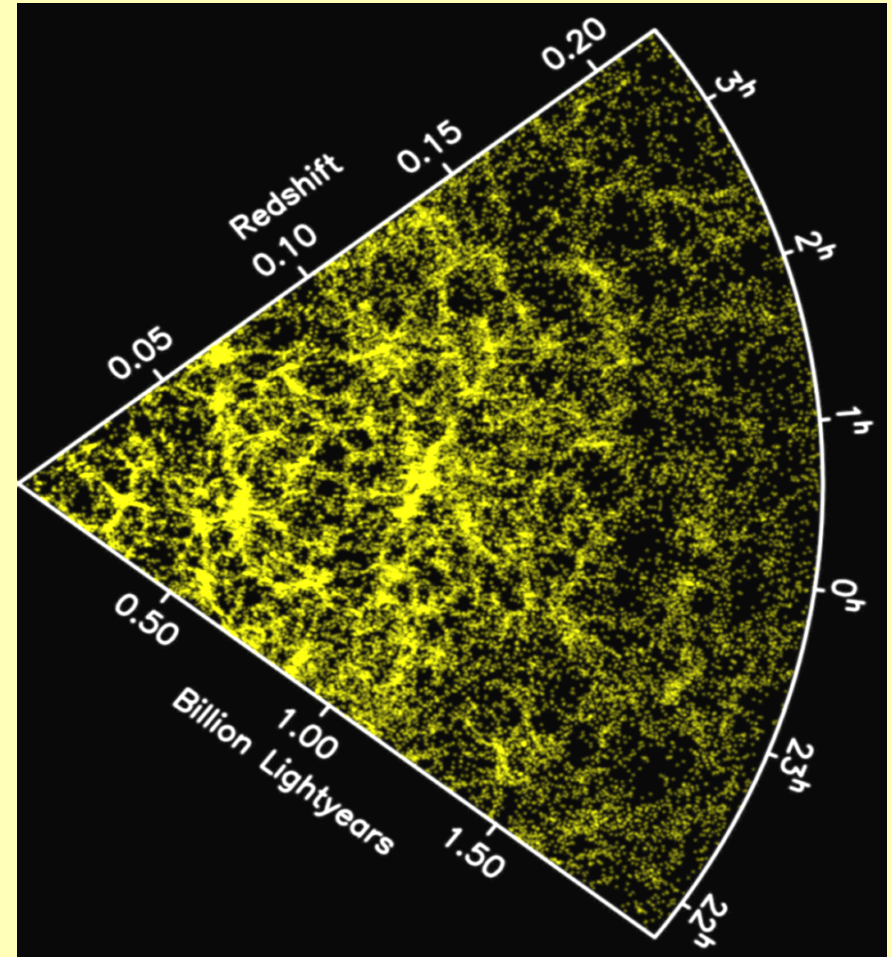
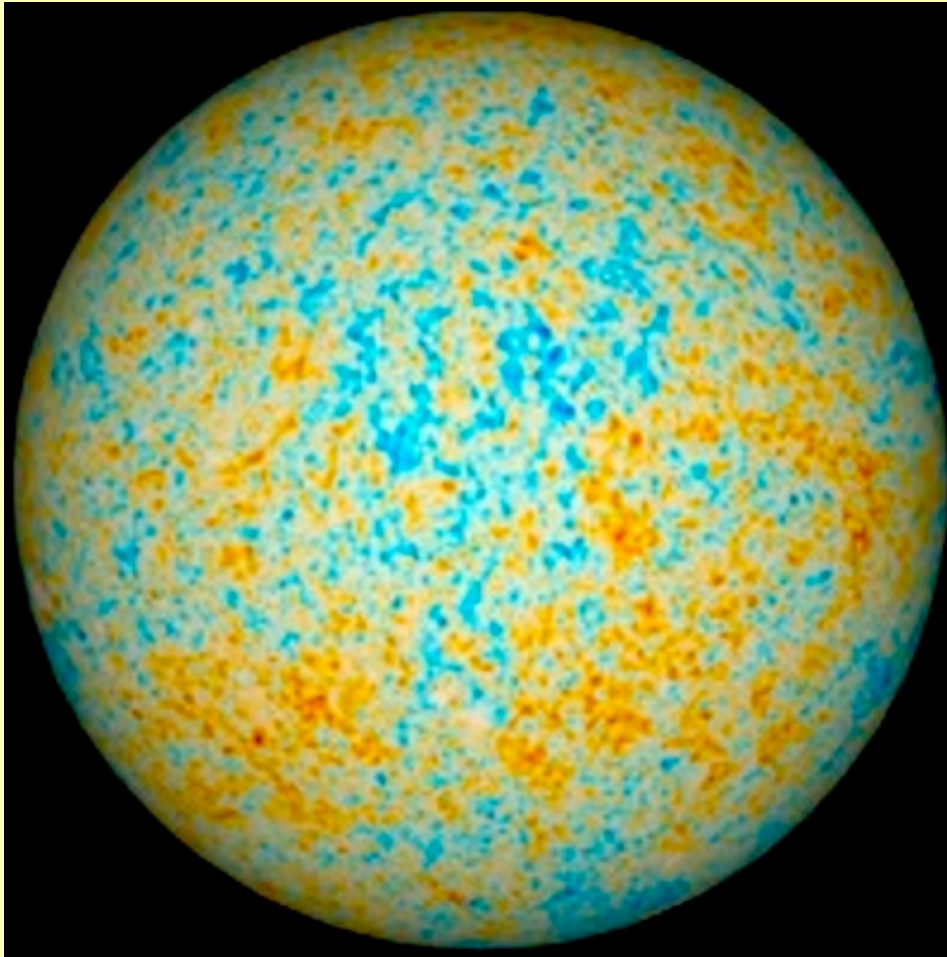
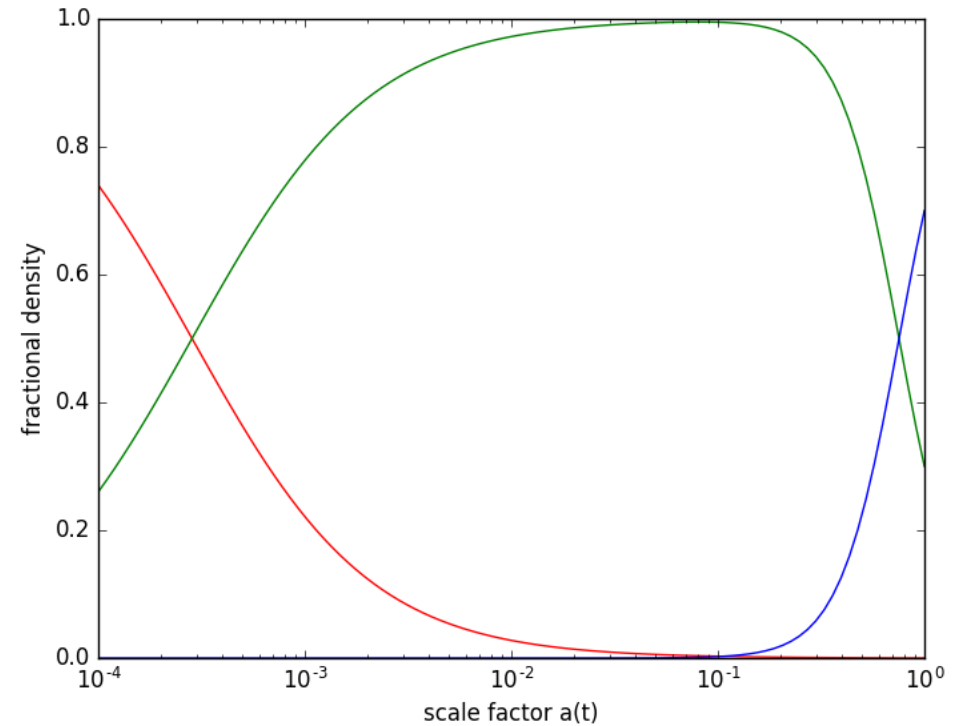
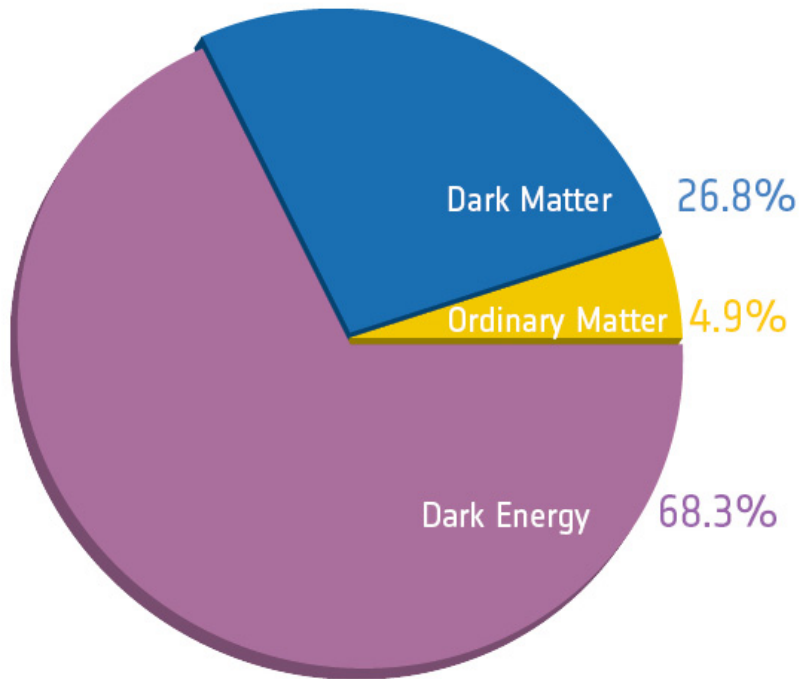


# Fundamental cosmology from the galaxy distribution



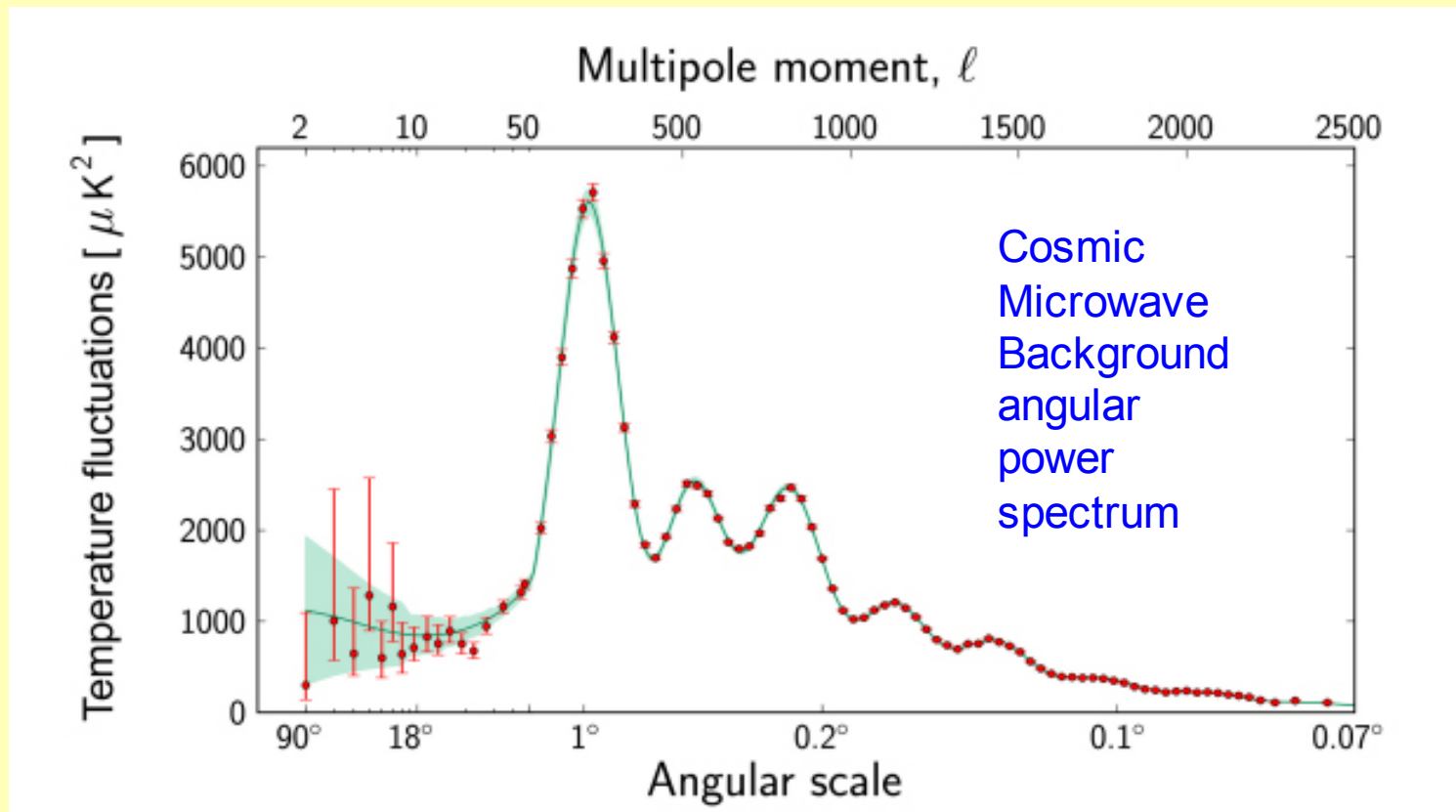
# The $\Lambda$ CDM standard model



Almost always dominated either by relativistic particles or by vacuum energy



# The big cosmological questions



- Cosmology has had inflationary  $\Lambda$ CDM as a standard model for structure formation for  $\sim 25$  years
  - Established during 1990s using galaxy clustering + CMB
  - Validated independently by SNe and BAO
  - Has survived huge improvements in data precision
- But we've never been happy

# The big cosmological questions

✓ = via galaxy surveys

- Nature of dark energy
  - Does it evolve? ✓
  - Does it fluctuate?
  - Is it a field that couples to dark matter? ✓
- Nature of dark matter
  - Thermal relic WIMP or scalar field?
  - Mass(es) and cross-sections?
  - Neutrino hierarchy? ✓
- Nature of gravity?
  - Distinctive non-Friedmann expansion history? ✓
  - Non-standard fluctuation growth? ✓

# The big cosmological questions

- Initial conditions
  - Did inflation happen?
  - Tensor modes?
  - Isocurvature modes?
  - Non-Gaussianity? ✓
- Fine tunings
  - Why are DM and baryon densities similar?
  - Why is the vacuum density so low?
  - Is there a multiverse? ✓



# Lecture plan

- Key elements of cosmological fluctuations
  - Power spectra and growth; diagnostics beyond density
- Galaxies as tracers of LSS
  - Galaxy bias and the halo model; environmental effects
- Geometrical cosmology
  - The BAO ruler; constraints from  $D(z)$  and  $H(z)$
- Peculiar velocities and redshift-space distortions
  - Growth rates from RSD
- The current situation
  - The  $H_0$  and normalization tensions
- Near-term outlook
  - Cosmological probes of the neutrino sector
- Longer-term outlook

**1:**  
**Key elements of cosmological  
fluctuations**

# Metric perturbations and gauges

Coordinate choice matters

$$x^\mu \rightarrow x'^\mu = x^\mu + \epsilon^\mu$$
$$\rho' = \rho - \epsilon^0 \dot{\rho}$$

$$d\tau^2 = a^2(\eta) \left\{ (1 + 2\phi)d\eta^2 + 2w_i d\eta dx^i - [(1 - 2\psi)\gamma_{ij} + 2h_{ij}] dx^i dx^j \right\}$$

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} 2\phi & -B_{,i} \\ -B_{,i} & 2[\psi\delta_{ij} - E_{,ij}] \end{pmatrix}$$

$$\Phi \equiv \phi + \frac{1}{a} [(B - E')a]'$$
$$\Psi \equiv \psi - \frac{a'}{a} (B - E')$$

Gauge-independent potentials. Simply obtained in Newtonian gauge

$$d\tau^2 = (1 + 2\Psi)dt^2 - (1 - 2\Phi)\gamma_{ij} dx^i dx^j$$

For perfect fluid matter source, both potentials are equal to Newtonian potential

Lensing: speed of light reduced by factor  $1 + \Psi + \Phi$ , so light is deflected as in glass, with GR factor 2 – or not??



# Linear Newtonian fluctuations

(valid inside horizon)

$$\rho = \bar{\rho}(1 + \delta)$$

$$\text{Continuity: } \dot{\rho} = -\nabla \cdot (\rho \mathbf{v}) \Rightarrow \dot{\delta} = -\nabla \cdot [(\mathbf{1} + \delta)\mathbf{u}]$$

$$\simeq -\nabla \cdot \mathbf{u}$$

$$\dot{\mathbf{u}} + 2\frac{\dot{a}}{a}\mathbf{u} = \frac{\mathbf{g}}{a}$$

$$\text{Poisson: } \nabla^2 \Phi / a^2 = 4\pi G \rho \delta$$

Homogeneous solution:  $\nabla \cdot \mathbf{u} = 0 \Rightarrow u \propto 1/a^2$  (vorticity)

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \rho \delta$$

$\Omega_m = 1$  Growing Solution:  $\delta \propto a \Rightarrow \Phi = \text{const}$

$$d \ln \delta / d \ln a \simeq \Omega_m(a)^{0.55}$$

$\Lambda$  damps  
primordial  
potential  
fluctuations

Radiation era: must include radiation pressure in e.o.m. and effect of pressure on Poisson. Result is again time-independent metric fluctuation on large scales

# Multi-component perturbation modes

- **Adiabatic:**
  - Compress matter & photons equally
  - $\delta_r = (4/3) \delta_m$
- **Isothermal:**
  - No radiation perturbation  $\delta$
- **Isocurvature (entropy perturbation)**
  - Initially isothermal
  - But evolution causes radiation perturbation
  - $\delta_r = (4/3)(\delta_m - \delta_{m_i})$
- Normally assume adiabatic, but can get isocurvature if matter generated after time of interaction with photons (e.g. curvaton) – strong constraints on isocurvature from CMB

# Fourier description of inhomogeneity

$$\delta(\mathbf{x}) = \sum \delta_k e^{-i\mathbf{k}\cdot\mathbf{x}}; k_x = n \frac{2\pi}{L} \text{ etc.}$$

$$\langle \delta^2 \rangle = \sum |\delta_k|^2 \rightarrow \frac{L^3}{(2\pi)^3} \int P(k) d^3k$$

$$= \int \Delta^2(k) d \ln k; \Delta^2(k) \equiv \frac{L^3}{(2\pi)^3} 4\pi k^3 P(k)$$

Dimensionless  
power per log  
scale

## N-point correlations



$$dP = \rho_0^n [1 + \xi^{(n)}] dV_1 \cdots dV_n$$

$$1 + \xi^{(n)} = \langle \prod_i (1 + \delta_i) \rangle$$

$$\text{e.g. } \xi^{(3)} = \xi(r_{12}) + \xi(r_{23}) + \xi(r_{31}) + \langle \delta_1 \delta_2 \delta_3 \rangle$$

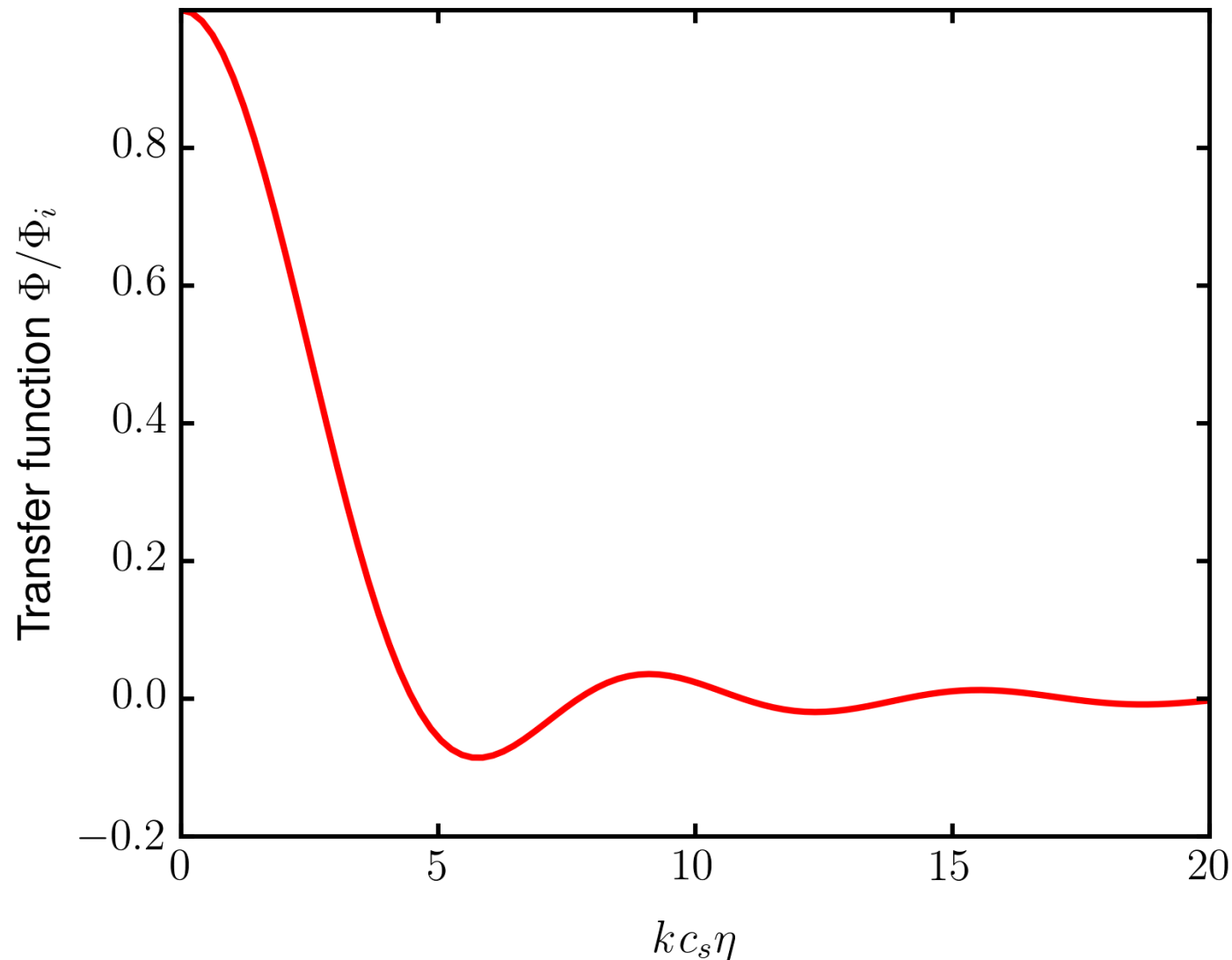
$$\xi(\mathbf{r}) = \sum_k |\delta_{\mathbf{k}}|^2 e^{-i\mathbf{k}\cdot\mathbf{r}} = \int \Delta^2(k) \frac{\sin(kr)}{(kr)} d \ln k$$

Note cosmological density fields are ergodic:  $\langle \rangle$  can be volume average or ensemble average



# Radiation era transfer functions

$$T(k) = 3(\sin x - x \cos x)/x^3 \quad x = k \eta c_s \quad d\eta = dt/a(t)$$



Pressure halts growth on small scales (Jeans length)

$\eta c_s$  comoving acoustic horizon

$T(k)$  Function of scale at one time – or time dependence for one scale

# The cosmological sound speed

$$\rho = \rho_b + \rho_\gamma; p = \rho_\gamma c^2 / 3$$

$$\frac{d\rho_\gamma}{\rho_\gamma} = \frac{4}{3} \frac{dV}{V} \Rightarrow dp = \frac{4}{9} \rho_\gamma c^2 \frac{dV}{V}; d\rho = (\rho_b + 4\rho_\gamma/3) \frac{dV}{V}$$

$$\Rightarrow c_s = \frac{c}{\sqrt{3}} \left( 1 + \frac{3}{2} \frac{\rho_b}{\rho_\gamma} \right)^{-1/2}$$

$$\Omega_\gamma h^2 = 2.488 \times 10^{-5}; \Omega_b h^2 = 0.0225$$

$$\Rightarrow c_s \simeq \frac{c}{\sqrt{3}} \left( 1 + \frac{600}{1+z} \right)^{-1/2}$$

Thompson scattering binds photons and baryons into a single fluid

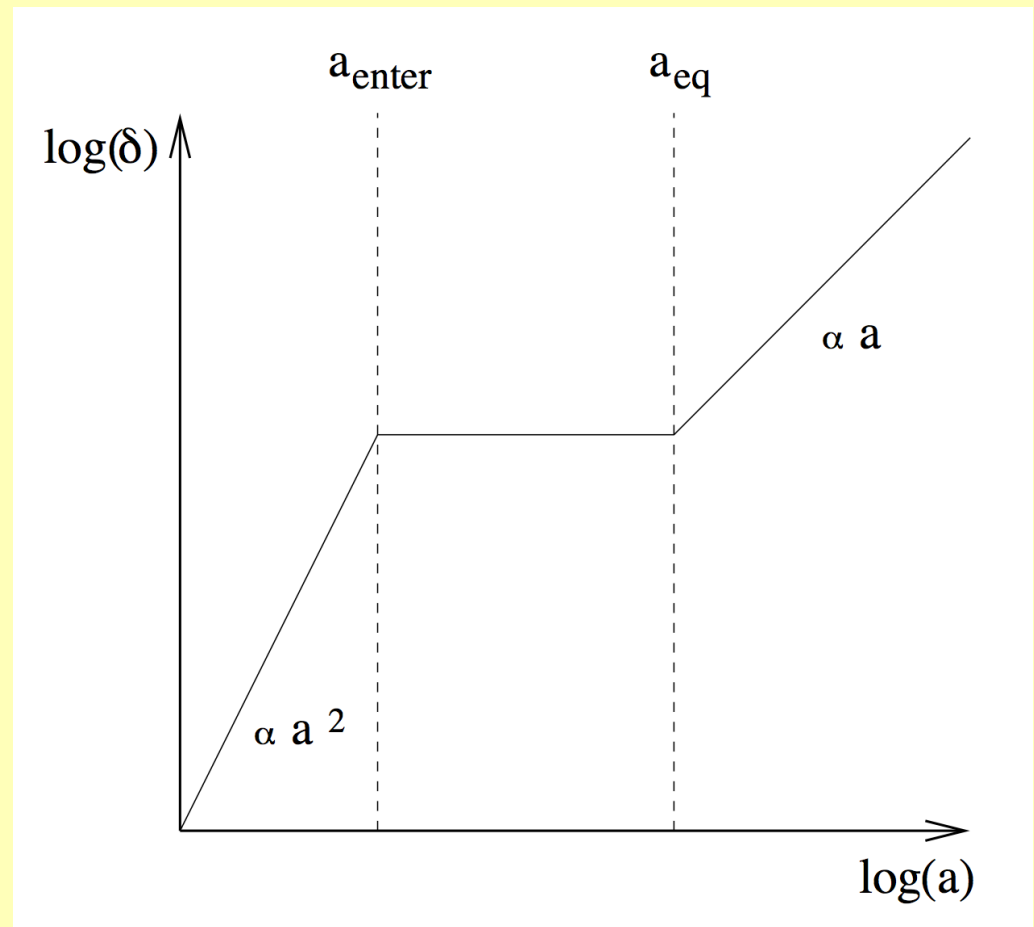
# CDM transfer function

$$\begin{aligned}\ddot{\delta}_c + 2H(a)\dot{\delta}_c &= 4\pi G (\delta\rho) \\ &= 4\pi G \bar{\rho} \delta_c \quad (\text{outside horizon}) \\ &= 4\pi G \bar{\rho} (\bar{\rho}_c/\bar{\rho}) \delta_c \quad (\text{inside horizon})\end{aligned}$$

CDM dynamics affected by gravity from photons+ baryons

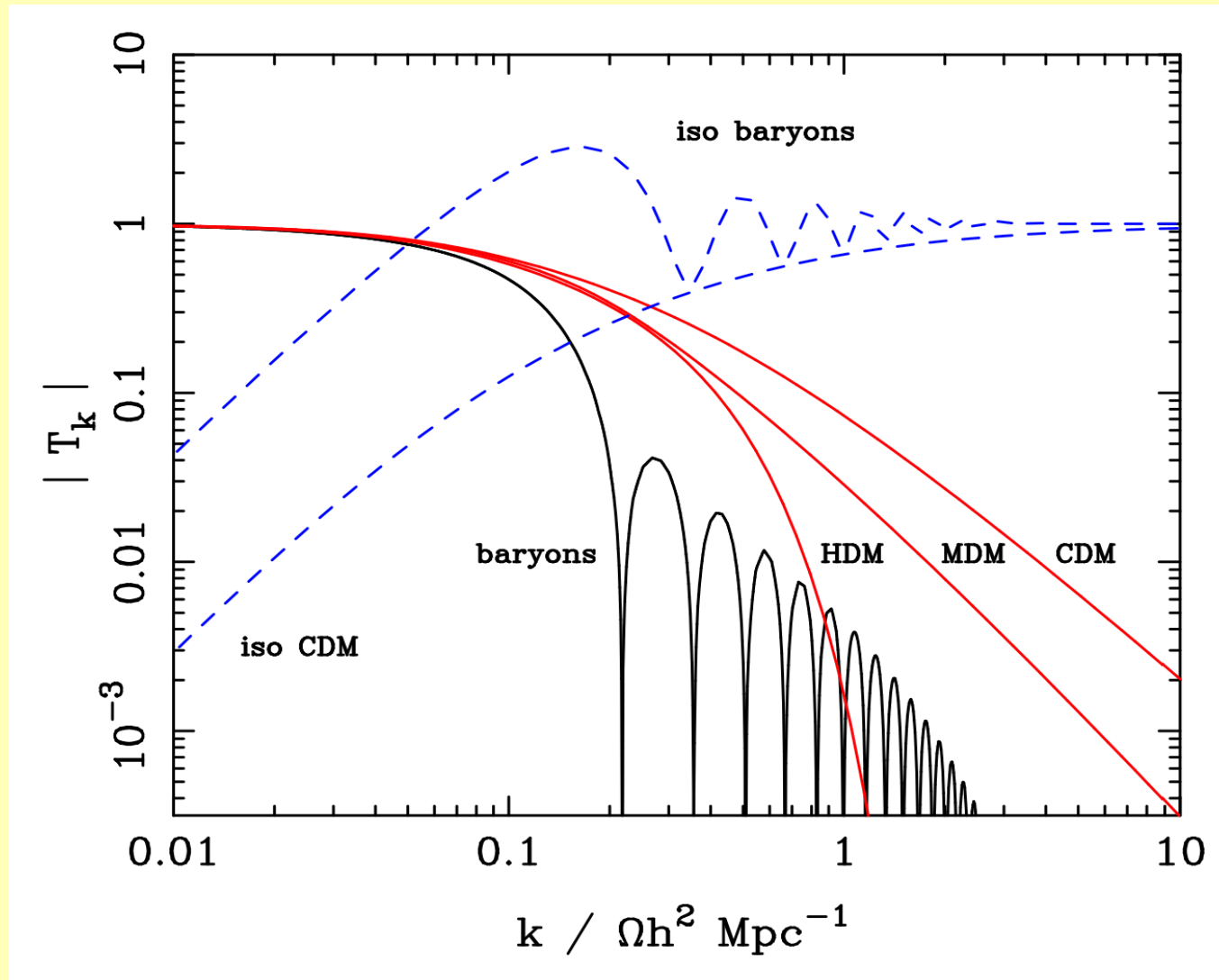
Reduces small-scale fluctuations in (coupled) photon-baryon fluid (acoustic oscillations without growth).

Then no gravity to drive growth in DM clustering inside horizon (Meszaros effect)





# Matter transfer functions

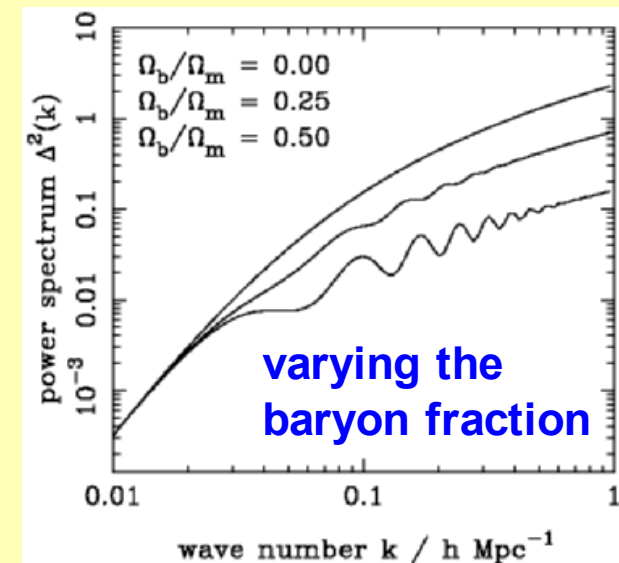
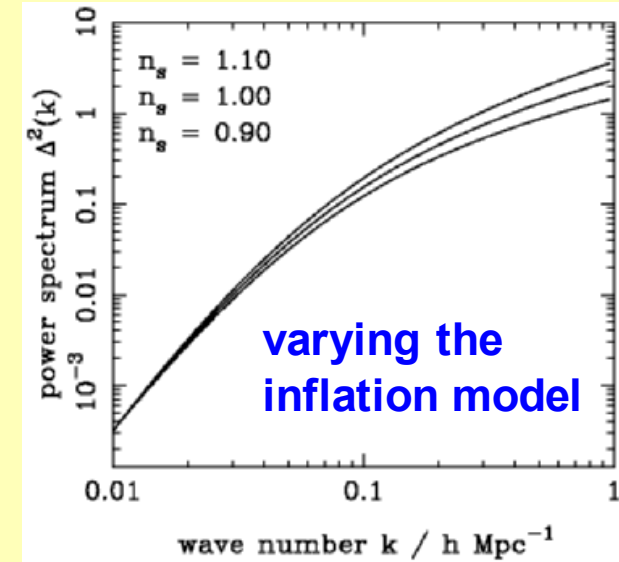
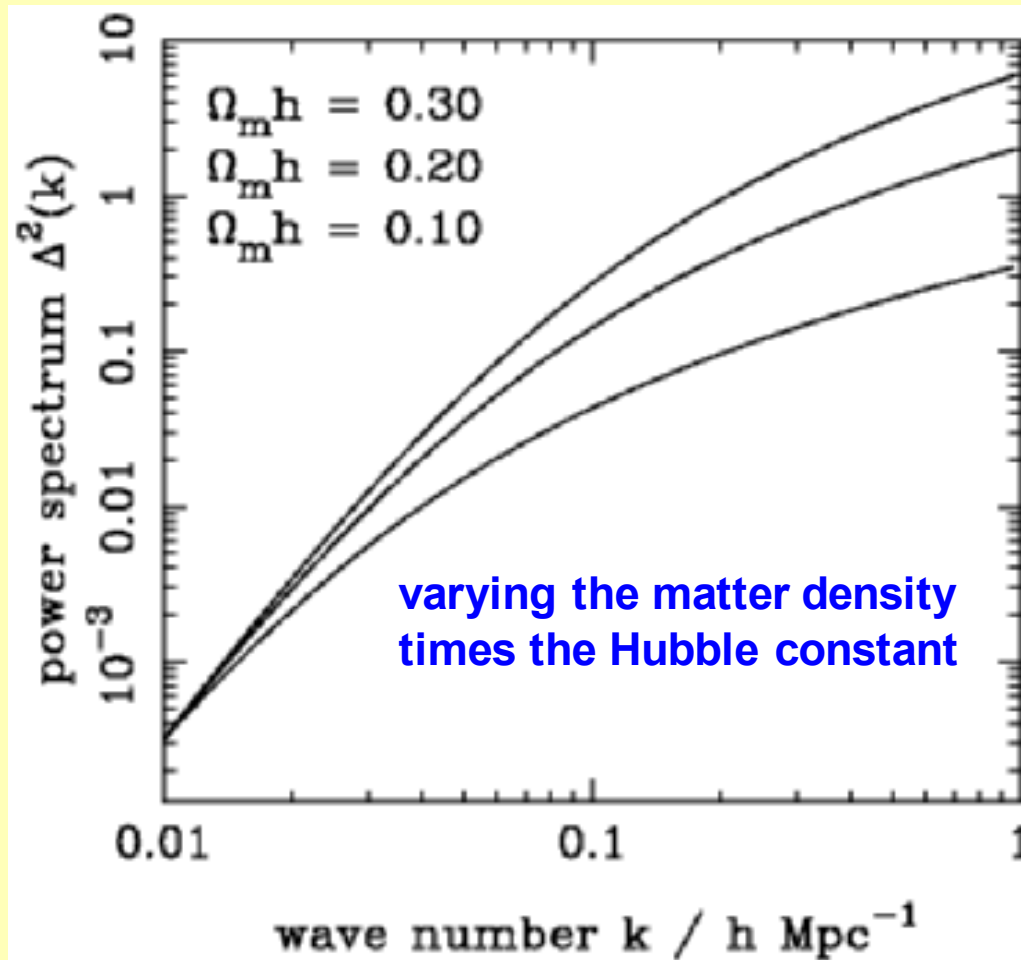


Break scale at horizon size at matter-radiation equality  
( $=16 (\Omega_m h)^{-1} h^{-1} \text{ Mpc}$ ). Potential to measure  $\Omega_m h$

# Model dependence of CDM linear P(k)

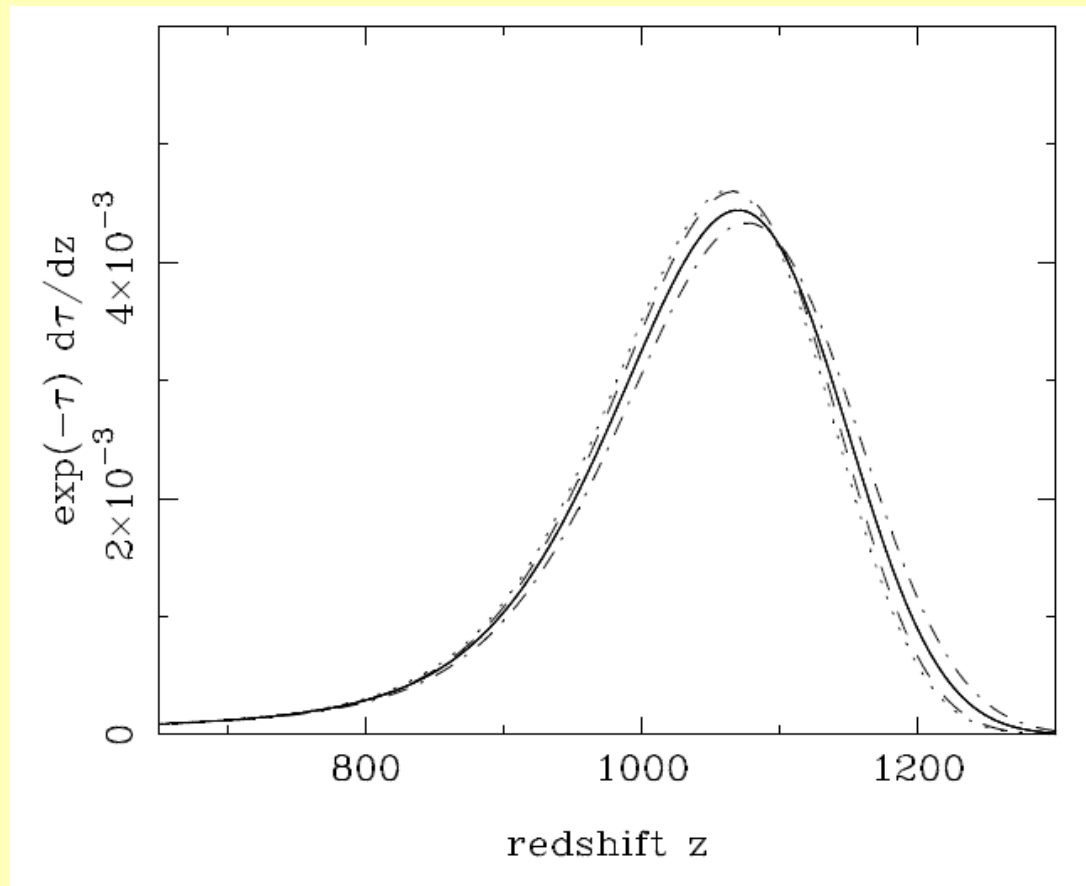
$$\delta = \sum \delta_k e^{(-ikx)} \quad |\delta_k|^2 = A k^n$$

$$\Delta^2(k) = d\sigma^2/d \ln k = |\delta_k|^2 \times (k^3/2\pi^2)$$



# Observing the initial conditions:

Furthest back we can see is the microwave background ( $z = 1100$ )

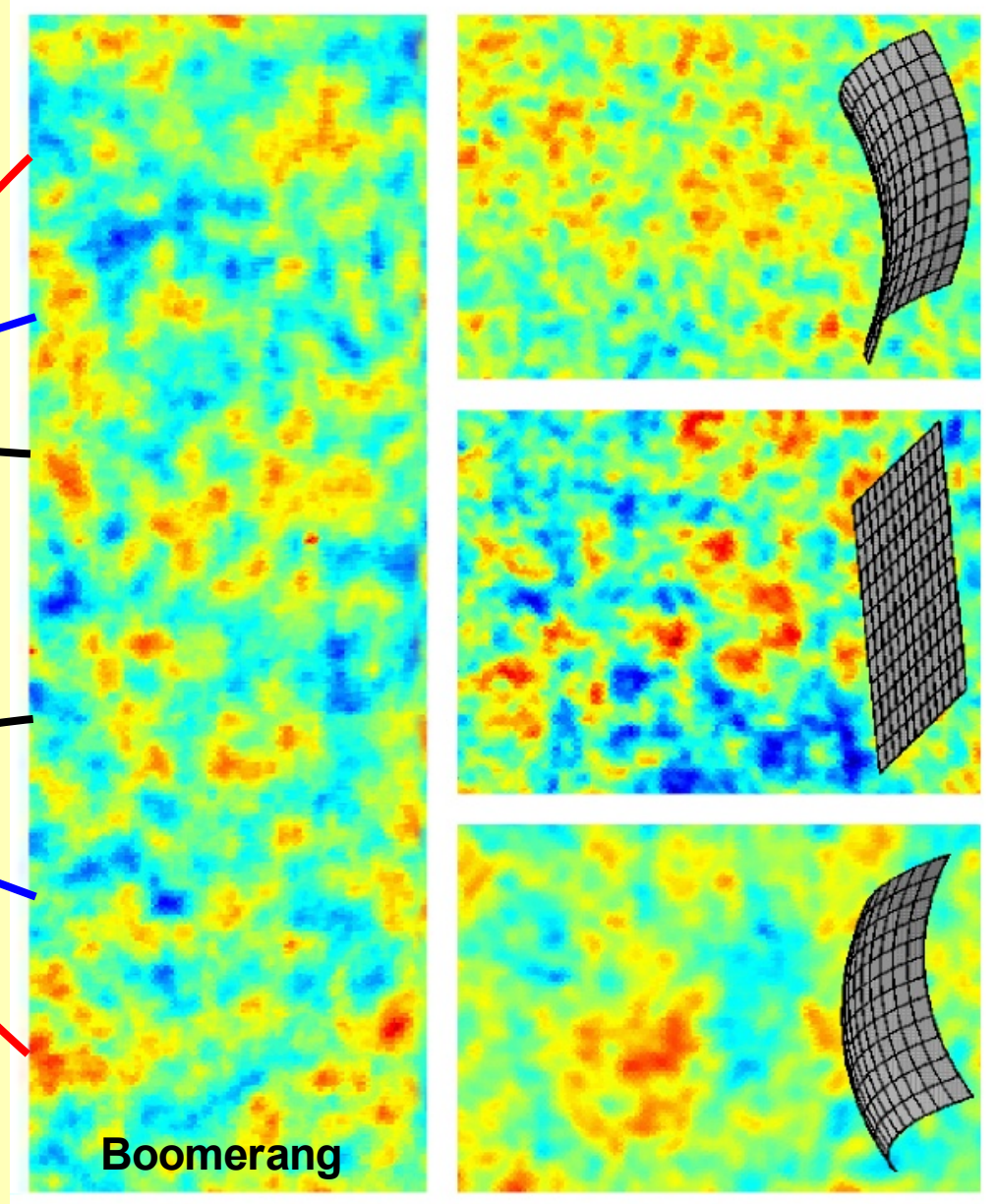
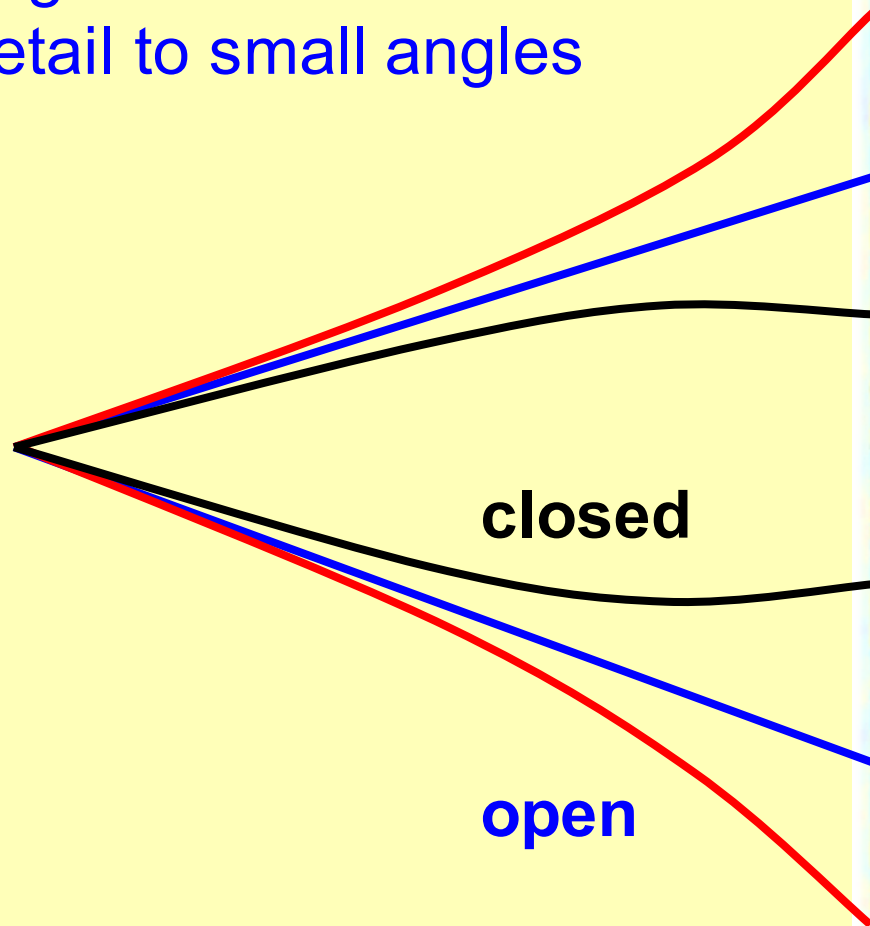


