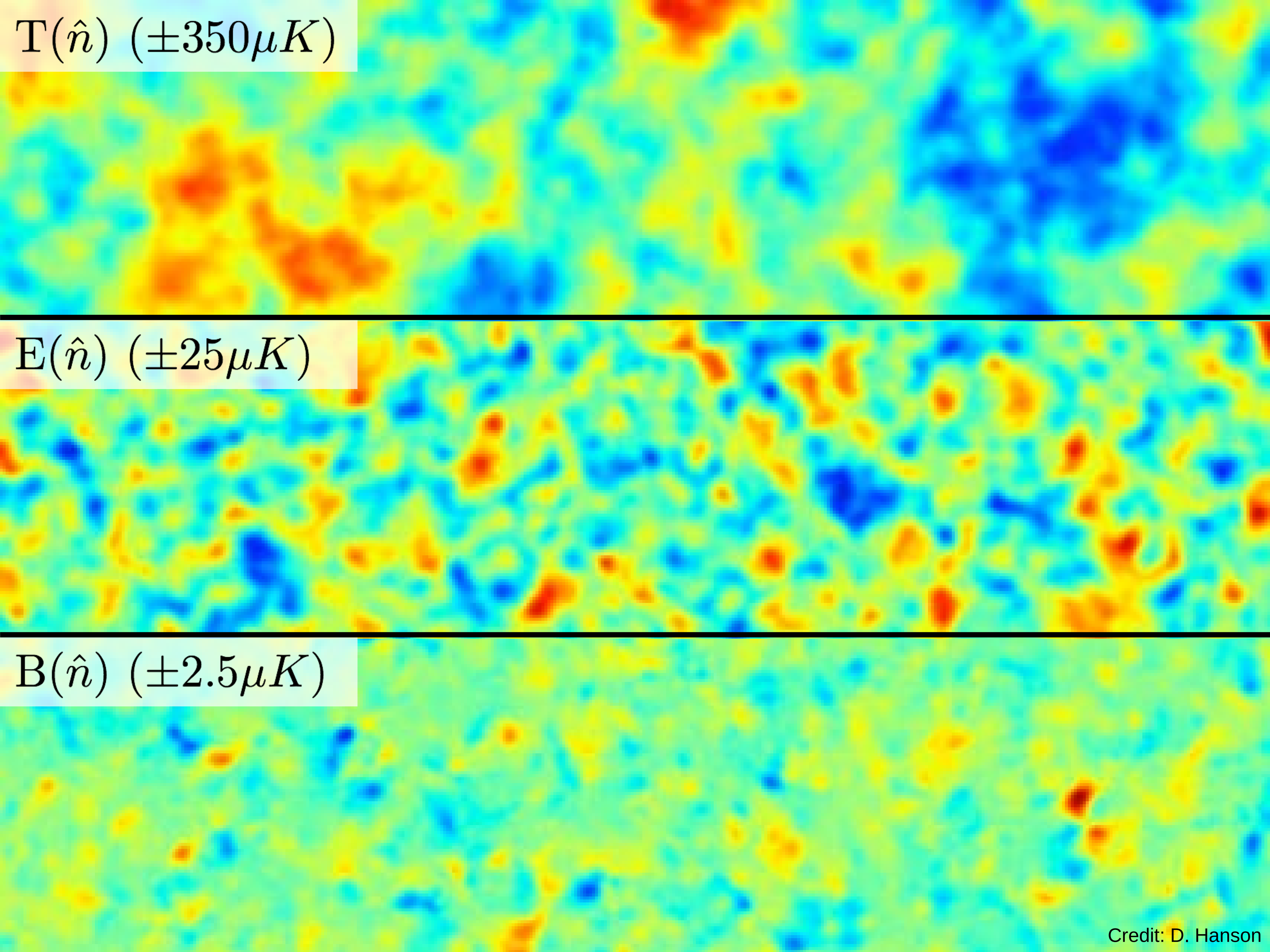
Not to scale!
All distances are comoving



Good approximation: CMB is single source plane at ~14 000 Mpc

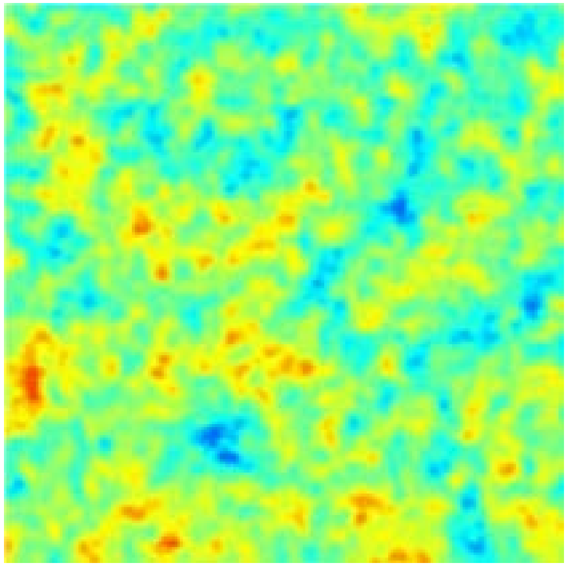


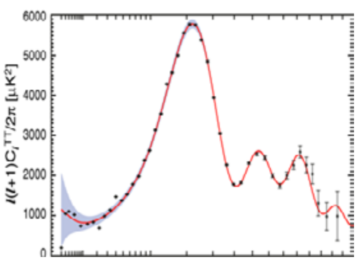
Credit: D. Hanson


$T(\hat{n})$ $(\pm 350\mu K)$
$E(\hat{n})$ $(\pm 25\mu K)$
$B(\hat{n})$ $(\pm 2.5\mu K)$
Credit: D. Hanson

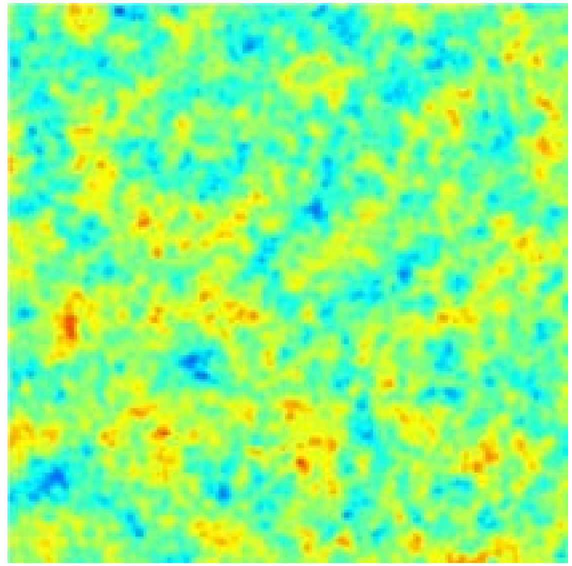
Local effect of lensing magnification on the power spectrum

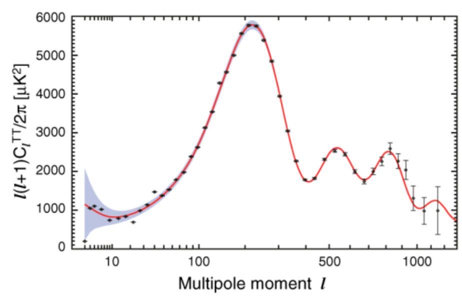
Magnified



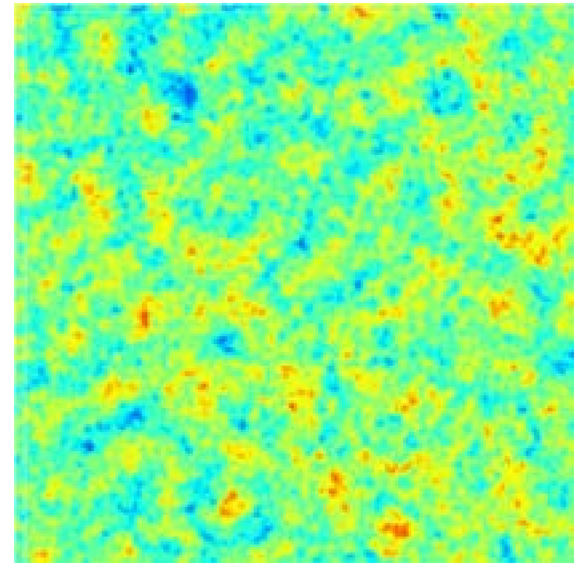


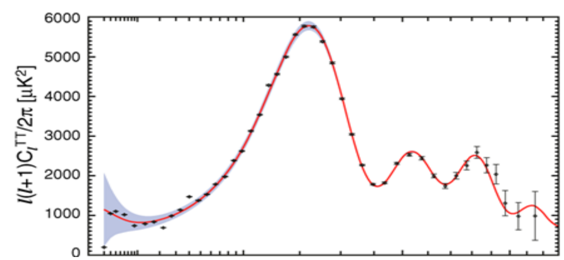
Unlensed



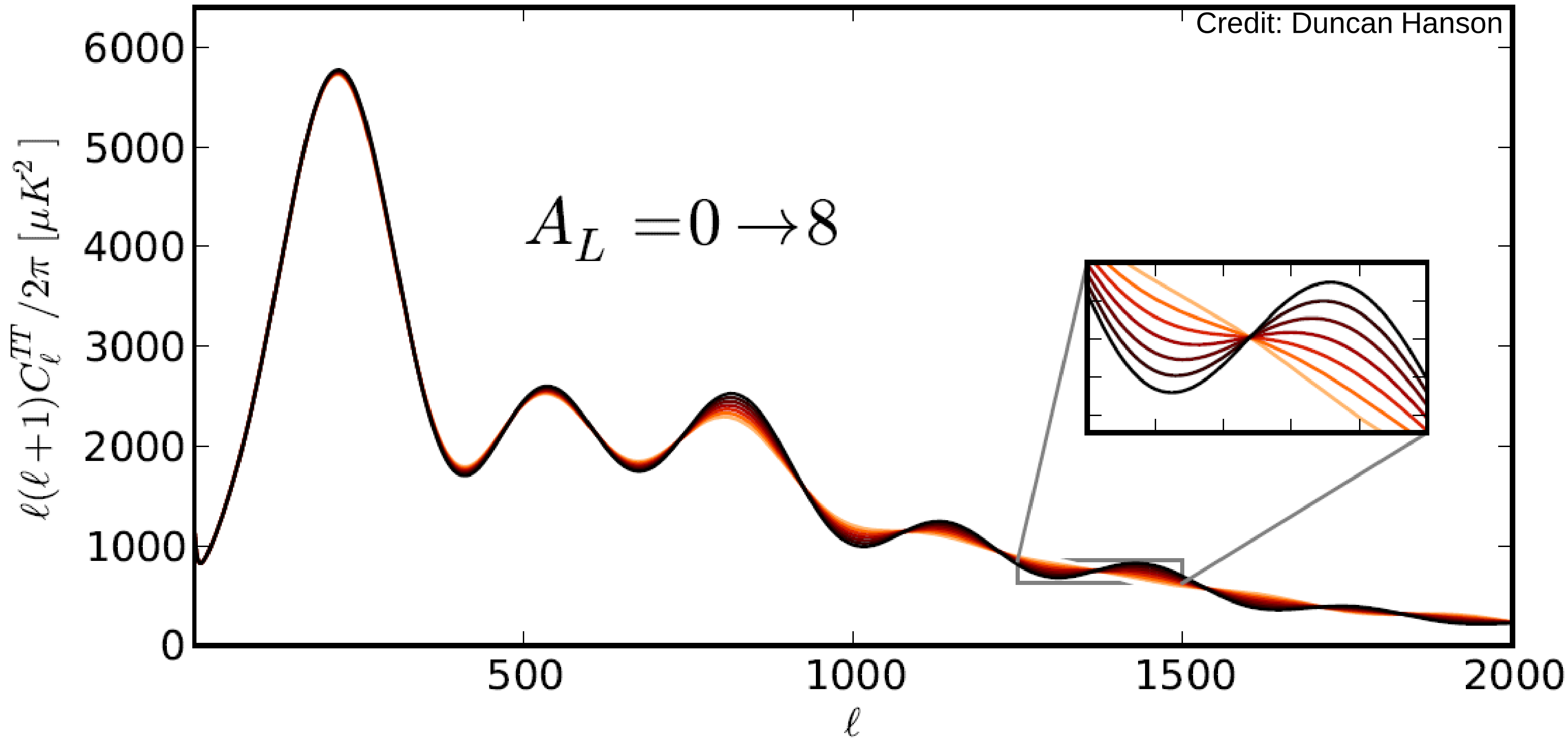


Demagnified





Averaged over the sky, lensing smooths out the power spectrum



Matter Power Spectrum

(in comoving gauge)

$\Delta = \delta\rho_m/\rho_m \qquad \langle \Delta(\mathbf{k},t)\Delta(\mathbf{k}',t)\rangle = \frac{2\pi^2}{k^3}\mathcal{P}(k,t)\delta(\mathbf{k}+\mathbf{k}')$

Large scales, $k \ll aH_{eq}$: Use Poisson equation $\bar{\Delta} = -(2/3)k^2\Phi/\mathcal{H}^2$

($\mathcal{H} = aH$ = comoving Hubble)

$$\mathcal{P}_{\bar{\Delta}}(\eta) \sim \frac{4}{9}\frac{k^4}{\mathcal{H}^4}\mathcal{P}_\Phi = \frac{4}{25}\frac{k^4}{\mathcal{H}^4}\mathcal{P}_\mathcal{R}$$

Small scales, $k \gg aH_{eq}$: $\mathcal{P}_{\bar{\Delta}} \sim \left(\frac{9\pi^2}{16}\right)^2 \frac{a^2}{a_{eq}^2}\left[1 + 2\ln\left(\frac{4k\eta_{eq}}{\pi 3\sqrt{3}}\right)\right]^2 \mathcal{P}_\mathcal{R}$

(+ matter domination)

Structure growth in matter domination $\frac{\delta\rho}{\rho} = \Delta \propto a$

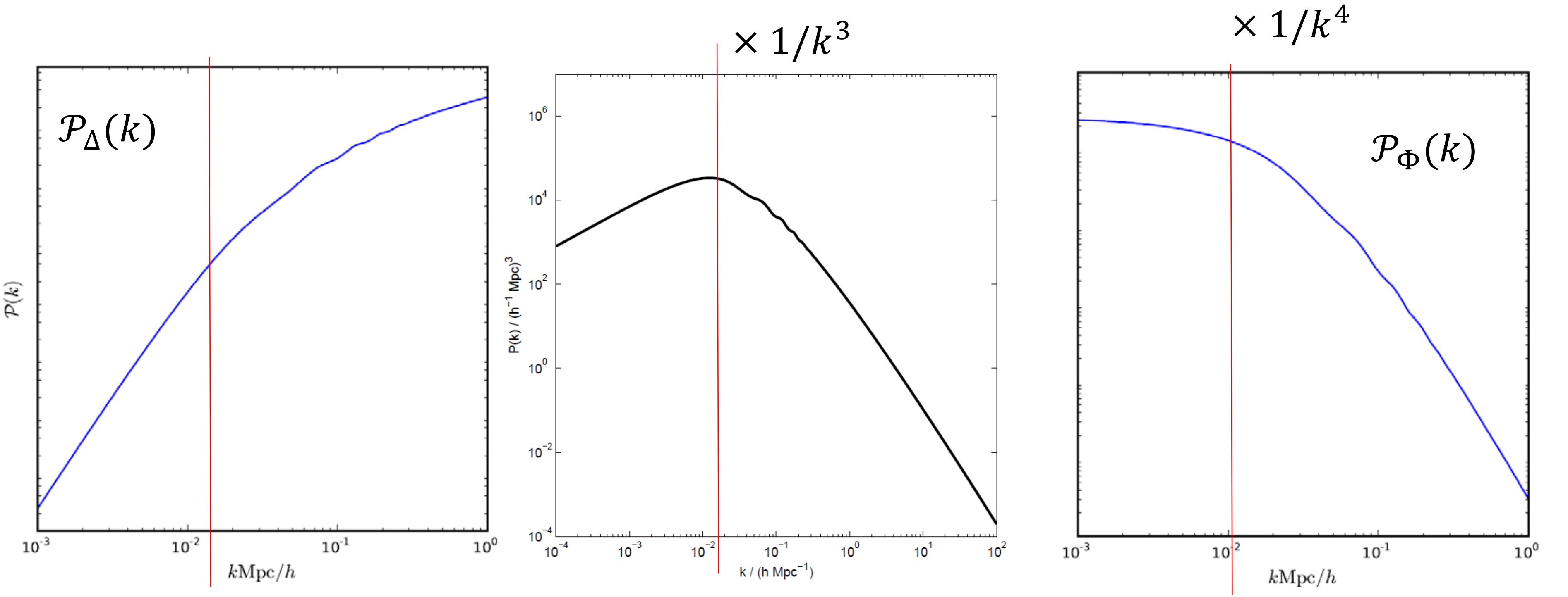
Growth during radiation domination.

Photon pressure stops growth: $\Phi \to 0$ due to expansion
⇒ no gravitational driving force, no acceleration
⇒ dark matter velocities redshift $\propto 1/a$
Integrate $v \propto 1/a$ to get density ⇒ $\ln(\eta)$ growth

For more details see notes at: http://cosmologist.info/teaching/EU/

Linear Matter Power Spectrum

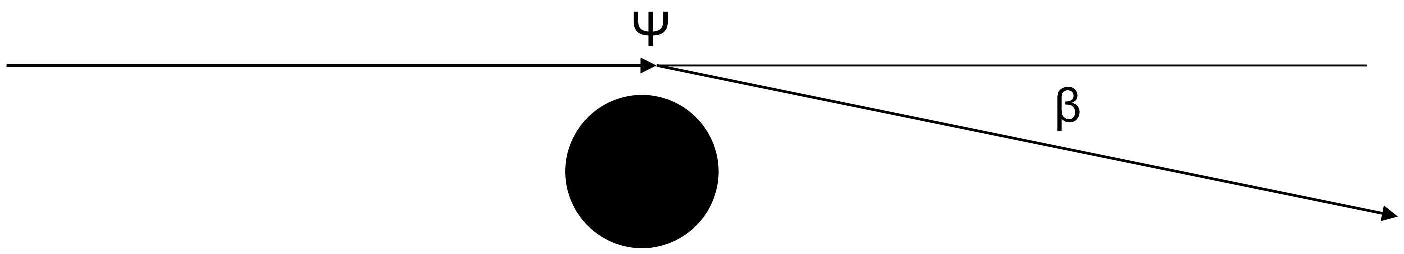
Note: $\Delta \propto a$ in matter domination, but $\nabla^2\Phi \propto a^2\rho\Delta$ is *constant*



Turnover in matter power spectrum at $k \sim 0.01 - 0.02$
(set by horizon size at matter-radiation equality)

More lenses ⇒ more lensing ⇒ most effect for small lenses for more along line of sight
Smallest lenses where potential has not decayed away $\sim 300\mathrm{Mpc}$

CMB lensing order of magnitudes



Newtonian argument: $\beta = 2\,\Psi$

General Relativity: $\beta = 4\,\Psi$ $\qquad$ ($\beta \ll 1$)

Potentials linear and approx Gaussian: $\Psi \sim 2 \times 10^{-5}$

$\beta \sim 10^{-4}$

Characteristic size from peak of matter power spectrum ~ 300Mpc

Comoving distance to last scattering surface ~ 14000 Mpc

pass through ~50 lumps $\quad$ total deflection $\sim 50^{1/2} \times 10^{-4}$

assume uncorrelated $\quad$ ~ 2 arcminutes

(neglects angular factors, correlation, etc.)

Why lensing is important

Relatively large $O(10^{-3})$ *not* $O(10^{-5})$ – GR lensing factor, many lenses along line of sight
[*NOT* because of growth of matter density perturbations, potentials are constant or decaying!]

- 2arcmin deflections: $l \sim 3000$

  - On small scales CMB is very smooth so lensing dominates the linear signal at high l

- Deflection angles coherent over 300/(14000/2) ~ 2°
  - comparable to CMB scales
  - expect 2arcmin/60arcmin ~ 3% effect on main CMB acoustic peaks

- Non-linear: observed CMB is non-Gaussian
  - more information
  - potential confusion with primordial non-Gaussian signals

- Does not preserve E/B decomposition of polarization: e.g. $E \rightarrow B$
  - Confusion for primordial B modes ("r-modes")
  - No primordial B ⇒ B modes clean probe of lensing

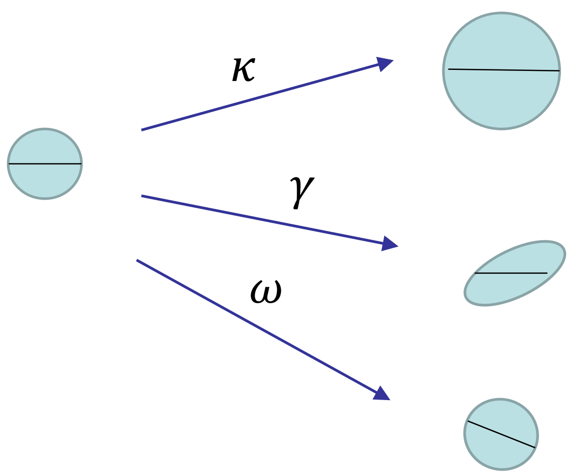
Deflection angle α

$$T(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

Bulk deflections unobservable (don't know unlensed CMB); only *differences* in deflection angle really matter. So sometimes instead use magnification matrix:

Shear γ_i , convergence κ , and rotation ω

$$A_{ij} \equiv \delta_{ij} + \frac{\partial}{\partial \theta_i}\alpha_j = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2+\omega \\ -\gamma_2-\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$



Convergence $\kappa = -\frac{1}{2}\nabla\cdot\boldsymbol{\alpha}$

Shear

Rotation

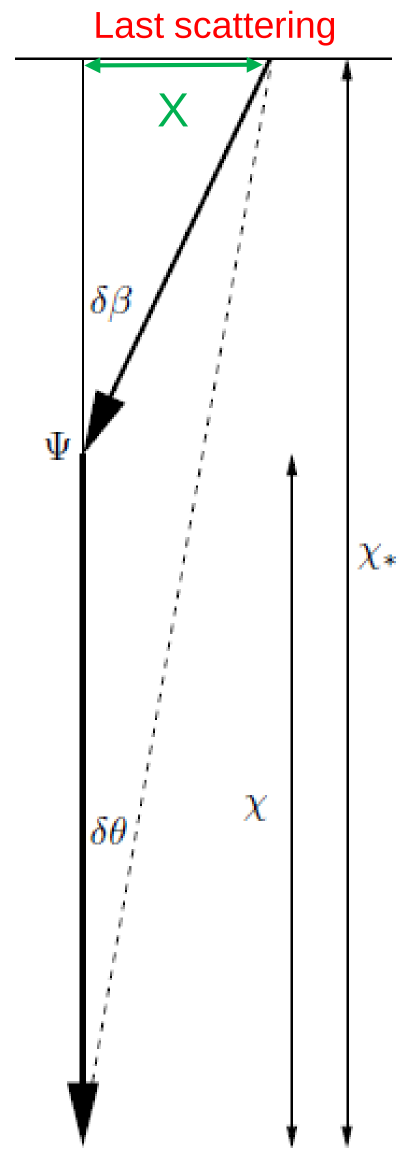
Rotation $\omega = 0$ from scalar perturbations in linear perturbation theory (because deflections from gradient of a potential)

Calculating the deflection angles

$$\frac{d\mathbf{e}}{d\eta} = -\nabla_\perp \underbrace{(\Phi + \Psi)}$$

Weyl Potential:
$\Psi_W \equiv (\Phi + \Psi)/2$
determines scalar
part of the Weyl tensor
Conformally invariant

$$\delta\beta = -2\delta\chi\nabla_\perp\Psi$$



FRW background: comoving angular diameter distance

$$f_K(\chi) = \begin{cases} K^{-1/2}\sin(K^{1/2}\chi) & \text{for } K > 0, \text{ closed,} \\ \chi & \text{for } K = 0, \text{ flat,} \\ |K|^{-1/2}\sinh(|K|^{1/2}\chi) & \text{for } K < 0, \text{ open.} \end{cases}$$

Write X, in two ways $\quad f_K(\chi_* - \chi)\delta\beta = f_K(\chi_*)\delta\theta$

$$\delta\theta_\chi = \frac{f_K(\chi_* - \chi)\delta\beta}{f_K(\chi_*)} = -\frac{f_K(\chi_* - \chi)}{f_K(\chi_*)}2\delta\chi\nabla_\perp\Psi$$

Observed deflection

Lensed temperature depends on deflection angle

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

Newtonian (Weyl) potential

$$\boldsymbol{\alpha} = \delta\theta = -2\int_0^{\chi^*} \mathrm{d}\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla_\perp \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi)$$

co-moving distance to last scattering

See lensing review for more rigorous spherical derivation

Lensing Potential

Deflection angle on sky given in terms of angular gradient of lensing potential $\boldsymbol{\alpha} = \nabla\psi$

$$\boldsymbol{\nabla}_\perp \Psi = (\nabla_{\hat{\mathbf{n}}}\Psi)/f_K(\chi)$$

$$\Rightarrow \quad \psi(\hat{\mathbf{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi) \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)}$$

$$\tilde{X}(\mathbf{n}) = X(\mathbf{n}') = X(\mathbf{n} + \nabla\psi(\mathbf{n}))$$

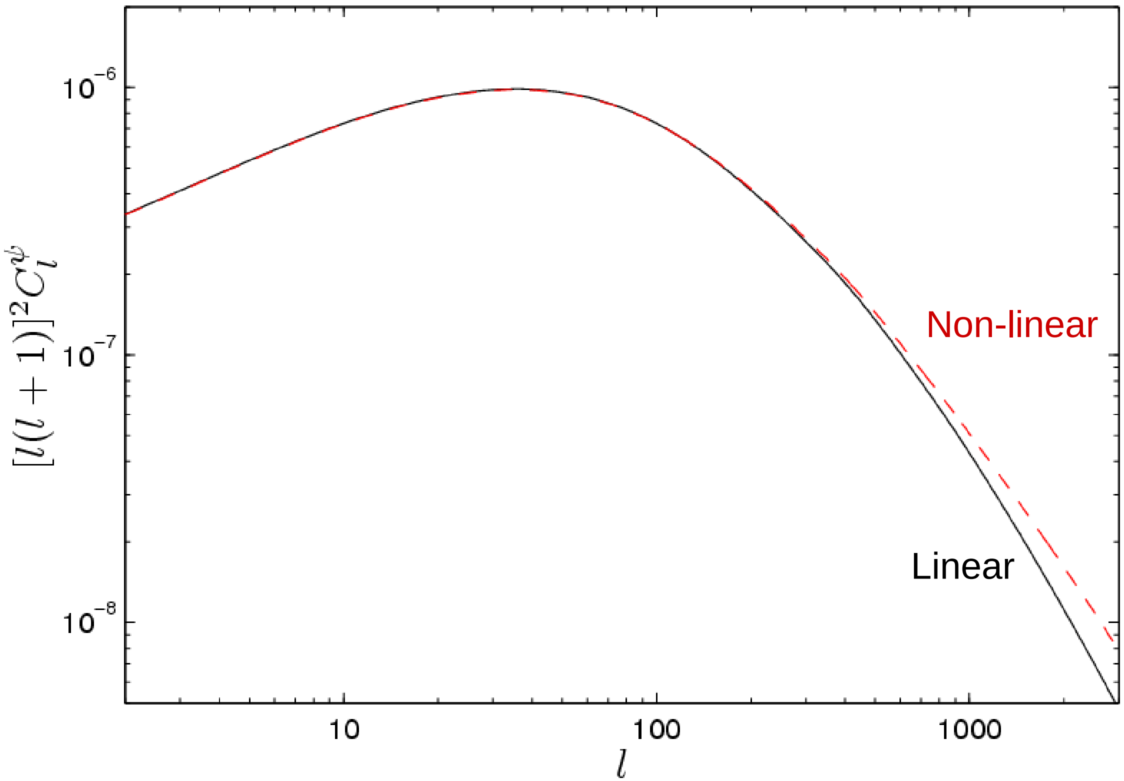
Deflection angle power spectrum

On small scales
(Limber approx, $k\chi \sim l$)

$$C_l^\psi \approx \frac{8\pi^2}{l^3}\int_0^{\chi_*}\chi \mathrm{d}\chi\, \mathcal{P}_\Psi(l/\chi;\eta_0-\chi)\left(\frac{\chi_*-\chi}{\chi_*\chi}\right)^2$$

(better: $l \to l + 1/2$)

Deflection angle power ~ $l(l+1)C_l^\psi$



Deflections O(10^{-3}), but coherent on degree scales → important!

