Super Sample Covariance - an uninvited guest

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Context: forecast analysis for *Euclid* photo

- Aim: to predict the *uncertainties* on the cosmological parameters for a given model, observables, survey specifics and systematic effects
- Probes investigated: photometric Galaxy Clustering, Weak Lensing Cosmic Shear and their combination (‘3x2pt’)
- Fisher Matrix analysis: *fast*, but limited to Gaussian posterior
- Recipe: construct a Likelihood around a fiducial cosmology and compute the expected value of the curvature around the maximum ($\boldsymbol{\theta}_{fid}$)
- If L is Gaussian $F_{\alpha\beta}$ depends only on the expected covariance of the data

$$F_{\alpha\beta} = \frac{\partial C_{ij}^{AB}(\ell)}{\partial \theta_\alpha} \mathrm{Cov}^{-1}\left[C_{ij}^{AB}(\ell), C_{mn}^{CD}(\ell)\right] \frac{\partial C_{mn}^{CD}(\ell)}{\partial \theta_\beta}$$

Super Sample Covariance (SSC)

- *Euclid* will resolve small angular scales, where nonlinear evolution becomes important
- SSC is believed to be the dominant source of non-Gaussian uncertainty for upcoming WL surveys (up to $\sim 100\%$ uncertainty increase: see, e.g., Barreira+ 2018, Upham+ 2021, Rizzato+ 2020)
- A complete characterization for the *Euclid* photometric survey is still missing

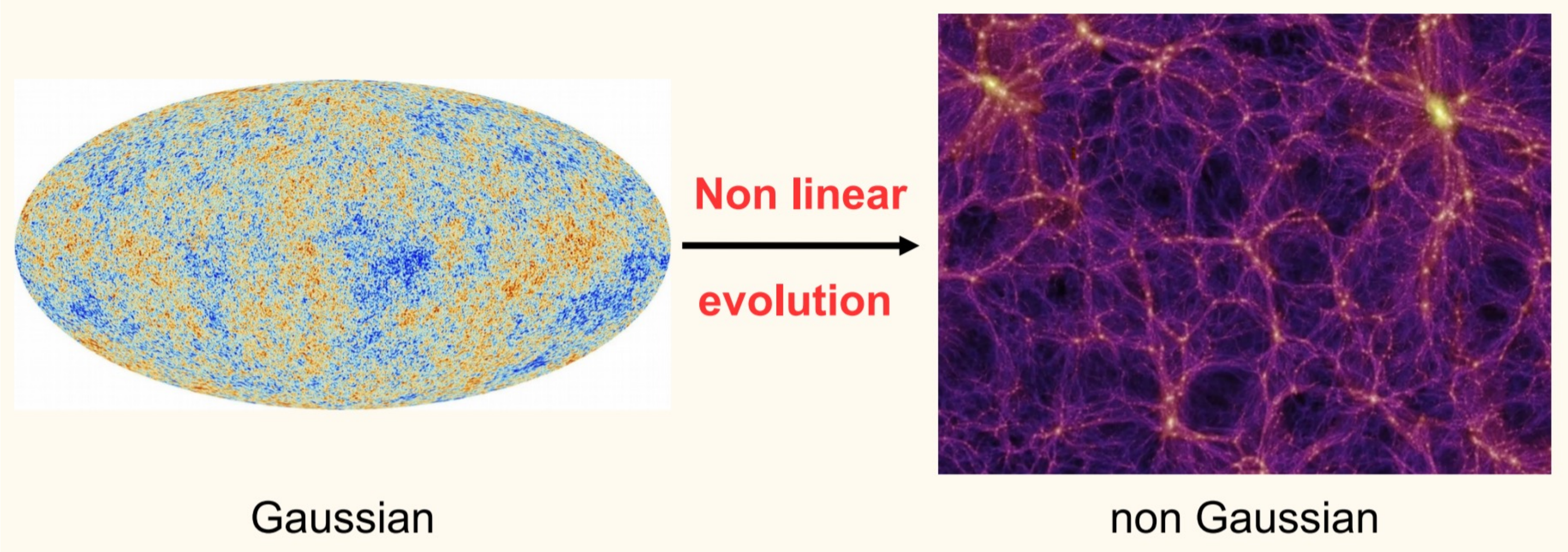
SSC

- *A form of sample variance*
- The volume of the universe observed is limited both in depth (z) and sky fraction