CMB photon 'visibility function' of last scattering

$T_0 = 2.725$  K, so last scattering at 3000 K

# CMB and cosmic geometry

Open geometries with negative curvature shift detail to small angles



# Weighing the universe with horizons

Growth of structure is affected by pressure on small scales

⇒ Horizon scale  $c_{\text{sound}} t \simeq c t = D_H$  leaves imprint in late-time structure

Three key eras:

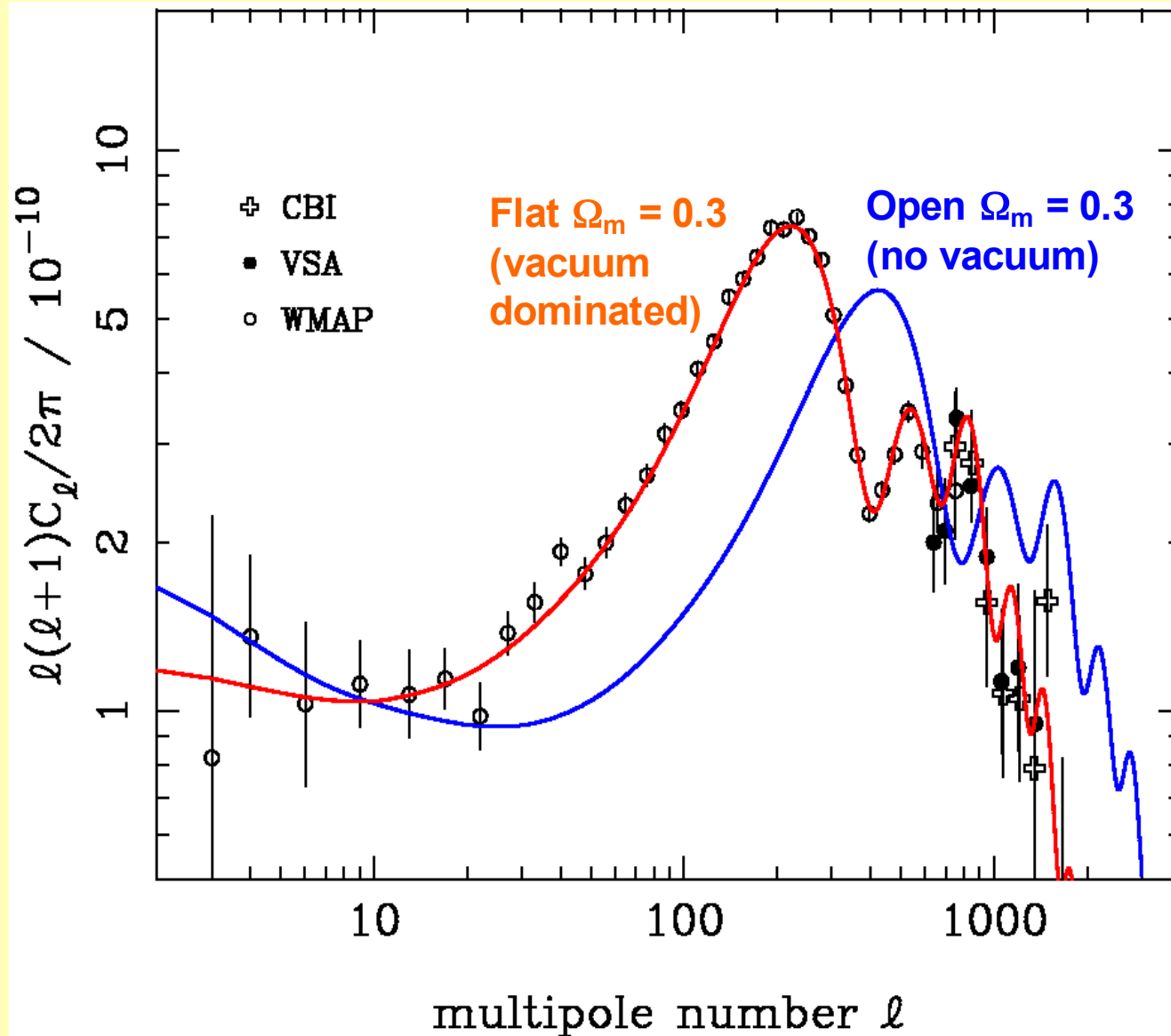
(1) Matter-radiation equality ( $z=23,900$   $\Omega_m h^2$ ):  $D_H = 16 (\Omega_m h^2)^{-1}$  Mpc

(2) Last scattering ( $z=1100$ ):  $D_H = 184 (\Omega_m h^2)^{-1/2}$  Mpc

(3) Today ( $z=0$ ):  $D_H = 6000 \Omega_m^{-0.4} h^{-1}$  Mpc (if flat)

- 100-Mpc 'break' in LSS from (1)
- $c_{\text{sound}}$  depends on baryon density: acoustic horizon gives extra info
- 1-degree scale on CMB sky from (2) / (3)

# Angular scales in the CMB



Horizon at last scatter:

$$184 \Omega_m^{-1/2} h^{-1} \text{ Mpc}$$

Present horizon if  $\Lambda=0$ :

$$6000 \Omega_m^{-1} h^{-1} \text{ Mpc}$$

$$\Rightarrow \text{angles} \propto \Omega_m^{1/2}$$

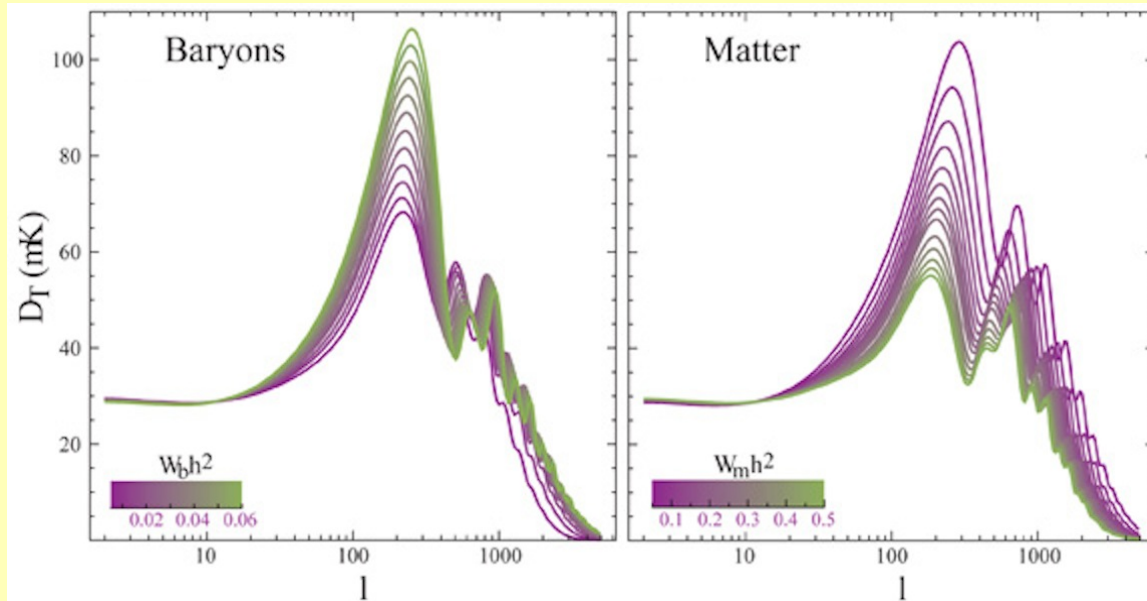
if  $\Lambda=0$

$$\theta_H \propto (\Omega_m h^{3.4})^{0.14}$$

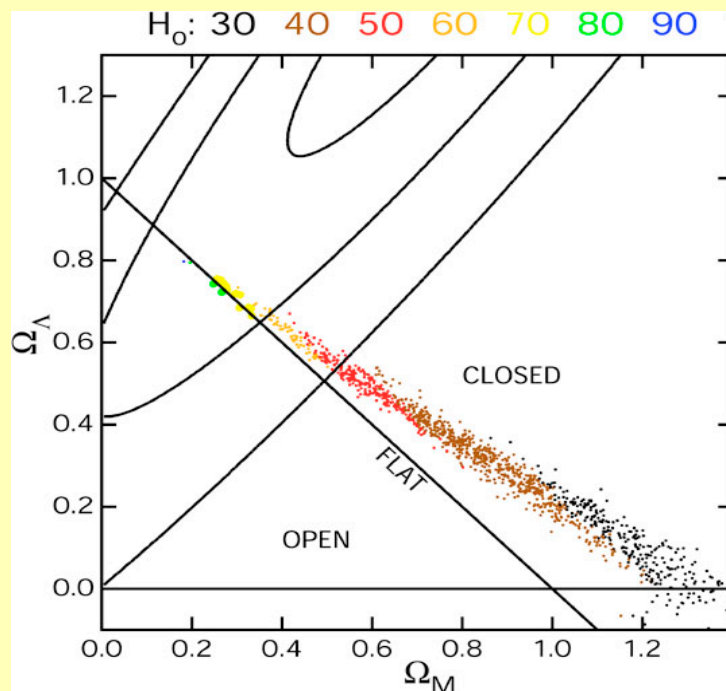
if flat ( $\Omega_m + \Omega_v = 1$ )



# Parameters from the CMB



Peak heights  
constrain matter  
content – via early  
ISW effect.  
Measures degree of  
radiation domination  
at LS



Geometrical degeneracy:  
peak location forbids open  
models, but closed allowed.  
Needs extra data (LSS or H)  
to force  $\Omega_m=0.3$  flat model

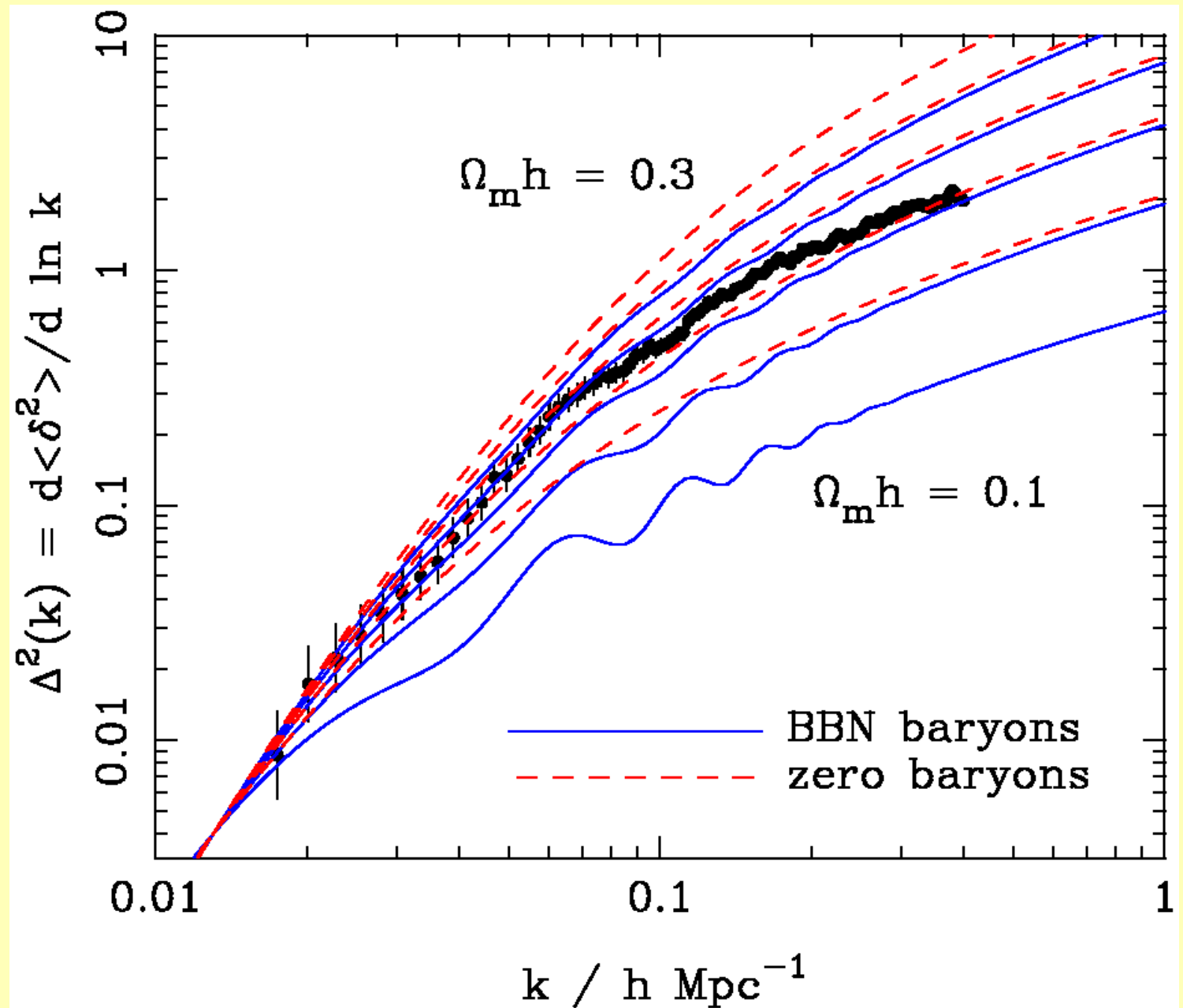
# 2dFGRS P(k): shape needs low density

Dimensionless power:

$\Delta^2(k) = d\langle\delta^2\rangle/d\ln k$   
(fractional variance in density) /  $d\ln k$

Note small degree of baryonic 'wiggles': direct evidence for collisionless DM

Percival et al.  
MNRAS 327,  
1279 (2001)





# The argument for $\Lambda$ : 1990 LSS + CMB limits $\Rightarrow$ low density but not open

## LETTERS TO NATURE

### The cosmological constant and cold dark matter

G. Efstathiou, W. J. Sutherland & S. J. Maddox

Department of Physics, University of Oxford, Oxford OX1 3RH, UK

THE cold dark matter (CDM) model<sup>1-4</sup> for the formation and distribution of galaxies in a universe with exactly the critical density is theoretically appealing and has proved to be durable, but recent work<sup>5-8</sup> suggests that there is more cosmological structure on very large scales ( $l > 10 h^{-1}$  Mpc, where  $h$  is the Hubble constant  $H_0$  in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) than simple versions of the CDM theory predict. We argue here that the successes of the CDM theory can be retained and the new observations accommodated in a spatially flat cosmology in which as much as 80% of the critical density is provided by a positive cosmological constant, which is dynamically equivalent to endowing the vacuum with a non-zero energy density. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant. As well as explaining large-scale structure, a cosmological constant can account for the lack of fluctuations in the microwave background and the large number of certain kinds of object found at high redshift.

We can, however, simply accept that  $\Omega_0 \approx 0.2$ , while retaining the key ingredients of the CDM model, namely a flat universe with scale-invariant, adiabatic initial fluctuations. This requires a positive cosmological constant and is compatible with inflation<sup>12</sup>. Furthermore, spatially flat scale-invariant CDM models with  $\Omega_0 h \approx 0.2$  are compatible with limits on the anisotropies of the microwave background radiation<sup>23</sup>, whereas equivalent low-density models with  $\Lambda = 0$  are firmly excluded by these limits<sup>14</sup>.

706

NATURE · VOL 348 · 20/27 DECEMBER 1990

cf. Perlmutter et al. 1996:  
SNe Ia  $\Rightarrow$   $\Lambda$ -dominated  
models excluded

# **2: Galaxies as tracers of LSS**

# Nonlinear evolution (comoving view)

redshift  $z=3$

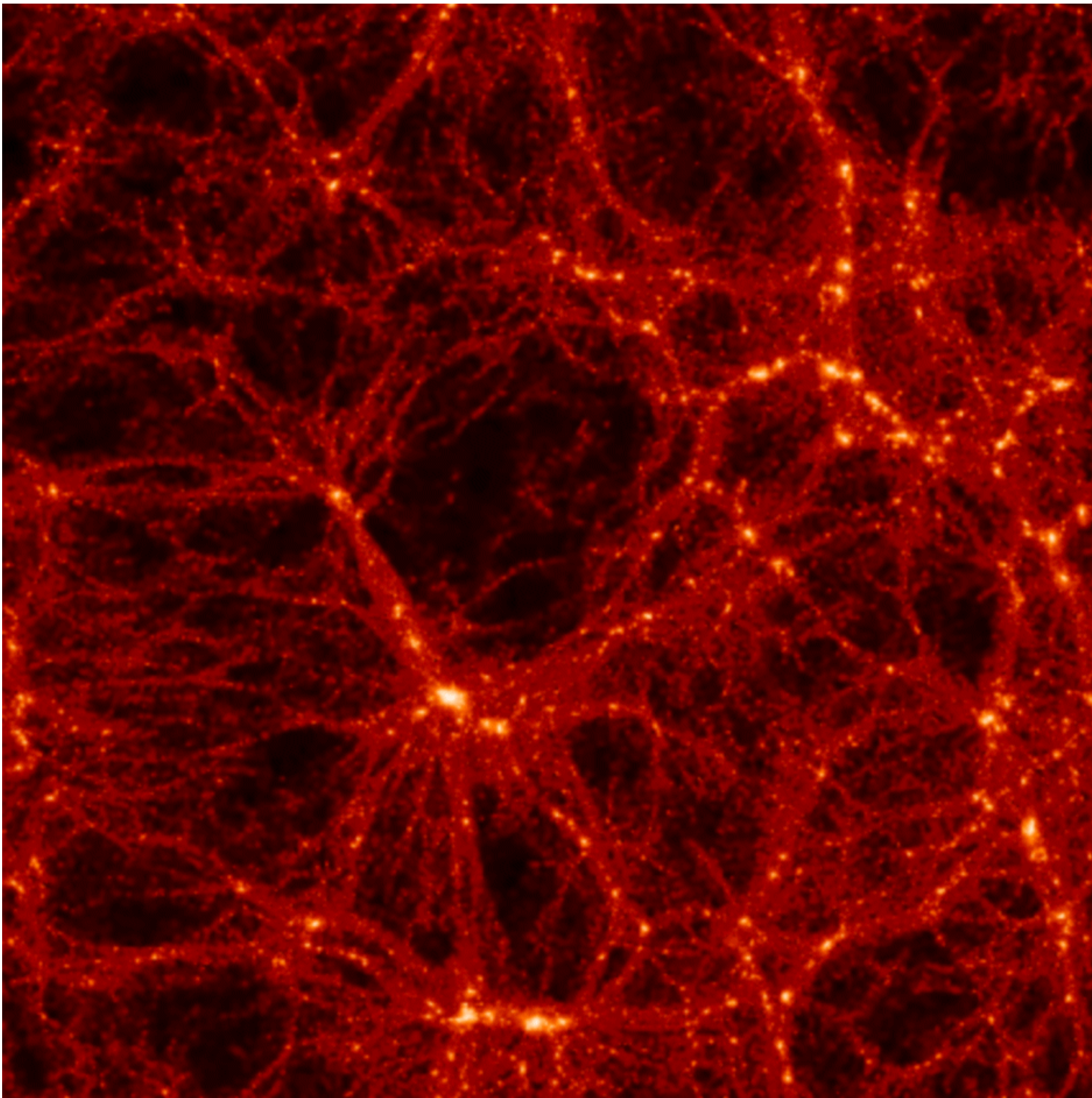
(1/4 present size)

redshift  $z=1$

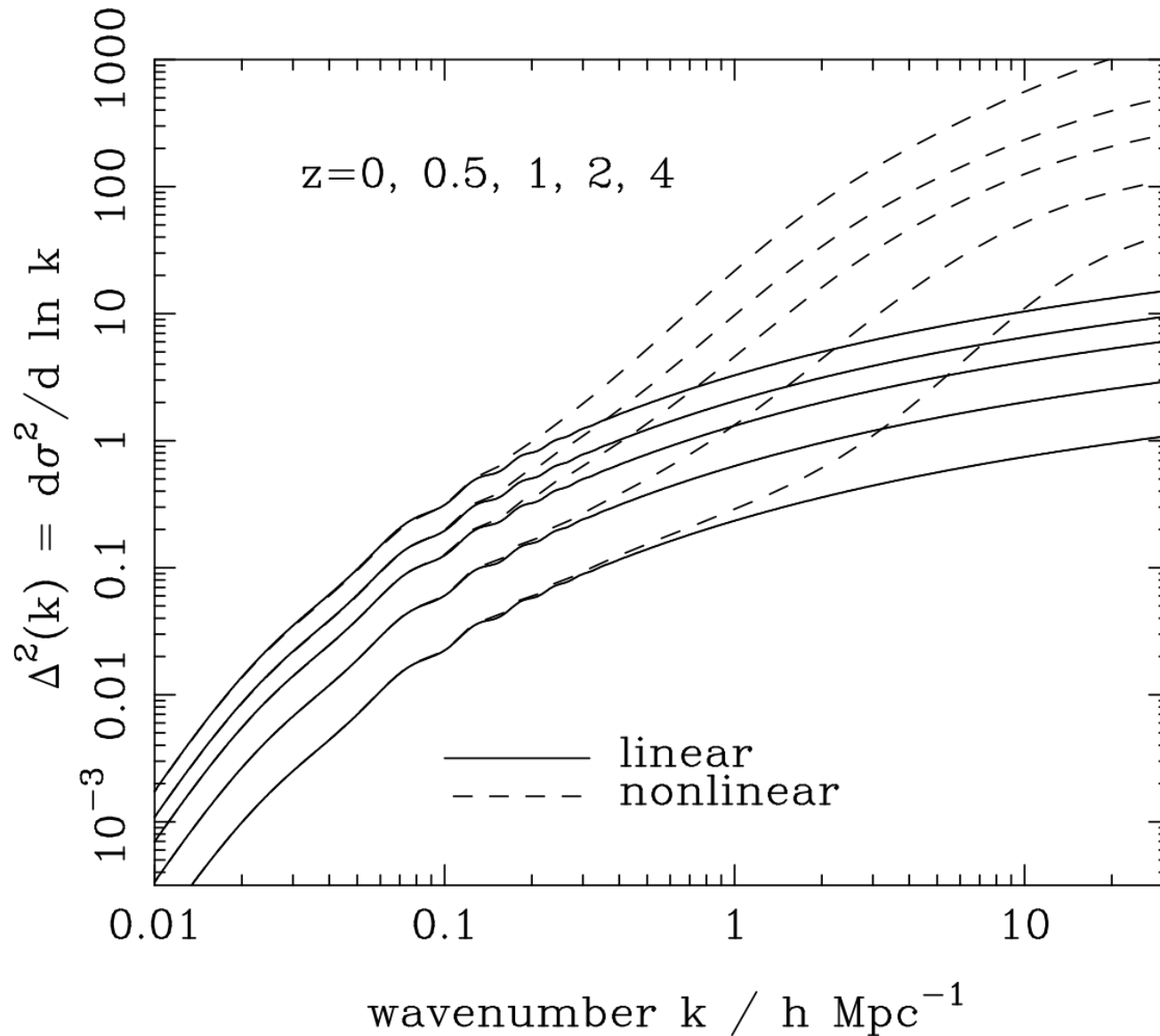
(1/2 present size)

Redshift  $z=0$

(today)



# Nonlinear evolution of P(k)

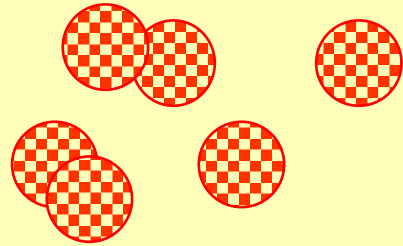


$$\Delta^2(k) \propto k^{n+3} T^2(k)$$
$$n \simeq 0.96$$

Nonlinear  
behaviour  
described by  
HALOFIT  
(Smith et al.  
2003) –  
derived from  
N-body data

# The halo model

Neyman Scott & Shane (1953): random clump model:  
correlations arise from pairs in the same clump



$$\rho \propto r^{-\alpha} \quad (r < R) \Rightarrow \xi \propto r^{-(2\alpha-3)}$$

$$\text{obs: } \xi \propto r^{-1.8} \Rightarrow \alpha = 2.4?$$

Application to CDM, following Benson et al. 2000:

Seljak 2000, Peacock & Smith 2000, Seljak 2002, Berlind et al. 2002

- ▶ Random halo distribution:

$$P(k) = 1/n \Rightarrow \Delta^2(k) = k^3 / (2\pi^2 n)$$

- ▶ Filtered by halo density profile

- ▶ Spectrum of masses:

$$\frac{1}{n_{\text{eff}}} = \frac{\int M^2 f(M) dM}{\left[ \int M f(M) dM \right]^2}$$

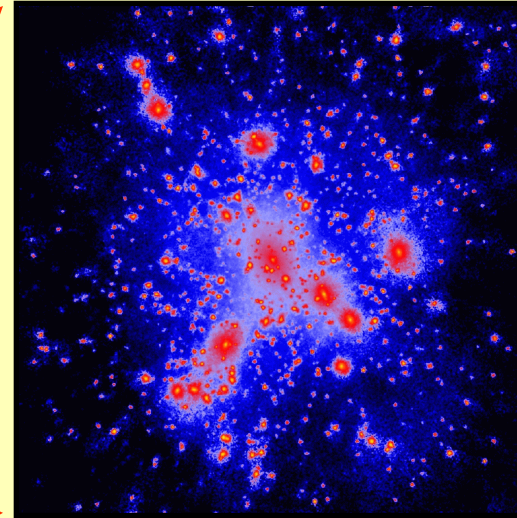
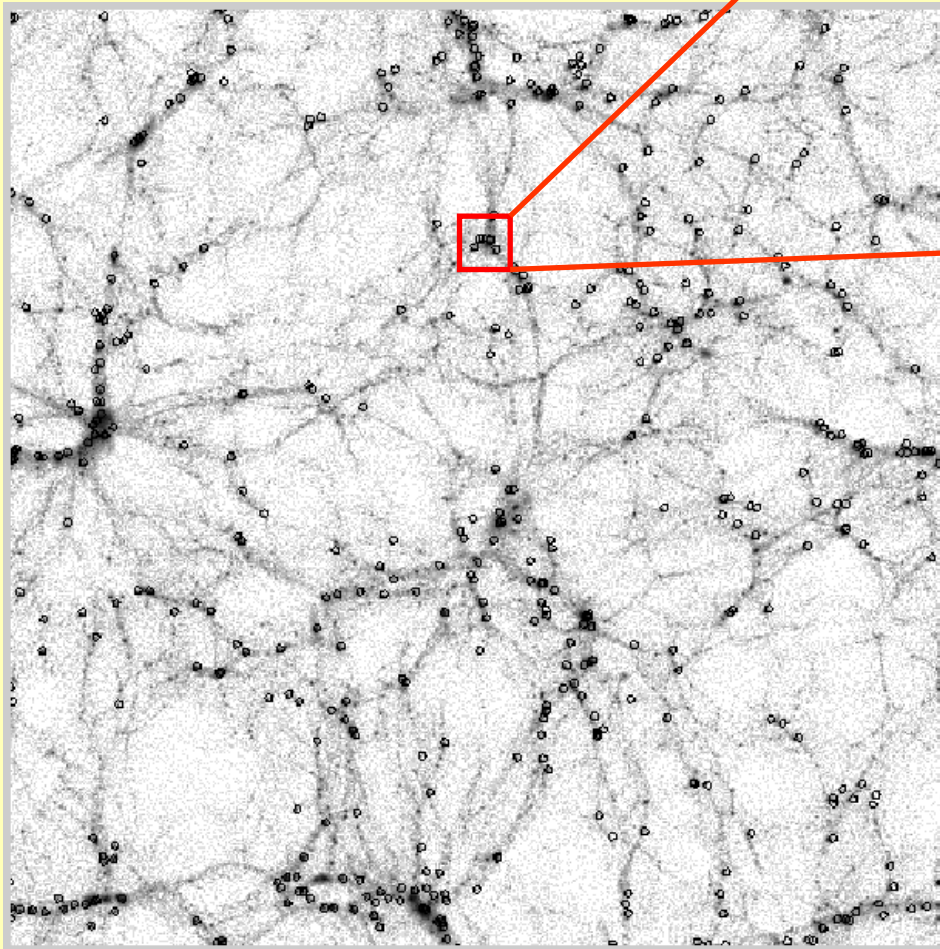
- ▶ 1-halo + 2-halo:

$$\Delta_{\text{tot}}^2 = \Delta_{\text{halo}}^2 + \Delta_{\text{linear}}^2$$

Note: random initial placement means haloes can overlap  
– better to use N-body halo catalogues



# CDM dark-matter halo profiles



Seek  
virialized  
objects of  
density  
contrast 200

N-body gives halo profile:

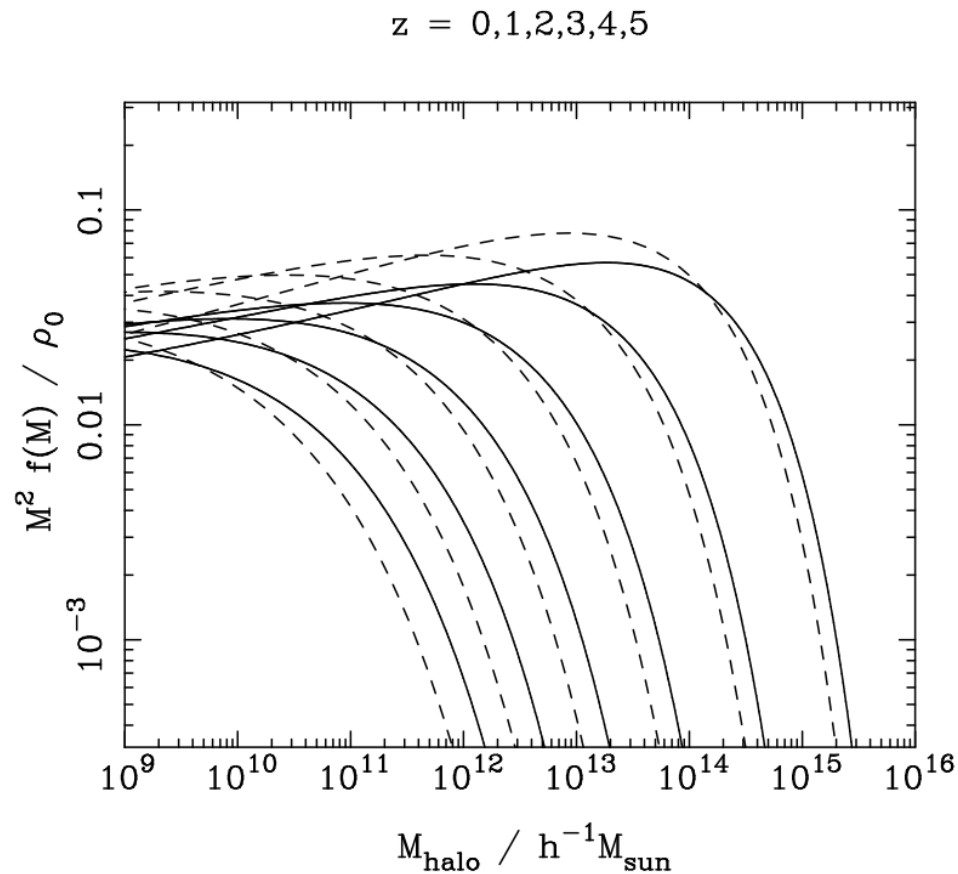
$$\rho = [y(1+y)^2]^{-1}; \quad y = r/r_c \quad (\text{NFW})$$

$$\rho = [y^{3/2}(1+y^{3/2})]^{-1}; \quad y = r/r_c \quad (\text{Moore})$$

$$\rho = \exp[-y^{1/4}]; \quad y = r/r_c \quad (\text{Einasto})$$

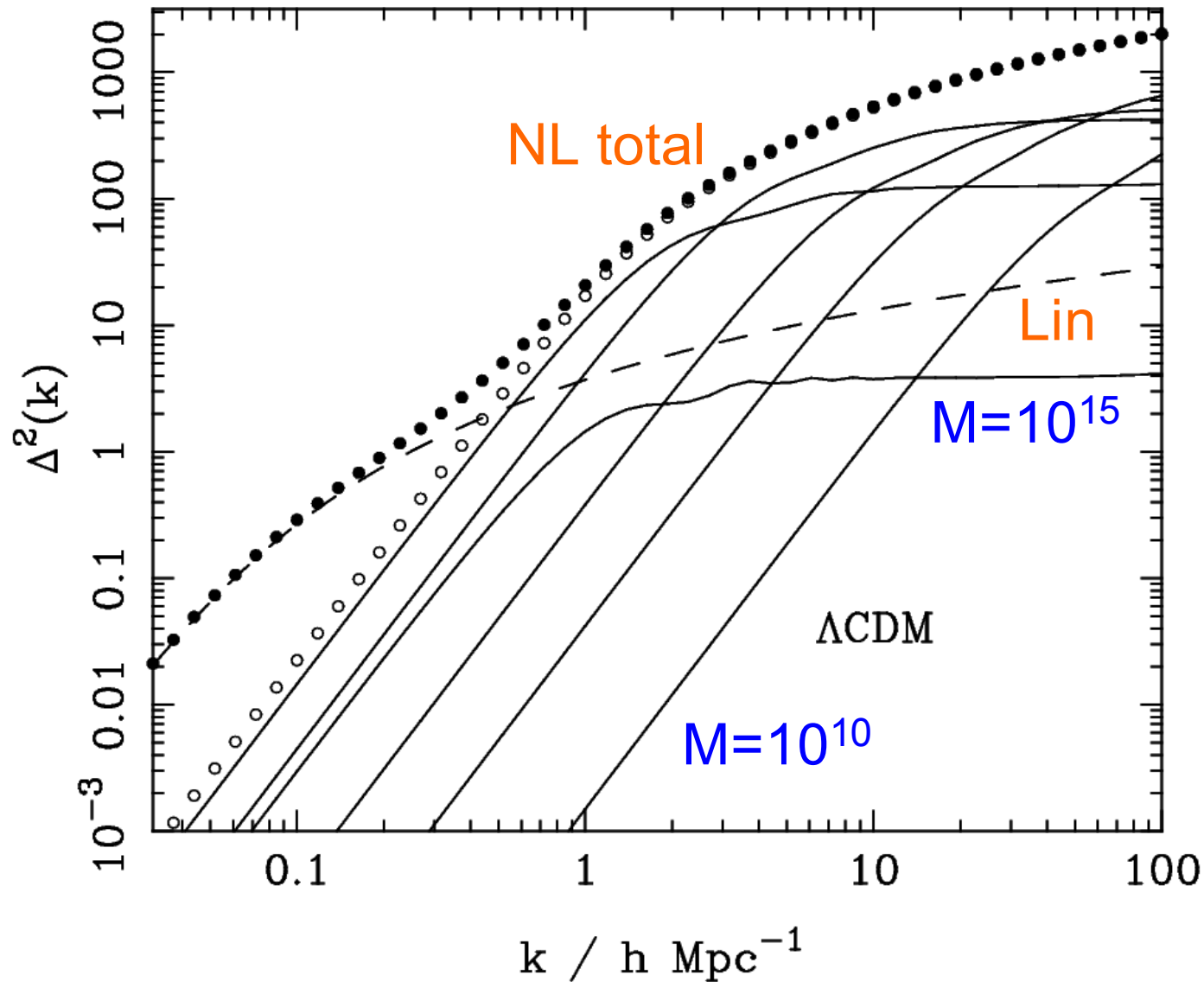
(cf. Isothermal sphere  $\rho = 1/y^2$ )

# The halo mass function



- ▶ Press-Schechter (1974): Collapse fraction from (2 times) positive half of Gaussian
- ▶ PS:  $F_{\text{coll}} = \text{erfc}(\nu/\sqrt{2})$
- ▶ 'exact':  $\nu \equiv \delta_c/\sigma(M)$
- ▶  $F_{\text{coll}} = (1 + a\nu^b)^{-1} \exp(-c\nu^2)$
- ▶  $(a, b, c) = (1.529, 0.704, 0.412)$
- ▶  $M = \frac{4\pi}{3} \bar{\rho} R^3$
- ▶  $Mf(M)/\rho_0 = |dF/dM|$

# The Halo Model view of nonlinearity

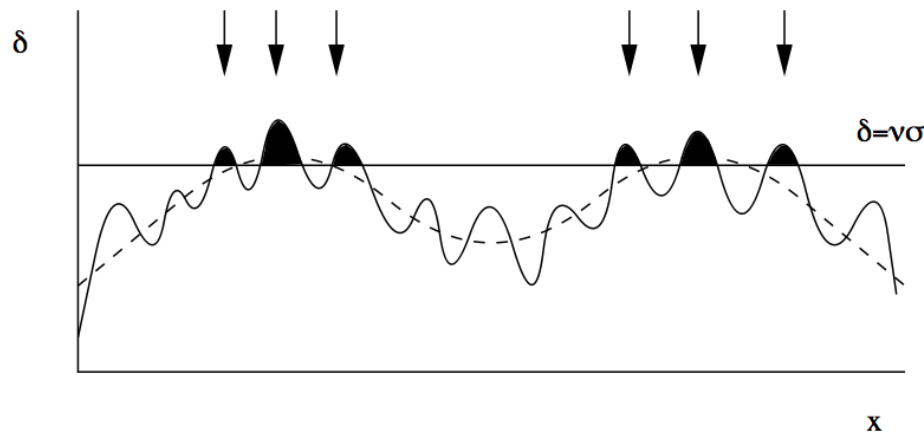


PS++ mass function and NFW++ halo profile gives correct small-scale clustering from random haloes.

Add linear large-scale power for complete model.