Can be computed with CLASS http://class-code.net or CAMB: http://camb.info

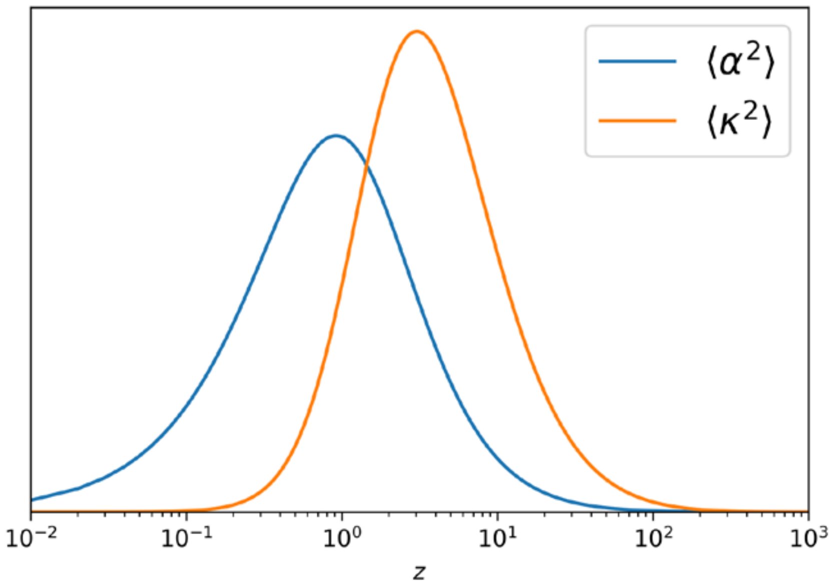
Redshift Dependence: broad redshift kernel all way along line of sight

Depends on l of interest. High z more important for higher l.

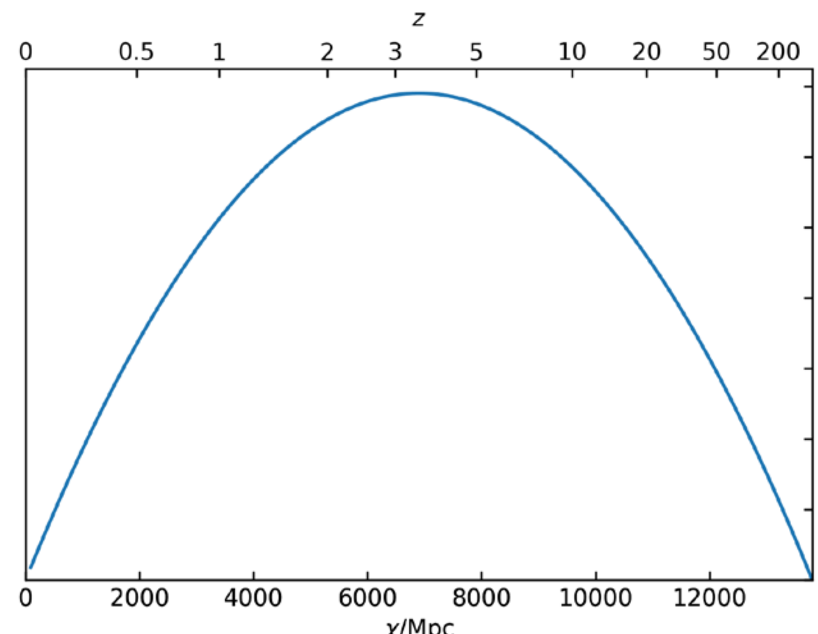
$$\langle\alpha^2\rangle \propto \int d\ln l \; C_l^\kappa \propto \int d\chi \left(1-\frac{\chi}{\chi_*}\right)^2 \int dk\, \mathcal{P}_\Psi\,(k, z(\chi))$$

$$\langle\kappa^2\rangle \propto \int d\chi \chi^2 \left(1-\frac{\chi}{\chi_*}\right)^2 \int dk\, k^2 \mathcal{P}_\Psi\,(k, z(\chi))$$

Redshift dependence of integrands per log z



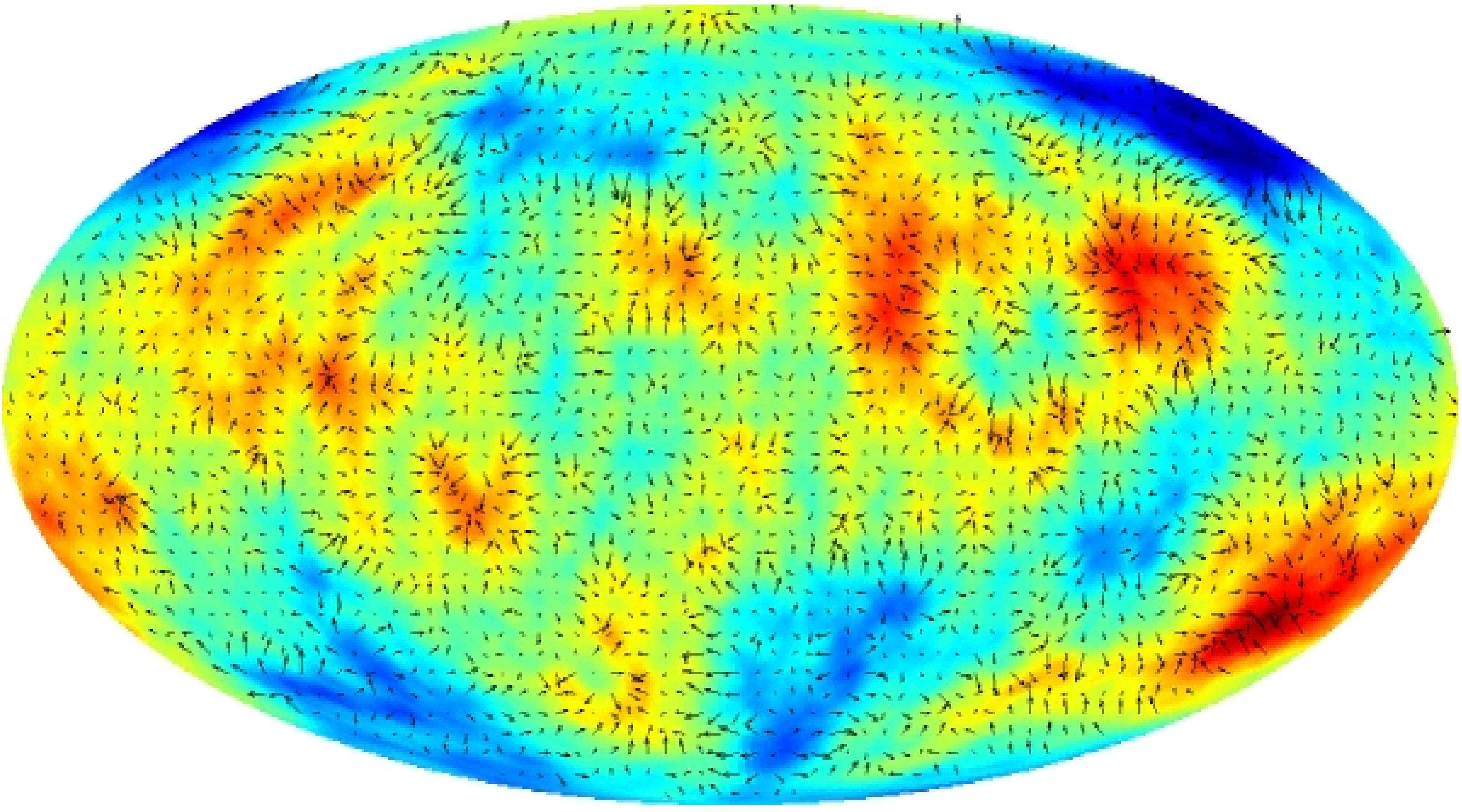
Convergence kernel $\propto \chi\left(1-\frac{\chi}{\chi_*}\right)$ for comparison with galaxy lensing:



(there are lots of different things you can plot and define as the “lensing kernel”)

Lensing potential and deflection angles: simulation

LensPix sky simulation code: http://cosmologist.info/lenspix (several others also available)



Note: my notation (and literature) is not very consistent: mix of ϕ, ψ for lensing potential

Lensed field: series expansion approximation

$$\begin{aligned}\tilde{\Theta}(\mathbf{x}) = \Theta(\mathbf{x}') &= \Theta(\mathbf{x} + \boldsymbol{\nabla}\psi) \\ &\approx \Theta(\mathbf{x}) + \nabla^a\psi(\mathbf{x})\nabla_a\Theta(\mathbf{x}) + \tfrac{1}{2}\nabla^a\psi(\mathbf{x})\nabla^b\psi(\mathbf{x})\nabla_a\nabla_b\Theta(\mathbf{x}) + \ldots\end{aligned}$$

(BEWARE: this is not an accurate approximation for power spectrum!
Better method uses correlation function)

Using Fourier transforms in flat sky approximation:

$$\boldsymbol{\nabla}\psi(\mathbf{x}) = i\int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\psi(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}, \qquad \boldsymbol{\nabla}\Theta(\mathbf{x}) = i\int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\Theta(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}$$

Then lensed harmonics then given by

use $\int d^2\mathbf{x}\, e^{i\mathbf{x}\cdot(\mathbf{l}_1 - \mathbf{l}_2 - \mathbf{l})} = (2\pi)^2\delta(\mathbf{l}_1 - \mathbf{l}_2 - \mathbf{l})$

$$\begin{aligned}\tilde{\Theta}(\mathbf{l}) \approx \Theta(\mathbf{l}) &- \int \frac{\mathrm{d}^2\mathbf{l}'}{2\pi}\,\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\psi(\mathbf{l}-\mathbf{l}')\Theta(\mathbf{l}') \\ &- \frac{1}{2}\int \frac{\mathrm{d}^2\mathbf{l}_1}{2\pi}\int \frac{\mathrm{d}^2\mathbf{l}_2}{2\pi}\,\mathbf{l}_1\cdot[\mathbf{l}_1 + \mathbf{l}_2 - \mathbf{l}]\,\mathbf{l}_1\cdot\mathbf{l}_2\Theta(\mathbf{l}_1)\psi(\mathbf{l}_2)\psi^*(\mathbf{l}_1 + \mathbf{l}_2 - \mathbf{l}).\end{aligned}$$

Lensed field still statistically isotropic: $\langle\tilde{\Theta}(\mathbf{l})\tilde{\Theta}^*(\mathbf{l}')\rangle = \delta(\mathbf{l}-\mathbf{l}')\tilde{C}_l^{\Theta}.$

with

$$\tilde{C}_l^{\Theta} \approx C_l^{\Theta} + \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}\left[\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\right]^2 C^{\psi}_{|\mathbf{l}-\mathbf{l}'|}C^{\Theta}_{l'} - C_l^{\Theta}\int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}(\mathbf{l}\cdot\mathbf{l}')^2 C^{\psi}_{l'}$$

Alternatively written as

$$\tilde{C}_l^{\Theta} \approx (1 - l^2 R^{\psi})C_l^{\Theta} + \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}\left[\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\right]^2 C^{\psi}_{|\mathbf{l}-\mathbf{l}'|}C^{\Theta}_{l'}.$$

where $$R^{\psi} \equiv \frac{1}{2}\langle|\nabla\psi|^2\rangle = \frac{1}{4\pi}\int \frac{\mathrm{d}l}{l}\, l^4 C_l^{\psi}, \qquad \sim 3\times 10^{-7}$$

(RMS deflection ~ 2.7 arcmin)

<u>Second term is like a convolution with the deflection angle power spectrum</u>

- smoothes out acoustic peaks
- transfers power from large scales into the damping tail

Small scales, large *l* limit:

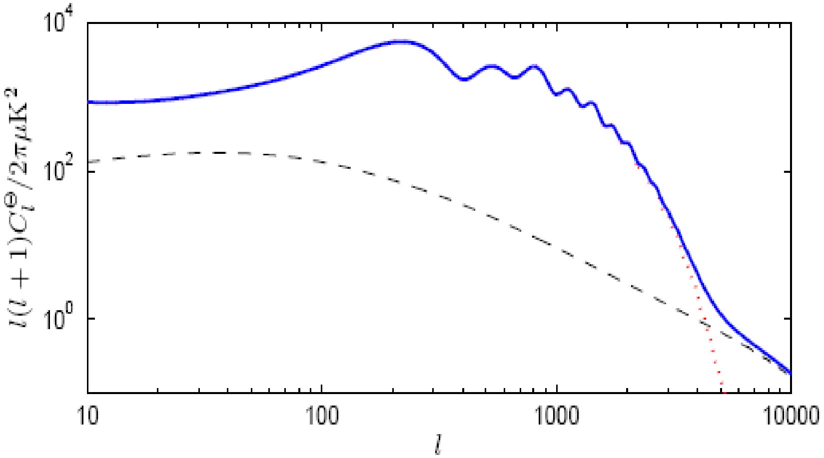
- unlensed CMB has very little power due to silk damping: $C_l^\Theta \sim 0$

$$\begin{aligned}\tilde{C}_l^\Theta &\approx \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}\left[\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\right]^2 C^\psi_{|\mathbf{l}'-\mathbf{l}|} C^\Theta_{l'} \\ &\approx C_l^\psi \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}\left[\mathbf{l}'\cdot\mathbf{l}\right]^2 C^\Theta_{l'} \\ &\approx l^2 C_l^\psi \int \frac{\mathrm{d}l_2}{l_2}\frac{l_2^4 C^\Theta_{l_2}}{4\pi} \\ &\approx l^2 C_l^\psi R^\Theta .\end{aligned}$$

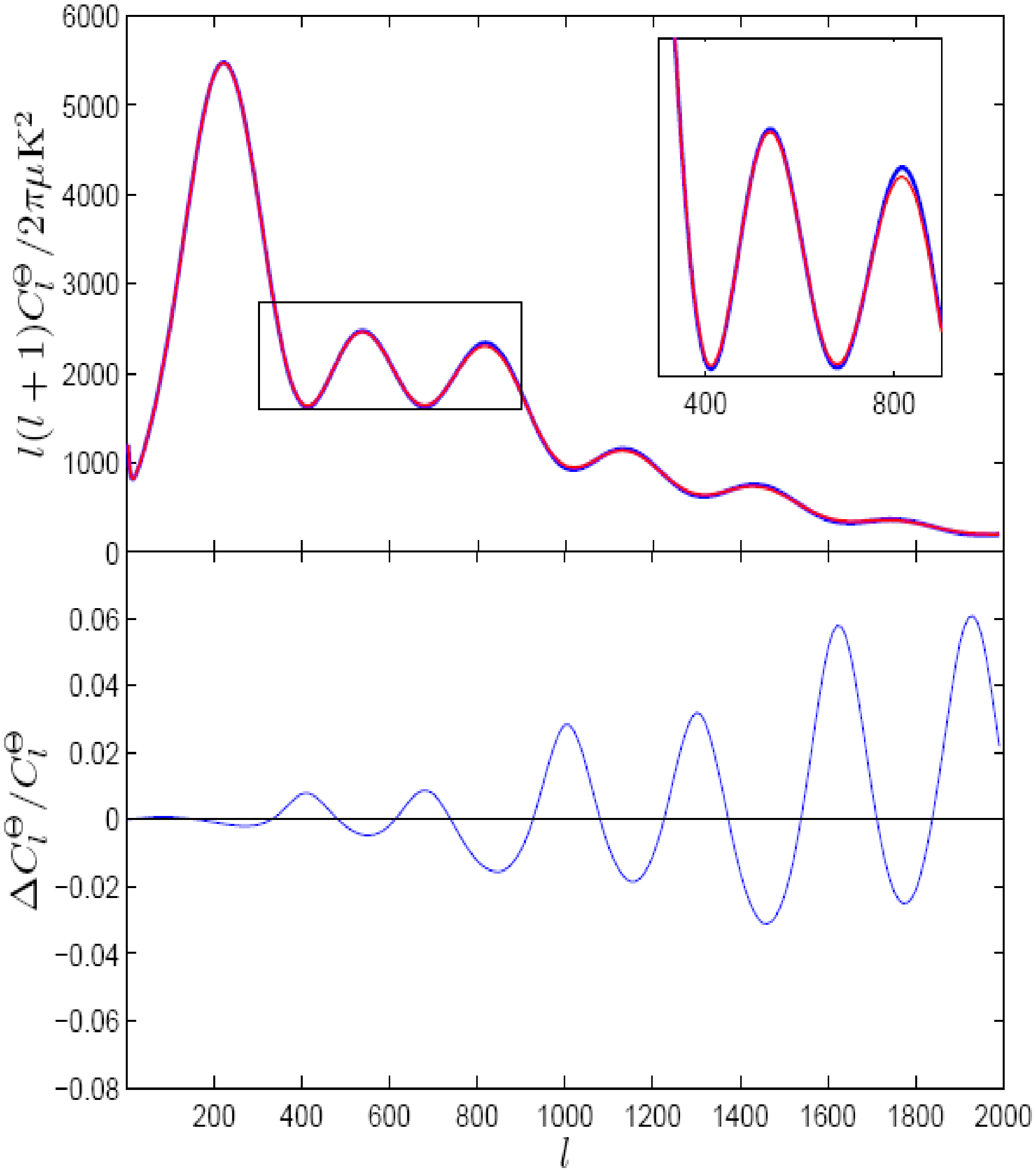
$$l' \ll l$$

$$R^\Theta \equiv \frac{1}{2}\langle|\nabla T|^2\rangle = \frac{1}{4\pi}\int\frac{\mathrm{d}l}{l}\, l^4 C_l^\Theta \sim 10^9 \mu\mathrm{K}^2$$

- Proportional to the deflection angle power spectrum and the (scale independent) power in the gradient of the temperature



Lensing effect on CMB temperature power spectrum



- Note: can only observe *lensed* sky

- Any bulk deflection is unobservable
  – degenerate with corresponding change in unlensed CMB:
  e.g.
  rotation of full sky
  translation in flat sky approximation

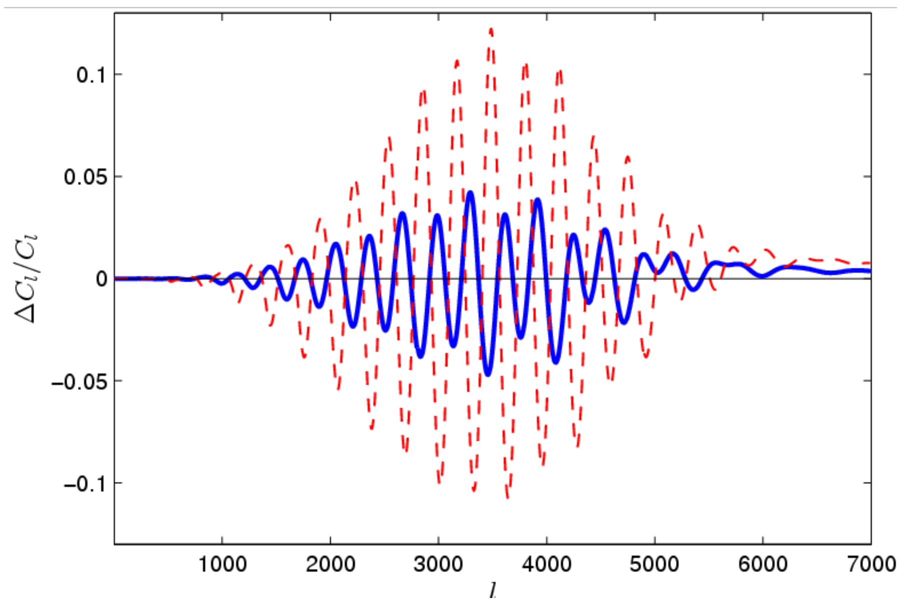
- Observations sensitive to *differences* of deflection angles
  - convergence and shear

Series expansion in deflection angle OK?

$$\tilde{\Theta}(\mathbf{x}) = \Theta(\mathbf{x}') = \Theta(\mathbf{x} + \boldsymbol{\nabla}\psi)$$
$$\approx \Theta(\mathbf{x}) + \nabla^a\psi(\mathbf{x})\nabla_a\Theta(\mathbf{x}) + \frac{1}{2}\nabla^a\psi(\mathbf{x})\nabla^b\psi(\mathbf{x})\nabla_a\nabla_b\Theta(\mathbf{x}) + \ldots$$

Only a good approximation when:
- deflection angle much smaller than wavelength of temperature perturbation
- OR, very small scales where temperature is close to a gradient

CMB lensing is a very specific physical second order effect; not accurately contained in 2nd order expansion – differs by significant 3rd and higher order terms



Error using series expansion:

temperature
E-polarization

Accurate lensed power spectrum calculation must non-perturbative correlation function method.

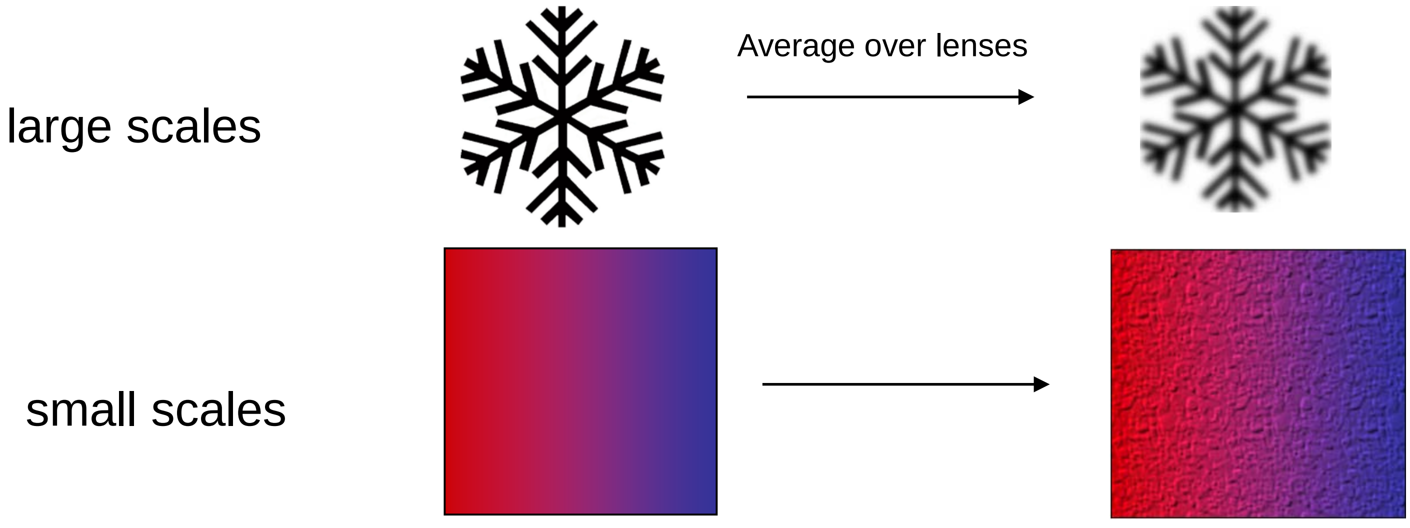
- See lensing review for details

Series expansion only good on large and very small scales – don't use for lensed C_l

Summary so far

- Deflection angles of ~ 3 arcminutes, but correlated on degree scales

- Lensing convolves TT with deflection angle power spectrum
  - Acoustic peaks slightly blurred
  - Power transferred to small scales



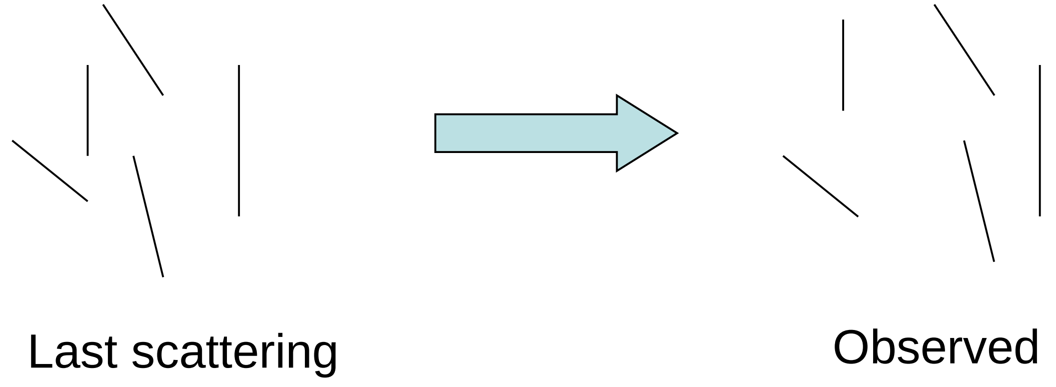
Comparison with galaxy lensing

- **Single source plane at known distance**
  (given cosmological parameters)
- **Statistics of sources on source plane well understood**
  - can calculate power spectrum; Gaussian linear perturbations
  - magnification and shear information equally useful - usually discuss in terms of deflection angle;
  - magnification analysis of galaxies much more difficult
- **Hot and cold spots are large, smooth on small scales**
  - ‘strong’ and ‘weak’ lensing can be treated the same way: infinite magnification of smooth surface is still a smooth surface
- **Source plane very distant, large nearly-linear lenses**
  - much less sensitive to non-linear modelling, baryon feedback, etc.
- **Full sky observations**
  - may need to account for spherical geometry for accurate results
- **Systematics completely different**
  - CMB/galaxy cross-correlations can be a good way to calibrate systematics

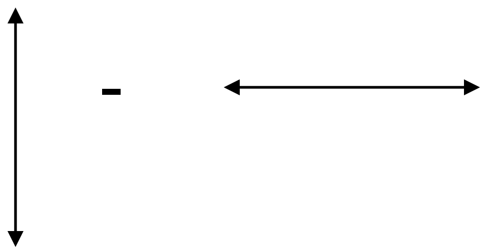
Lensing of polarization

- Polarization not rotated w.r.t. parallel transport (vacuum is not birefringent; higher order post-Born rotation also negligible)
- Q and U Stokes parameters simply re-mapped by the lensing deflection field

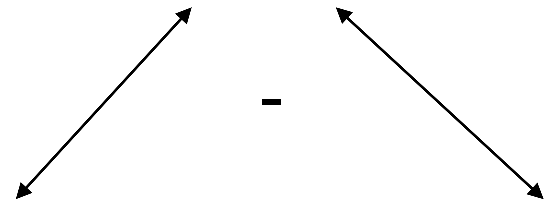
e.g.



