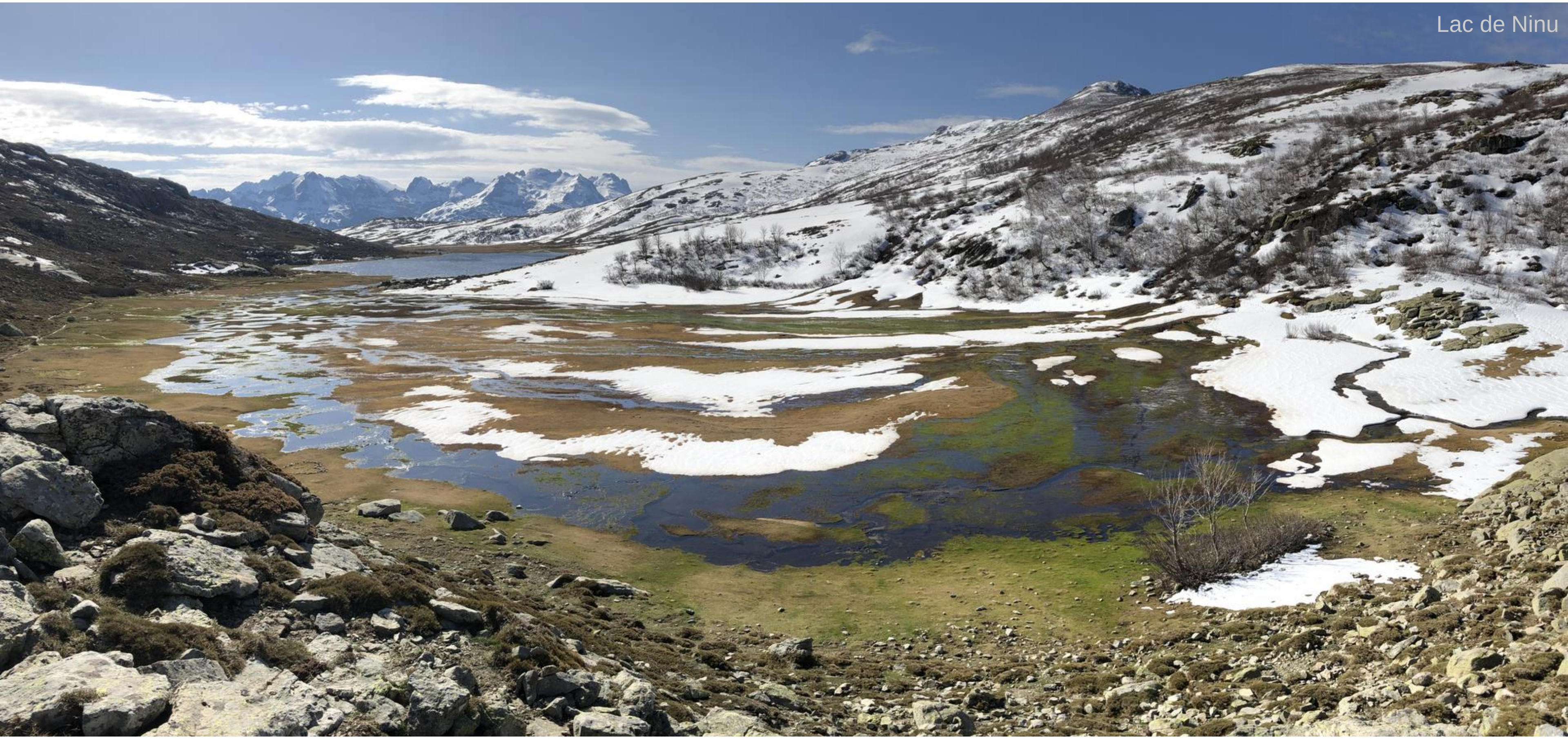
CMB and Hubble Parameter

Antony Lewis



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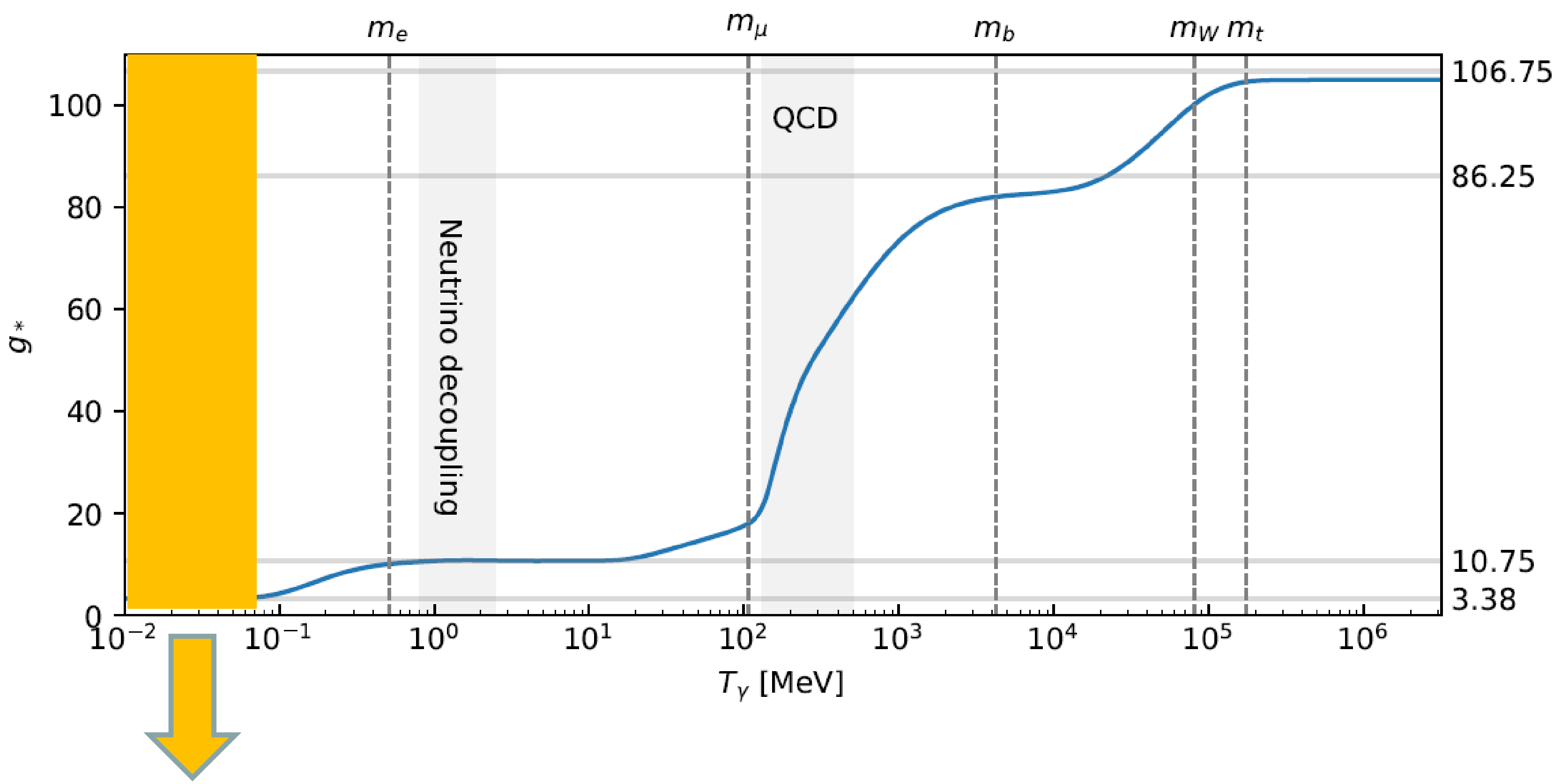
Thermal history after the hot big bang



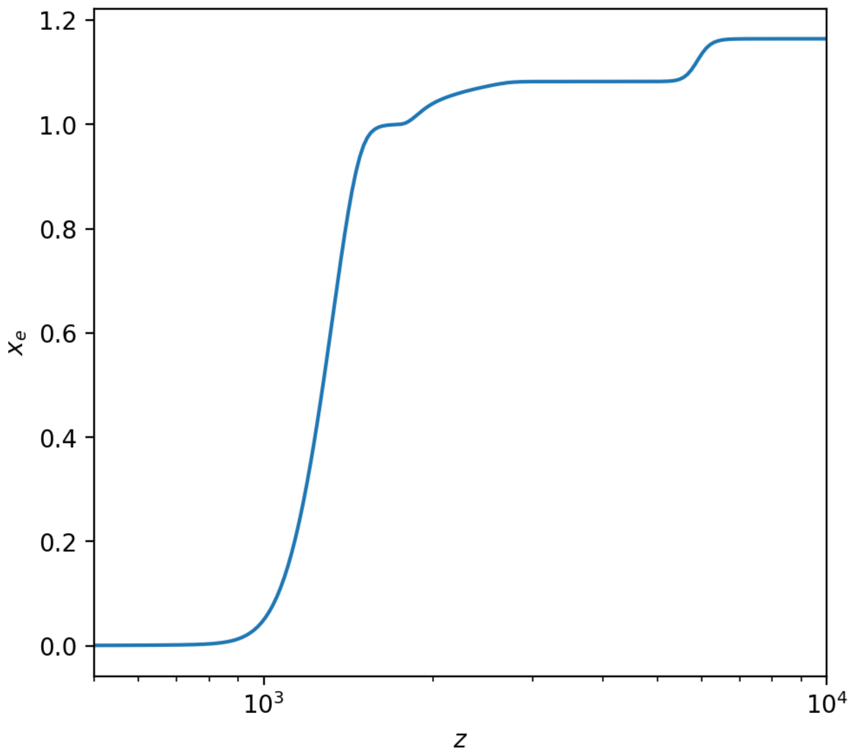
Just photons, protons, electrons, neutrons, and decoupled neutrinos left + CDM (assumed non-interacting by this time)

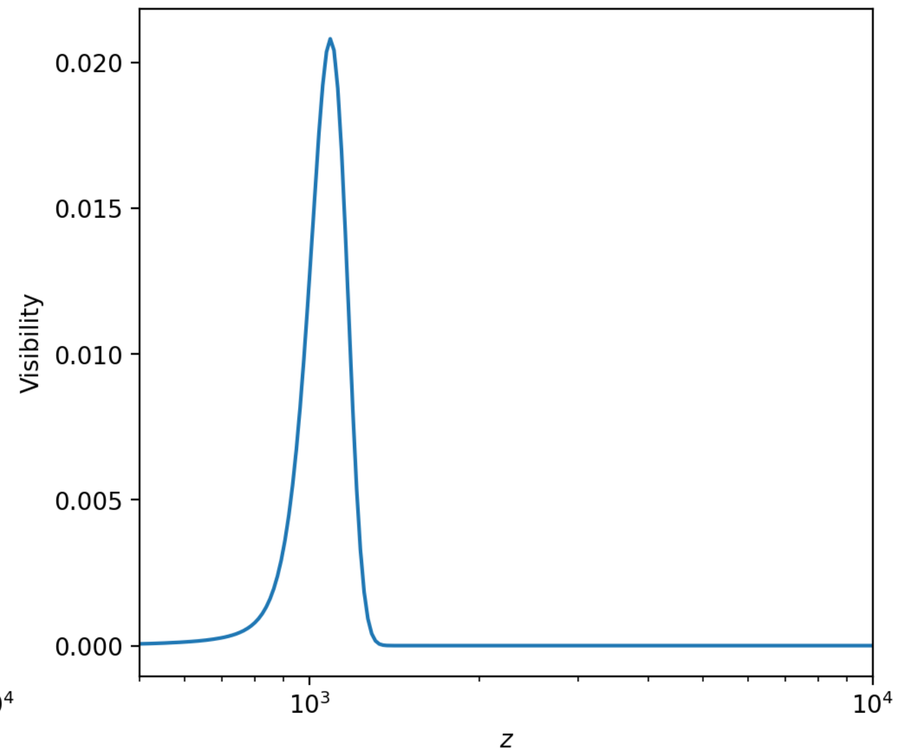
Neutrinos too low energy to easily detect today

Recombination

As the Universe expanded it cooled, eventually the temperature was low enough that neutral atoms could form; this is the epoch of recombination.

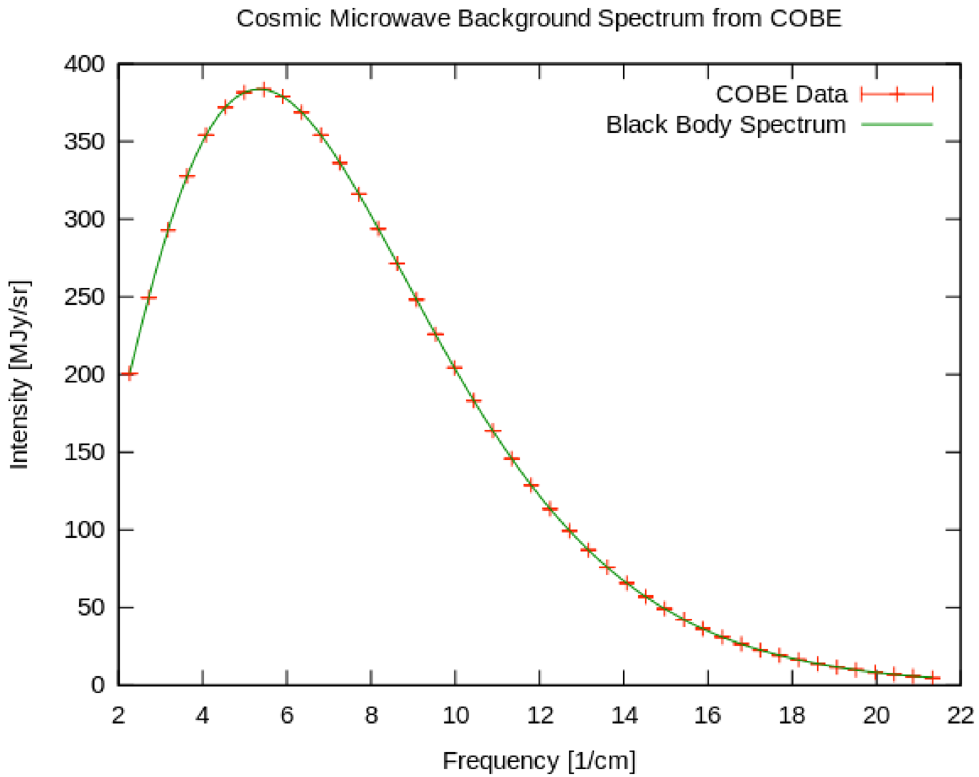
After recombination photons could travel (mostly) unimpeded and should be observable today. We see the last-scattering surface.





BACKGROUND RADIATION

COBE MISSION



$$T_{\mathrm{CMB},0} = T_{\gamma,0} \approx (2.7255 \pm 0.0006)\ \mathrm{K}$$

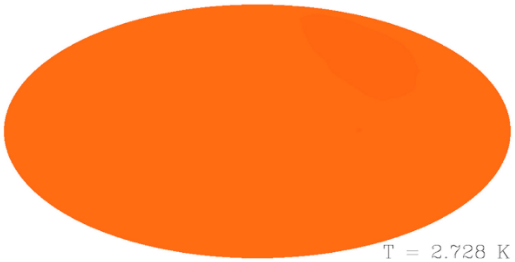
[Fixsen et al]

error bars are too small to see – very accurate fit to black body thermal spectrum

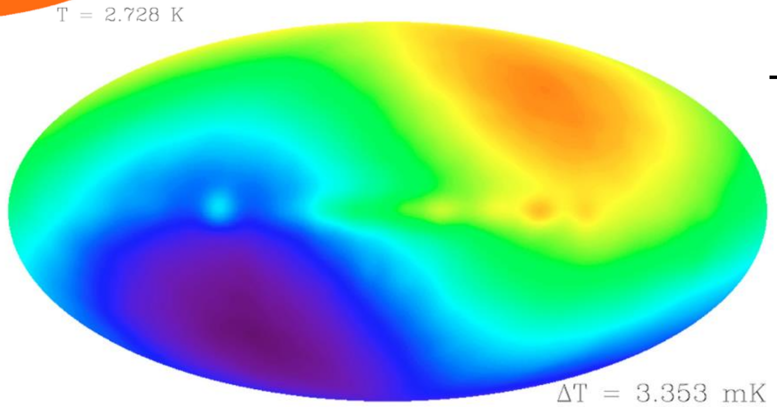
$\text{Recombinaton } \mathrm{T}_* \approx 3000K$

Using $(1+\mathrm{z})T_{\mathrm{CMB},0} = \mathrm{T}_*$ with $T_{\mathrm{CMB},0} \sim 2.7\ K$

$\Rightarrow\ z_* \sim 1100$ (well after the epoch of matter-radiation equality)

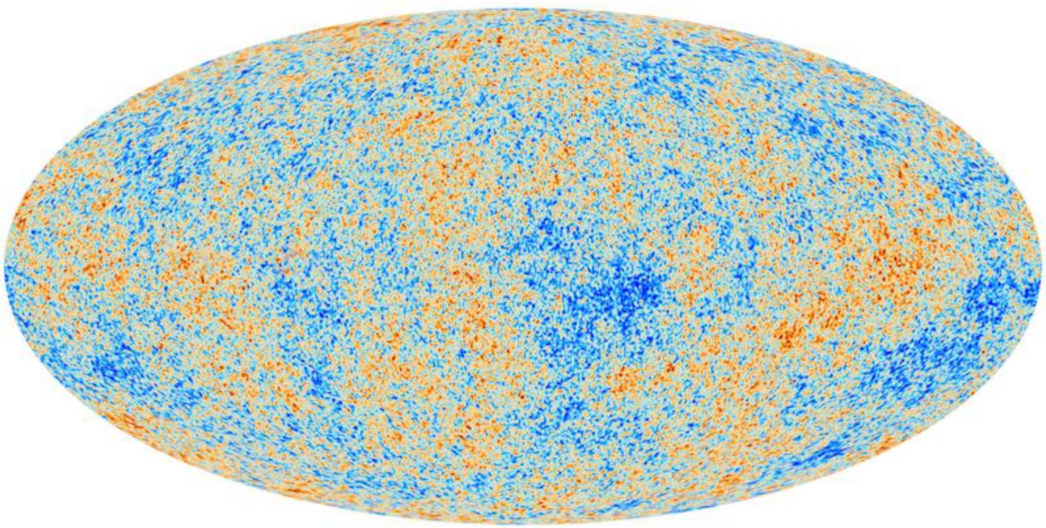


(almost) uniform 2.726K blackbody



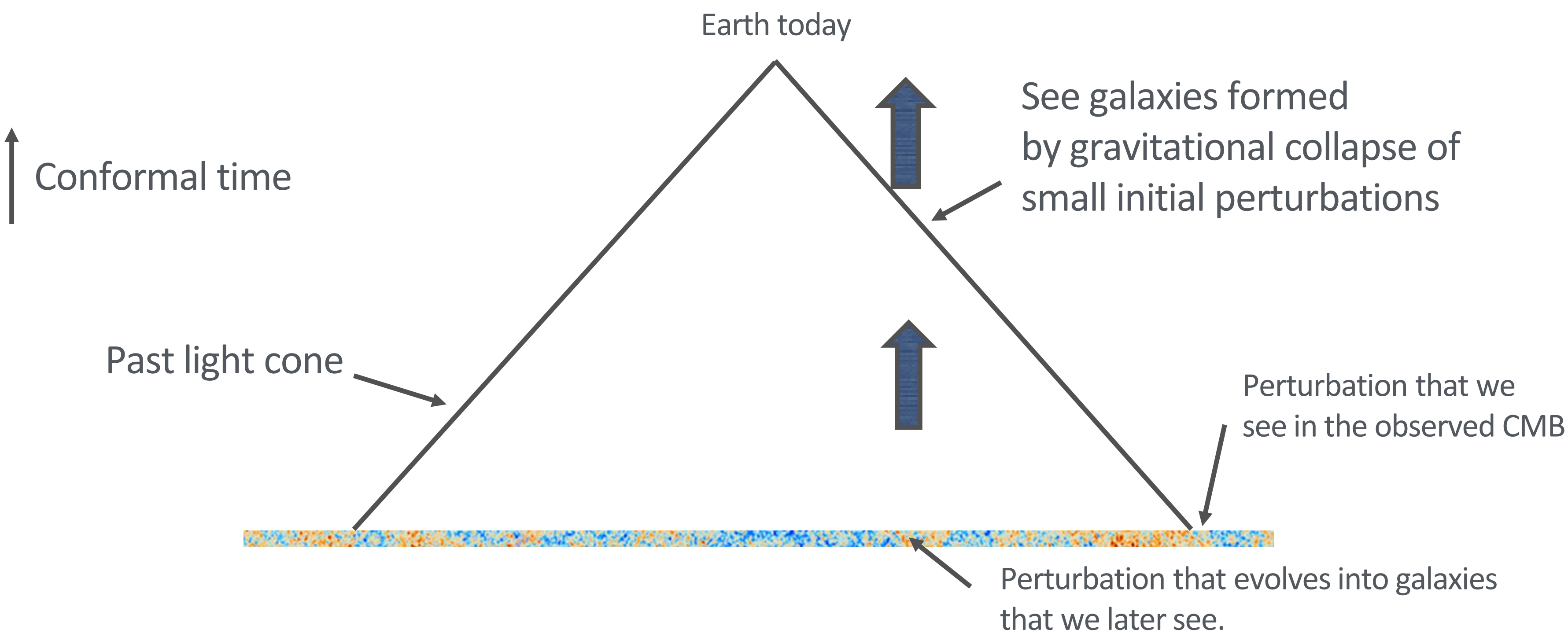
+ Dipole (local motion Doppler shift)

+ O(10^{-5}) perturbations



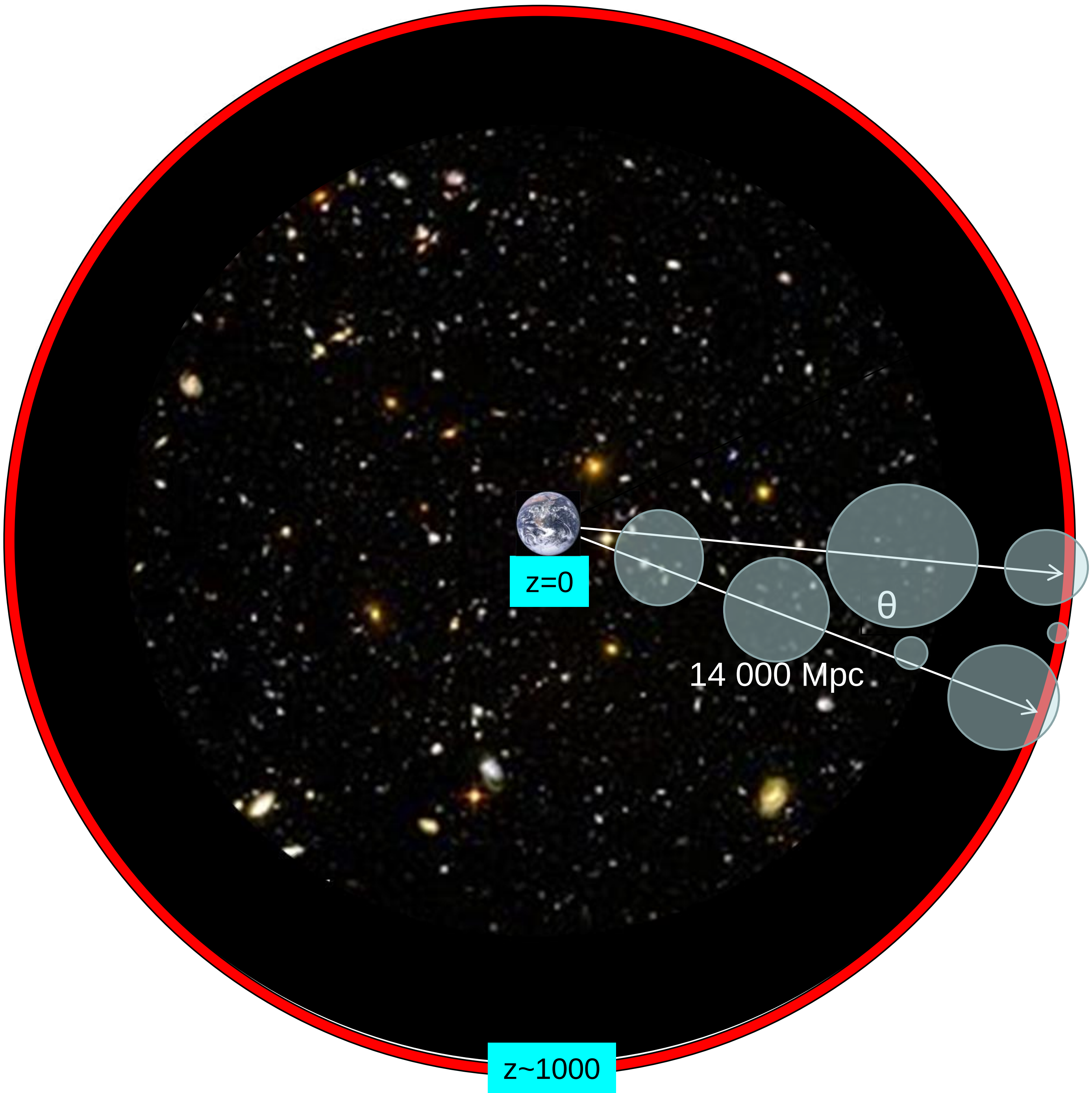
0^{th} order (uniform 2.726K) + 1^{st} order perturbations (*anisotropies*)