  → matter density fluctuations with wavelength λ larger than the linear survey size L are not sampled

SSC - II

- Large Scale Structures have evolved through non-linear dynamics (gravity)
  - Non-linear structure formation **breaks homogeneity** and produces non-Gaussian matter density field
  - Density perturbations with different λ are **coupled** by non-linear evolution
- SSC comes from the non-linear coupling of observed (“sub-survey”) modes with unobservable (“super-survey”) modes
- Modulation of the observables by unobservable density perturbation



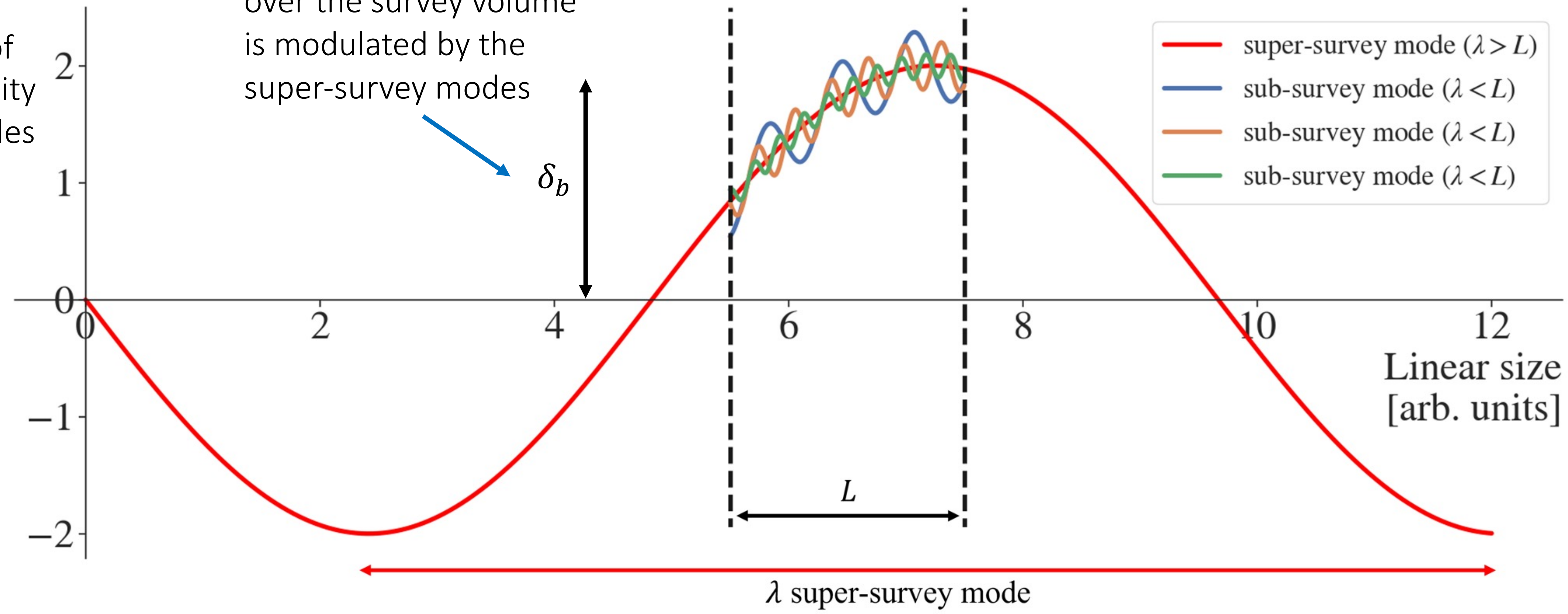
Source: F. Lacasa, shorturl.at/doJOQ

Visualizing the SSC

$L = V_s^{1/3}$ = linear size of the survey
λ = density mode's wavelength

bg. density averaged over the survey volume is modulated by the super-survey modes

