Extension of Evolution Mapping to Velocity Statistics

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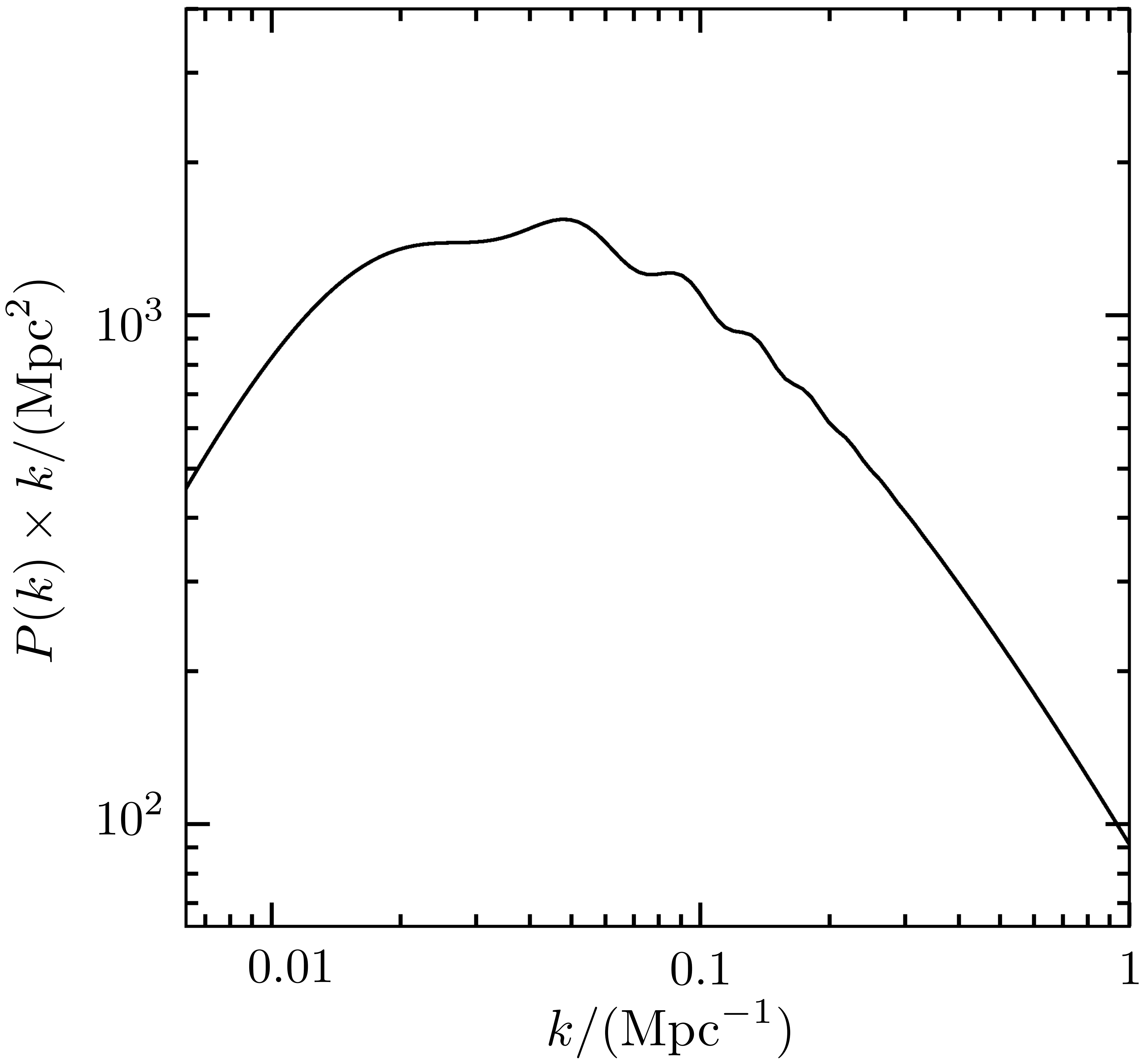
Evolution mapping: linear $P(k)$