# Peak-background split and halo bias



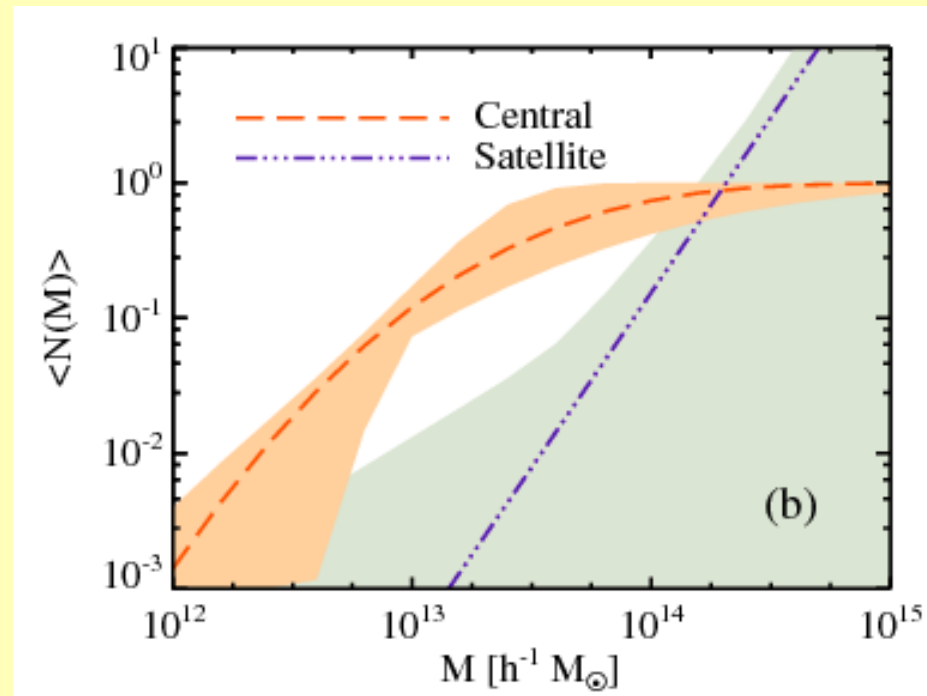
- ▶ Very large wavelength modes effectively shift  $\delta_c$
- ▶  $\delta \rightarrow \delta + \epsilon \Rightarrow \delta'_c = \delta_c - \epsilon$
- ▶  $f(M) \rightarrow f(M) - \frac{df}{d\delta_c} \epsilon$
- ▶  $\Rightarrow b_{\text{Lagrange}} = -\frac{d \ln f}{d\delta_c}$
- ▶  $b_{\text{tot}} = 1 - \frac{d \ln f}{d\delta_c}$
- ▶ e.g. Press-Schechter:  
 $b(\nu) = 1 + \frac{\nu^2 - 1}{\delta_c}$

- Kaiser (1984)
- Sheth & Tormen (1999)
- Hence biased halo clustering:  $\delta_{\text{halo}} = b(M) \delta_{\text{mass}}$



# Galaxies: halo occupation numbers

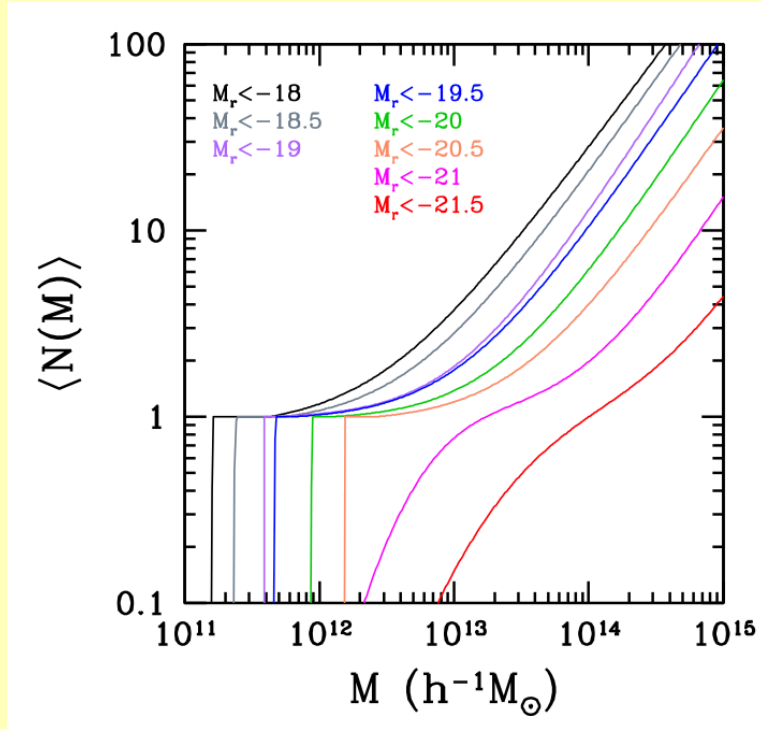
Understand galaxy bias by assigning one central plus Poissonian satellite galaxies as function of halo mass. Simple  $N(M)$  fixed via known  $n$



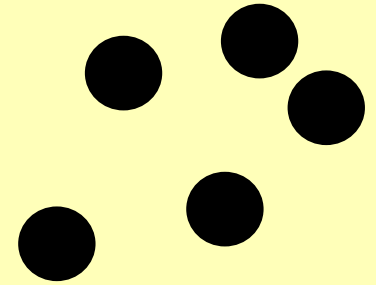
Galaxy bias is weighted mean of halo bias factors

$$b_{\text{tot}} = 1 + \frac{\int_{\nu}^{\infty} b(\nu) w(\nu) \frac{dF}{d\nu} d\nu}{\int_{\nu}^{\infty} w(\nu) \frac{dF}{d\nu} d\nu}$$

# The halo model in SDSS

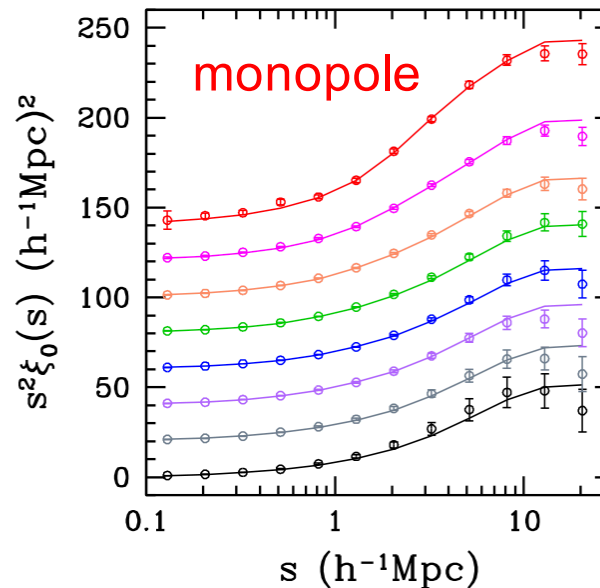
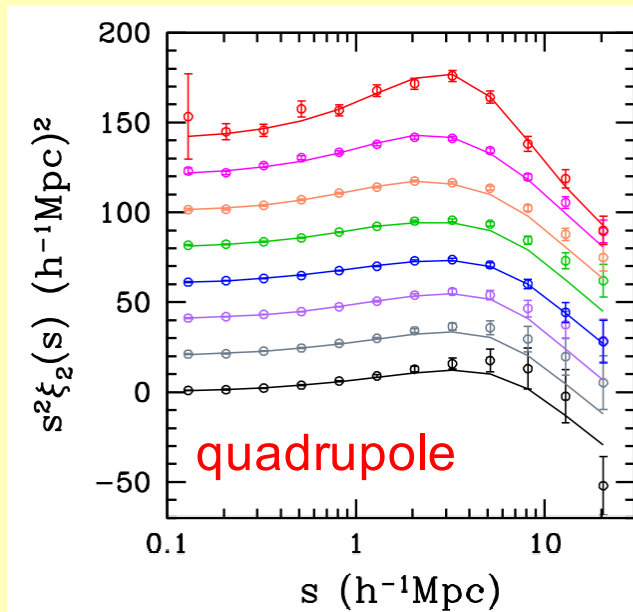


Fitting SDSS:  
Guo et al.  
1505.07861



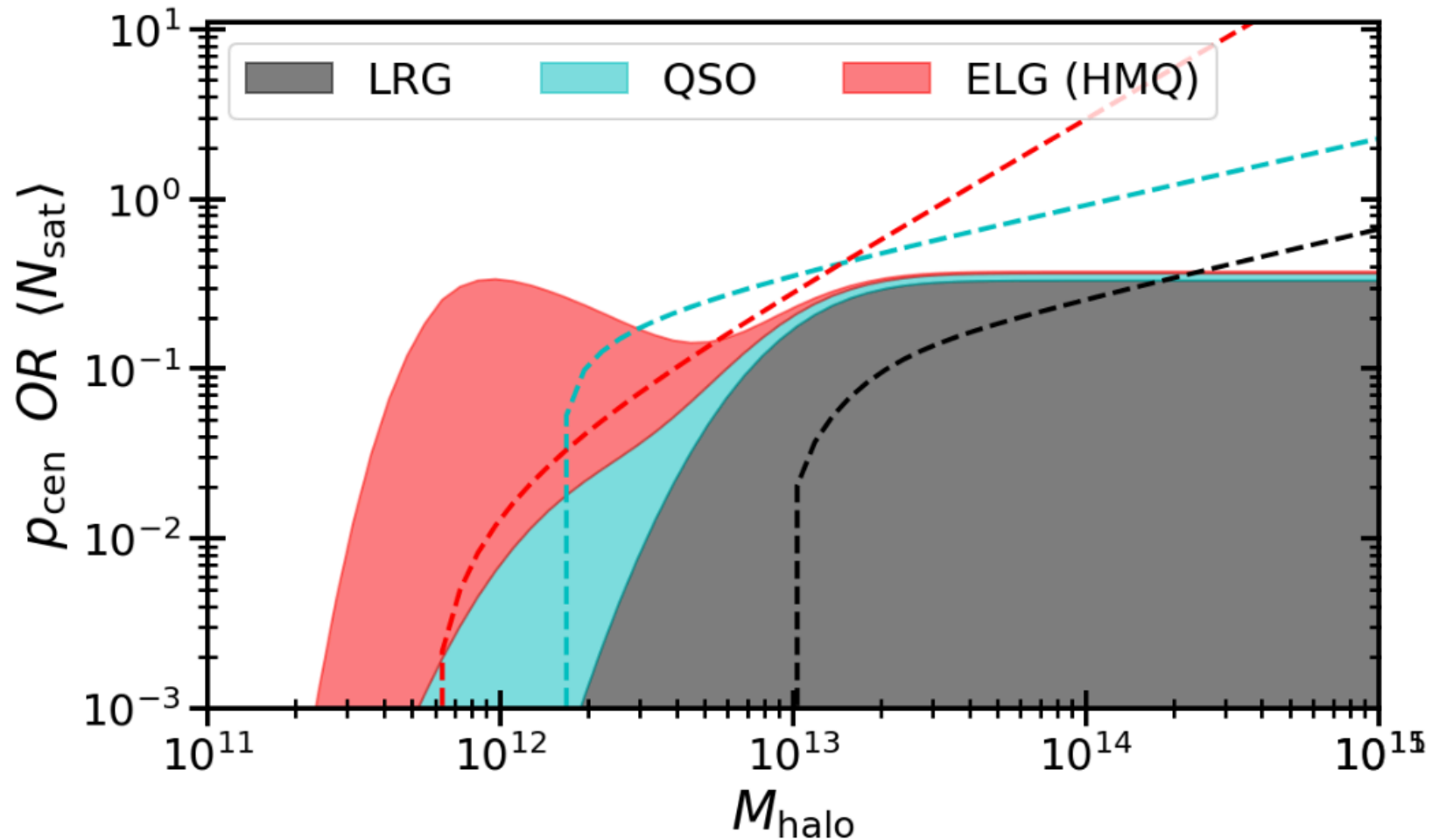
Halo model:

$$\rho = \bullet + \bullet$$



Basis of best  
method for  
rapid mock  
galaxy  
catalogues

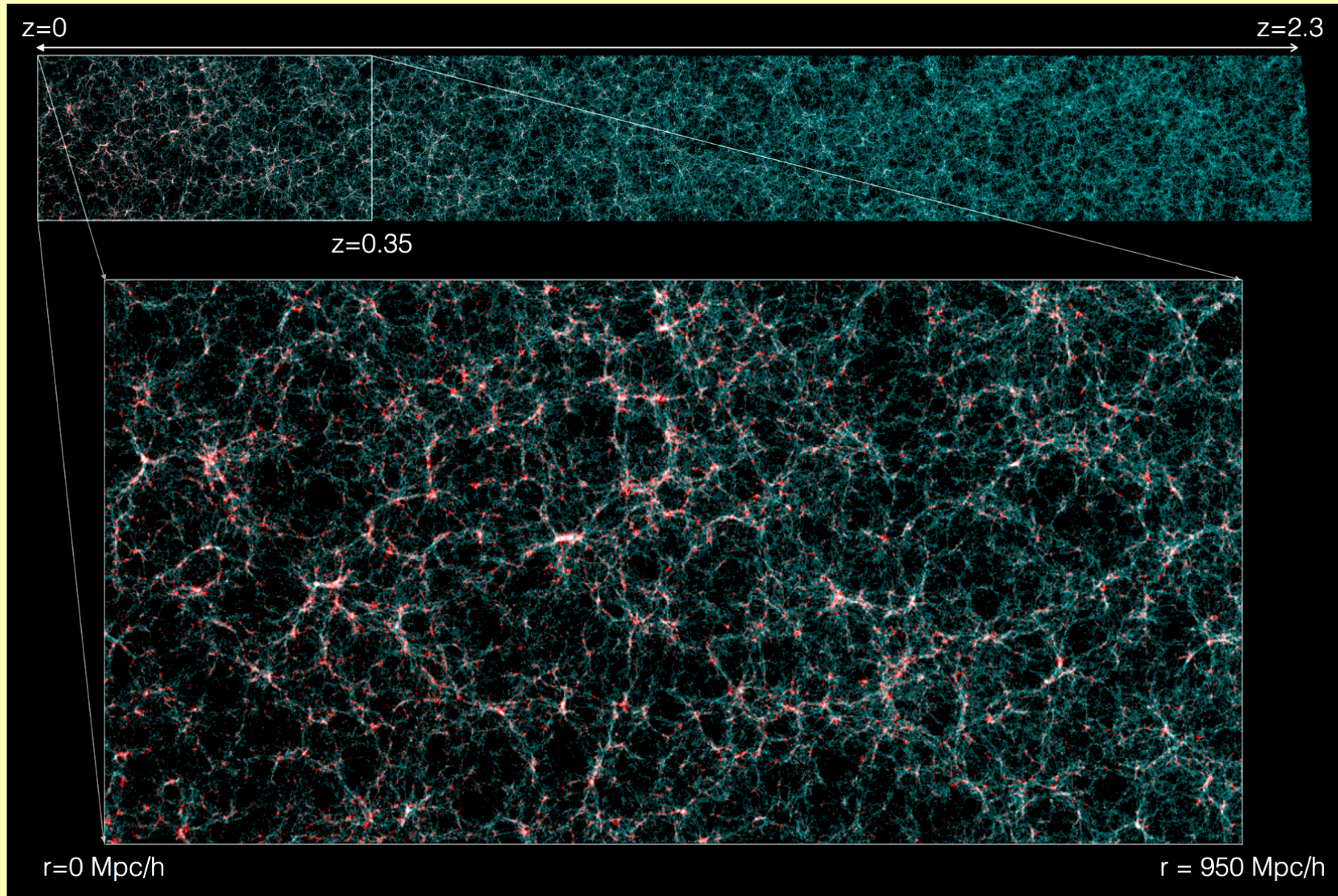
# Multitracer HOD



Need to allow for simultaneous presence of different galaxy types (Alam et al. 1910.05095)



# Euclid Flagship Mock



2 trillion particles; 2 billion 'galaxies' from halo model  $N_g(M_{\text{halo}})$

# Haloes vs perturbation theory

$$\begin{aligned}
 P^{hh}(k) = & \int d^3q e^{i\mathbf{k}\cdot\mathbf{q}} \exp \left[ -\frac{1}{2} k_i k_j A_{ij}^{\text{lin}} \right] \left\{ 1 - \frac{1}{2} k_i k_j A_{ij}^{1\text{-loop}} - \frac{i}{6} k_i k_j k_l W_{ijk}^{1\text{-loop}} \right. \\
 & - b_1 \left( k_i k_j A_{ij}^{10} - 2i k_i U_i^{(1)} \right) + b_1^2 \left( \xi_L + i k_i U_i^{11} - k_i k_j U_i^{(1)} U_j^{(1)} \right) \\
 & + b_2 \left( i k_i U_i^{20} - k_i k_j U_i^{(1)} U_j^{(1)} \right) + b_1 b_2 \left( 2i k_i U_i^{(1)} \xi_L \right) + b_2^2 \left( \frac{1}{2} \xi_L^2 \right) \\
 & - b_{s^2} \left( k_i k_j A_{ij}^{20} - 2i k_i V_i^{10} \right) + b_1 b_{s^2} \left( 2i k_i V_i^{12} \right) + b_2 b_{s^2} \chi^{12} + b_{s^2}^2 \zeta_L \\
 & \left. - \frac{1}{2} \alpha_\xi k^2 + i 2 b_{\nabla^2} \left( k_i \frac{\nabla^2}{\Lambda_L^2} U_i^{(1)} \right) + 2 b_1 b_{\nabla^2} \left( \frac{\nabla^2}{\Lambda_L^2} \xi_L \right) + \dots \right\} + \text{“stochastic”},
 \end{aligned}$$

EFT programme: supplement perturbation expansion with general terms of correct symmetry. Even for matter, hard to get beyond  $k = 0.2h \text{ Mpc}^{-1}$ .

# $N(M_{+++})?$ Assembly bias

- **Not** just that haloes collapsing early are more clustered
  - Always present in Kaiser (1984)
  - Halo model averages over such effects:

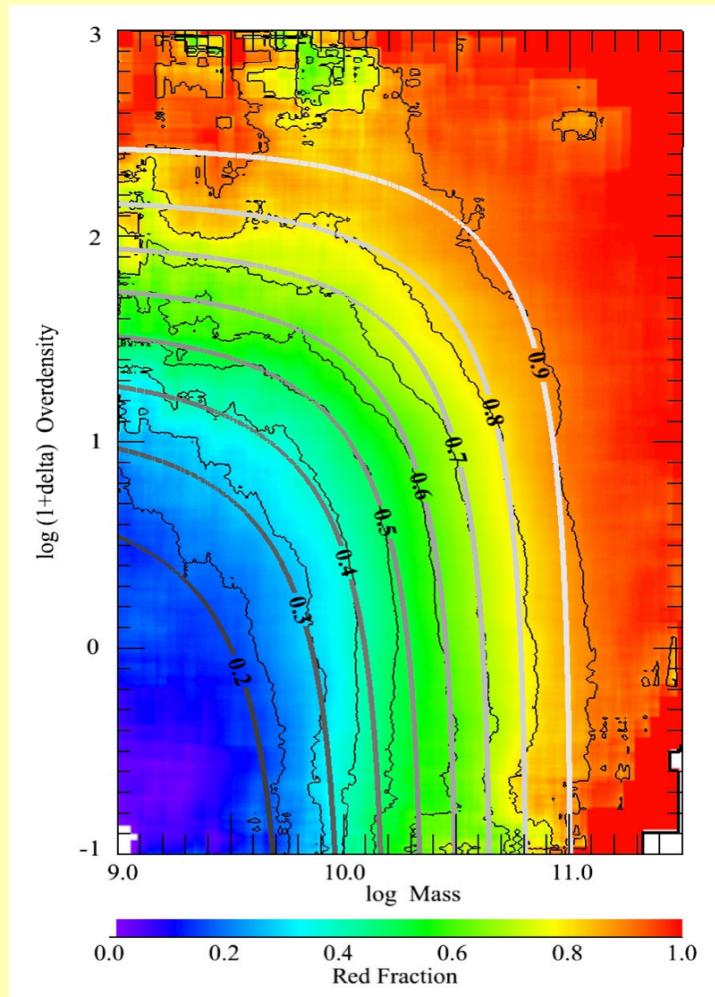
$$b(M, z_f) + N(M): \langle b N \rangle = \langle b \rangle \langle N \rangle$$

- **But galaxy contents(M) can couple to formation z:**
  - Early formation yields older stars
  - But deeper potential: harder to quench?
  - Early formation gives fewer subhaloes (= satellites)

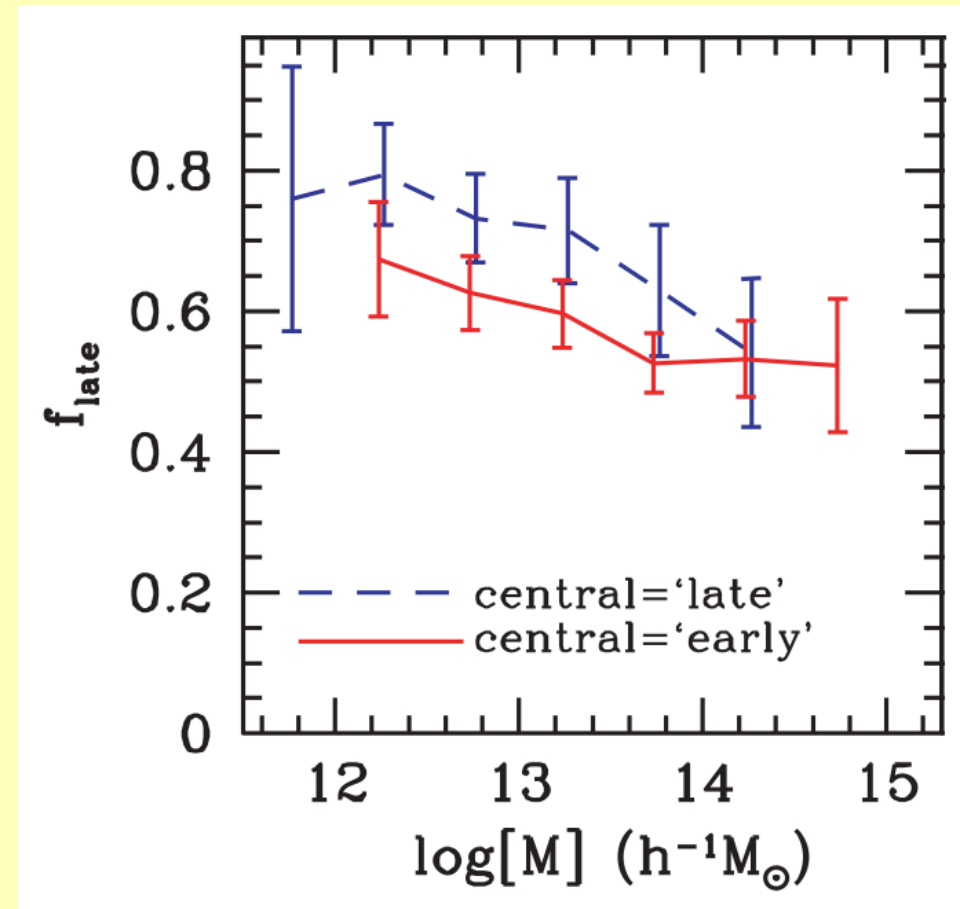
$$b(M, z_f) + N(M, z_f): \langle b N \rangle \neq \langle b \rangle \langle N \rangle$$



# Environment and galaxy formation



Quenching empirically relates to environment (Peng et al. 2010)



Whole-halo phenomenon: 'galactic conformity' as sign of assembly bias (Weinmann et al. 2006)



# Cosmic variance and survey design

Gaussian field: real and imaginary parts of  $\delta_k$  have Gaussian distributions.  
Hence power for one mode has an exponential distribution:

$$\text{prob}(P > X) = \exp[-X/\langle P \rangle]$$

Hence error in power is  $\delta P = (P + P_{\text{shot}}) / (N_{\text{modes}})^{1/2}$

Take 2 volumes with different number densities. Weight power by reciprocal variance – i.e. by  $1/(1+P_{\text{shot}}/P)^2 = 1/(1+1/nP)^2$

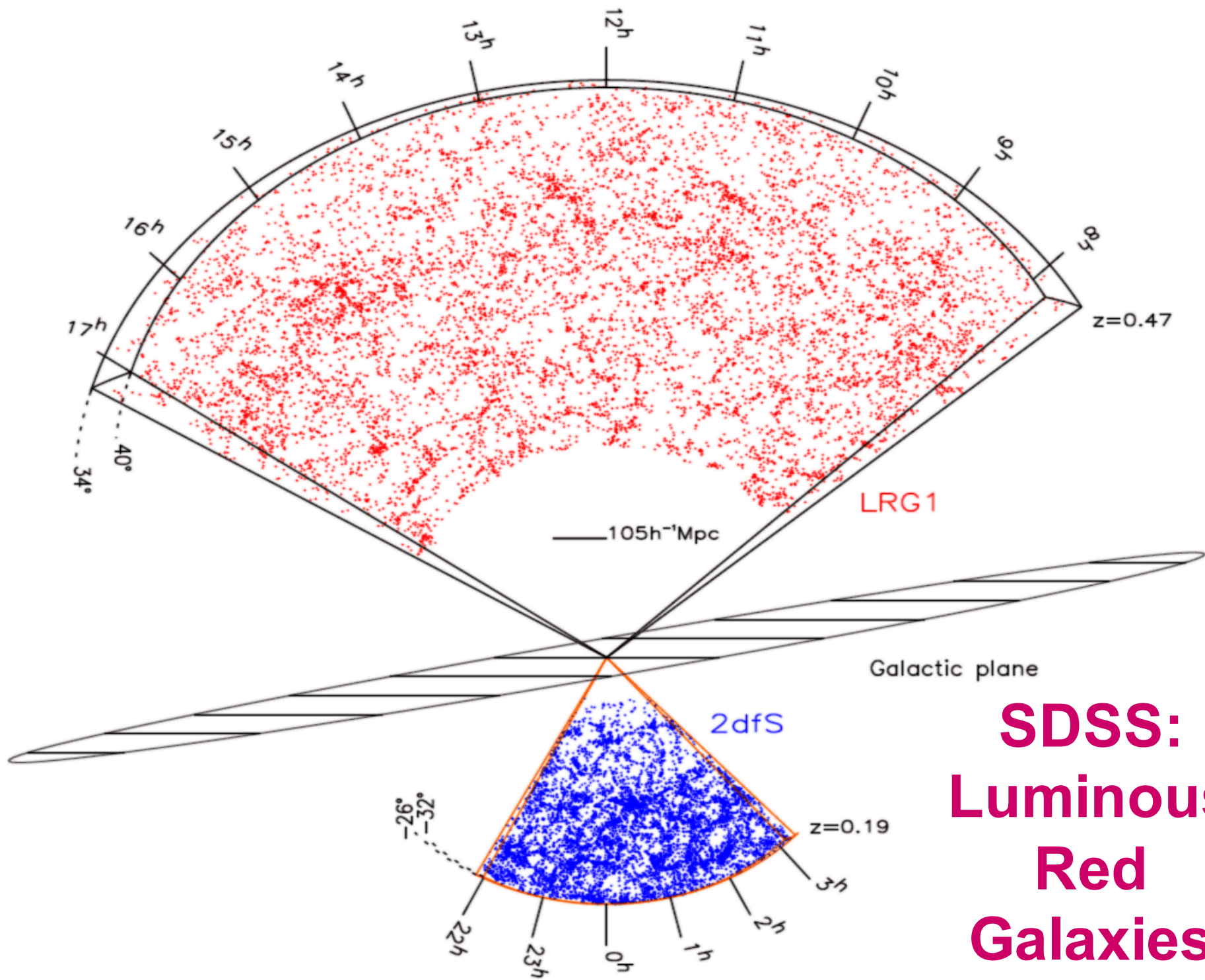
Hence weight  $\delta$  by  $= 1/(1+1/nP)$  – i.e. weight each galaxy by  $1/(1+nP)$

FKP weight (1994):

equal wt per galaxy at low  $n$ ; equal weight per volume at high  $n$

For fixed survey time, volume  $\propto 1/n$ , so  $N_{\text{modes}} \propto 1/n$

Hence  $nP = 1$  for optimum target density.



**SDSS:  
Luminous  
Red  
Galaxies**

# Non-Gaussianity

Potentially deepest impact of LSS on initial conditions

Dalal et al., Matarrese &  
Verde, Slozar et al., 2008

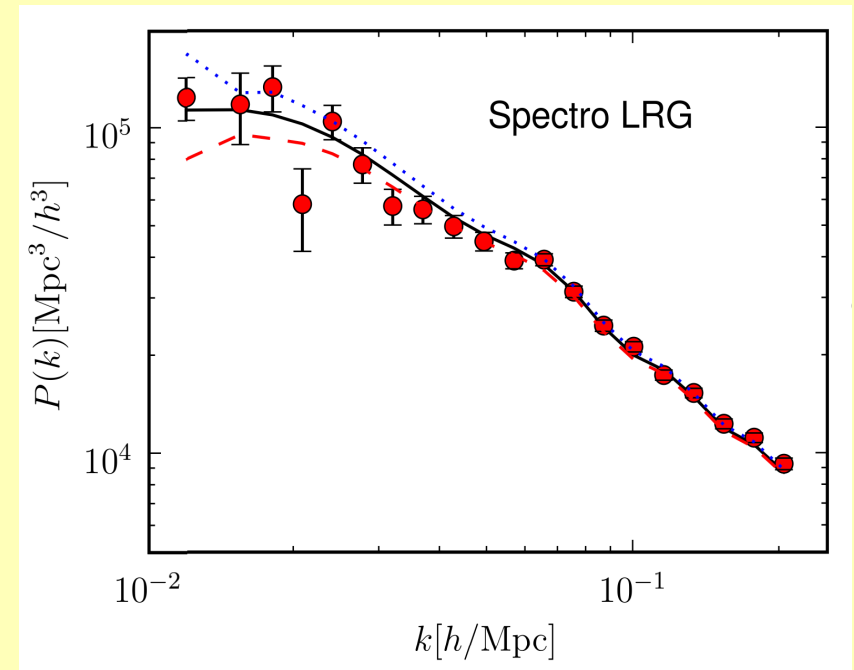
$$\Phi \rightarrow \Phi + f_{\text{NL}}(\Phi^2 - \langle \Phi^2 \rangle)$$

$$\text{Poisson : } \delta \rightarrow \delta(1 + 2f_{\text{NL}}\Phi)$$

$$\text{Threshold : } \delta_c \rightarrow \delta_c(1 + 2f_{\text{NL}}\Phi)$$

$$\text{Same effect as long-wavelength } \delta_+ = 2f_{\text{NL}}\delta_c\Phi$$

$$\text{Extra bias } \Delta b = b_L\delta_+/\delta = 2b_Lf_{\text{NL}}\delta_c\Phi/\delta \sim f_{\text{NL}}\left(\frac{kc}{H}\right)^{-2}$$

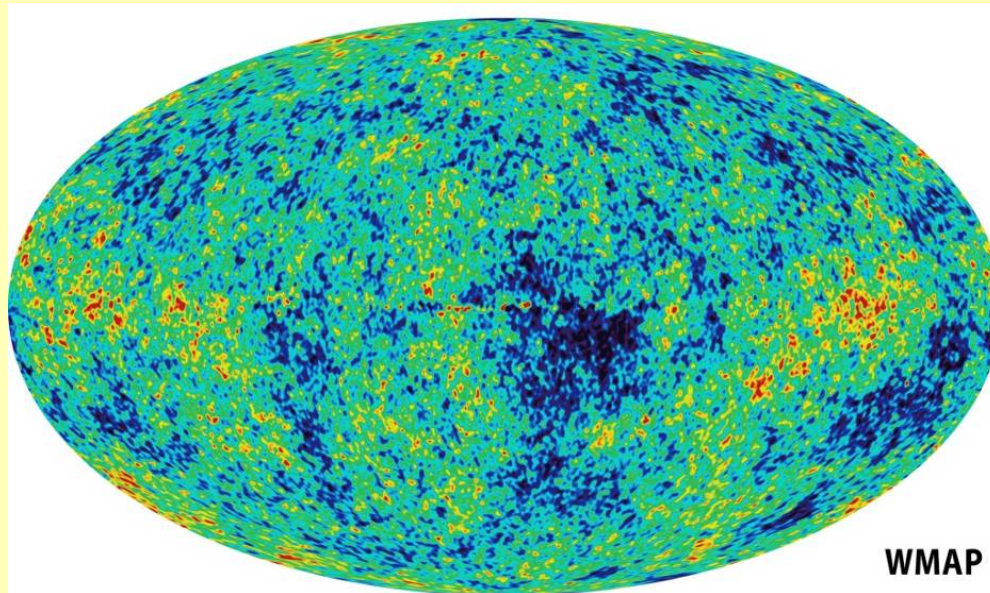


Scale-dependent bias limits  $f_{\text{NL}}$  with precision  $\sim 25$

– less strong than Planck, but DESI/Euclid should reach  $f_{\text{NL}} \sim 1$

# **3: Geometrical cosmology**

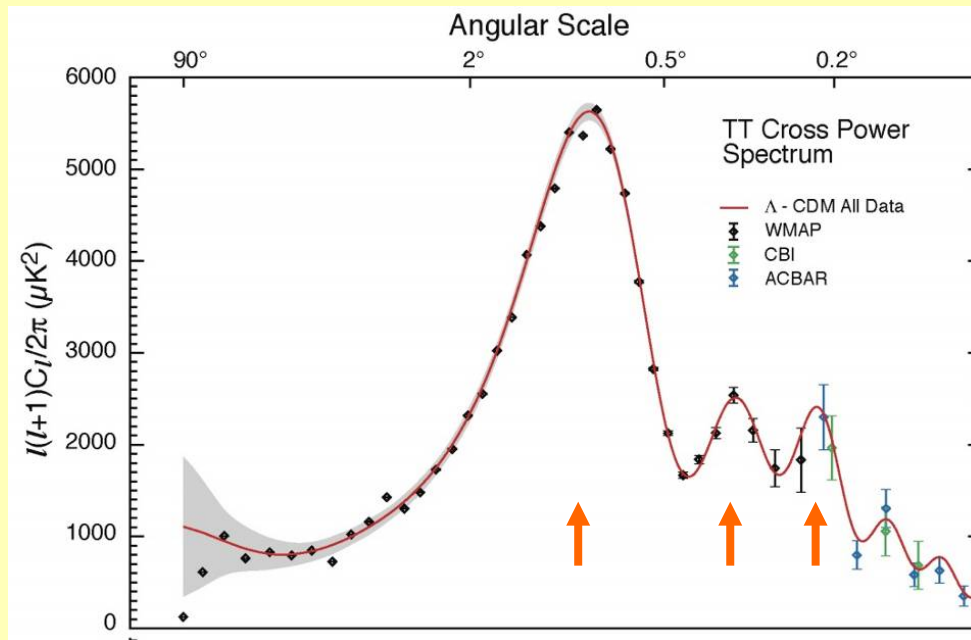
# Baryon Acoustic Oscillations



The (comoving) distance that sound waves travel by recombination sets the length of the BAO cosmic ruler at  $t = 380,000$  years:

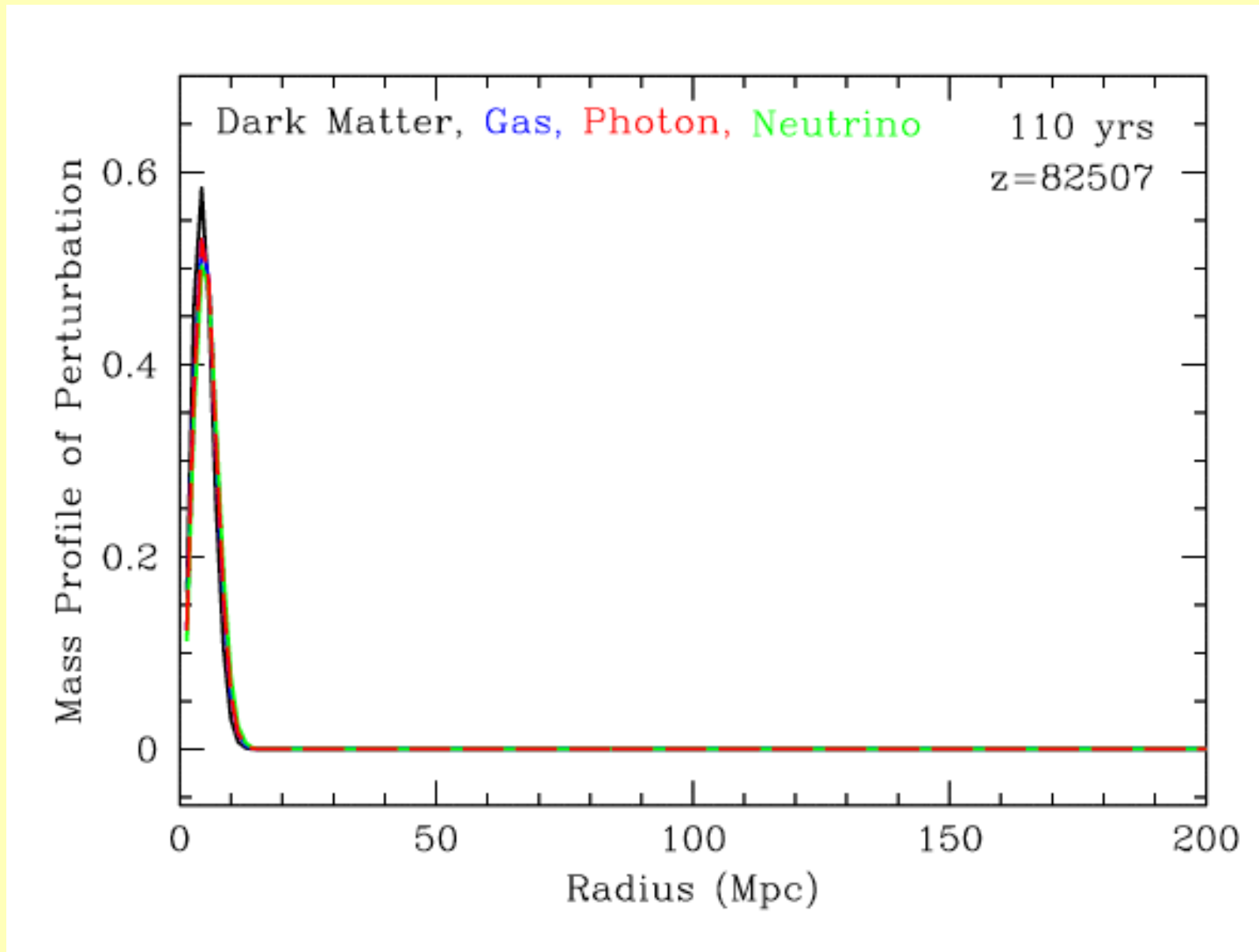
$$l_{\text{BAO}} = \int_0^{t_{\text{rec}}} \frac{c_s}{a} dt \approx \frac{c}{\sqrt{3}} \frac{t_{\text{rec}}}{a_{\text{rec}}}$$

$$a_{\text{rec}} = 1/1100$$



‘Baryon wiggles’ at 1 degree (& 0.3, 0.2, 0.1...): **150 Mpc at 13 Gpc** Oscillations of baryonic gas and radiation before decoupling

# BAO Green's function



Eisenstein, Seo, and White (2007). Nice, but fluctuations were not created by propagation of sound waves

# Two acoustic scales

(1) For CMB structure we need the acoustic horizon at  $z=1080$ :

$$s = 145.0 (\Omega_m h^2 / 0.140)^{-0.25} (\Omega_b h^2 / 0.0225)^{-0.08} \text{ Mpc}$$

(2) But for LSS we need the "drag redshift" where decoupling from Thomson scattering becomes total. According to (6) of Eisenstein & Hu 1998, this is  $z_d=1020$  for Planck parameters – so the final BAO horizon is slightly larger than for CMB. Since 2014, standard  $s$  is 2.6% smaller than EH98 (see 1312.4877 & 1411.1074):

$$s = 147.7 (\Omega_m h^2 / 0.140)^{-0.26} (\Omega_b h^2 / 0.0225)^{-0.13} \text{ Mpc}$$

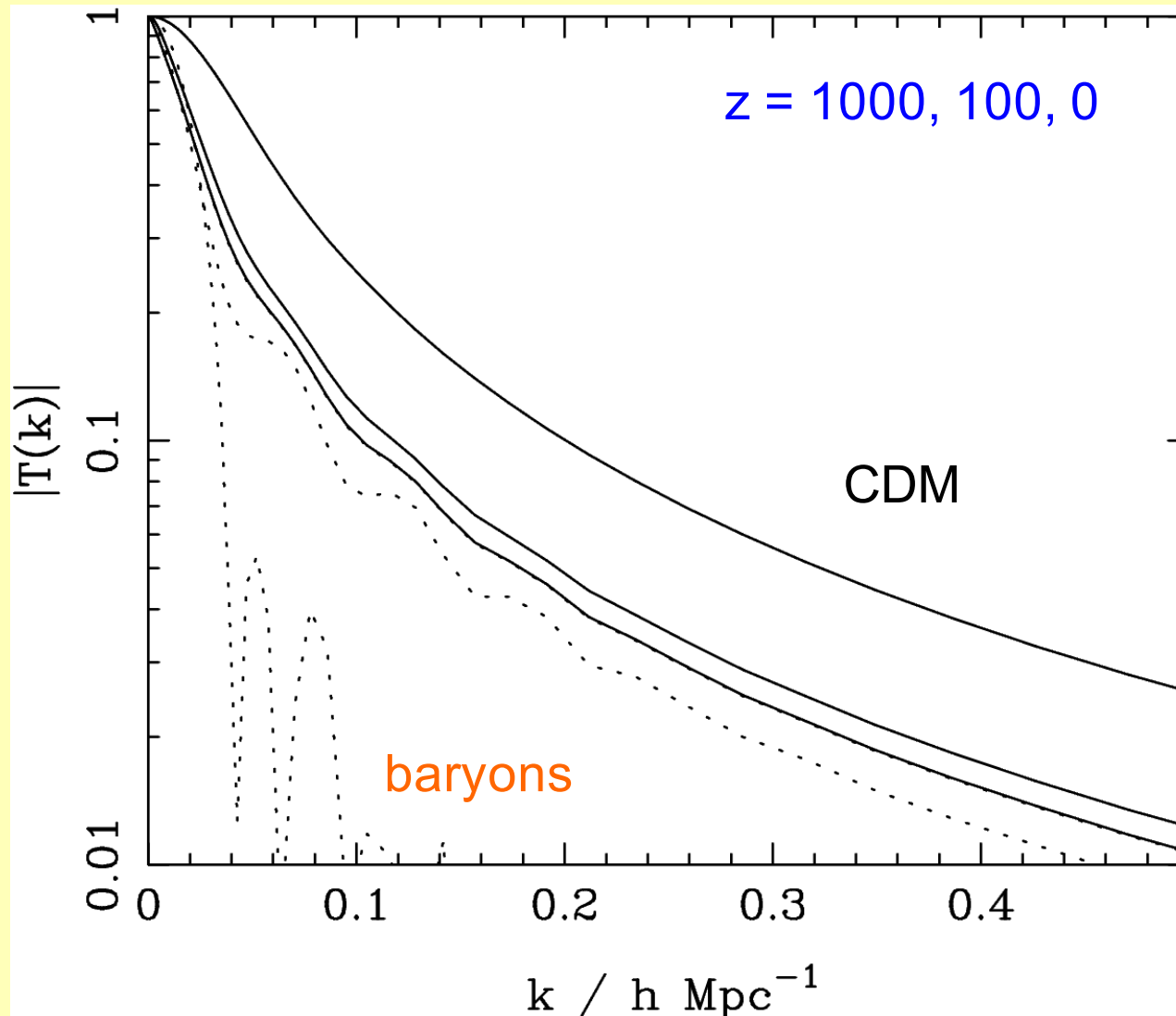
(3) The BAO peak in  $\xi(r)$  is at  $105 h^{-1}$  Mpc, empirically

consistency with (1) requires  $h = 0.71$



# Evolution of transfer functions

Radiation era:  $T(k) = 3(\sin x - x \cos x)/x^3$     $x = k \eta c_s$     $d\eta = dt/a(t)$



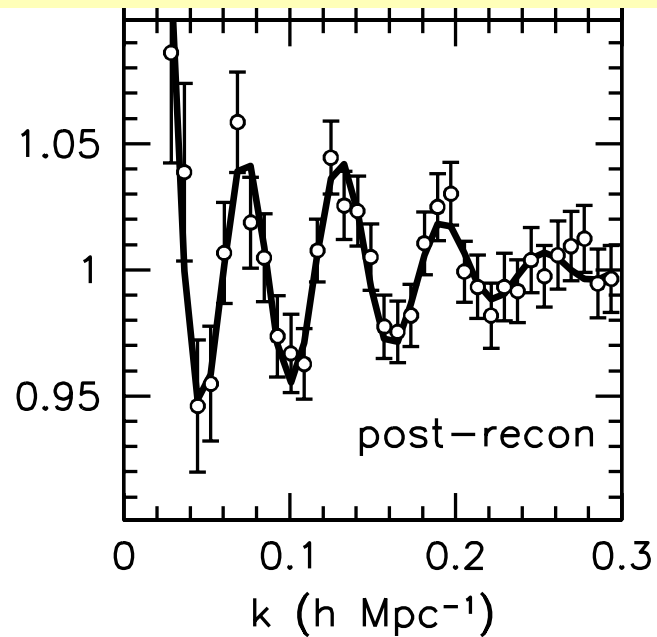
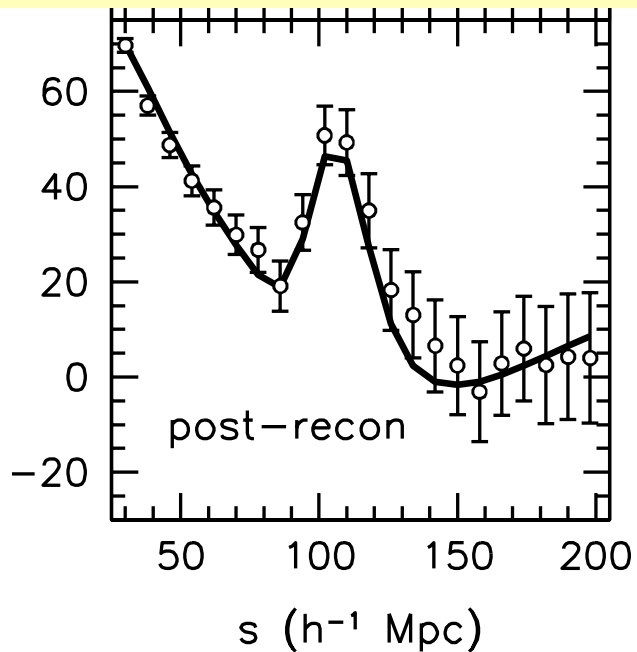
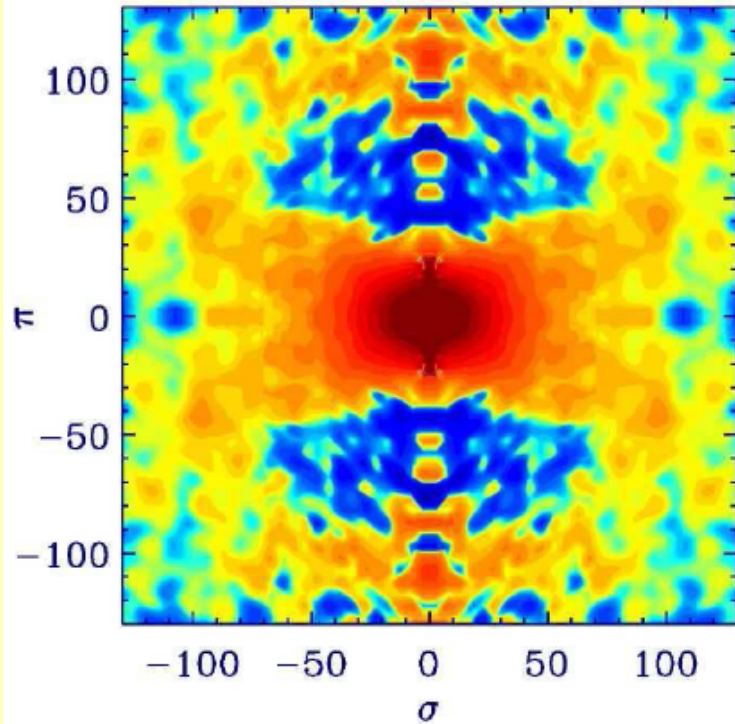
Wiggles in CDM arise after last scattering, and are in place only at  $z=50$