Amplitude of matter density Fourier modes [arb. units]



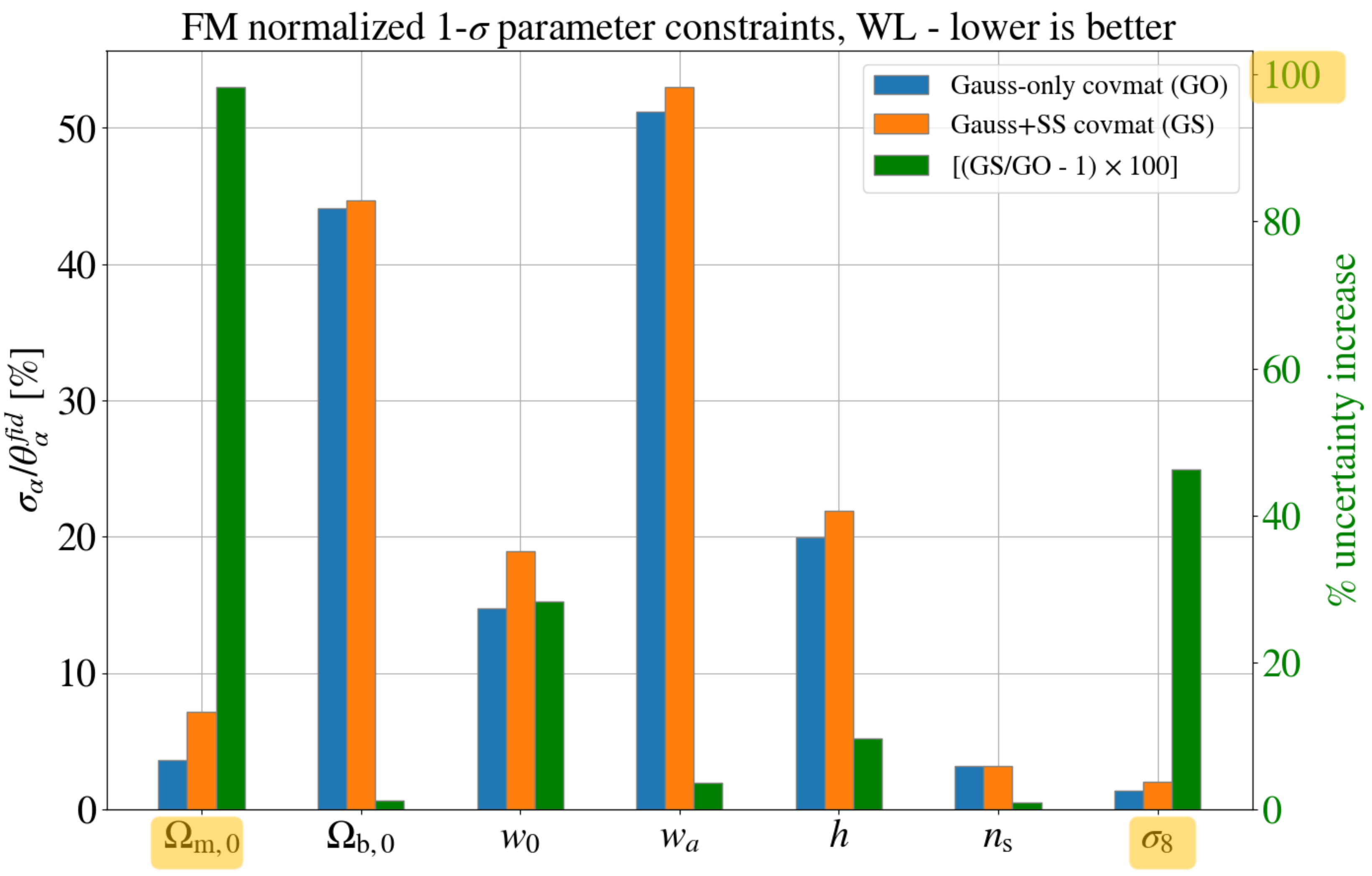
SSC – analytical expression

- Expression for two projected observables in different z bins is given by[1]

$$\mathrm{Cov}_{\mathrm{SSC}}[O_1, O_2] = \iint \mathrm{d}\,V_1 \mathrm{d}\,V_2 \frac{\partial o_1}{\partial \delta_b}(z_1) \frac{\partial o_2}{\partial \delta_b}(z_2) \sigma^2(z_1, z_2)$$