Observed Stokes' Parameters



Q



U

Q $\rightarrow$ -Q, U $\rightarrow$ -U under 90 degree rotation

Q $\rightarrow$ U, U $\rightarrow$ -Q under 45 degree rotation

Measure E field perpendicular to observation direction $\hat{\boldsymbol{n}}$

Intensity matrix defined as $$\mathcal{P}_{ab} = C\langle E_a E_b^* \rangle = P_{ab} + \frac{1}{2}\delta_{ab} I + V_{[ab]}$$

Linear polarization + Intensity + circular polarization

CMB only linearly polarized. In some fixed basis $$P_{ij} = \frac{1}{2}\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$

Alternative complex representation

Define complex vectors $\mathbf{e}_\pm = \mathbf{e}_1 \pm i\mathbf{e}_2$ e.g. $\mathbf{e}_\pm = \mathbf{e}_x \pm i\mathbf{e}_y$

And complex polarization

$$P \equiv \mathbf{e}_+^a \mathbf{e}_+^b P_{ab} = Q + iU$$

$$P^* = \mathbf{e}_-^a \mathbf{e}_-^b P_{ab} = Q - iU.$$

Under a rotation of the basis vectors

$$\begin{aligned} \mathbf{e}_\pm \equiv \mathbf{e}_x \pm i\mathbf{e}_y &\to \mathbf{e}_x' \pm i\mathbf{e}_y' \\ &= (\cos\gamma\, \mathbf{e}_x - \sin\gamma\, \mathbf{e}_y) \pm i(\sin\gamma\, \mathbf{e}_x + \cos\gamma\, \mathbf{e}_y) \\ &= e^{\pm i\gamma}(\mathbf{e}_x \pm i\mathbf{e}_y) = e^{\pm i\gamma}\mathbf{e}_\pm. \end{aligned}$$

$$P' = \mathbf{e}_+^a{}' \mathbf{e}_+^b{}' P_{ab} = e^{2i\gamma} P.$$

- spin 2 field

Series expansion

Similar to temperature derivation, but now complex spin-2 quantities:

$$\tilde{P}(\mathbf{x}) = P(\mathbf{x}+\nabla\psi) \sim P(\mathbf{x}) + \nabla^a\psi\nabla_b P(\mathbf{x}) + \frac{1}{2}\nabla^c\psi\nabla^d\psi\nabla_c\nabla_d P(\mathbf{x})$$

Unlensed B is expected to be very small. Simplify by setting to zero.
Expand in harmonics

$$\tilde{E}(\mathbf{l}) \pm i\tilde{B}(\mathbf{l}) \approx E(\mathbf{l}) - \int \frac{\mathrm{d}^2\mathbf{l}'}{2\pi}\, \mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')e^{\pm 2i(\phi_{\mathbf{l}'}-\phi_{\mathbf{l}})}\psi(\mathbf{l}-\mathbf{l}')E(\mathbf{l}')$$
$$-\frac{1}{2}\int \frac{\mathrm{d}^2\mathbf{l}_1}{2\pi}\int \frac{\mathrm{d}^2\mathbf{l}_2}{2\pi}\, e^{\pm 2i(\phi_{\mathbf{l}'}-\phi_{\mathbf{l}})}\mathbf{l}_1\cdot[\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}]\,\mathbf{l}_1\cdot\mathbf{l}_2 E(\mathbf{l}_1)\psi(\mathbf{l}_2)\psi^*(\mathbf{l}_1+\mathbf{l}_2-\mathbf{l})$$

First order terms are

$$\tilde{E}(l) = E(l) - \int \frac{d^2l'}{2\pi} l'\cdot(l-l')\cos\left(2\left[\phi_{l'}-\phi_l\right]\right)\psi(l-l')E(l')$$

$$\tilde{B}(l) = -\int \frac{d^2l'}{2\pi} l'\cdot(l-l')\sin\left(2\left[\phi_{l'}-\phi_l\right]\right)\psi(l-l')E(l')$$

Lensed spectrum: lowest order calculation

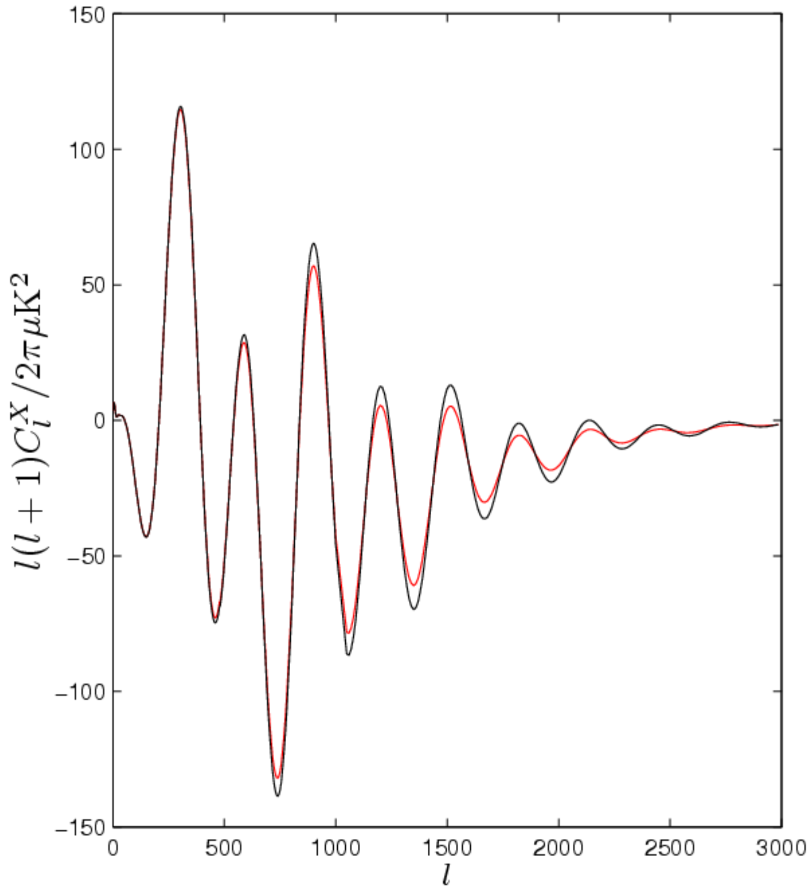
Need second order expansion for consistency with lensed E: 0th x 2nd order + 1st x 1st order :

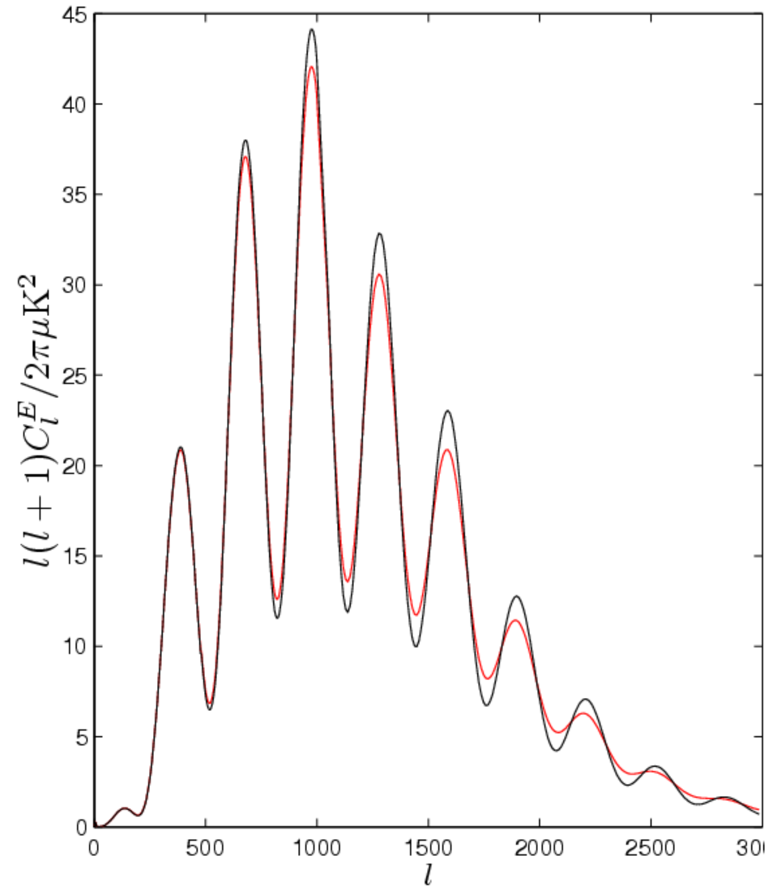
$$\tilde{E}(\mathbf{l}) \pm i\tilde{B}(\mathbf{l}) \approx E(\mathbf{l}) - \int \frac{d^2\mathbf{l}'}{2\pi} \mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}') e^{\pm 2i(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})} \psi(\mathbf{l} - \mathbf{l}') E(\mathbf{l}')$$
$$- \frac{1}{2} \int \frac{d^2\mathbf{l}_1}{2\pi} \int \frac{d^2\mathbf{l}_2}{2\pi} e^{\pm 2i(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})} \mathbf{l}_1 \cdot [\mathbf{l}_1 + \mathbf{l}_2 - \mathbf{l}]\, \mathbf{l}_1 \cdot \mathbf{l}_2 E(\mathbf{l}_1) \psi(\mathbf{l}_2) \psi^*(\mathbf{l}_1 + \mathbf{l}_2 - \mathbf{l})$$

Calculate power spectrum. Result is

$$\tilde{C}_l^E = (1 - l^2 R^\psi) C_l^E + \int \frac{d^2\mathbf{l}'}{(2\pi)^2} [\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')]^2 C^\psi_{|\mathbf{l}-\mathbf{l}'|} C^E_{|\mathbf{l}'|} \cos^2 2(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})$$
$$\tilde{C}_l^B = \int \frac{d^2\mathbf{l}'}{(2\pi)^2} [\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')]^2 C^\psi_{|\mathbf{l}-\mathbf{l}'|} C^E_{|\mathbf{l}'|} \sin^2 2(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})$$
$$\tilde{C}_l^X = (1 - l^2 R^\psi) C_l^X + \int \frac{d^2\mathbf{l}'}{(2\pi)^2} [\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')]^2 C^\psi_{|\mathbf{l}-\mathbf{l}'|} C^X_{|\mathbf{l}'|} \cos 2(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}}).$$

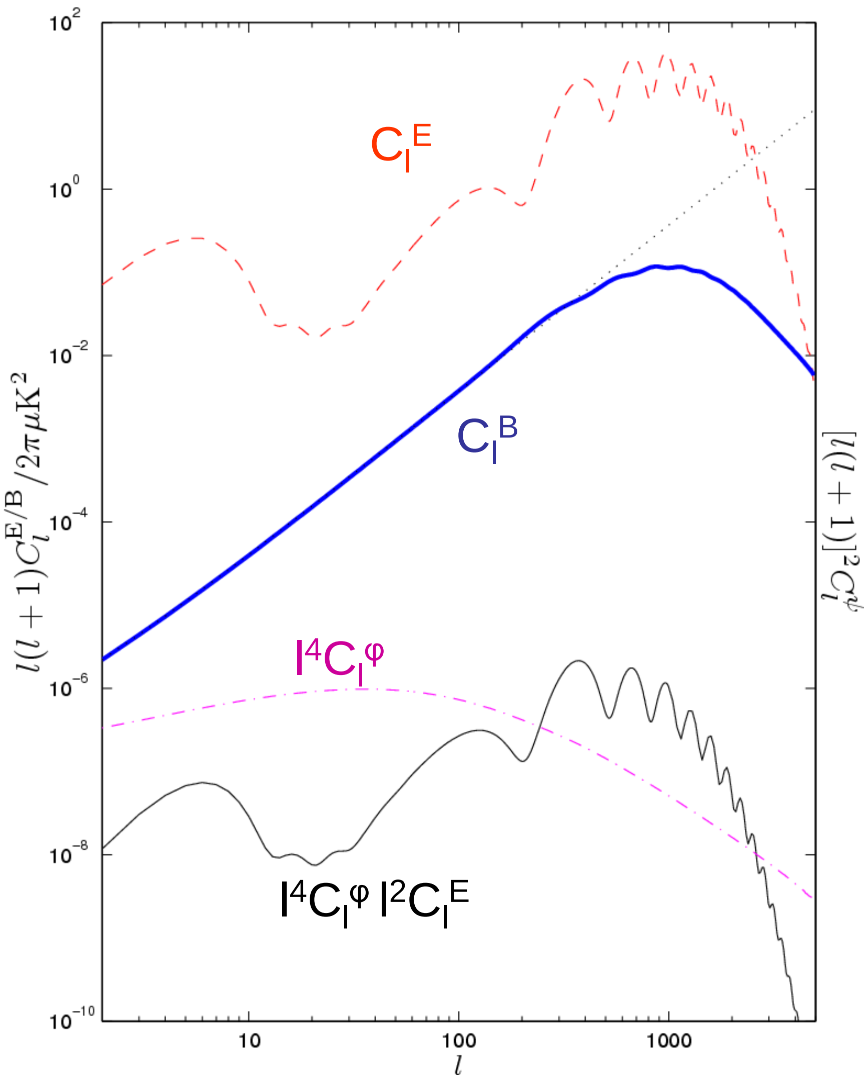
Effect on EE and TE similar to temperature: convolution smoothing + transfer of power to small scales





Polarization lensing power spectra

BB generated by lensing even if unlensed B=0



On large scales, $|\mathbf{l}| \ll |\mathbf{l}'|$ lensed BB given by

$$\tilde{C}_l^B \sim \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2} l'^4 C_{l'}^{\psi} C_{l'}^{E} \sin^2 2(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})$$
$$= \frac{1}{4\pi} \int \frac{\mathrm{d}l'}{l'} \, l'^4 C_{l'}^{\psi} \; l'^2 C_{l'}^{E},$$

Nearly white spectrum on large scales (power spectrum independent of l)

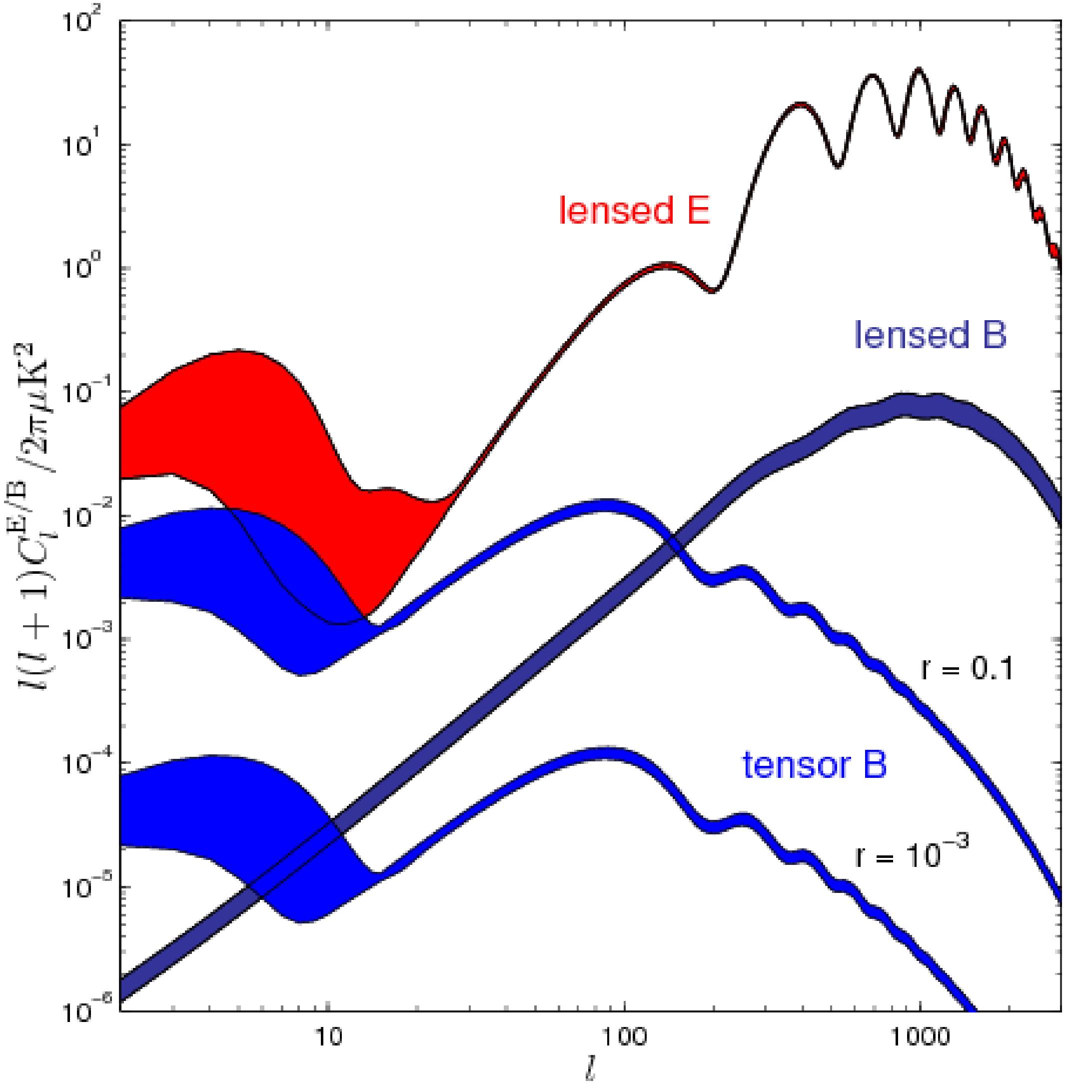
$$\tilde{C}_l^B \sim 2 \times 10^{-6} \mu\mathrm{K}^2$$

- unless removed, acts like an effective white-noise of $5\ \mu\mathrm{K\ arcmin}$

Can also do more accurate calculation using polarization correlation functions

Polarization power spectra

Current 95% indirect limits for LCDM given WMAP+2dF+HST



Warm up quiz: some answers

✗ 1) Overdensities focus light rays, so the CMB looks hotter where there are overdensities along the line of sight

✗ 2) Even in linear theory lensing is mostly at low redshift because density perturbations grow with time

✓ 3) The lensing potential is nearly Gaussian because there are many lenses along the line of sight and perturbations nearly linear

✗ 4) Lensing rotates polarization, partly turning E modes into B modes

Lecture 2

Non-Gaussianity, statistical anisotropy
and reconstructing the lensing field

$\tilde{T}(\mathbf{x}) = T(\mathbf{x} + \nabla\psi)$

$P(T,\psi) \approx$ Gaussian; on small scales $\langle T\psi \rangle = 0 \Rightarrow P(T,\psi) = P(T)P(\psi)$

In pixels this is a remapping $\tilde{T}_i = [\Lambda(\psi)]_{ij}T_j$: Linear in T; non-linear in ψ

1. Marginalize over (unobservable) lensing and unlensed temperature fields:

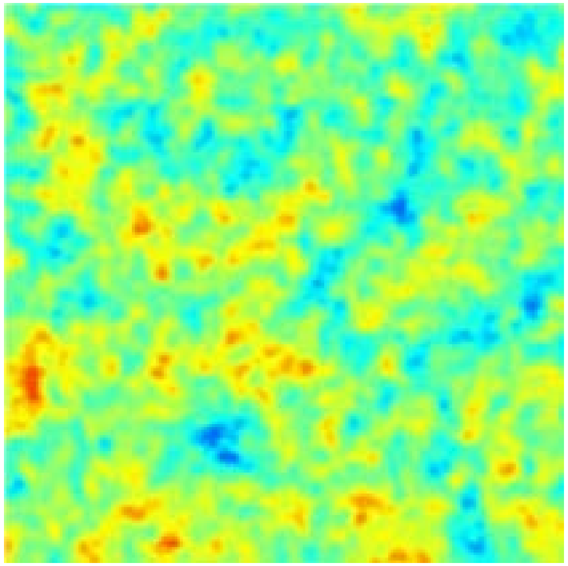
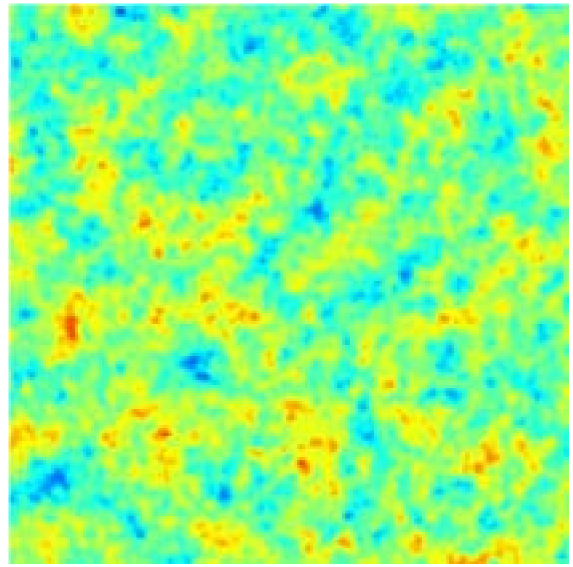
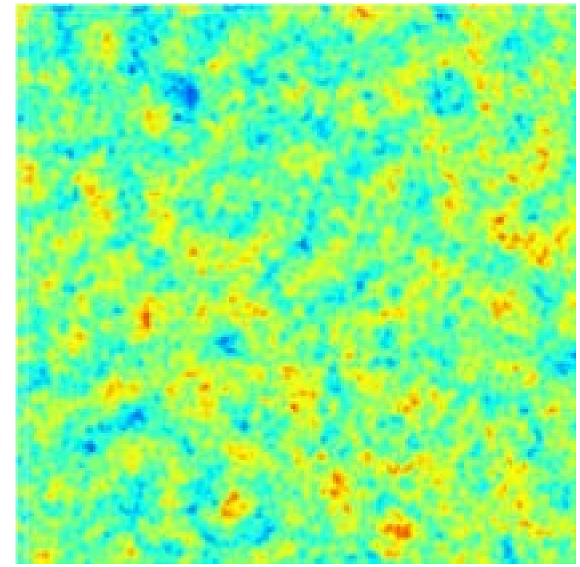
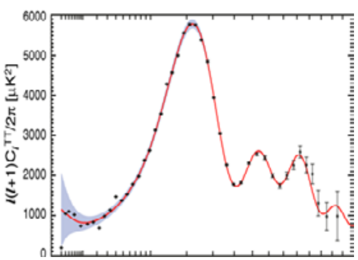
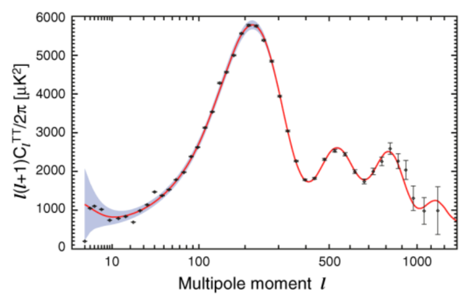
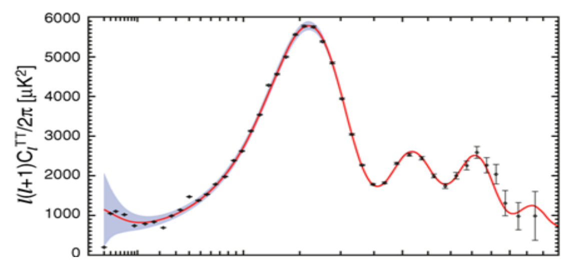
$\Rightarrow$ Non-Gaussian statistically isotropic lensed temperature distribution

2. For the given (fixed) lensing field in our universe think about $\tilde{T} \sim P(\tilde{T}|\psi)$:

$\tilde{T} = \Lambda T$ is a *linear* function of T for fixed ψ:

$\Rightarrow$ Anisotropic Gaussian lensed temperature distribution

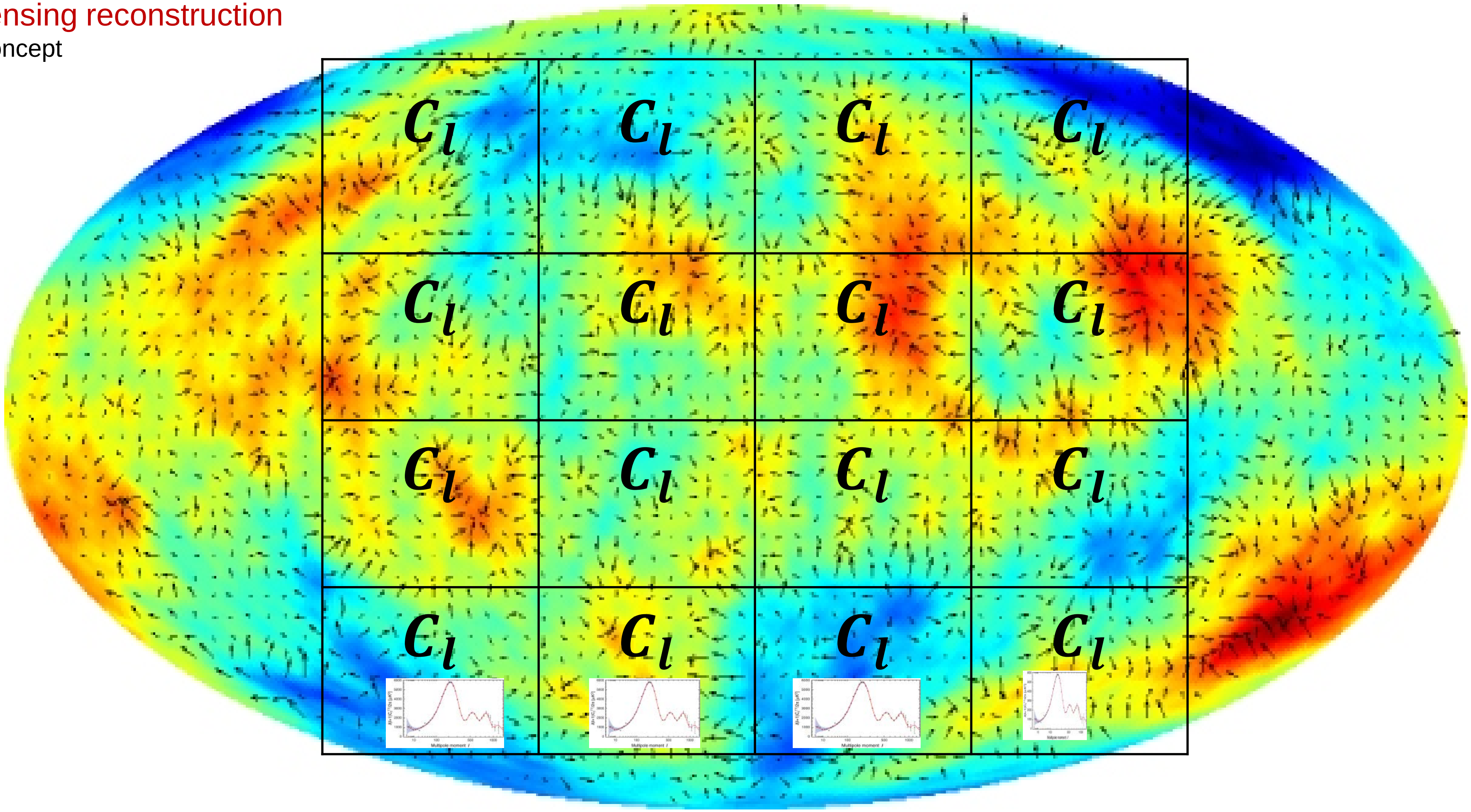
Think about ‘squeezed’ configuration: big nearly constant lenses, much smaller lensed T

Magnified	Unlensed	Demagnified
		
		

Fractional magnification ~ convergence $\kappa = -\boldsymbol{\nabla} \cdot \frac{\boldsymbol{\alpha}}{2}$ + shear modulation:

Lensing reconstruction

-concept



Variance in each C_l measurement $\propto 1/N_{\mathrm{modes}}$

$N_{\mathrm{modes}} \propto l_{\mathrm{max}}^2$ - dominated by smallest scales

$\Rightarrow$ measurement of angular scale ($\Rightarrow \kappa$) in each box nearly independent

$\Rightarrow$ Uncorrelated variance on estimate of magnificantion κ in each box

$\Rightarrow$ Nearly white ‘reconstruction noise’ $N_l^{(0)}$ on κ , with $N_l^{(0)} \propto 1/l_{\mathrm{max}}^2$

For fixed ψ: Gaussian anisotropic distribution $\Rightarrow \langle \Theta(\mathbf{l})\Theta(\mathbf{l}')\rangle \neq C_l \delta(\mathbf{l}-\mathbf{l}')$

Use series expansion:

$$\tilde{\Theta}(\mathbf{l}) \approx \Theta(\mathbf{l}) - \int \frac{\mathrm{d}^2\mathbf{l}'}{2\pi} \mathbf{l}' \cdot (\mathbf{l}-\mathbf{l}')\psi(\mathbf{l}-\mathbf{l}')\Theta(\mathbf{l}')$$

(higher order terms are important, but bias can be corrected for later)

Average over unlensed CMB Θ:

$$\langle \tilde{\Theta}(\mathbf{l})\tilde{\Theta}^*(\mathbf{l}-\mathbf{L})\rangle_\Theta = \quad \delta(\mathbf{L})\, C_l^\Theta + \frac{1}{2\pi}\left[(\mathbf{L}-\mathbf{l})\cdot \mathbf{L}\, C^\Theta_{|\mathbf{l}-\mathbf{L}|} + \mathbf{l}\cdot\mathbf{L}\, C_l^\Theta\right]\psi(\mathbf{L}) + \mathcal{O}(\psi^2)$$

Off-diagonal correlation $\propto \psi(L)$ – use to measure ψ!