# BAO: the acoustic horizon in SDSS

Acoustic horizon at drag era  
( $z=1020$ ):

$$s = 147 (\Omega_m h^2 / 0.142)^{-0.26} (\Omega_b h^2 / 0.0225)^{-0.13} \text{ Mpc}$$

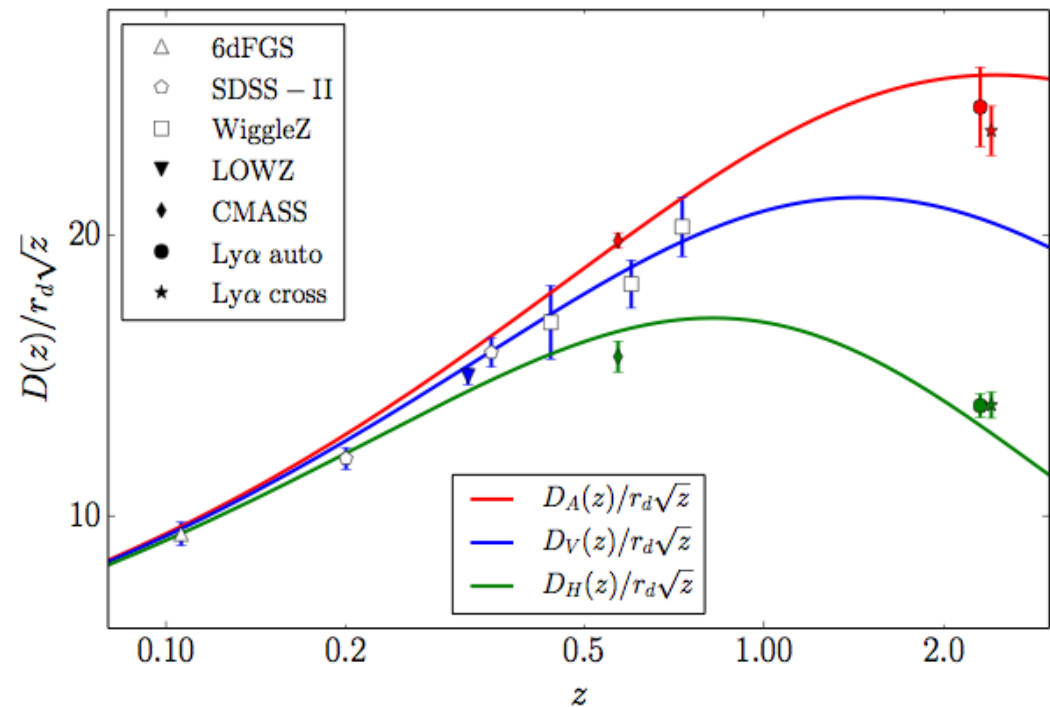
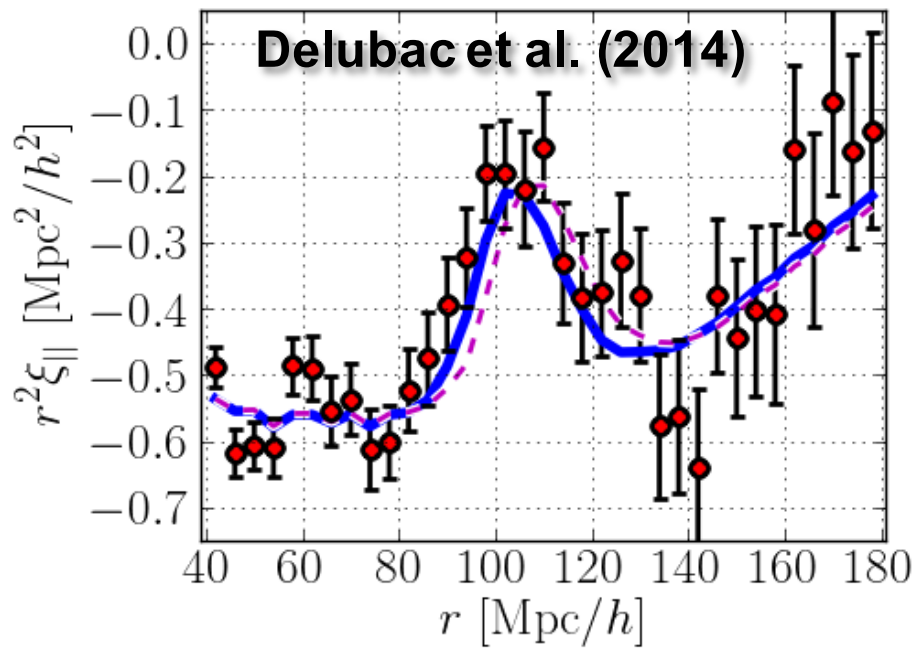


Measure  
transversely and  
radially:

=>  $D(z)$  &  $H(z)$

Anderson et al.  
(2014)

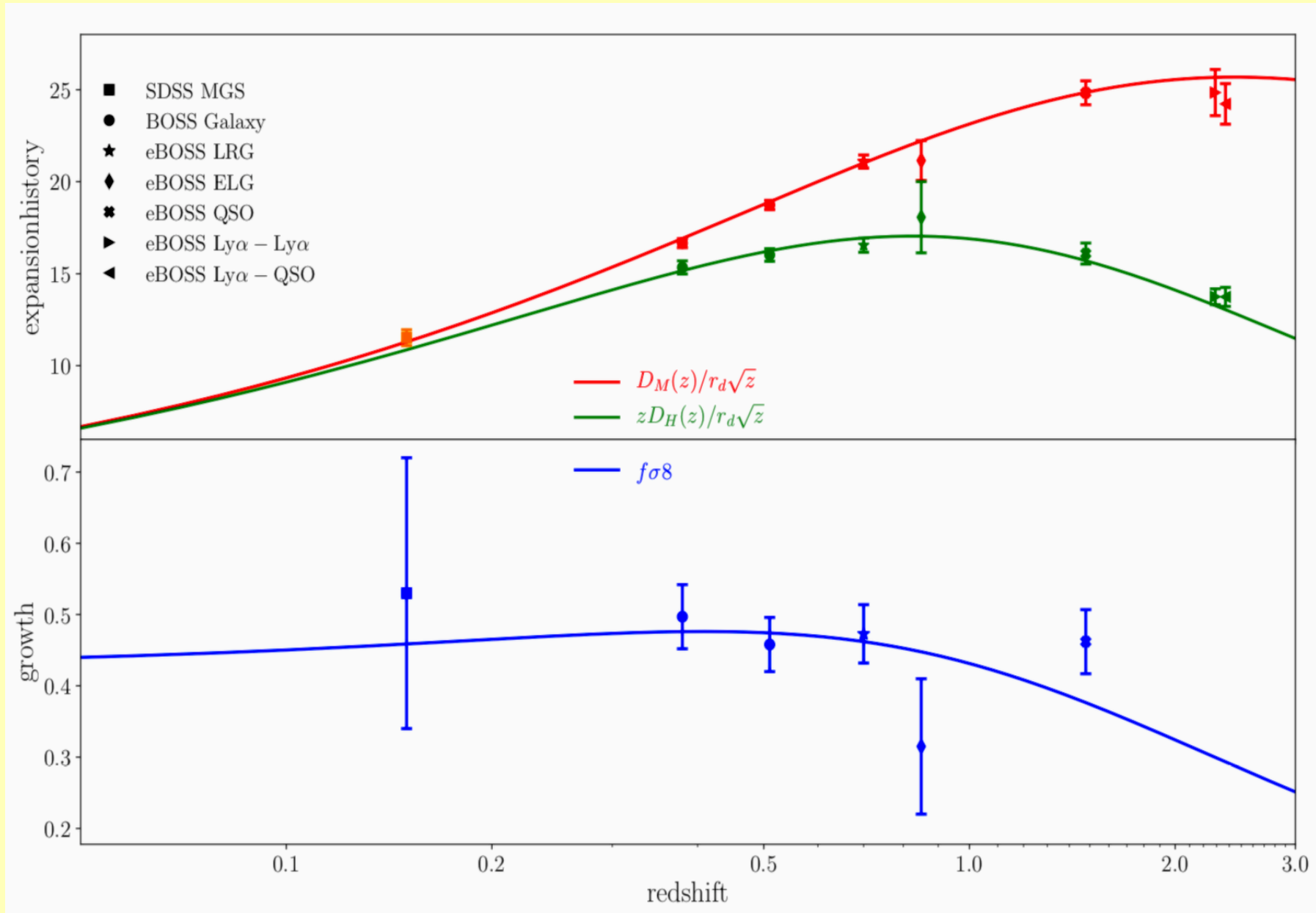
# BAO in the Forest



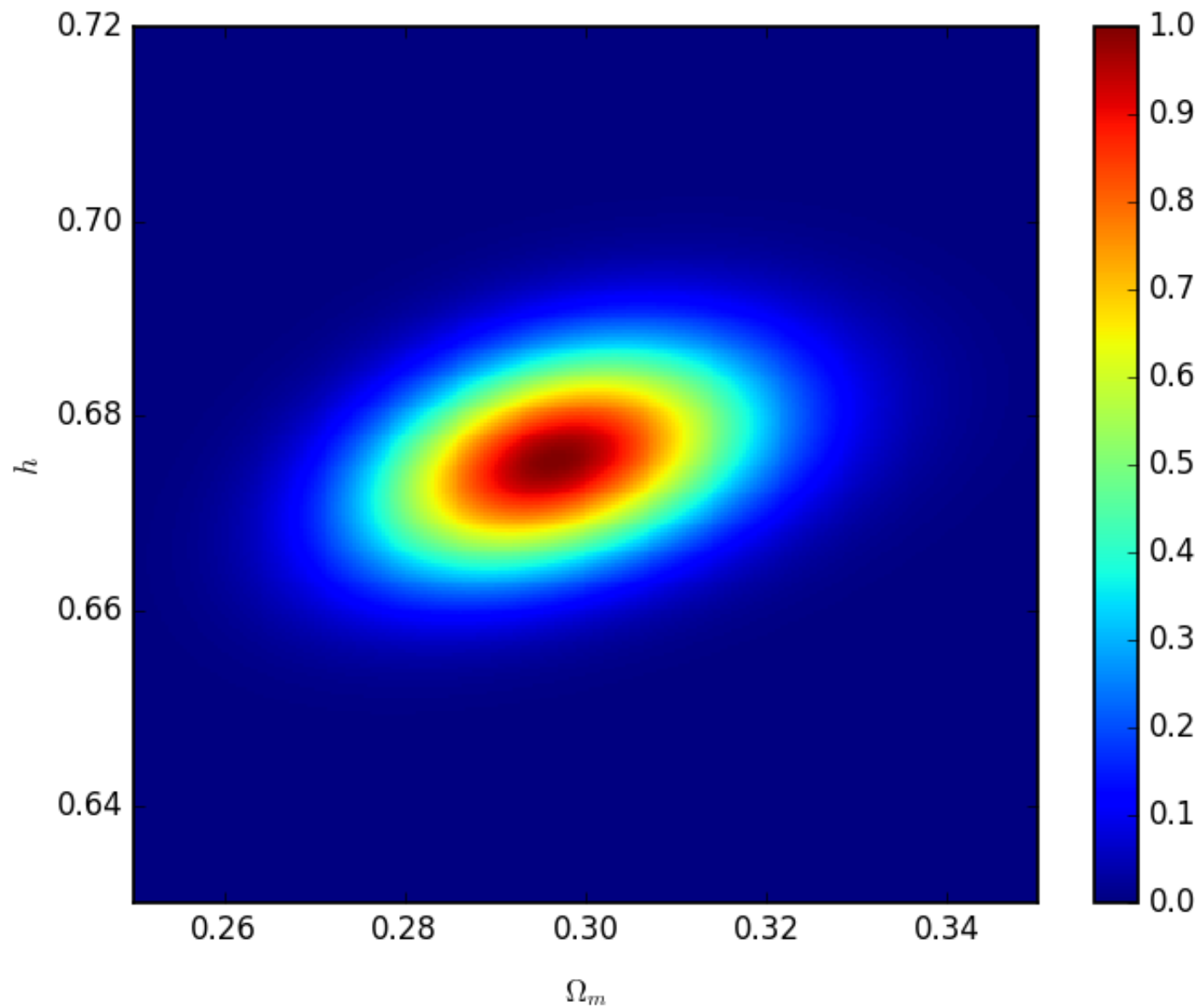
BAO detection transversely and radially from correlations between 140,000  $z > 2$  quasar spectra:

Busca et al.; Slosar et al.; Delubac et al.; Font-Ribera et al.

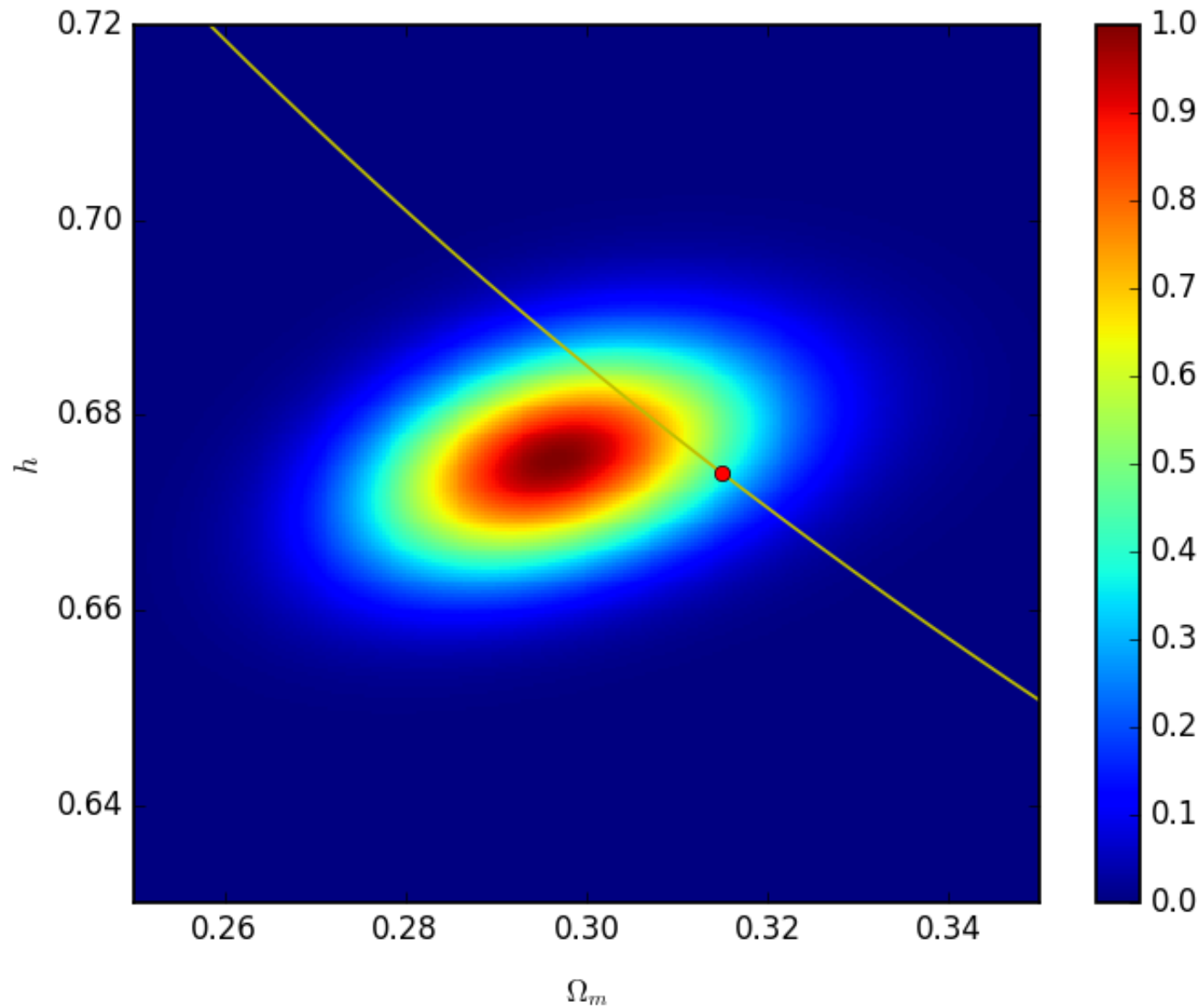
# 20 years of SDSS (2007.08991)



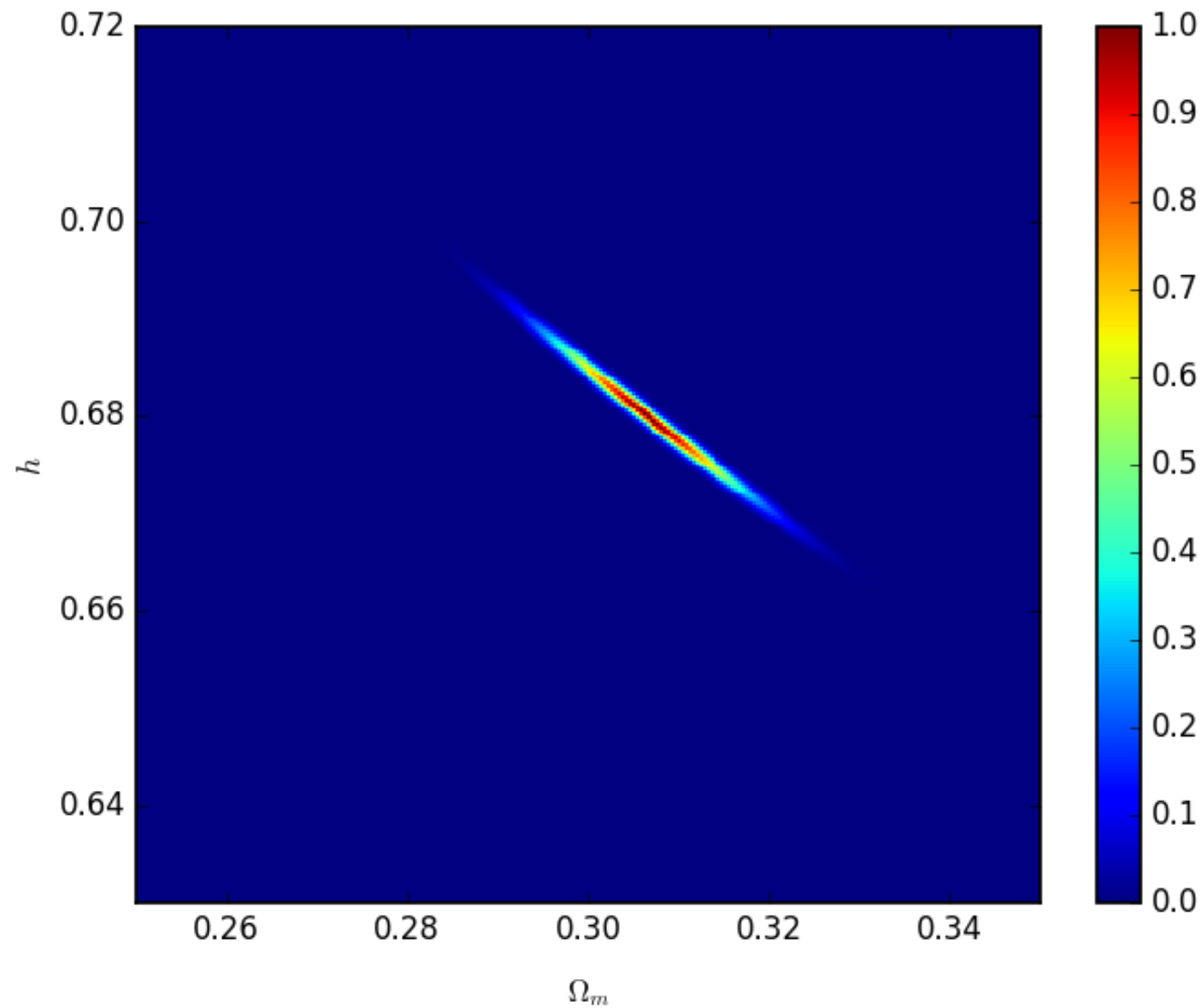
# BAO + BBN flat constraints



# BAO + BBN flat constraints



# BAO + BBN flat constraints





# Sensitivity to Dark Energy

$$D(z) = \frac{c}{H_0} \int_0^z \frac{dz}{[\Omega_v(1+z)^{3+3w} + \Omega_m(1+z)^3 + \Omega_k(1+z)^2]^{1/2}}$$

Dark Energy affects  $H(z)$ ,  $D(z)$  and perturbation growth  $g(z)$

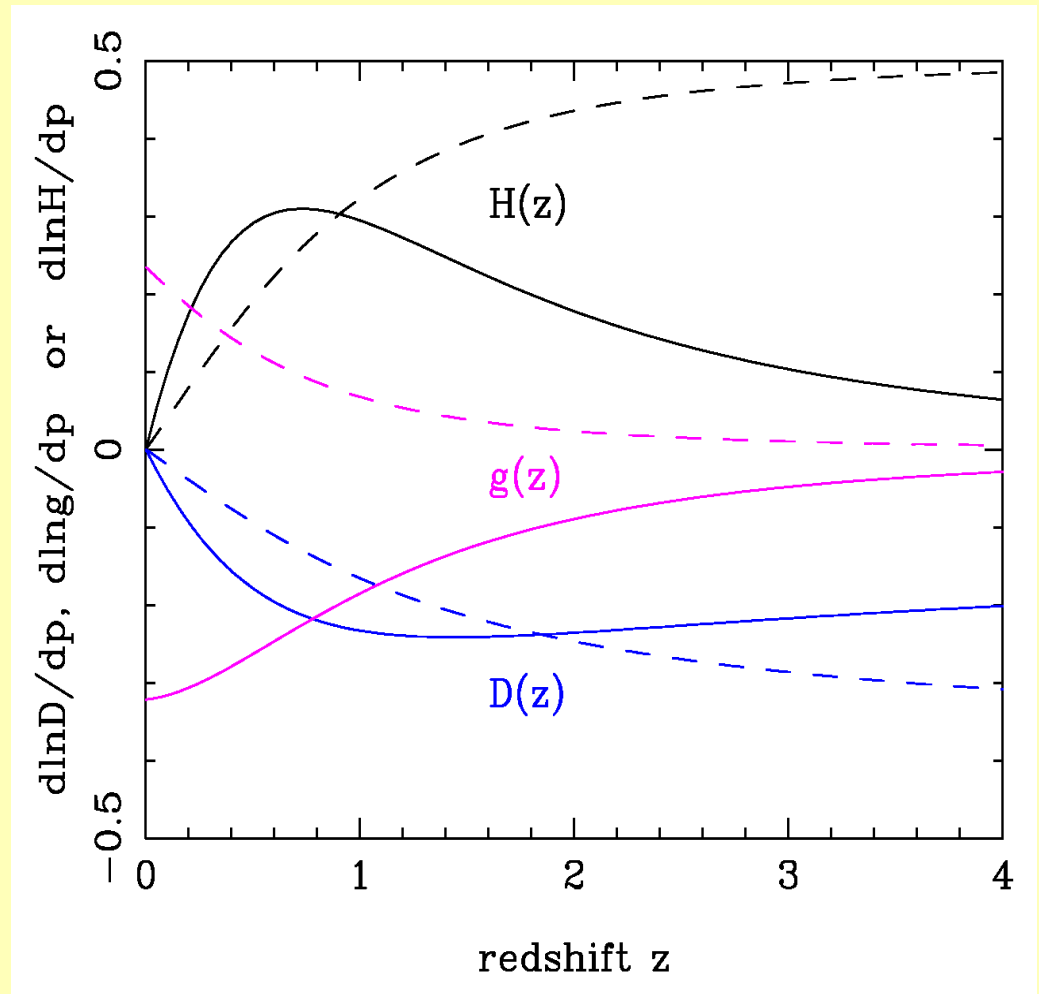
Effects of  $w$  are:

(1) Small (need  $D$  to 1% for  $w$  to 5%)

**Rule of 5**

(2) Degenerate with changes in  $\Omega_m$

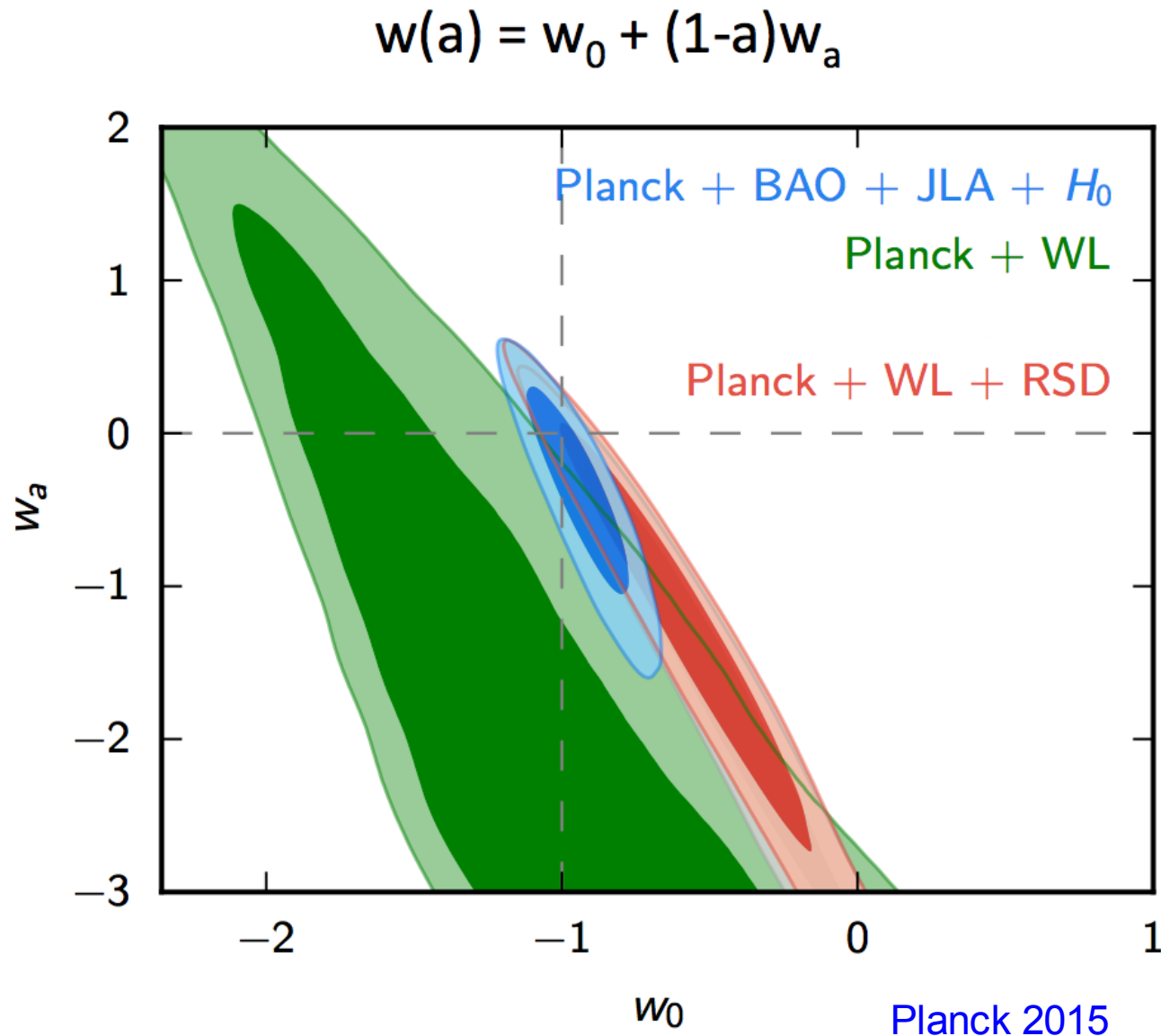
Future target should be  $<1\%$  on BAO scale, requiring much larger redshift surveys



Solid: vary  $w$

Dashed: vary  $\Omega_m$

# BAO limits on DE equation of state ( $w = P / \rho c^2$ )



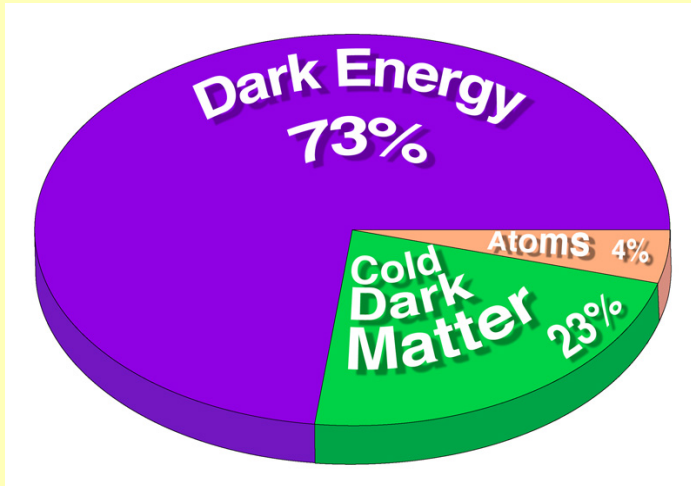
$w = -1 \pm 0.06$   
if unevolving:  
DE looks like  
cosmological  
constant

Future probes  
need to achieve  
<1% accuracy in  
 $D(z)$

**4:**

# **Peculiar velocities and RSD**

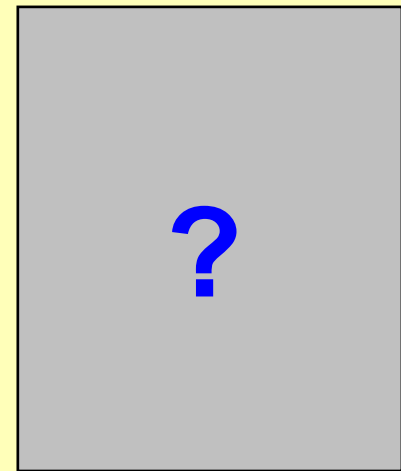
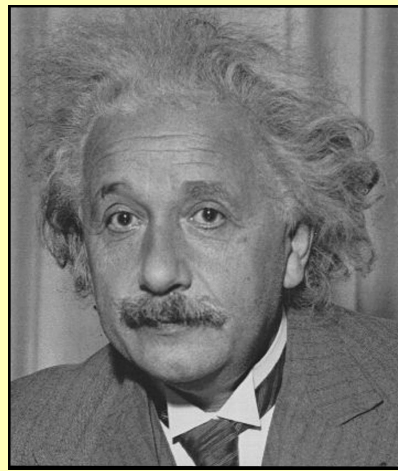
# Dark energy or modified gravity?



Dark energy: inferred assuming  $H(z)$  comes from standard Friedmann equation.

Focus on equation of state  $w = P / \rho c^2 (= -1?)$  assumes DE is a real substance – but is it?

$$H^2(z) = H_0^2 \left[ \underbrace{(1-\Omega)}_{\text{Curvature}} (1+z)^2 + \underbrace{\Omega_M}_{\text{matter}} (1+z)^3 + \underbrace{\Omega_R}_{\text{radiation}} (1+z)^4 + \underbrace{\Omega_{DE}}_{\text{extra term from non-Einstein?}} (1+z)^{3(1+w)} \right]$$



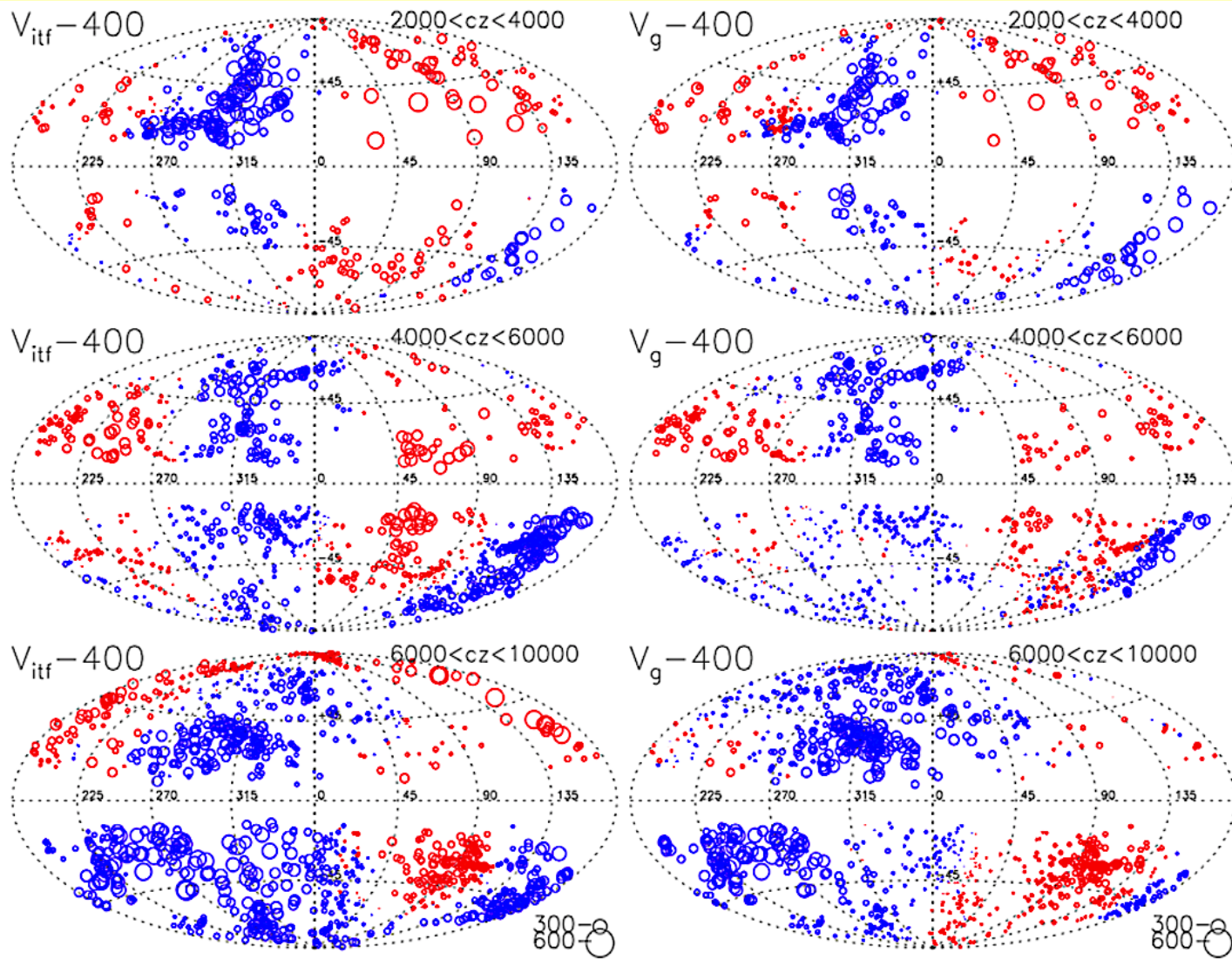
## How can we tell?

Measure gravity on intermediate scales, using the rate of growth of density fluctuations and the ‘peculiar velocities’ (deviations from uniform expansion) associated with this structure formation

# Peculiar velocity surveys

Davis & Nusser  
1410.7622:  
estimate  
velocities from  
TF etc.  
distances.

Excellent  
match with  
peculiar gravity



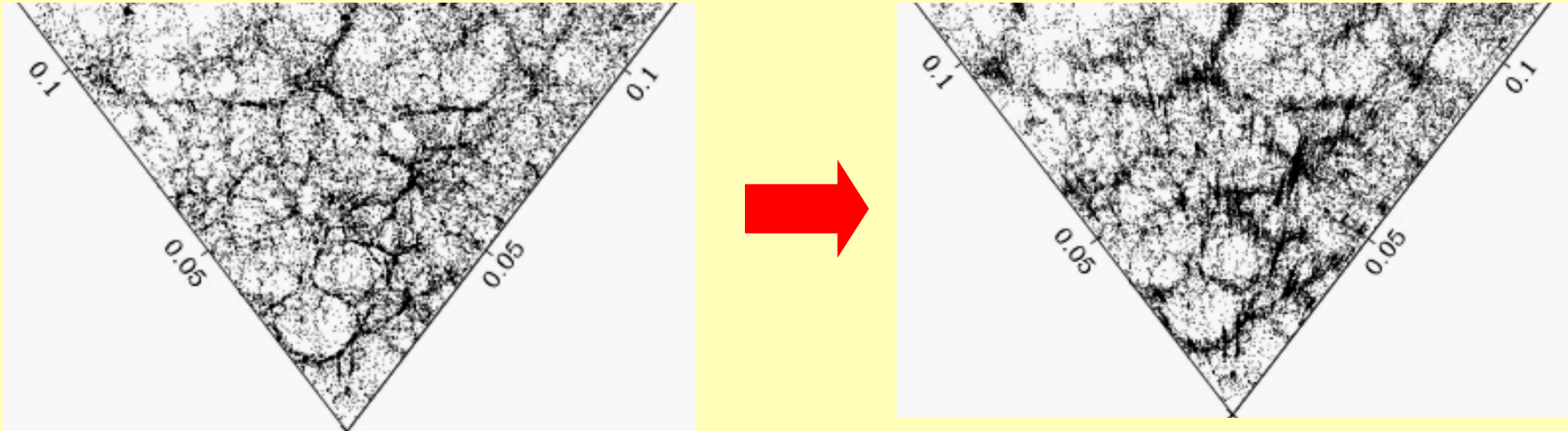
$$\mathbf{v}_g(\mathbf{r}) = \frac{2f(\Omega)}{3H_0\Omega} \mathbf{g}(\mathbf{r})$$

$$\mathbf{g}(\mathbf{r}) = G\bar{\rho} \int d^3\mathbf{r}' \delta_\rho(\mathbf{r}') \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^3}$$



# Redshift-space distortions as a probe of gravity

$$D \simeq cz/H \rightarrow (cz_{\text{cos}} + \delta v)/H$$



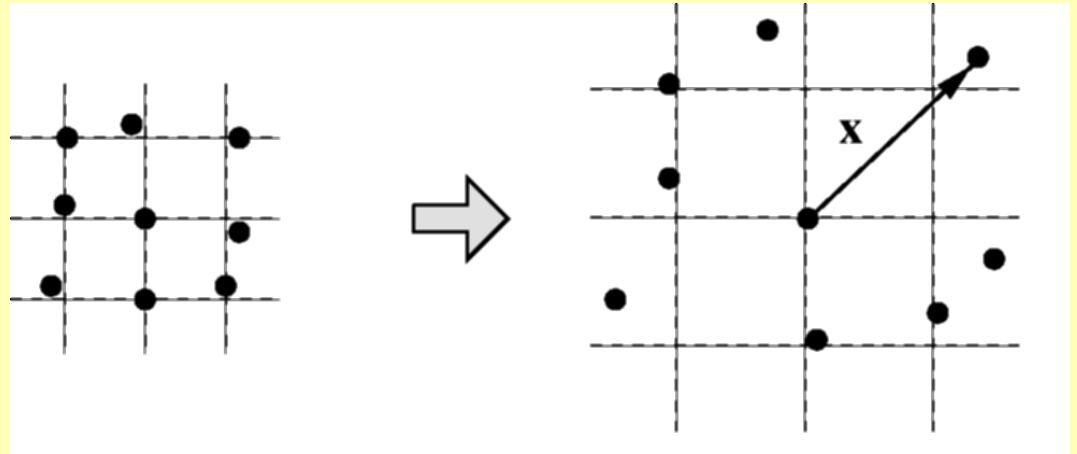
Mass: measure  $f_g \equiv d \ln \delta / d \ln a$  ( $\simeq \Omega_m^{0.55}$  for standard gravity)

Galaxies: measure  $\beta \equiv f_g/b$ ;  $b$  unknown, but  $f_g \sigma_8$  observable

$P(k)$  approximately Kaiser-Lorentz:  $P(k, \mu) = P_{\text{real}}(1 + \beta\mu^2)^2(1 + k^2\sigma_p^2/2)^{-1}$

Infer  $\beta$  from quadrupole-to-monopole ratio in anisotropic power spectrum  
Use simulations to assess deviations from simple distortion model (and to assign errors)

# Linear redshift-space distortions



Displacement field:  $\mathbf{x} = \mathbf{q} + \mathbf{D}(\mathbf{q})$

Density contrast via Jacobian:  $1 + \delta = |\partial \mathbf{q} / \partial \mathbf{x}| = |\delta_{ij} + \partial D_i / \partial x_j|^{-1} \simeq 1 - \nabla \cdot \mathbf{D}$

Peculiar velocity and continuity:  $\mathbf{u} = \dot{\mathbf{D}} \quad \dot{\delta} = -\nabla \cdot \mathbf{u}$

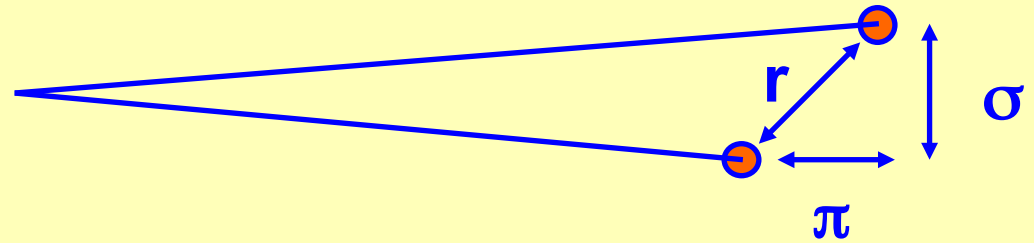
Linear growth:  $\delta \propto g(t)$     Linear bias:  $\delta_g = b\delta_m$

Apparent displacement:  $b\mathbf{D} + H^{-1}(\mathbf{u} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}}$

