

# Squeezing the bispectrum up to the non-linear regime



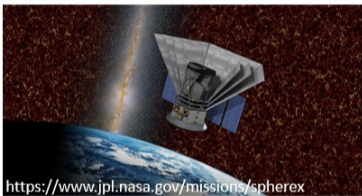
Lina Julieth Castiblanco Tolosa



Future Cosmology  
April 2023



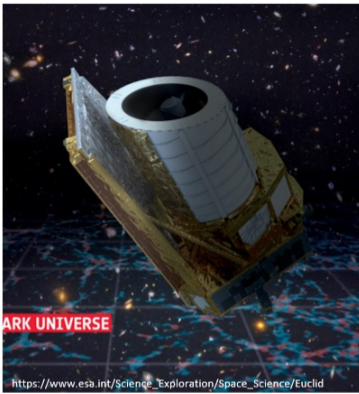
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<https://www.physics.ox.ac.uk/research/group/square-kilometre-array-ska>

**We are in the era of  
precision cosmology**

<https://mapoftheuniverse.net/>

**Extracting the maximum information  
from the observations is a challenge!**

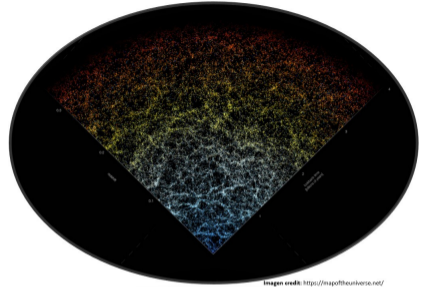
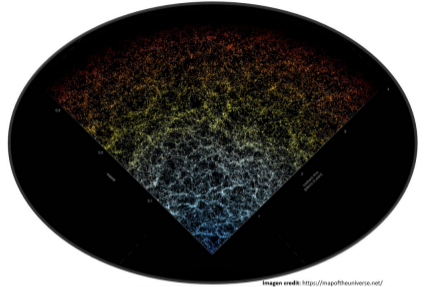


Image credit: <http://insightfuluniverse.net/>

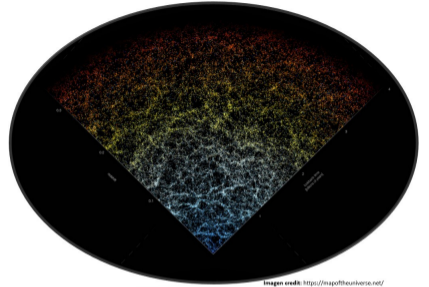
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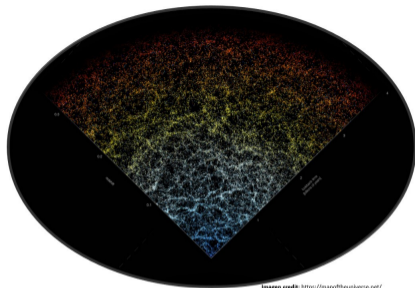
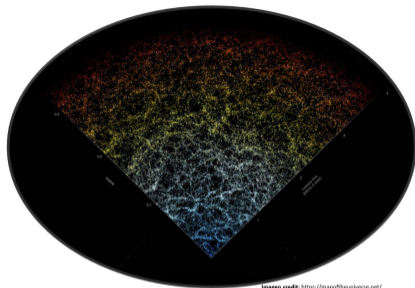


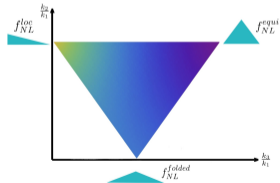
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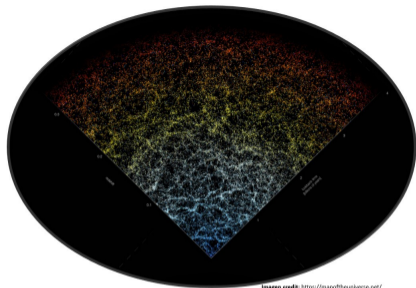
Squeezed bispectrum model valid at small scales.



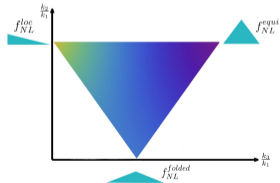
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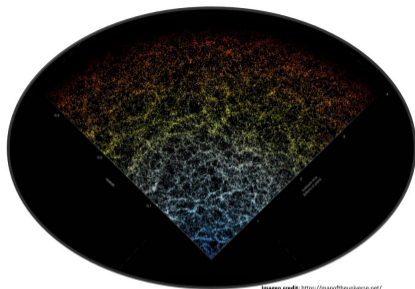
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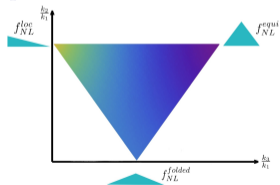


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Response functions approach.