We can classify cosmological parameters according to their impact on $P(k)$



$$P_L(k,z) = P(k|\underline{A_s}, n_s) \times T(k) \times \underline{D(z)}$$

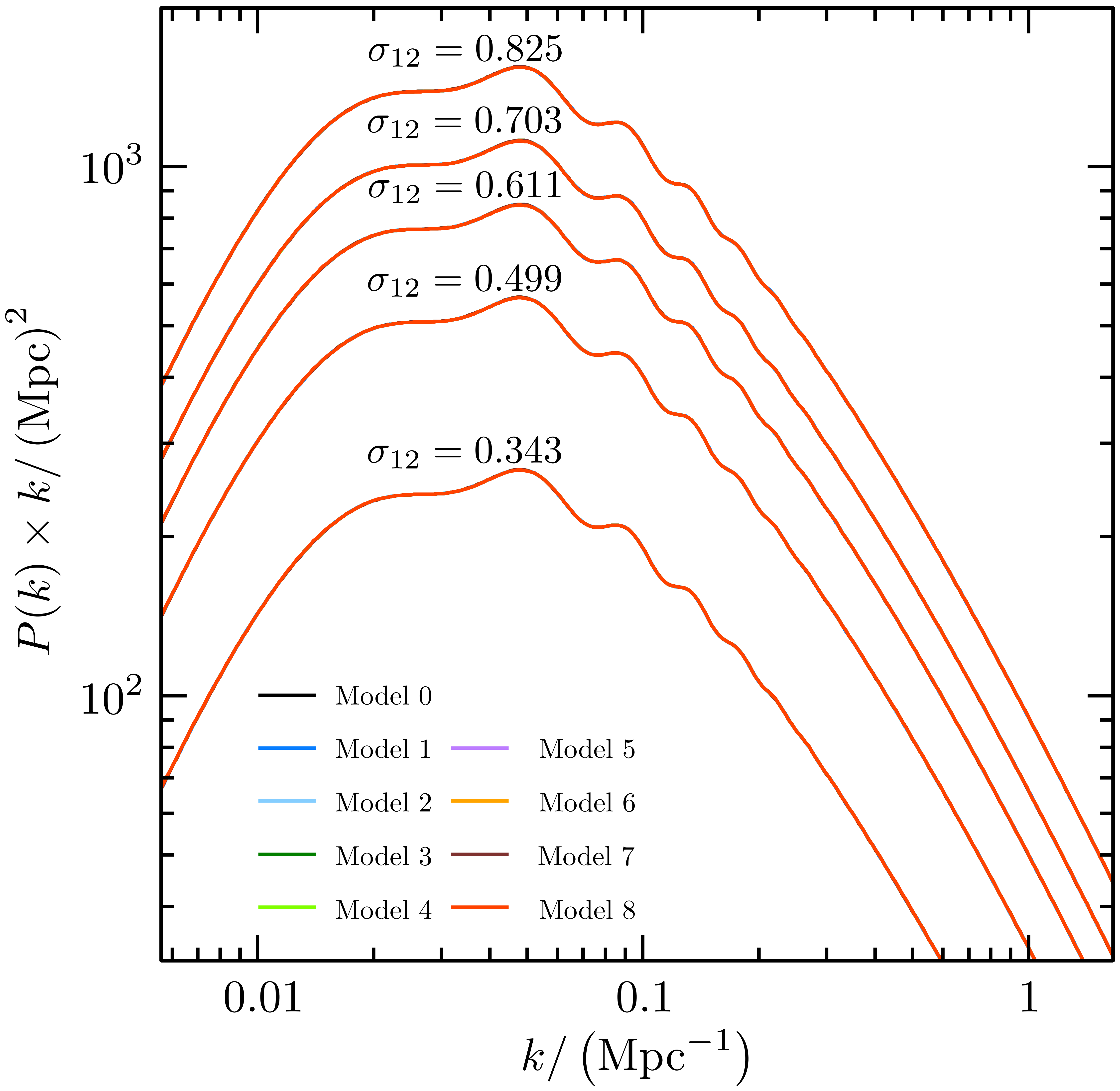
Cosmological parameters

Θ_s: ω_c ω_b n_s

$\underline{\Theta_e}$: ω_K A_s ω_{DE} $\omega_{\mathrm{DE}}(a)$

Sánchez (2020)

Evolution mapping: linear $P(k)$

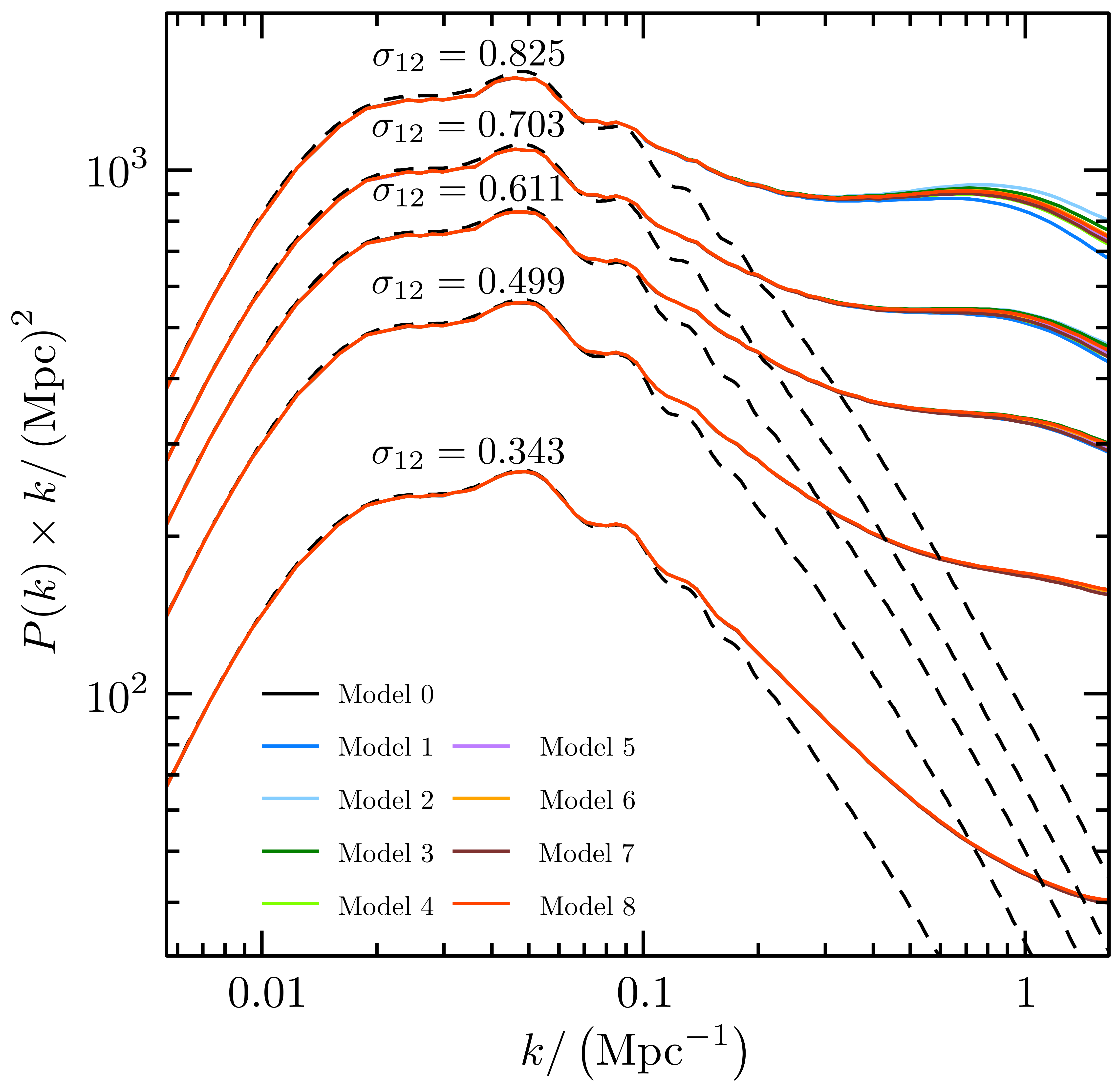


$$P_L(k|z, \Theta_s, \Theta_e) = P_L(k|\Theta_s, \sigma_{12}(z, \Theta_s, \Theta_e))$$

- Impact of evolution parameters on $P(k)$ can be fully described by $\sigma_{12}(z)$, the rms linear variance of the density field in spheres with R = 12 Mpc.
- Lost when using quantities that explicitly depend on h (σ_8, Ω_i).

