Multipoles in the deceleration parameter: A comparison between different approaches

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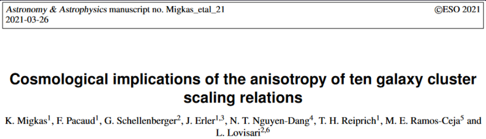
UNIVERSITY
OF THESSALONIKI

Why do we care?

Evidence for anisotropies in the Hubble parameter

Evidence for anisotropies in the Hubble parameter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2021-03-26

Cosmological implications of the anisotropy of ten galaxy cluster scaling relations

K. Migkas[1], F. Pacaud[1], G. Schellenberger[2], J. Erler[1,3], N. T. Nguyen-Dang[4], T. H. Reiprich[1], M. E. Ramos-Ceja[5] and L. Lovisari[2,6]

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Hints of FLRW Breakdown from Supernovae

Chethan Krishnan,[1,*] Roya Mohayaee,[2,†] Eoin Ó Colgáin,[3,4,‡] M. M. Sheikh-Jabbari,[5,§] and Lu Yin[3,4,¶]

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PHYSICAL REVIEW D **107,** 023507 (2023)

Multipole expansion of the local expansion rate

Basheer Kalbouneh,[*] Christian Marinoni,[†] and Julien Bel[‡]
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PAPER

A new way to test the Cosmological Principle: measuring our peculiar velocity and the large-scale anisotropy independently

Tobias Nadolny[1], Ruth Durrer[1], Martin Kunz[1] and Hamsa Padmanabhan[1]

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arXiv > astro-ph > arXiv:2212.13569

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 27 Dec 2022]

Potential signature of a quadrupolar Hubble expansion in Pantheon+ supernovae

Jessica A. Cowell, Suhail Dhawan, Hayley J. Macpherson

And many others...

Evidence for a dipole in the deceleration parameter

Hints of FLRW Breakdown from Supernovae

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Physics of the Dark Universe
Volume 40, May 2023, 101224

Testing ΛCDM cosmology in a binned universe: Anomalies in the deceleration parameter

Erick Pastén, Víctor H. Cárdenas

A&A 631, L13 (2019)
Letter to the Editor

Evidence for anisotropy of cosmic acceleration*

Jacques Colin[1], Roya Mohayaee[1], Mohamed Rameez[2] and Subir Sarkar[3]

Do supernovae indicate an accelerating universe?

Roya Mohayaee, Mohamed Rameez & Subir Sarkar

The European Physical Journal Special Topics **230**, 2067–2076 (2021) | Cite this article

More analysis required...

Three different theoretical approaches

FLRW formalism

we all know about it...

Generalized time-like formalism

new + parts inspired on the expansion tensor definition (see Ellis, Maartens & MacCallum, *Relativistic Cosmology*).

Null formalism

Kristian & Sachs *Astrophys. J.* 143 379 (1966); MacCallum & Ellis *Commun. Math. Phys.* 19 31-64 (1970); Clarkson PhD Thesis (2000); Heinesen *JCAP* 05 008 (2021)

The FLRW formalism

In order to obtain a $d_L(z)$ relation we must first expand the redshift either in terms of t, τ or λ. In the FLRW case, we have:

$$1 + z = \frac{a(t)}{a(t_0)} = z(t) \, .$$

By making use of the photon travelled distance ($D = c \int \mathrm{d}t = D(t)$) and its relation to the luminosity distance, we get:

$$d_L(z) = \frac{z}{H_0} + \frac{1}{2}\frac{(1 - q_0)}{H_0} \, z^2 + \cdots \, ,$$

where the Hubble and deceleration parameters are defined as[1]:

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \qquad q(t) = -\frac{a \, \ddot{a}(t)}{\dot{a}^2(t)} = -\left(1 + \frac{\dot{H}}{H^2}\right) \, .$$

[1] M. Visser, Class. Quantum Grav.21(2004) 1–13

The FLRW formalism

Advantages:

- Directly connected to observations;
- Few parameters.

Drawbacks:

- Cannot account for possible spatial anisotropies;
- Doesn't allow directional dependence;
- Cannot handle the influence of local effects into data.