# Large-scale structure consistency relations

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$$\langle \delta_{\mathbf{q}} \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_n} \rangle'_{q \rightarrow 0} = -P_{\delta_L}(q, \tau) \sum_{a=1}^n \frac{D_+(\tau_a)}{D_+(\tau)} \frac{\mathbf{q} \cdot \mathbf{k}_a}{q^2} \langle \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_n} \rangle'$$

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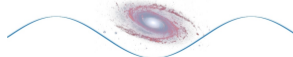
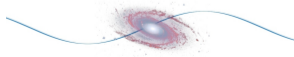
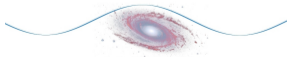
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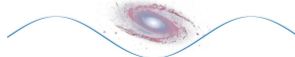
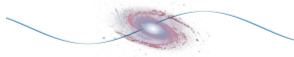
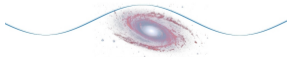
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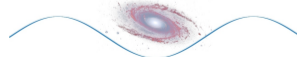
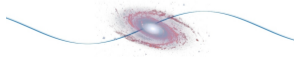
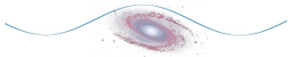
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*Are valid at any order in perturbations theory.*





# Power spectrum response function

A Barreira, F Schmidt ( 2017)

P Valageas (2013)

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*Effect of long wavelength perturbations on small scales  $n$ -point correlation functions.*

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- "Bias-like" expansion in terms of local gravitational observables.

$$\frac{P_m(\mathbf{k}|\mathbf{x})}{P_m(k)} - 1 = \sum_{\mathcal{O}} R_{\mathcal{O}}(k) \left( \hat{k}^i \hat{k}^j \dots \right) \mathcal{O}_{ij\dots}(\mathbf{x})$$

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## Response function expansion at the field level.

*Response of the matter density at small scales to the change of the long wavelength gravitational potential*

- ① Rotational invariance

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### Squeezed Bispectrum

$$\lim_{q \rightarrow 0} \langle \delta(\mathbf{q}) \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \rangle' = P_m(q) P_m(k_1) \left[ \frac{\mathbf{k}_1 \cdot \mathbf{q}}{q^2} + R_1(k_1) + R_\theta(k_1) (\hat{k}_1 \cdot \hat{q})^2 \right] + (1 \leftrightarrow 2)$$

$$R_i(k) = \frac{\langle \delta(\mathbf{k}) \Delta_i(-\mathbf{k}) \rangle'}{P_m(k)}$$

# Response coefficients from perturbation theory

M Biagetti, J Calles, L Castiblanco, K Gonzalez, J Noreña. (2022)

# Response coefficients from perturbation theory

## Tree level bispectrum

$$\delta(\mathbf{k})|_{\Phi_L} = \delta_\ell(\mathbf{k}) + 2F_2(\mathbf{q}, \mathbf{k} - \mathbf{q})\delta_\ell(\mathbf{q})\delta_\ell(\mathbf{k} - \mathbf{q}) + \mathcal{O}(\delta_\ell^3, q/k\delta_\ell(q)\delta_\ell(k))$$

$$\text{Isotropic response: } \Delta_1(\mathbf{k}) = \frac{10}{7}\delta_\ell(\mathbf{k})$$

$$\text{Angular response: } \Delta(\mathbf{k}) = \frac{4}{7}\delta_\ell(\mathbf{k})$$

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## One loop bispectrum

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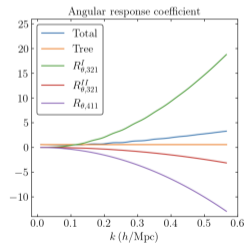
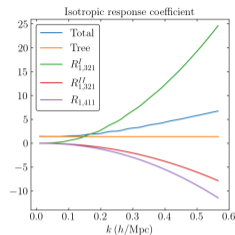
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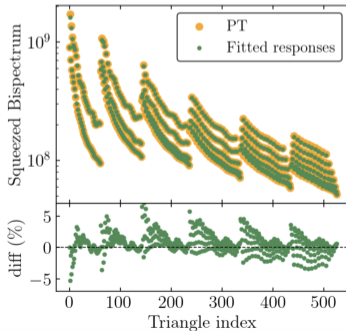
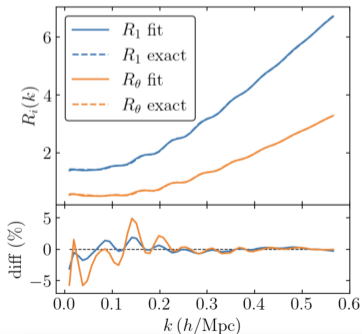