PERTURBATIONS



BUT: Universe recombines at same temperature everywhere; recombination is an *equal temperature surface* (at T_*) in the gas rest frame even in a perturbed universe

– why do we see temperature variations at all?



z=0
θ
14 000 Mpc
z~1000

- Linear modes cause anisotropic redshifting along the line of sight
  - 0th order equal-temperature last scattering surface modulated by 1st order perturbations

⇒ generates linear CMB anisotropies

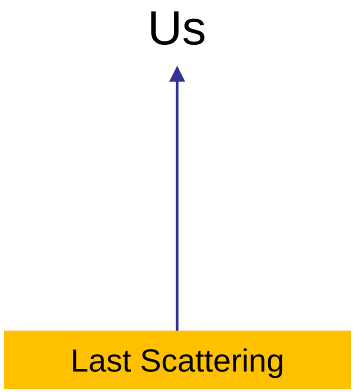
- linear perturbations are observed in perturbed universe:
  -1st order small-scale perturbations are modulated by the effect 1st order large-scale (and smaller-scale) modes

⇒ non-linear CMB anisotropies, mainly CMB lensing (2nd order and higher)

For simplicity consider recombination to give sharp visibility (instantaneously opaque → transparent)

⇒ In the background, CMB photons come from single spherical shell about us at background conformal time η_*

Need to use geodesic equation to see how photon energy changes along line of sight



$$\frac{dp^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0$$

Affine parameter λ

4-momentum $p^\mu = \frac{dx^\mu}{d\lambda}$

(this defines choice of normalization of λ)

Use linear perturbation theory with $ds^2 = a(\eta)^2 \left[(1+2\Psi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2\right]$

- Conformal Newtonian Gauge (CNG) [scalar perturbations]

Linear CMB anisotropies

Note: perturbations Φ and Ψ are functions of time and position

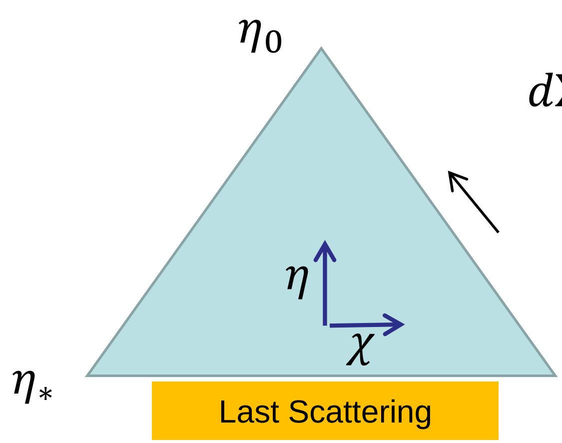
Zero component of geodesic equation in the Conformal Newtonian Gauge:

$$\frac{dp^0}{d\lambda} + \left(\frac{a'}{a} + \Psi'\right) p^0 p^0 + 2 p^0 p^i \frac{\partial \Psi}{\partial x^i} + \left(\frac{a'}{a}(1 - 2\Psi - 2\Phi) - \Phi'\right) \delta_{ij} p^i p^j = 0$$

$$x^0 = \eta$$
$$x^r = \chi$$

Null geodesic:

$$d\eta = -d\chi$$



$$d\mathrm{X} = \frac{\partial \mathrm{X}}{\partial \eta}\, d\eta + \frac{\partial \mathrm{X}}{\partial \chi}\, d\chi$$

$$\Rightarrow \frac{d\mathrm{X}}{\mathrm{d}\eta} = \frac{\partial \mathrm{X}}{\partial \eta} - \frac{\partial \mathrm{X}}{\partial \chi}$$

$$p^0 = E(1-\Psi)/a$$

$$\frac{d\eta}{d\lambda} = p^0 = \frac{E}{a}(1-\Psi)$$

$$\frac{d(aE)}{d\eta} = aE\left(\frac{\partial\Psi}{\partial\chi} + \Phi'\right) = aE\left(-\frac{d\Psi}{d\eta} + \Psi' + \Phi'\right)$$

Integrate along light cone between time η and today (η_0), rearrange

$$E(\eta_0) = a(\eta)E(\eta)\left[1 + \Psi(\eta) - \Psi_0 + \int_\eta^{\eta_0} d\eta(\Psi' + \Phi')\right]$$

All photons redshift the same way, so $kT \sim E$.

Recombination at fixed temperature T_* in gas rest frame.
Also add Doppler effect:

$$\begin{aligned} T(\hat{\mathbf{n}}, \eta_0) &= (a_* + \delta a)T_*\left[1 + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}}\cdot(\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \\ &= T_0\left[1 + \frac{\delta a}{a_*} + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}}\cdot(\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \end{aligned}$$

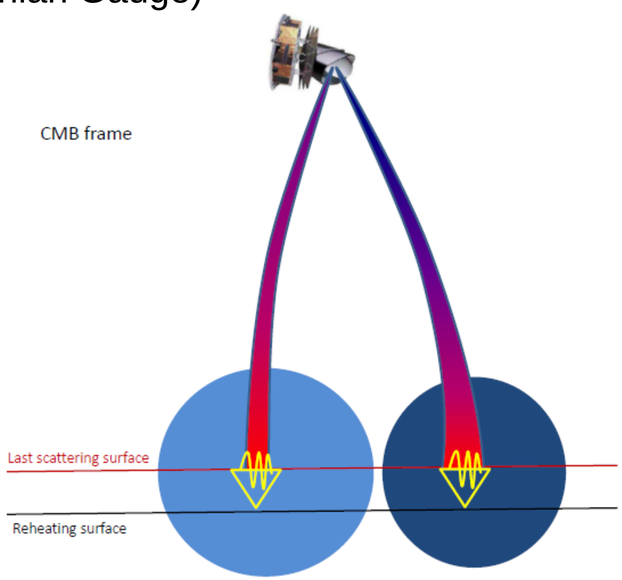
More details https://cosmologist.info/teaching/EU/notes_structure.pdf

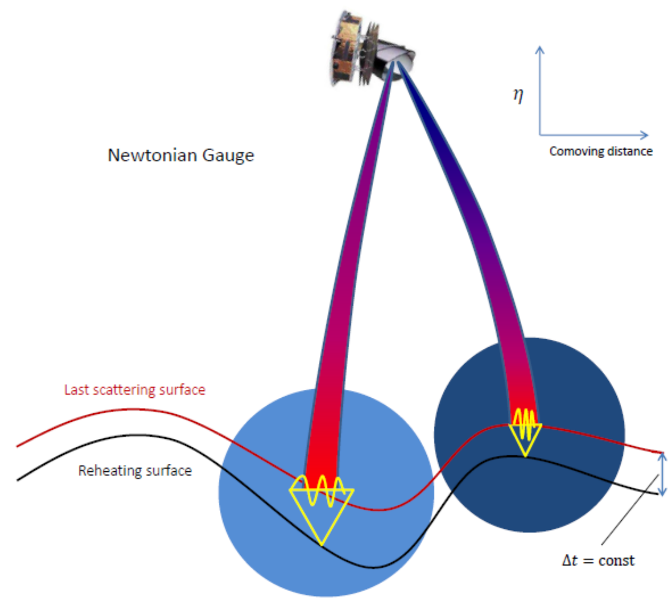
$\rho_\gamma(\eta\,,\boldsymbol{x}) = \rho_{\gamma*} + \rho_\gamma'\delta\eta + \delta\rho_\gamma = \rho_{\gamma*}$ (perturbated and unperturbed LSS at same temperature/density)

$\Rightarrow \delta\eta = -\frac{\delta\rho_\gamma}{\rho_\gamma'} = \frac{\Delta_\gamma a}{4a'} \Rightarrow \frac{\delta a}{a} = \frac{a'}{a}\delta\eta = \frac{\Delta_\gamma}{4}$

$$\Rightarrow \quad \frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \frac{\Delta_\gamma(\eta_*)}{4} + \underbrace{\Psi(\eta_*) - \Psi_0}_{\text{Sachs-Wolfe}} + \underbrace{\hat{\mathbf{n}}\cdot(\mathbf{v}_o - \mathbf{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')}_{\text{ISW}}$$

Temperature perturbation at recombination
(Newtonian Gauge)





On large scales (super-horizon for matter-dominated recombination): $\frac{\Delta_\gamma}{4} + \Psi = \frac{\Psi}{3}$

In general need to calculate Δ_γ, v_b at recombination numerically

In practice recombination visibility is also not sharp
⇒ also need to integrate over source planes through last scattering

Visibility $g = \frac{dP}{d\eta} = -\tau' e^{-\tau}$ Optical depth $\tau \equiv \int_\eta^{\eta_0} a n_e \sigma_T d\eta'$

Calculation of theoretical perturbation evolution

Perturbations O(10^{-5})

⇒ Simple linearized equations are very accurate (except small scales)

Can use real or Fourier space

Fourier modes evolve independently: simple to calculate accurately

Physics Ingredients

•Thomson scattering (non-relativistic electron-photon scattering)
   - tightly coupled before recombination: 'tight-coupling' approximation
  (baryons follow electrons because of very strong e-m coupling)
•Background recombination physics (Saha/full multi-level calculation)
•Linearized General Relativity
•Boltzmann equation (how angular distribution function evolves with scattering)

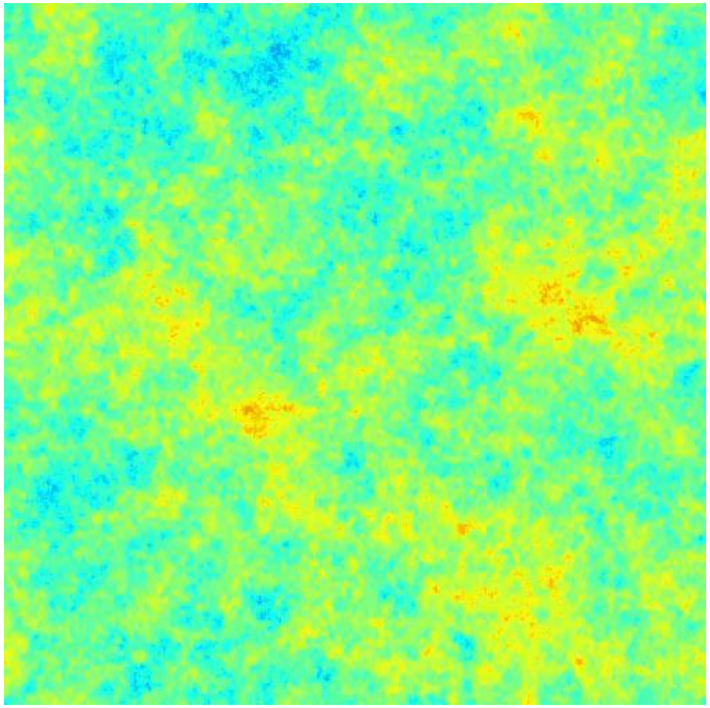
To calculate power spectrum from statistically homogeneous and isotropic perturbations do *not* need to evolve realisations (unlike in large-scale structure simulations)

Linearity: $X(\boldsymbol{k}, \eta) = X(\boldsymbol{k}, 0)T(k, \eta)$

- only need to evolve **transfer function** $T(k, \eta)$, tells you how *all* perturbations with same **|k|** evolve

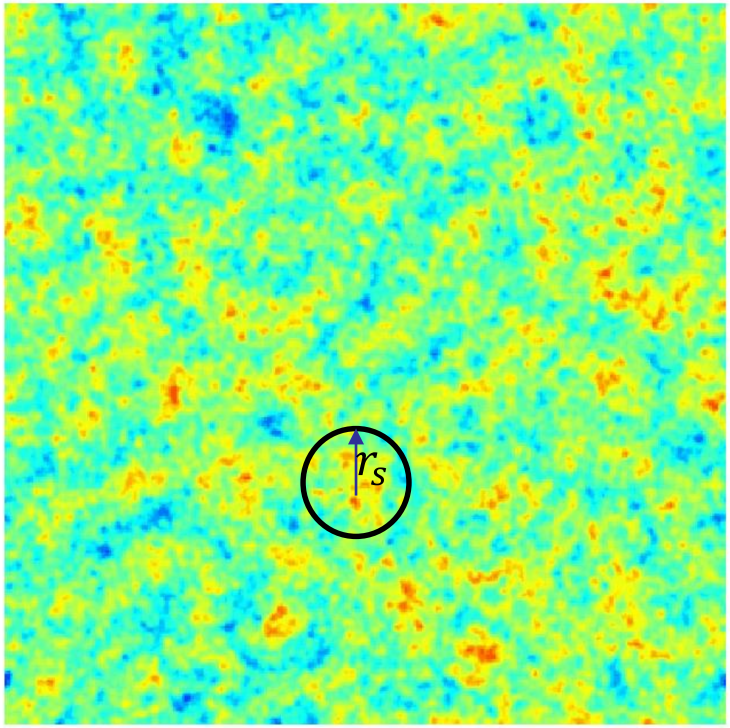
Perturbation evolution

Perturbations: start of hot big bang

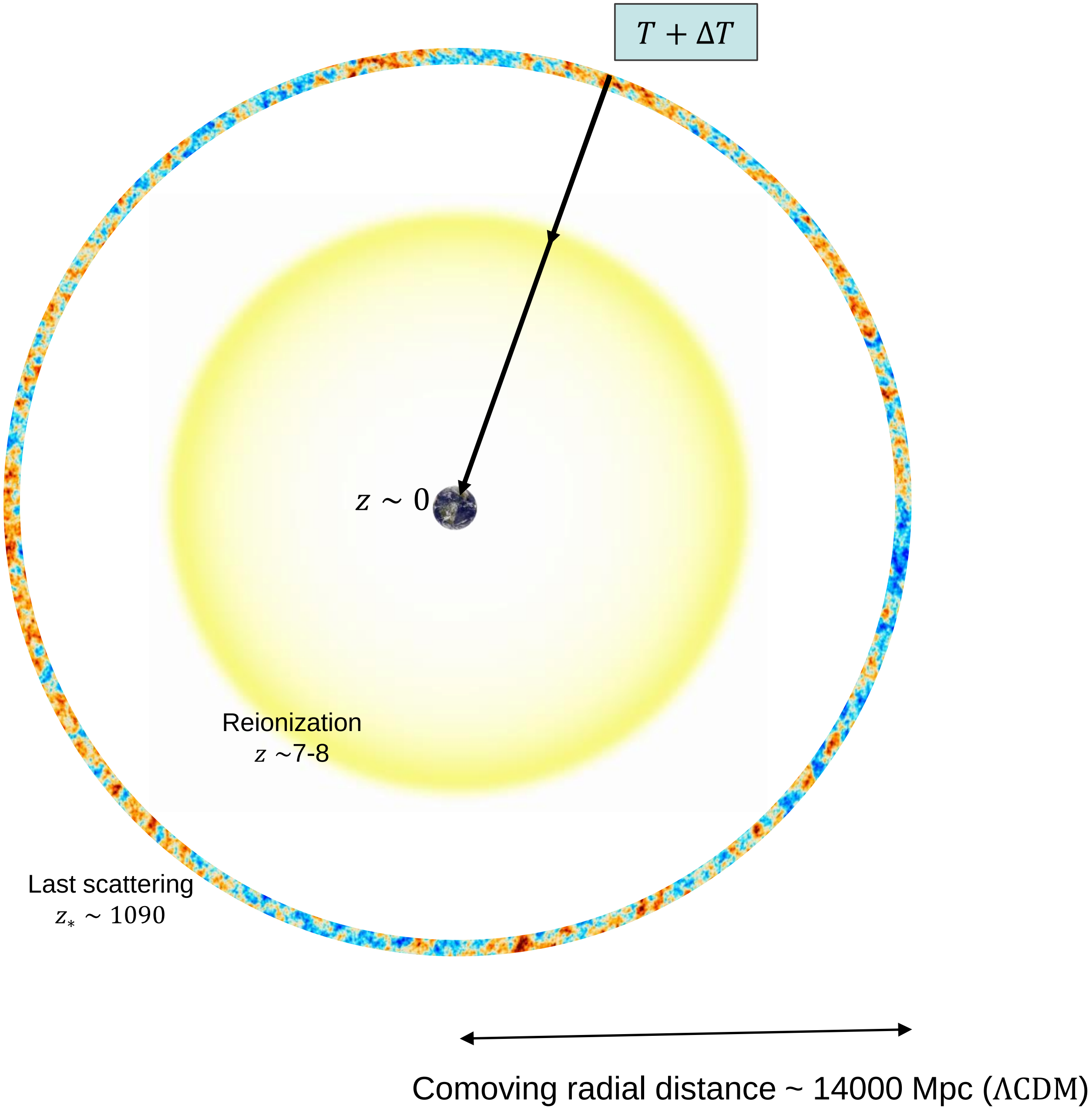


gravity+
pressure+
diffusion

Perturbations: Last scattering surface



In comoving distance



CMB power spectrum C_l

- Theory: Linear physics + Gaussian primordial fluctuations

$$a_{lm} \equiv \int d\Omega \Delta T \, Y_{lm}^*$$

Random variates, zero mean ($\langle \Delta T \rangle = 0$)

Theory prediction $C_l \equiv \langle |a_{lm}|^2 \rangle$

- variance (average over all possible sky realizations)

- statistical isotropy implies independent of *m*

- for Gaussian statistically-isotropic fluctuations, C_l contains all the information (fully describes statistics)

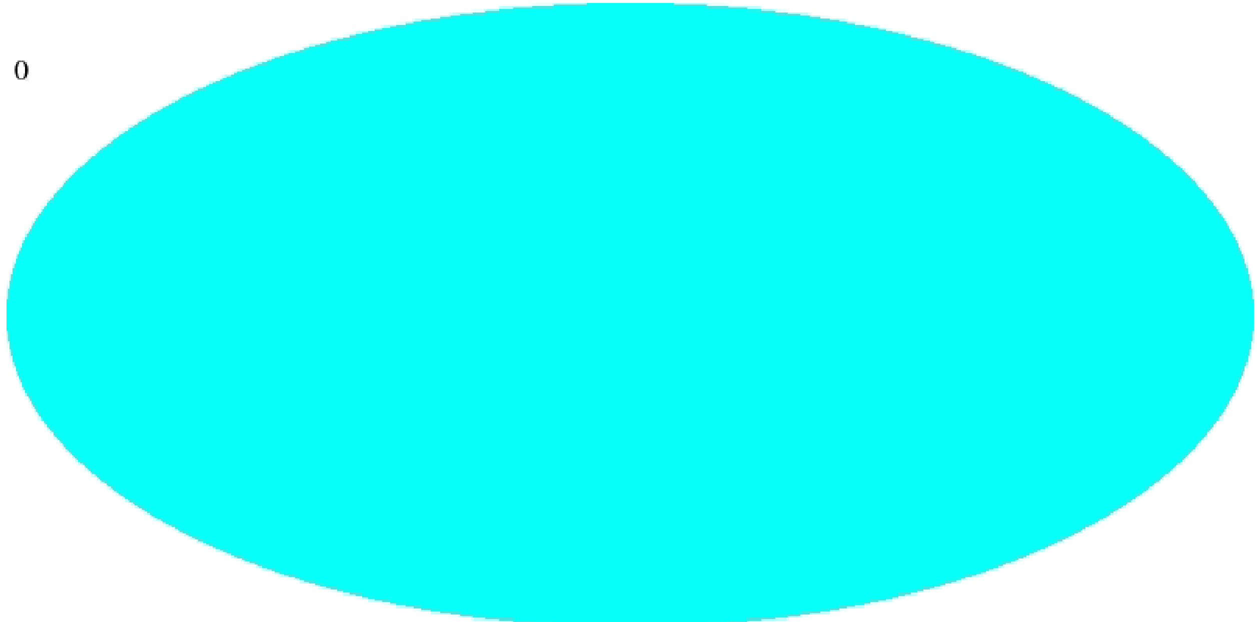
Initial conditions
+ cosmological parameters

$\xrightarrow[\text{}]{\text{linearized GR + Boltzmann equations}}$ C_l

CAMB: "pip install camb" https://camb.info
CLASS https://lesgourg.github.io/class_public/class.html

Sum over l

$$\sum_{lm} a_{lm} Y_{lm}$$

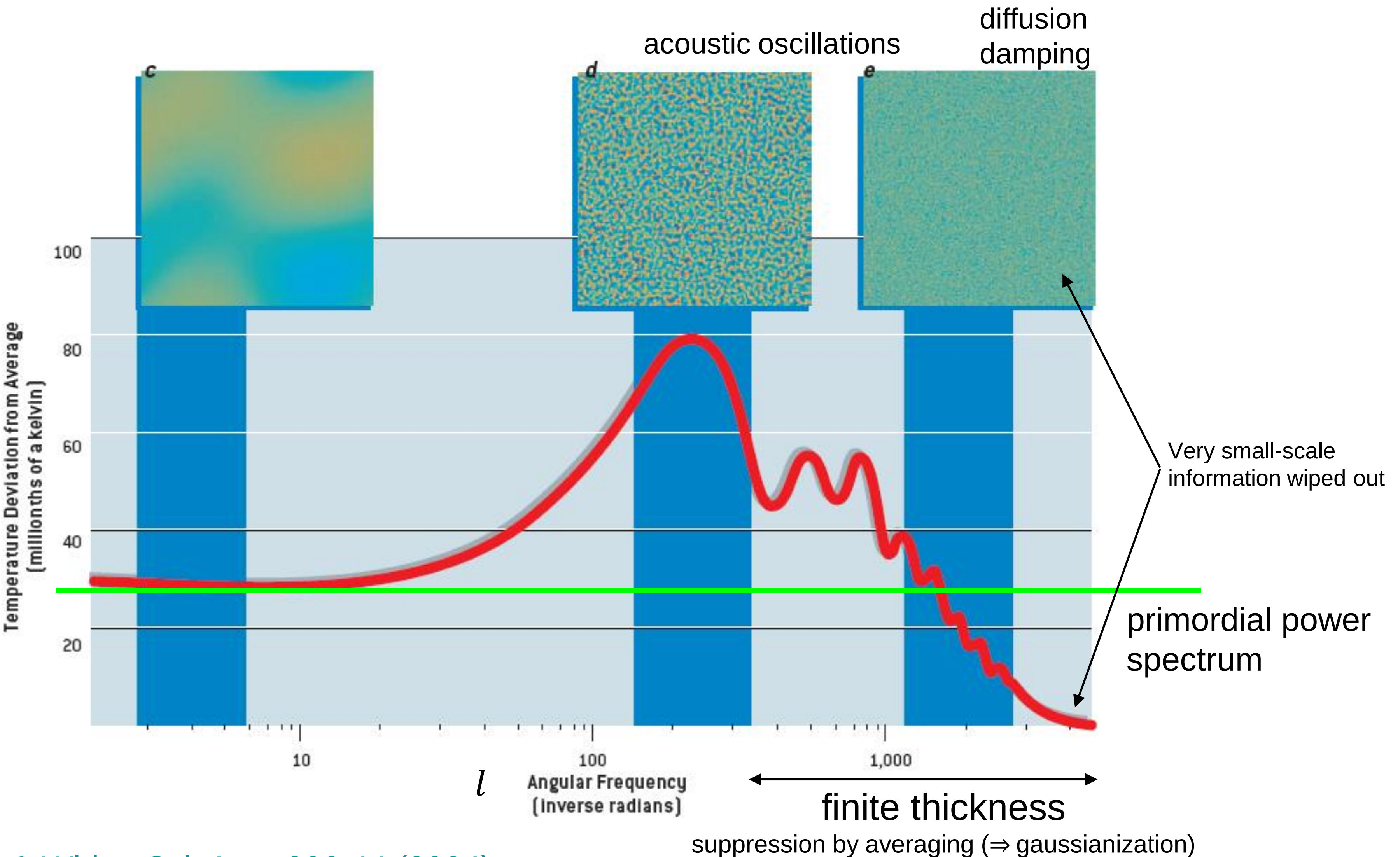


Single l

$$\sum_{m} a_{lm} Y_{lm}$$

CMB temperature power spectrum

Primordial perturbations + later physics



Hu & White, Sci. Am., 290 44 (2004)

Why C_l oscillations?