Sánchez (2020); Sánchez et al. (2022)

Evolution mapping: non-linear $P(k)$



- Given Θ_s, all Θ_e and z leading to the same σ_{12} give indistinguishable $P_{\mathrm{L}}(k)$.
- Evolution mapping is a good approximation also for the $P_{\mathrm{NL}}(k)$.
- Deviations are larger at high k and increase with σ_{12}.

Sánchez (2020); Sánchez et al. (2022)

Evolution mapping: velocity field

- Extend evolution mapping to the statistics of the velocity field which are an essential ingredient for the modelling of the redshift space power spectrum.

$$\vec{v} = a\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = \boxed{\frac{\mathrm{d}\vec{x}}{\mathrm{d}\ln\sigma_{12}}}\,\boxed{a\frac{\mathrm{d}\ln\sigma_{12}}{\mathrm{d}t}}$$

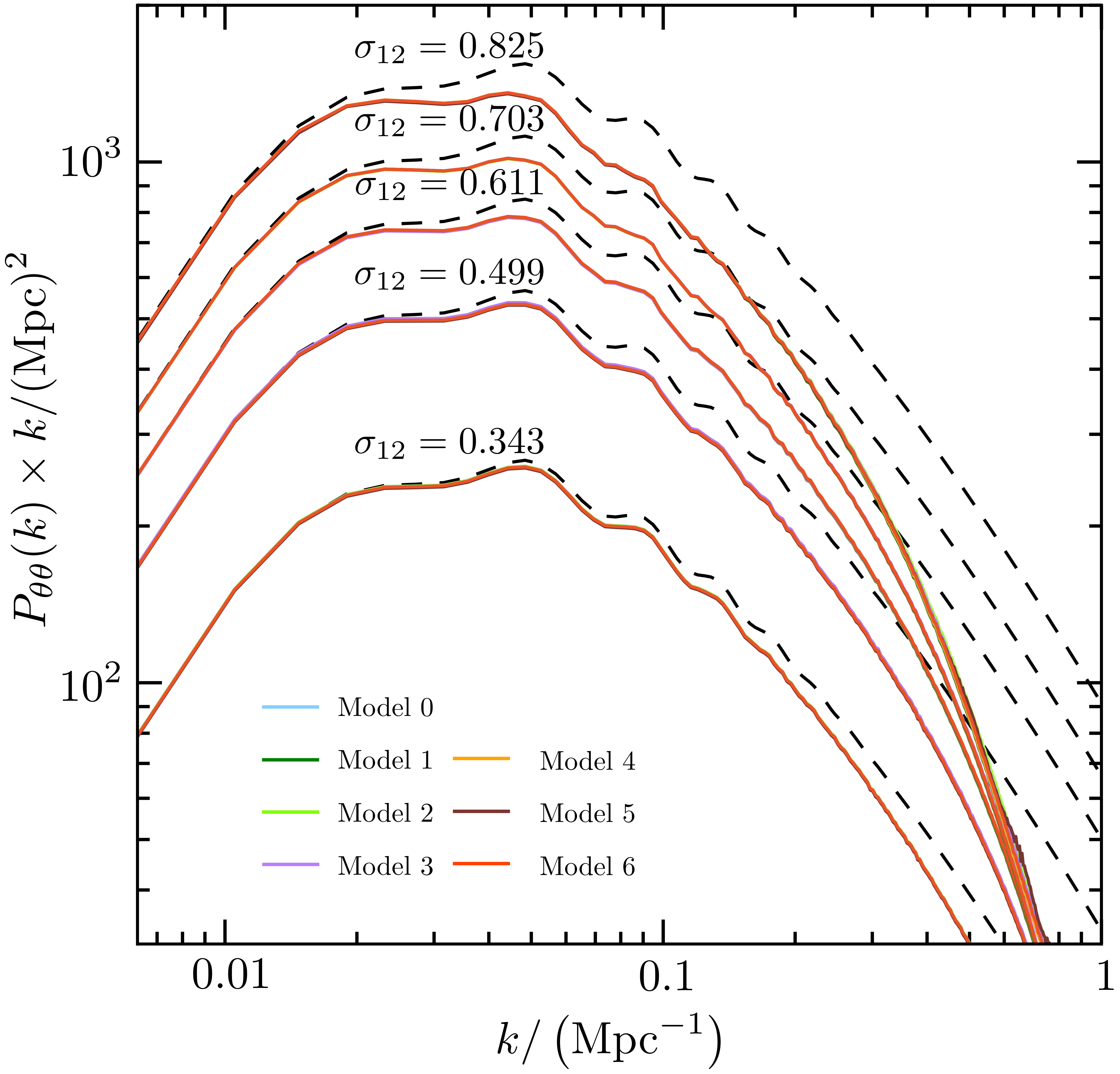
$$a\frac{\mathrm{d}\ln\sigma_{12}}{\mathrm{d}t} = faH$$

$$\theta \equiv \frac{\nabla\cdot\vec{v}}{faH}$$

Trajectories of particles in terms of σ_{12} are almost identical in cosmologies that only differ in shape parameters.

Can be calculated from linear theory and used to map one cosmology to another.

Evolution mapping: velocity divergence $P(k)$



- Evolution mapping is a good approximation also for the $P_{\theta\theta}(k)$.
- Deviations on small scales are smaller than for the $P_{\delta\delta}(k)$.
- This approach can be applied in general to the full particle phase-space and thus to all kind of statistics.