# Fitting function

$$R_1(k) = \frac{10}{7} + \sum_{n=1} S_n^1 k^n + \left( \sum_{m=0} O_m^1 k^m \right) P_{\text{nw}}(k) e^{-\Sigma^2 k^2} \sin(\omega k + \phi)$$

$$R_s(k) = \frac{4}{7} + \sum_{n=1} S_n^\theta k^n + \left( \sum_{m=0} O_m^\theta k^m \right) P_{\text{nw}}(k) e^{-\Sigma^2 k^2} \sin(\omega k + \phi)$$

Zvonimir Vlah, Uros Seljak, Man Yat Chu, Yu Feng (2016)



$$q \sim (0.009, 0.057) \text{h/Mpc}$$

$$k \sim (0.094, 0.565) \text{h/Mpc}$$

## Comparison with simulations **in the deep non-linear regime**

- $\ln \mathcal{L} = -\frac{1}{2}(\mathbf{D} \cdot \mathbf{C}^{-1} \cdot \mathbf{D})$

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- $\ln \mathcal{L} = -\frac{1}{2}(\mathbf{D} \cdot \mathbf{C}^{-1} \cdot \mathbf{D})$
- $C_{ij}^B \simeq \frac{\delta_{ij}}{k_f^3 N_{tr}^i} P(q^i) P(k_1^i) P(k_2^i) + \frac{k_f^2}{N_q} B(q^i, k_1^i, k_2^i) B(q^j, k_1^j, k_2^j)$

M.Biagetti, L.C, J.Noreña, E. Sefusatti (2021)

## Comparison with simulations in the deep non-linear regime

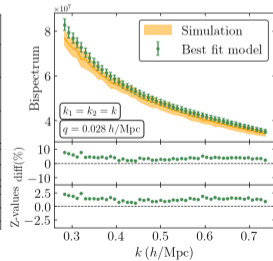
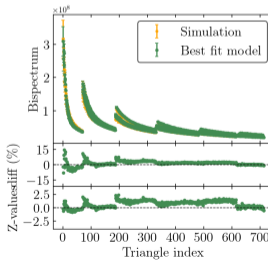
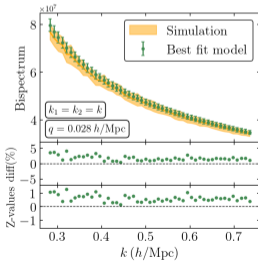
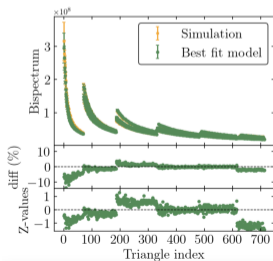
- $\ln \mathcal{L} = -\frac{1}{2}(\mathbf{D} \cdot \mathbf{C}^{-1} \cdot \mathbf{D})$
- $C_{ij}^B \simeq \frac{\delta_{ij}}{k_f^3 N_{tr}^i} P(q^i) P(k_1^i) P(k_2^i) + \frac{k_f^2}{N_q} B(q^i, k_1^i, k_2^i) B(q^j, k_1^j, k_2^j)$   
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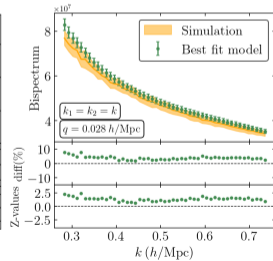
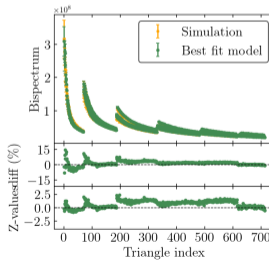
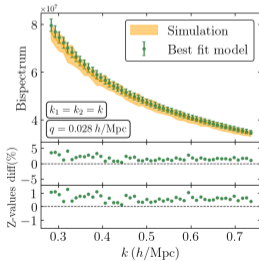
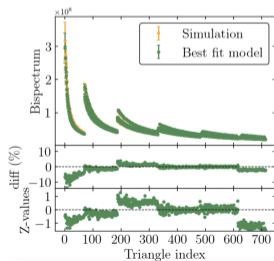
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The consistency relation improves the fitting!

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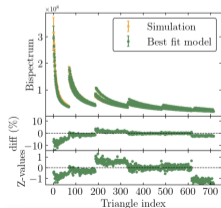
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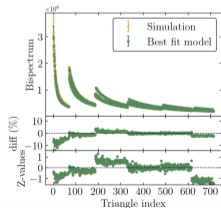


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- The model can be extended to describe the squeezed galaxy bispectrum in redshift space.

*Thank  
you*