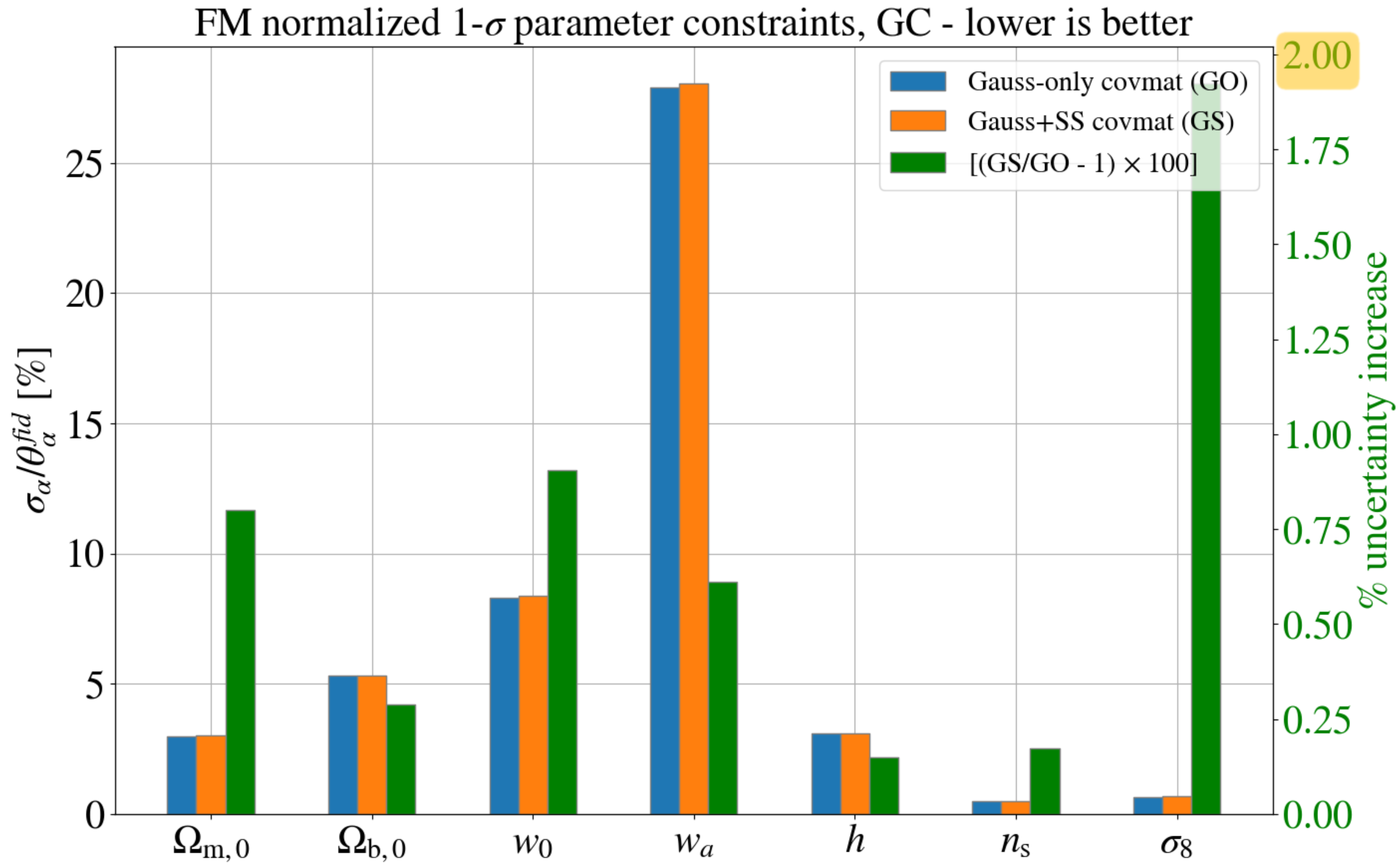
- 2 main ingredients needed:
  - *probe response*: change in the observable o_i due to a long-wavelength density perturbation δ_b
  - covariance of the background density δ_b due to super-survey modes
- note: it is difficult to compute SSC from simulations (simulation volume must be >> than the survey volume to capture super-survey modes)