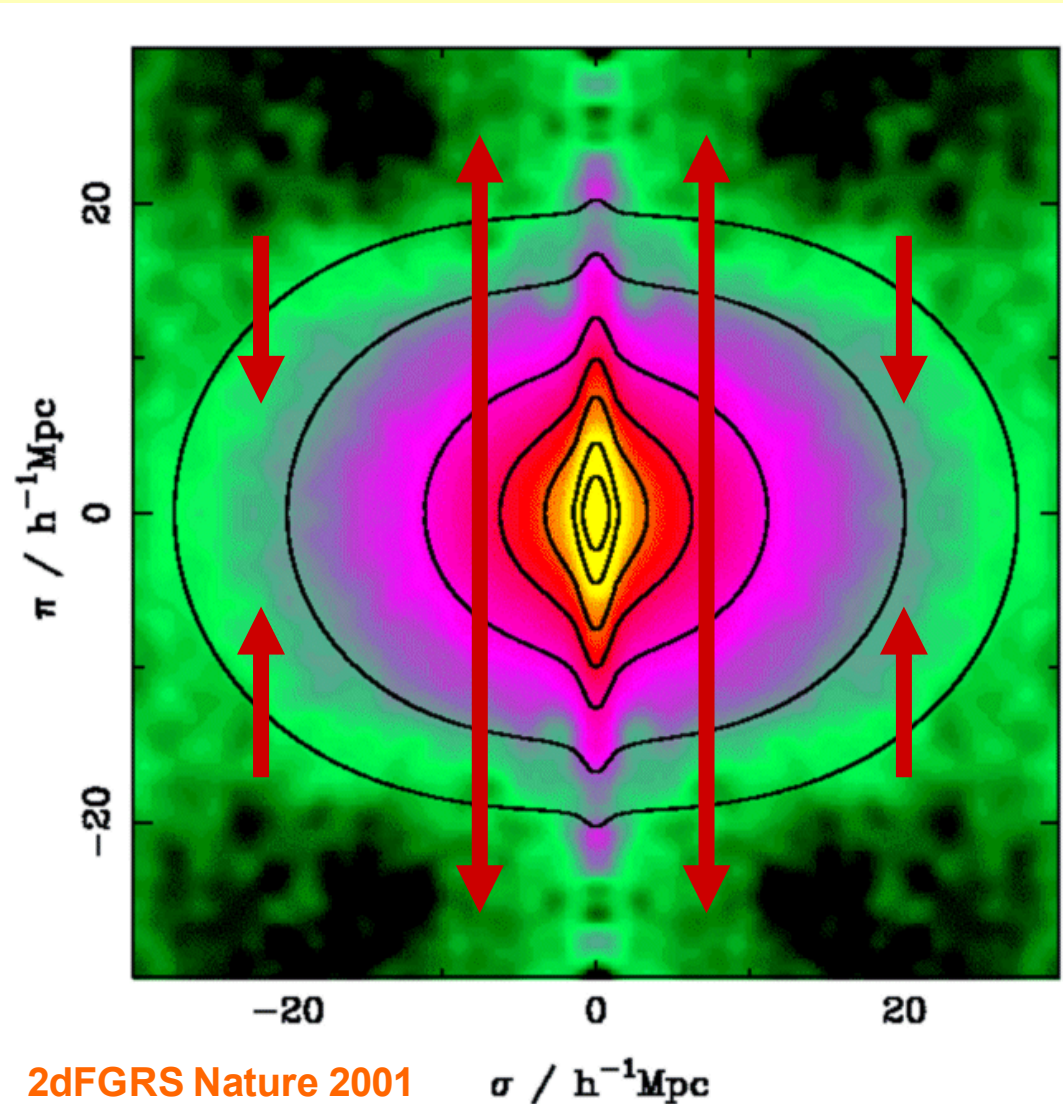
Fourier space:  $\mathbf{D}$  and  $\mathbf{u}$  parallel to  $k$

$$\Rightarrow \delta_s = (b + f\mu^2)\delta = b(1 + \beta\mu^2)\delta; \quad \mu \equiv \hat{\mathbf{r}} \cdot \hat{\mathbf{k}} \quad f \equiv H^{-1}\dot{g}/g = d \ln g / d \ln a$$

# Redshift-Space Correlations

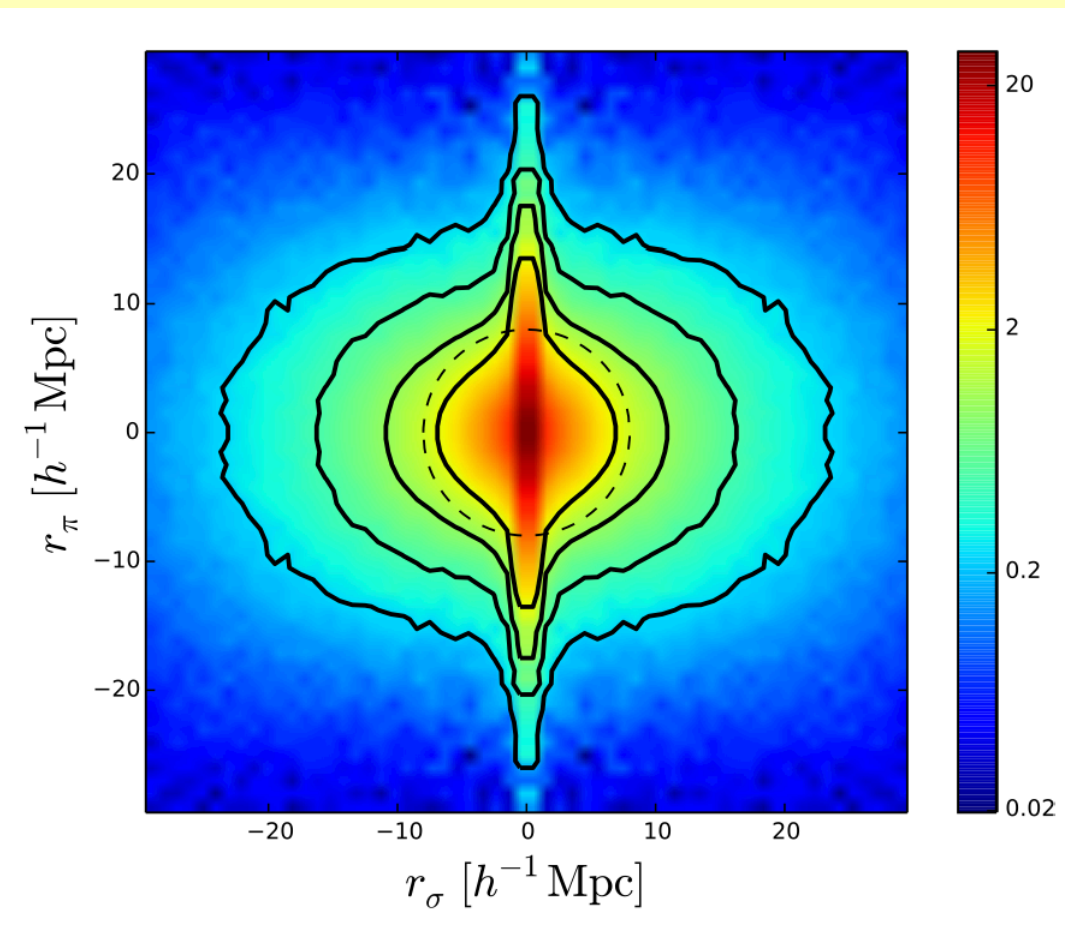
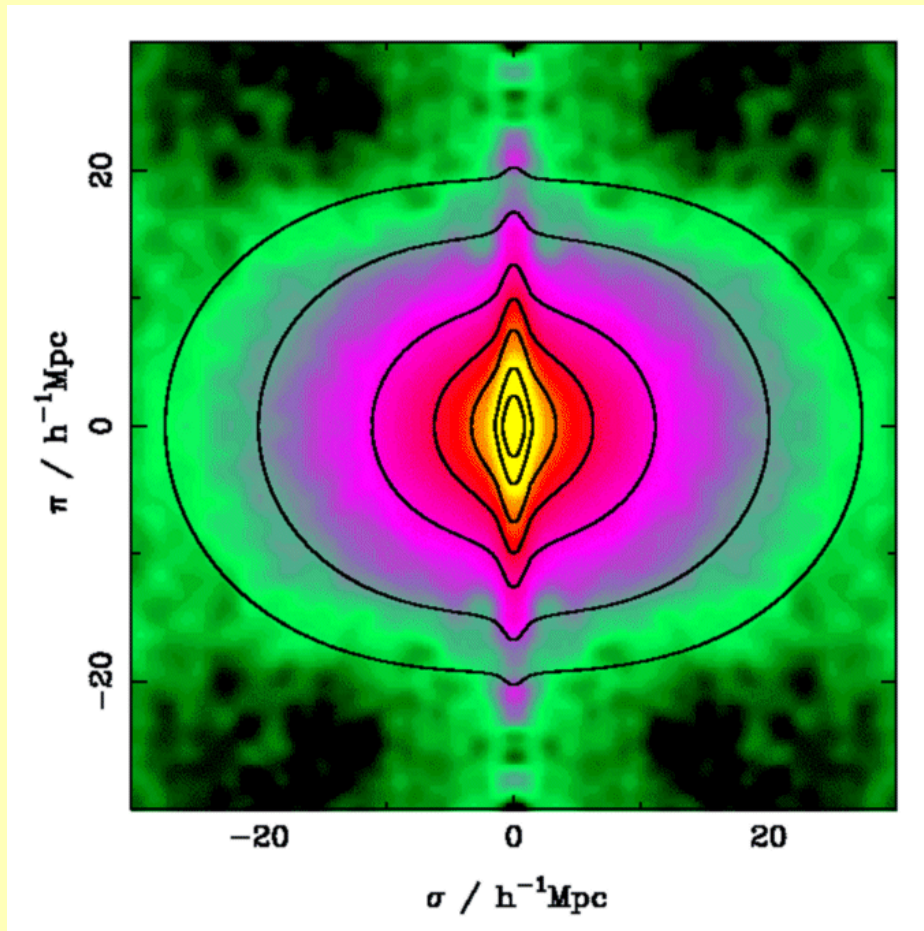


- RSD due to peculiar velocities are quantified by correlation fn (excess fraction of pairs)  $\xi(\sigma, \pi)$
- Two effects visible:
  - Small separations on sky: ‘Finger-of-God’;
  - Large separations on sky: flattening along line of sight.



# 2 decades of RSD

Split 2-point correlations in transverse and radial directions

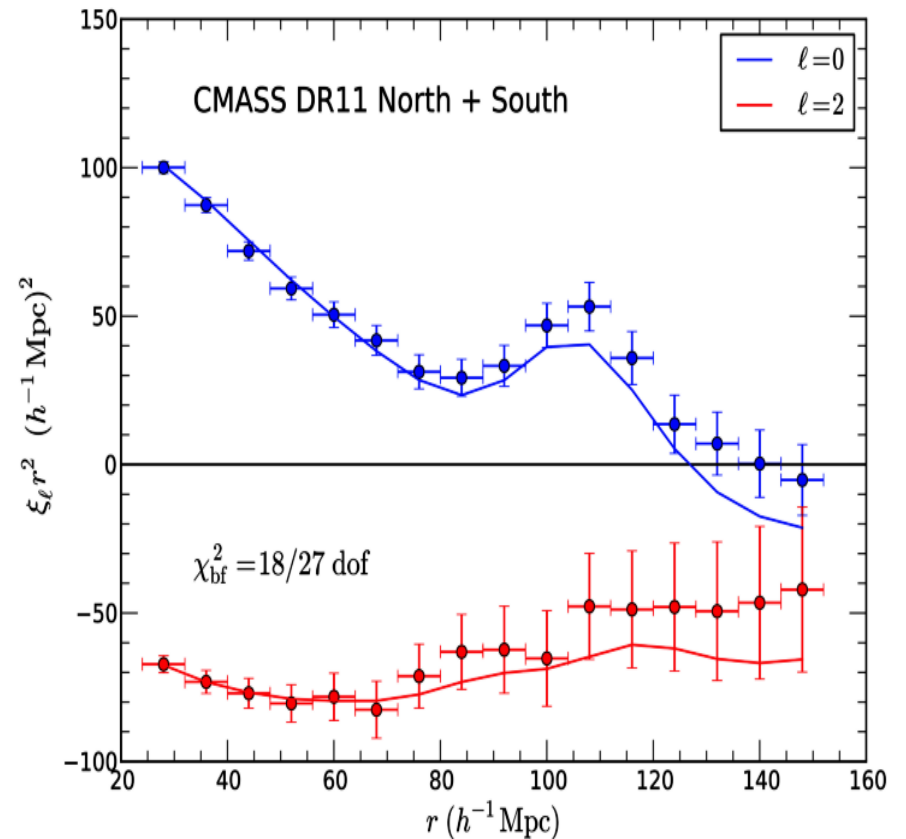
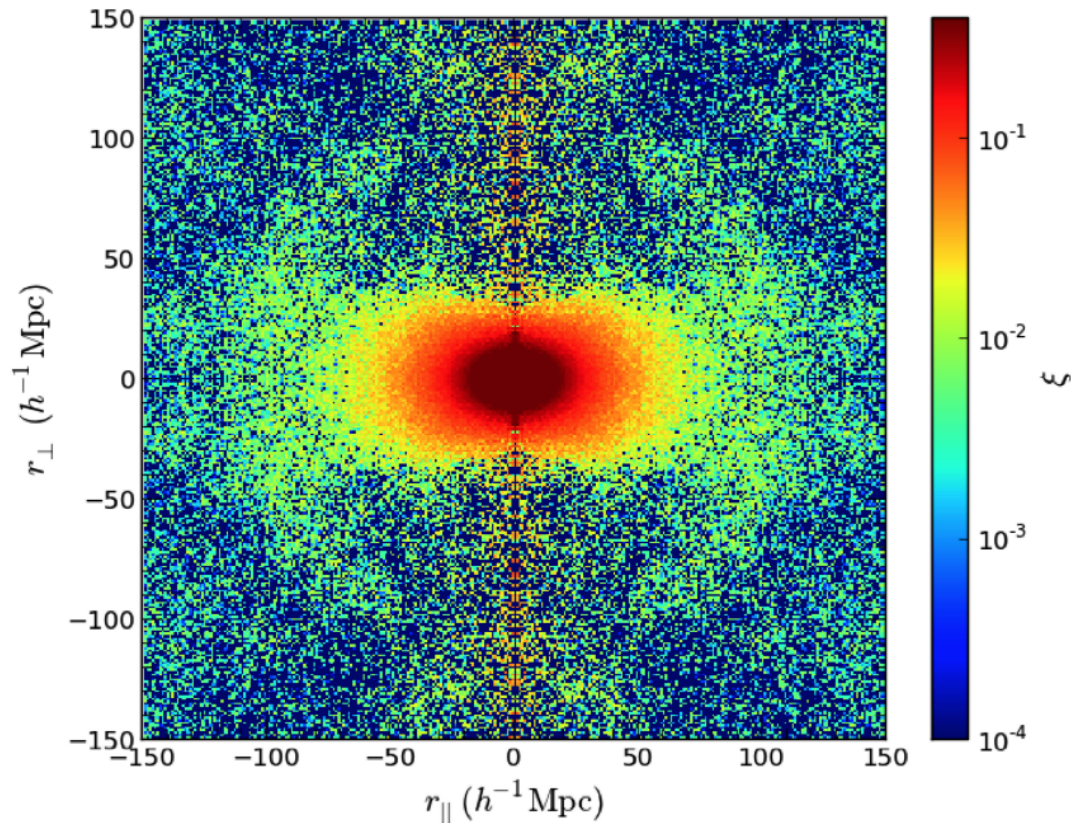


2001: 2dFGRS 8% on  $f_g \sigma_8$

2014: SDSS LRG 2.5% on  $f_g \sigma_8$



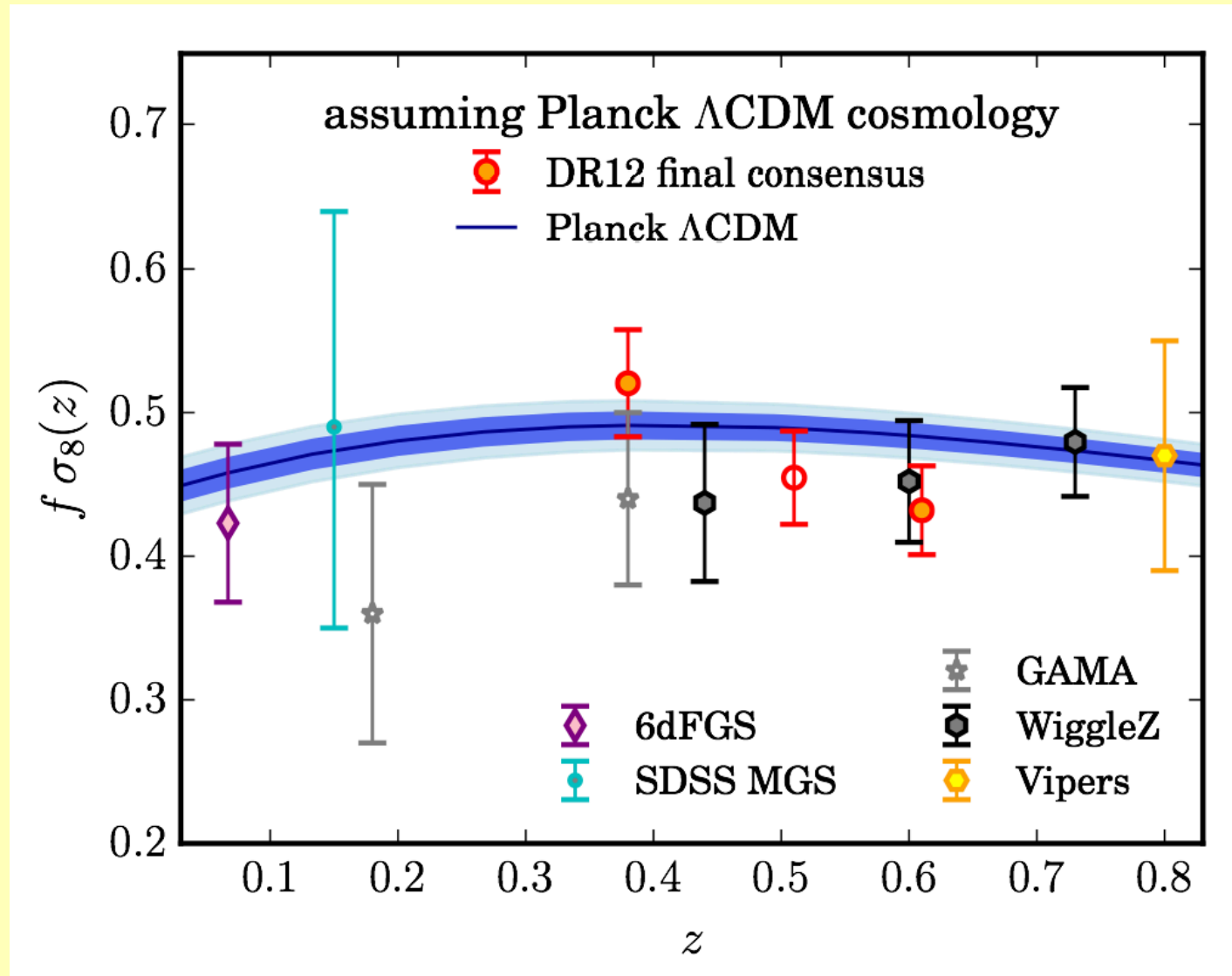
# BOSS DR11 (Samushia et al. 1312.4899)



690826 galaxies over 8498 deg<sup>2</sup> ( $V=6.0$  Gpc<sup>3</sup>)

Growth rate:  $f \sigma_8 = 0.447 \pm 0.028$  (6%)

# Growth rate: Einstein OK at 10%

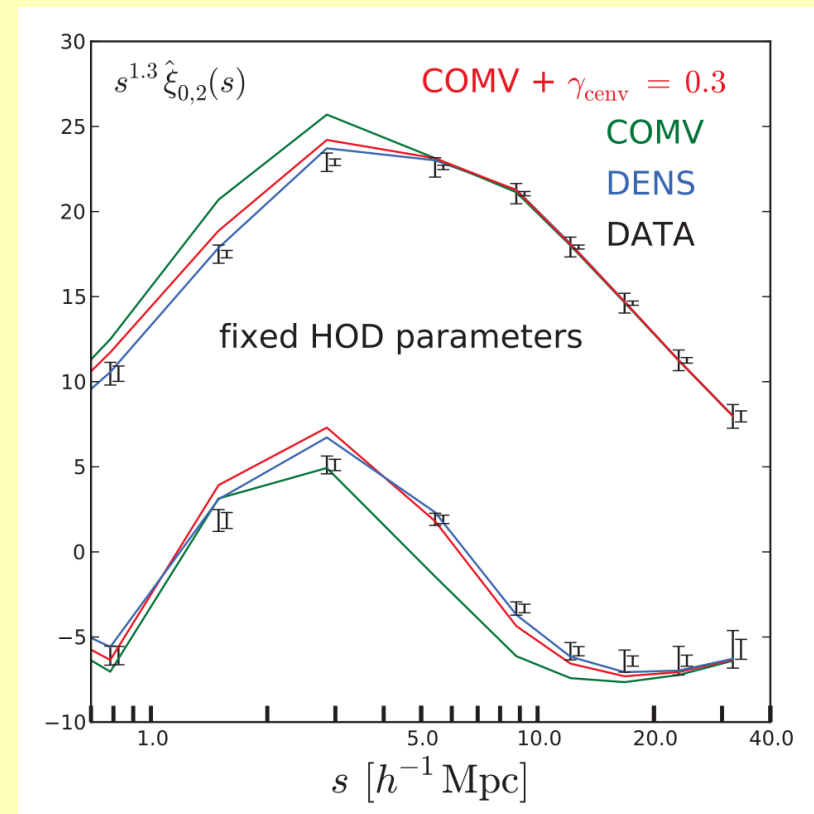
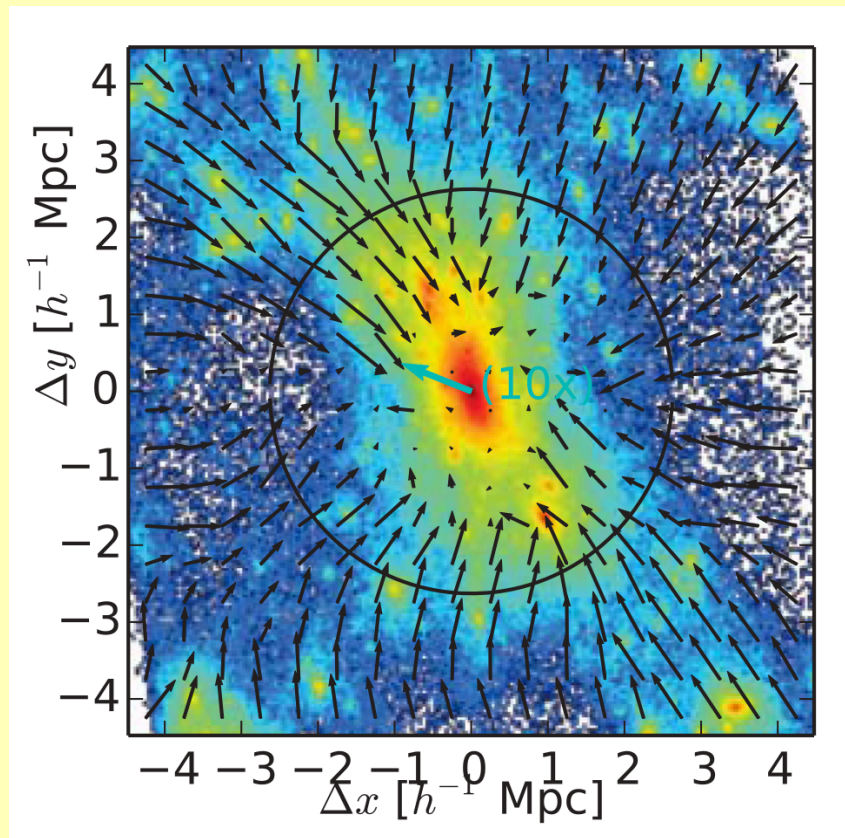


1607.03155

DESI, Euclid will push towards <1% precision at higher  $z$



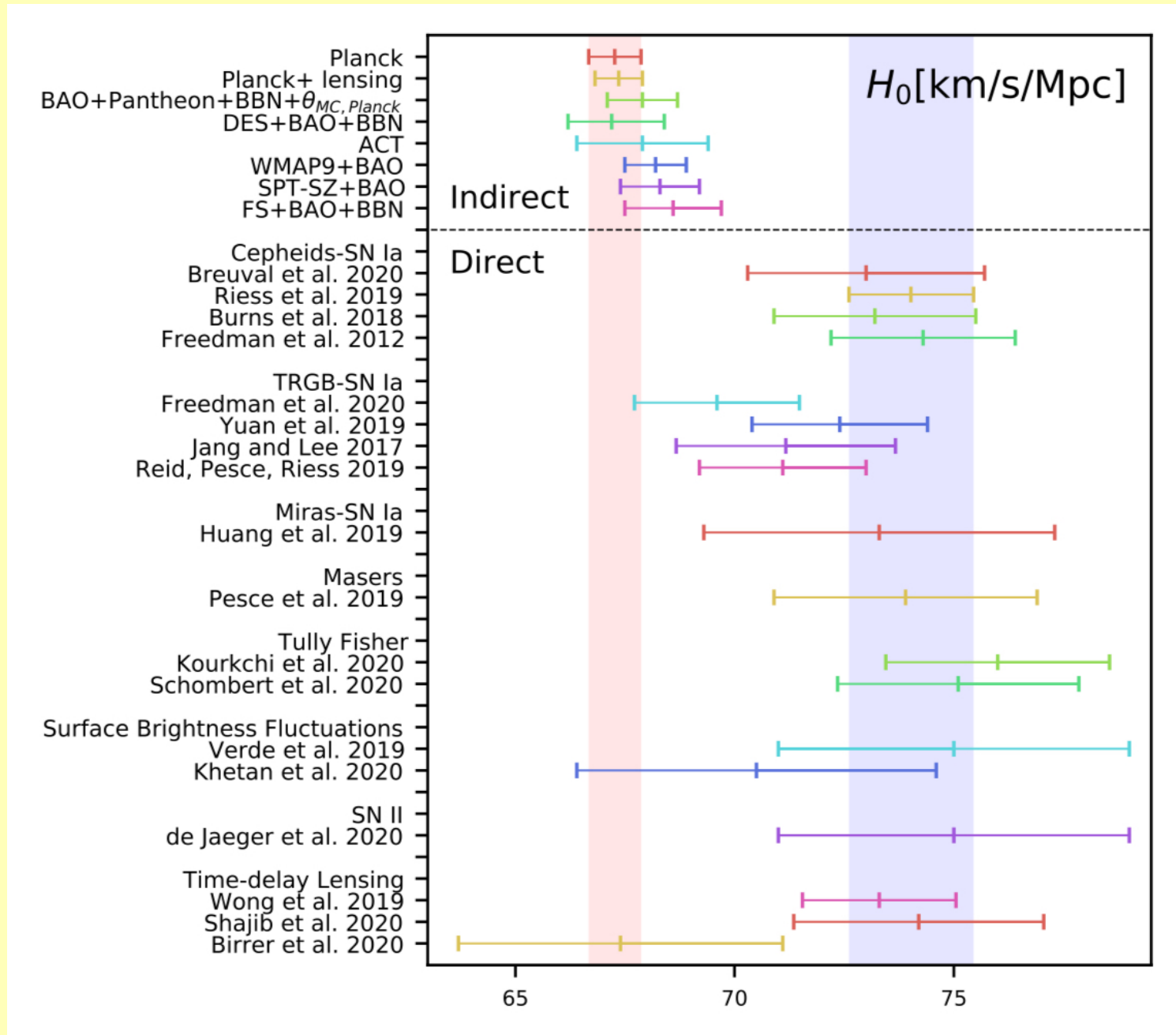
# RSD and fine details of velocity field



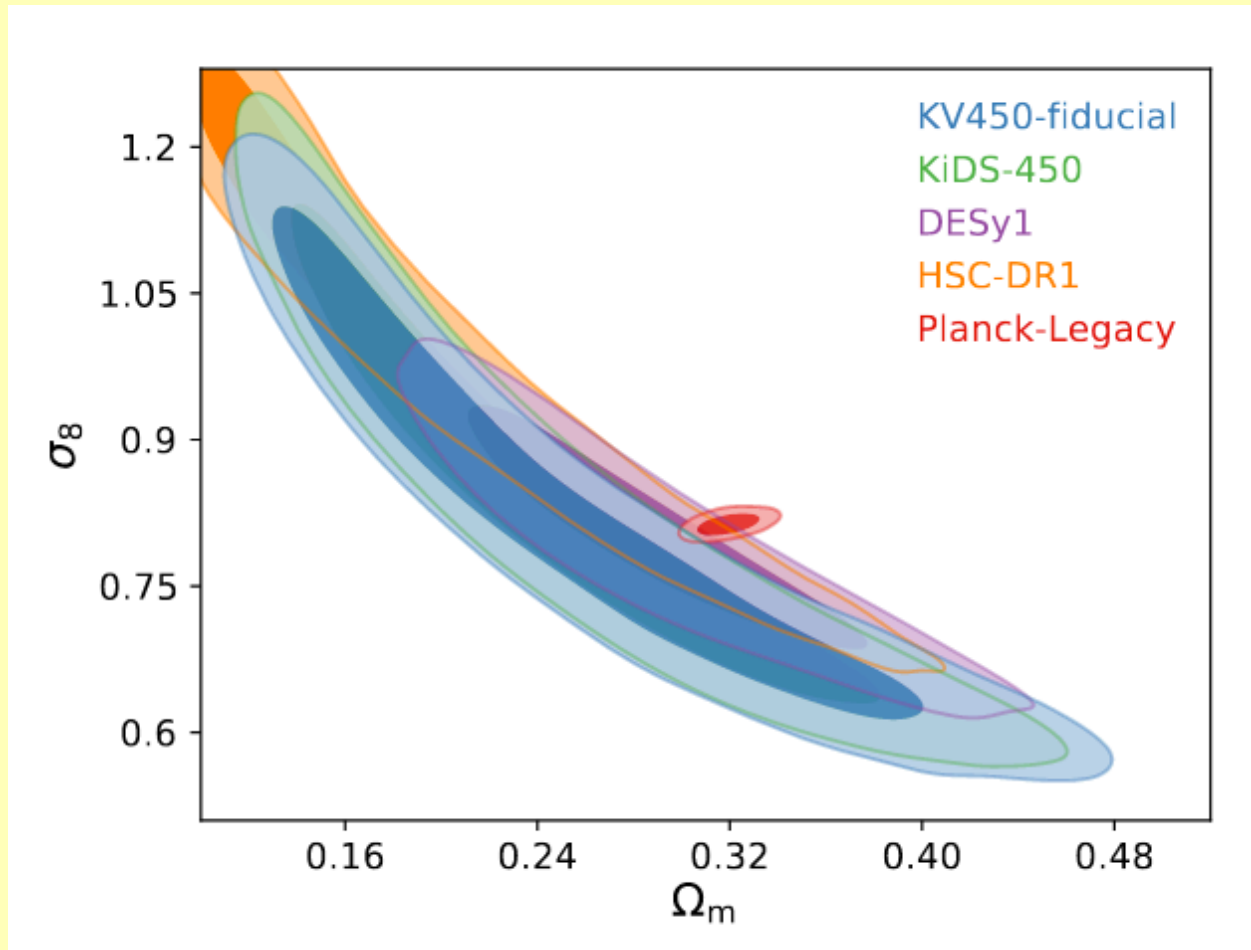
e.g. Reid et al. (2014): central galaxy velocity offset matters in RSD modelling at % level

**5:**  
**The current situation and  
cosmological tensions**

# The H0 tension



# The lensing-CMB tension

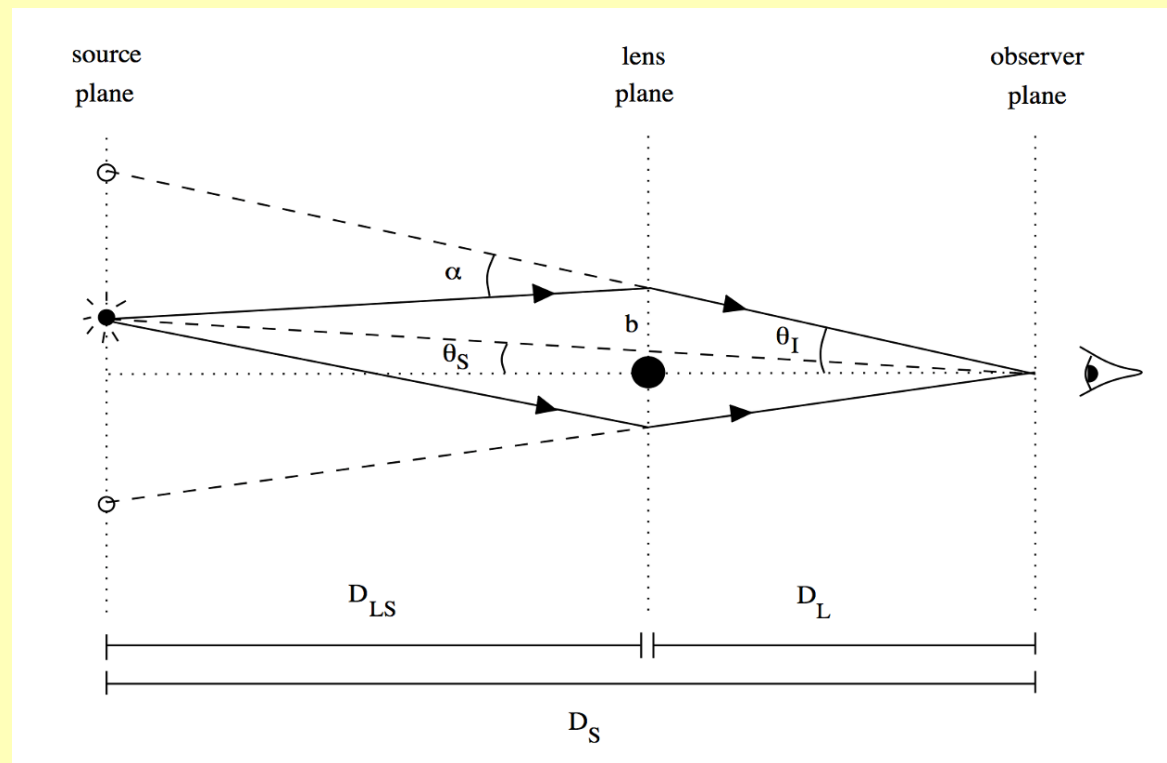


Lensing banana:  $S_8 = \sigma_8(\Omega_m/0.3)^{0.5} = 0.766 \pm 0.020$  (KiDS + DESy1)  
 $= 0.917 \pm 0.024$  times Planck prediction (= 0.835)

– very well consistent with DESy3  $0.772 \pm 0.017$

# Gravitational lensing basics

Sky plane or image plane: where extrapolation of observed rays meets source plane.



Lensing deflection:  $\theta_I - \theta_S = \nabla_{\theta} \psi$

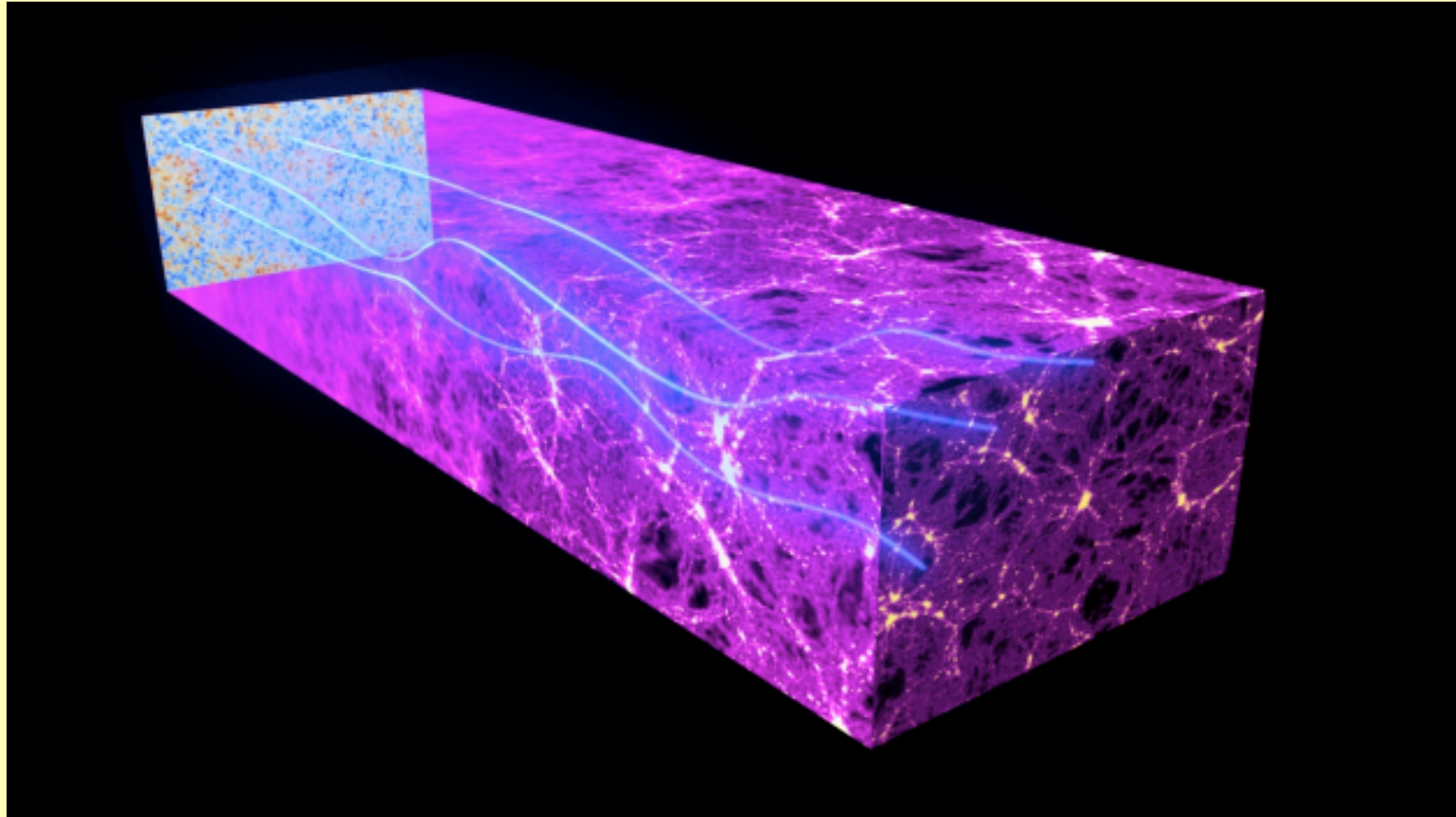
Lensing potential:  $\psi = 2 \int \frac{D_{LS}}{D_L D_S} \Phi dl$

Lensing convergence:  $\nabla_{\theta}^2 \psi = 2\kappa \quad \kappa = 4\pi G \int \frac{D_L D_{LS}}{D_S} \rho dl$

Weak shear: galaxy ellipticities from potential gradients

Naïve signal scales as  $\Omega_m \sigma_8$  but actually  $\Omega_m^{0.5} \sigma_8$

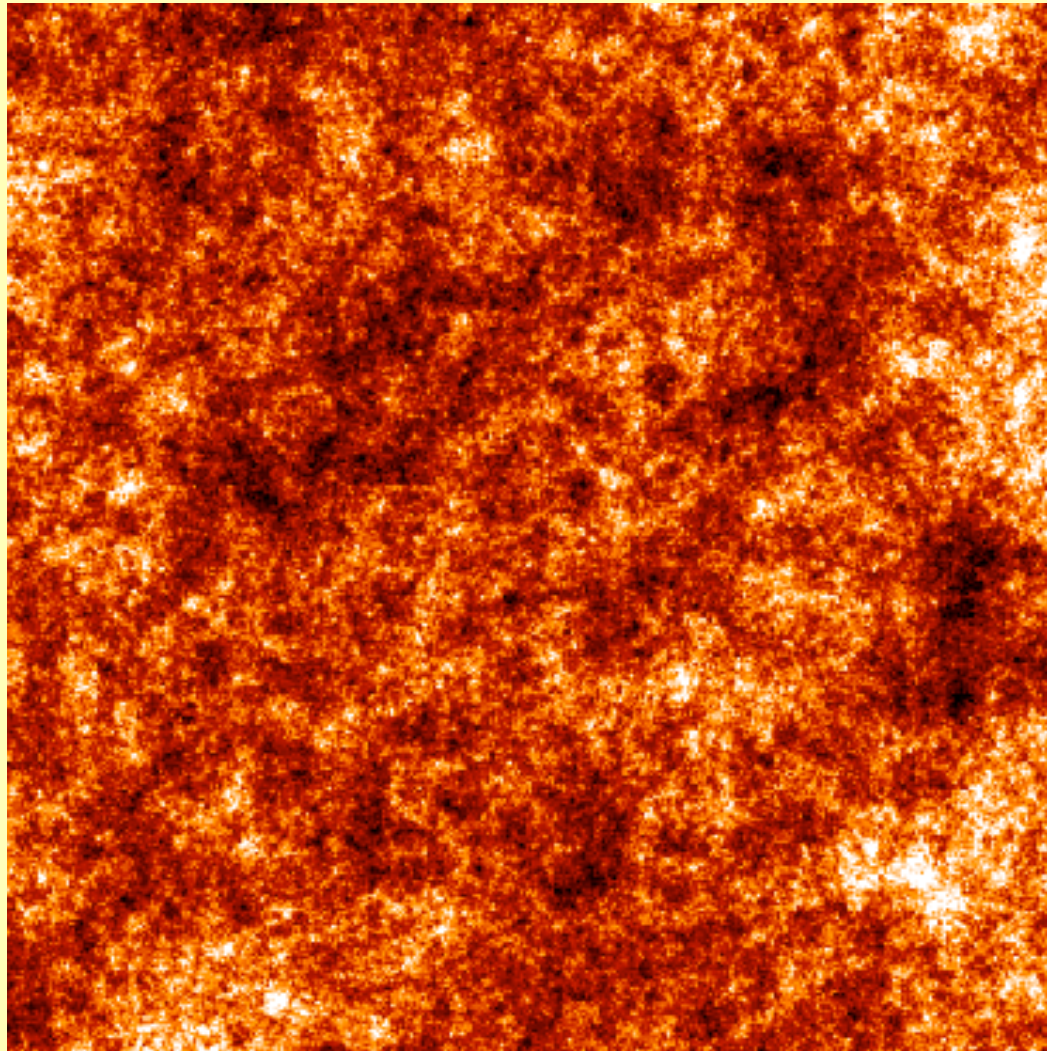
# Gravitational lensing of the CMB



Foreground matter fluctuations deflect light and distort apparent CMB sky map

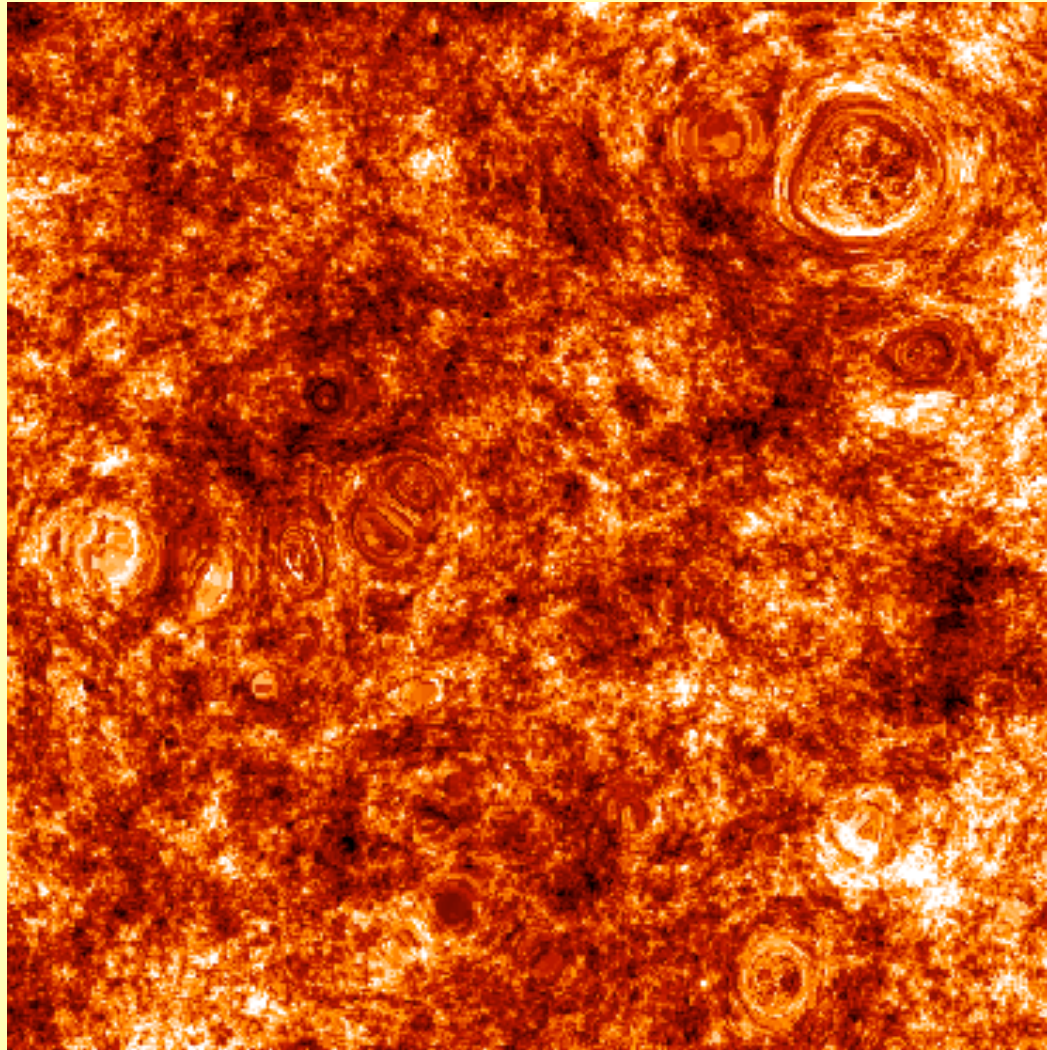


## Unlensed CMB: 6 arcmin image (MPIA)





## Lensed CMB: 6 arcmin image (MPIA)



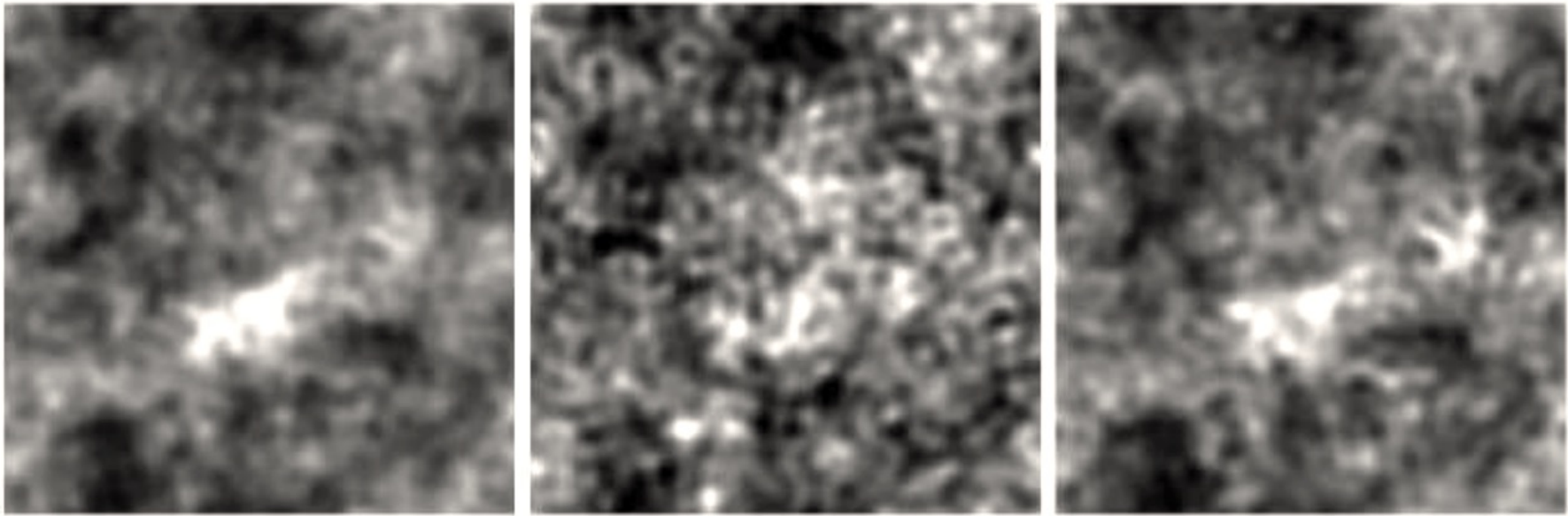
Imprinted non-Gaussian signature allows a map of foreground structure to be made