Radiation perturbation evolution

- Comoving Poisson equation: $\nabla^2\Phi = 4\pi G\delta\bar{\rho} \Rightarrow \left(\frac{k}{a}\right)^2 \Phi = -4\pi G\rho\,\bar{\Delta}$

  - potentials approx constant on super-horizon scales
  - radiation domination ρ ~ 1/a^4

  → $\bar{\Delta} \sim k^2 a^2 \Phi$

  → since Φ ~ constant, super-horizon comoving density perturbations grow ~ a^2

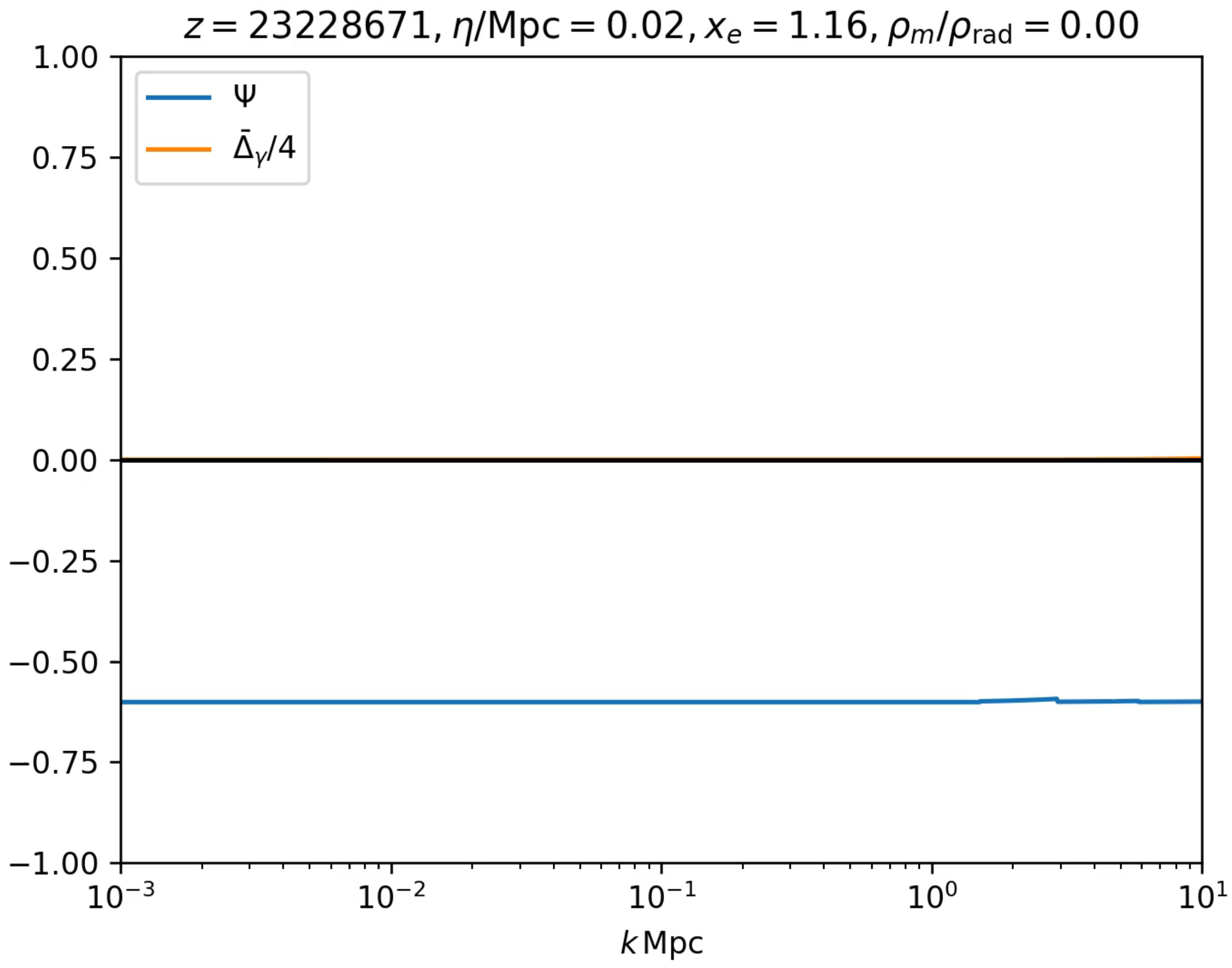
+ pressure ($P = \frac{\rho}{3}$ for radiation ⇒ $c_s \sim \frac{c}{\sqrt{3}}$ for waves in radiation fluid)
  (pressure support/oscillation ⇒ stop collapse of radiation perturbations)

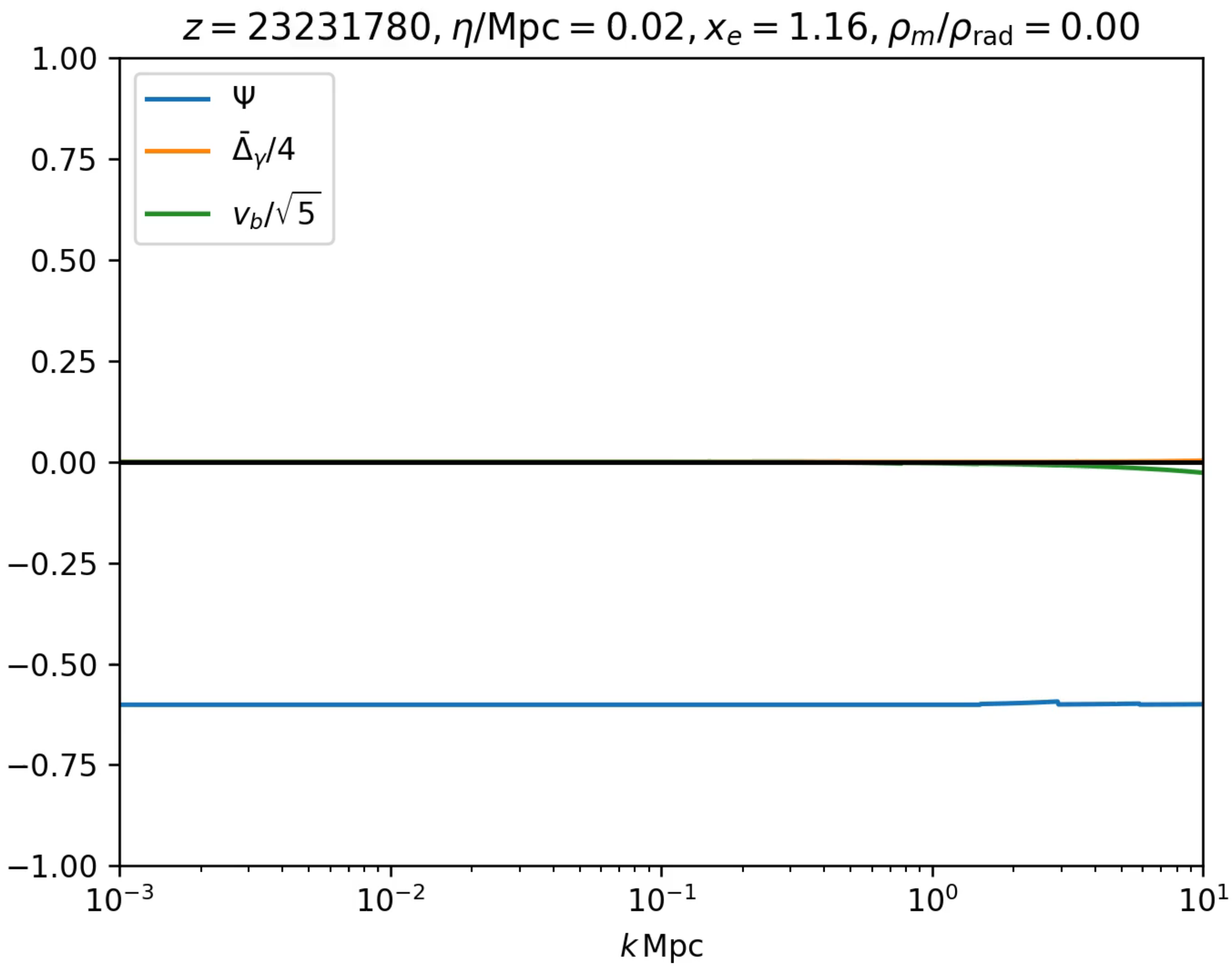
more generally:
$$c_s^2 = \frac{1}{3}\left(\frac{4\rho_\gamma}{4\rho_\gamma + 3\rho_b}\right)$$

+ expansion
($\Psi \propto \frac{GM}{r} \to 0$ as physical size r of perturbation increases after pressure stops growth)

⇒ nearly free SHM oscillations during radiation domination driven only by initial collapse
- "sound" waves with speed c_s

Evolve 1D grid of k values

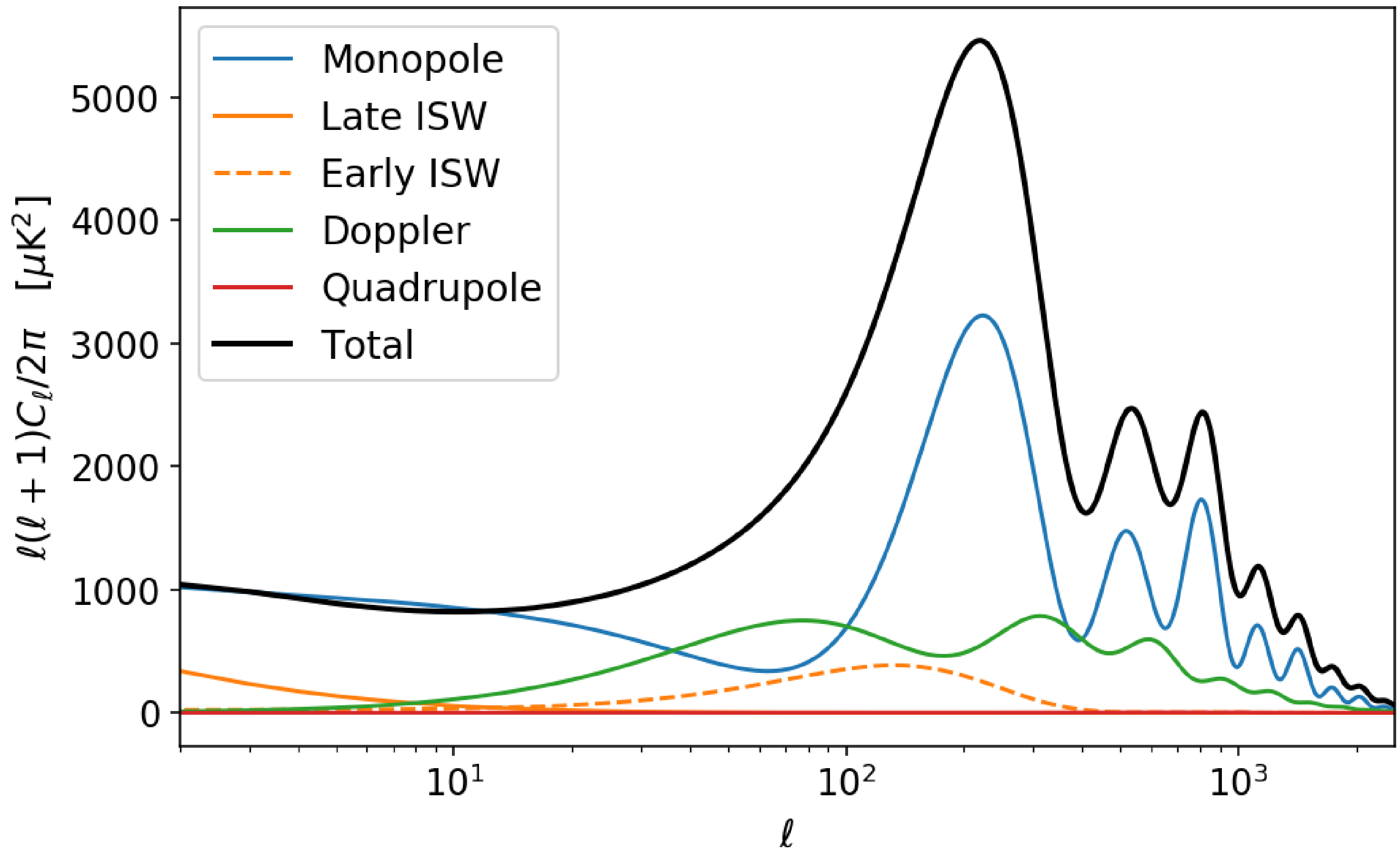



$z = 23231780, \eta/\mathrm{Mpc} = 0.02, x_e = 1.16, \rho_m/\rho_{\mathrm{rad}} = 0.00$
Ψ
$\bar{\Delta}_\gamma/4$
$v_b/\sqrt{5}$
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
$k\,\mathrm{Mpc}$

Contributions to temperature C_l

$$\frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \underbrace{\frac{\Delta_\gamma(\eta_*)}{4} + \Psi(\eta_*)}_{\text{Monopole}} \underbrace{- \Psi_0}_{\text{Const.}} + \underbrace{\hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta (\Psi' + \Phi')}_{\text{ISW}} \text{ + other}$$

On small scales $l \sim k\, \chi_*$ ($\chi_* \sim 14000$ Mpc in LCDM)

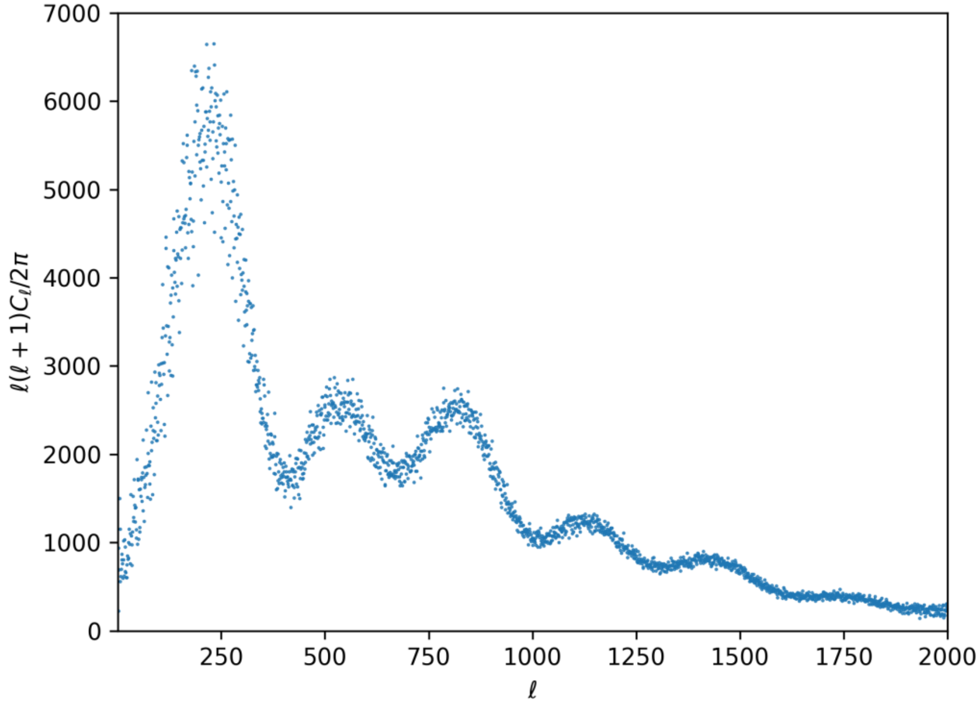


Cosmic Variance: only one sky, spectrum same at all frequencies (blackbody)

Use estimator for variance: $$\widehat{C}_l = \frac{1}{2l+1}\sum_m |a_{lm}^2|$$

"Cosmic Variance"

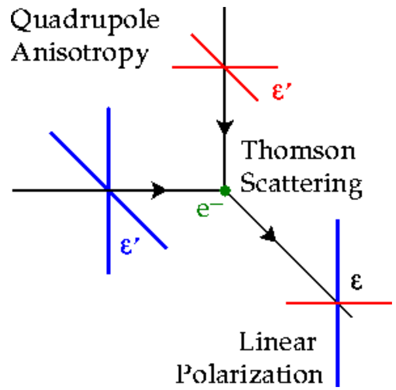
$$\left\langle \left|\widehat{C}_l - C_l\right|^2 \right\rangle = \frac{2C_l^2}{2l+1}$$



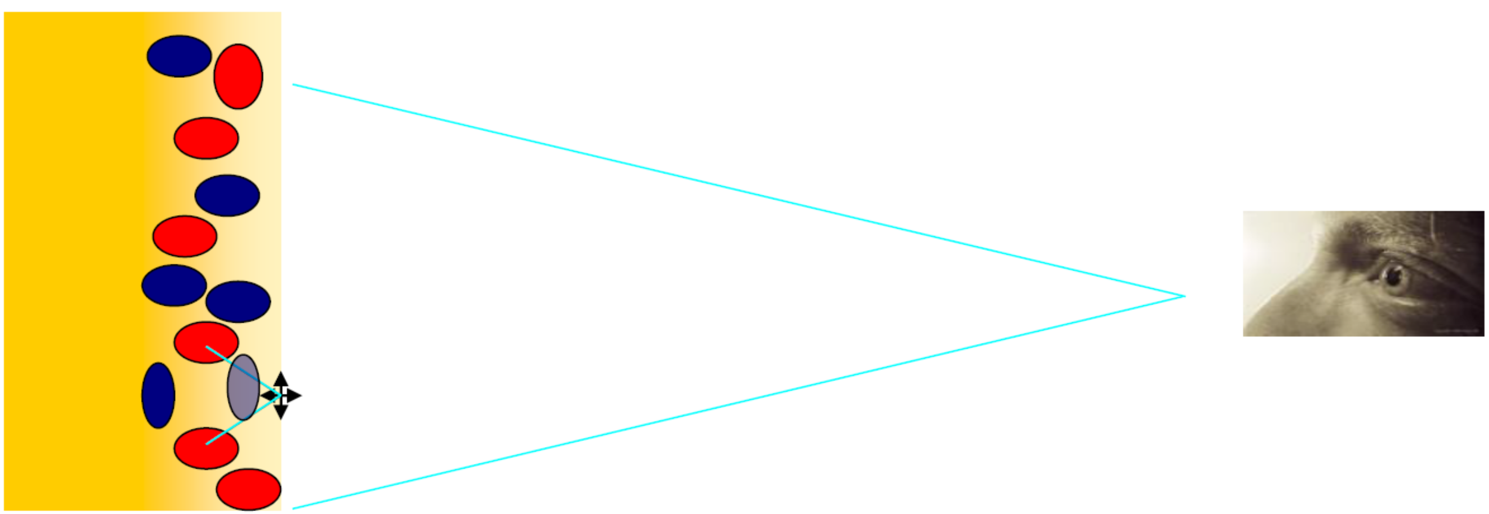
Cosmic variance gives fundamental limit on how much we can learn from CMB
- smaller errors at high l – most information from the small-scale spectrum

CMB Polarization

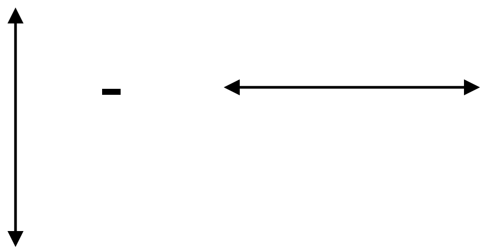
Generated during last scattering (and reionization) by Thomson scattering of anisotropic photon distribution



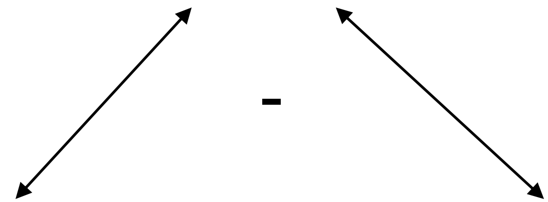
Hu astro-ph/9706147



Observed Stokes' Parameters



Q



U

Q → -Q, U → -U under 90 degree rotation

Q → U, U → -Q under 45 degree rotation

Measure E field perpendicular to observation direction $\hat{\boldsymbol{n}}$
Intensity matrix defined as $$\mathcal{P}_{ab} = C\langle E_a E_b^* \rangle = P_{ab} + \frac{1}{2}\delta_{ab} I + V_{[ab]}$$

Linear polarization + Intensity + circular polarization

CMB only linearly polarized. In some fixed basis $$P_{ij} = \frac{1}{2}\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$

Alternative complex representation

Define complex vectors $\mathbf{e}_\pm = \mathbf{e}_1 \pm i\mathbf{e}_2$ e.g. $\mathbf{e}_\pm = \mathbf{e}_x \pm i\mathbf{e}_y$

And complex polarization

$$P \equiv \mathrm{e}_+^a \mathrm{e}_+^b P_{ab} = Q + iU$$

$$P^* = \mathrm{e}_-^a \mathrm{e}_-^b P_{ab} = Q - iU.$$

Under a rotation of the basis vectors

$$\begin{aligned}\mathrm{e}_\pm \equiv \mathrm{e}_x \pm i\mathrm{e}_y \to \mathrm{e}_x' \pm i\mathrm{e}_y' &= (\cos\gamma\,\mathrm{e}_x - \sin\gamma\,\mathrm{e}_y) \pm i(\sin\gamma\,\mathrm{e}_x + \cos\gamma\,\mathrm{e}_y) \\ &= e^{\pm i\gamma}(\mathrm{e}_x \pm i\mathrm{e}_y) = e^{\pm i\gamma}\mathrm{e}_\pm.\end{aligned}$$

$$P' = \mathrm{e}_+^a{}'\mathrm{e}_+^b{}' P_{ab} = e^{2i\gamma}P.$$ - spin 2 field

(Exactly analogous to shear in cosmic shear)

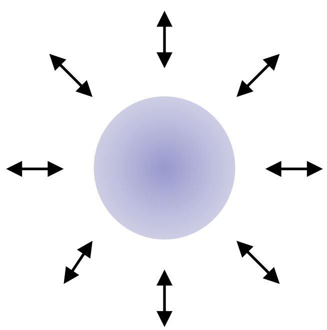
E and B polarization

$$\mathcal{P}_{ab} = \nabla_{\langle a}\nabla_{b\rangle} P_E - \epsilon^{c}{}_{\langle a}\nabla_{b\rangle}\nabla_c P_B$$