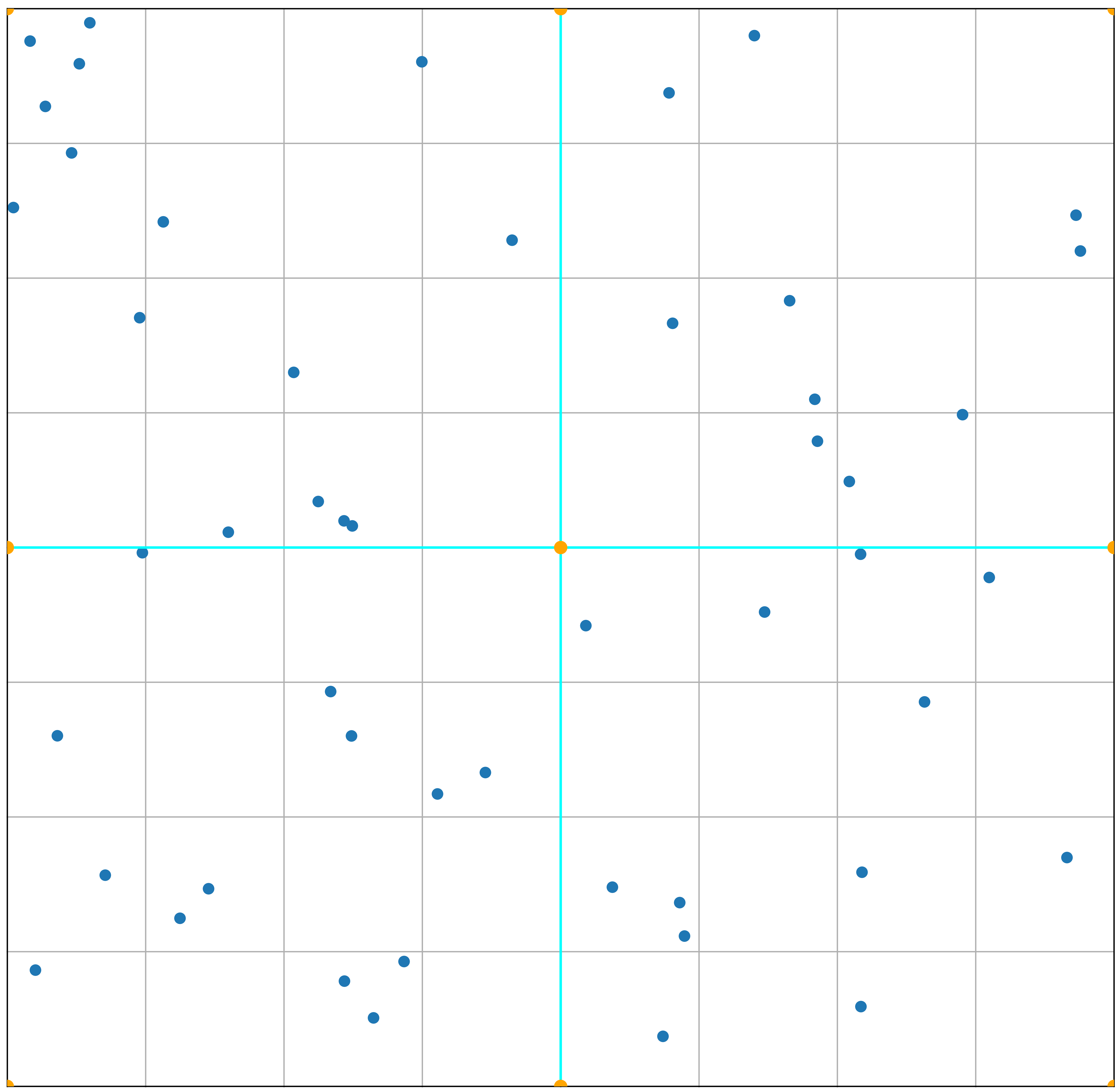

That's all Folks!

Backup slides

The algorithm

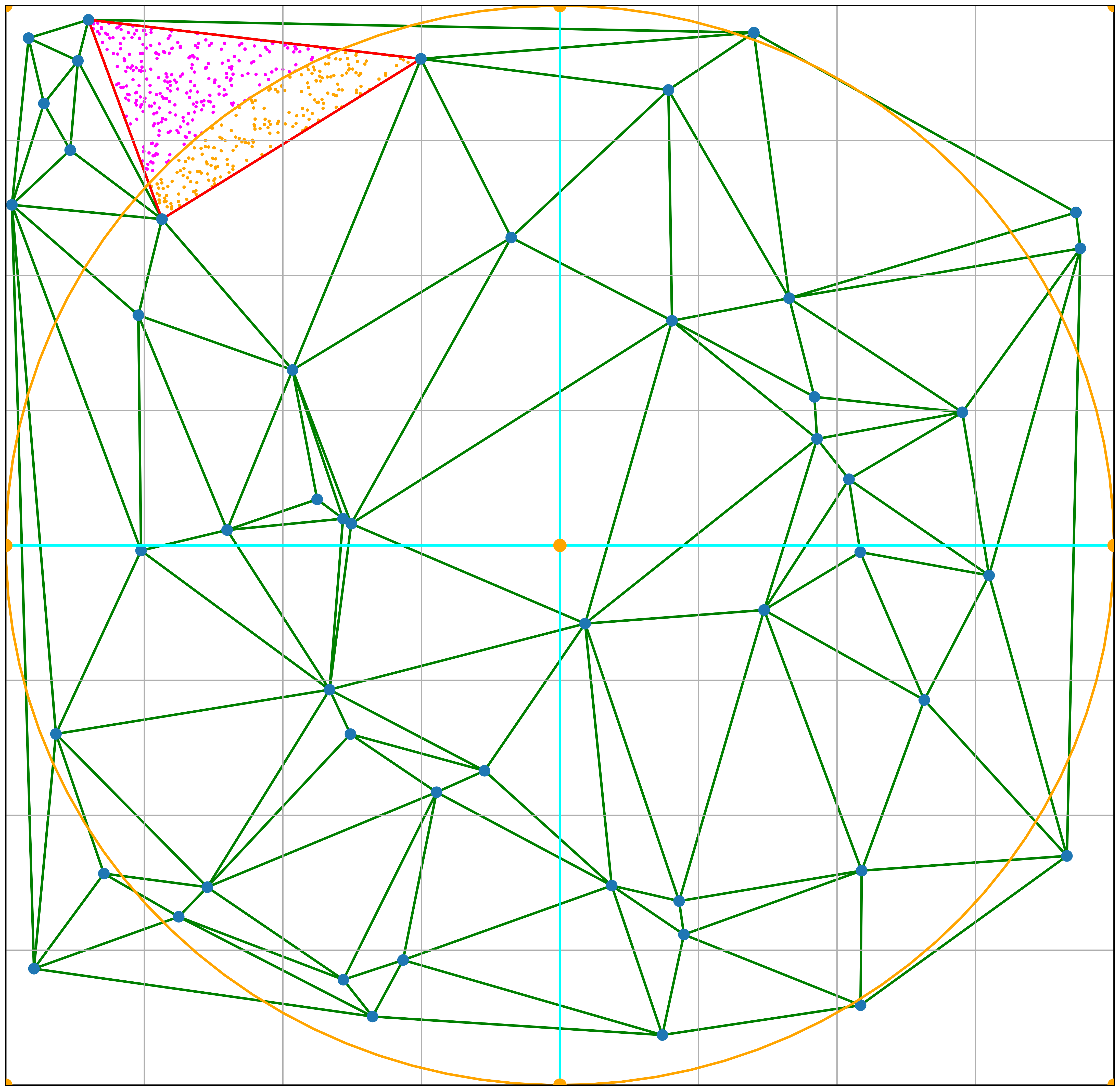
- Assign the particles to cubical cells (cyan lines)
- Loop over the grid points (orange dots):
  - Center a sphere on the grid point
  - Gather the particles in the cubical cells that have the center of the sphere as one of their vertices
  - Draw a Delaunay tessellation on these particles