# Reconstructing lensing from the CMB map

$$T'(x) = T(x + \nabla\psi) \simeq T(x) + \nabla\psi \cdot \nabla T$$

$$\Rightarrow \langle T\nabla T \rangle \neq 0 \quad (\text{non-Gaussian})$$

$\nabla \cdot (T\nabla T)$  gives an estimator of  $\kappa$  (with suitable filtering)



(a) true |deflection|

(b) reconstructed from T

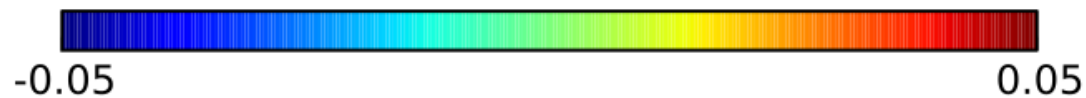
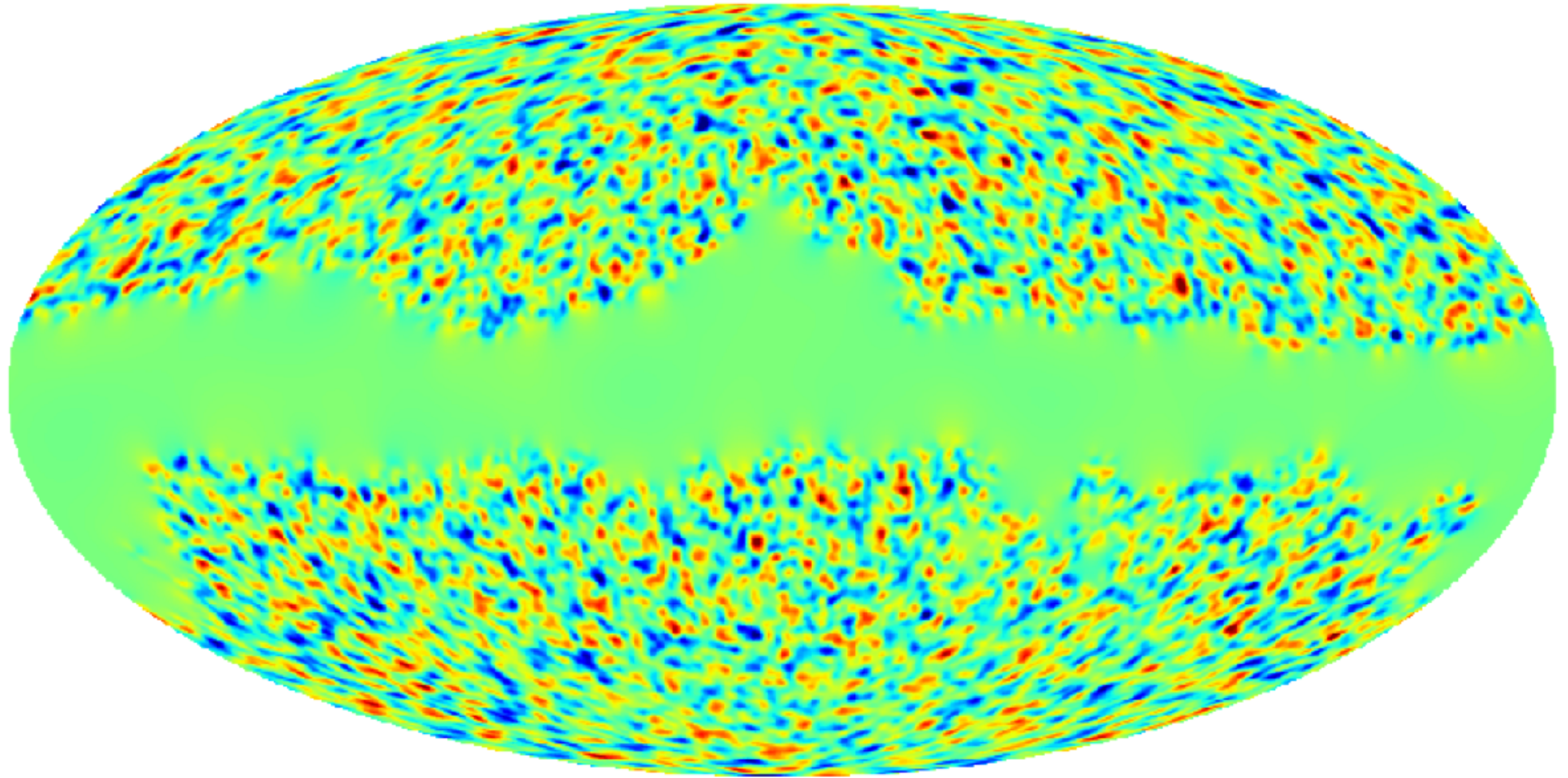
(c) reconstructed from TEB

See Lewis & Challinor 2006 arXiv:astro-ph/0601594



# Planck lensing map – 2013

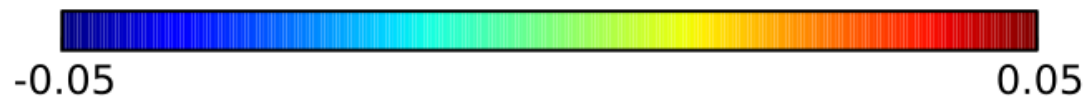
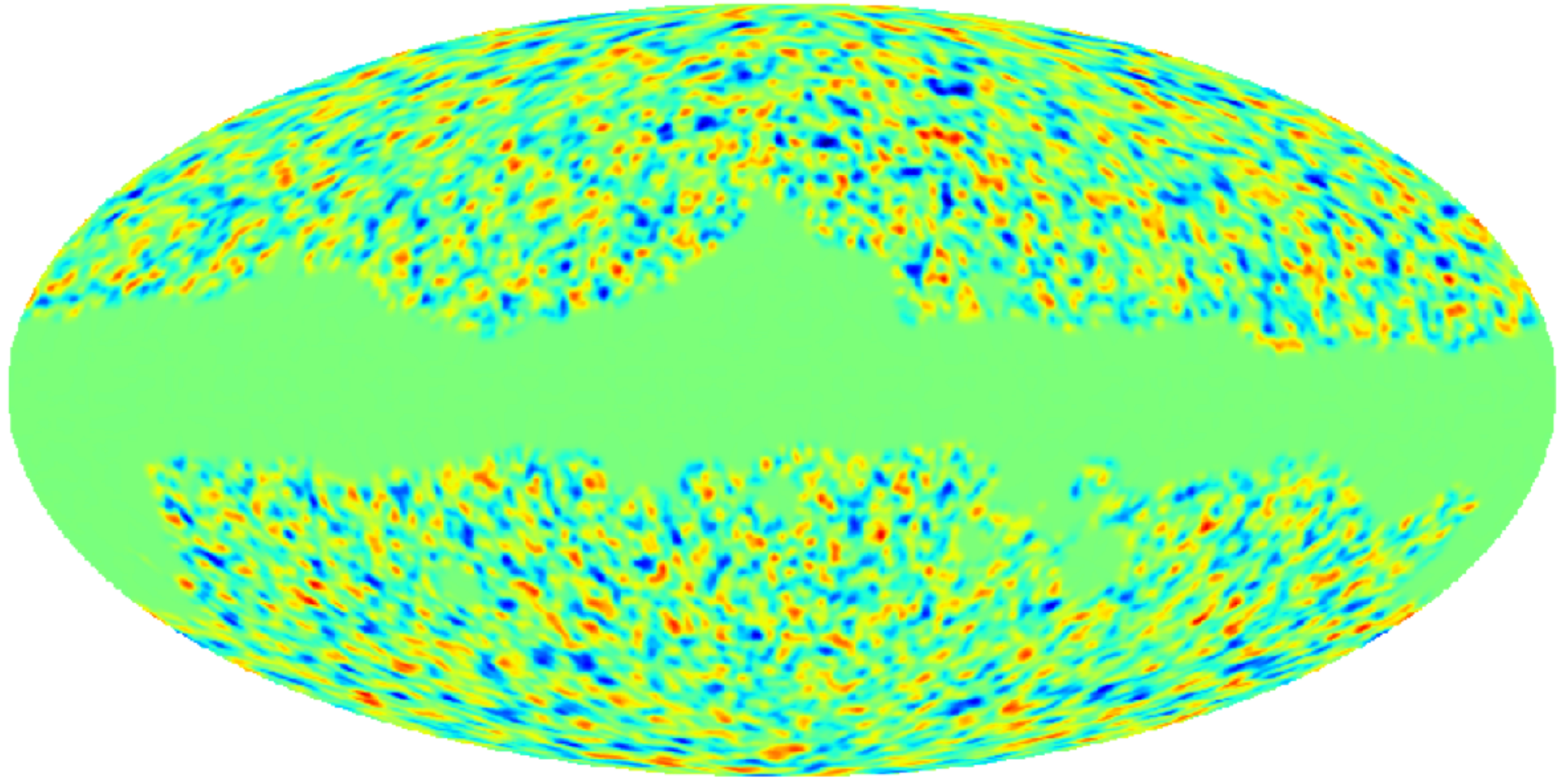
Lensing year 1: FWHM 2 degrees



Lensing convergence: projected mass distribution back to  $z=1100$

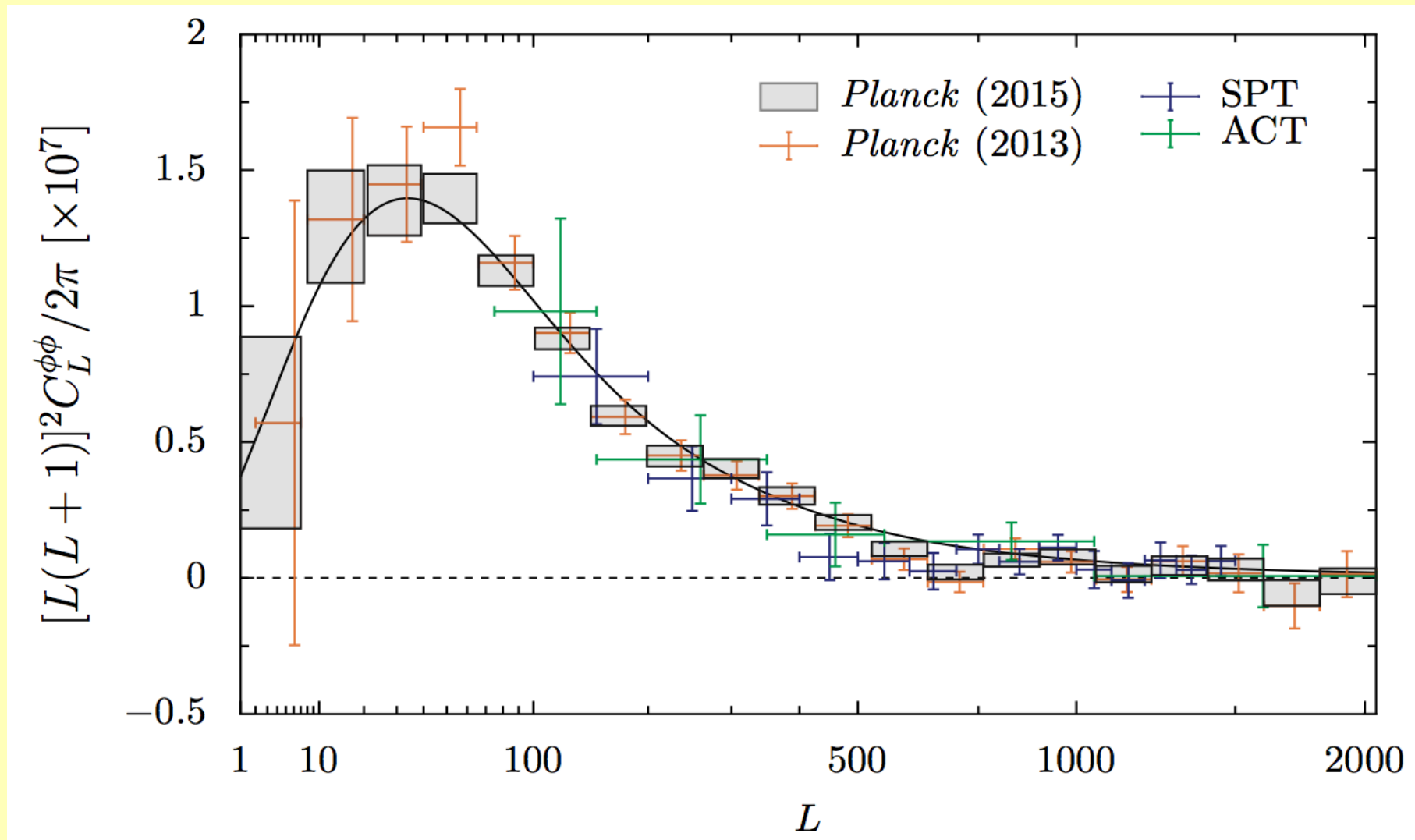
# Planck lensing map – 2015

Lensing year 2: FWHM 2 degrees



Lensing convergence: projected mass distribution back to  $z=1100$

# Planck lensing power spectrum

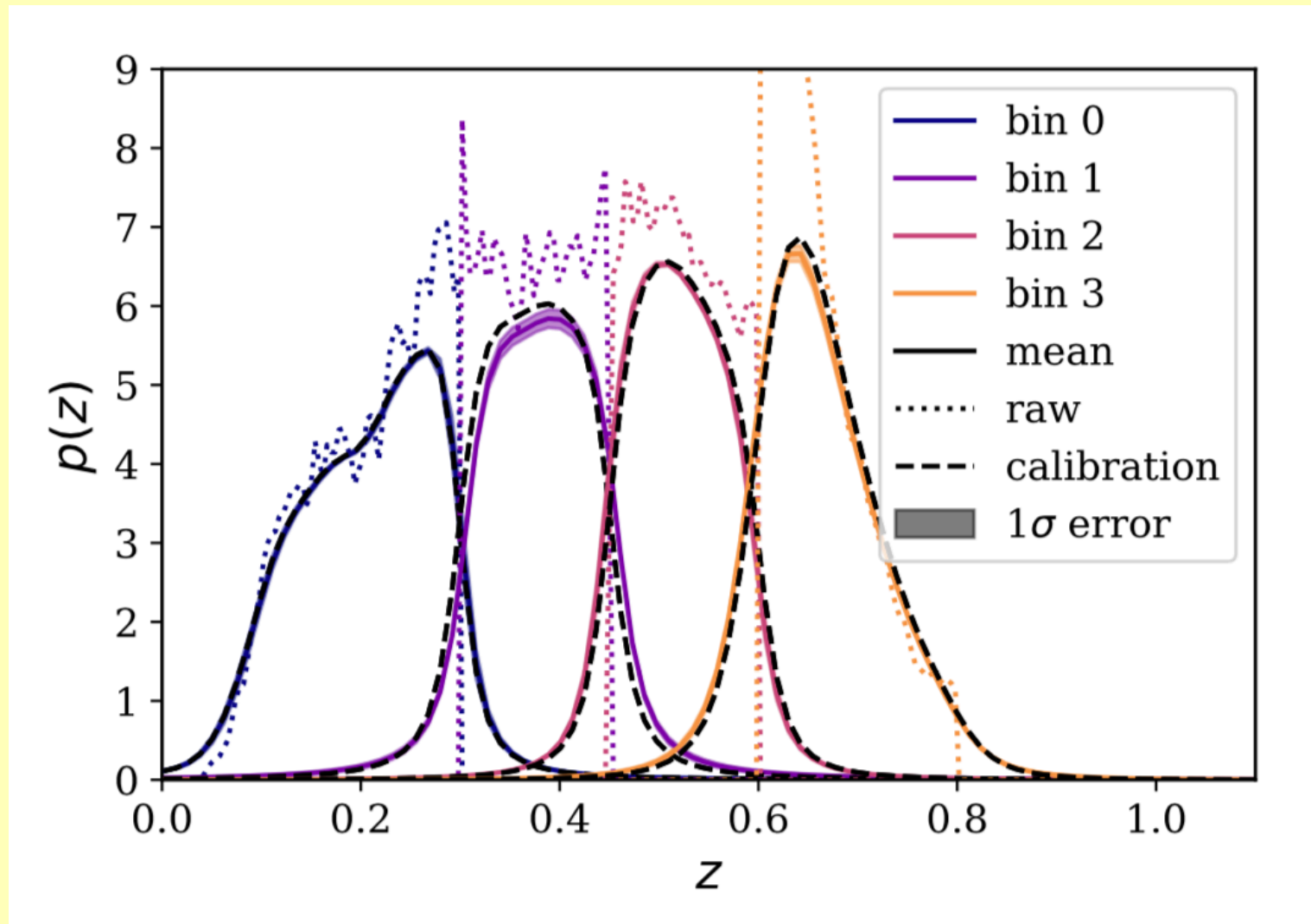


Noise corrected: noise dominates beyond multipole 100

Closely consistent with Planck best TTTEEE model



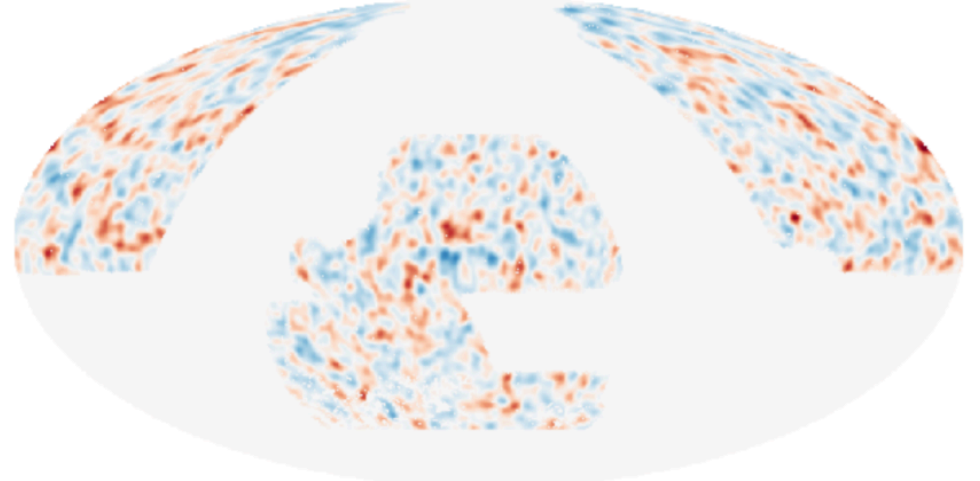
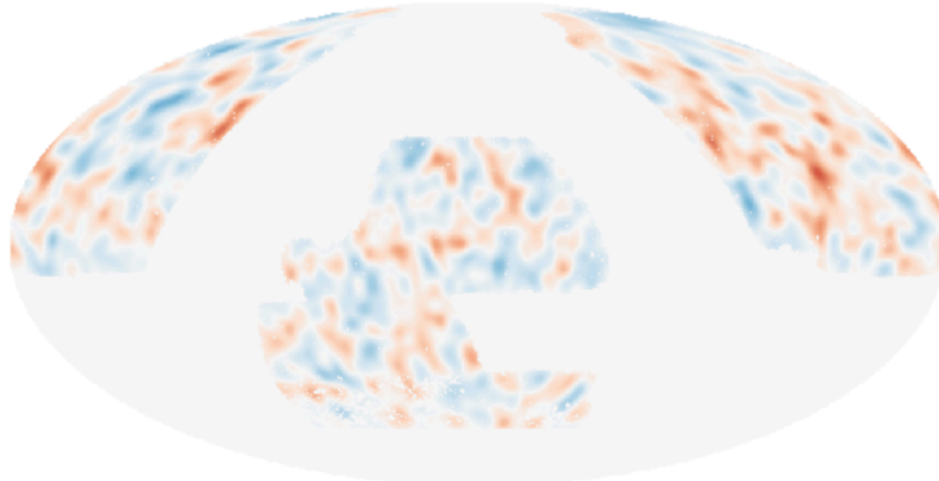
# 2010.00466: photo-z tomography from DESI legacy survey



# Sky maps: 49M objects over 17k deg<sup>2</sup>

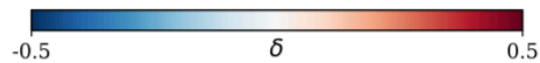
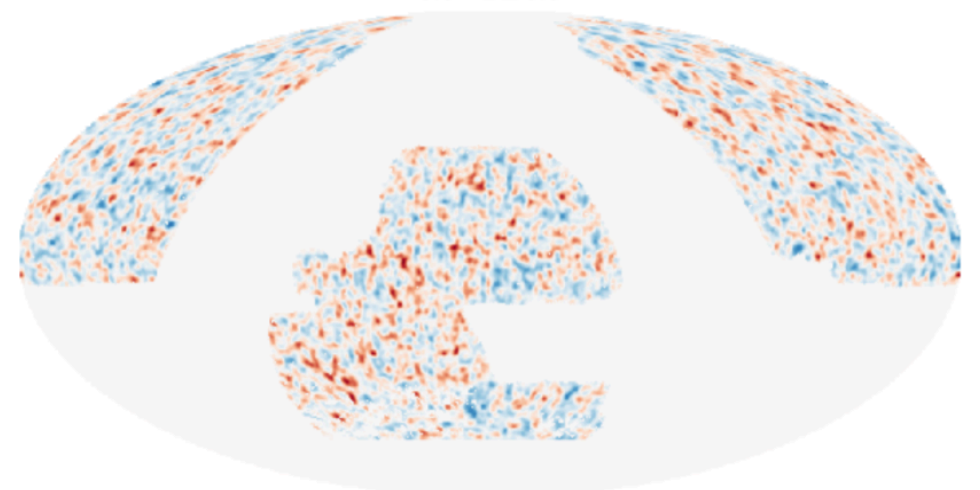
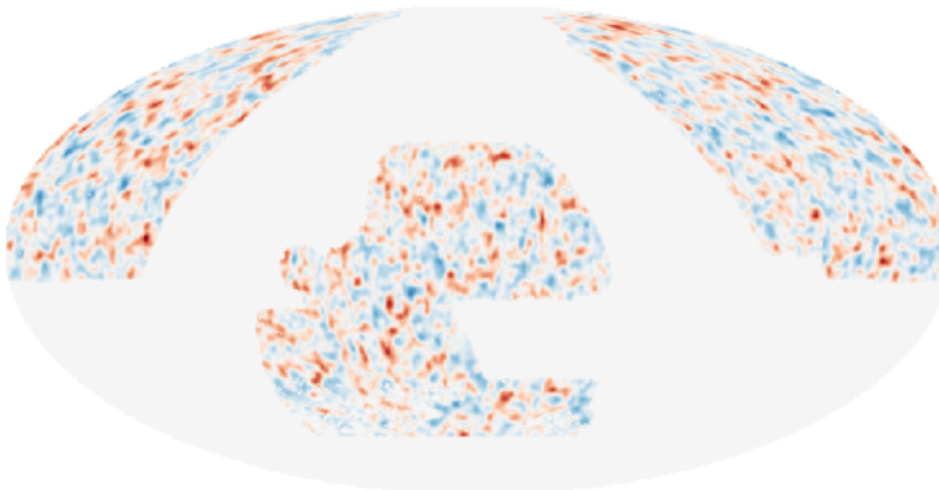
$0 < z \leq 0.3$

$0.3 < z \leq 0.45$



$0.45 < z \leq 0.6$

$0.6 < z \leq 0.8$





# Projected clustering

Project with a kernel:  $\delta_p = \int \delta(r) K(r) dr$ .

$K = N(z)dz/dr$  for clustering

$K = (3\Omega_m H_0^2 / 2c^2) r(r_{\text{LS}} - r) / ar_{\text{LS}}$  for lensing  $\kappa$

Angular spectrum:  $\delta_p = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\theta, \phi)$

Angular correlation:  $w(\theta) = (1/4\pi) \sum_{\ell} (2\ell + 1) C_{\ell} P_{\ell}(\cos \theta)$

Limber approx (survey thick wrt correlation length):

$$w(\theta) = \int K^2(y) dy \int \xi \left( \sqrt{x^2 + y^2 \theta^2} \right) dx$$

$$w(\theta) = \int K^2(r) dr \int \pi \Delta^2(k) J_0(kr\theta) dk / k^2$$

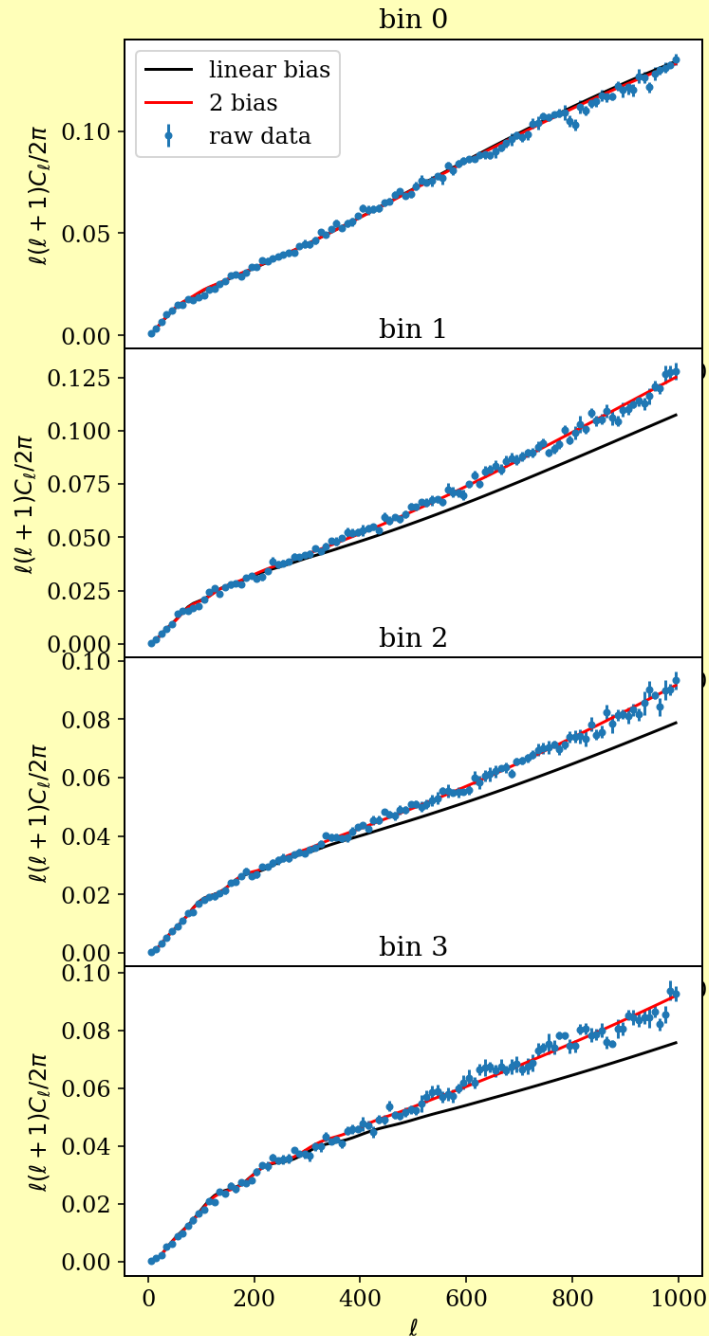
Kaiser: better in harmonic space (x-corr)

$$C_{ab}(\ell) = (\pi/\ell) \int \Delta^2(\ell/r) K_a(r) K_b(r) r dr$$

Early work: broad flux-limited kernel

Today: often used in tomography with bands defined by photo-z

# Predicting cross-power



Cross-correlate galaxy and CMB maps in harmonic space. Galaxy autocorrelation fixes bias very precisely, so cross-power with CMB can be predicted for fiducial Planck cosmology

$$f = \sum a_{\ell m} Y_{\ell m}(\theta, \phi); \quad g = \sum b_{\ell m} Y_{\ell m}(\theta, \phi)$$

$$C_{fg} = a_{\ell m} b_{\ell m}^*$$

$$C_{gg}(\ell) = \int b^2 P(k = \ell/r, z) K(r) p^2(z) dz$$

$$C_{g\kappa}(\ell) = \int b P(k = \ell/r, z) K_{\kappa}(r) p(z) dz$$

$$C_{gT}(\ell) = \int b P(k = \ell/r, z) K_T(r) p(z) dz$$

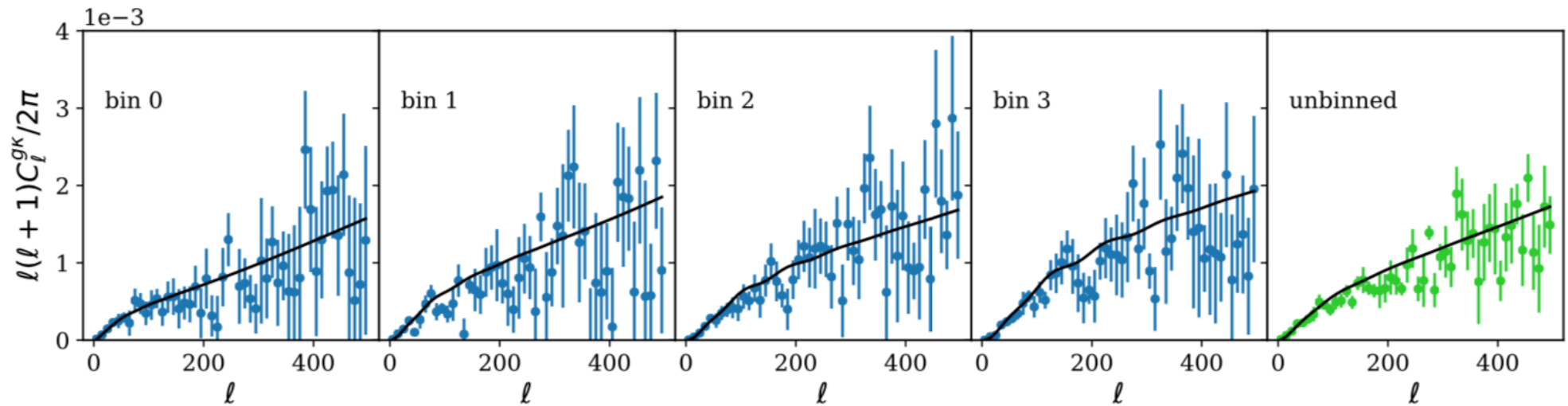
linear bias:  $P_g = b^2 P_{\text{lin}}$

nonlinear bias:  $P_g = b^2 P_{\text{nonlin}}$

halo-model bias:  $P_g = b_1^2 P_{2\text{-halo}} + b_2^2 P_{1\text{-halo}}$

**Kaiser-Limber small angle approximation**

# Galaxy-lensing cross-power



Signal is consistently low compared to fiducial Planck cosmology ( $\Omega_m = 0.315$ ,  $\sigma_8 = 0.811$ ):

$$A_\kappa = 0.901 \pm 0.026$$

# Implications of low signal

$$\frac{\ell(\ell + 1)}{2\pi} C_{\ell}^{g\kappa} = \frac{\pi}{\ell} \int b\Delta^2(k = \ell/r, z) p(z) K(r) r dz,$$

where the lensing kernel is given by

$$K(r) = \frac{3H_0^2 \Omega_m}{2c^2 a} \frac{r(r_{\text{LS}} - r)}{r_{\text{LS}}}.$$

Bias from galaxy autocorrelation

$C_{g\kappa} \propto \Omega_m \sigma_8$  at low  $z$

Nonlinear at higher  $z$ :  $C_{g\kappa} \propto \Omega_m^{0.78} \sigma_8$  (cf. galaxy shear  $S_8$ )

$$\Omega_m^{0.78} \sigma_8 = 0.297 \pm 0.009$$

# A conservative solution (2010.00466)

Total CMB lensing fits Planck:

$$\Omega_m^{0.25} \sigma_8 = 0.589 \pm 0.020$$

Local CMB lensing is also low:

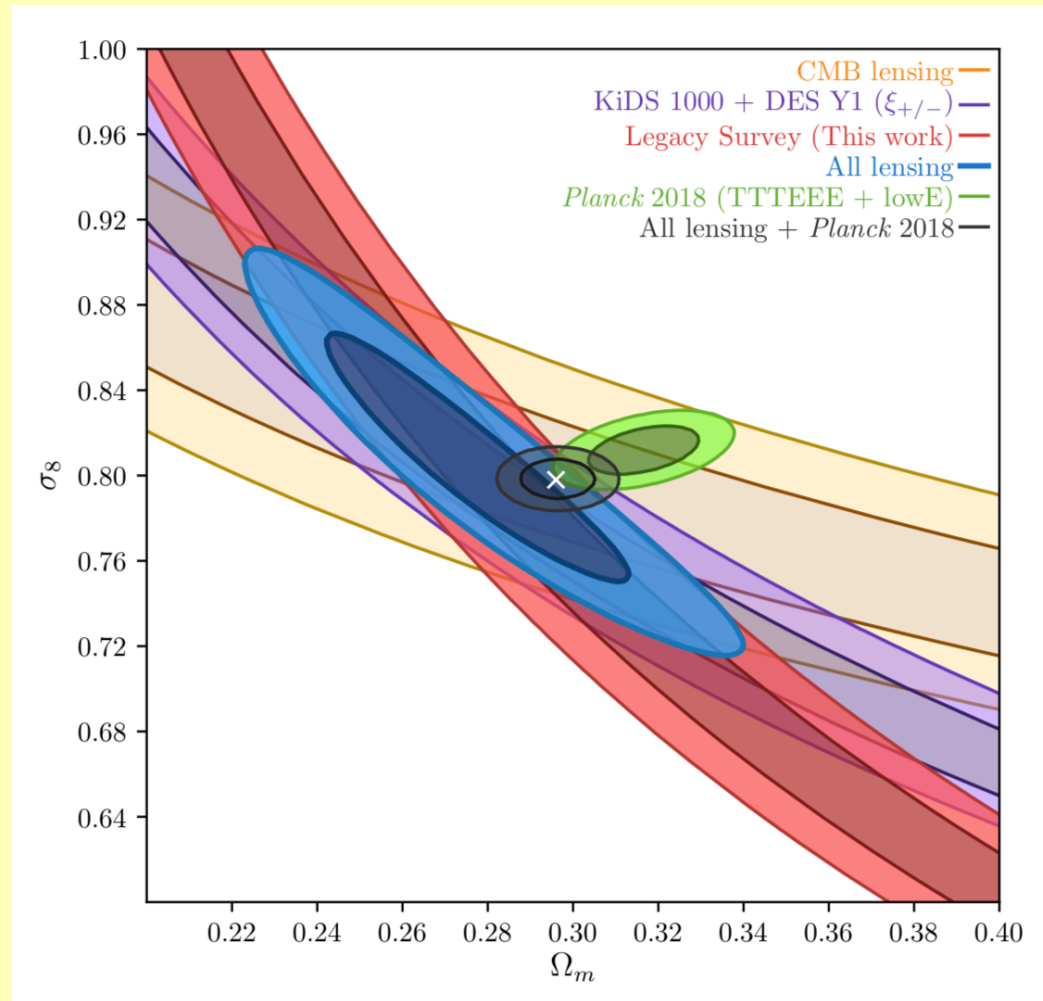
$$\Omega_m^{0.78} \sigma_8 = 0.297 \pm 0.009$$

Lensing is consistent, and needs lower density than Planck:

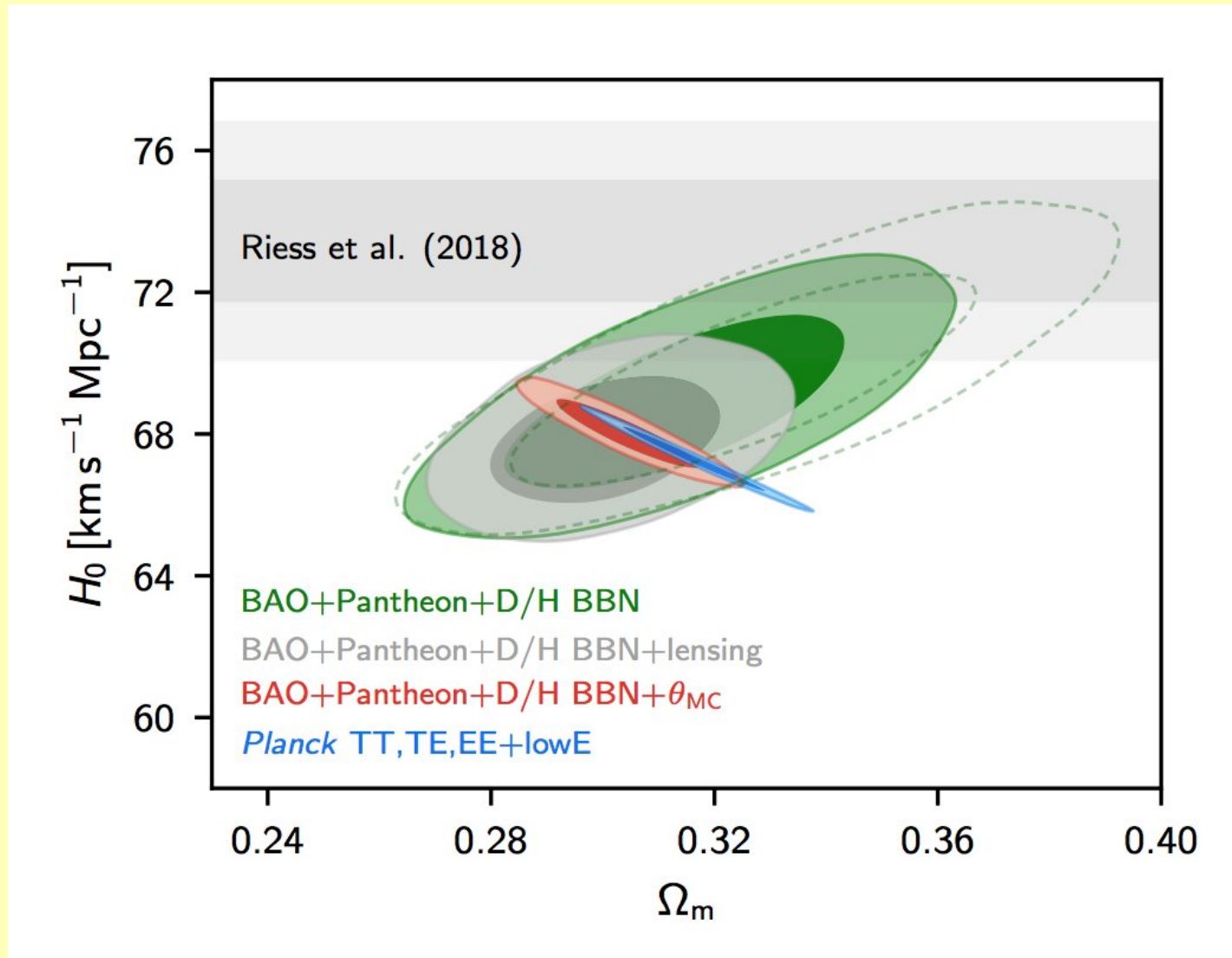
$$\Omega_m = 0.274 \pm 0.024$$

Formal combination with Planck just consistent with both constraints at 95%

$$\Omega_m = 0.296$$
$$\sigma_8 = 0.798$$



# Implications for the $H_0$ tension



# Implications for the $H_0$ tension

CMB most robustly measures  $\Omega_m h^3$  – from acoustic scale

– so lower density inevitably means higher  $h$ :

$$\Omega_m = 0.296: h = 0.69$$

$$\Omega_m = 0.274: h = 0.71$$

– lower density (from lensing only) removes  $H_0$  tension

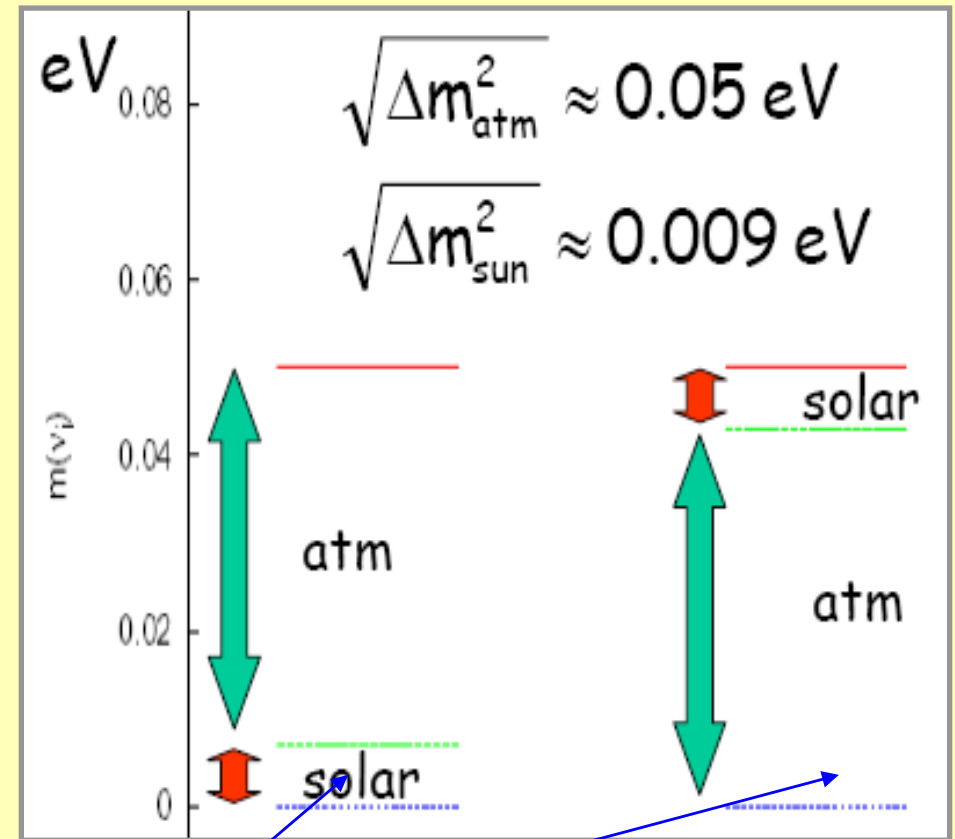
– Conservative view: tensions reflect small systematics