The Generalized time-like formalism

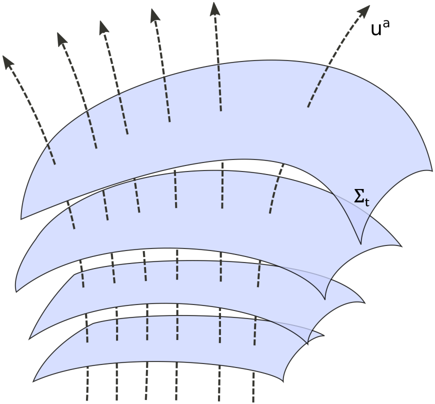
We need to define a congruence of observers with four-velocity u^a.

This allows us to realize a 3+1 split, where the metric on each Σ_t surgace is given by:

$$h_{ab} = g_{ab} + u_a u_b \ ,$$

also known as the projection operator.

Vectors living on Σ_t are represented as e^a, with $e^a e_a = 1$ and $e^a u_a = 0$.

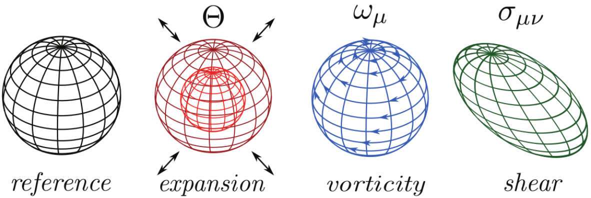


The Generalized time-like formalism

How this congruence varies along all 4 space-time directions can be used to infer the dynamical properties of the universe.

$$\nabla_b u_a = \frac{1}{3}\Theta h_{ab} + \omega_{ab} + \sigma_{ab} - A_a u_b \ ,$$

where Θ, ω_{ab} and σ_{ab} are the *expansion*, *vorticity* and *shear*, respectively. $A^a = 0$ for geodesic congruences (matter frame).



Rezzolla & Zanotti (2013)

The Generalized time-like formalism

We can define the generalized time-like expansion (or Hubble) parameter as:

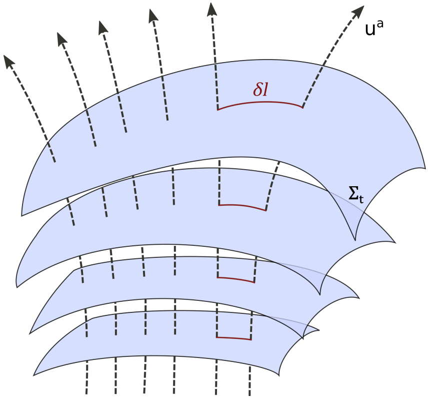
$$\mathrm{H} = \frac{\dot{\delta l}}{\delta l} = e^a e^b \nabla_a u_b = \frac{\Theta}{3} + \sigma_{ab} e^a e^b \ .$$

Extending the previous definition for the deceleration parameter

$$\mathrm{Q} = -1 - \frac{\dot{\mathrm{H}}}{\mathrm{H}^2} \ ,$$

in terms of δl we have:

$$\mathrm{QH}^2 = -\frac{\ddot{\delta l}}{\delta l} \ .$$



The Generalized time-like formalism

Arranjing in a multipole expanded way, we have:

$$\mathrm{QH}^2 = -\frac{\dot{\Theta}}{3} - \frac{\Theta^2}{9} - \frac{8}{15}\sigma^{ab}\sigma_{ab} - e^a e^b \left[\dot{\sigma}_{\langle ab\rangle} + \frac{2\Theta}{3}\sigma_{\langle ab\rangle} - 2\,\sigma^c_{\langle a}\omega_{b\rangle c} + \frac{10}{7}\,\sigma^c_{\langle a}\sigma_{b\rangle c}\right] + e^a e^b e^c e^d \left[\sigma_{\langle ab}\sigma_{cd\rangle}\right] \,.$$

Keeping only linear order terms, we have:

$$\mathrm{QH}^2 = -\frac{\dot{\Theta}}{3} - \frac{\Theta^2}{9} - e^a e^b \left[\dot{\sigma}_{\langle ab\rangle} + \frac{2\Theta}{3}\sigma_{\langle ab\rangle}\right] \,.$$

Several new terms compared to the FLRW result — yet, *no dipole*.