- Loop over the tetrahedra (green lines) and check if they are:
  - **Fully outside the sphere:** skip the step;
  - **Fully inside the sphere**: calculate the mean velocity interpolated inside the tetrahedron and assign it its volume as a weight
  - **Intersecting the sphere (red triangle):**

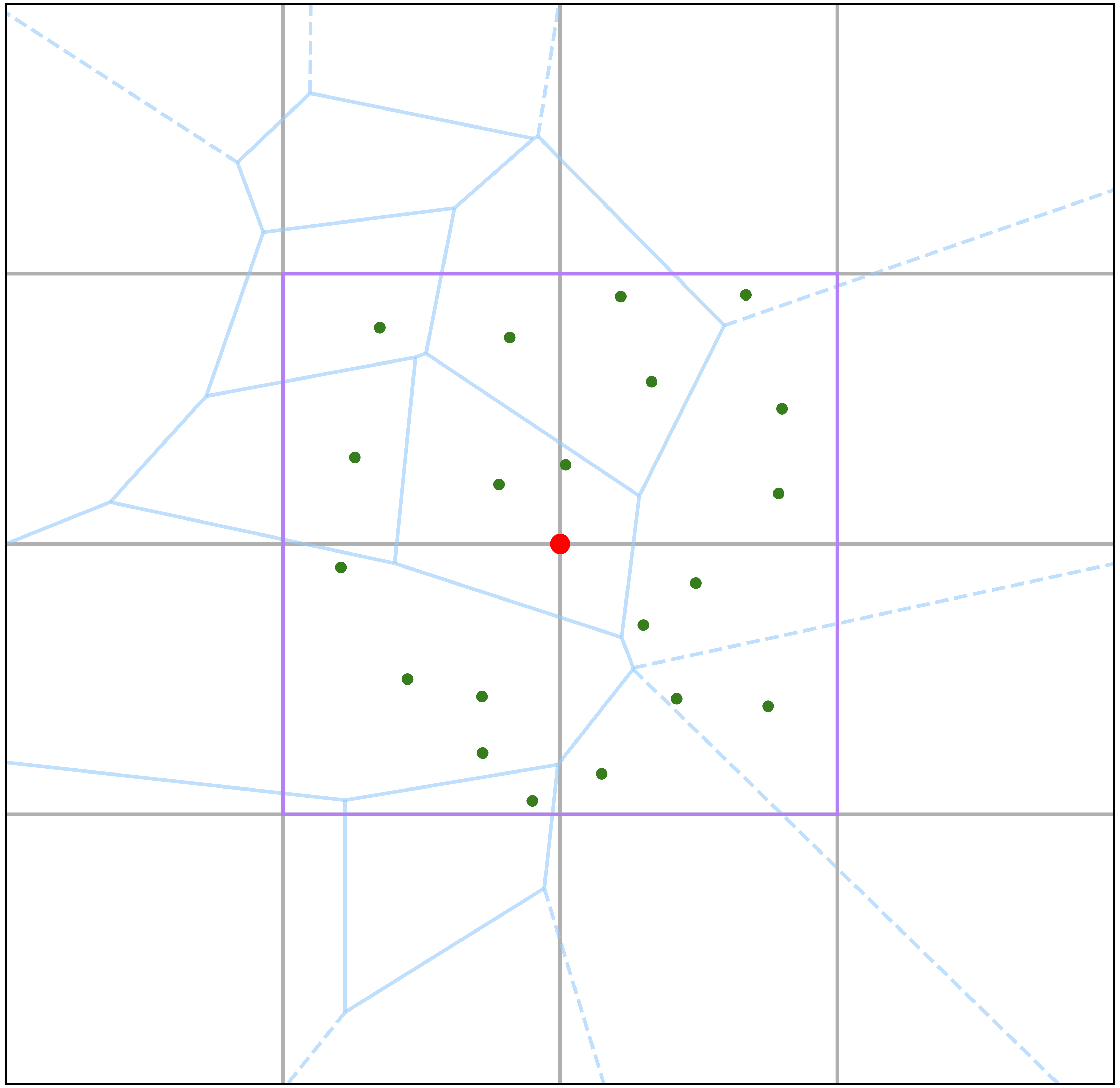


The algorithm

- **Intersecting the sphere (red triangle):**
  - Draw random points inside the tetrahedra
  - Linearly interpolate the velocity for those points that fall inside the sphere (orange dots)
  - Use those points to calculate the average velocity and the volume of the intersection with MC



A simpler approximation



- The Delaunay approach assumes a constant gradient of the field inside each tetrahedron.
- The simplest approximation is to assume the field to be constant inside each Voronoi cell.
- We can use a glass of random points to sample the field in this approximation.
- Faster and comparable results.