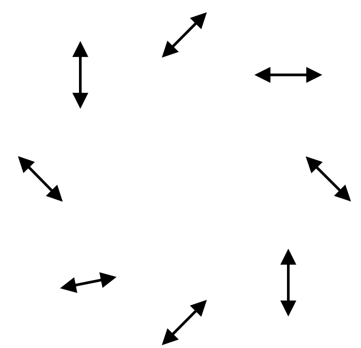
“gradient” modes
E polarization

“curl” modes
B polarization

e.g.



e.g. cold spot



CMB Polarization Signals

Average over possible realizations (statistically isotropic):

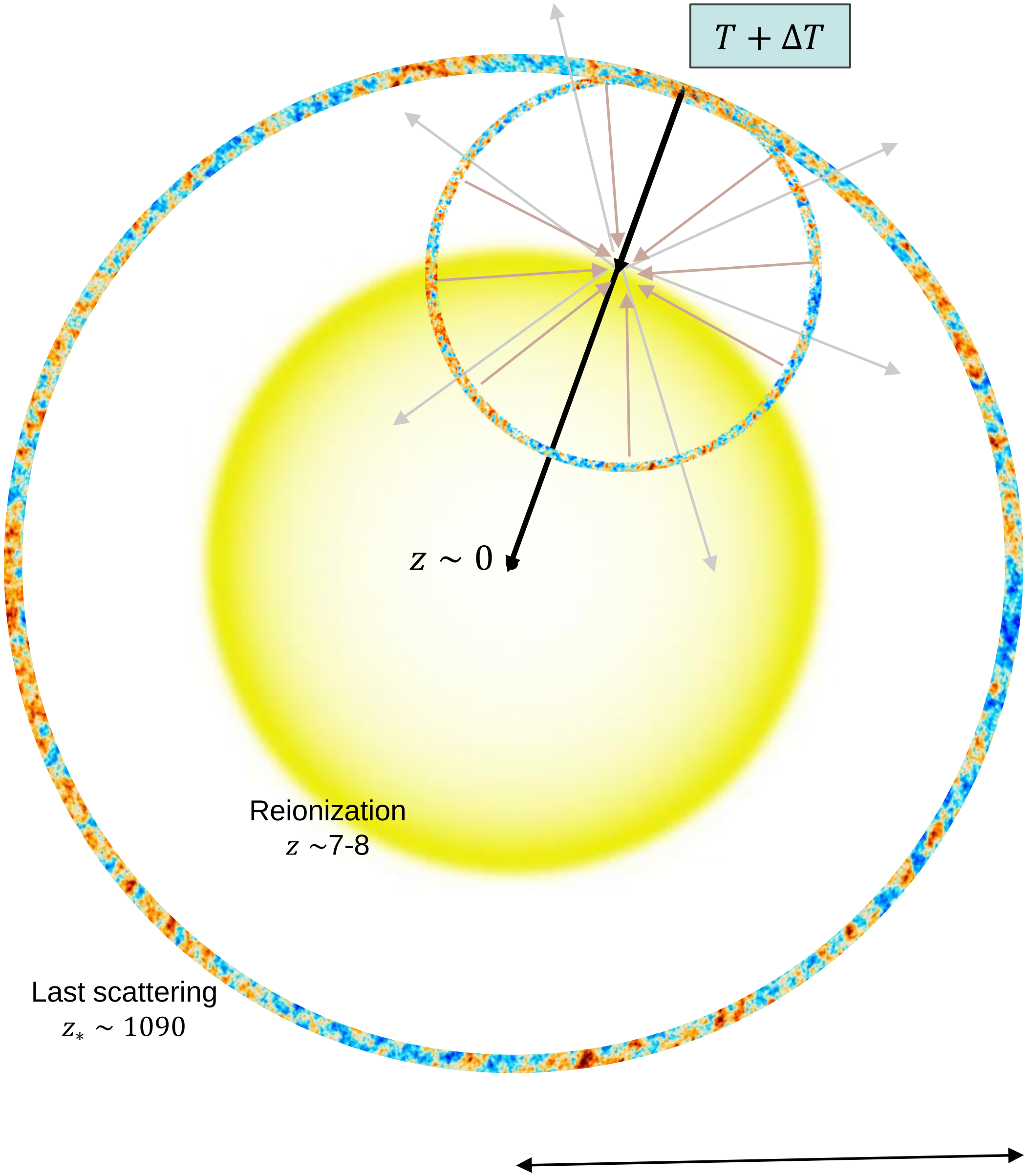
$$\langle E^*_{l'm'} E_{lm} \rangle = \delta_{l'l}\delta_{m'm} C_l^{EE} \qquad \langle B^*_{l'm'} B_{lm} \rangle = \delta_{l'l}\delta_{m'm} C_l^{BB}$$

Parity symmetric ensemble: $\langle E^*_{l'm'} B_{lm} \rangle = 0$

Also cross-correlation $\langle E^*_{lm} T_{lm} \rangle = C_l^{TE}$, Parity symmetric $\Rightarrow C_l^{TB} = 0$

Power spectra contain all the useful information if the field is Gaussian

Reionization scattering



Comoving radial distance ~ 14000 Mpc (ΛCDM)

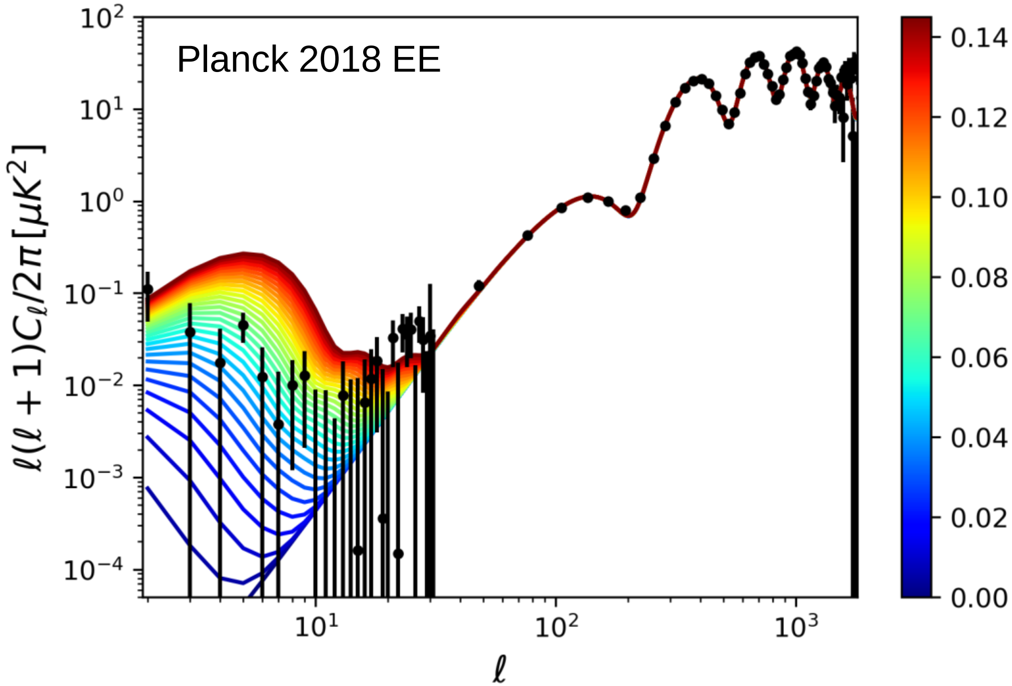
Optical depth τ

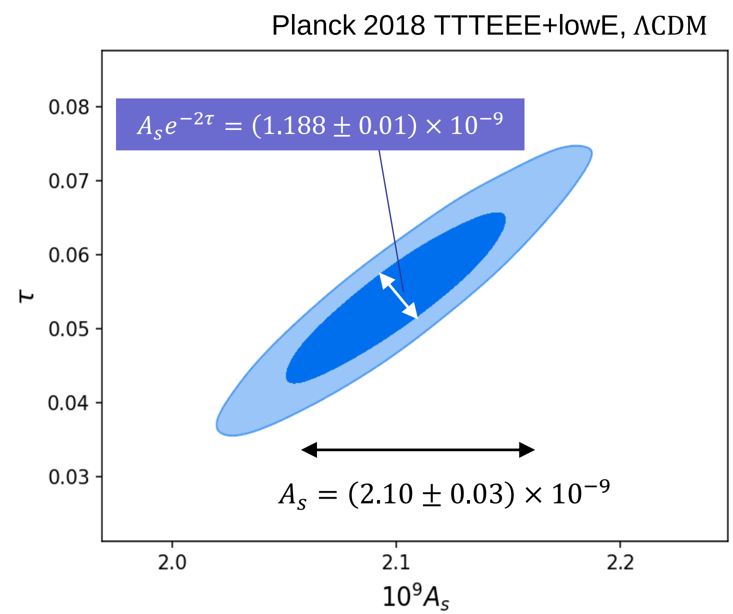
$$\boxed{\frac{T + e^{-\tau}\Delta T}{1 + z_*}}$$

+ large-scale polarization

Planck optical depth constraint

Large-scale E polarization (+foregrounds, systematics…)





Planck 2018

$$\tau = 0.054 \pm 0.007$$

Pagano et al, arXiv:1908.09856
('SRoll2' Planck HFI reanalysis)

$\tau = 0.057 \pm 0.006$

Planck 2020, arXiv:2007.04997
('NPIPE' Planck HFI reanalysis)

$\tau = 0.051 \pm 0.006$

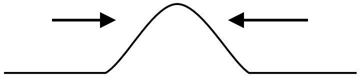
Belsunce et al., arXiv:2103.14378
(`SRoll2'' reanalysis)

$\tau = 0.058 \pm 0.0055$

Perturbations O(10^{-5})

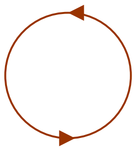
- Linear evolution
- Fourier k mode evolves independently
- Scalar, vector, tensor modes evolve independently

Scalar modes: Density perturbations, potential flows



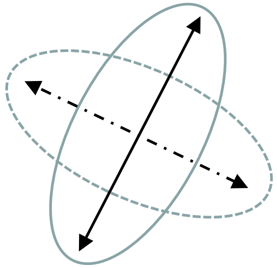
$\delta\rho, \nabla\delta\rho, etc$

Vector modes: Vortical perturbations



velocities, *v* $(\nabla \bullet v = 0)$

Tensor modes: Anisotropic space distortions
– gravitational waves



B modes only from vectors or tensors in linear theory. + non-linear.

Primordial Gravitational Waves
(tensor modes)

- Well motivated by some inflationary models

  - Amplitude measures inflaton potential at horizon crossing

  $$P_T \approx \frac{2}{3\pi^2}\frac{V_*}{M_P^4}$$

  - distinguish models of inflation ("small-field" vs "large-field"; detection ⇒ some symmetry that protects $|\Delta\phi| \sim M_{\mathrm{pl}}$)

- Observation would rule out some other models for origin of structure

- Usually constrain $r \equiv P_T/P_S$ at some pivot scale

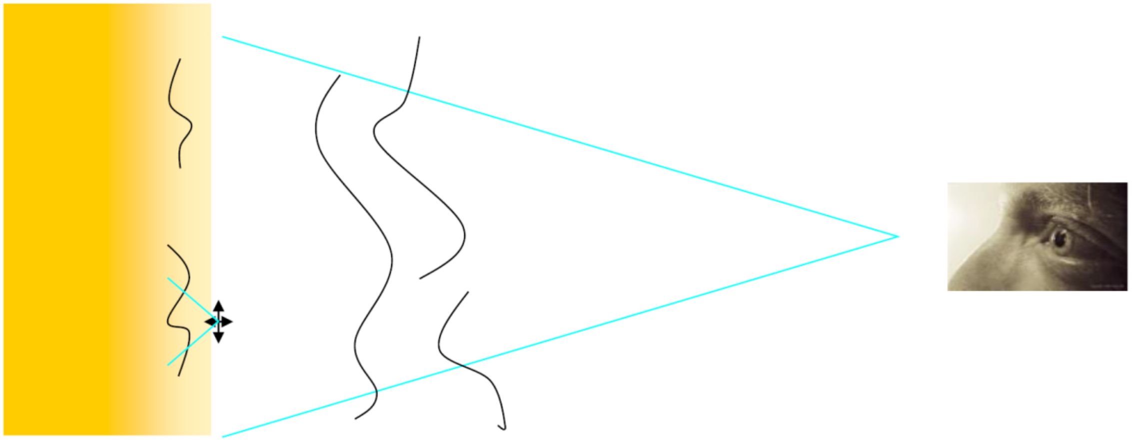
  $$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\mathcal{R}} = 16\epsilon_V$$

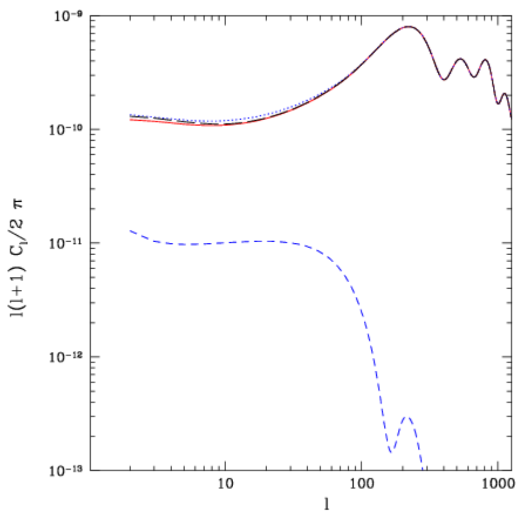
  $r \sim O(1 - n_s)$ probed by current observations

  $$\epsilon_V(\phi) \equiv \frac{M_P^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2$$

  $r \sim O\left([1 - n_s]^2\right)$ a target for future observations

Gravitational waves anisotropically redshift CMB photons as they pass through
Waves decay after they enter the horizon ⇒ signal from horizon entry only

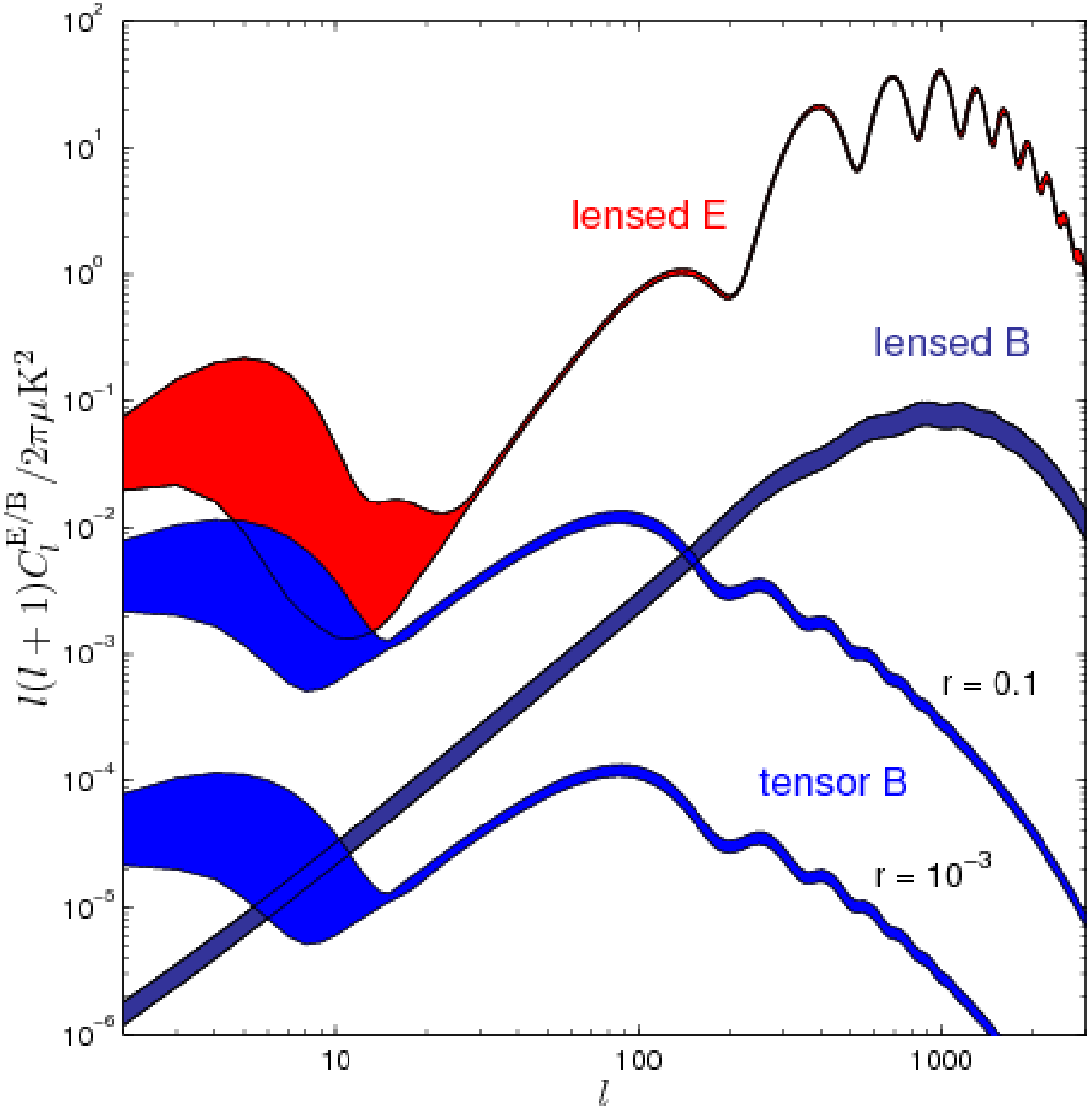




Temperature:
- Anisotropic redshifting of 0^{th} order last scattering by 1^{st} order gravitational waves along the line of sight
- cosmic variance limited to 10%
- degenerate with other parameters (tilt, reionization, etc)

Look at CMB polarization:
B-mode "smoking gun"

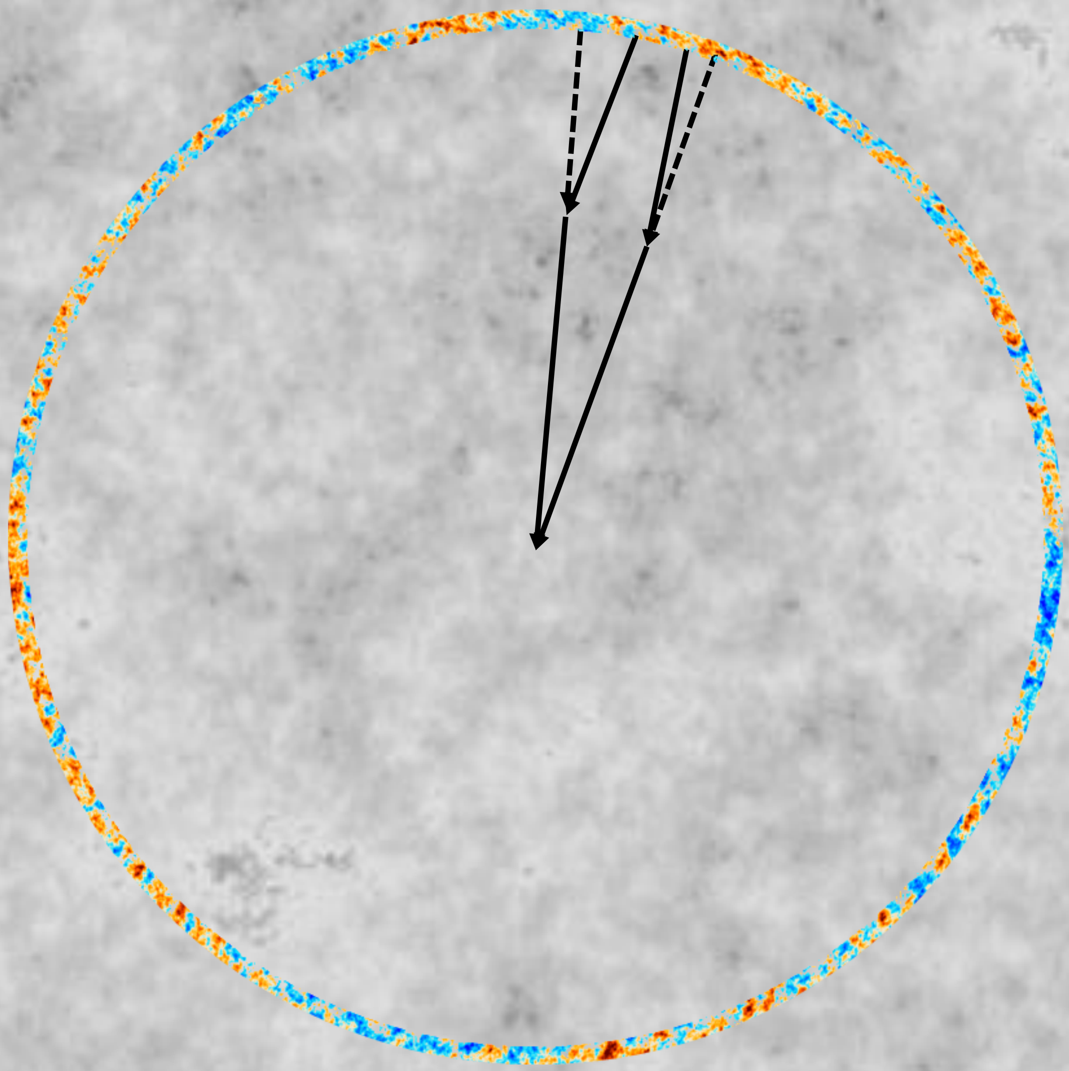
Polarization power spectra



Current B-mode constraint: $r_{0.05} < 0.036$ (95%, Bicep/Keck+Planck)