For $L \geq 1$ define quadratic estimator by summing up with weights $g(\boldsymbol{l},\boldsymbol{L})$

$$\hat{\psi}(\mathbf{L}) \equiv N(\mathbf{L}) \int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\, \tilde{\Theta}(\mathbf{l})\tilde{\Theta}^*(\mathbf{l}-\mathbf{L})g(\mathbf{l},\mathbf{L}),$$

Zaldarriaga & Seljak, Hu 2001+

Reconstruction 'Noise' $N(L)$ – from random fluctuations of the unlensed CMB
Turns out to be the same as the normalization $N(L)$

$$\delta(\mathbf{0})\langle|\hat{\psi}(\mathbf{L})|^2\rangle^{-1} = N(\mathbf{L})^{-1} = \int \frac{d^2\mathbf{l}}{(2\pi)^2} \frac{\left[(\mathbf{L}-\mathbf{l})\cdot\mathbf{L}\, C^{\Theta}_{|\mathbf{l}-\mathbf{L}|} + \mathbf{l}\cdot\mathbf{L}\, C^{\Theta}_{l}\right]^2}{2\tilde{C}^{\mathrm{tot}}_{l}\tilde{C}^{\mathrm{tot}}_{|\mathbf{l}-\mathbf{L}|}}$$

(in limit of no lensing – there are higher order corrections)

On large scales (large lenses), $L \ll l$, with no instrumental noise

$$\frac{1}{L^4 N(L)} \approx \frac{1}{16\pi}\int l\,dl \left(\underbrace{\left[\frac{d\ln l^2 C_l}{d\ln l}\right]^2}_{\text{Convergence}} + \frac{1}{2}\underbrace{\left[\frac{d\ln C_l}{d\ln l}\right]^2}_{\text{Shear}}\right)$$ constant

Want $\langle \hat{\psi}(\mathbf{L}) \rangle_\Theta = \psi(\mathbf{L})$

$$\Rightarrow \quad N(\mathbf{L})^{-1} = \int \frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2} \left[(\mathbf{L}-\mathbf{l}) \cdot \mathbf{L}\, C^\Theta_{|\mathbf{l}-\mathbf{L}|} + \mathbf{l} \cdot \mathbf{L}\, C^\Theta_l \right] g(\mathbf{l}, \mathbf{L})$$

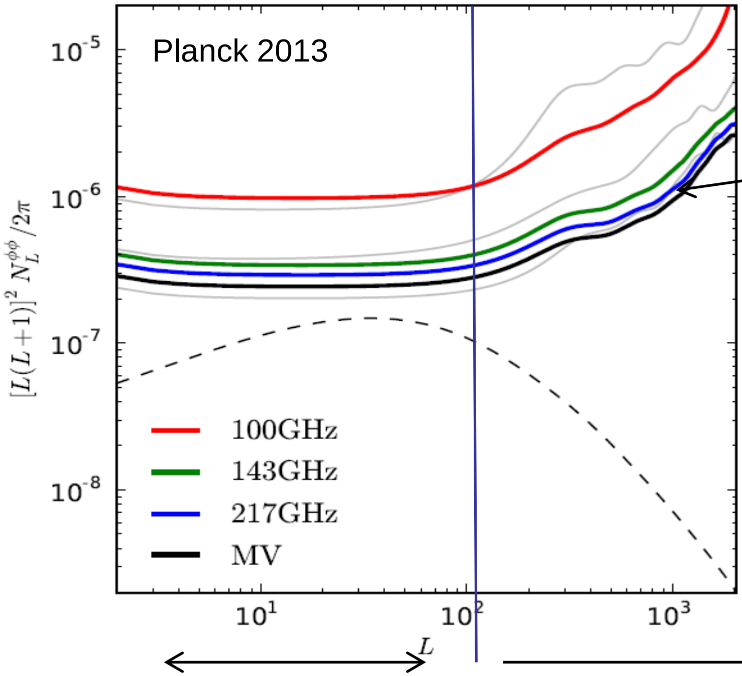
Want the best estimator: find weights g to minimize the variance

$$\langle \hat{\psi}^*(\mathbf{L}) \hat{\psi}(\mathbf{L}') \rangle = \delta(\mathbf{L}-\mathbf{L}') 2N(\mathbf{L})^2 \int \frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2} \tilde{C}^{\mathrm{tot}}_l \tilde{C}^{\mathrm{tot}}_{|\mathbf{l}-\mathbf{L}|} [g(\mathbf{l}, \mathbf{L})]^2 + \mathcal{O}(C^\psi_l)$$

$$\tilde{C}^{\mathrm{tot}}_l = \tilde{C}^\Theta_l + N_l$$

$$\Rightarrow \quad g(\mathbf{l}, \mathbf{L}) = \frac{(\mathbf{L}-\mathbf{l}) \cdot \mathbf{L}\, C^\Theta_{|\mathbf{l}-\mathbf{L}|} + \mathbf{l} \cdot \mathbf{L}\, C^\Theta_l}{2\tilde{C}^{\mathrm{tot}}_l \tilde{C}^{\mathrm{tot}}_{|\mathbf{l}-\mathbf{L}|}}$$

$\propto C_l^\kappa$



Reconstruction Noise
$N_L^{(0)}$

$N_l^{(0)} \sim$ const

Beam, noise, shape of C_l and $l \sim l_{\max}$ effects

Lensing reconstruction information mostly in the *smallest scales* observed

- Need high resolution and sensitivity
- Almost totally insensitive to large-scale T (so only *small-scale* foregrounds an issue)

Practical fast way to do it, using FFT:

$$\hat{\psi}(\mathbf{L}) = N(\mathbf{L})\,\mathbf{L}\cdot\int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\frac{\mathbf{l}C_l^{\Theta}\tilde{\Theta}(\mathbf{l})}{\tilde{C}_l^{\mathrm{tot}}}\,\frac{\tilde{\Theta}(\mathbf{L}-\mathbf{l})}{\tilde{C}_{|\mathbf{L}-\mathbf{l}|}^{\mathrm{tot}}}$$

- Looks like convolution: use convolution theorem

$$\begin{aligned}\hat{\psi}(\mathbf{L}) &= -iN(\mathbf{L})\,\mathbf{L}\cdot\int \frac{\mathrm{d}^2\mathbf{x}}{2\pi}\,e^{-i\mathbf{L}\cdot\mathbf{x}}F_1(\mathbf{x})\nabla F_2(\mathbf{x}) \\ &= -N(\mathbf{L})\int \frac{\mathrm{d}^2\mathbf{x}}{2\pi}\,e^{-i\mathbf{L}\cdot\mathbf{x}}\nabla\cdot\underbrace{[F_1(\mathbf{x})\nabla F_2(\mathbf{x})]}_{\text{Easy to calculate in real space: multiply maps}}\end{aligned}$$

$$F_1(\mathbf{l}) \equiv \frac{\tilde{\Theta}(\mathbf{l})}{\tilde{C}_l^{\mathrm{tot}}} \qquad F_2(\mathbf{l}) \equiv \frac{\tilde{\Theta}(\mathbf{l})C_l^{\Theta}}{\tilde{C}_l^{\mathrm{tot}}}$$

- fast and easy to compute in harmonic space

- Can make similar argument on full sky and for polarization

Alternative more general derivation (works for cut-sky, anisotropic noise)
For fixed lenses, sky is Gaussian but anisotropic:

$$-\mathcal{L}(\hat{\Theta}|\alpha) = \frac{1}{2}\hat{\Theta}^T \left(\hat{C}^{\hat{\Theta}\hat{\Theta}}\right)^{-1} \hat{\Theta} + \frac{1}{2}\ln\det(\hat{C}^{\hat{\Theta}\hat{\Theta}}),$$

Find the maximum-likelihood estimator for the lensing potential/deflection angle

$$\frac{\delta\mathcal{L}}{\delta\alpha_i(\mathbf{x})} = \frac{1}{2}\hat{\Theta}^T(\hat{C}^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta\hat{C}^{\hat{\Theta}\hat{\Theta}}}{\delta\alpha_i(\mathbf{x})}(\hat{C}^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} - \frac{1}{2}\mathrm{Tr}\left[(\hat{C}^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta\hat{C}^{\hat{\Theta}\hat{\Theta}}}{\delta\alpha_i(\mathbf{x})}\right] = 0$$

Trick: $Tr(A) = Tr\left(AC^{-1}\langle xx^T\rangle\right)$ where x has covariance $\boldsymbol{C}$

$= \langle x^T A C^{-1} x\rangle$ - rewrite trace as "mean field" average

Can show that first-step Newton-Raphson maximum likelihood solution is as before, but with

$$\frac{1}{C_l^{tot}} \to (\hat{C}^{\hat{\Theta}\hat{\Theta}})^{-1} = (S+N)^{-1}$$

$$\hat{\psi} \to \hat{\psi} - \langle\hat{\psi}\rangle$$

weights optimally for cuts/noise and subtracts average signal from noise inhomogeneity and cuts

Sets to zero in cut, downweights high noise (also need *lensed* C_l)

"mean field" calculated from expectation from simulations

astro-ph/0209489, arXiv:0908.0963

Lensing potential power spectrum

$$\langle \hat{\psi}(L)\hat{\psi}(L') \rangle = \delta(L+L')\left(C_L^{\psi} + N_0(L) + \text{other biases}\right)$$

$\hat{\psi}$ is *quadratic* in T $\Rightarrow$ $\hat{C}^{\psi} \propto \langle \hat{\psi}\hat{\psi} \rangle$ is *quartic* in T: measure of 4-point function

Other biases include:

- Other sources of connected 4-point function (e.g. point sources)
- instrumental/observational complications
- N1

N1 comes from ‘non-primary’ contractions that depend on ϕ:

$$\left\langle C^{\hat{\phi}} \right\rangle \sim \langle T_1T_2T_3T_4 \rangle = C^{\psi} + \underbrace{\langle T_1T_3\rangle\langle T_2T_4\rangle + \langle T_1T_4\rangle\langle T_2T_3\rangle}_{\text{N0+N1}}$$

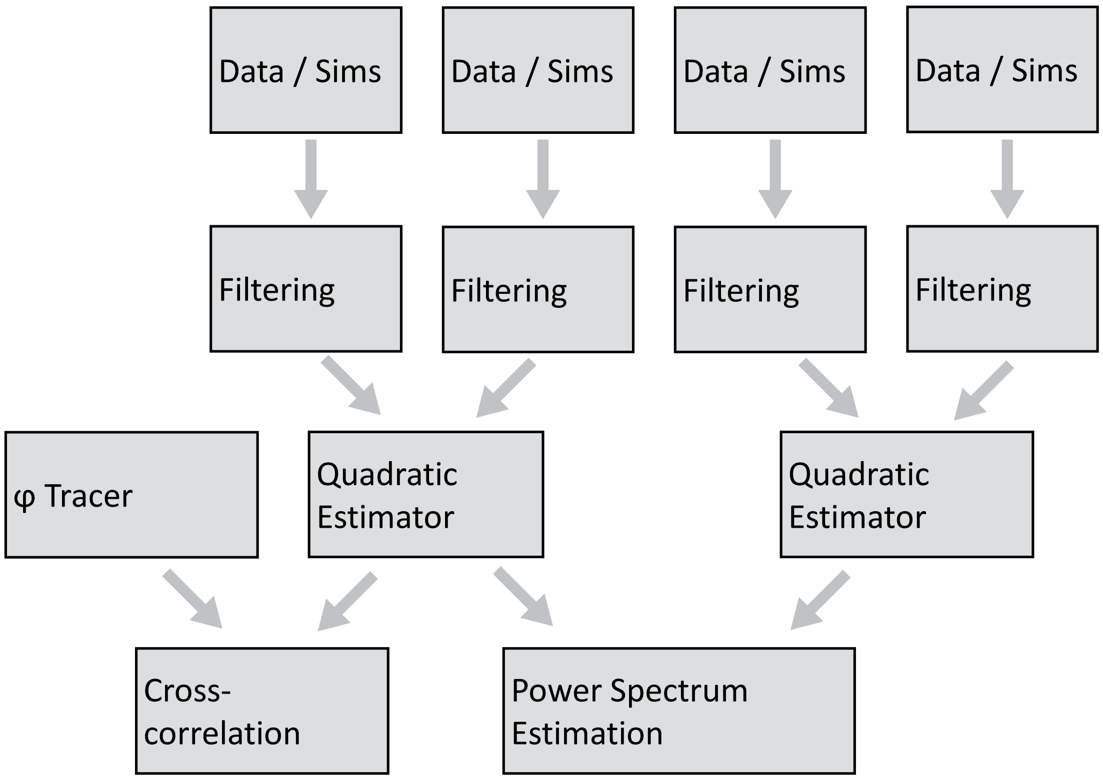
N0= independent of C^{ψ}:
N1=$O(C^{\psi})$ from off-diagonal correlations of T(l)T(l’)

Lens Reconstruction Pipeline

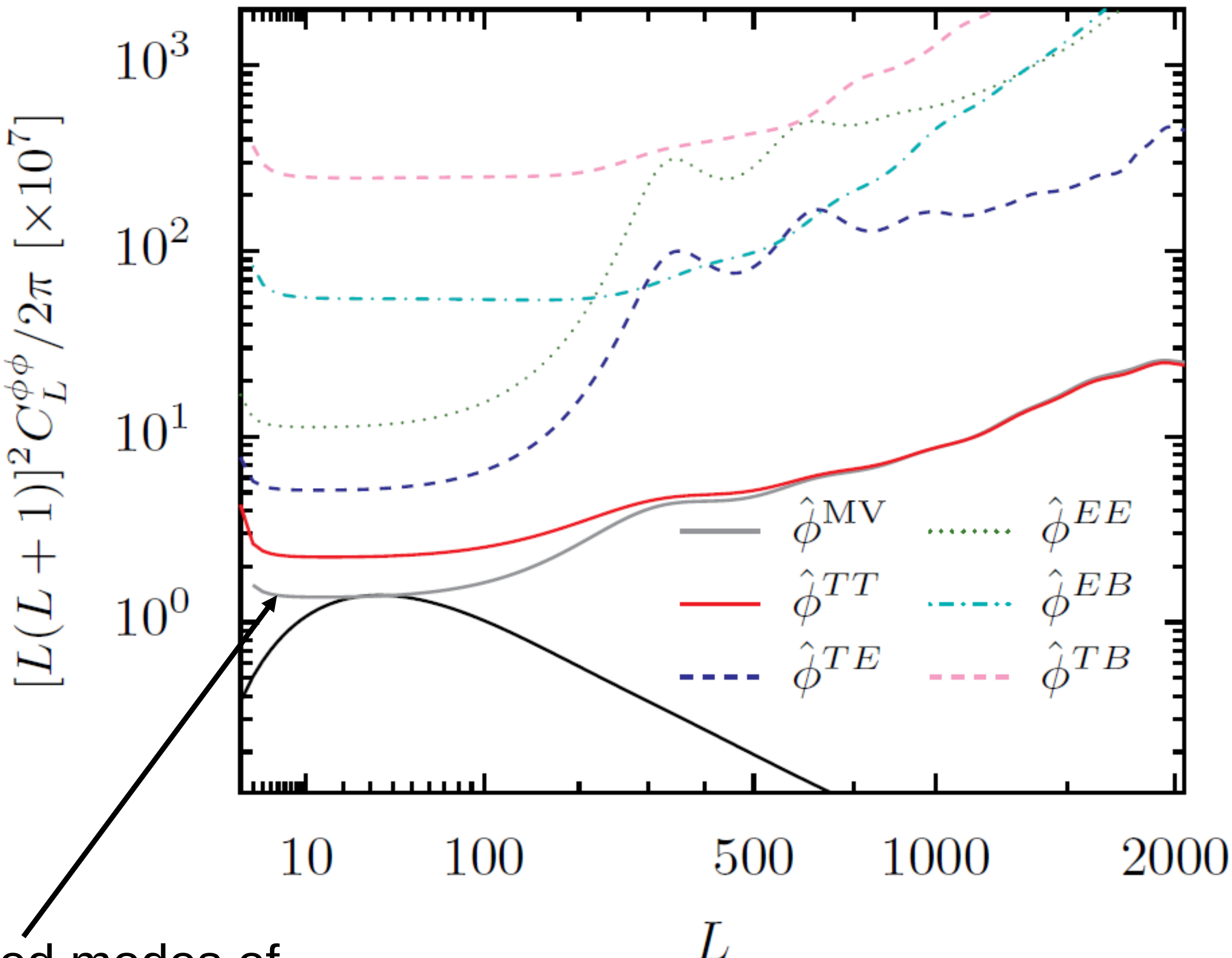
→ process input maps

→ estimate lensing potential from anisotropic 2-point

→ estimate lensing power spectrum.

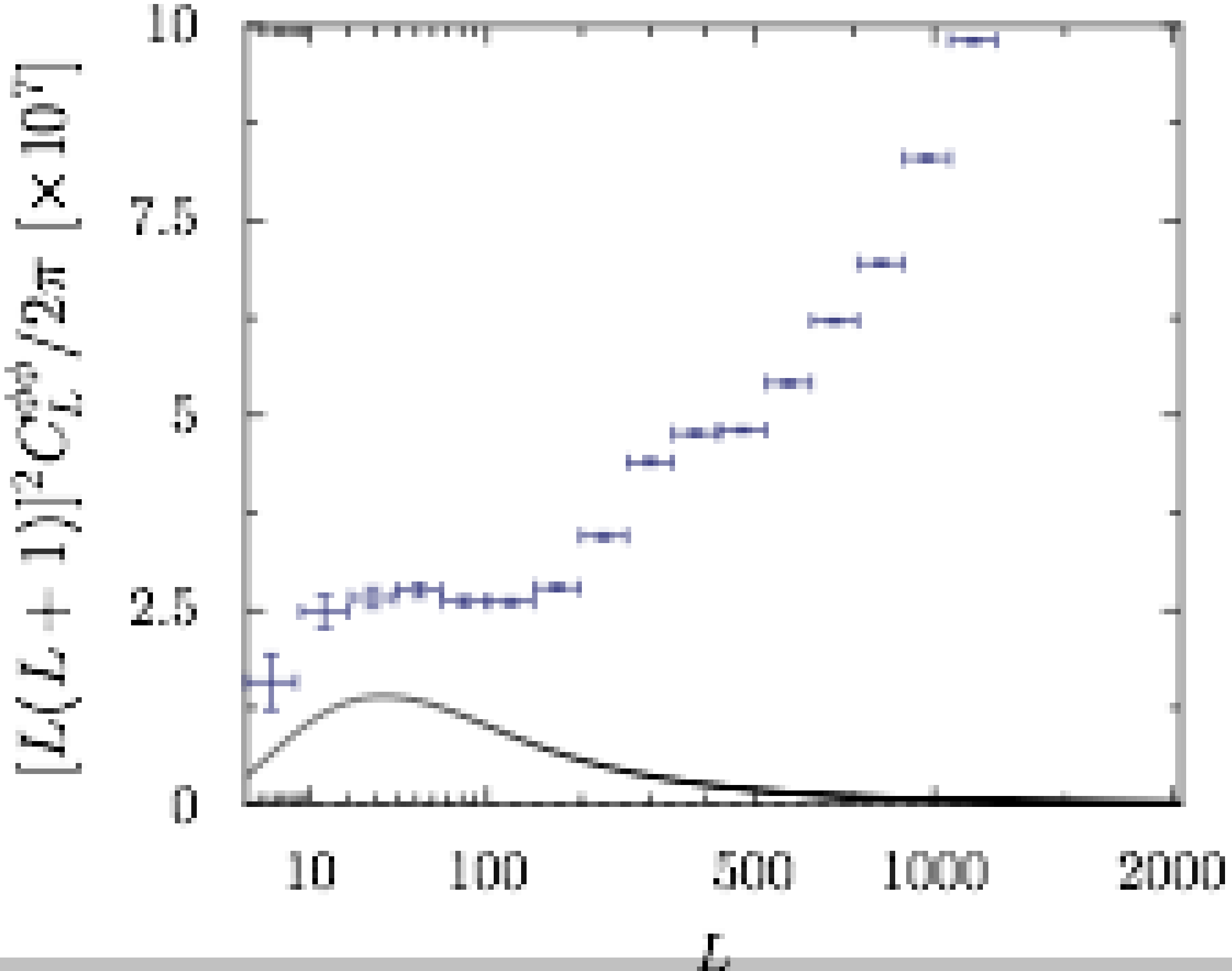


Planck noise power spectra for lensing estimators.



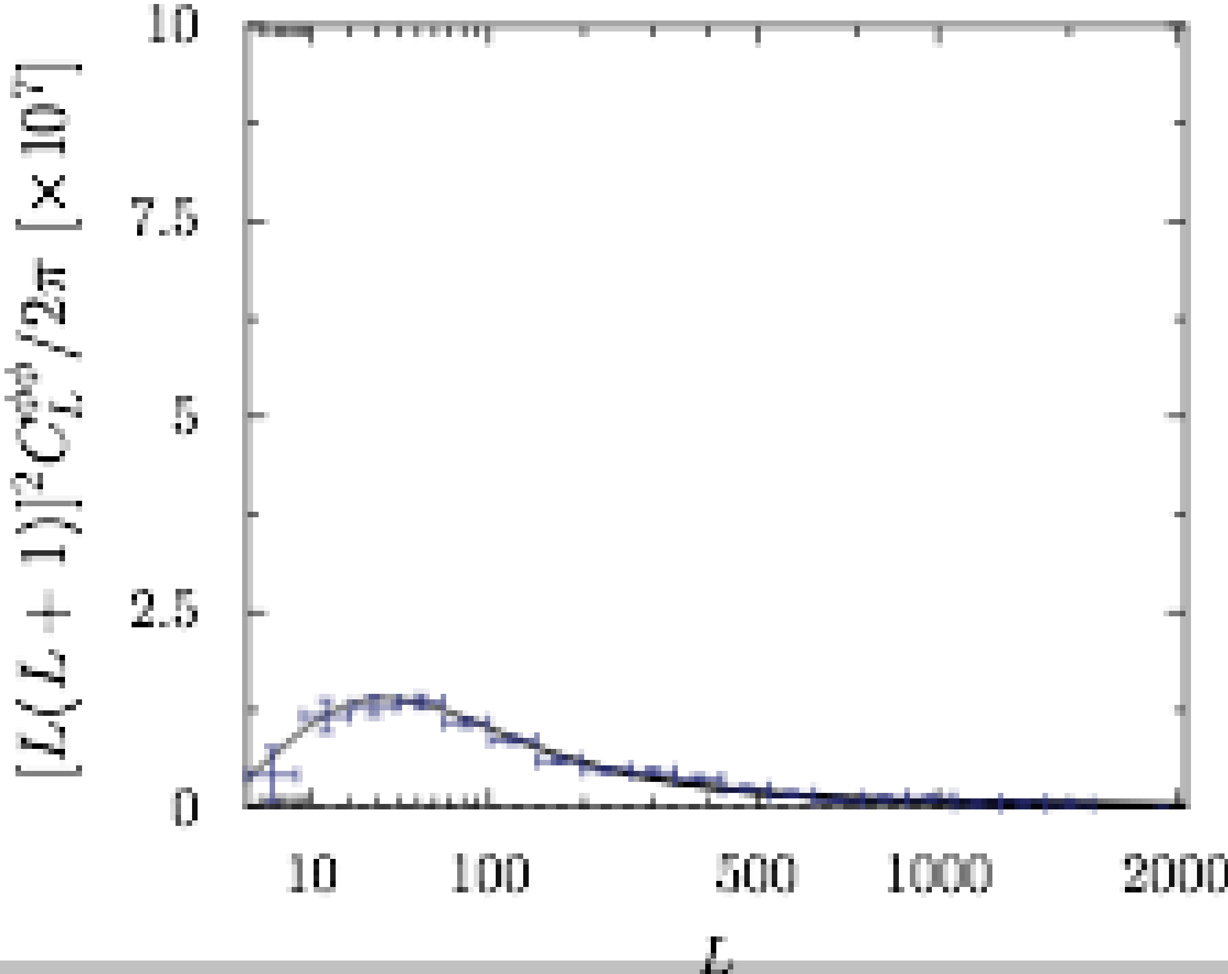
Best measured modes of
MV estimator have S/N=1.