**6:**  
**Cosmological measurements of  
massive neutrinos**

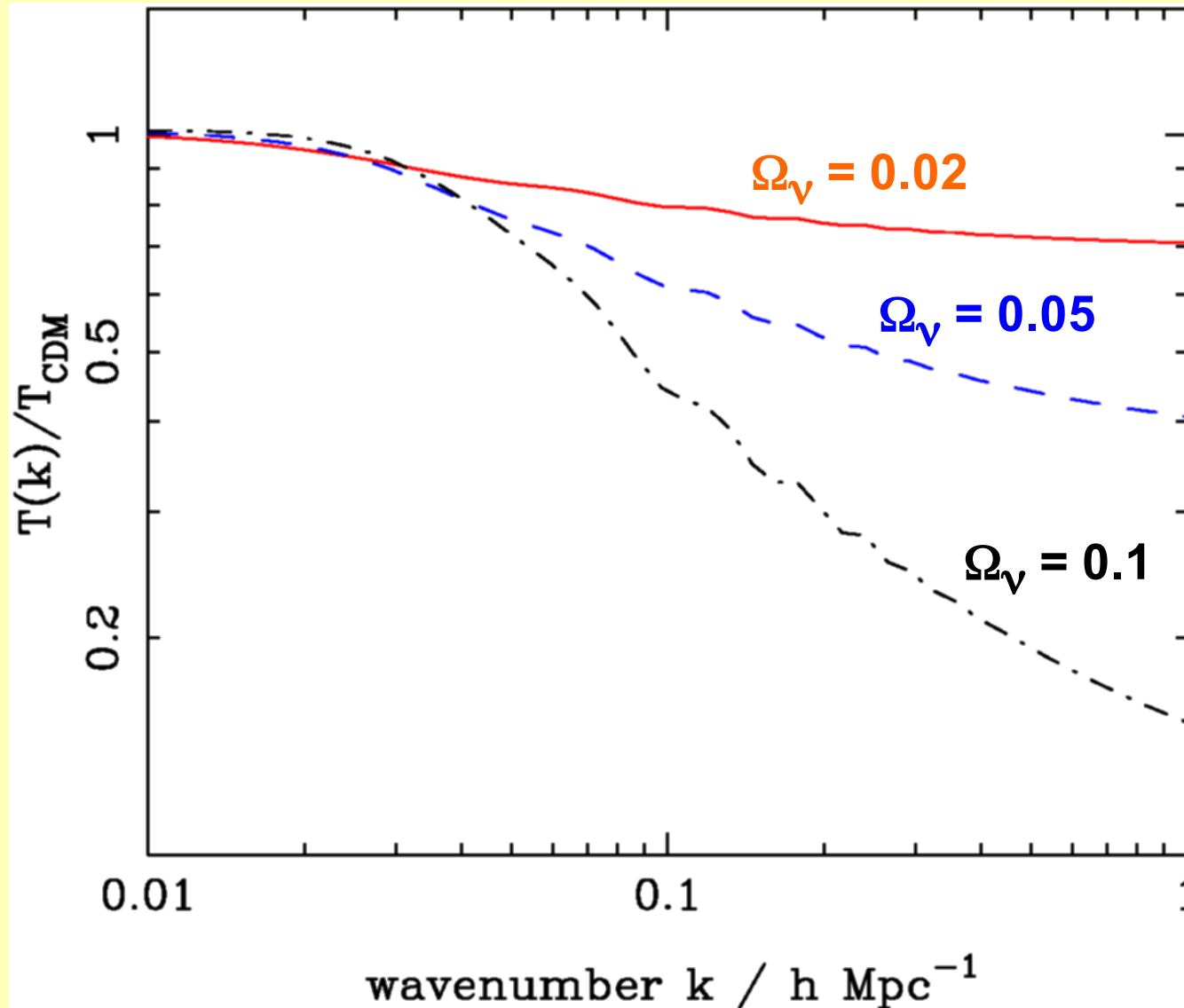
# Neutrino mixing results

- Three neutrinos (known from PP  $\Gamma_W$ )
  - 2 mass diff's:  $\Delta m_{12}$ ,  $\Delta m_{23}$  & 3 phases:  $\theta_{12}, \theta_{23}, \theta_{13}$
- MSW effect ...an added complication
  - In matter, coherent scattering induces an 'effective mass'
  - Matter enhanced oscillations (only effects  $\nu_e$  because matter only has  $e^-$ )
- Recent best fits
  - Solar neutrinos:
    - LMA Solution
    - $\Delta m_{12}^2 \sim 5 \times 10^{-5} \text{ eV}^2$ ,  $\tan^2 \theta_{12} \sim 0.34$
  - Atmospheric neutrinos:
    - Maximal mixing
    - $\Delta m_{23}^2 = 3.5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} = 1$
    - $\theta_{13} \dots$  (small:  $< 0.1$ )
- Hierarchy unknown: ( $m_1=0$ ,  $m_3=0.05 \text{ eV}$ , or degenerate?)



Offset unknown, but  $< 1 \text{ eV}$

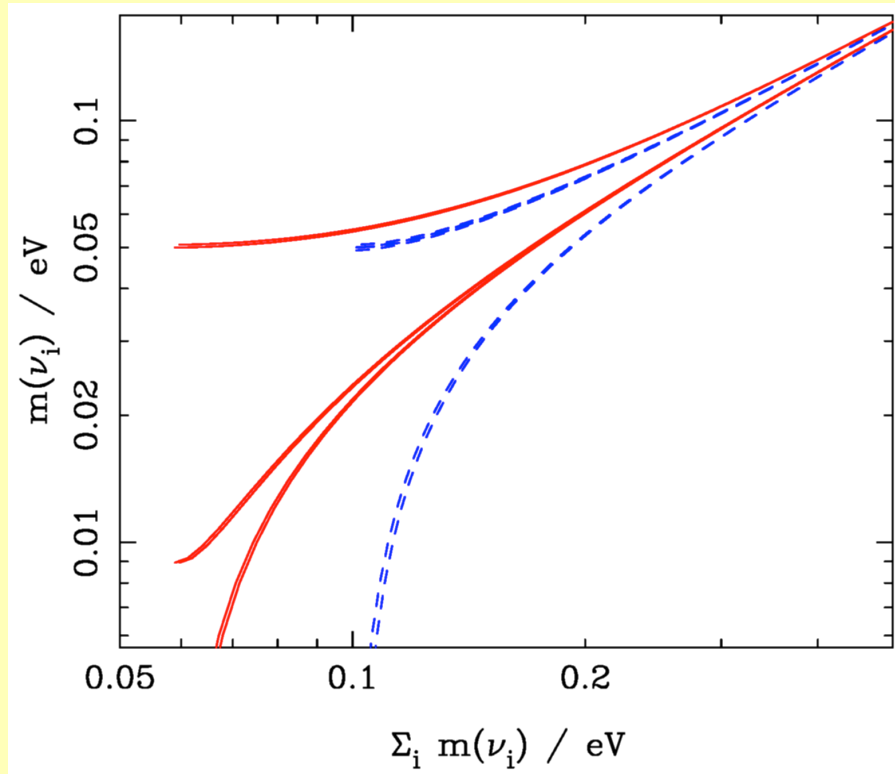
# Effect of massive neutrinos



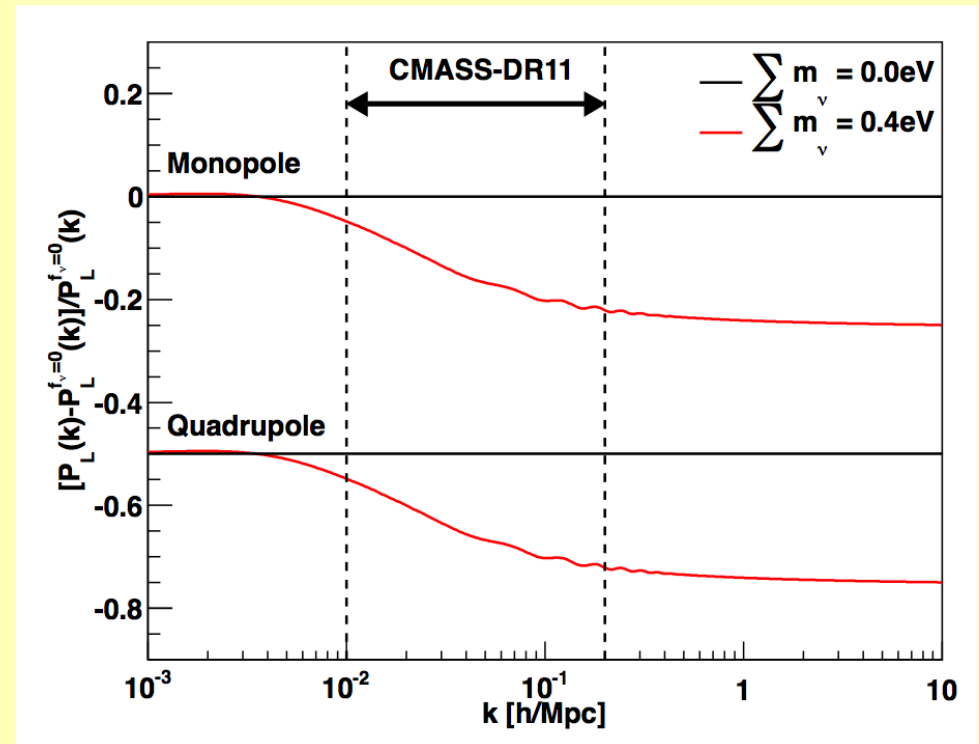
Free-stream length:  
 $80 (M/\text{eV})^{-1} \text{ Mpc}$   
( $\Omega_m h^2 = M / 93.5 \text{ eV}$ )

$M \sim 1 \text{ eV}$  causes  
lower power at almost  
all scales – main  
constraint is from low- $z$   
amplitude of mass  
fluctuations (e.g. from  
redshift-space  
distortions)

# Neutrinos



Normal or inverted hierarchies fit oscillation data



Free-streaming erases neutrino fluctuations

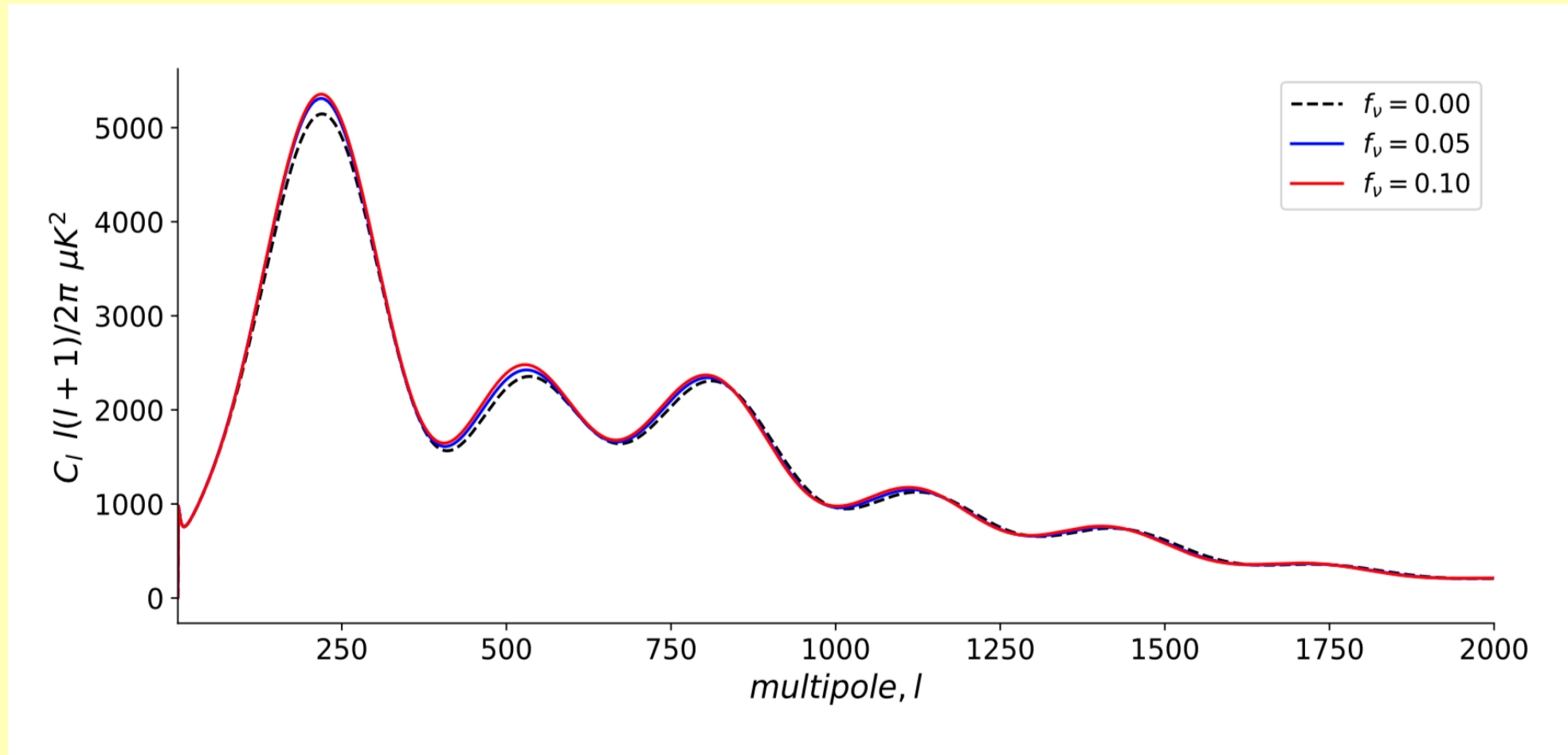
Nearly degenerate as lightest mass increases

Reduced growth rate for  $k > \sim 0.05$  – reduced  $\sigma_8$

Claims of detection at  $m = 0.36 \pm 0.10 \text{ eV}$  (1403.4599)

Planck++ 2018:  $m < 0.12 \text{ eV}$  (0.06 eV smallest possible)

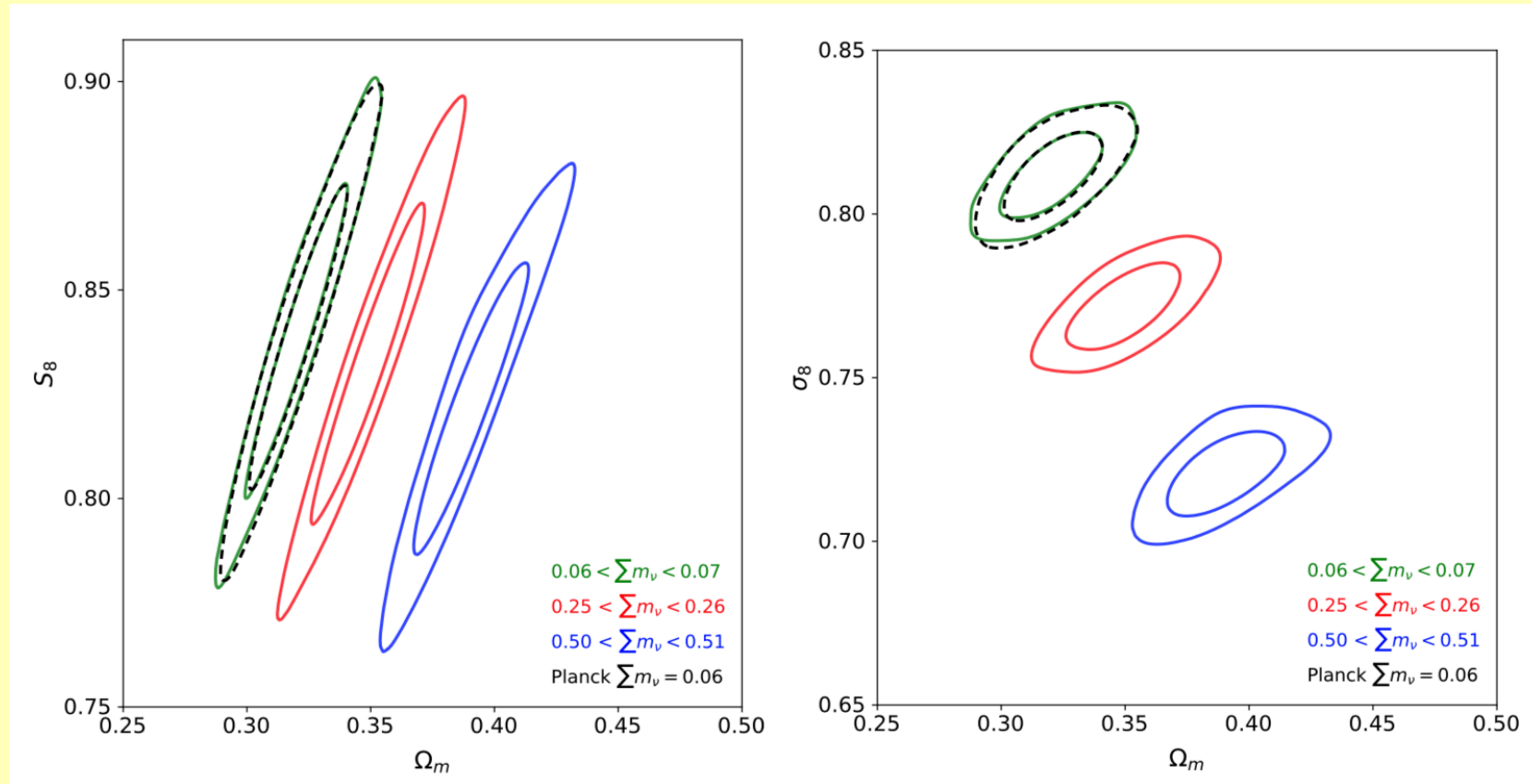
# Neutrinos: impact on cosmology



$$s = 145.0 (\Omega_m h^2 / 0.140)^{-0.25} (\Omega_b h^2 / 0.0225)^{-0.08} \text{ Mpc}$$

Now no neutrino contribution. Shift to large scales as  $(1-f)^{-0.25}$

# Neutrinos: impact on cosmology



$$\Omega_m \propto (1 - f_\nu)^{-6}$$
$$\sigma_8 \propto (1 - f_\nu)^{-3}$$

Tension with lensing formally favours negative neutrino mass. Need to resolve role of systematics in tensions before neutrinos can be detected.

# 7: Outlook



# DESI



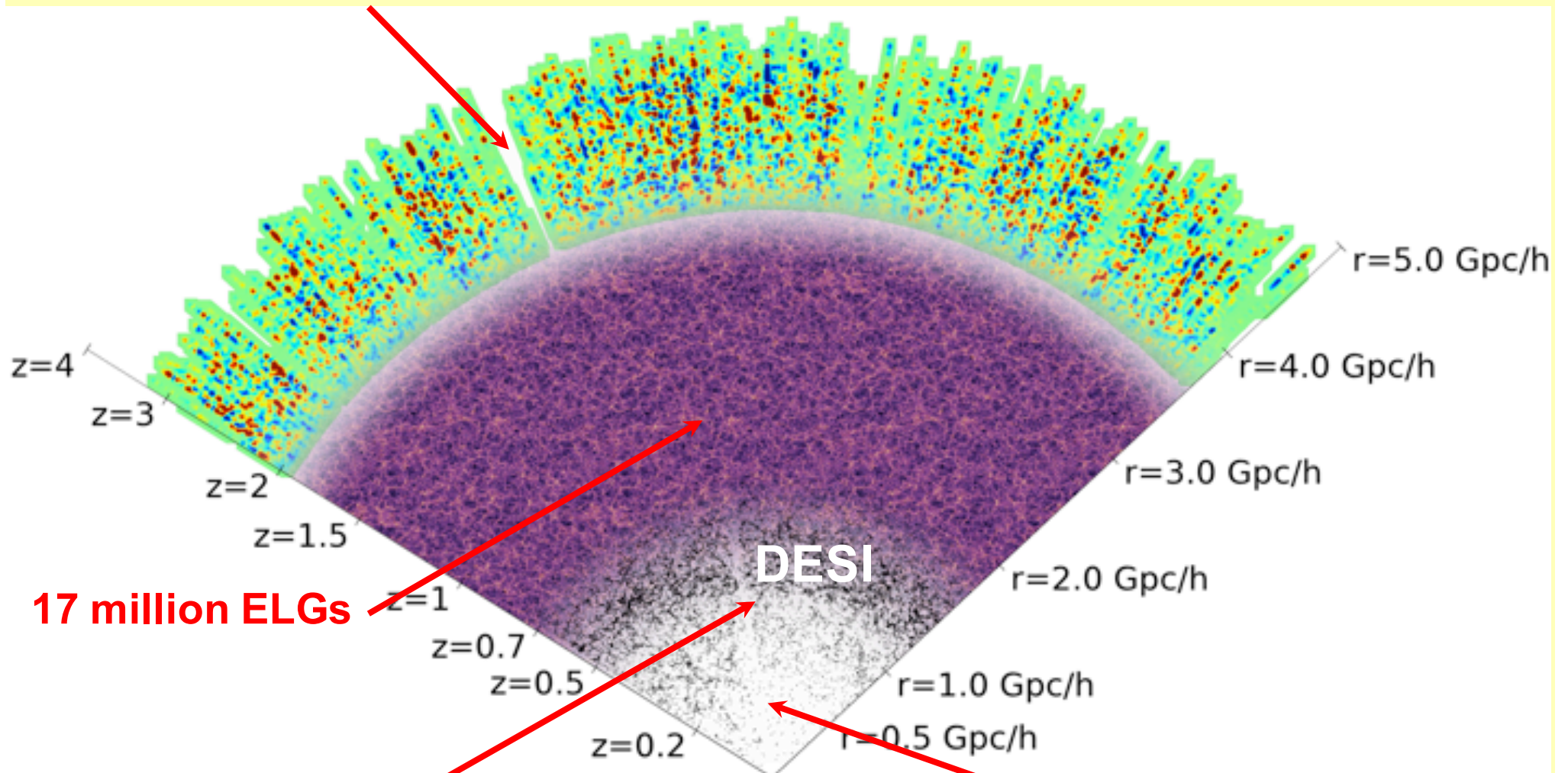
DOE project for KPNO 4m  
over 2019-2024:

5000 Fibres; 3-deg field  
30M galaxies

- LRGs to  $z = 0.9$
- OII ELGs to  $z = 1.7$
- QSOs to  $z = 3$

# DESI redshift coverage

3 million QSOs

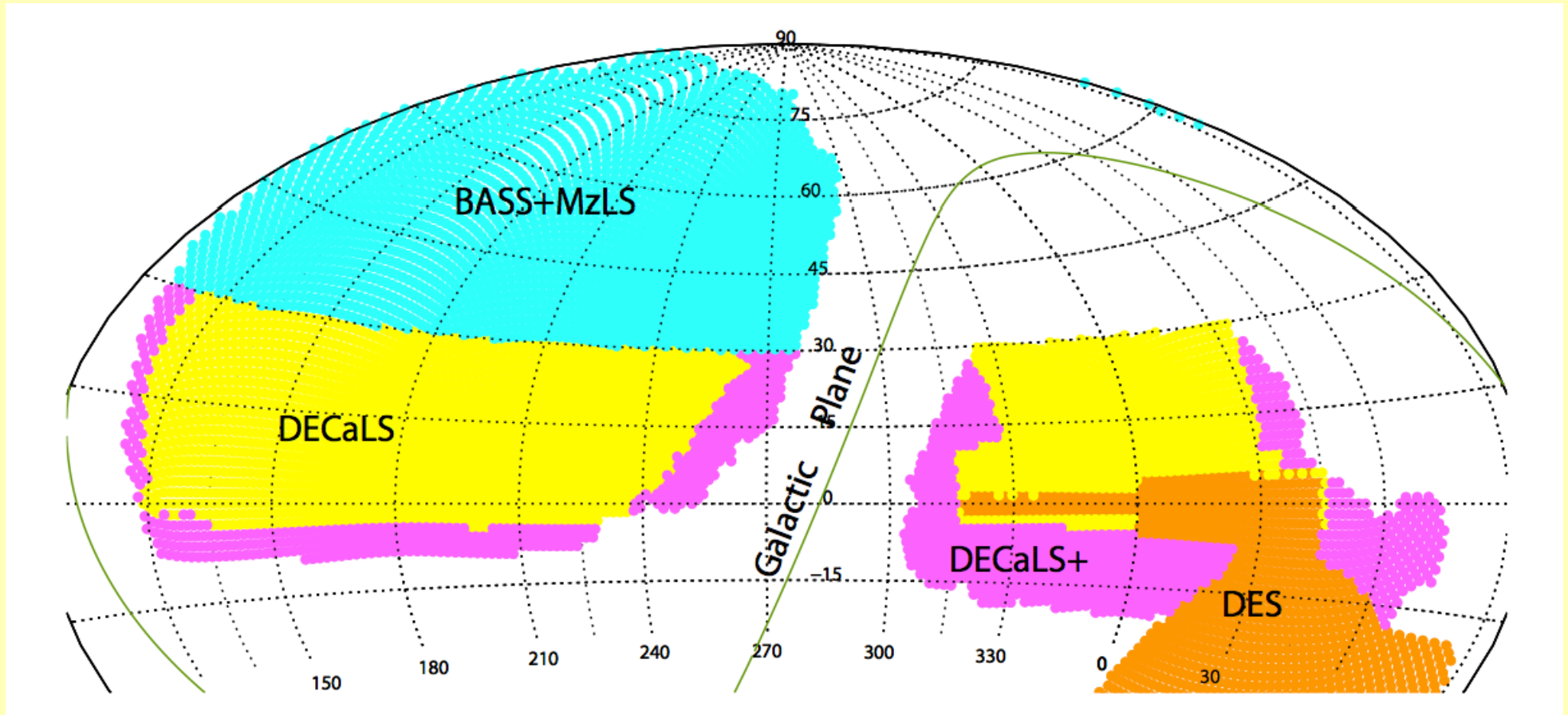


17 million ELGs

4 million LRGs

10 million BGs  
( $r < 19.5$ )

# DESI target photometry



14,000 deg<sup>2</sup> in grz to 24.0, 23.4, 22.5 – nearly complete



# Public data: legacysurvey.org

RA,Dec = 187.2688, 2.0486, zoom 15

+

-

30 arcsec

Jump to object:

Custom catalog upload (FITS table; RA,Dec,[name]):  
 No file selected.

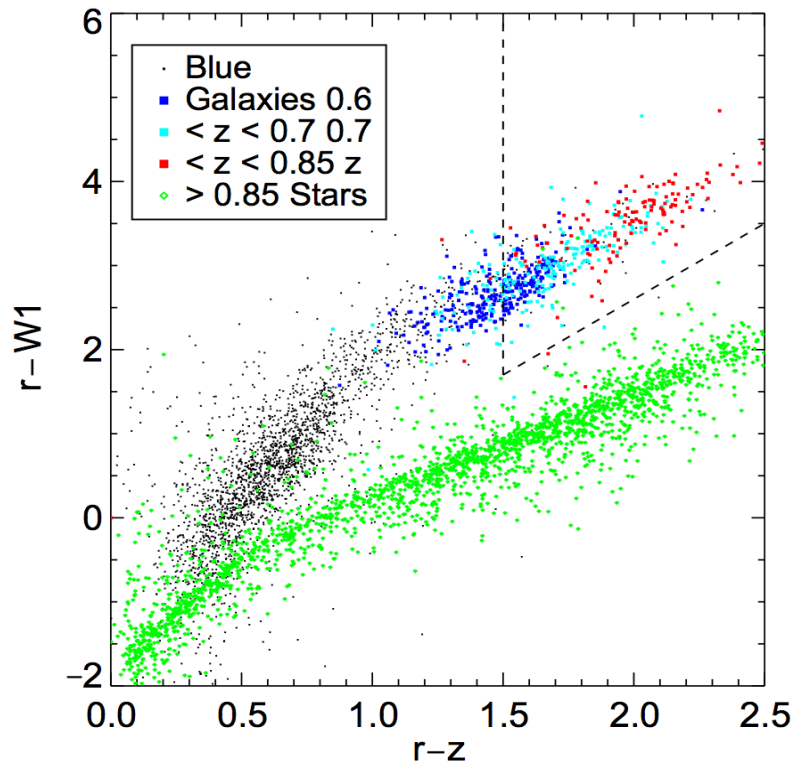
- DECaLS DR5 images
- DECaLS DR5 models
- DECaLS DR5 residuals
- DECaPS images
- DECaPS models
- DECaPS residuals
- MzLS+BASS DR4 images
- MzLS+BASS DR4 models
- MzLS+BASS DR4 residuals
- DECaLS DR3 images
- DECaLS DR3 models
- DECaLS DR3 residuals
- SDSS images
- unWISE W1/W2
- unWISE W1/W2 NEO2
- SFD dust map
- Halpha map

- DECaLS DR3 Catalog loading 6...
- MzLS+BASS DR4 Catalog
- DECaLS DR5 Catalog
- DECaLS DR3 CCDs
- MzLS+BASS DR4 CCDs
- DECaLS DR5 CCDs
- DECaLS DR3 Exposures
- DECaLS DR5 Exposures
- DECaLS Bricks
- SDSS CCDs
- NGC/IC galaxies
- Bright stars
- Tycho-2 stars
- Gaia DR1 sources
- SDSS Spectra
- SDSS Spectro Plates
- DEEP2 Spectra

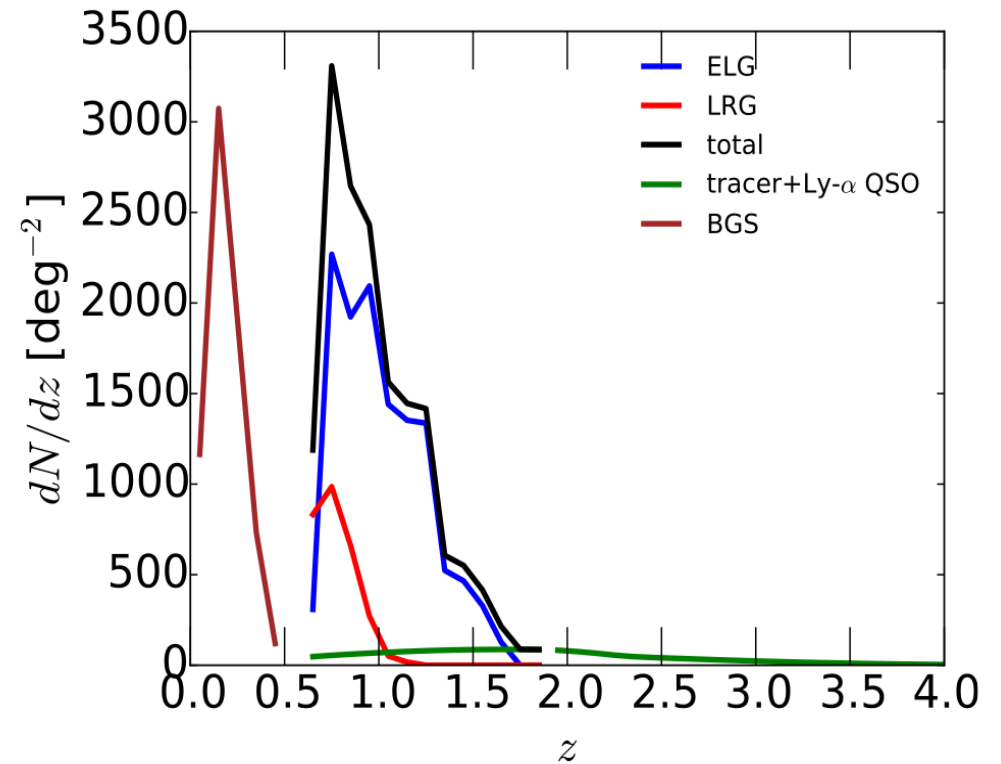
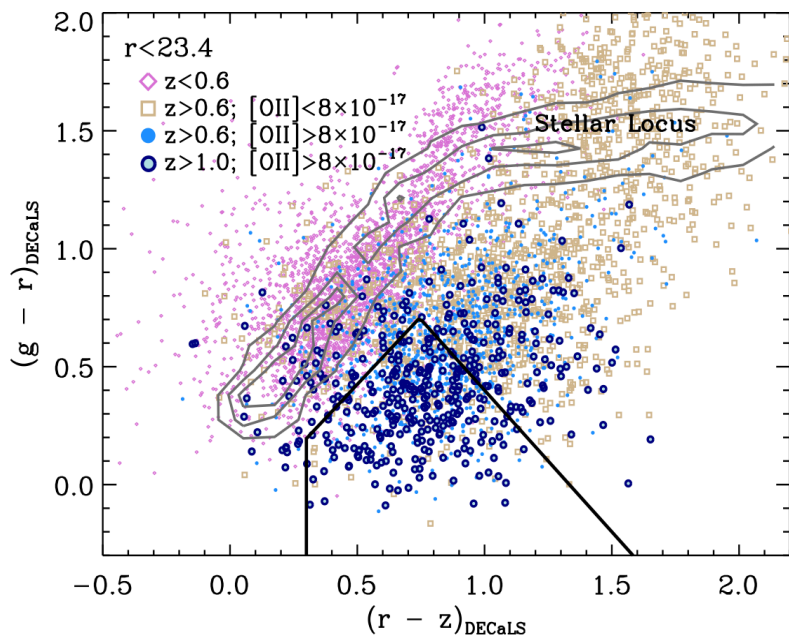
LRG