Results – Weak Lensing



- σ_8, $\Omega_{m,0}$ most affected
- They have a large impact on the amplitude of the matter PS
- → SSC ~ unknown shift in bg. density
- Similar results found in Barreira+ 2018 (for a 5-parameter cosmology)

Results – photometric Galaxy Clustering



Lower impact for GCph:

- Probe response is lower than WL's
- The effect is diluted by an uncertainty increase in the bias (nuisance) parameters

Conclusions

- SSC **must** be included in upcoming analyses: it significantly degrades parameter constraints for the *Euclid* WL survey and for the 3x2pt
- No variation in the survey strategy shows significant potential to mitigate the effect
- The FoM decreases significantly for the full 3x2pt photometric analysis
- SSC relative impact decreases for more realistic forecasts (*i.e.*, including more systematic effects), because of the larger overall error budget

Thank you

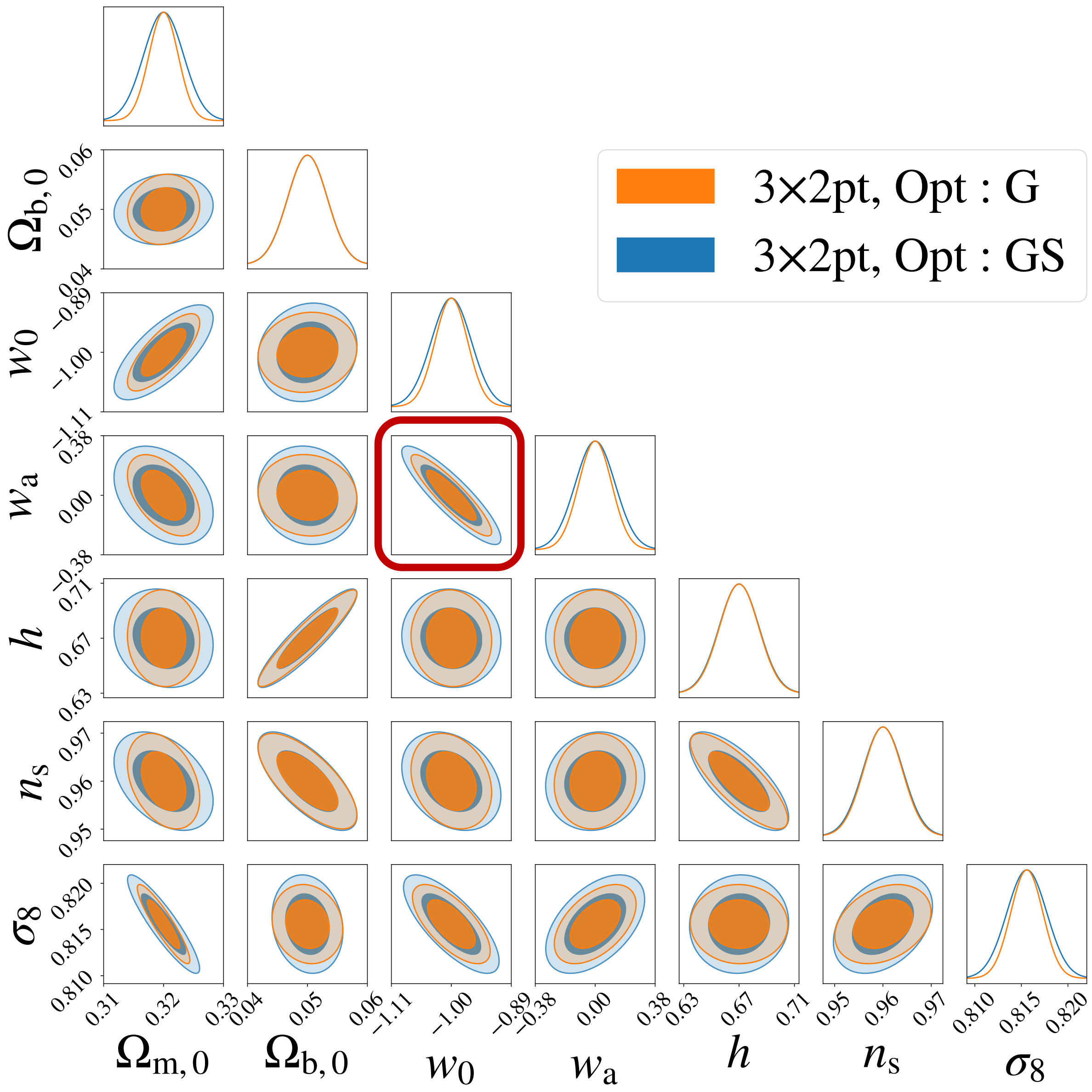
Additional slides

Computing the SSC: probe response

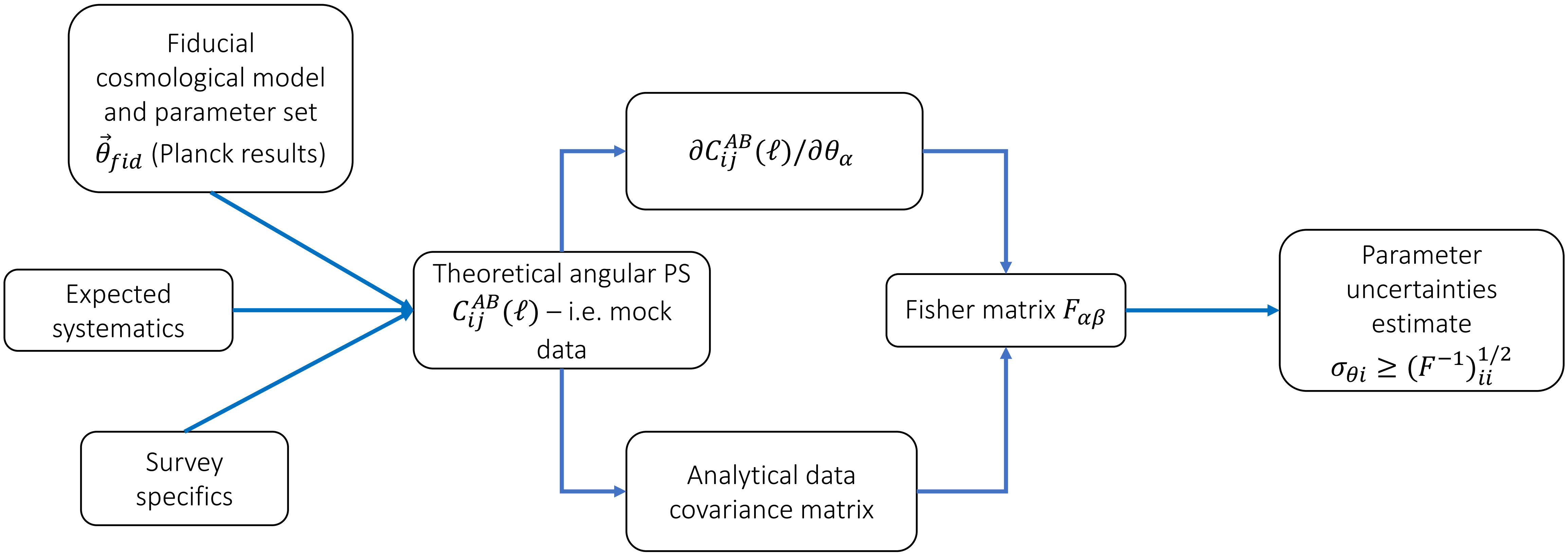
- We build upon the work of Lacasa+ 2018 by taking into account the dependence of the response on scale, redshift and probe (as opposed to a constant response)
- Two possible approaches:
  - Semi-analytical prescription, e.g. using the Halo Model
  - Separate universe technique: the region observed by the survey is equivalent to a universe with different background density
    → N-body simulations with external homogeneous overdensity imposed
- We choose the second approach, following Wagner+ 2014 and Barreira+ 2018
- We develop an analytical mapping of the matter into the galaxy PS response and project the responses in 2D