Comparison to fitting functions

- Bel et al. (2019) proposed a set of N-body calibrated parametric equations for $P_{\theta\theta}(k,z)$ and $P_{\delta\theta}(k,z)$.

$$P_{\theta\theta}(k) = P^{\mathrm{L}}_{\theta\theta}(k)\, \mathrm{e}^{-k(a_1+a_2k+a_3k^2)}$$

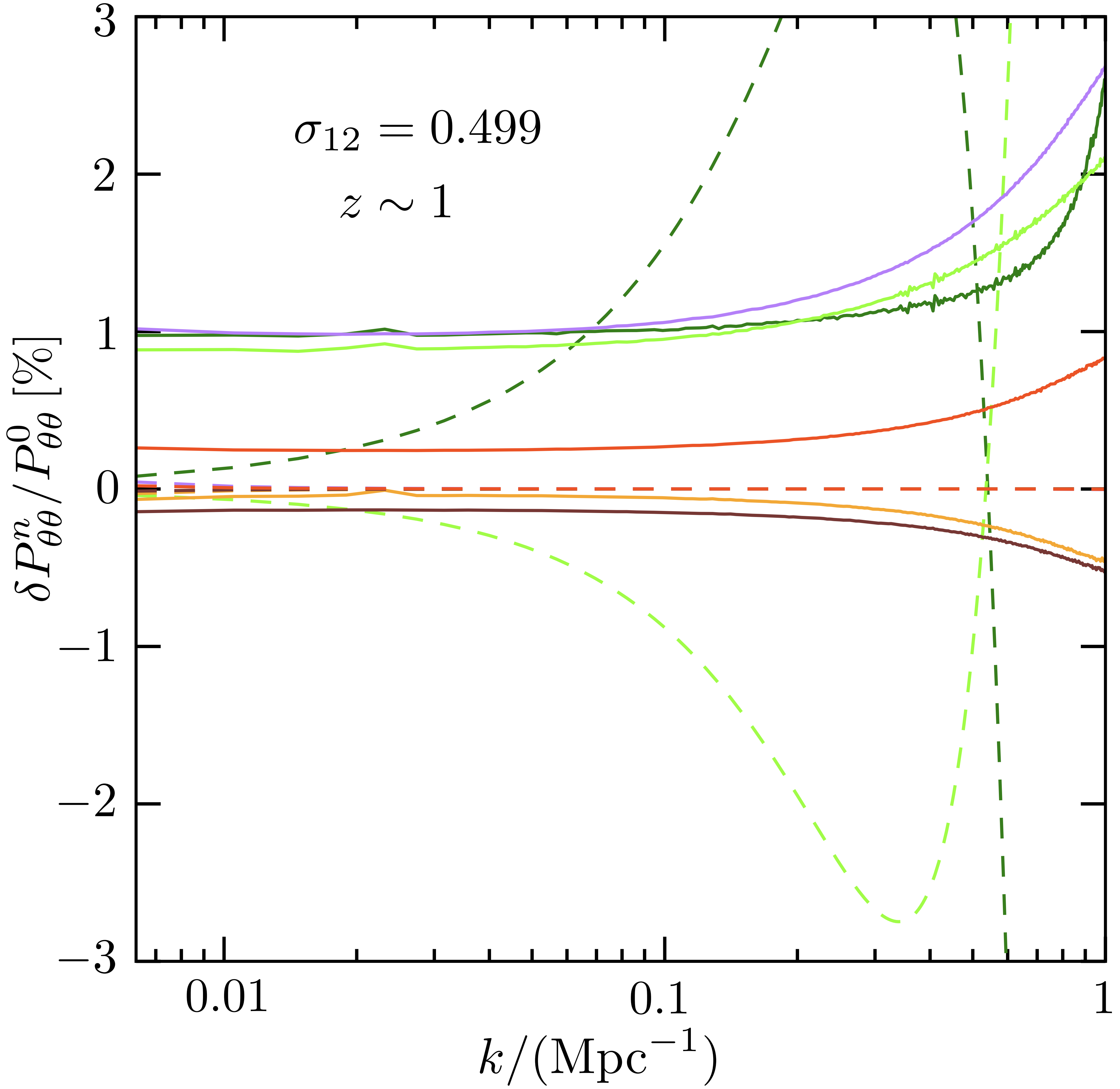
$$P_{\delta\theta}(k) = \left[P^{\mathrm{NL}}_{\delta\delta}(k)\, P^{\mathrm{L}}_{\theta\theta}(k)\right]^{\frac{1}{2}} \mathrm{e}^{-\frac{k}{k_\delta}-bk^6}$$

$$\begin{cases} a_1 = -0.817 + 3.198\,\sigma_8(z) \\ a_2 = 0.877 - 4.191\,\sigma_8(z) \\ a_3 = -1.199 + 4.629\,\sigma_8(z) \\ b = 0.091 + 0.702\,\sigma_8^2(z) \\ 1/k_\delta = -0.048 + 1.917\,\sigma_8^2(z) \end{cases}$$

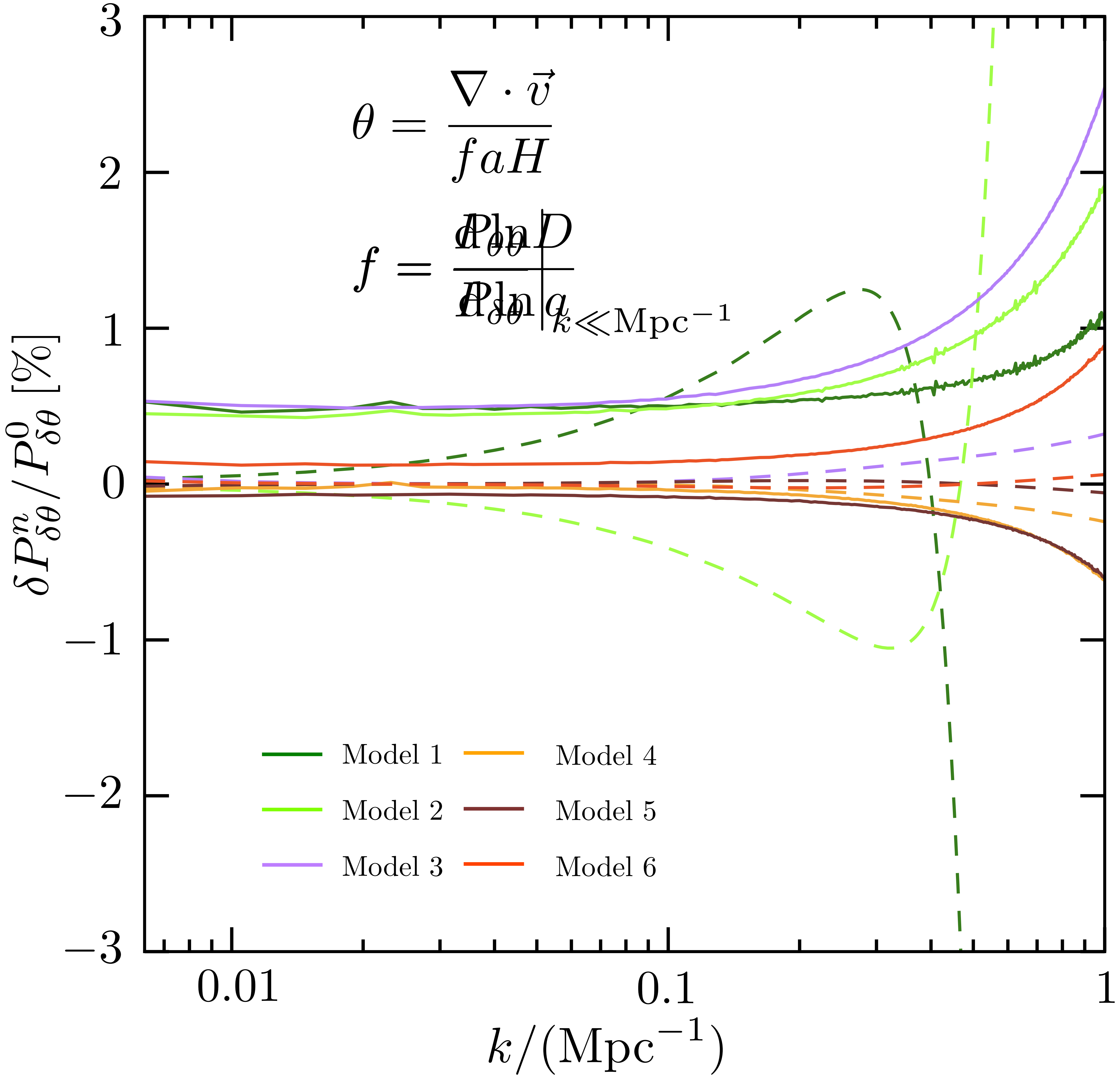
- Usage of Mpc$/h$ units in σ_8, introduces an implicit dependence on h, making the fitting functions unreliable when varying this parameter.