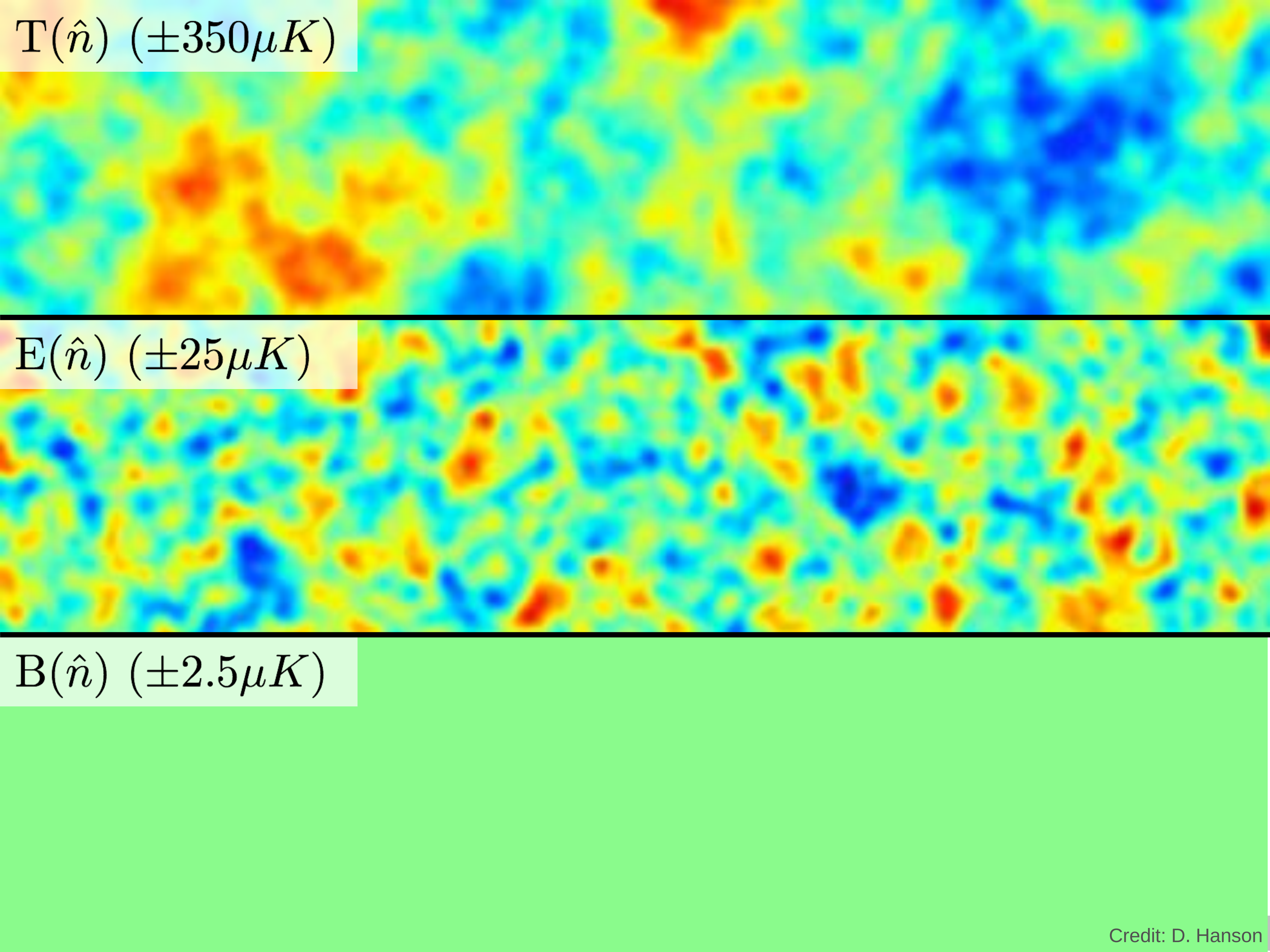
arXiv:2110.00483

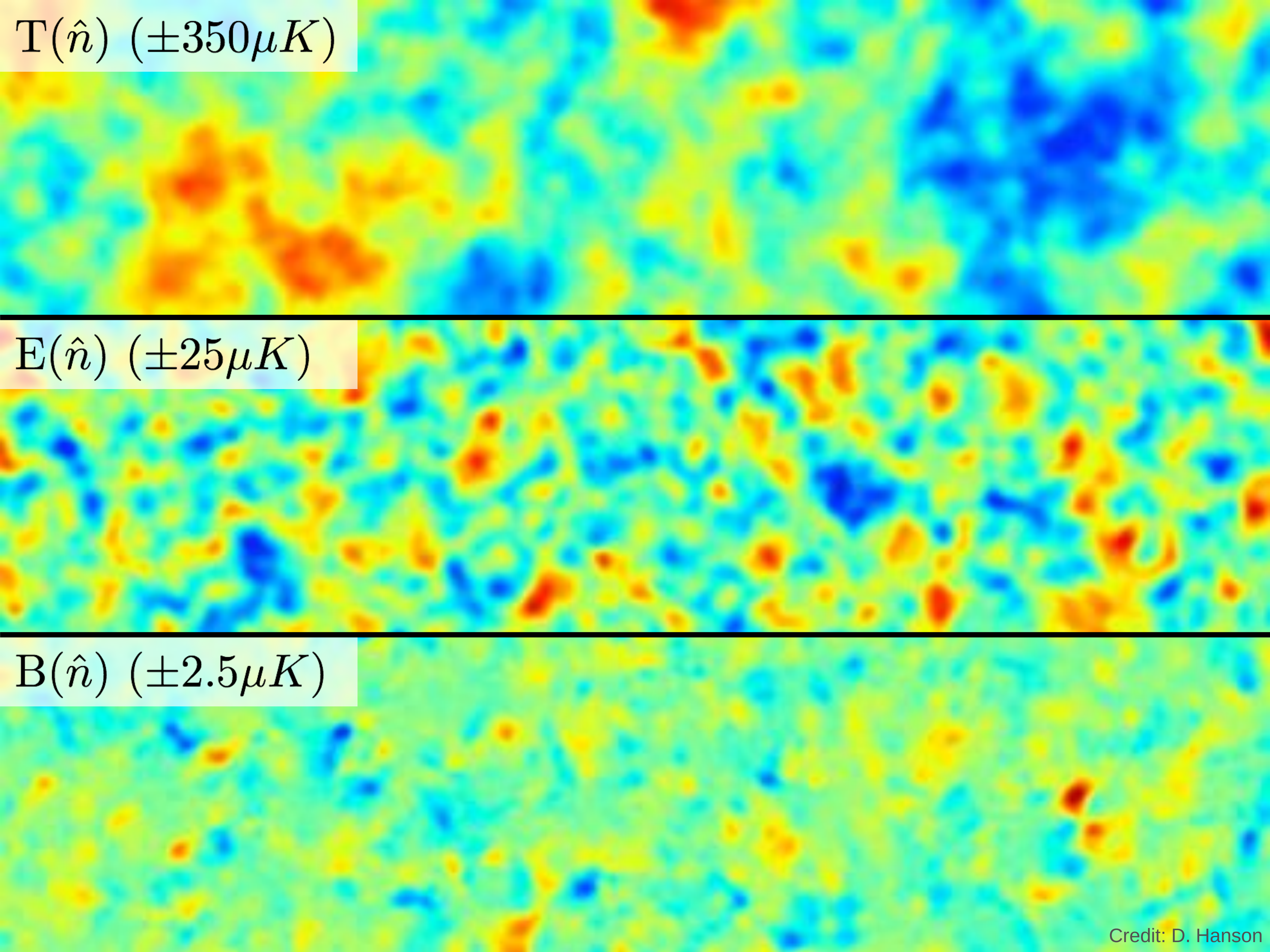
CMB Lensing: 1st order light propagation



Spatially varying gravitational potentials: high-z kernel, mostly linear

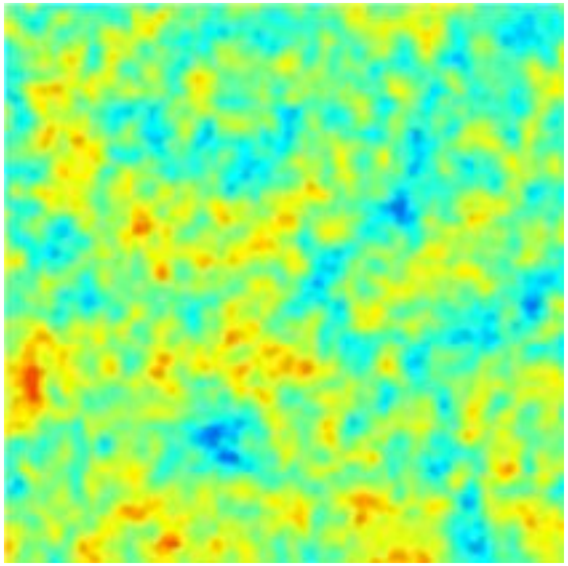
(perturbations here not to scale)

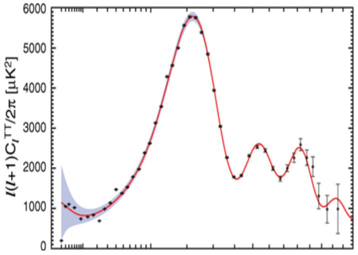

$T(\hat{n})$ $(\pm 350\mu K)$
$E(\hat{n})$ $(\pm 25\mu K)$
$B(\hat{n})$ $(\pm 2.5\mu K)$
Credit: D. Hanson


$T(\hat{n})$ $(\pm 350\mu K)$
$E(\hat{n})$ $(\pm 25\mu K)$
$B(\hat{n})$ $(\pm 2.5\mu K)$
Credit: D. Hanson

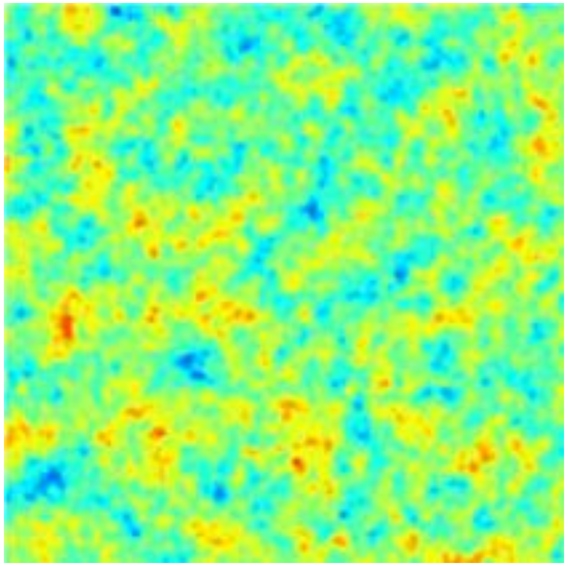
Local effect of lensing magnification on the power spectrum

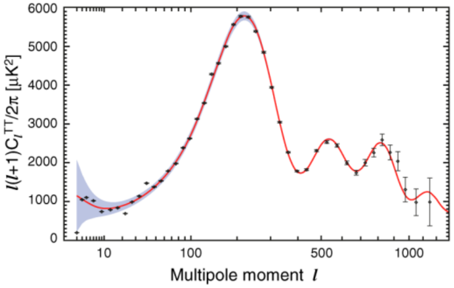
Magnified



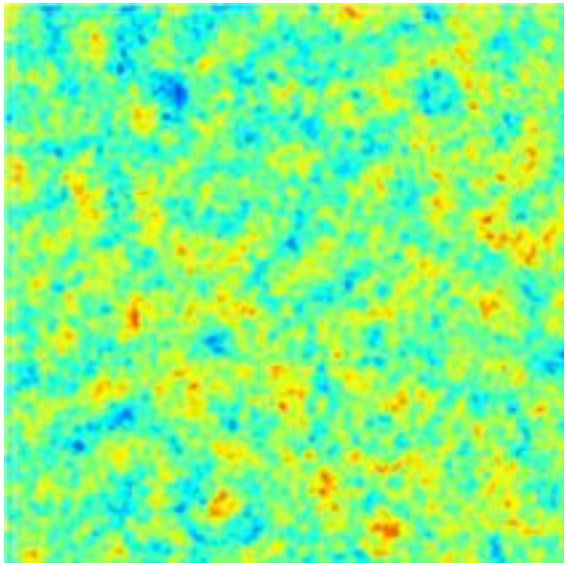


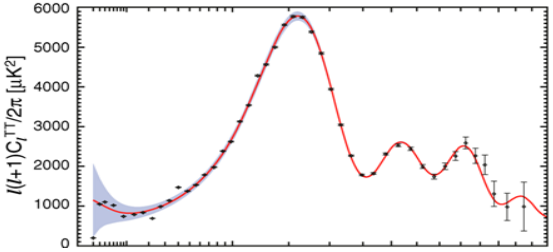
Unlensed



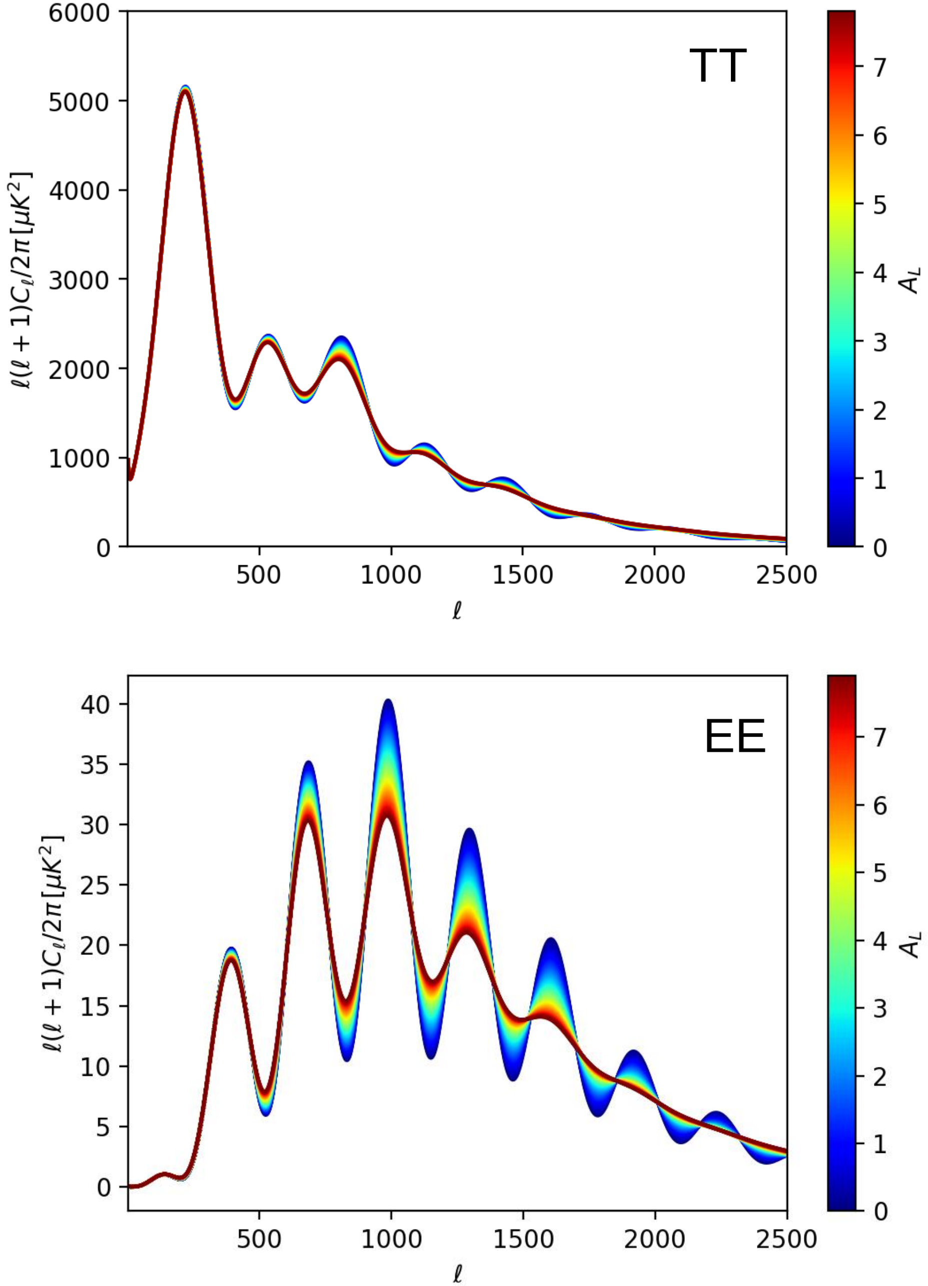


Demagnified





Averaged over the sky, lensing smooths out the power spectrum



Amount of lensing
($A_L = 1$ is actual level)

Lensed temperature depends on deflection angle

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

Newtonian potential

$$\boldsymbol{\alpha} = \delta\theta = -2\int_0^{\chi^*} \mathrm{d}\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla_\perp \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi)$$

co-moving distance to last scattering

Lensing Potential

Deflection angle on sky given in terms of angular gradient of lensing potential $\boldsymbol{\alpha} = \nabla\psi$

$$\psi(\hat{\mathbf{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi \, \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi) \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)}$$

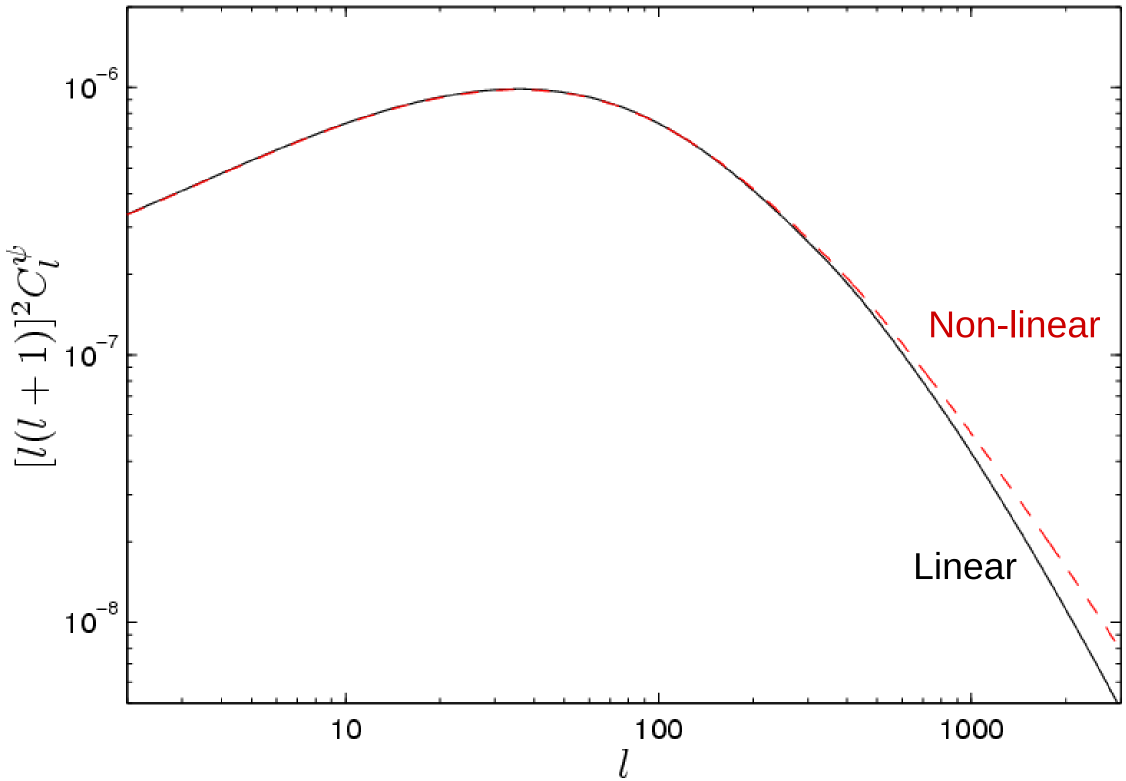
$$\bar{X}(\mathbf{n}) = X(\mathbf{n}') = X(\ \mathbf{n} + \nabla\psi(\mathbf{n})\ )$$

Deflection angle power spectrum

On small scales (Limber approx) $$C_l^\psi \approx \frac{8\pi^2}{l^3}\int_0^{\chi_*} \chi \mathrm{d}\chi\, \mathcal{P}_\Psi(l/\chi; \eta_0 - \chi)\left(\frac{\chi_* - \chi}{\chi_* \chi}\right)^2$$

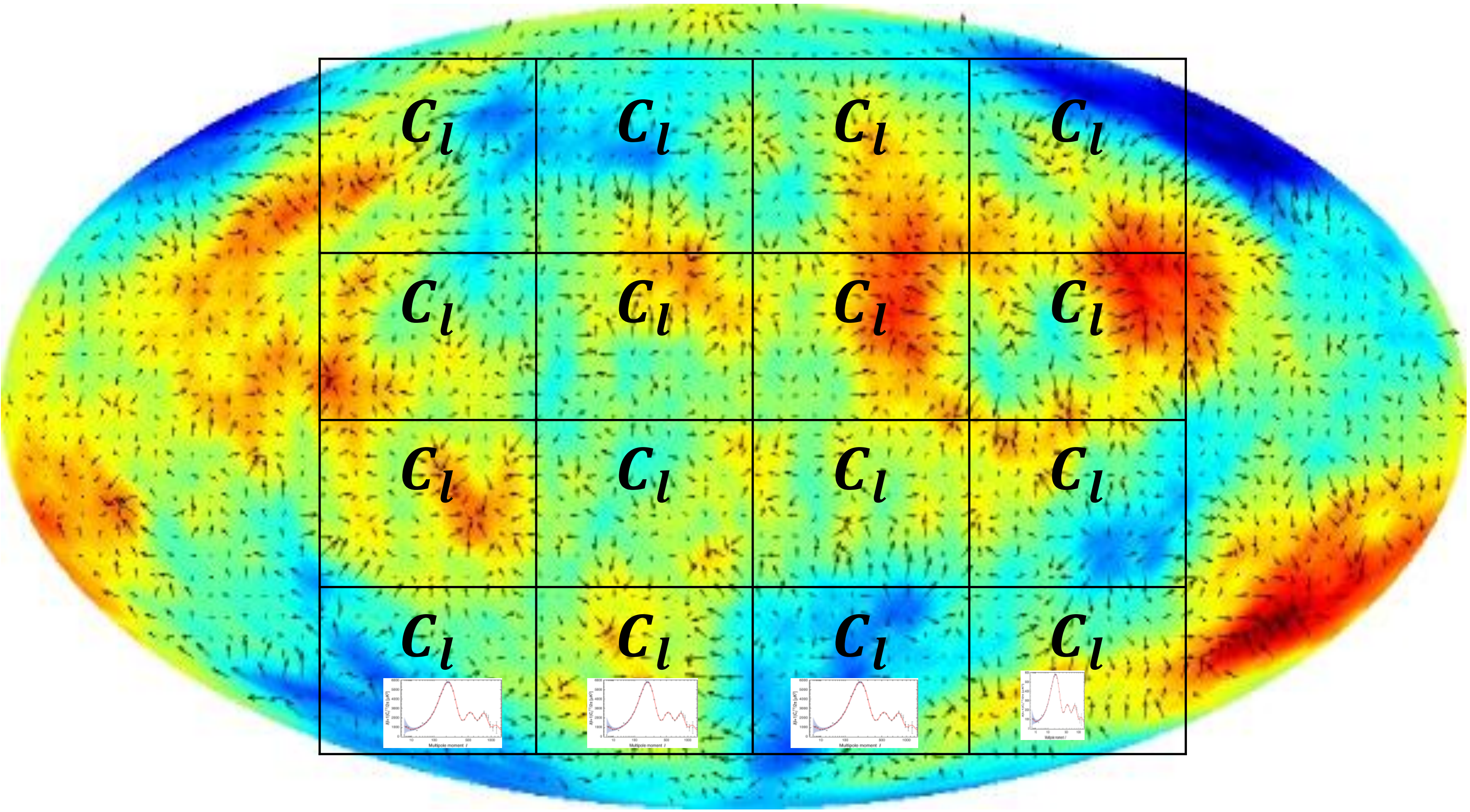
Deflection angle power ~ $l(l+1)C_l^\psi$

Convergence power $C_l^\kappa = [l(l+1)]^2 C_l^\psi / 4$



Deflections O(10^{-3}), but coherent on degree scales → important!