# DESI targets

Multicolour grz selection including WISE new data

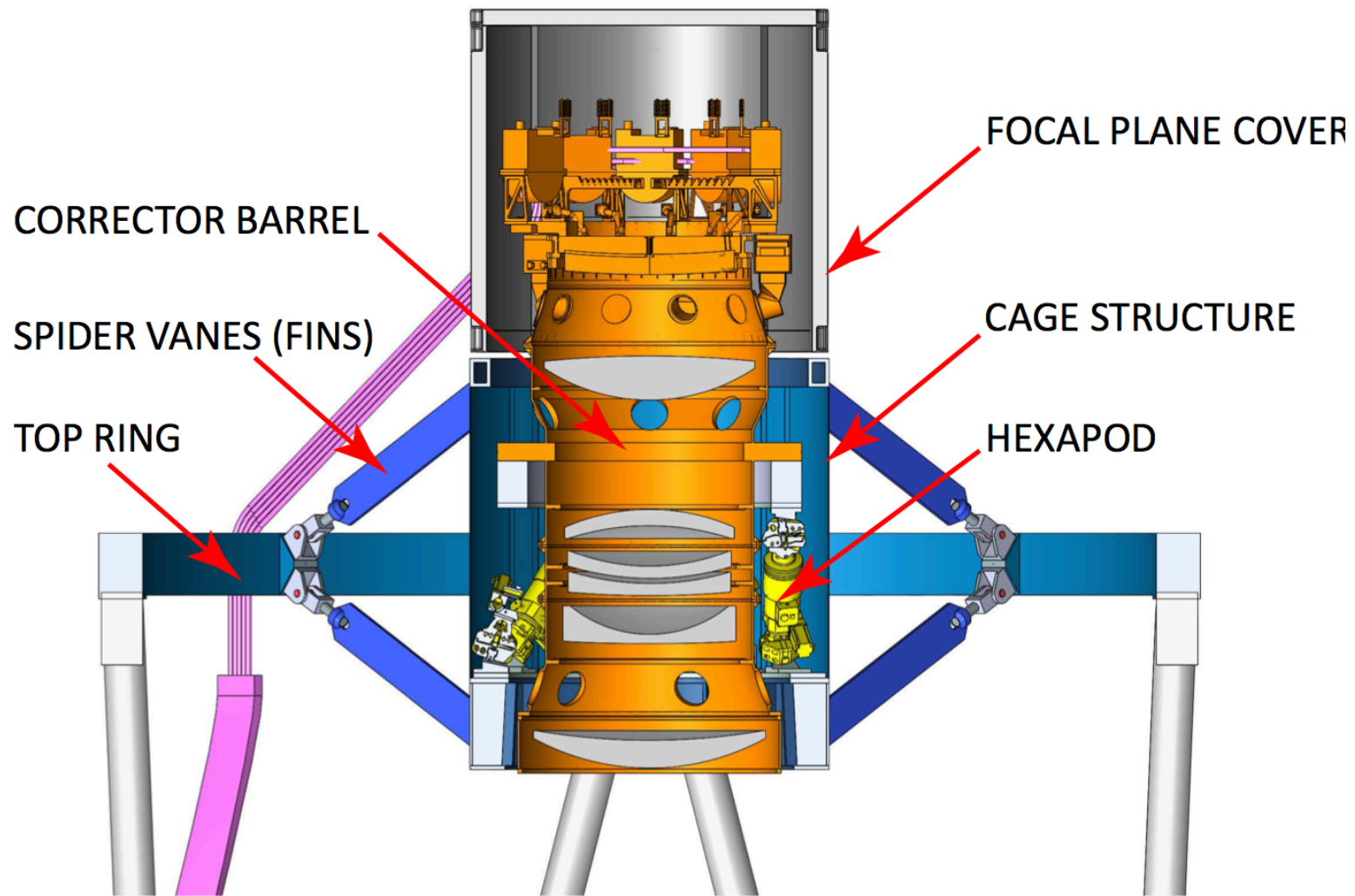


ELG





# DESI corrector and positioner

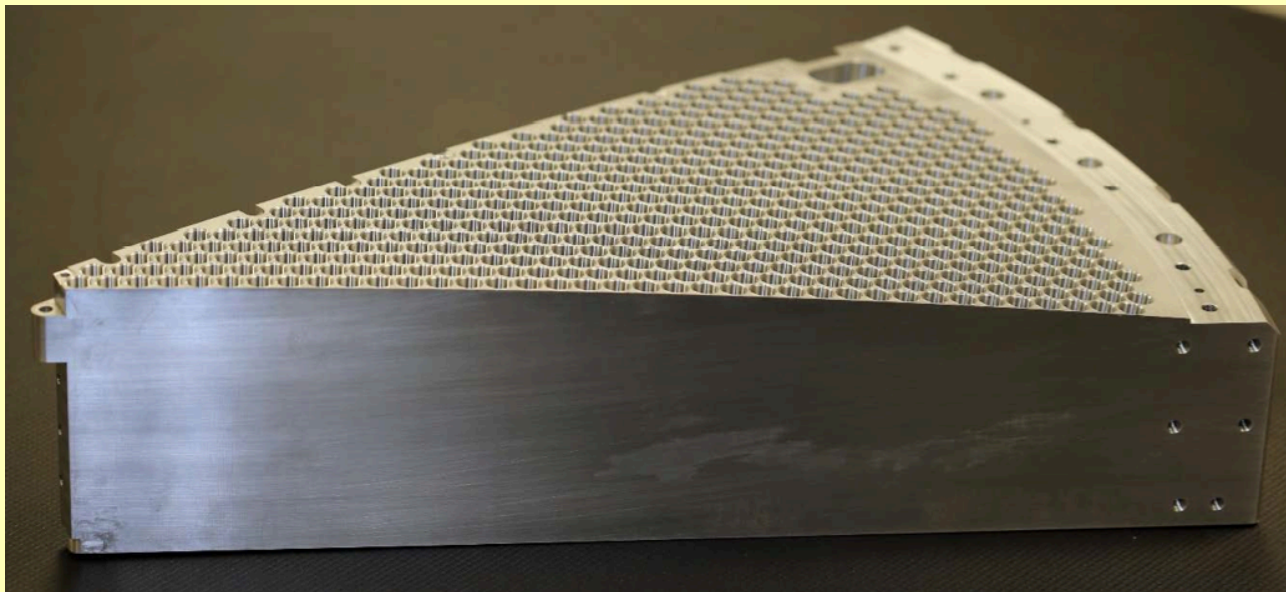




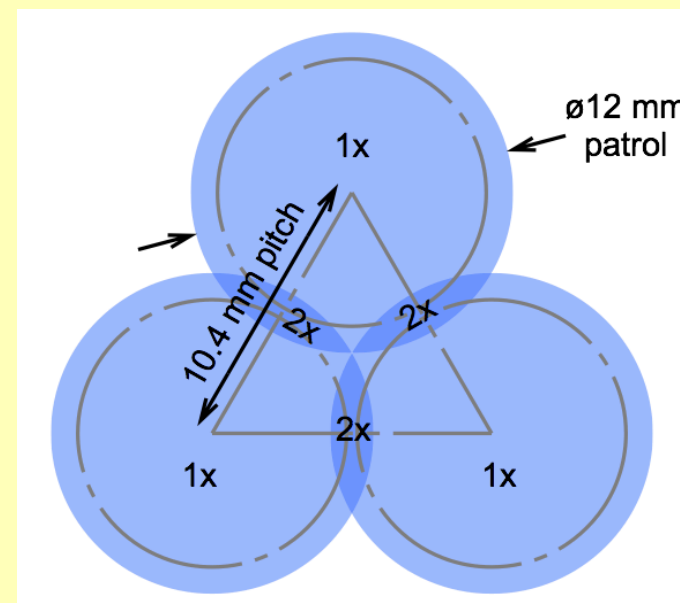
# DESI optics



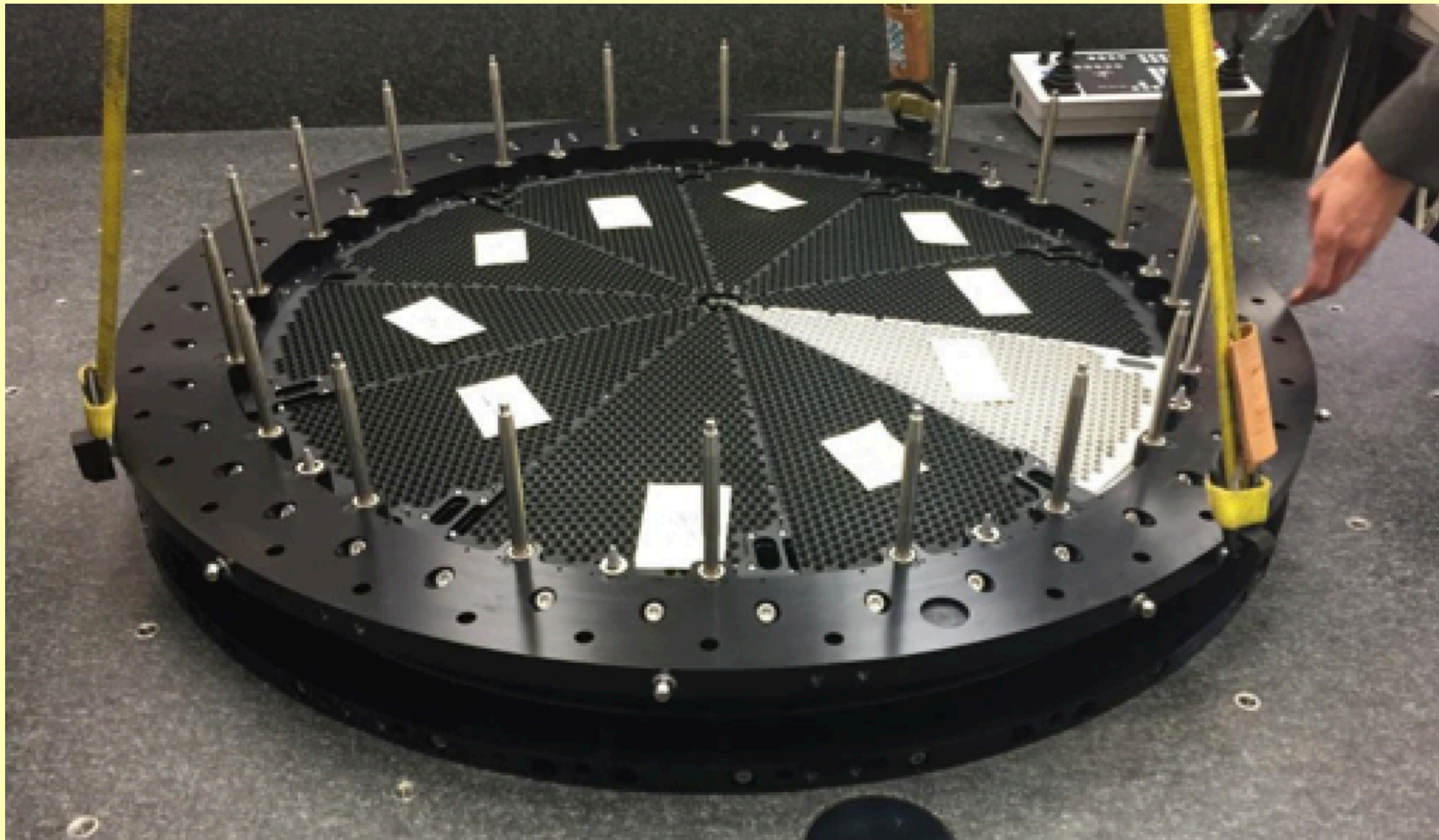
# DESI positioner



5000 twin r-theta epicyclic positioners, mounted in petals

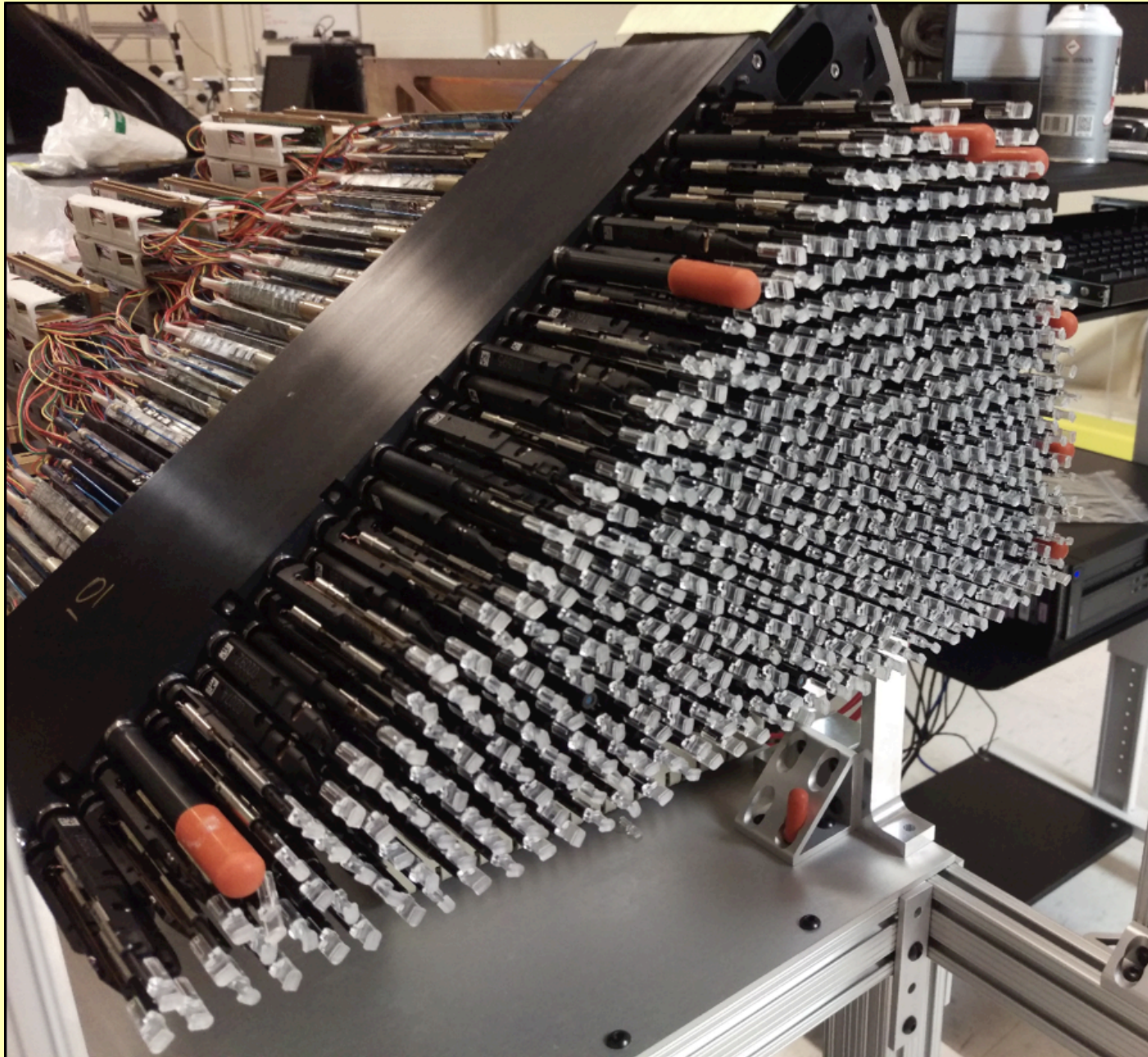


# DESI positioner

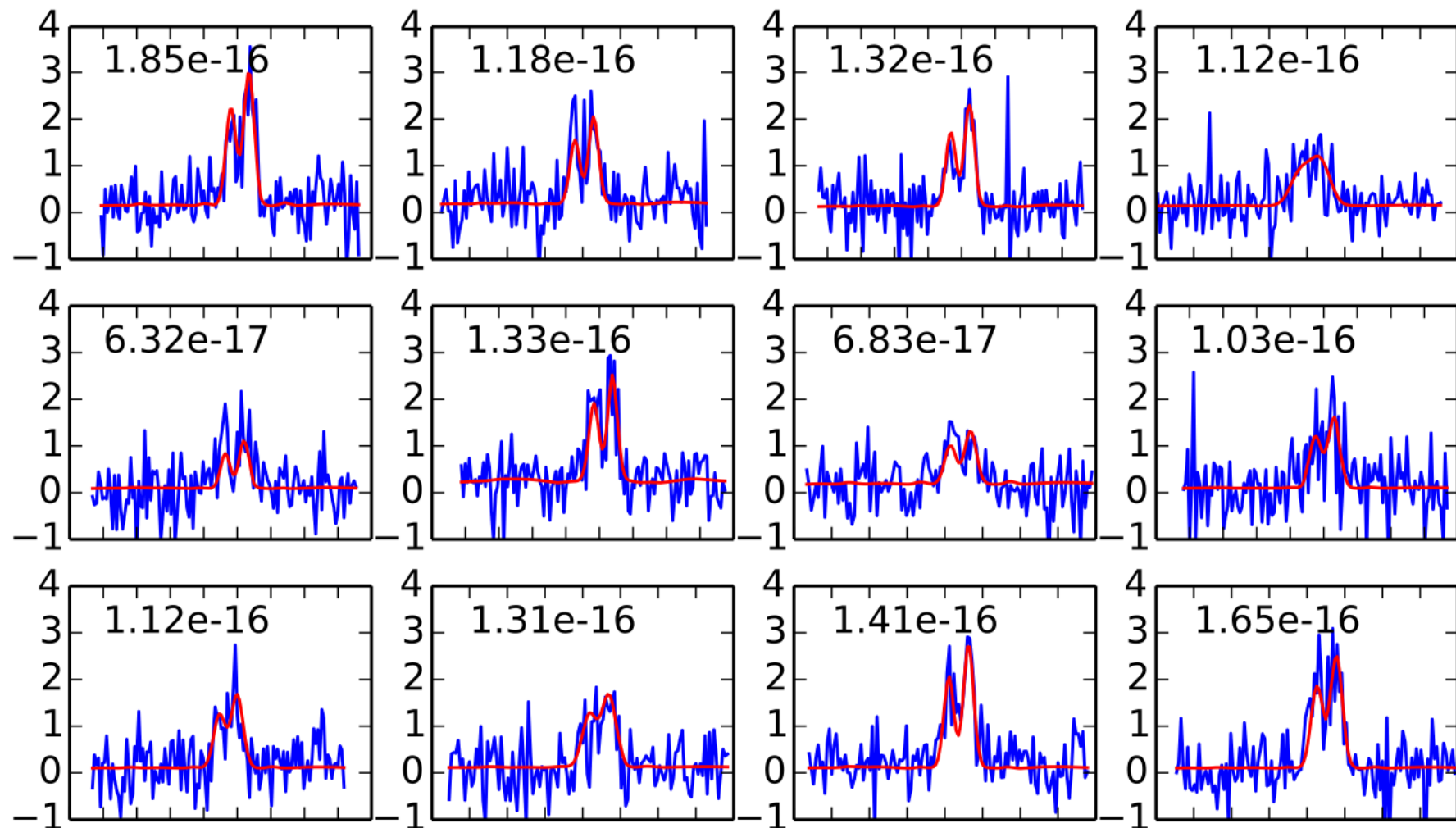




# DESI positioner



# DESI spectra ( $R \sim 3000$ )



OII flux limit  $8 \times 10^{-17}$  cgs in 20-min exposures (5m for BGS)

# DESI Schedule

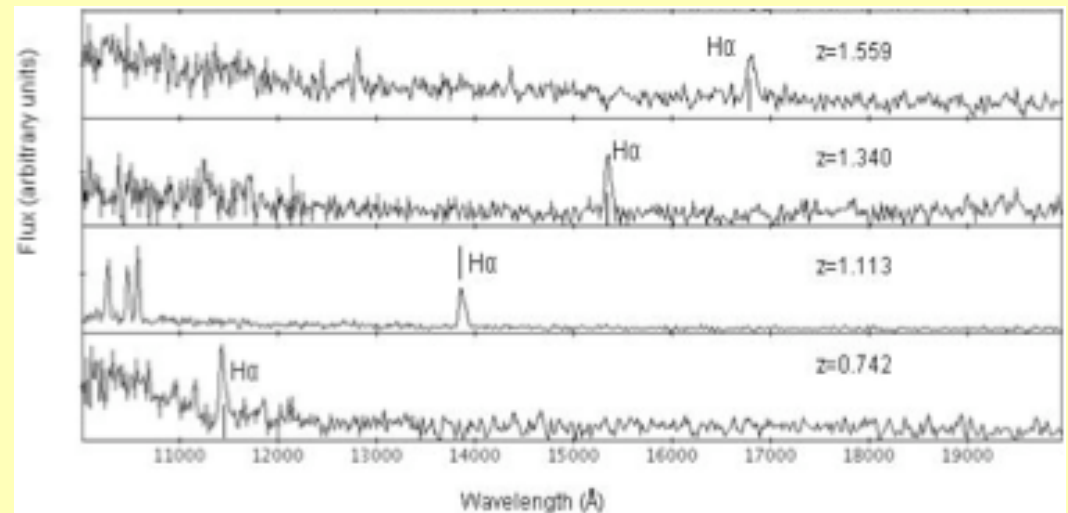
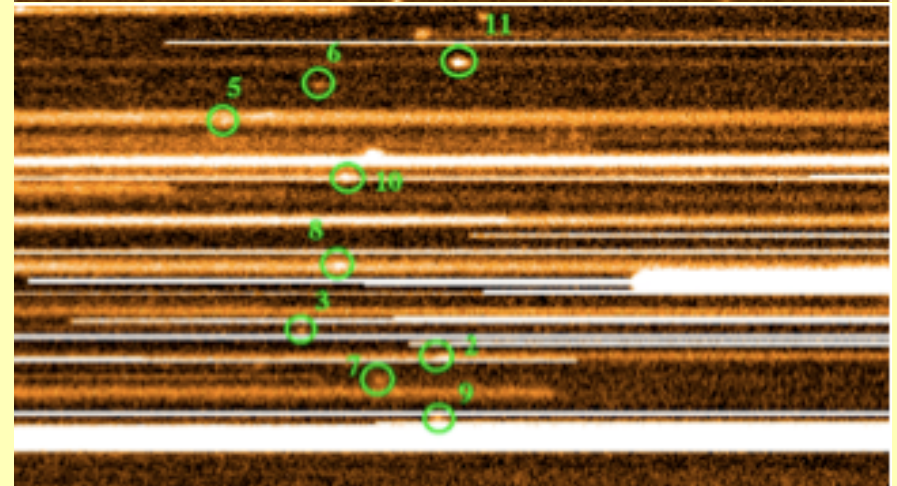
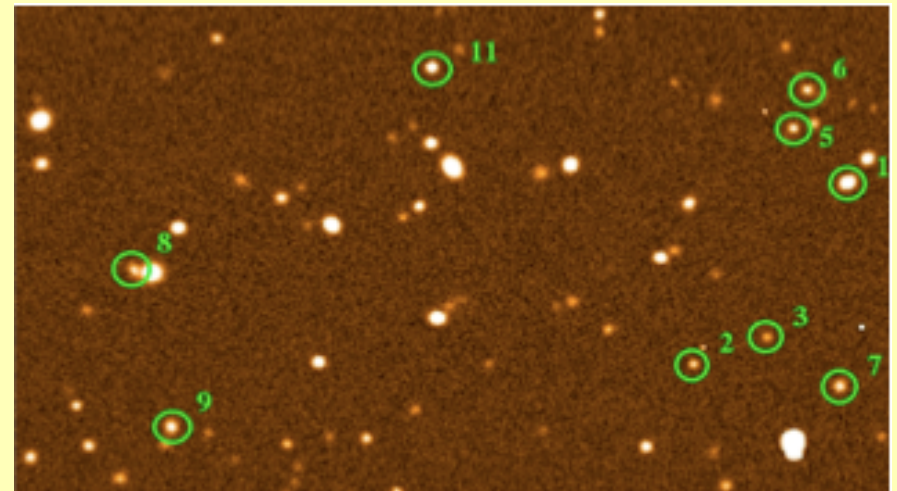
- April 2019: Commissioning starts
- ....Covid....
- May 2021: start of main survey operations
  - Complete in 5 years
  - Currently ~50% (>10M z's)
  - Data release Summer 2023 (2M z's)
  - First key science papers early 2024



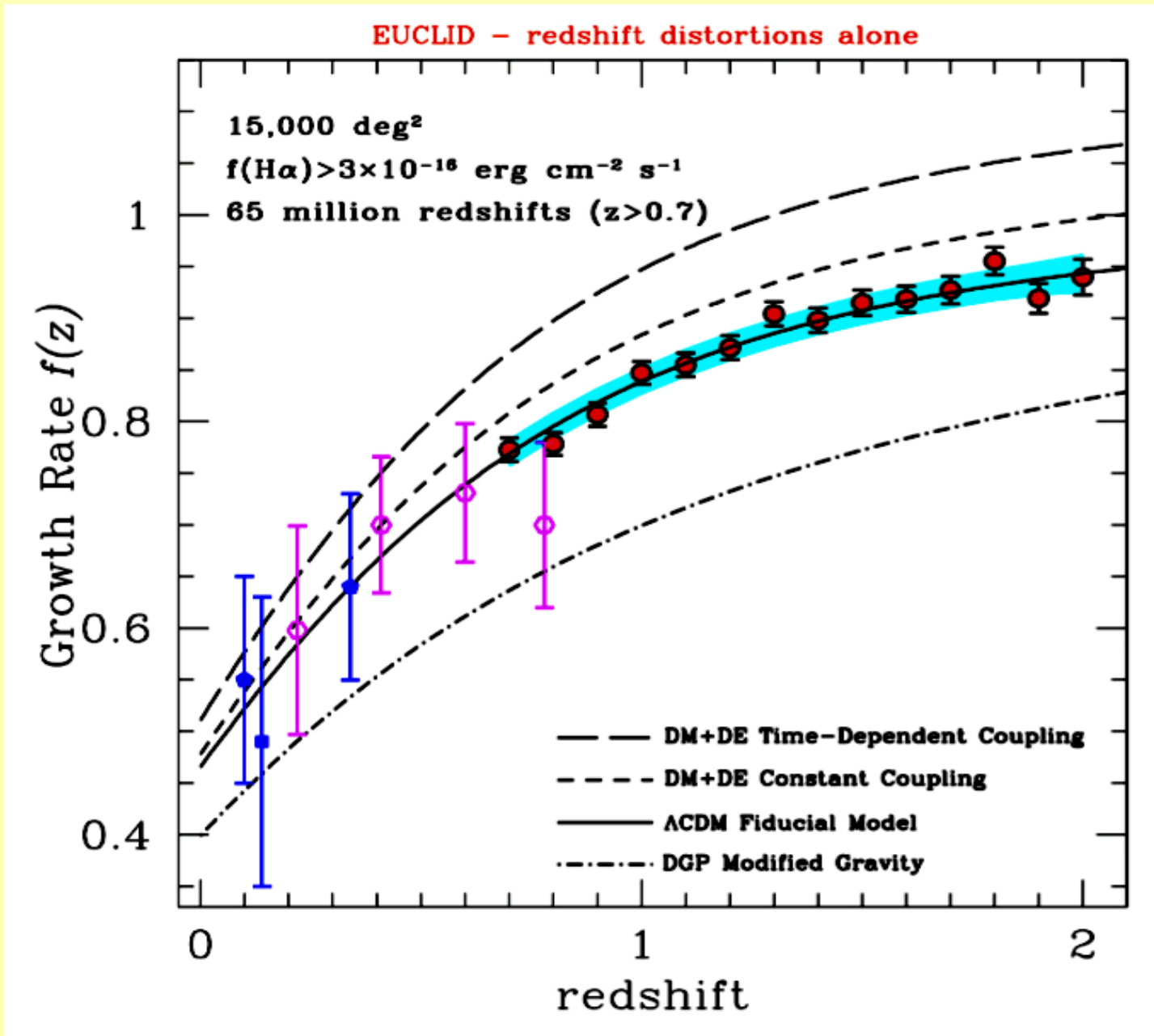
# Euclid slitless spectroscopy

NIS Instrument:

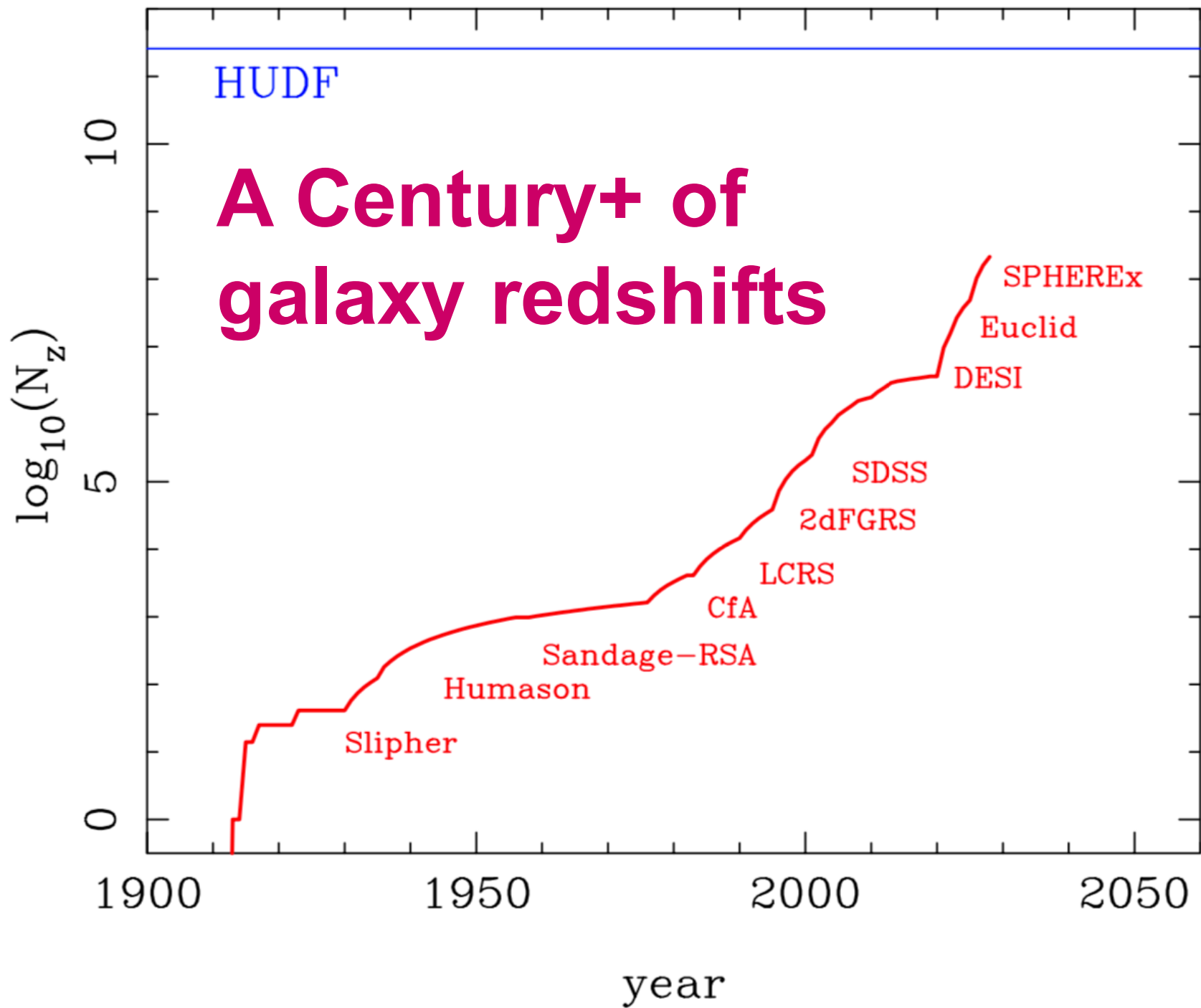
- ~ 25M redshifts to  $z \sim 2$
- 15,000  $\text{deg}^2$
- $H < 19.5$



# Euclid (2023-)

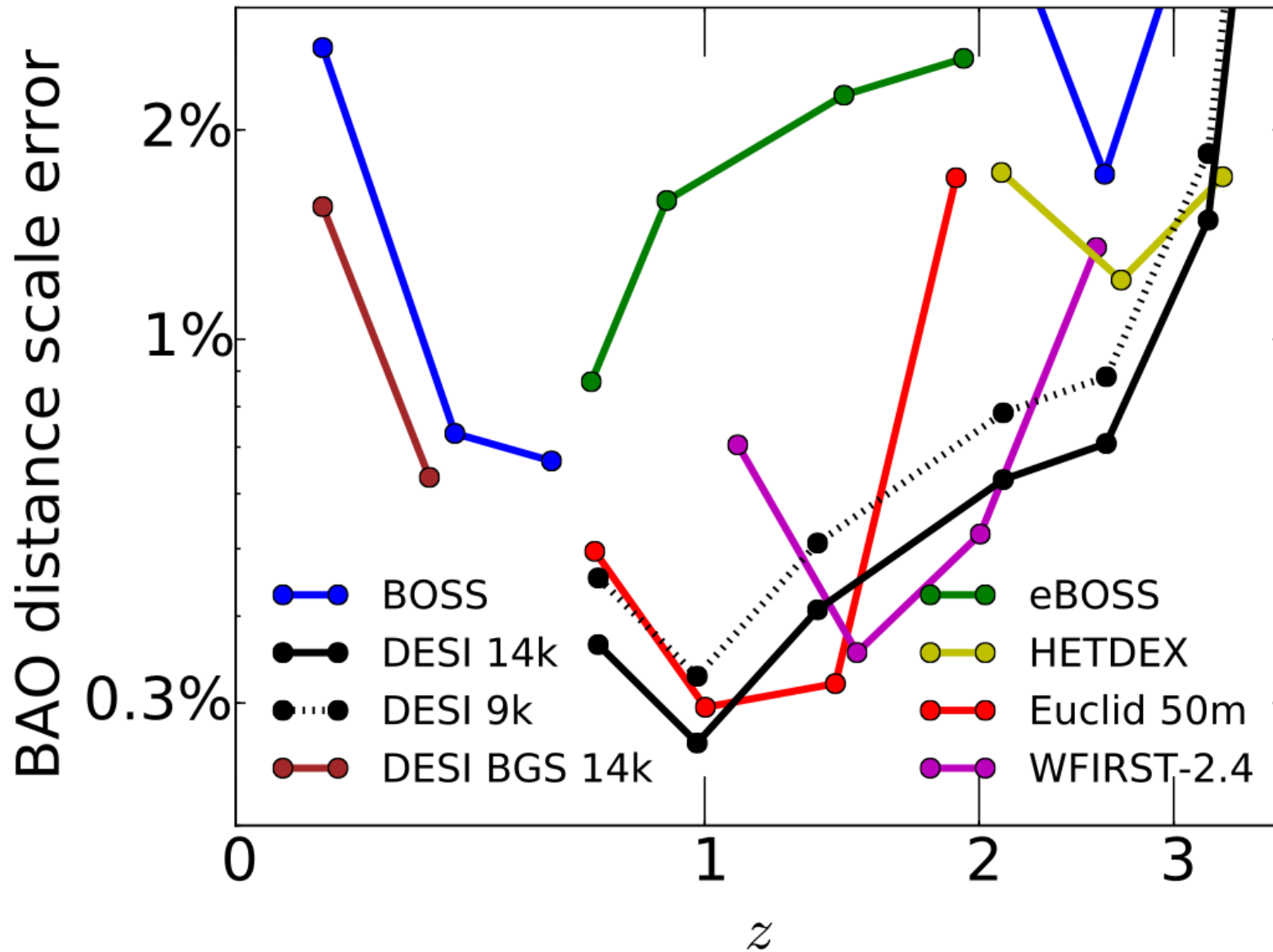


Need sub-%  
accuracy  
modelling

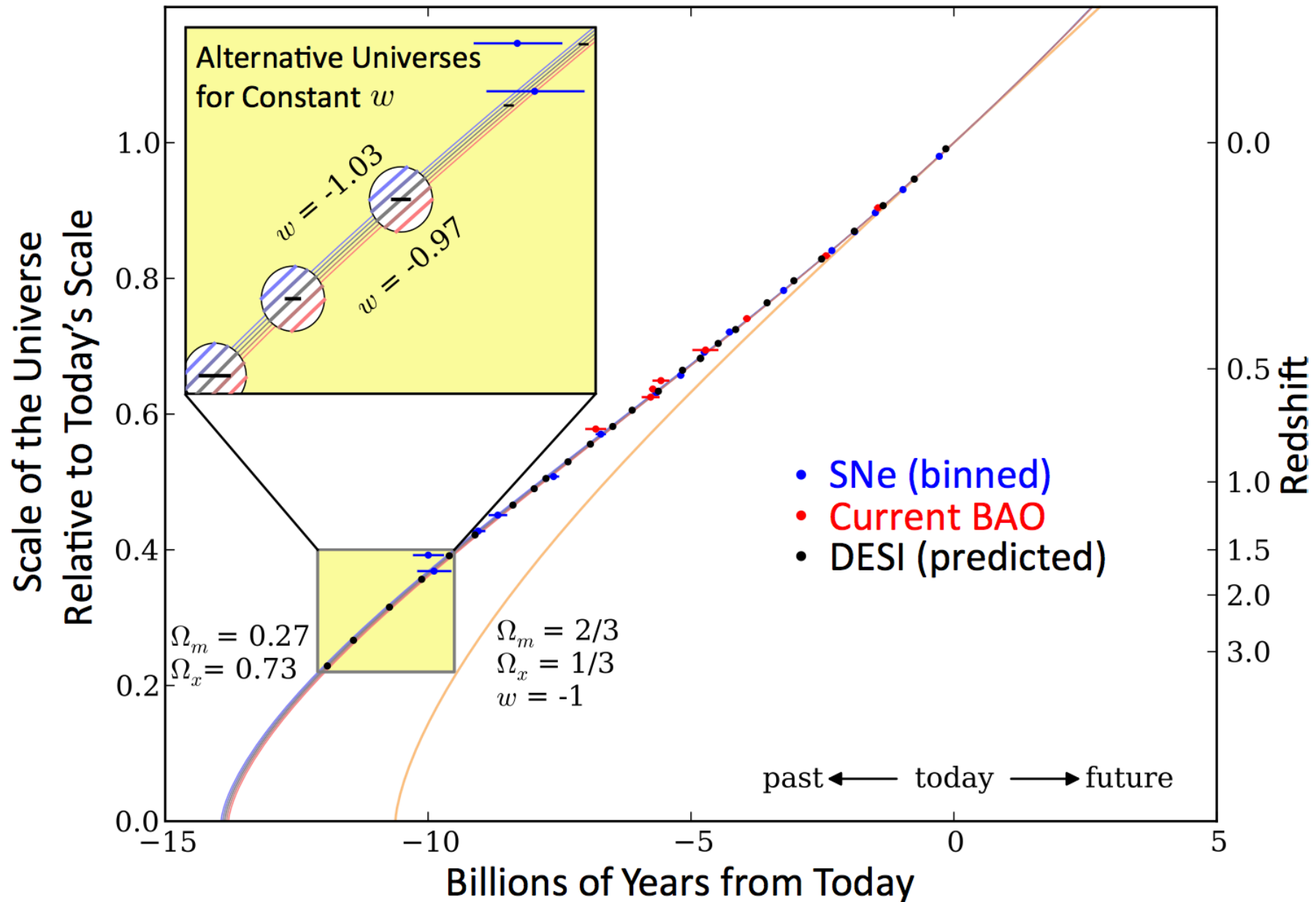


# A Century+ of galaxy redshifts

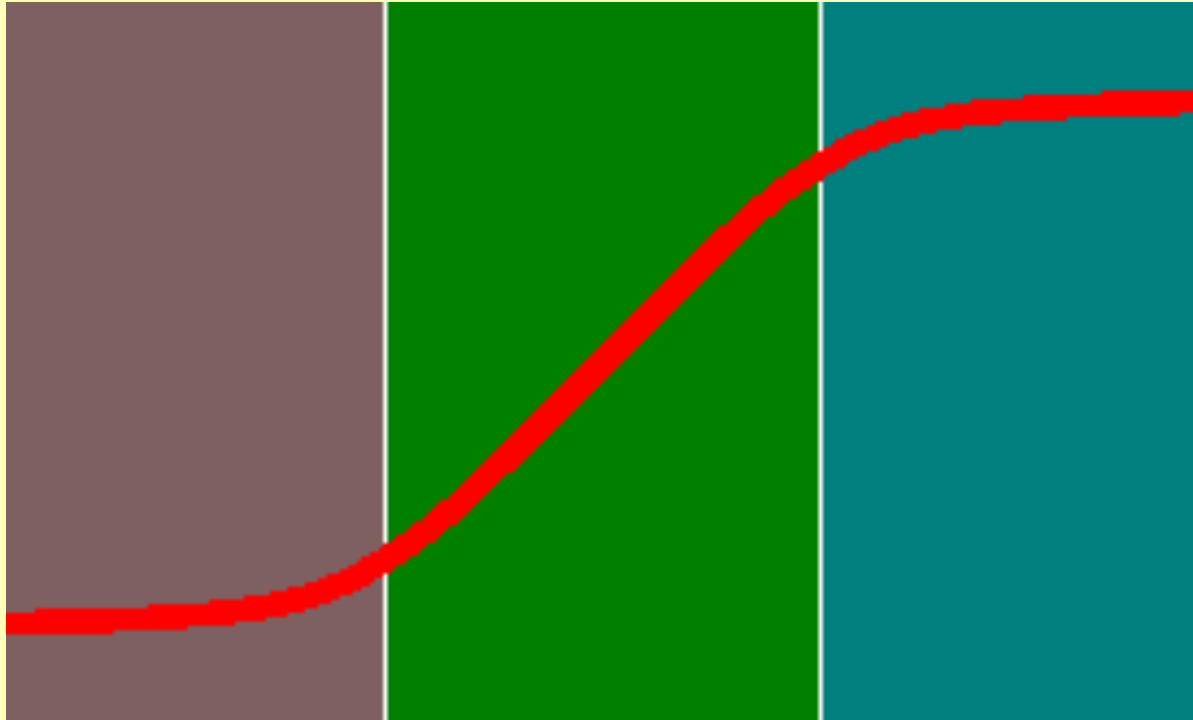
# Outlook: 0.1% cosmology



# Precision is challenging



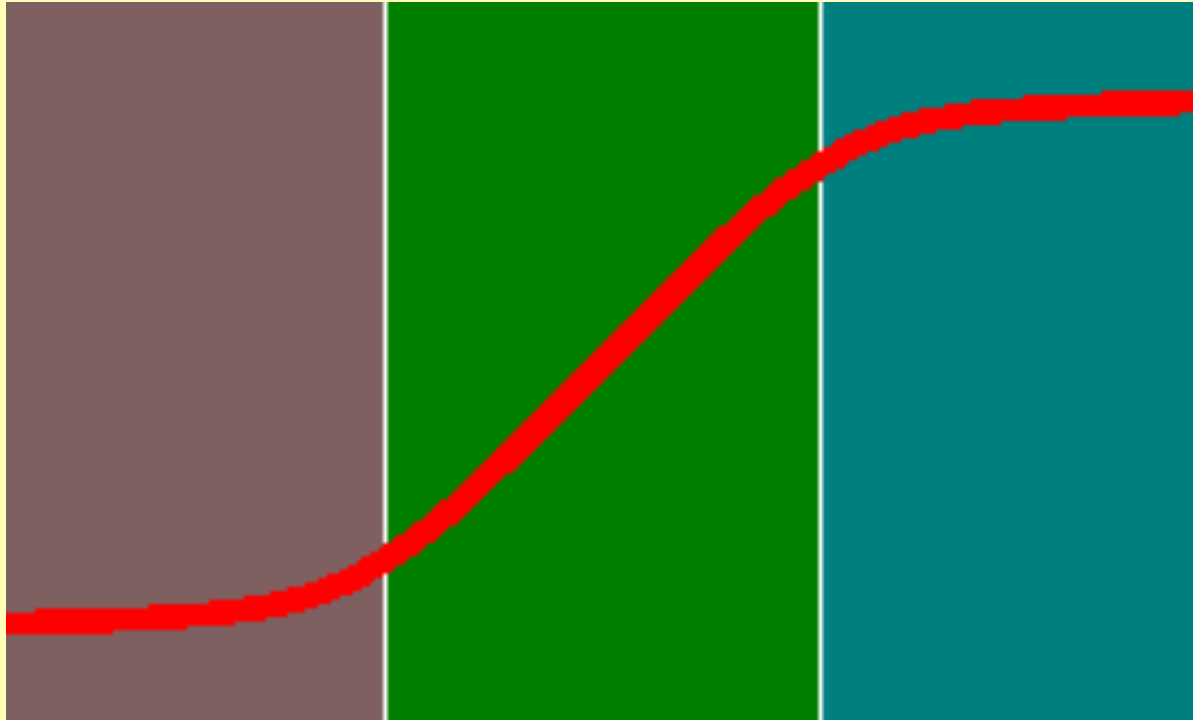
# The statistical demographic transition



Data  
hopeless



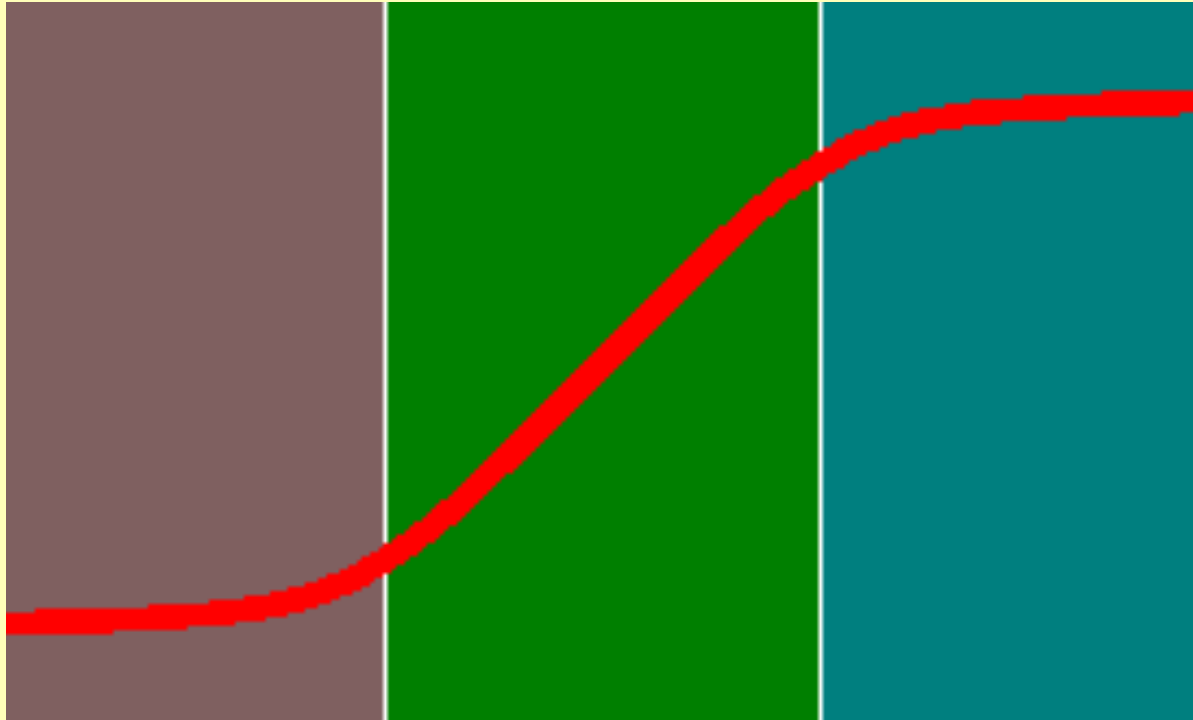
# The statistical demographic transition



Data  
hopeless

Careful  
statistics  
can give  
results

# The statistical demographic transition



Data  
hopeless

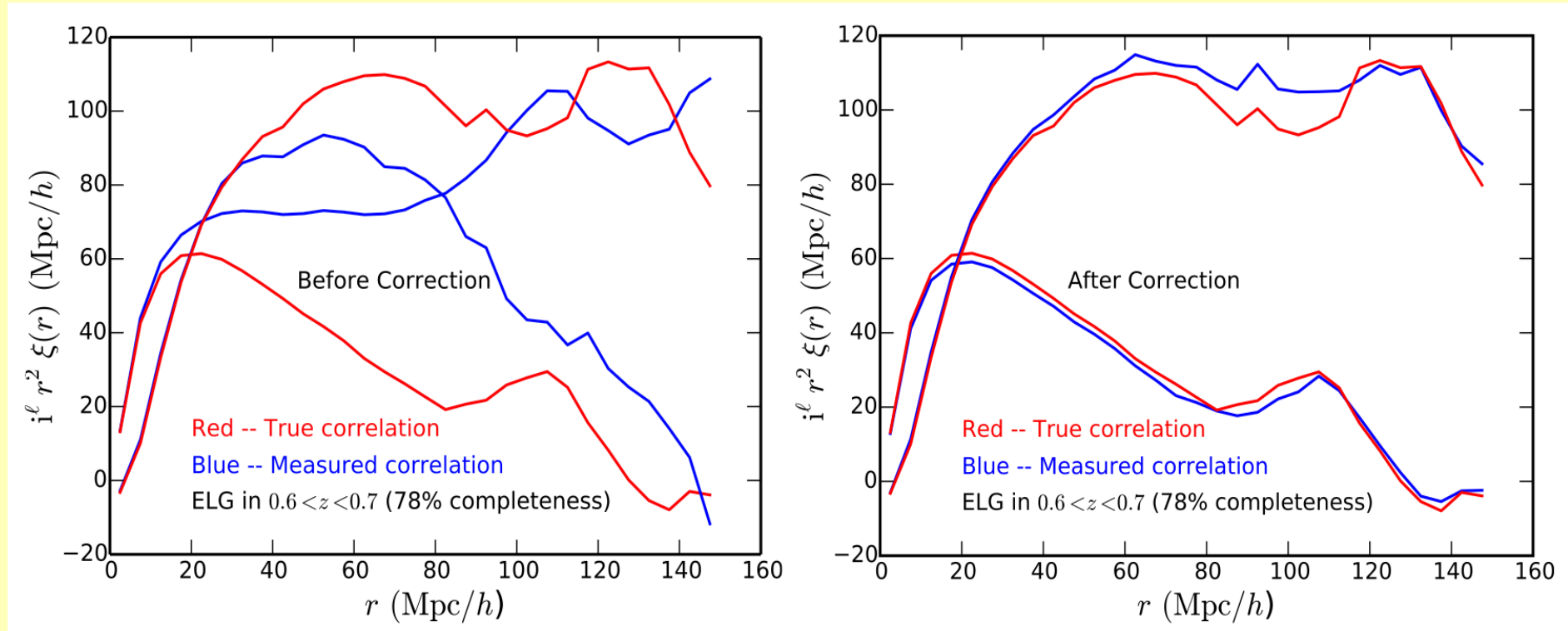
Careful  
statistics  
can give  
results

Systematics  
dominated

# Issues with systematics

- Internal consistency
  - Essential to pass null tests between data subsets
  - If cosmic variance dominates, can rule out many data systematics
  - But if noise dominates, systematics at  $1\sigma$  level are undetectable
  - cf. Planck results at  $\ell < 1000$  vs  $\ell > 1000$
- External consistency
  - But consistency doesn't prove no systematics (1803.04470):
    - True posterior has non-Gaussian wings for 'unknown unknowns'
    - Naïve standard errors only work with many consistent experiments
    - Important role for independent techniques of moderate precision

# Vulnerability to mocks



Observational strategy causes  $O(1)$  raw systematics, which must be corrected to 0.1% precision

# Vulnerability to Bayes

Will we believe any detections of new ingredients?

$$P(\text{model} | \text{data}) \sim L(\text{data} | \text{model}) P(\text{model})$$

- Moderate prior belief in simplest neutrino hierarchy
- Strong prior belief in unevolving  $\Lambda$
- Even stronger prior belief in Einstein gravity

Already plenty of ‘detections’ that are ignored: e.g.  $\Lambda$  in 1990s; Bean 2009 GR disproof; 2014 Beutler et al. massive neutrino detection.

# Conclusions & outlook

- Cosmology has had  $\Lambda$ CDM as a standard model for structure formation for  $\sim 25$  years
  - First established using galaxy clustering + CMB
  - Has survived huge improvements in data precision
- The level of vacuum energy is a deep puzzle for the  $\Lambda$ CDM model
  - But so far 'dark energy' looks just like  $\Lambda$
- Problem for field: no definite predicted non- $\Lambda$ CDM signal
- Need to understand systematics better if the model is ever to be rejected