The Generalized time-like formalism

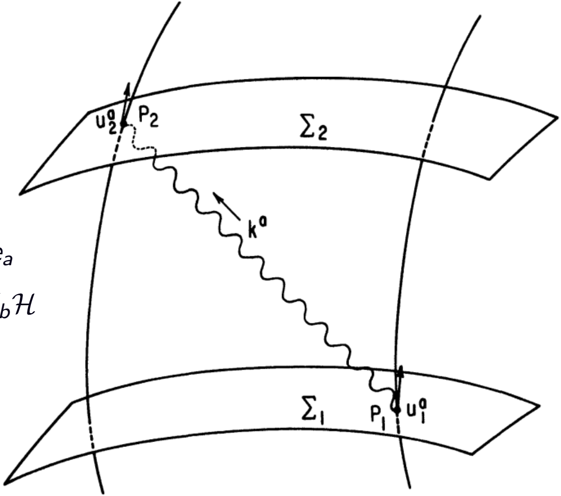
Advantages:

- No need to pre-define a metric;
- Allows for the presence of spatial anisotropies;
- Allows directional dependence;
- Can account for the influence of local effects into data.

Drawbacks:

- Not directly connect to observations.

The null formalism



$$K_a = -u_a + e_a$$

$$K^a \nabla_a K_b = -K_b \mathcal{H}$$

Wald, R., General Relativity (1984)

The null formalism

Like in the FLRW case, the Hubble and deceleration parameters for the null case are obtained via an expansion of d_L in z:

$$d_L(z,e) = \frac{z}{\mathcal{H}_0} + \frac{1}{2}\frac{(1-\mathcal{Q}_0)}{\mathcal{H}_0}\, z^2 + \cdots \ ,$$

where the null Hubble parameter is given by

$$\mathcal{H} = K^a K^b \nabla_a u_b = \frac{\Theta}{3} - A_a e^a + \sigma_{ab} e^a e^b \ ,$$

while the deceleration parameter is defined as:

$$\mathcal{Q} = \frac{K^a K^b K^c \nabla_a \nabla_b u_c}{(K^a K^b \nabla_b u_a)^2} - 3 = -1 - \frac{\mathcal{H}'}{\mathcal{H}^2} \ .$$

Here prime represents a derivative w.r.t $K_f^a = u^a - e^a$.

The null formalism

Expanding all the variables and taking the traces, we have:

$$\mathcal{Q}\mathcal{H}^2 = \mathfrak{q}^0 + e^a \mathfrak{q}^1_a - e^a e^b \mathfrak{q}^2_{ab} + e^a e^b e^c \mathfrak{q}^3_{abc} - e^a e^b e^c e^d \mathfrak{q}^4_{abcd} \, .$$

At linear order for geodesic fluids, we have:

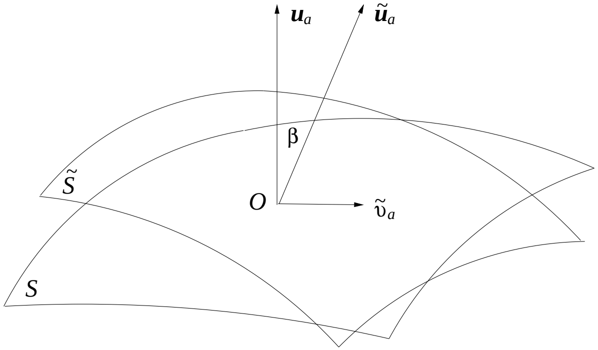
$$\begin{aligned} \mathcal{Q}\mathcal{H}^2 = & -\left(\frac{\Theta^2}{9} + \frac{\dot{\Theta}}{3}\right) + e^a \left(\frac{1}{3} D_a \Theta + \frac{2}{5} D_b \sigma^b_a\right) \\ & - e^a e^b \left(\frac{2}{3} \Theta \sigma_{ab} + \dot{\sigma}_{\langle ab \rangle}\right) + e^a e^b e^c (D_{\langle a} \sigma_{bc \rangle}) \end{aligned}$$

Now we have dipole and octopole terms!

But are they less significant?
Where do they actually come from?
Why should we care?

The null formalism

$$\mathcal{Q} = -1 - \frac{\left(\dot{\mathcal{H}} - e^a \nabla_a \mathcal{H}\right)}{\mathcal{H}^2} = \mathcal{Q}_\tau + \frac{e^a \nabla_a \mathcal{H}}{\mathcal{H}^2} \; .$$



The null formalism

Advantages:

- No need to pre-define a metric;
- Allows for the presence of spatial anisotropies;
- Allows directional dependence;
- Can account for the influence of local effects into data;
- Directly connected to observations.

Drawbacks:

- Requires more data and computational power.

Thank you!