Lensing reconstruction (concept)



Measure spatial variations in magnification and shear

Use assumed unlensed spectrum, and unlensed statistical isotropy

Lensing Reconstruction – Quadratic Estimators

- Fixed lenses introduce statistically-anisotropic correlations:

$$\Delta\langle X_{l_1m_1}Y_{l_2m_2}\rangle_{\mathrm{CMB}} = \sum_{LM}(-1)^M \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix} \mathcal{W}^{XY}_{l_1l_2L}\phi_{LM}$$

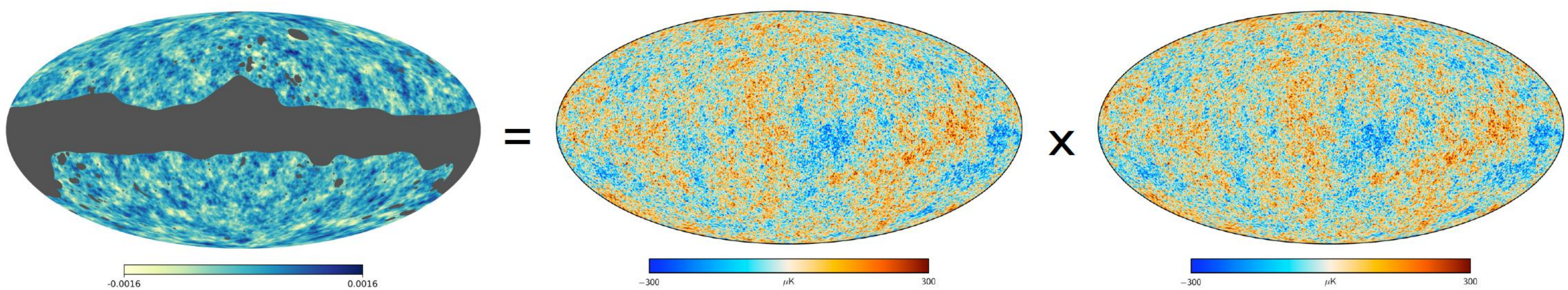
- Noisy lensing estimates from quadratic CMB combinations:

$$\hat{\phi}_{LM} = \frac{(-1)^M}{2}\frac{1}{\mathcal{R}^{XY}_L}\sum_{l_1m_1,l_2m_2}\begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix}\left[\mathcal{W}^{XY}_{l_1l_2L}\right]^* \bar{X}_{l_1m_1}\bar{Y}_{l_2m_2}$$

Normalisation

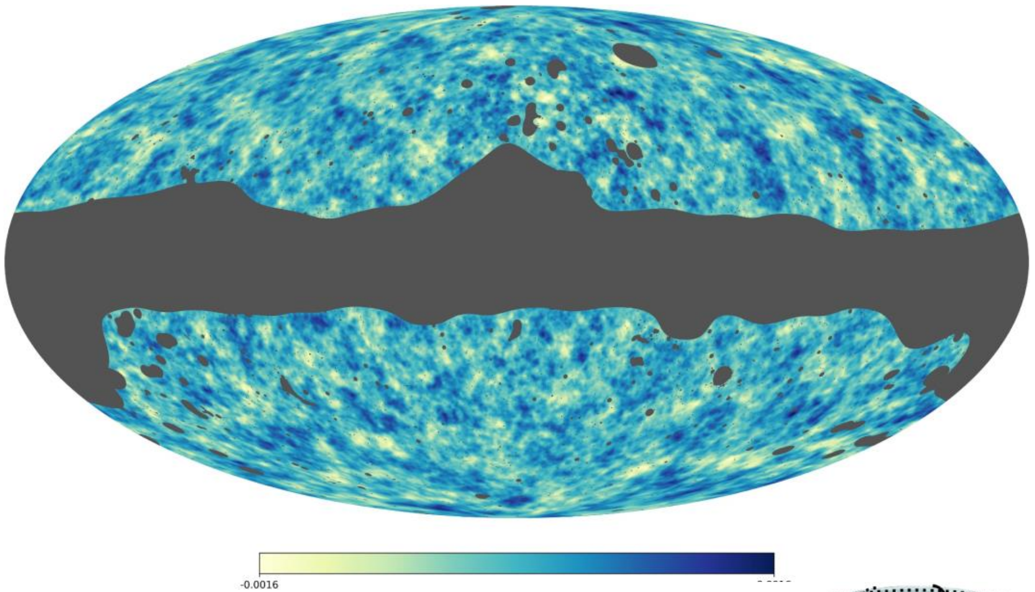
Known lensing-induced correlations

Inverse-variance-weighted CMB fields



Credit: Anthony Challinor

Map of the gradient-mode lensing

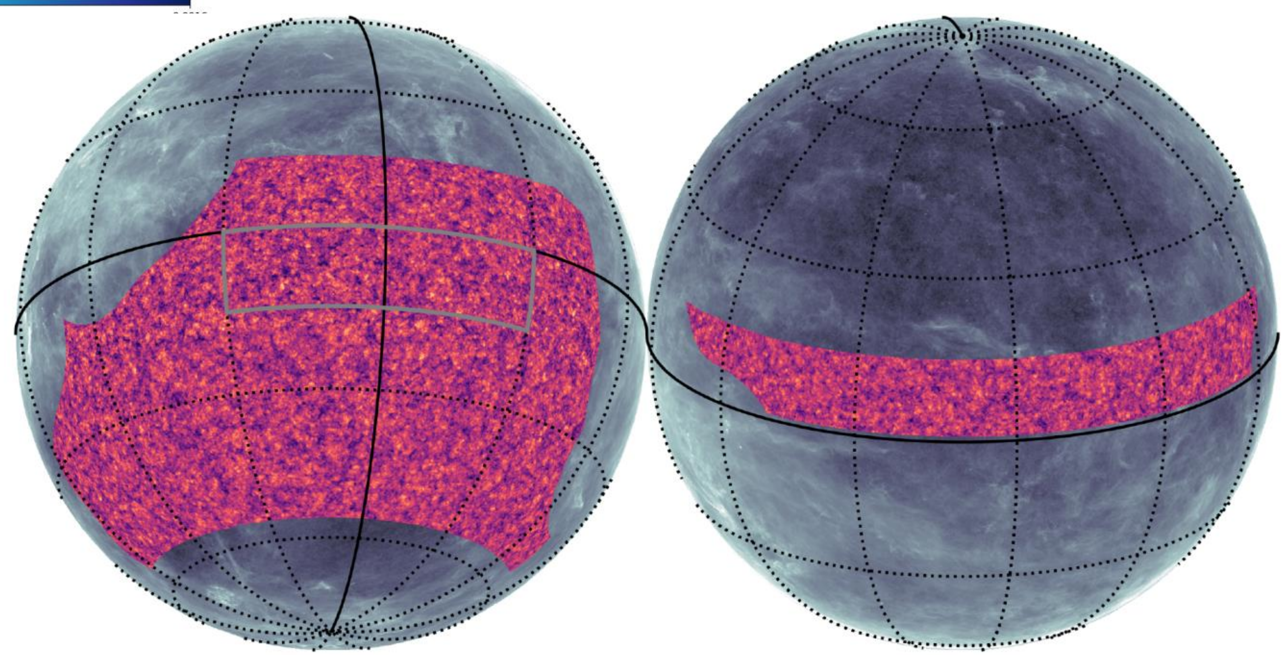


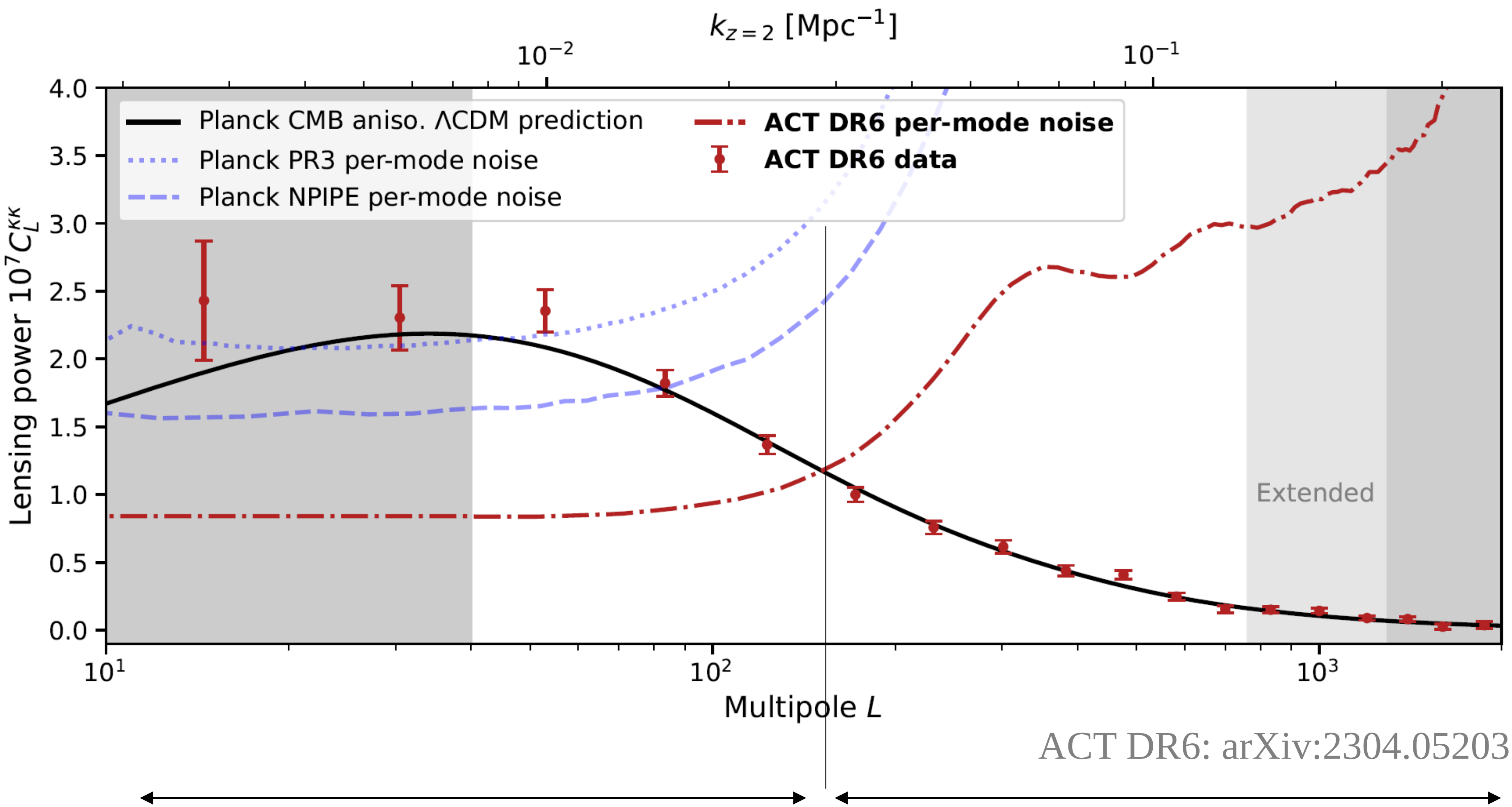
Planck 2018

70% of the sky
Only largest lensing modes resolved

ACT DR6 (2023)

23% of the sky
But higher resolution



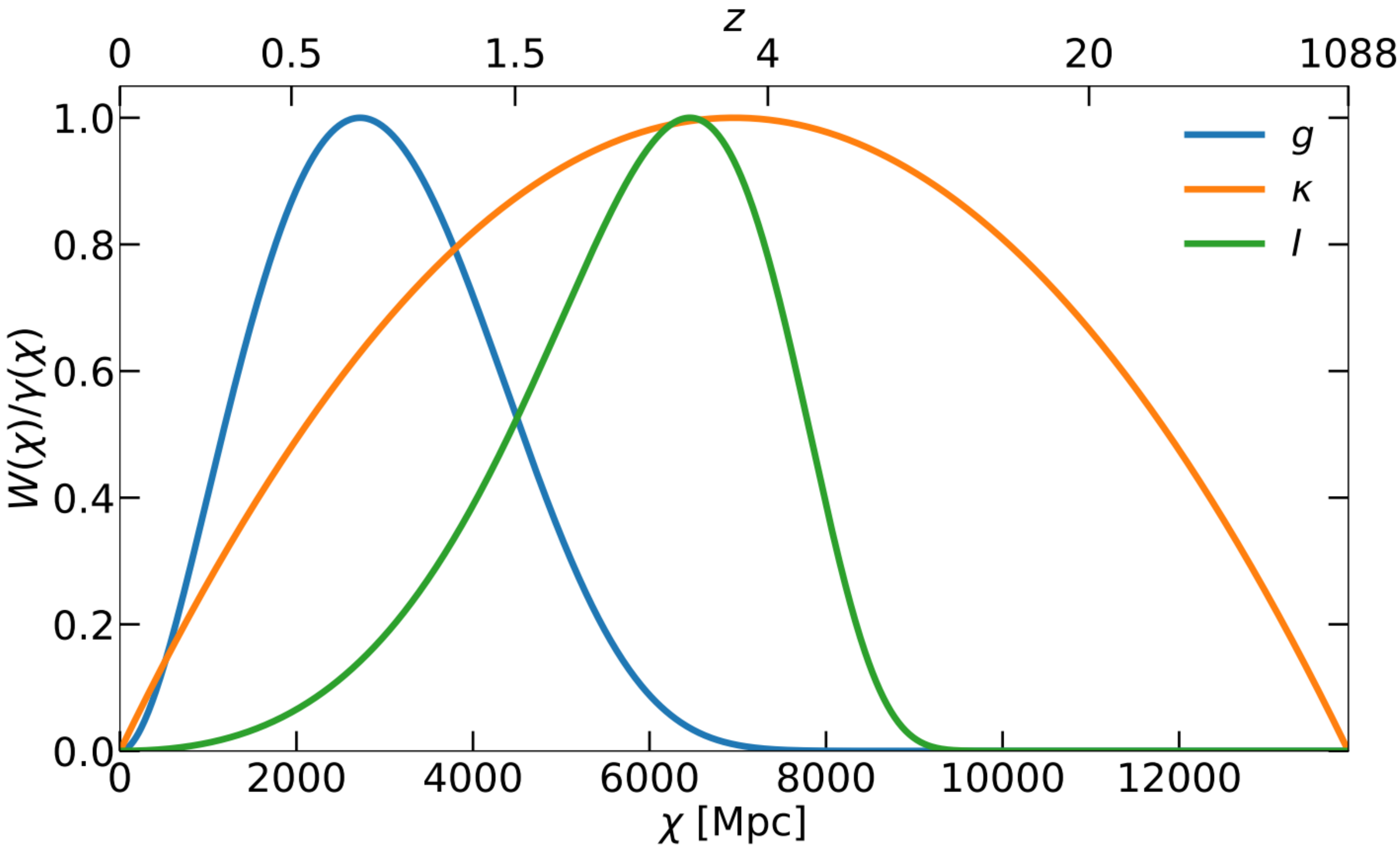


ACT DR6: arXiv:2304.05203

Lens cosmic variance/$f_{\rm sky}$ limited

Reconstruction "noise" limited

+ cross-correlations with many other large-scale structure probes (can do tomographic cross-correlation)

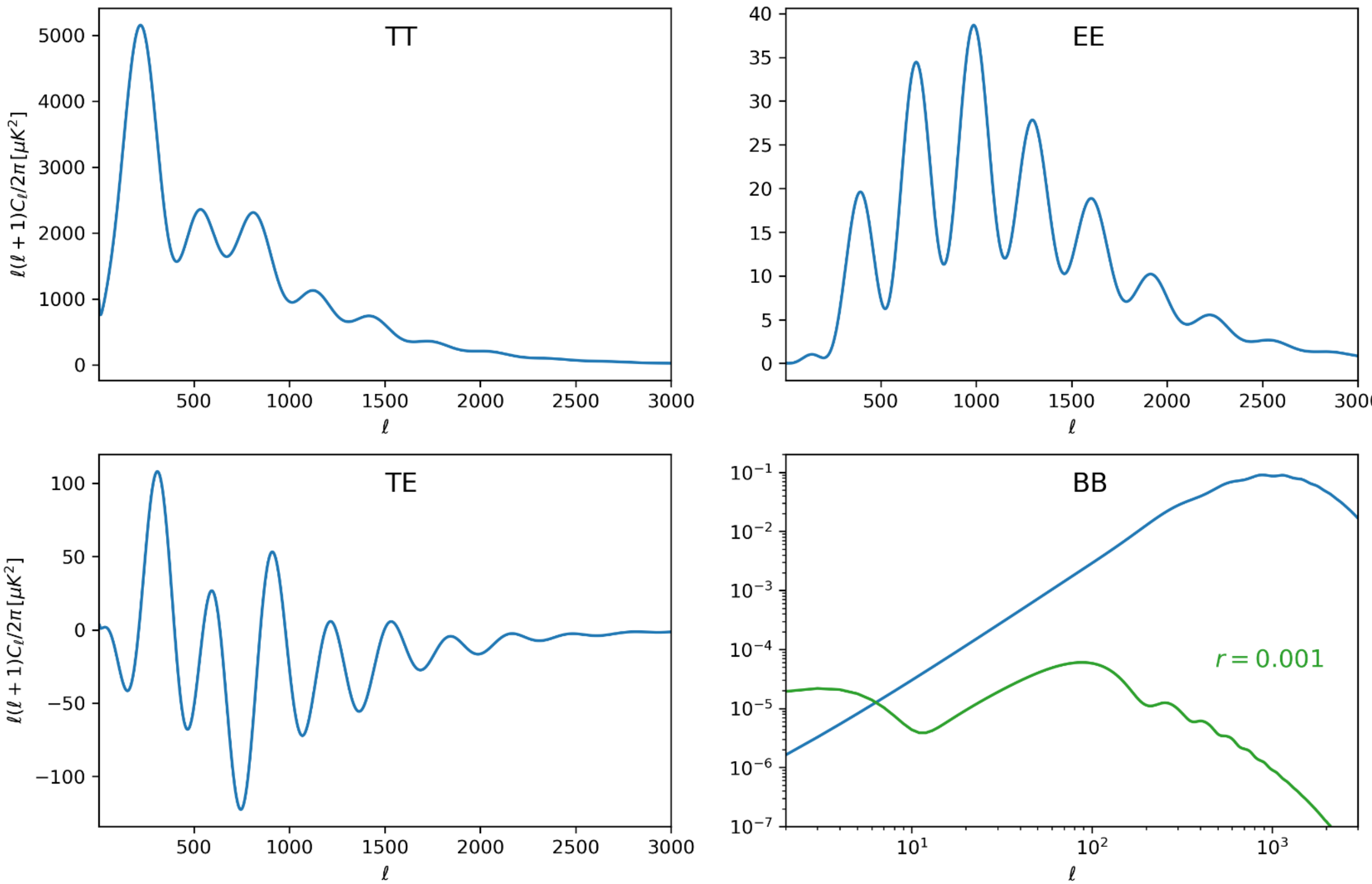


arXiv:2303.13313

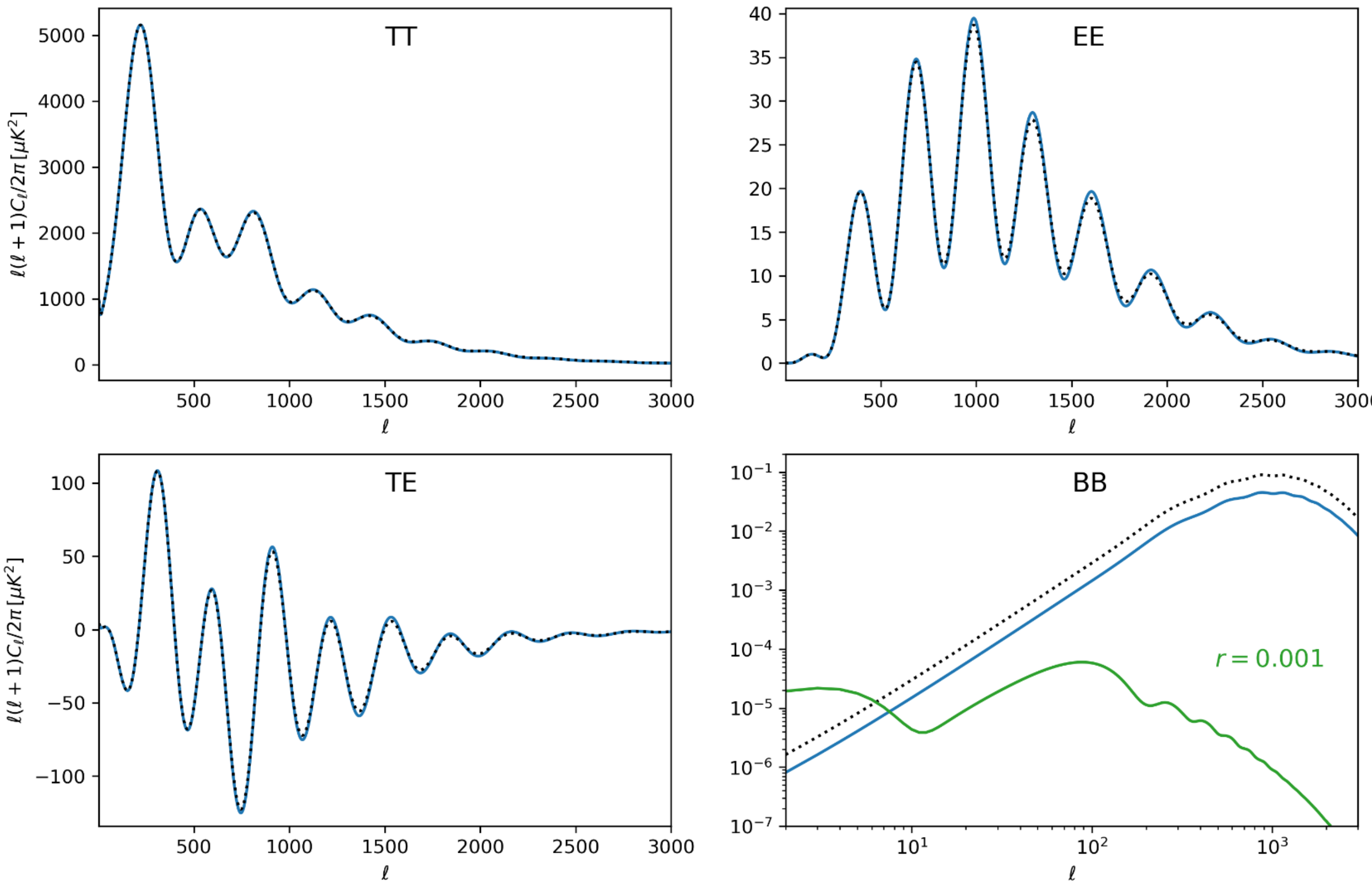
Comparison with galaxy lensing

- **Single source plane (known distance given cosmological model)**
  - limited information on low-redshift dark energy
- **Statistics of sources on source plane well understood**
  - can calculate power spectrum; Gaussian linear perturbations
  - magnification and shear information equally useful - usually discuss in terms of deflection angle;
  - magnification analysis of galaxies much more difficult
- **Hot and cold spots are large, smooth on small scales**
  - 'strong' and 'weak' lensing can be treated the same way: infinite magnification of smooth surface is still a smooth surface
- **Source plane very distant, large nearly-linear lenses**
  - much less sensitive to non-linear modelling, baryon feedback, etc.
- **Noise-dominated on small scales**
  - but smaller scales more limited by non-linear modelling anyway
- **Nearly full sky, high redshift kernel $\Rightarrow$ some sensitivity to matter turnover scale**
- **Systematics completely different**
  - CMB/galaxy cross-correlations can be a good way to calibrate bias/systematics