Power Spectrum Estimation



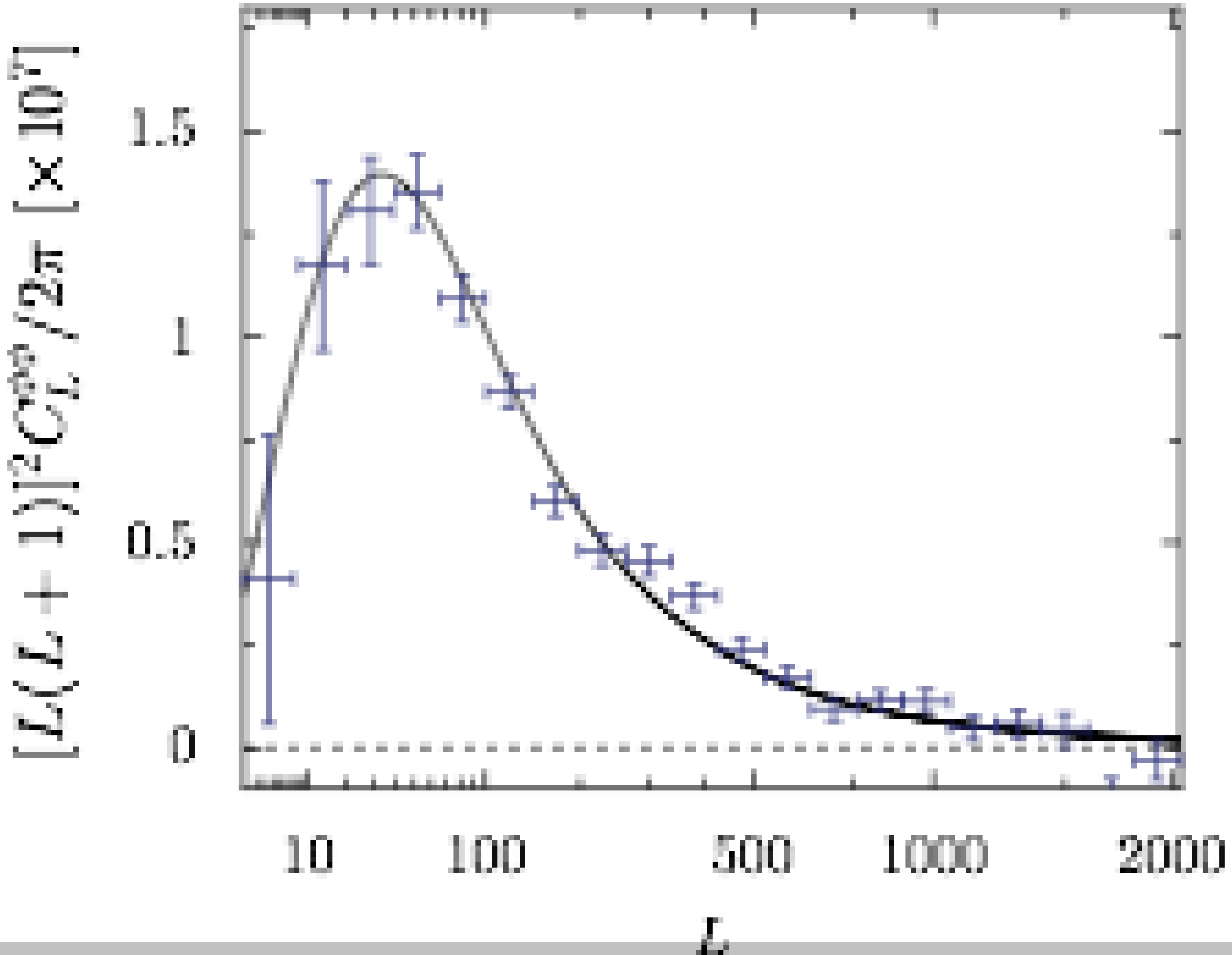
1) Raw power spectrum of quadratic estimates.

Power Spectrum Estimation



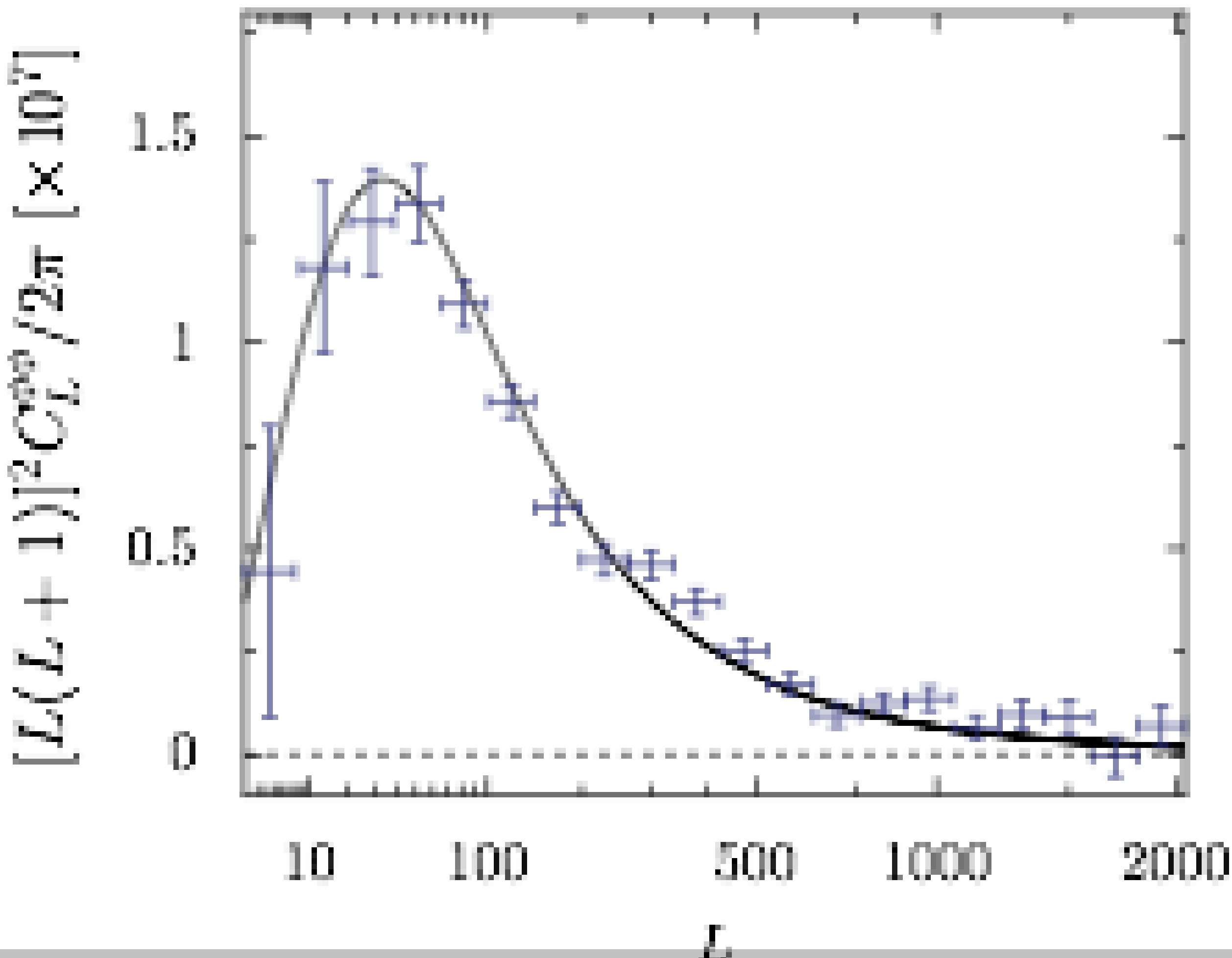
2) Correct for N0 noise bias estimated from sims.

Power Spectrum Estimation



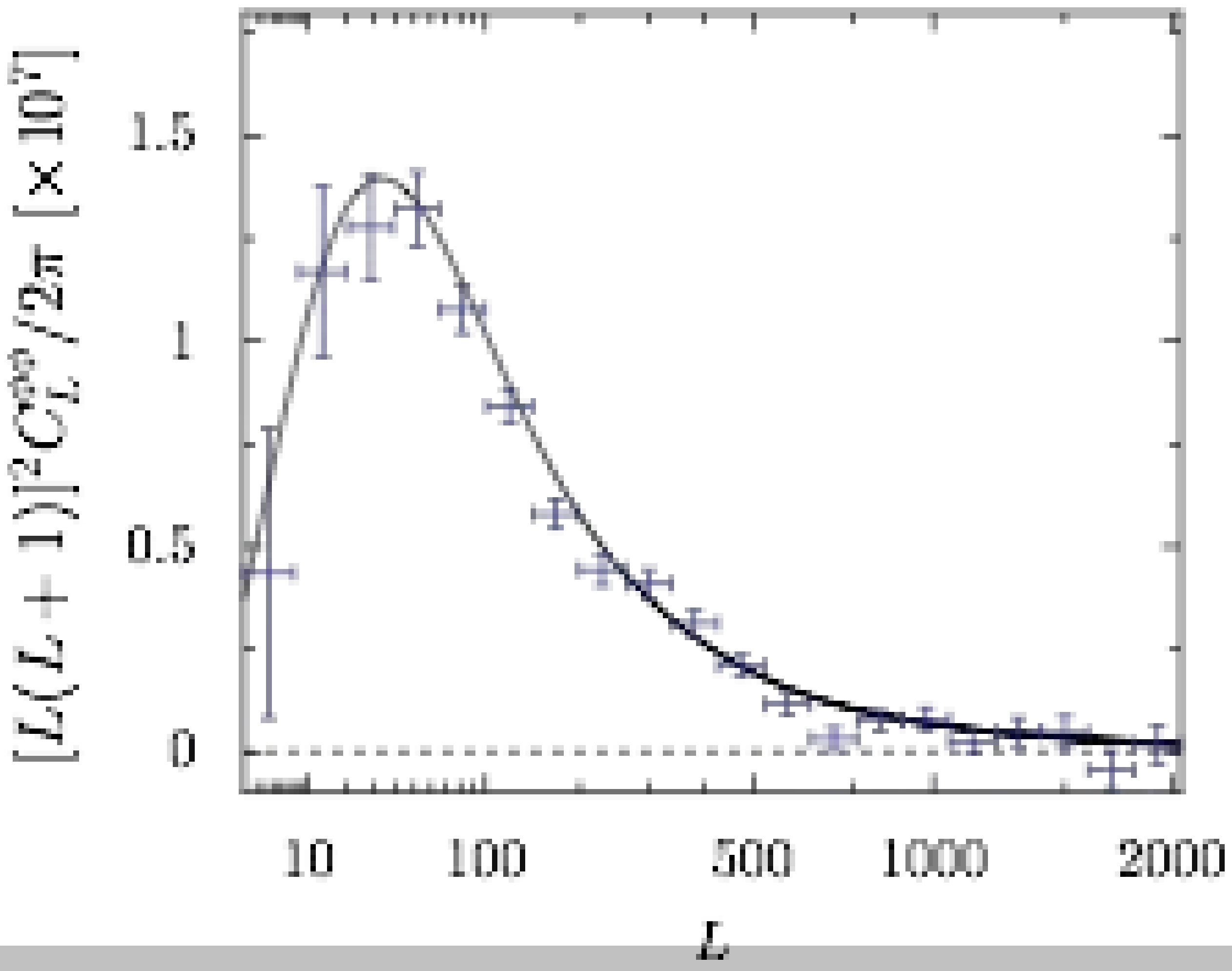
2) Correct for noise bias estimated from sims.

Power Spectrum Estimation



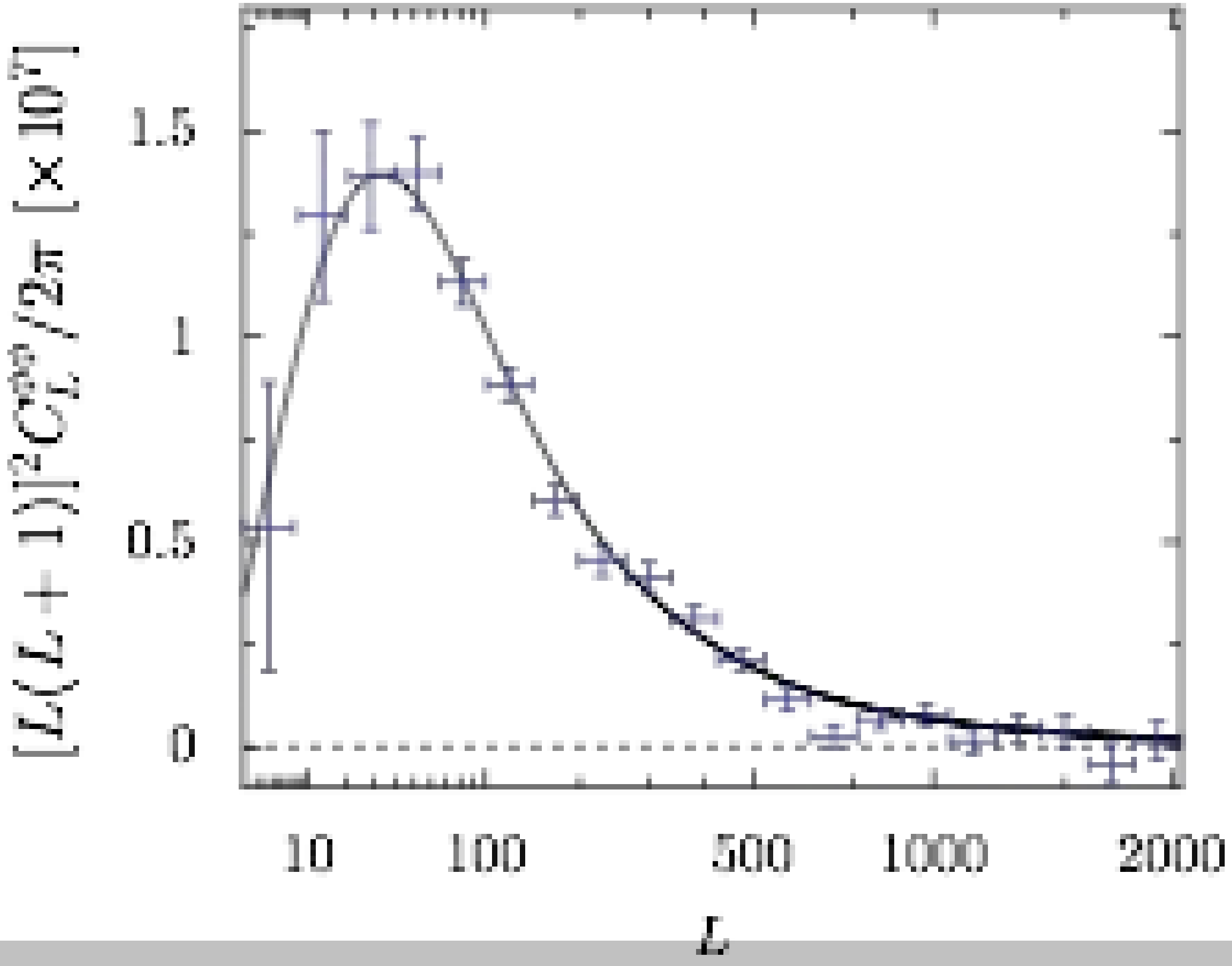
3) Apply further data-based estimate of noise bias to reduce sensitivity to inaccuracy of sims (‘RDN0’).

Power Spectrum Estimation



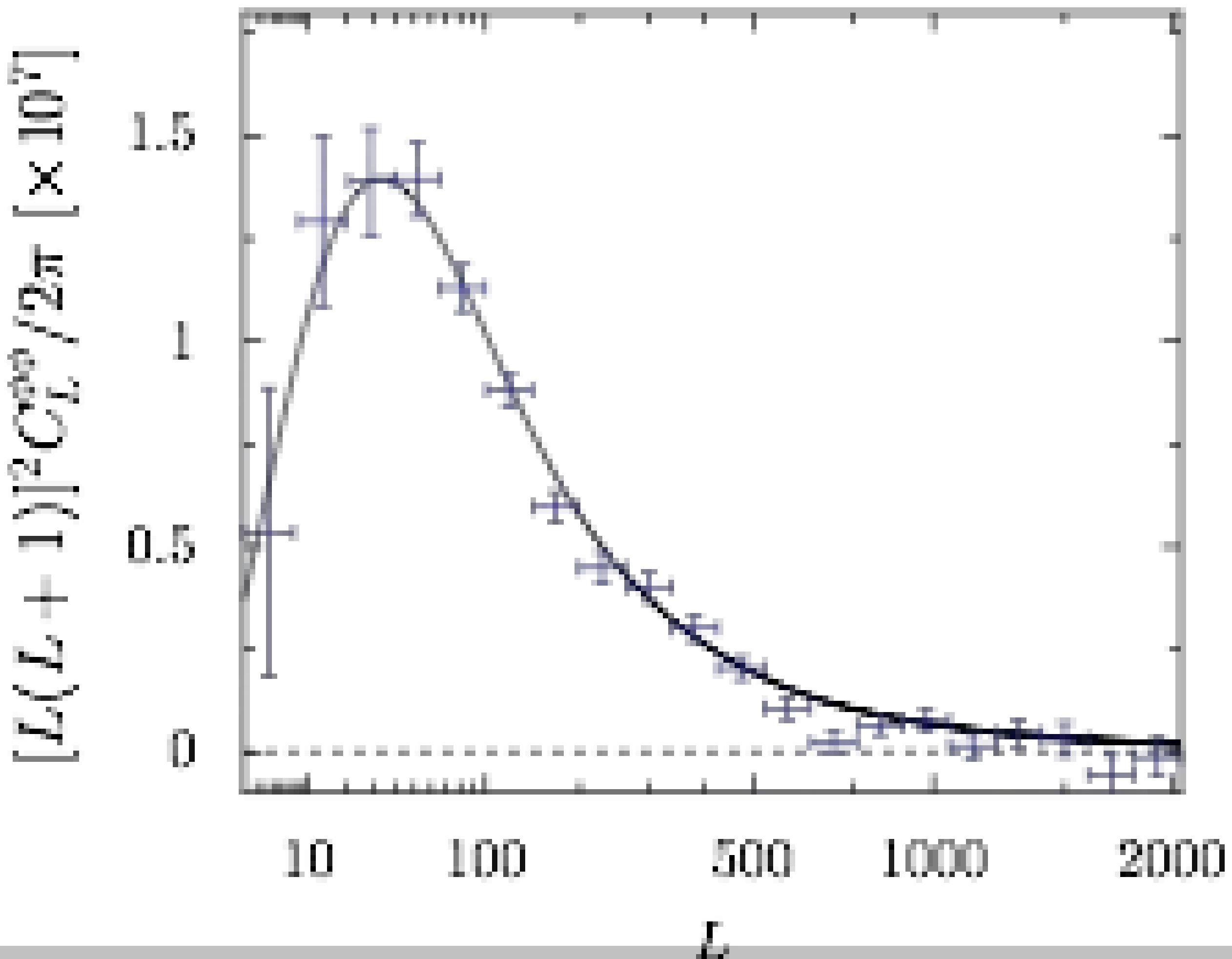
4) Correct for N1 bias.

Power Spectrum Estimation



5) MC correction for mode mixing / inaccuracies in normalization.

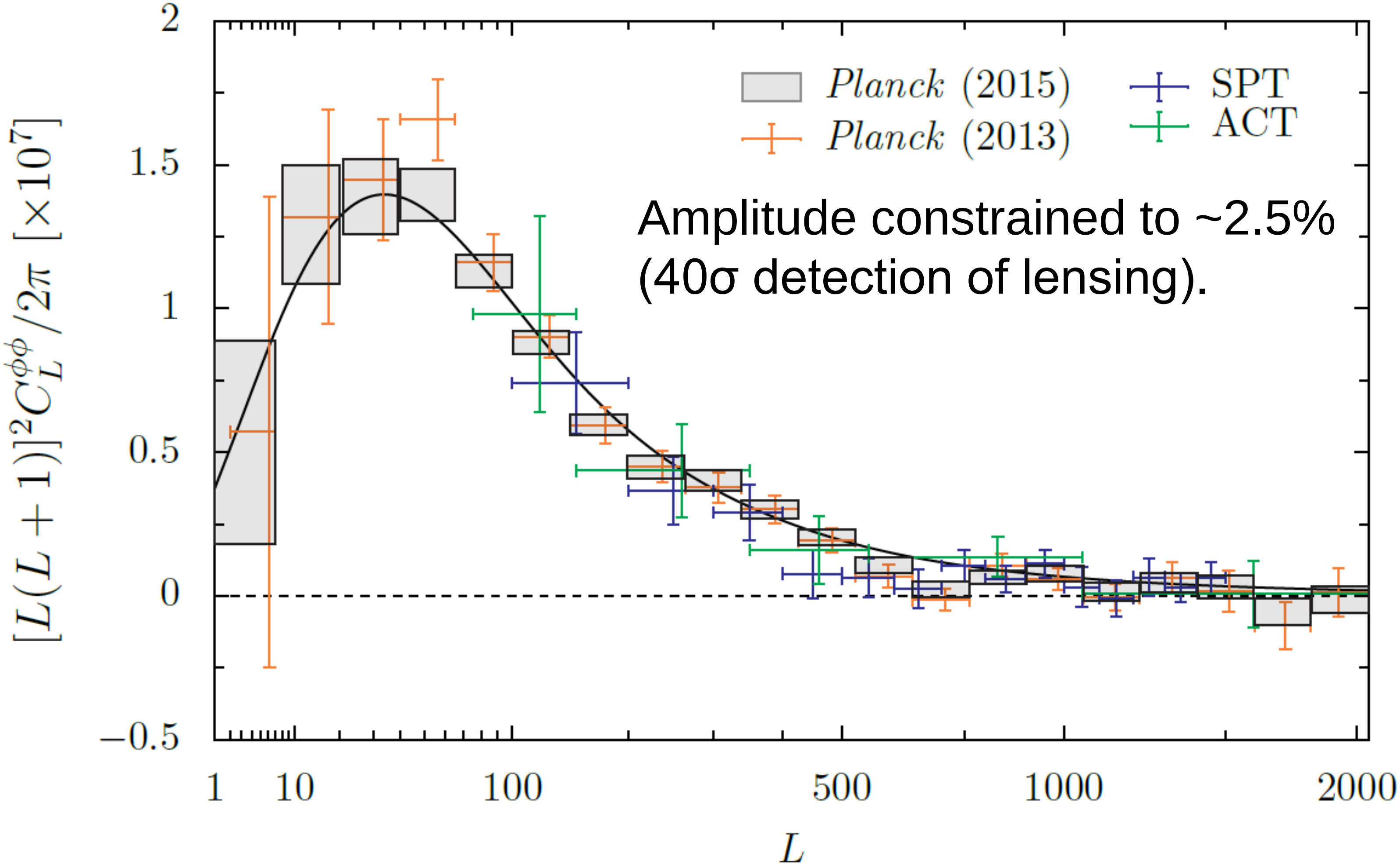
Power Spectrum Estimation



6) Correct for point source bias.

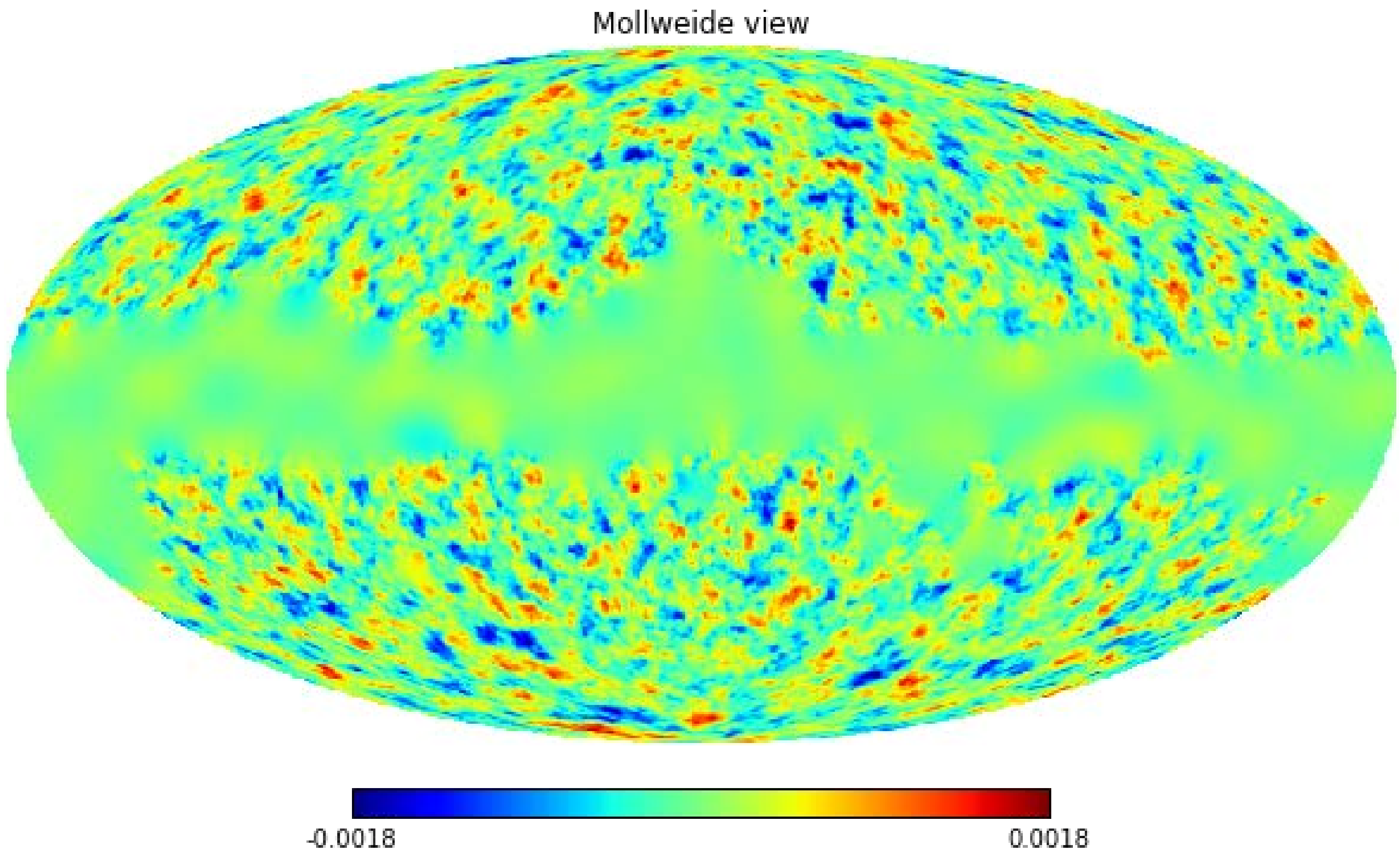
Done!