Results – ‘3x2pt’

- WL + GGL + GCph: three 2-point correlation functions
- 1σ and 2σ contours, representing projections of the likelihood in parameter space
- Figure of Merit shrinks by about 50%: from 958 to 494 (optimistic case)



Fisher Forecast pipeline



- DE EOS: $p = w_{DE}\rho$; $w_{DE}(z) = w_0 + w_a \frac{z}{1+z}$; $w_{DE}(z=0) = w_0$
- Gauss Only covariance:

$$\mathrm{Cov}\left[C_{ij}^{AB}(\ell), C_{kl}^{A'B'}(\ell')\right] = \frac{\left[C_{ik}^{AA'}(\ell) + N_{ik}^{AA'}(\ell)\right]\left[C_{jl}^{BB'}(\ell') + N_{jl}^{BB'}(\ell')\right] + \left[C_{il}^{AB'}(\ell) + N_{il}^{AB'}(\ell)\right]\left[C_{jk}^{BA'}(\ell') + N_{jk}^{BA'}(\ell')\right]}{(2\ell+1) f_{\mathrm{sky}} \Delta\ell} \delta_{\ell\ell'}^{\mathrm{K}}.$$

- SSC: dominates SNR above $\ell \gtrsim 300$ at *z = 1*

$$\mathrm{Cov}_{\mathrm{SSC}}\left(C_\ell^{AB}(i_z, j_z), C_{\ell'}^{CD}(k_z, l_z)\right) \approx R_\ell^{AB}\, C_\ell^{AB}(i_z, j_z)\, R_{\ell'}^{CD}\, C_{\ell'}^{CD}(k_z, l_z) \times S_{i_z, j_z; k_z, l_z}^{A,B;C,D}, \quad (4)$$

$$\mathrm{Cov}_{\mathrm{SSC}}(\mathcal{O}_1, \mathcal{O}_2) = \iint \mathrm{d}V_1 \mathrm{d}V_2 \frac{\partial \mathfrak{o}_1}{\partial \delta_b}(z_1) \frac{\partial \mathfrak{o}_2}{\partial \delta_b}(z_2)\, \sigma^2(z_1, z_2)$$

$$S_{i_z, j_z; k_z, l_z}^{A,B;C,D} = \int \mathrm{d}V_1\, \mathrm{d}V_2 \frac{W_{i_z}^A(z_1)\, W_{j_z}^B(z_1)}{I^{AB}(i_z, j_z)} \frac{W_{k_z}^C(z_2)\, W_{l_z}^D(z_2)}{I^{CD}(k_z, l_z)}\, \sigma^2(z_1, z_2),$$

$$\sigma^2(z_1, z_2) = \frac{1}{2\pi^2} \int k^2\, \mathrm{d}k\, P_{\mathrm{m}}(k|z_{12})\, j_0(kr_1) j_0(kr_2)$$