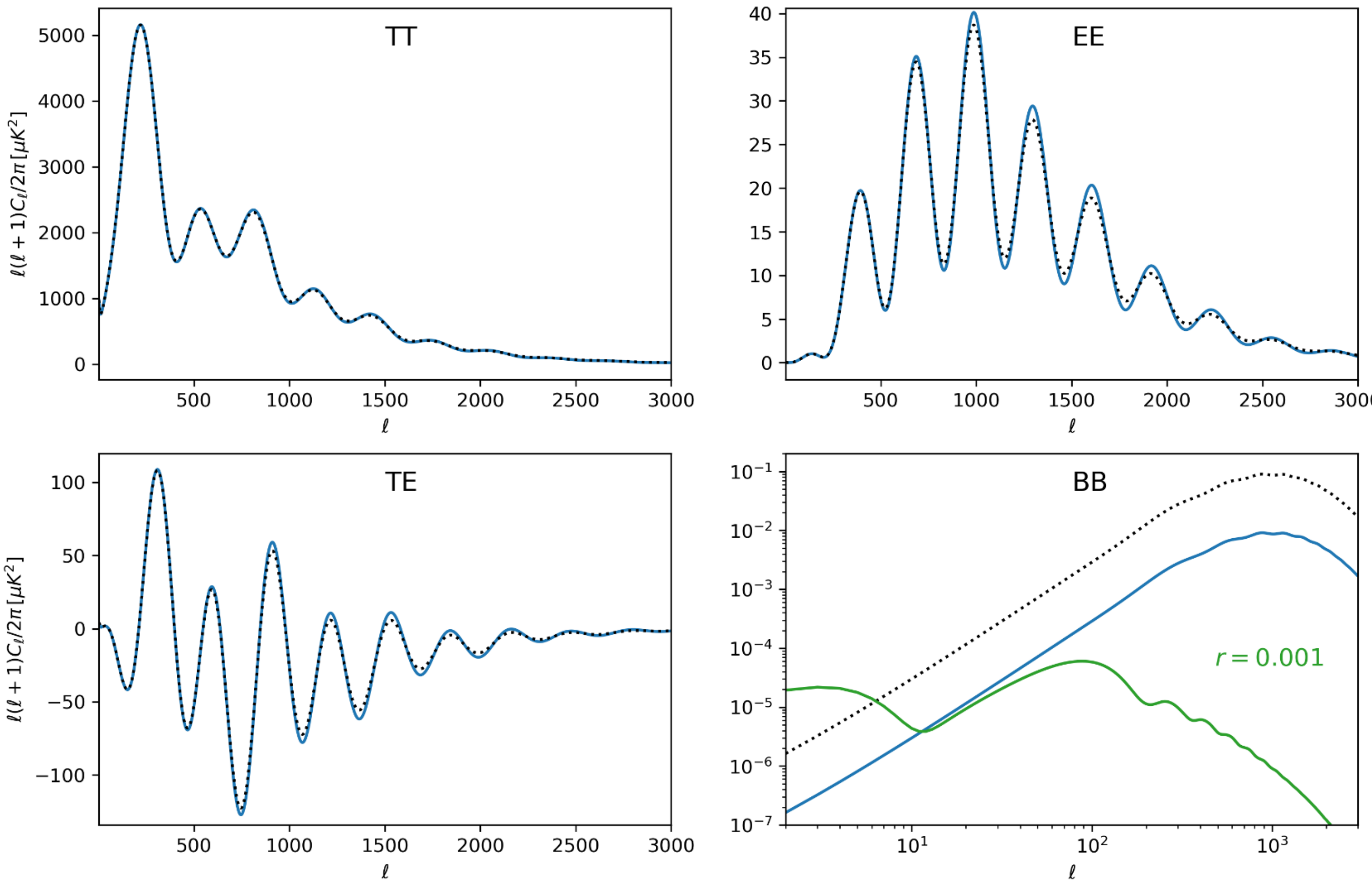
Delensing ($A_L = 1$)



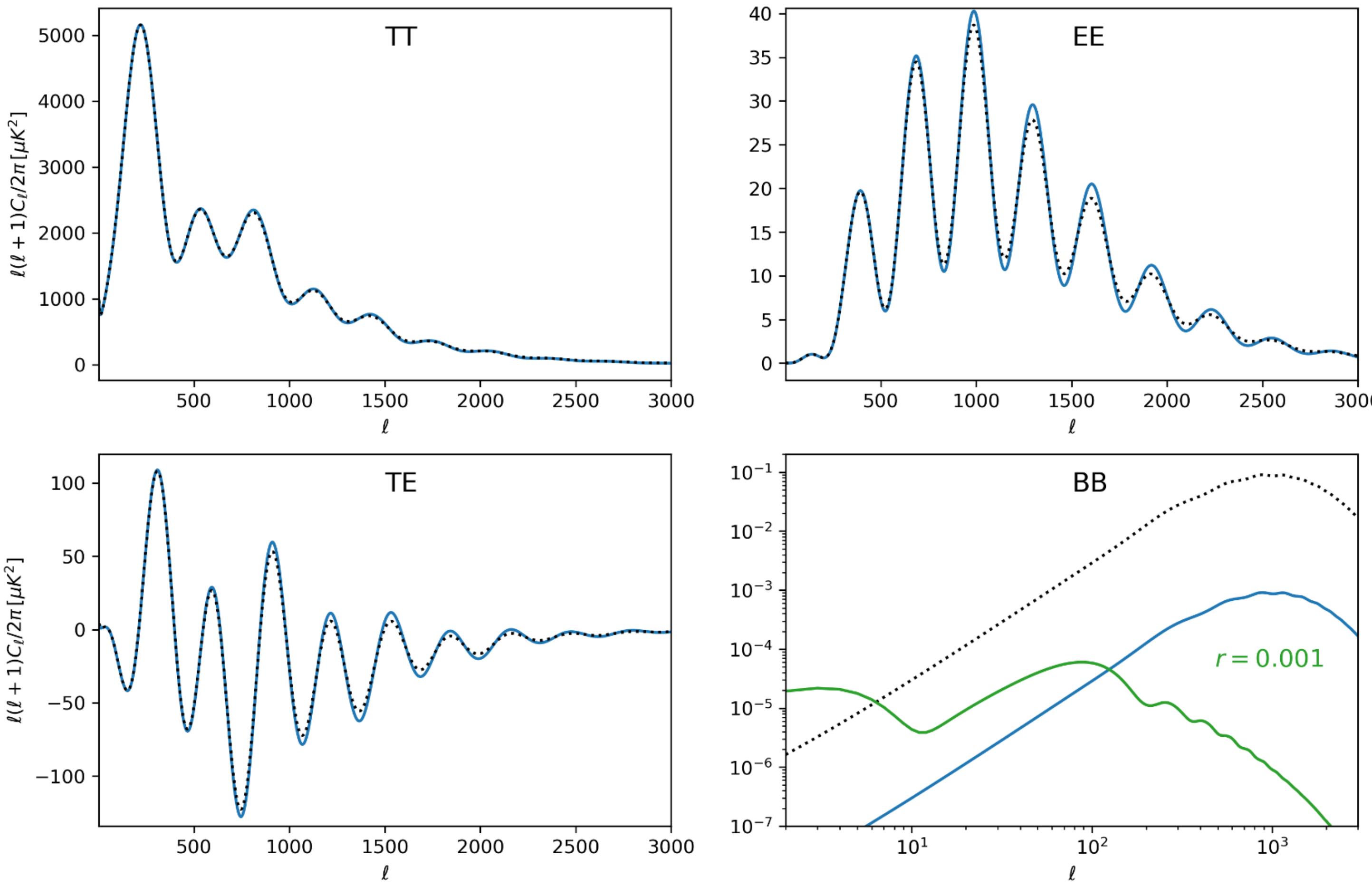
Delensing ($A_L = 0.5$)



Delensing ($A_L = 0.1$)



Delensing ($A_L = 0.01$)



Analogous to reconstruction in BAO analysis: sharpens peaks, cleans non-linear B modes

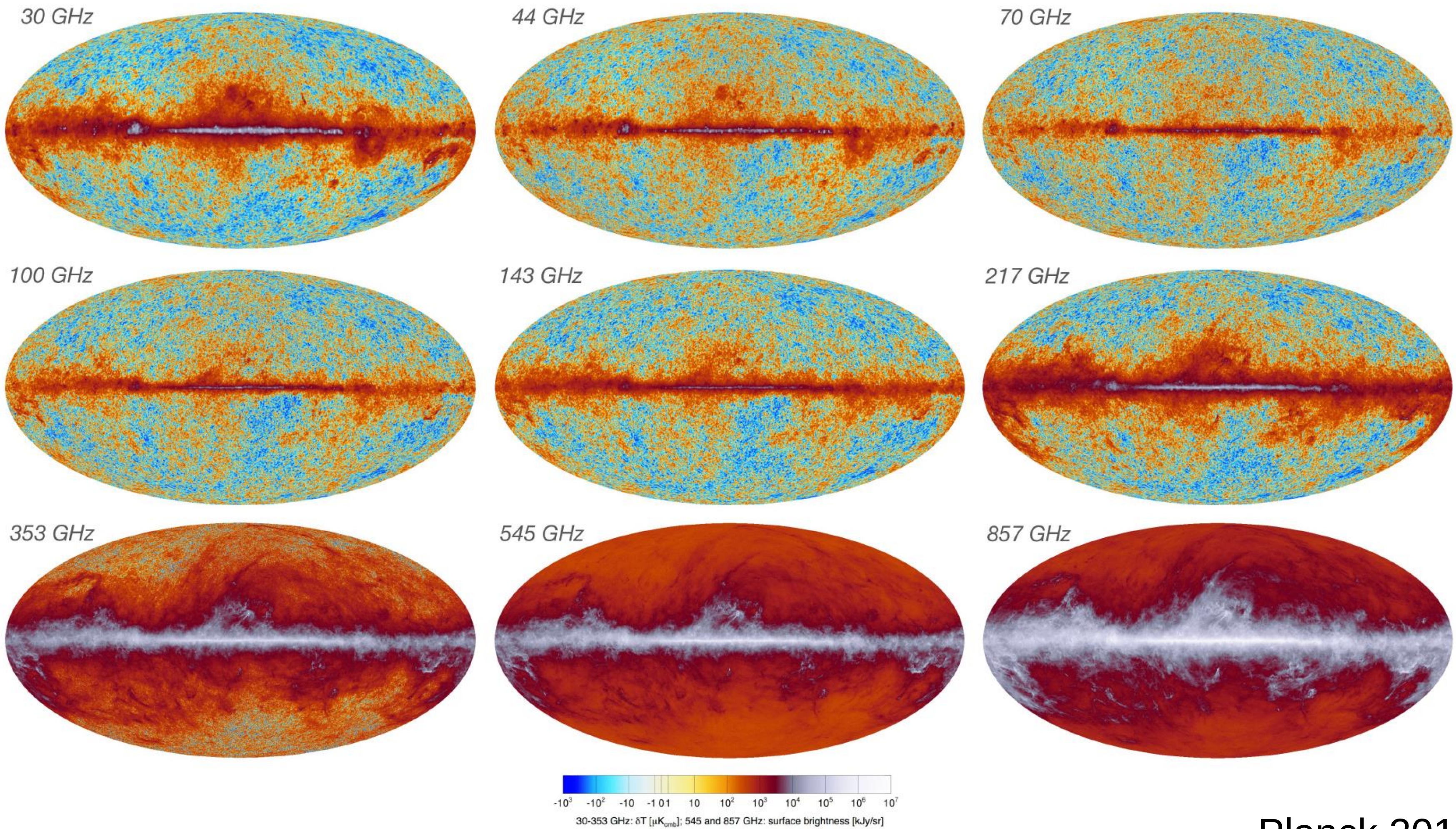
Other non-linear effects

- Thermal Sunyaev-Zeldovich
  Doppler from electron velocity dispersion in hot gas: frequency dependent signal: probe of clusters (strongly non-linear)

- Kinetic Sunyaev-Zeldovich (kSZ)
  Doppler from bulk motion at low redshift; (almost) frequency independent signal (more linear signal)

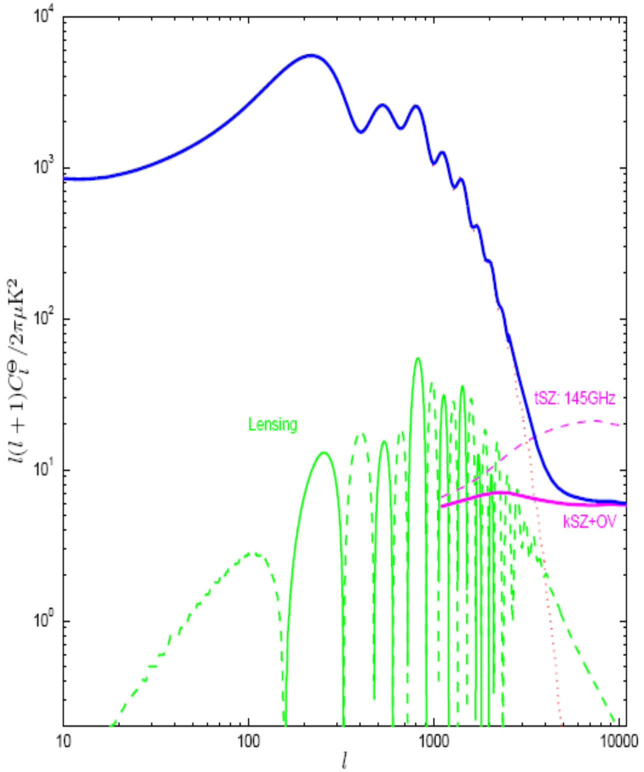
- + others mostly small

Foregrounds: synchrotron → Dust/Cosmic Infrared Background (CIB)



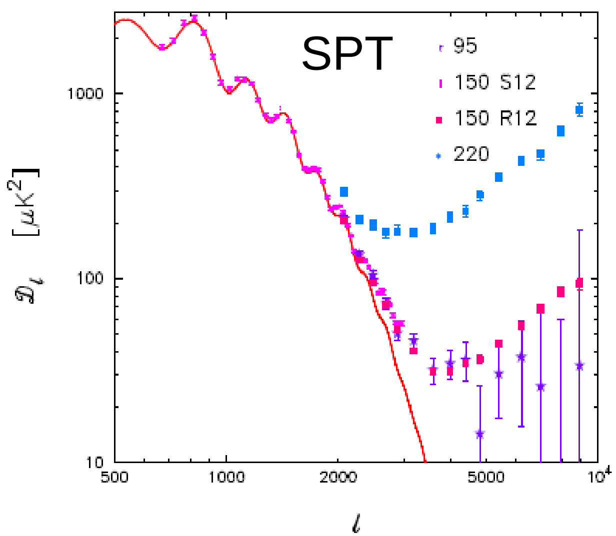
Planck 2018

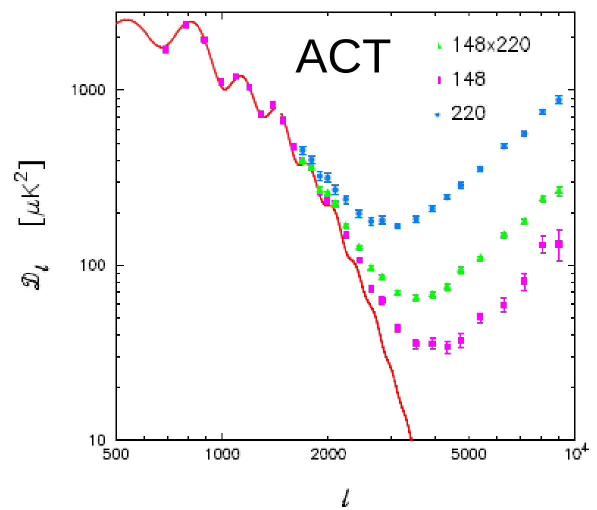
Less of an issue for polarization (except low ℓ)



Lensing important at $500 < l < 3000$
Dominated by SZ, CIB etc. on small scales

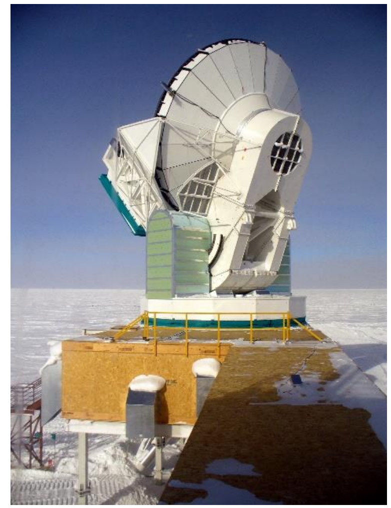
+ foregrounds
- actually dominate at $l \gg 2000$





Current CMB projects

New CMB observations will measure smaller scales than Planck, and much improve measurements of CMB polarization (and hence lensing/B modes)



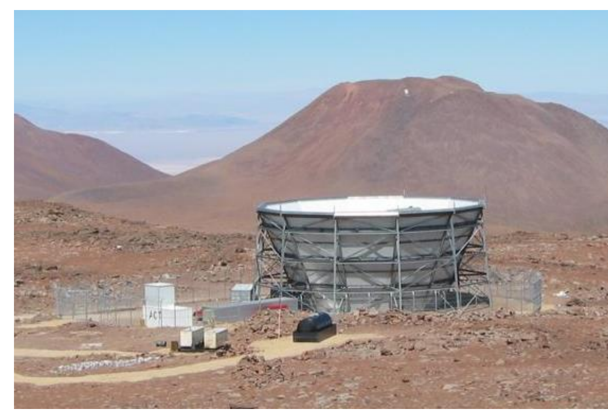
South Pole Telescope

22°57'31" SOUTH, 67°47'15" WEST / ELEVATION: 5190 m

SIMONS OBSERVATORY

PARQUE ASTRONÓMICO ATACAMA, CHILE

Atacama Cosmology Telescope



+ B modes, $r \sim 0.01$

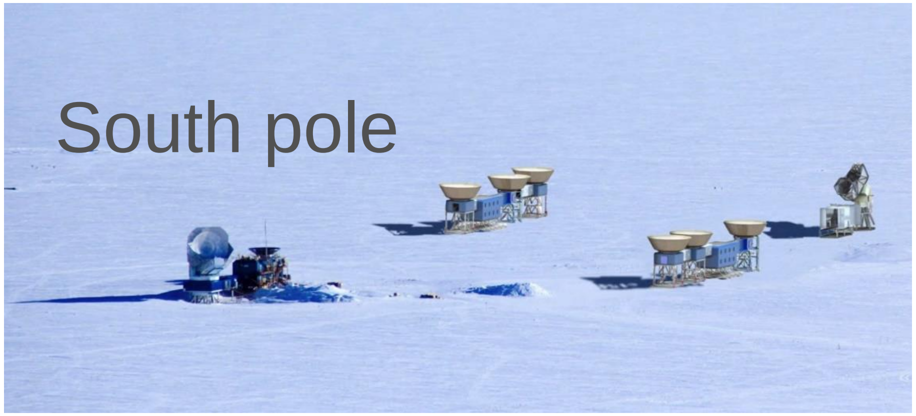
BICEP array

+ others..

CMB-S4

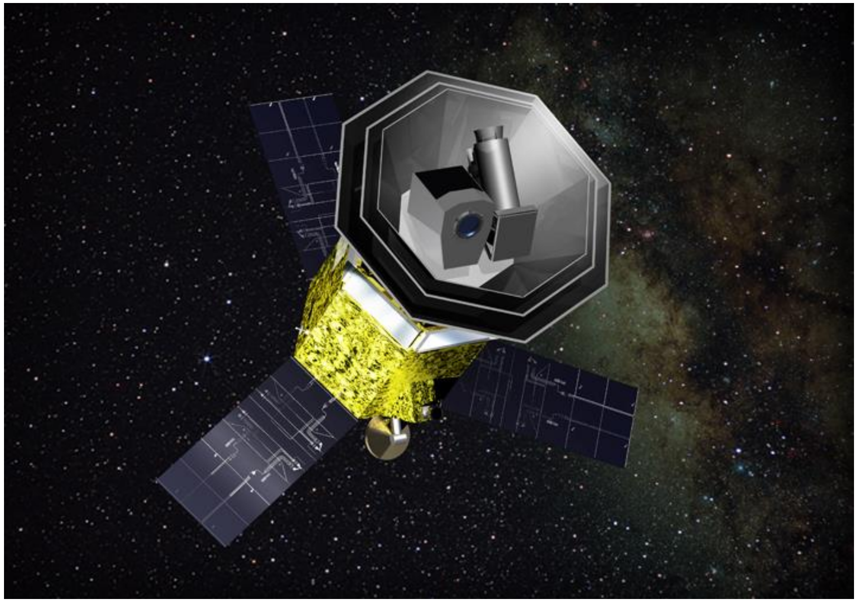
21 telescopes, 500000+ detectors, 7 years

+



LiteBird

Satellite, 15 frequencies, low res



+ other proposals

Ali CMB, CMB Bharat, CMB HD, …

Quiz: true or false?

1) CMB anisotropies probe density perturbations, so a larger matter density leads to larger CMB power spectrum

2) Overdensities focus light rays, so the CMB looks hotter where there are overdensities along the line of sight

3) Even in linear theory lensing is mostly at low redshift because density perturbations grow with time

4) The observed blackbody CMB anisotropies are non-Gaussian at high significance

5) A linear comoving radius 1000 Mpc overdensity at recombination appears as a large hot spot in the observed CMB anisotropies

Lecture 2

Quiz: true or false?

1) CMB anisotropies probe density perturbations, so a larger matter density leads to larger CMB power spectrum

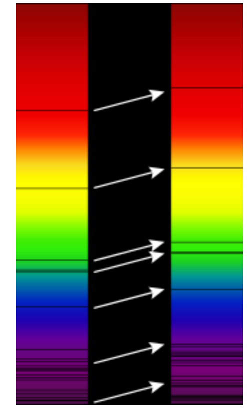
2) Overdensities focus light rays, so the CMB looks hotter where there are overdensities along the line of sight

3) Even in linear theory lensing is mostly at low redshift because density perturbations grow with time

4) The observed blackbody CMB anisotropies are observed to be non-Gaussian at high significance

5) A linear comoving radius 1000 Mpc overdensity at recombination appears as a large hot spot in the observed CMB anisotropies

Hubble parameter and distance measures



Measure redshift: $z \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm lab}} - 1$

Define recession velocity: $v = c\, z$

Nearby ($z \ll 1$): $v = H_0 D \Rightarrow \; H_0 = \frac{cz}{D}$

BUT: D is not observable. Only see photons and angles on the sky today.

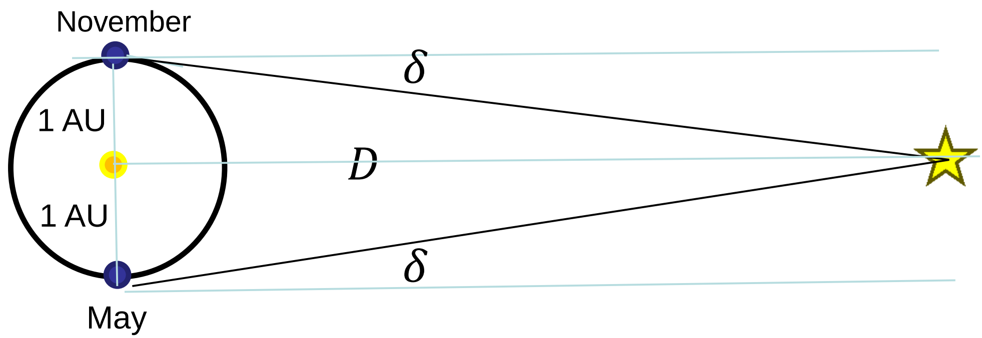
Redshifts are “easy”:

Measuring D(z) $\Rightarrow$ Measuring H_0

$$H_0 \propto 1/D$$

How to measure distance?