Lensing Power Spectrum

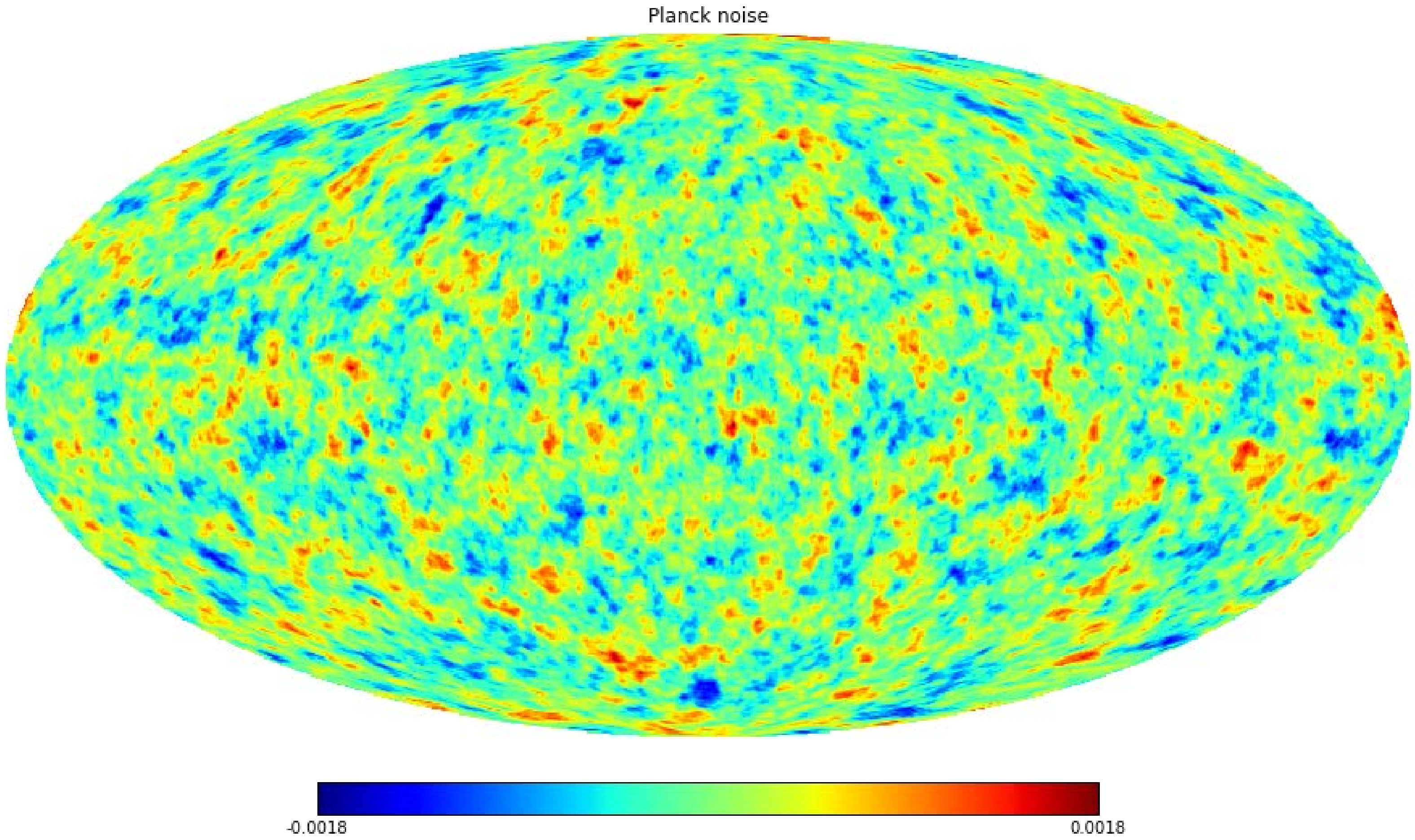


Amplitude constrained to ~2.5% (40σ detection of lensing).

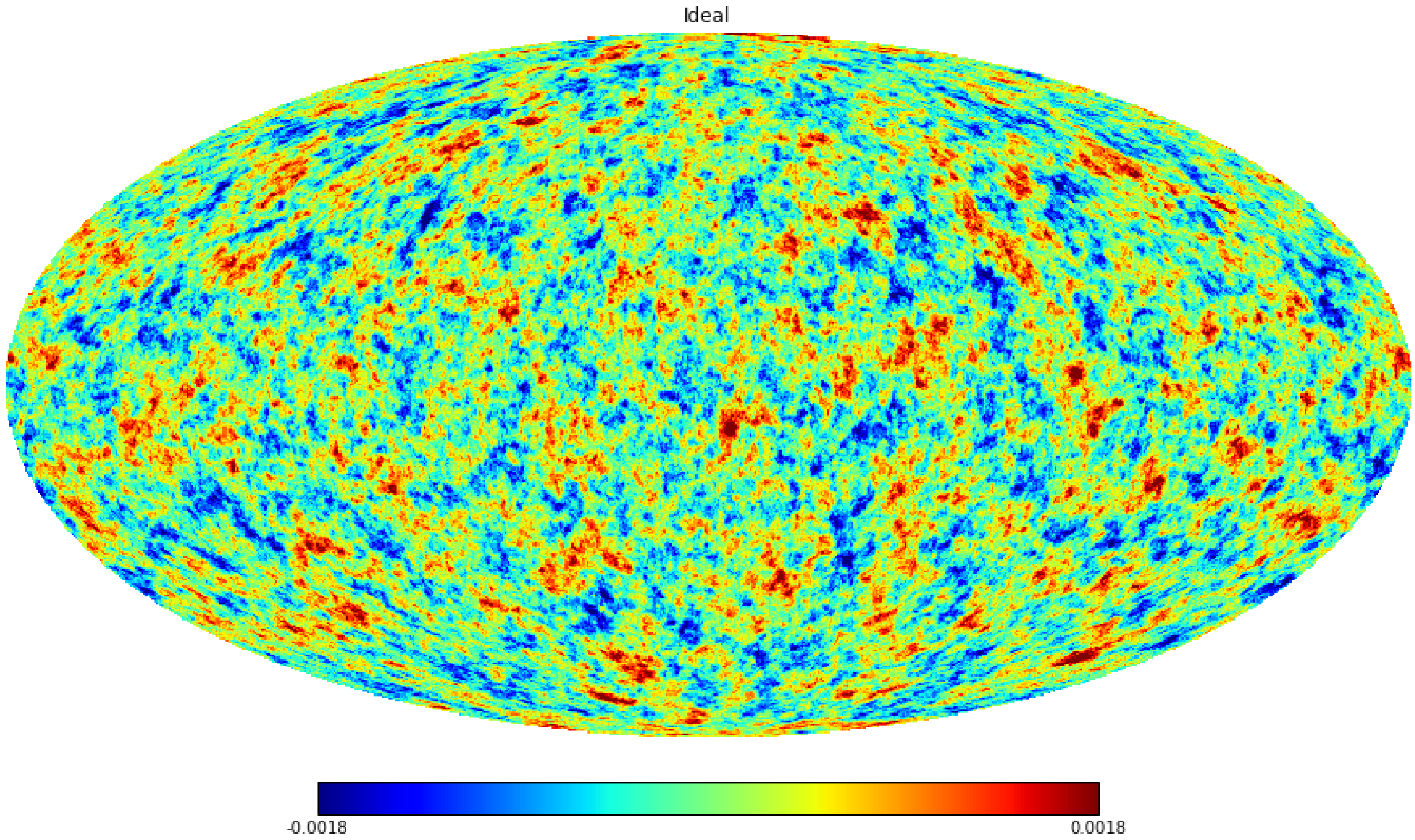
Planck 2015 lensing reconstruction ($E_{\nabla\Phi}$)



Simulated Planck lensing reconstruction

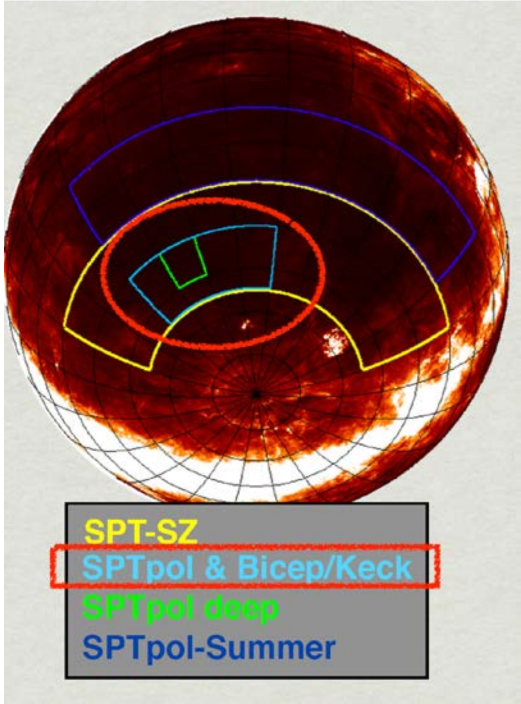


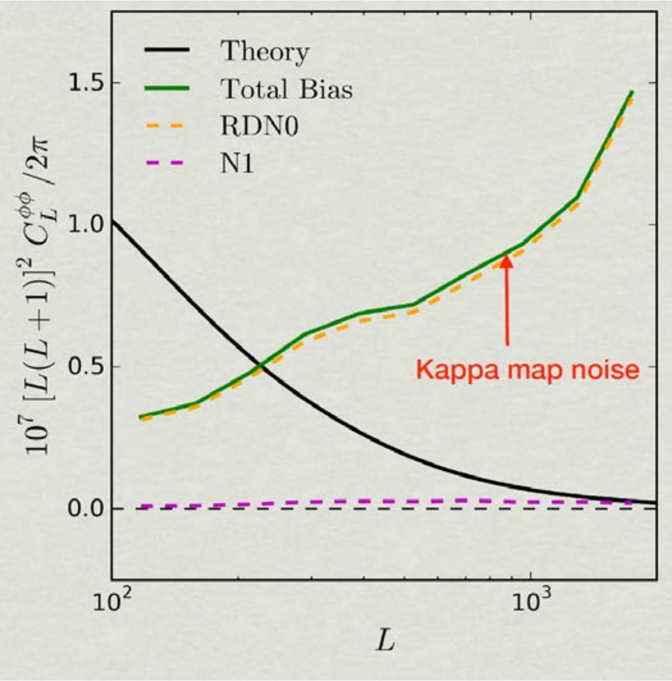
True simulation input

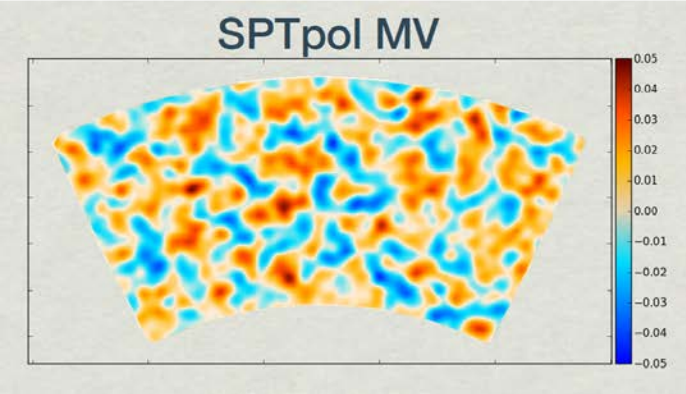


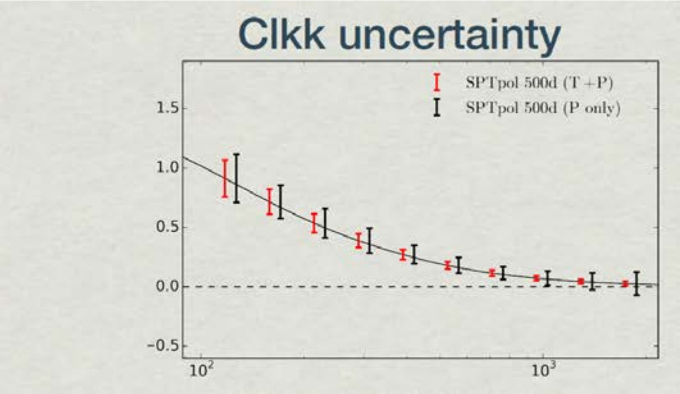
High-resolution ground-based observations can measure smaller sky area with much higher S/N

e.g. SPTpol



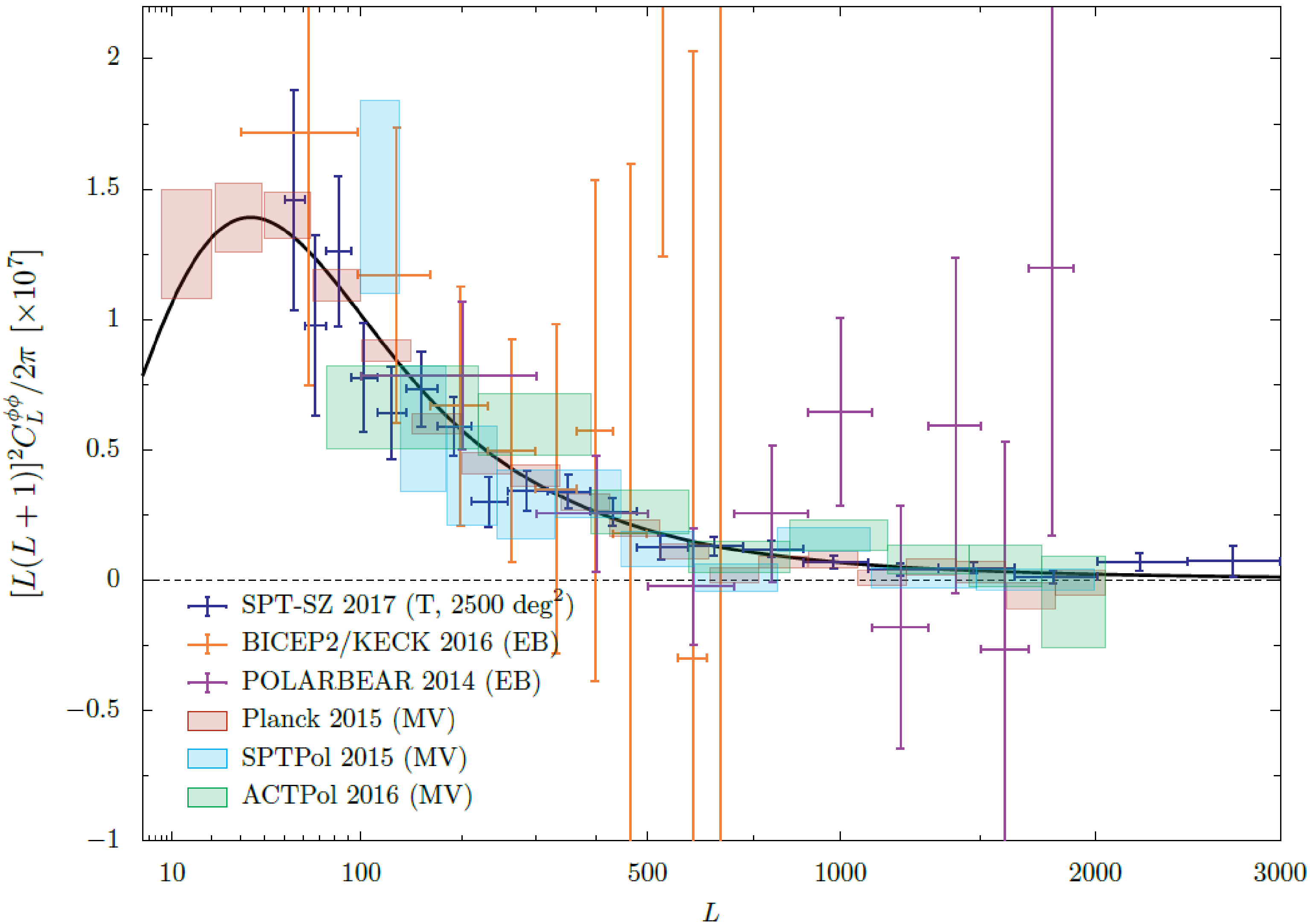






cosmo-nordita.fysik.su.se/talks/w3/d5/kstory_spt_lensing_stockholm_201707.pdf

Current lensing reconstruction power spectra

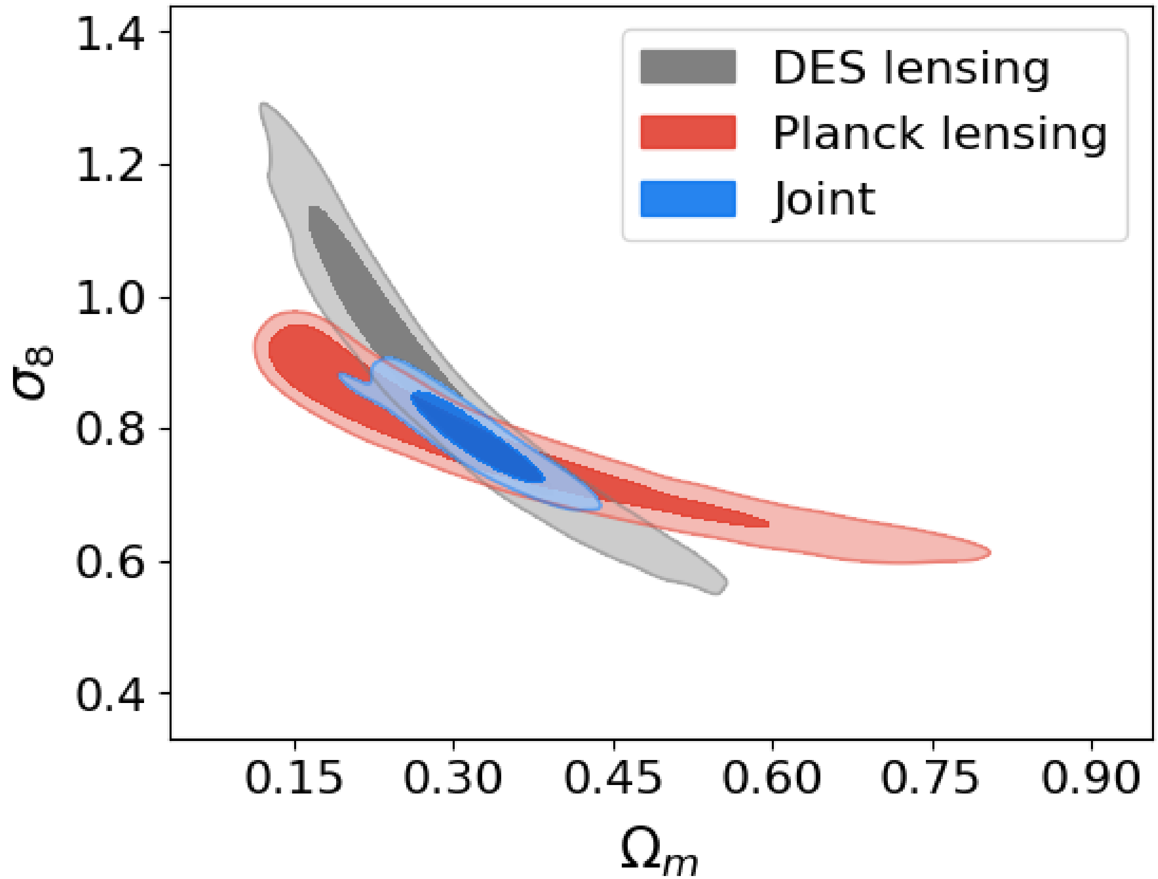


Challinor et al. arXiv:1707.02259

CMB lensing currently competitive with galaxy lensing

Probes higher redshift ⇒ constrains $\Omega_m \sigma_8^{0.25}$ vs. galaxy $\Omega_m \sigma_8^{0.5}$

DES 1YR +Planck lensing only LCDM forecast



DES 1Yr has 10 nuisance parameters, conservative cuts: limited by modelling not statistics
CMB lensing currently limited by low S/N (and only one source redshift plane)

Warm up quiz answers

1) Overdensities focus light rays, so the CMB looks hotter where there are overdensities along the line of sight

2) Even in linear theory lensing is mostly at low redshift because density perturbations grow with time

3) The lensing potential is nearly Gaussian because there are many lenses along the line of sight and perturbations nearly linear

4) Lensing rotates polarization, partly turning E modes into B modes

5) The CMB lensing power spectrum peaks at $L \sim 60$, so is sensitive to large-scale galactic temperature foregrounds

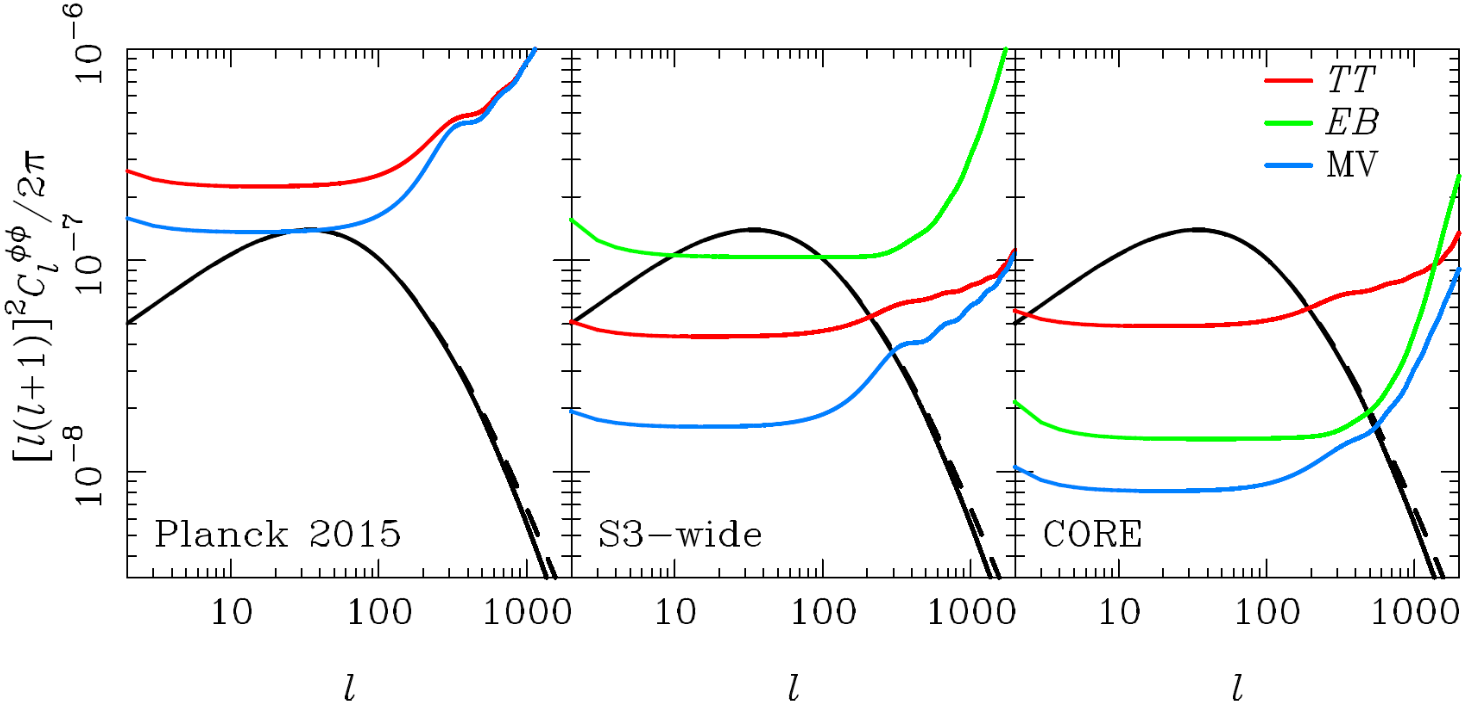
*Large-scale reconstruction information comes from large-scale **variations** (shear/magnification) of **small-scale** temperature modes*

Lensing reconstruction with polarization

- Expect *no* primordial small-scale B modes (r-modes only large scales $l < \sim 300$)

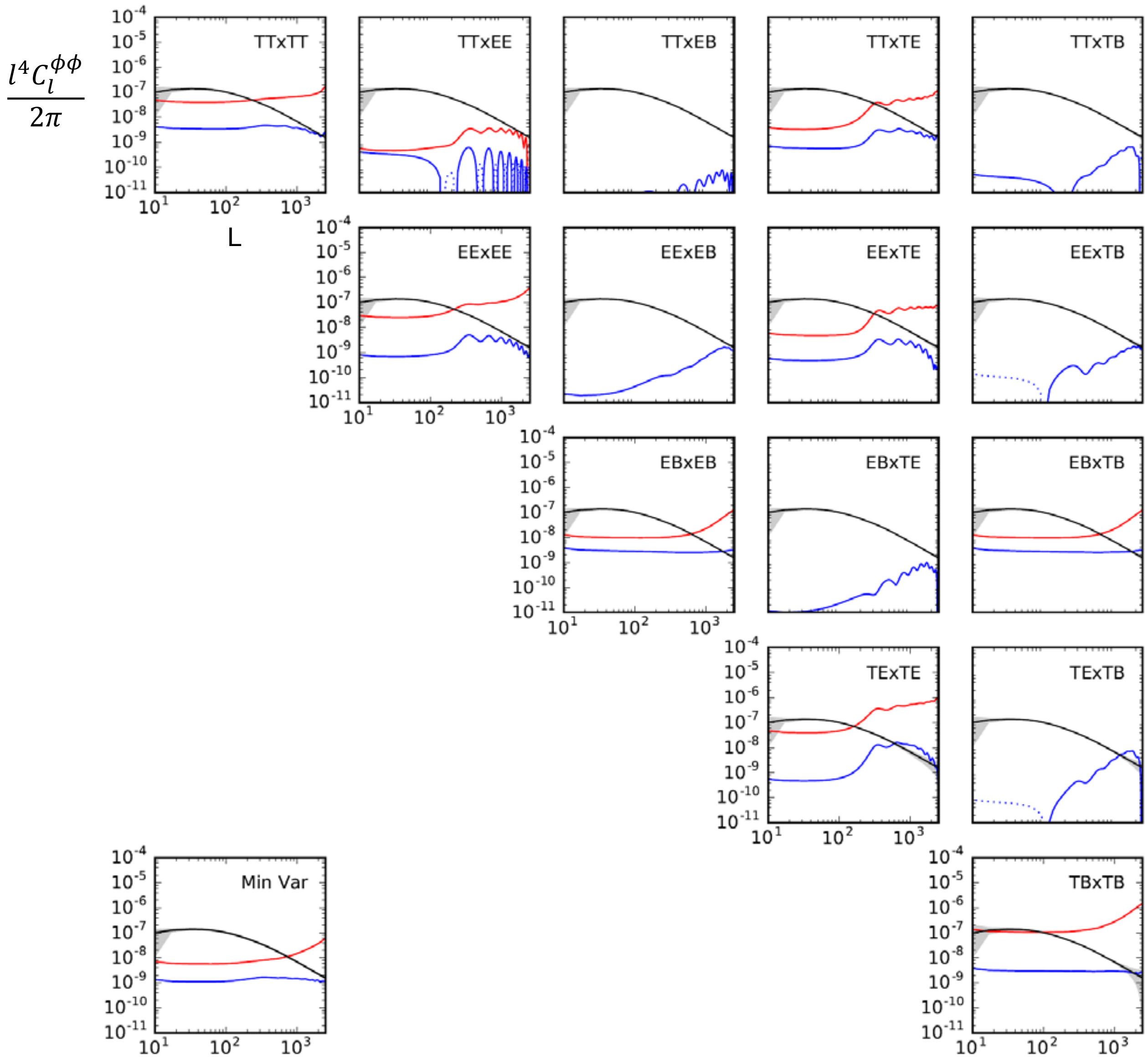
- *All* small-scale B-mode signal is lensing: *no* cosmic variance confusion with primordial signal as for E and T

- Polarization data does *much* better than temperature if sufficiently good S/N (mainly EB estimator).