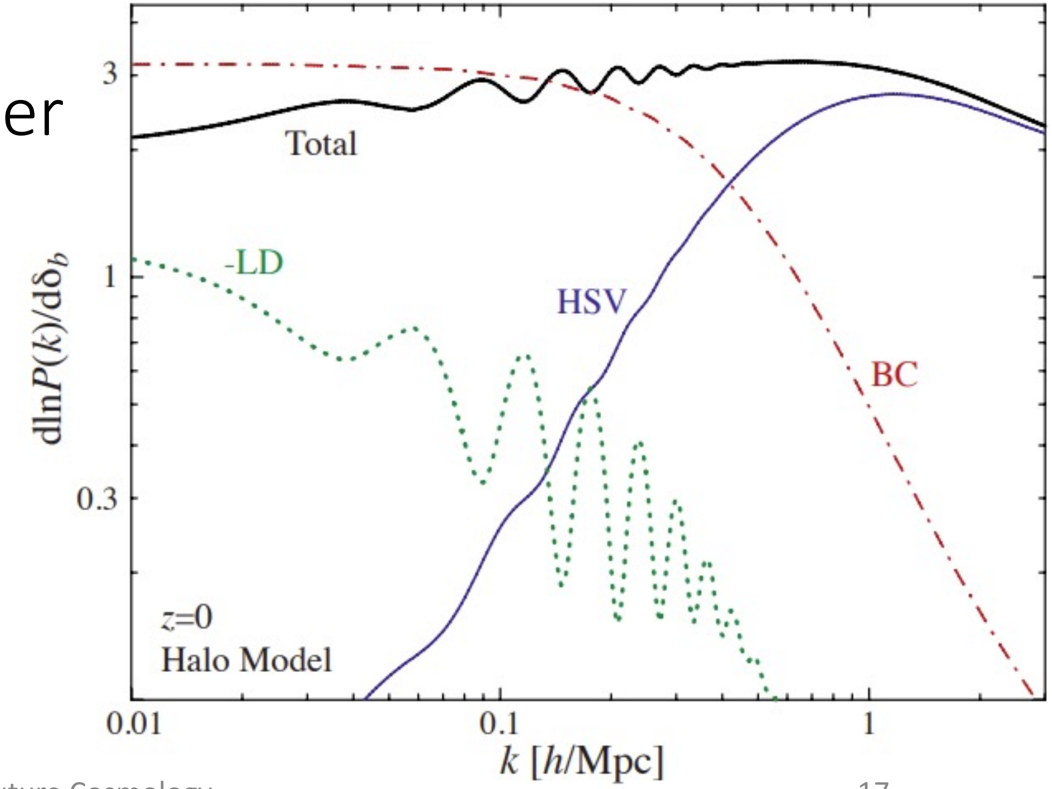
$$I^{AB}(i_z, j_z) = \int \mathrm{d}V_1\, W_{i_z}^A(z_1)\, W_{j_z}^B(z_1).$$

$\sigma^2(z_1, z_2)$ = variation of background density on a given survey volume due to super survey modes modulations

Approximation used: the response of the probe, which amounts how a given probe varies with changes of the background density δ_b , vary slowly with redshift compared to $\sigma^2(z_1, z_2)$

Probe response II

- Example of probe response evaluated using the Halo Model
- beat coupling ("BC"): growth of a short wavelength perturbation is enhanced in a large scale overdensity
- halo sample variance ("HSV"): halo number densities are also enhanced in such a region
- Linear dilation ("LD"): long wavelength mode changes the local expansion factor and hence the physical size of a mode that would have comoving wavelength k in its absence



Matter Pk response – Li+2014

- σ_8: amplitude of the (linear) matter power spectrum smoothed on the scale of 8 Mpc/h with a top-hat filter; it describes the amplitude of matter density fluctuations in spheres of 8 Mpc/h.
- n_s= scalar spectral index, measures the deviation (tilt) from scale invariance (n_s = 1) of dimensionless primordial power spectrum of the curvature perturbation ζ generated by inflation. and $k_0$is a pivot scale, A_s the amplitude

$$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

- Cosmic variance: variance arising from the fact that only one realization of the universe is available to us → when computing the $C(\ell)$, the average over the $a_{\ell m}$ is done over m: $\hat{C}(\ell) = \frac{1}{2\ell+1}\sum_m a^*_{\ell m}\, a_{\ell m}$, but at low ℓ few m modes are available → relative uncertainty of $\frac{\Delta C(\ell)}{C(\ell)} = \frac{\sqrt{2}}{\sqrt{2\ell+1}} \times [\frac{4\pi}{A}]$ (term in square brackets is sample variance)