Comparison to fitting functions

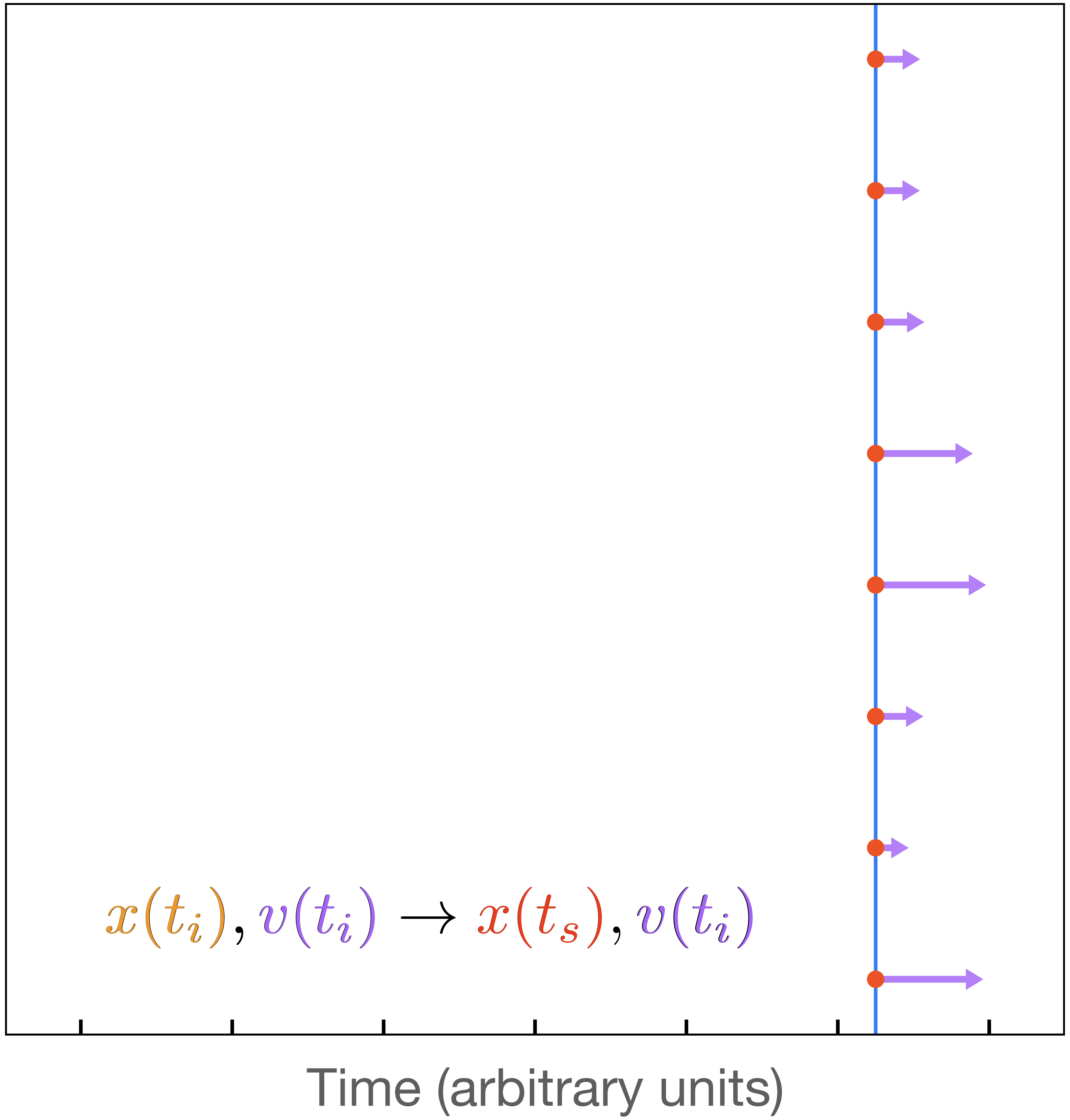
$$P_{\theta\theta}(k) = P^{\mathrm{L}}_{\theta\theta}(k)\,\mathrm{e}^{-k(a_1+a_2k+a_3k^2)}$$



$$P_{\delta\theta}(k) = \left[P^{\mathrm{NL}}_{\delta\delta}(k)\,P^{\mathrm{L}}_{\theta\theta}(k)\right]^{\frac{1}{2}}\,\mathrm{e}^{-\frac{k}{k_\delta}-bk^6}$$



A problem with velocities



- Gadget4 produces snapshots at the closest synchronisation point to the required z.
- An optional setting forces snapshots at a specific z at the cost of biased velocities.
- The particle positions get synchronised with a drift to the required redshift while velocities remain unchanged.
- This produces (at first order) an overall shift in the amplitude of the $P_{\theta\theta}(k, z)$.