Challinor et al. arXiv:1707.02259

Large set of possible estimators, e.g. for S4 several nearly-independent probes

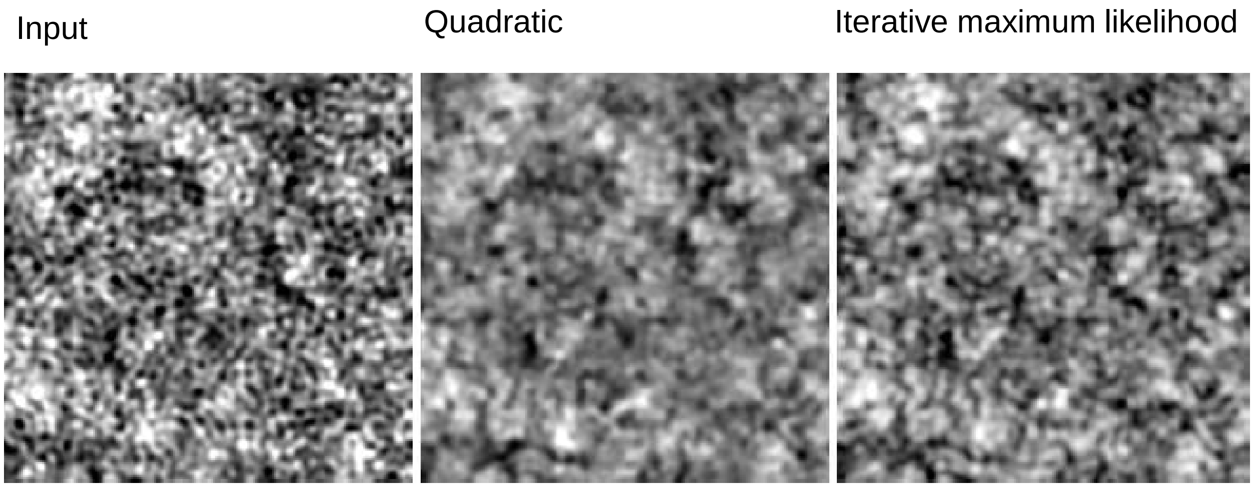


Noise

N1

Optimal polarization lensing reconstruction

- Maximum likelihood techniques much better than quadratic estimators for polarization when noise levels low enough (astro-ph/0306354)



- Carron & Lewis 2017: public code that can be used in practice (1704.08230)
(efficient handling of anisotropic noise, beams, sky cuts..)

LensIt:
https://github.com/carronj/LensIt (by Julien Carron)

Is the lensing reconstruction useful?

Neutrino mass from high S/N lensing reconstruction

Neutrinos suppress late-time perturbations

Lensing measures late-time perturbations

CMB measures $A_s e^{-2\tau}$, if we know τ well enough, know early perturbation amplitude

⇒ can measure neutrino mass

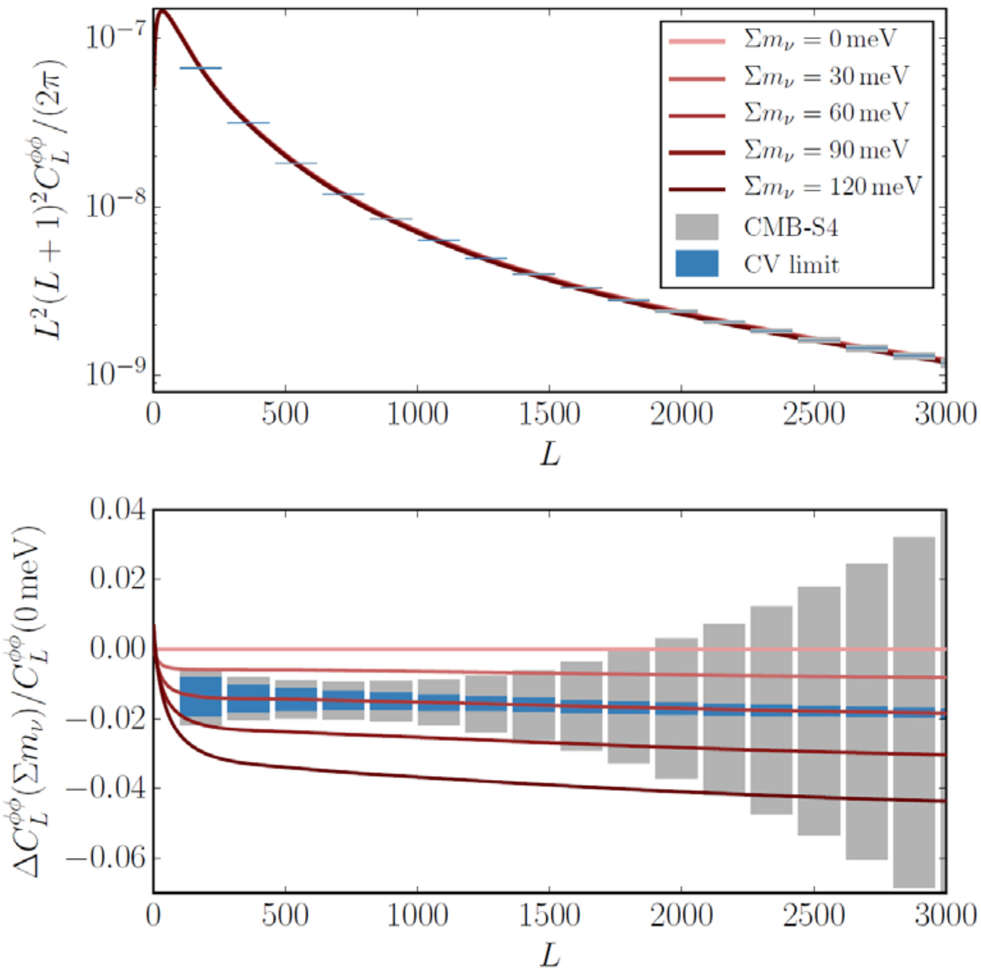


Figure 49. *Constraining neutrino mass with CMB-S4. Top: lensing power spectra for multiple neutrino masses (curves) together with forecasted errors for S4. Bottom: residual from curve at zero neutrino mass. Error boxes are shown centered at the minimal value of 60 meV. S4 will be targeted to resolve differences in neutrino mass of 20 meV.*

S4 Science Book

CMB lensing to calibrate shear for galaxy lensing

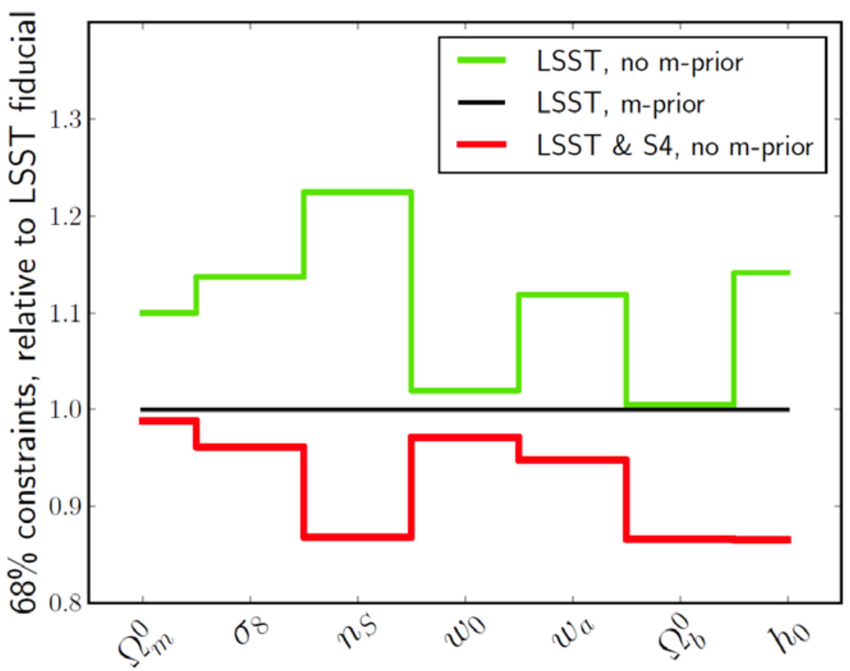
Galaxy lensing surveys measure (roughly) galaxy ellipticity e_g.
Hard to relate directly to lensing shear γ_{lens}.

$$e_{\text{g}} \sim (1+m)\gamma_{\text{lens}}$$

m could mimic different dark energy models.

Cross-correlation with CMB lensing can measure m

e.g. S4 to calibrate LSST



Schaan et al. arXiv: 1607.01761

Valuable for EUCLID, WFIRST, LSST, etc.
- more robust prior-independent constraints on dark energy

CMB lensing and cross-correlations to measure bias

- Lensing probes dark matter distribution directly

- Galaxy number counts perturbations (g) depend on bias b

- Very roughly:

$$g(z) \sim b(z)\sigma_8(z)$$

$$\langle g(z)g(z)\rangle \propto b(z)^2, \langle g(z)\phi\rangle \propto b(z)$$

$\Rightarrow$ estimate b(z) from $\frac{\langle g(z)g(z)\rangle}{\langle g(z)\phi\rangle}$

- Can help with parameters and non-Gaussianity from scale-dependent bias
  (with many redshifts can be limited by shot noise and reconstruction noise, not cosmic variance as lensing and galaxies probe same underlying mass perturbations)

See e.g. arXiv:1710.09465 and refs therein

Delensing

$$X^{\rm len}(\boldsymbol{n}) = X^{\rm unl}(\boldsymbol{n} + \boldsymbol{\alpha}(\boldsymbol{n}))$$

Measure ψ (hence $\alpha = \nabla\psi$)

Re-map back lensed fields *or* subtract perturbative lensing signal

To measure ψ:

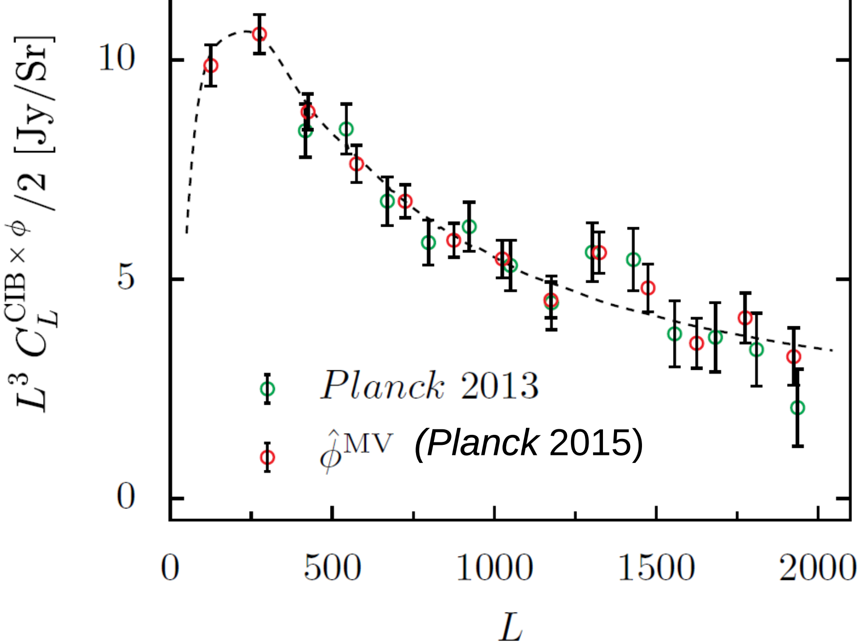
1. Use external tracer of matter,
   e.g. CIB (Cosmic Infrared Background).
   (Sherwin et al 2015, Larsen et al. 2016)

2. Use internal lensing reconstruction

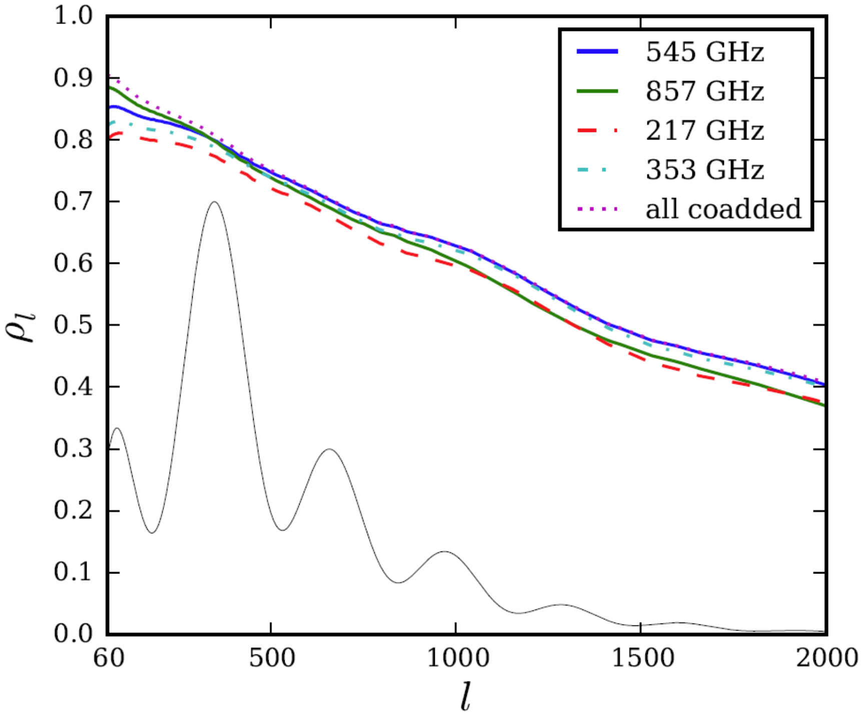
(or use both!)

CIB-lensing cross-correlation

Detected at ~50σ.



Correlation coefficient

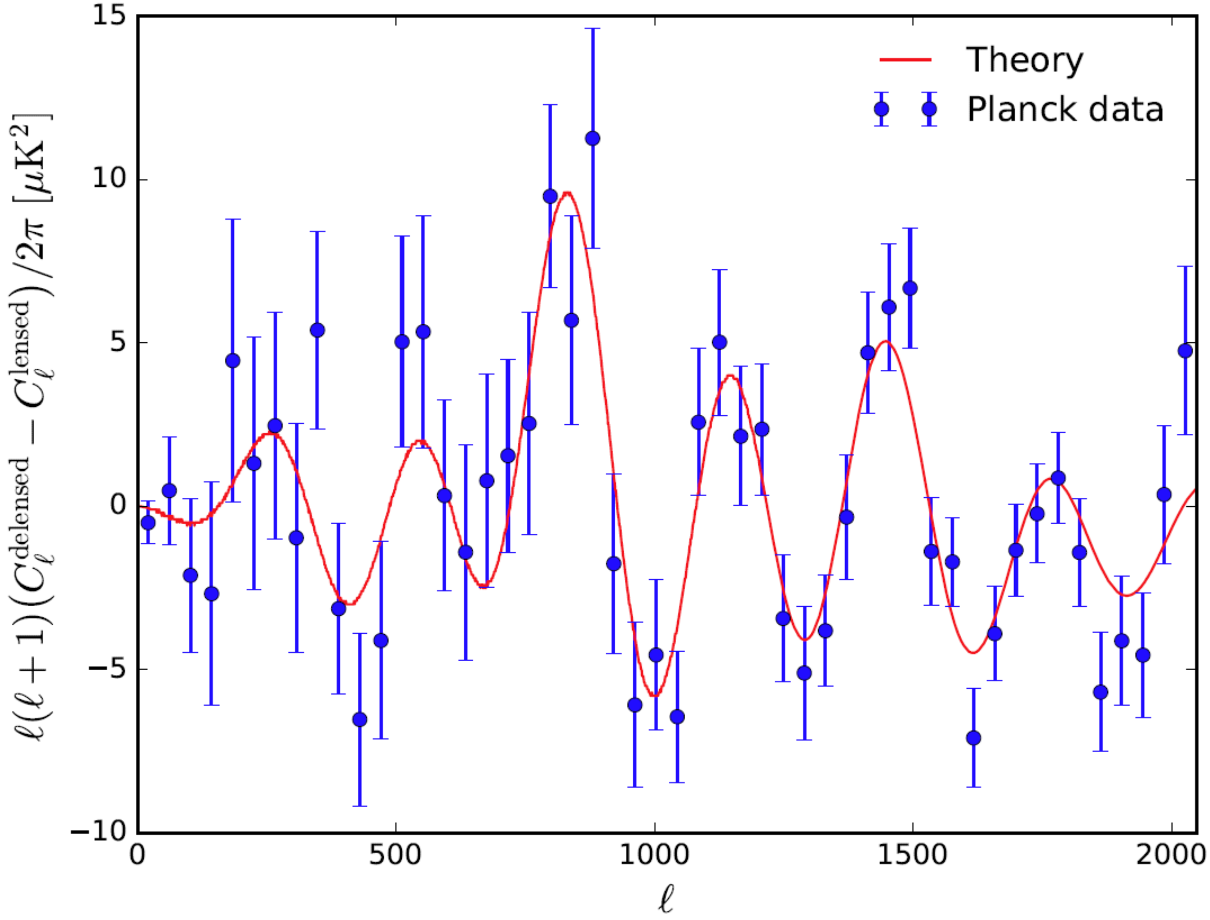


[1502.05356](1502.05356)

CIB provides an independent, quite high S/N probe of ϕ - good for delensing

CIB can already by used for delensing

Detection of delensing of TT power spectrum:
Partly undo peak-smoothing effect - peaks get sharper

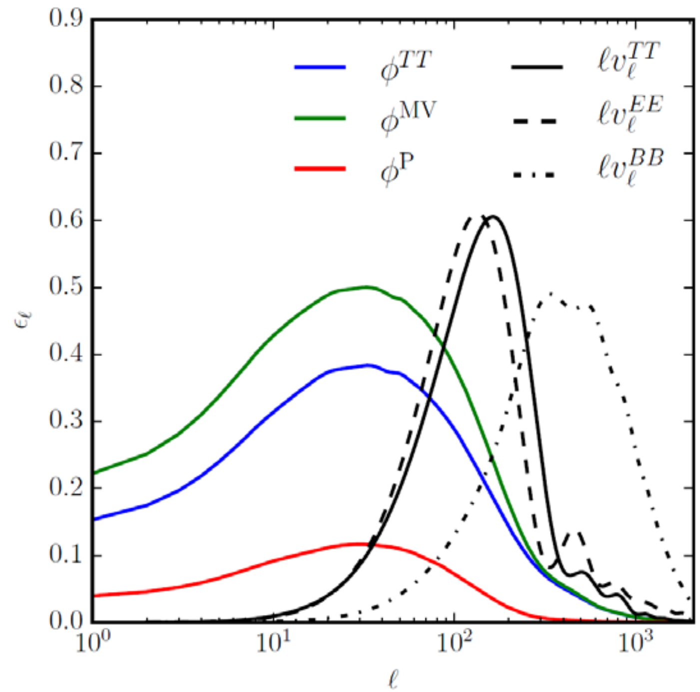


Larsen et al. 1607.05733

Internal delensing: ϕ from lensing reconstruction

Current (Planck) delensing efficiencies currently limited by reconstruction noise

$\epsilon_\ell = 1 - C^{\phi\phi}_{\ell,\mathrm{del}}/C^{\phi\phi}_\ell$

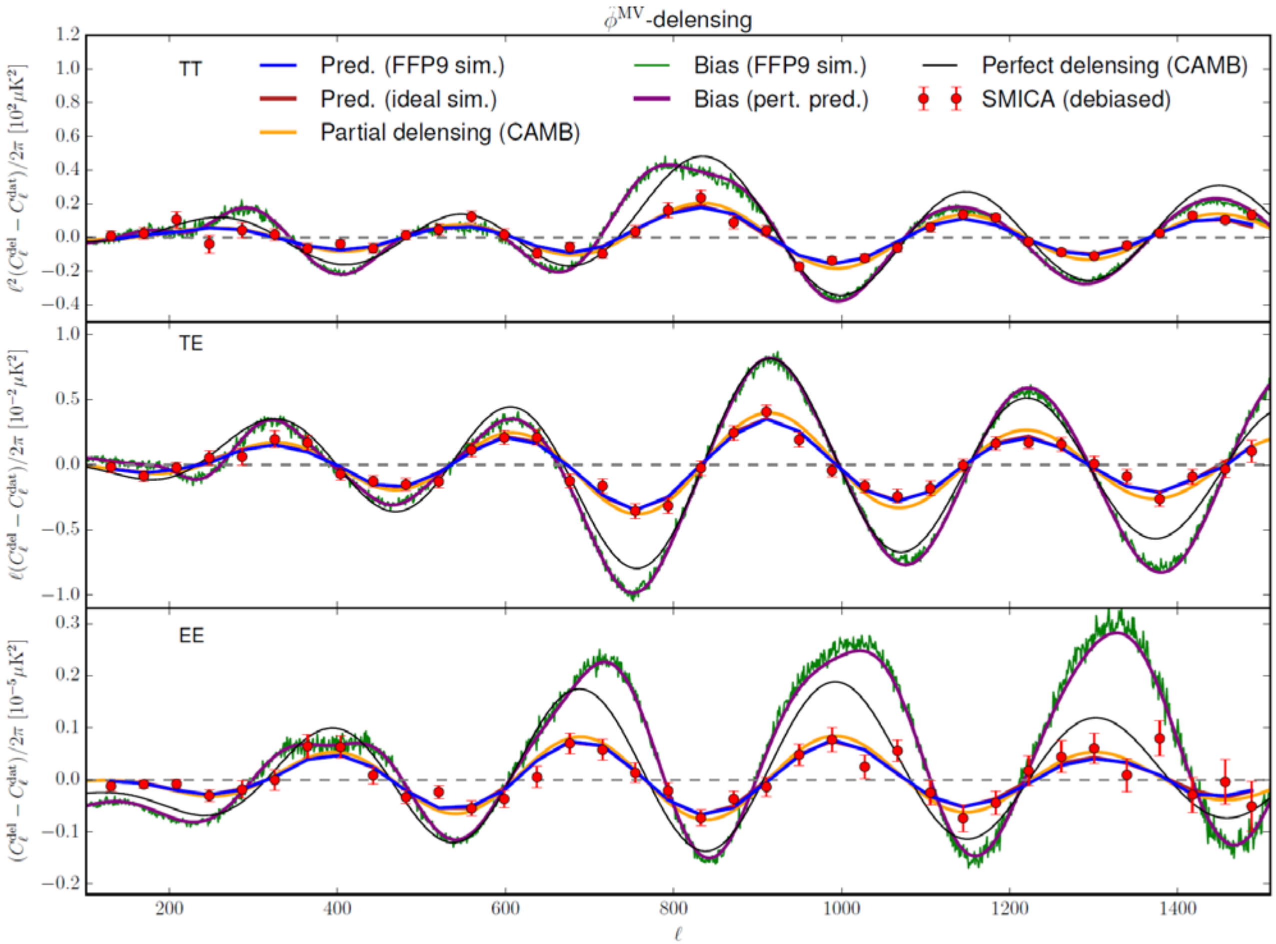


Carron, Lewis, Challinor arXiv:1701.01712

(But in future will be much better)

Planck internal delensing of TT, TE, EE

Carron, Lewis, Challinor arXiv:1701.01712



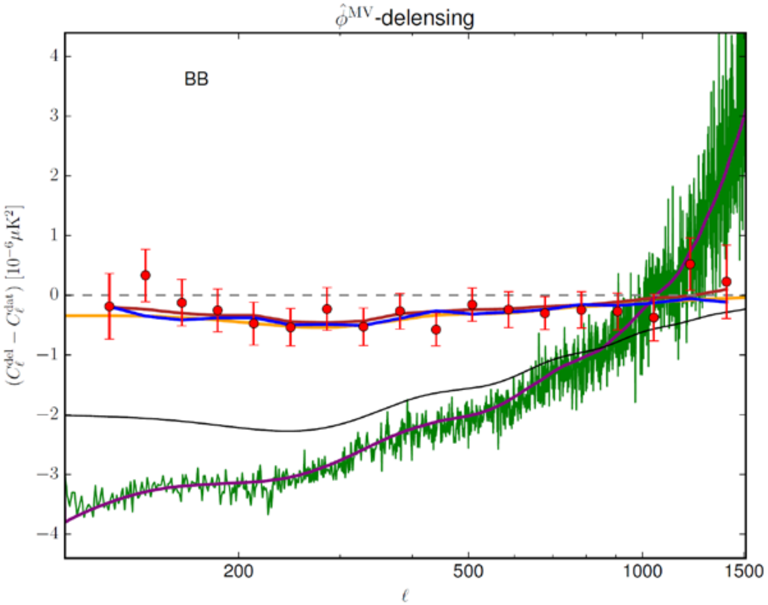
$\sim 25\sigma$ detection of TT delensing, 20σ of polarization delensing; consistent with expectations

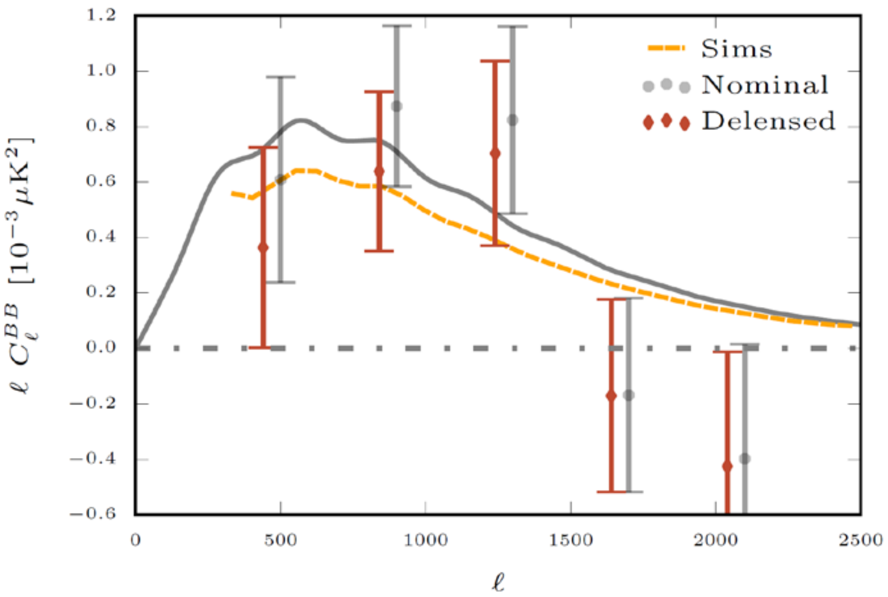
(S4 EE/TE delensing could sharpen peaks and decrease errors by up to ~ 20%, arXiv: **1609.08143)**

Current status of B-Mode Delensing

Planck internal
First detection of delensing of B-mode polarization at 4.5 σ

SPTpol and Herschel
28% delensing detected at 6.9σ





Carron, Lewis, Challinor arXiv:1701.01712

Manzotti et al. arXiv:1701.04396

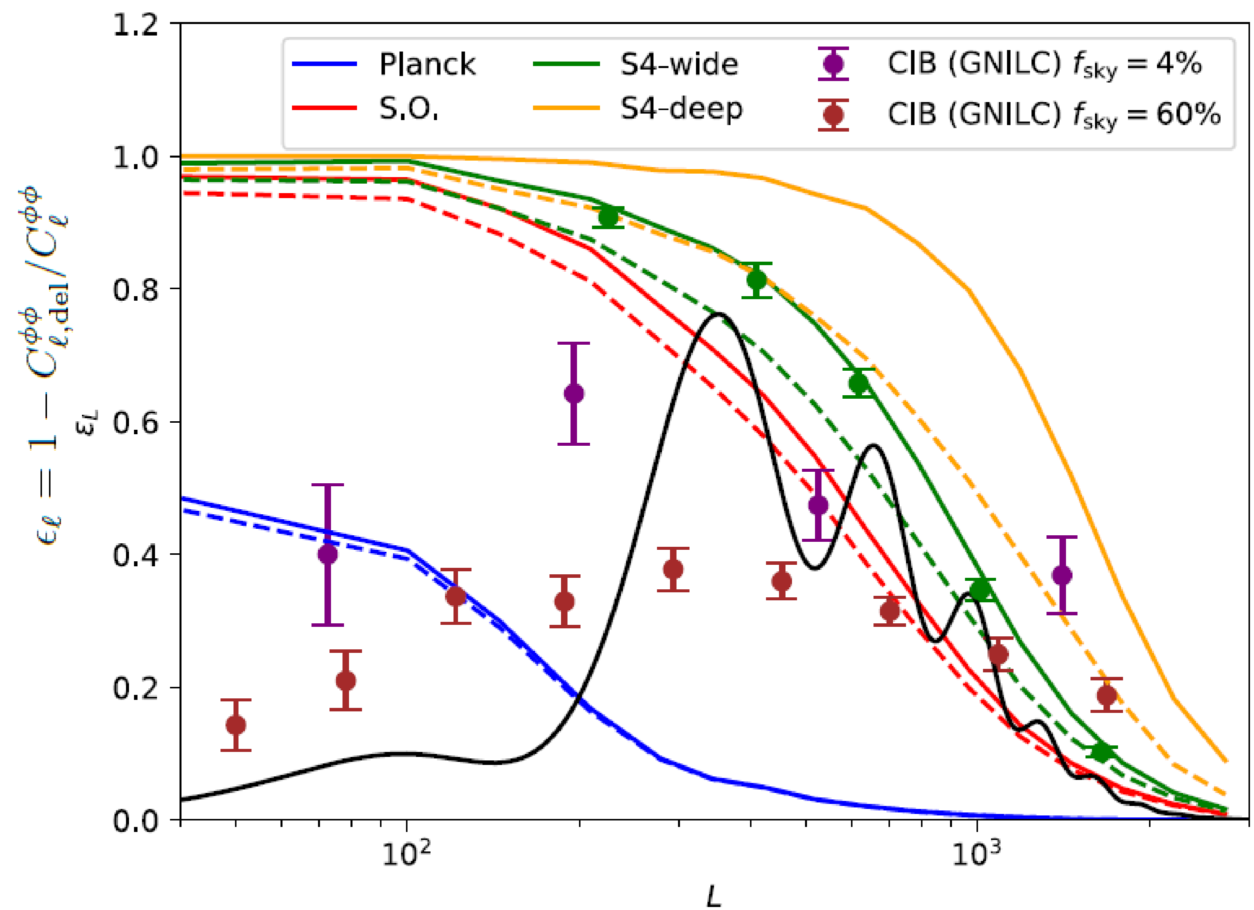
(*Proof of principle only*: noise high, so does not yet help at all with tensor r constraint)

Future lensing reconstruction efficiency

quadratic estimator (dashed)

Iterative estimator (solid)

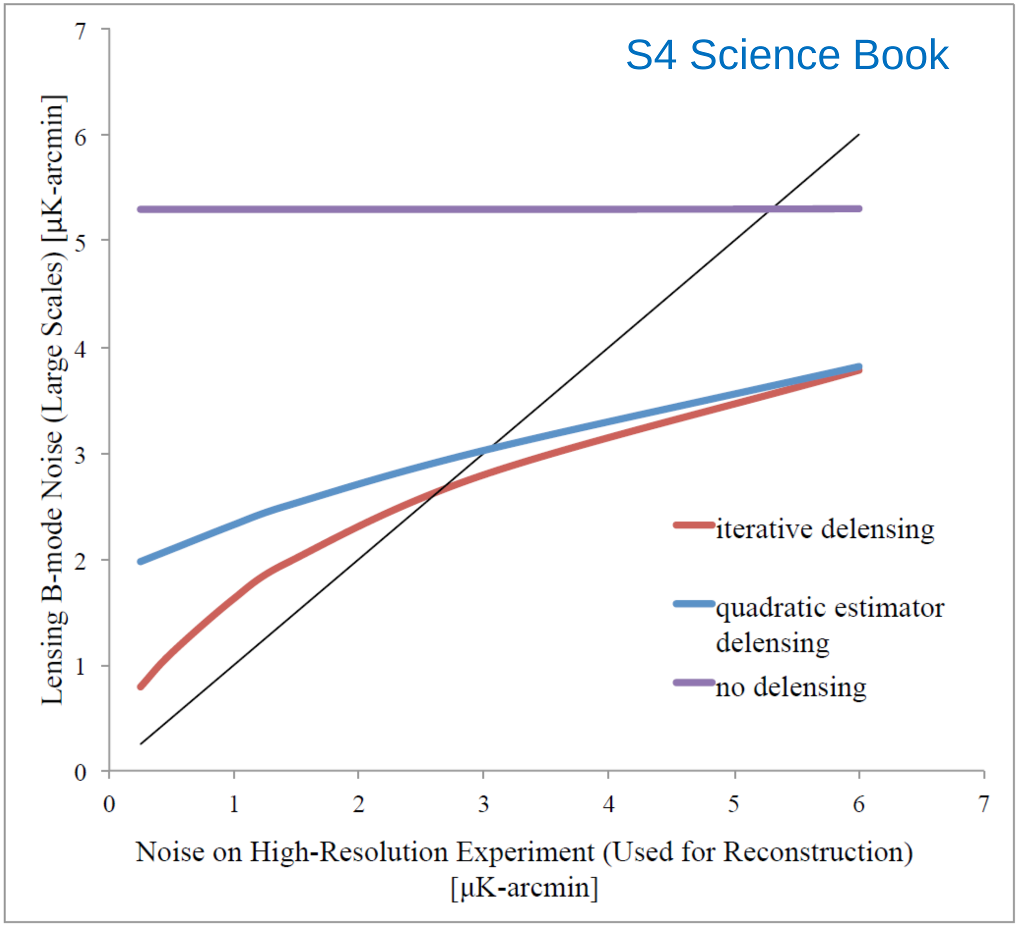
CIB not competitive for S4, though may help (reddish points)



Carron & Lewis, 1704.08230

Without delensing, r limited by lensing not instrumental noise smaller than $\sim 5\ \mu K$ arcmin

Future high-resolution data could remove most of large-scale B-mode “noise” on r



How well can we delens in principle?

Standard lensing remapping approximation:

$$\tilde{P}_{ab}(\hat{\boldsymbol{n}}) = P_{ab}(\hat{\boldsymbol{n}} + \boldsymbol{\nabla}\phi)$$

Observed		Theory	
$\tilde{E}, \tilde{B}$	$\xleftarrow{\text{lensing}}$ $\xrightarrow[\text{Delensing?}]{}$	E, ϕ	(almost all modes small-scale, unlensed $B = 0$)
2 d.o.f.		2 d.o.f.	

<u>Perfect lensing reconstruction, hence delensing, if only 2 d.o.f.</u>

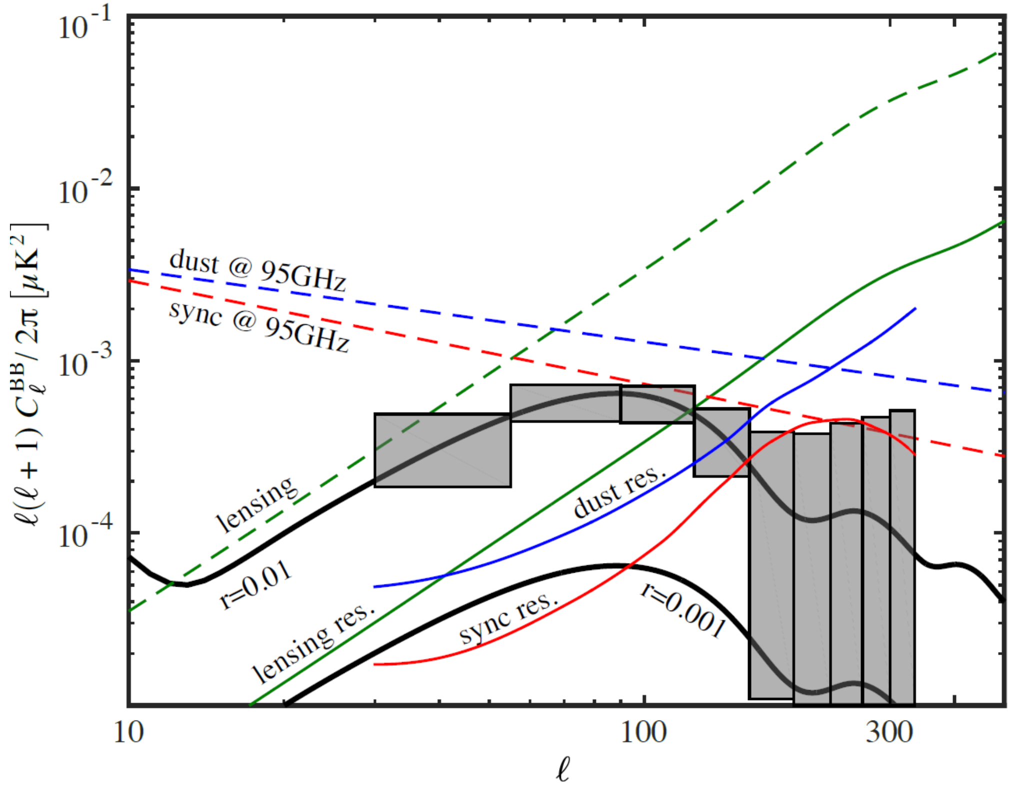
Hirata & Seljak 2003

Breaks down eventually even in principle:
Second order effects from reionization ($\mathrm{r} \sim 10^{-4}$, **1709.01395)**, time-delay/emission angle ($\mathrm{r} \sim 10^{-6}$), non-linear recombination ($\mathrm{r} \sim 10^{-7}$), …

Optimal internal reconstruction in principle only limited by noise down to $r \sim 10^{-4} - 10^{-6}$

But: residual foregrounds in B may limit nearer $r \sim 10^{-3}$ for planned observations

e.g. S4 Science Book

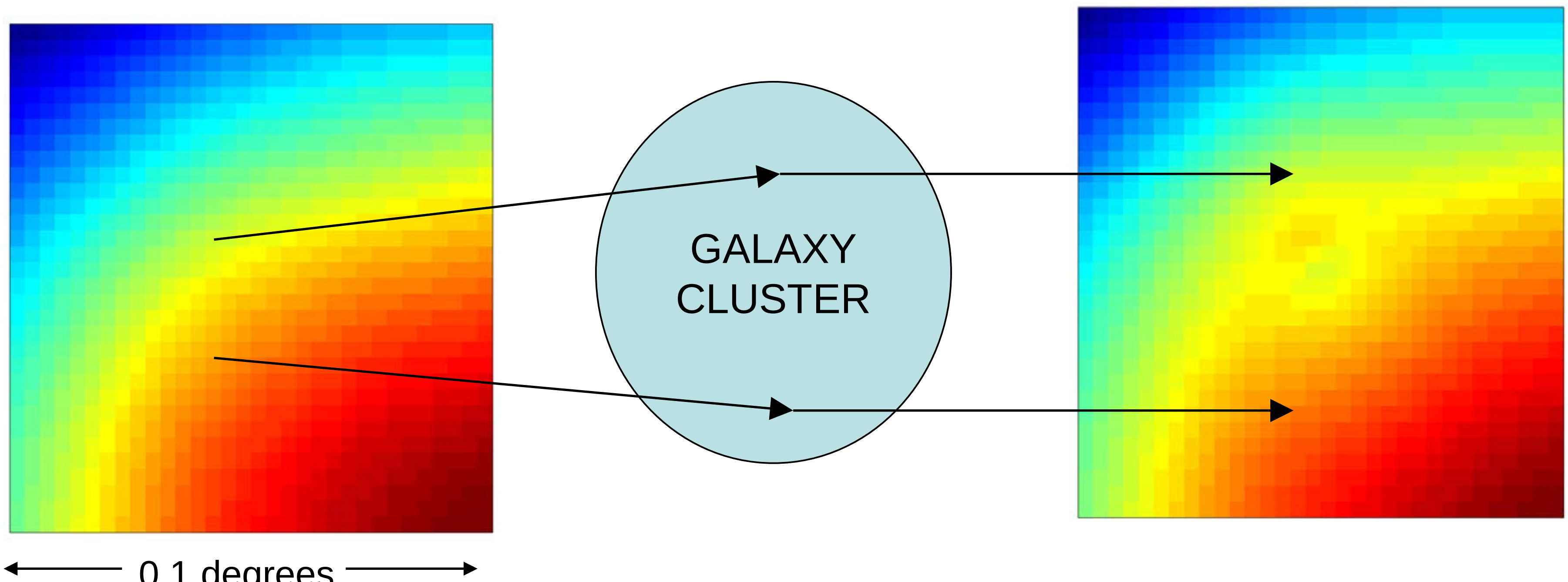


Cluster CMB lensing

CMB very smooth on small scales: approximately a gradient

Last scattering surface

What we see

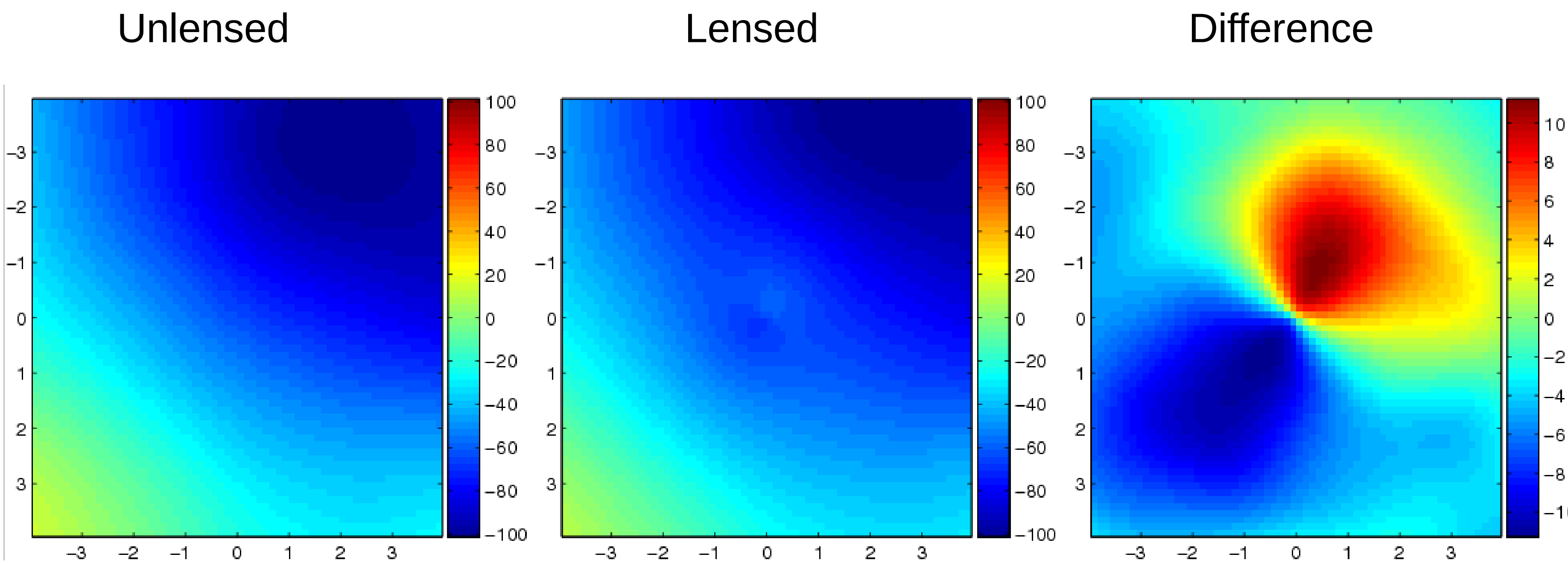


Need sensitive ~ arcminute resolution observations

RMS gradient ~ 13 μK / arcmin
deflection from cluster ~ 1 arcmin

Lensing signal ~ 10 μK

BUT: depends on CMB gradient behind a given cluster

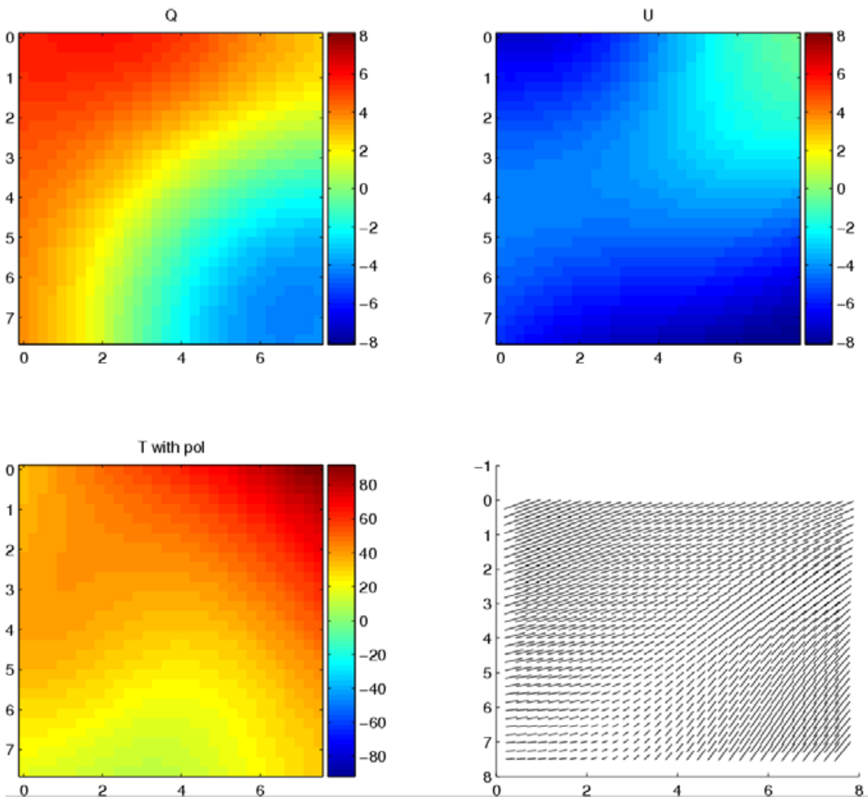


Unlensed CMB unknown, but statistics well understood (background CMB Gaussian) :

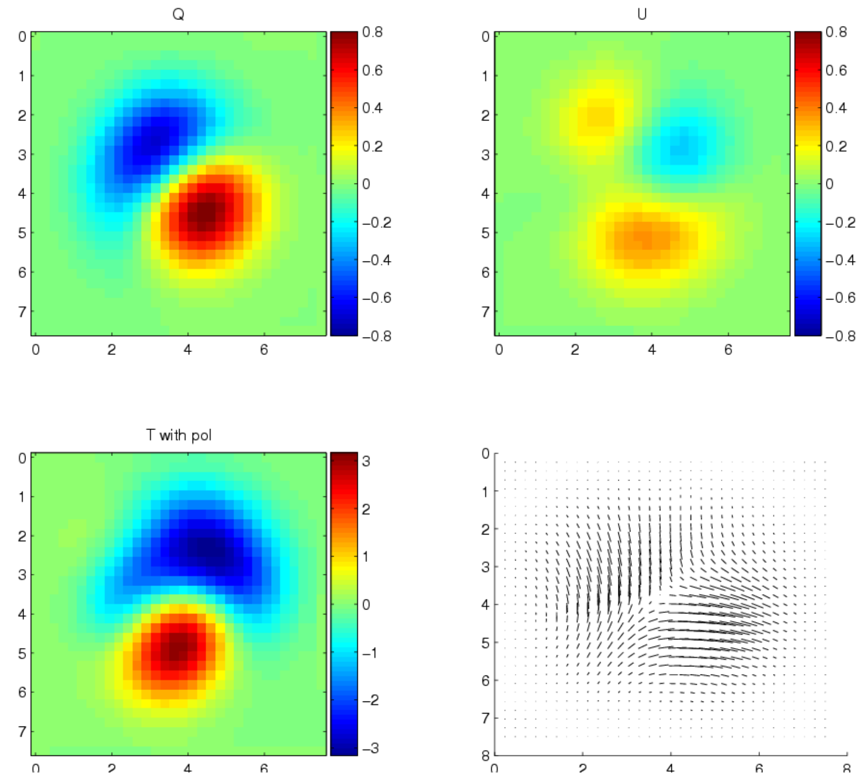
can compute likelihood of given lens (e.g. NFW parameters) essentially exactly

Add polarization observations?

Unlensed T+Q+U



Difference after cluster lensing



Less sample variance – but signal ~10x smaller: need 10x lower noise

Note: E and B equally useful on these scales; gradient could be either e.g. high-sensitivity, high-resolution CMB can calibrate mass of 1000 stacked clusters to a few percent

S4 Science Book

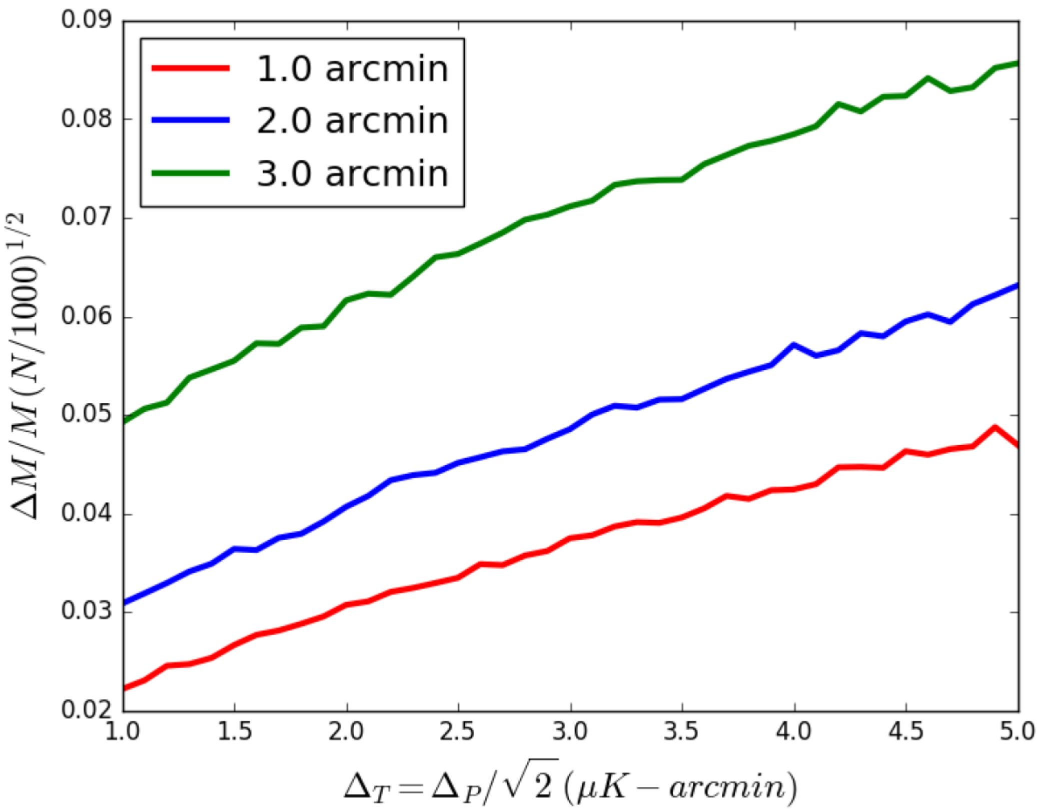


Figure 53. *Mass uncertainty from CMB halo lensing measurements stacking* 10^3 *halos of mass* $M_{180\rho_{m0}} \approx 5 \times 10^{14} M_\odot$, *as a function of instrumental noise and varying instrumental resolution.*

Complications

- Temperature

  - Thermal SZ, dust, etc. (frequency subtractable)
  - Kinetic SZ (cluster rotation, can average in stacking)
  - Moving lens effect (velocity Rees-Sciama, dipole-like)
  - Background Doppler signals
  - Other lenses

- Polarization

  - Quadrupole scattering
  ($< 0.1\mu K$)
  - Re-scattered thermal SZ (freq)
  - Kinetic SZ (higher order)
  - Other lenses

  Generally much cleaner

But usually galaxy lensing does much better, esp. for low redshift clusters

Conclusions

Lensing the leading secondary effect on the CMB anisotropies
- *Smooths acoustic peaks*
- *Transfers power to small scales*
- *Introduces non-Gaussianity*
- *Makes B-mode polarization by lensing E*

Can be used to reconstruct the lensing potential
- *map integrated density of universe on largest scales*
- *very different systematics to galaxy lensing*
  (cross-correlation can be used for calibration)

Test LCDM, constrain parameters, $\sum m_\nu$, dark energy, bias, etc.

Delensing important for future tensor mode searches if r small
- just starting to be possible