Emulating the non-linear $P(k)$

Standard approach: $\boldsymbol{\Theta} = (\overbrace{\omega_{\rm c}, \omega_{\rm b}, n_{\rm s}}^{\text{shape}}, \overbrace{\omega_K, \omega_{\rm DE}, w_0, w_a, \ldots}^{\text{evolution}}, z)$

Evolution mapping: $\boldsymbol{\Theta} = (\omega_{\rm c}, \omega_{\rm b}, n_{\rm s}, \sigma_{12})$

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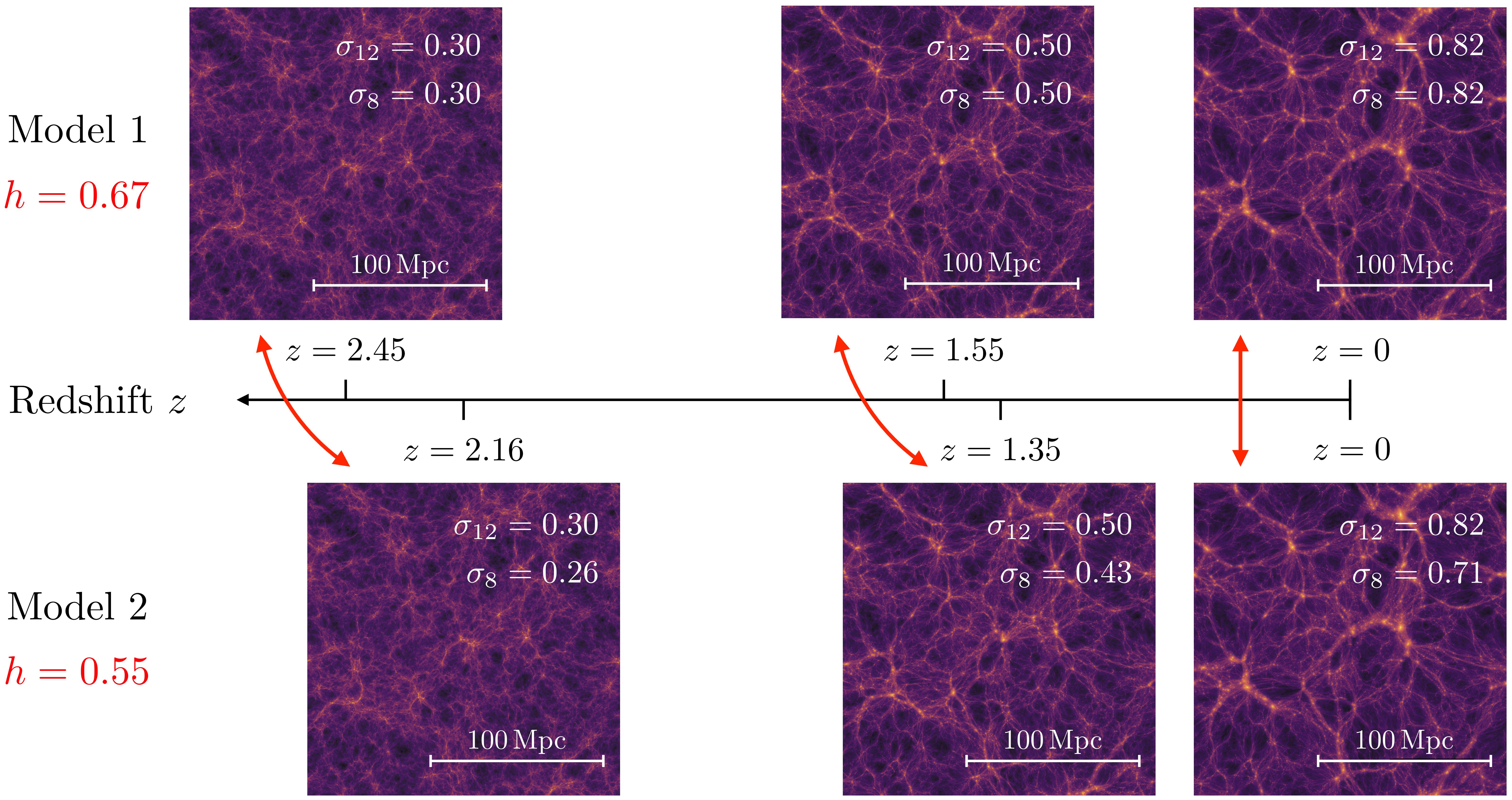

Evolution mapping reduces the required number of parameters to describe $P(k|z)$.

Emulator results must be corrected by $\Delta g(\sigma_{12})$

Linear level

$$v_j(k,a) = i\frac{k_j}{k^2}\mathcal{H}f(a)\delta_{\mathrm{m}}(k,a).$$

$$\Theta_k \equiv \frac{ik_jv_j(k,a)}{Ha}$$

$$P_{\Theta\Theta}(k,a) = f^2\boxed{P_m(k,a)}$$

$$\frac{\mathrm{d}\sigma_{12}}{\mathrm{d}t} = \sigma_{12}(z)\frac{\mathrm{d}D(z)}{\mathrm{d}a}\frac{\mathrm{d}a}{\mathrm{d}t} = f(a)H(a)\sigma_{12}$$

$$v = a\frac{\mathrm{d}x}{\mathrm{d}t} = a\underbrace{\boxed{\frac{\mathrm{d}x}{\mathrm{d}\sigma_{12}}}}_{\tilde{v}}\frac{\mathrm{d}\sigma_{12}}{\mathrm{d}t} = \tilde{v}\sigma_{12}faH$$

$$\tilde{\Theta}_k \equiv ik_j\tilde{v}_j(k,a) = -\frac{\delta_m(k,a)}{\sigma_{12}}$$

$$P_{\tilde{\Theta}\tilde{\Theta}}(k,a) = \frac{\boxed{P_m(k,a)}}{(\sigma_{12})^2}$$