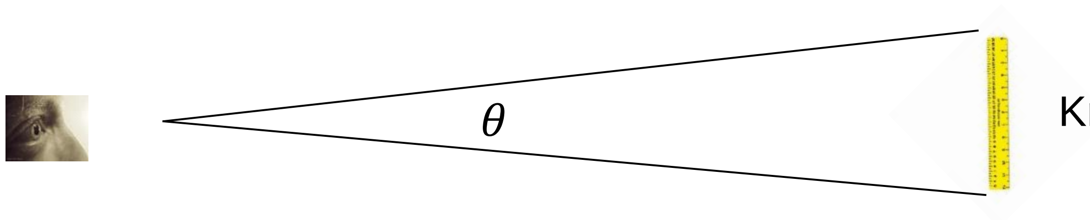
Parallax



$$D = 1\ \mathrm{AU}/\delta$$

Large D impossible

Standard Rulers

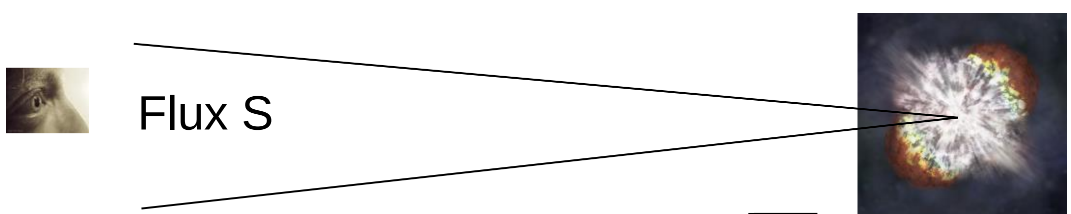


Known size r

$$D = r/\theta$$

Large D: $D \rightarrow d_A/d_M$

Standard Candles



Known Luminosity L

$$S = \frac{L}{4\pi D^2} \Rightarrow \quad D = \sqrt{\frac{L}{4\pi S}}$$

Large D: $D \rightarrow d_L$

MEASURING DISTANCES

PARALLAX METHOD

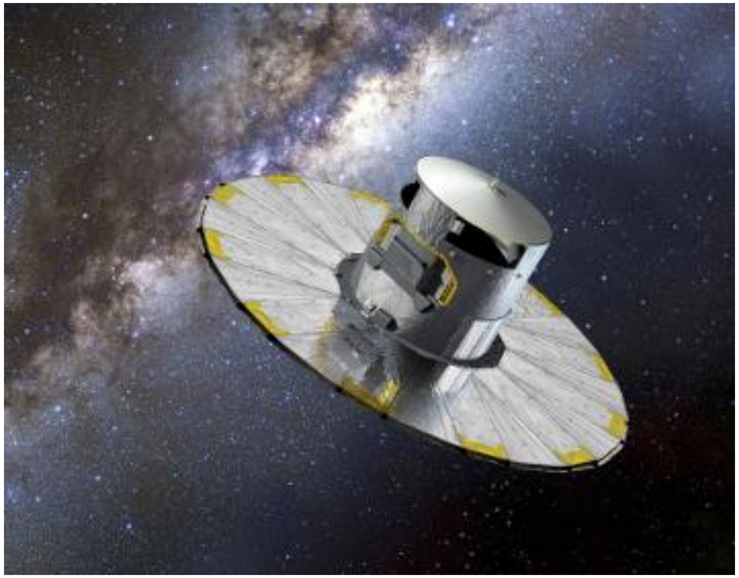
Parallax technique for measuring the distances to stars:

$$d\,[\mathrm{pc}] = \frac{1[\mathrm{AU}]}{\theta['']}$$

parallax in arc-seconds

The state of the art today is the ESA Gaia satellite (launched in 2013). Gaia is able to measure the parallax down to 10 micro-arc-seconds for bright stars.

$$d\,[\mathrm{pc}] = \frac{1[\mathrm{AU}]}{0.00001['']} \approx 100,000\,\mathrm{pc}$$



This covers about the entire Milky Way from the Earth.

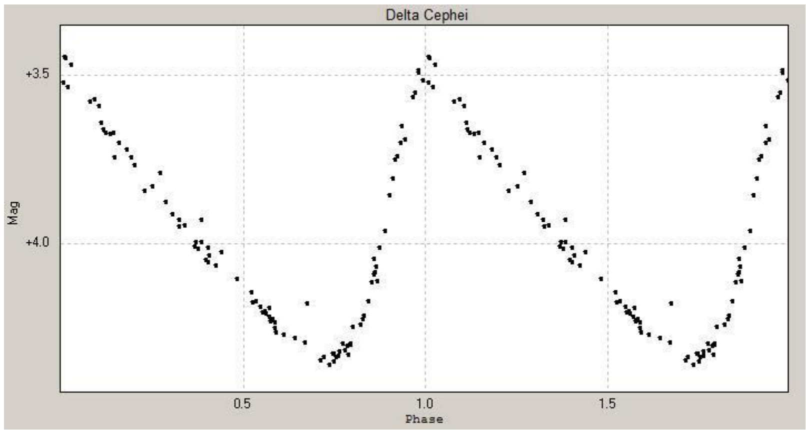
MEASURING DISTANCES

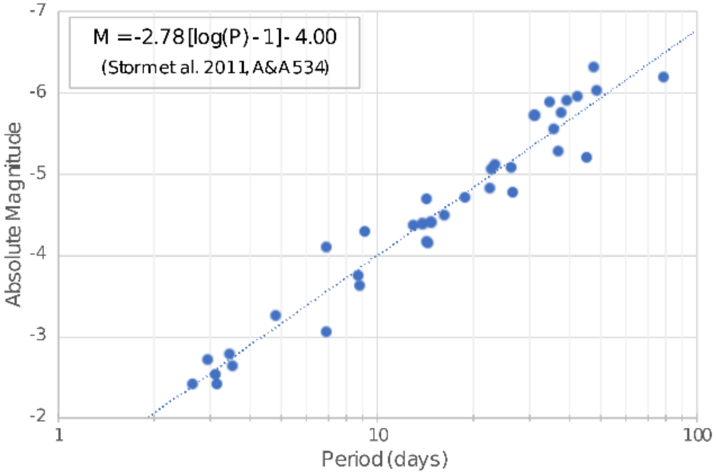
CEPHEID VARIABLE STANDARD CANDLE

Cepheid variables are a class of very luminous variable stars.

Henrietta Swan Leavitt, an American astronomer working at the turn of the 20th century, was studying variable stars in the Magellanic clouds and noticed that the brighter Cepheid variables had longer periods.

Period/luminosity relation (Leavitt law)





MEASURING DISTANCES

CEPHEID DISTANCE LADDER

Distances to galactic cepheids can be measured by parallax, hence calibrating the absolute luminosity from observations of the pulsation period and the flux.

Cepheids outside the galaxy can then be used to infer larger extra-galactic distances (assuming nearby cepheids in the Milky Way are similar to those elsewhere).

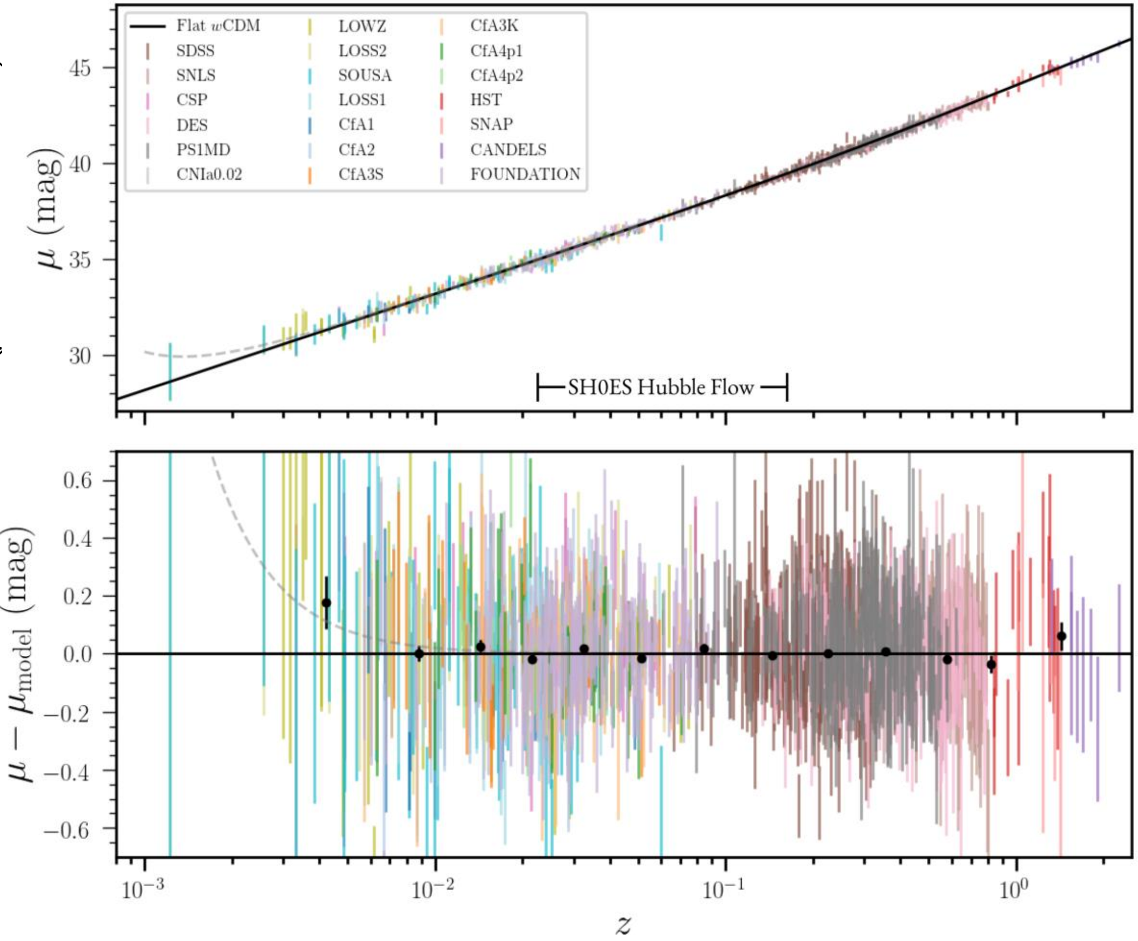
But need objects on cosmological distance (in the *Hubble flow*) to measure H_0

E.g. Supernova standard(izable) candles

measure L_{SN}/d_L^2- *but* intrinsic luminosity L_{SN} unknown
$\Rightarrow$ constrain *relative* redshift evolution very well, $d_L(z) \times \mathrm{const}$

Brout et al. Pantheon+, arXiv:2202.04077

(μ = distance modulus)

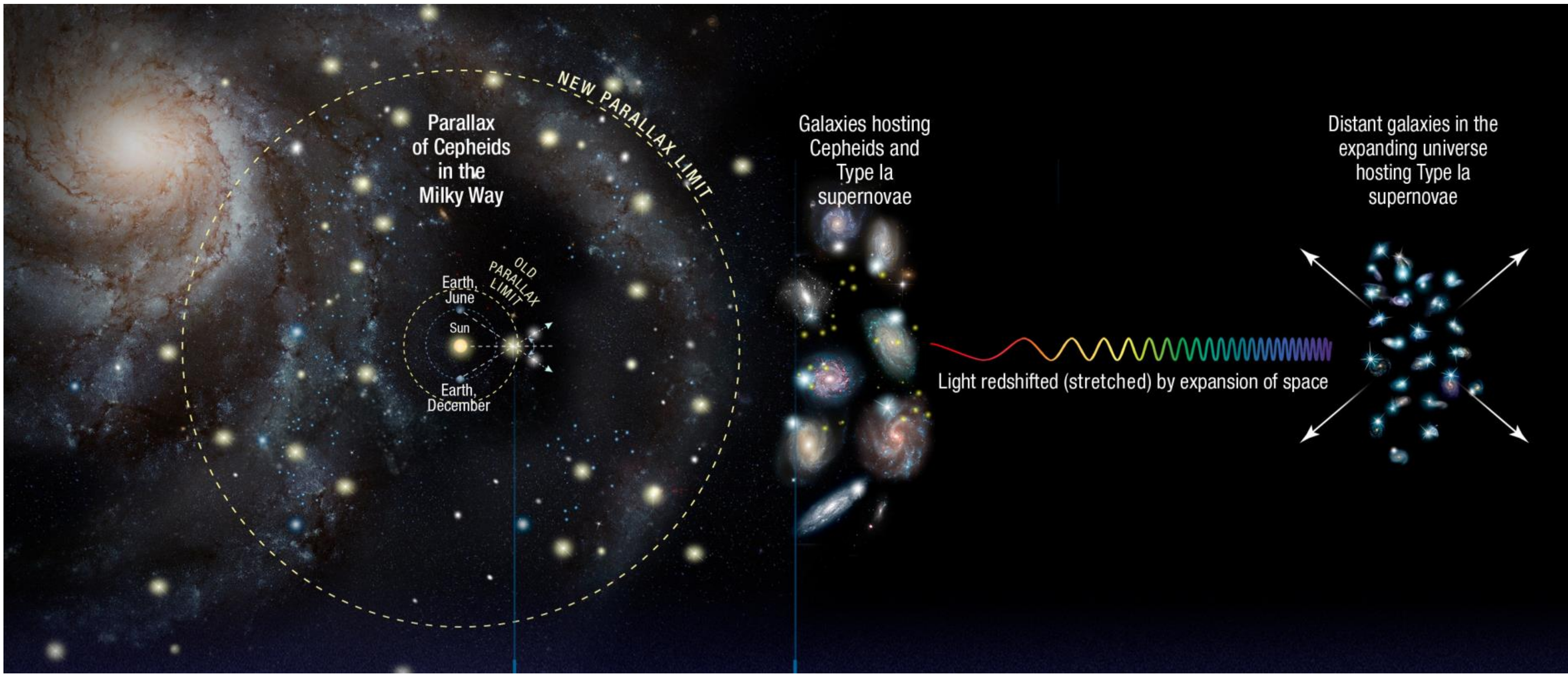


Unknown $L_{\mathrm{SN}} \Rightarrow$ no direct constraint on H_0

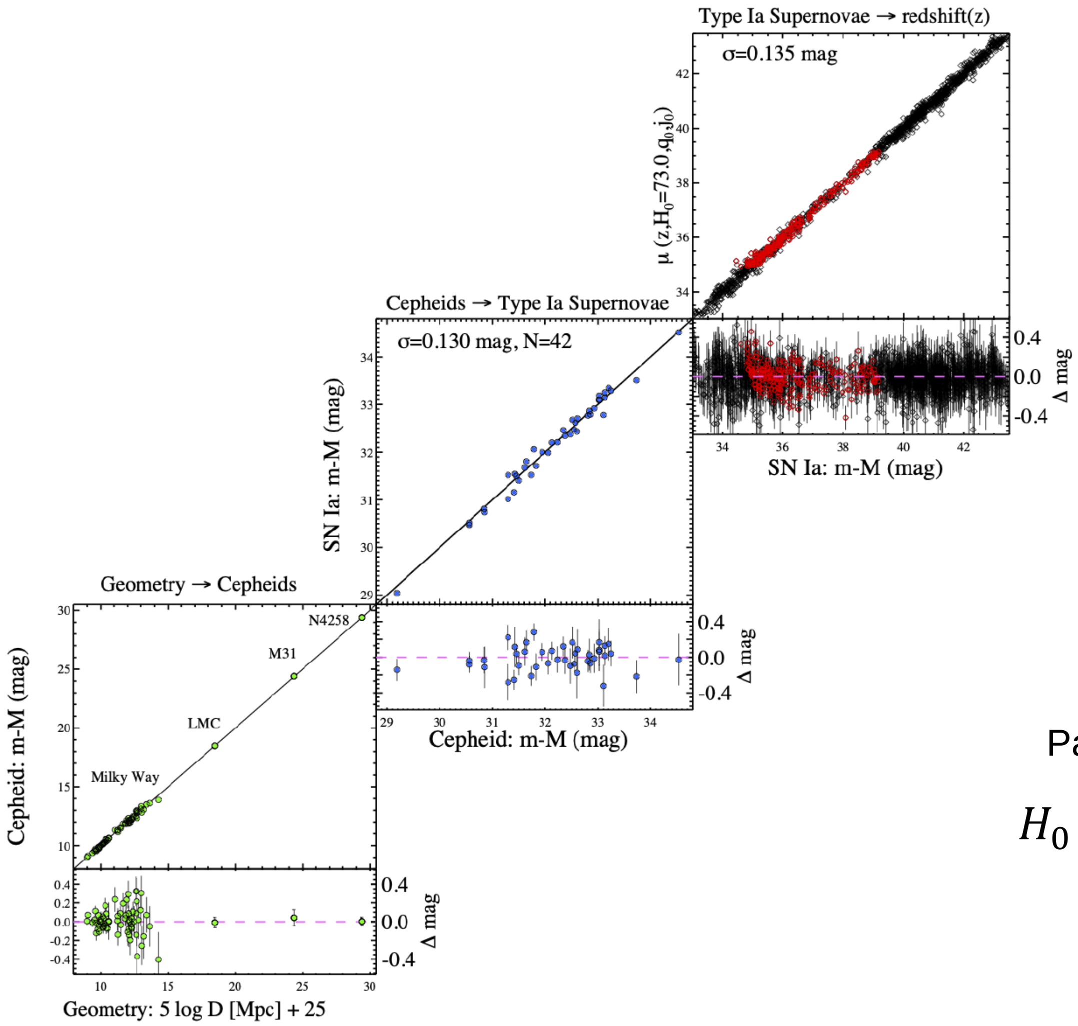
(can measure $\Omega_{\mathrm{m}}, \Omega_K$)

DISTANCE LADDER

LUMINOSITY CALIBRATION LADDER



https://www.spacetelescope.org/news/heic1611/



⇒ can measure H_0 well

Parallax+cepheids+SN (+ megamaser)

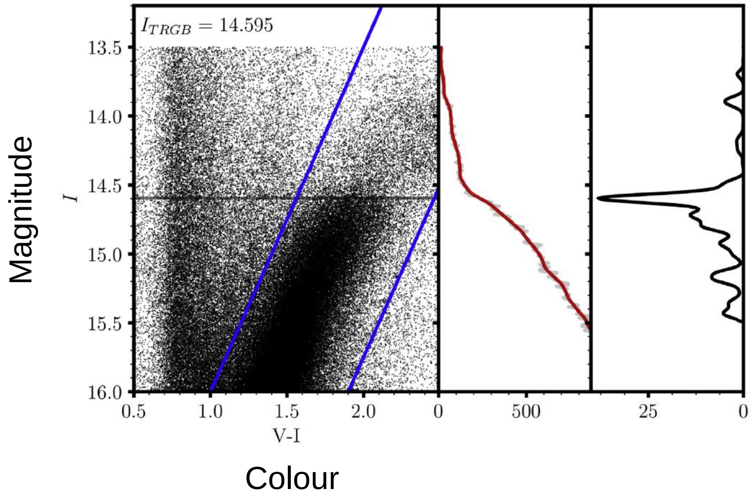
$$H_0 = (73.04 \pm 1.04)\ \mathrm{km\ s^{-1} Mpc^{-1}}$$

Riess et al. arXiv: 2112.04510

E.g. Other standardizable candles

Tip of the Red Giant branch (TRGB)

Freedman et al. arXiv:1907.05922



Also needs calibrating, no *direct* H_0 measurement, but can replace cepheids

$$H_0 = 69.8 \pm 1.9 \text{ km s}^{-1}\text{Mpc}^{-1}$$

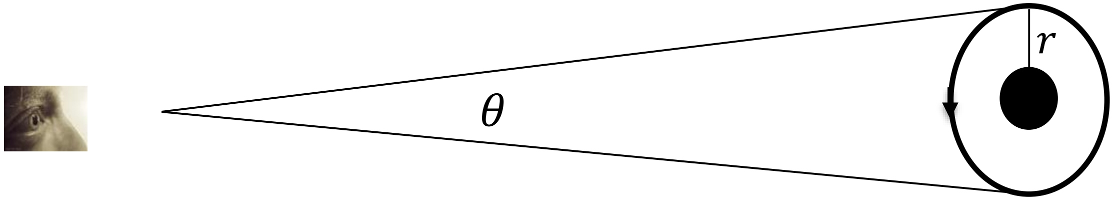
Freedman et al. arXiv:1907.05922, 2002.01550

$$H_0 = 73.2 \pm 2.1 \text{ km s}^{-1}\text{Mpc}^{-1}$$

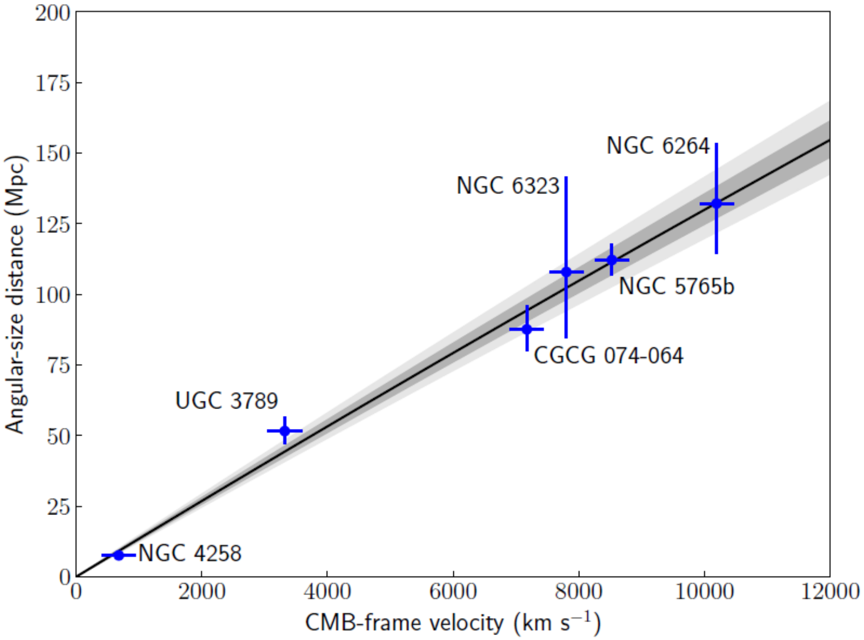
Scolnic et al. arXiv:2304.06693
(with Pantheon+ SN)

Alternative: standard ruler
E.g. Orbital standard ruler (megamaser)

NGC 4258



r inferred from fits to detailed observations of orbits



$$H_0 = (73.9 \pm 3.0)\ \mathrm{km}\ s^{-1}\ \mathrm{Mpc}^{-1}$$

Megamaser Cosmology Project
Pesce et al, arXiv: 2001.09213

Independent of ladder results

Other forward distance ladders

+ other several results using other local calibrators, all giving broadly consistent results

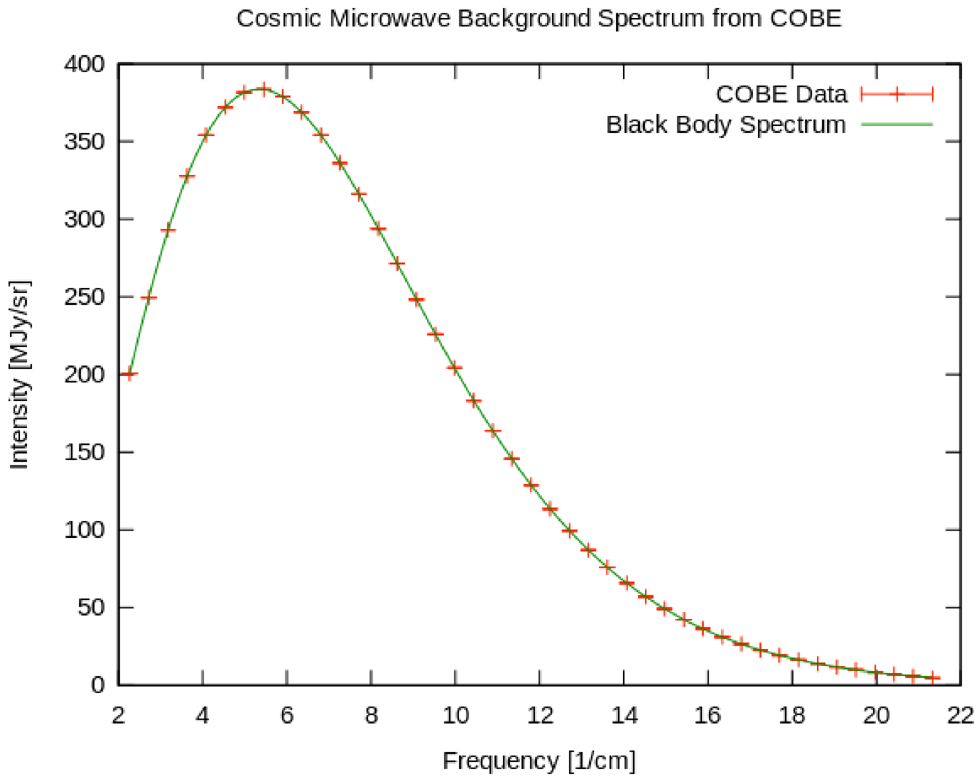
- Nearly independent of the cosmological model

Cosmology

“Inverse distance ladder”

Model early universe (e.g. early-LCDM)
– use observations to infer free parameters (e.g. ρ_R, ρ_b, ρ_c)
– use model to calculate standard ruler/candle (e.g. r_s)

CMB standard backlight



$$T_{\mathrm{CMB},0} = T_{\gamma,0} \approx (2.7255 \pm 0.0006)\ \mathrm{K}$$

[Fixsen et al]

$$\rho_\gamma \propto T_\gamma^4$$

$$\Rightarrow \rho_{\gamma,0} \Rightarrow \rho_\gamma = \frac{\rho_{\gamma,0}}{a^4}$$

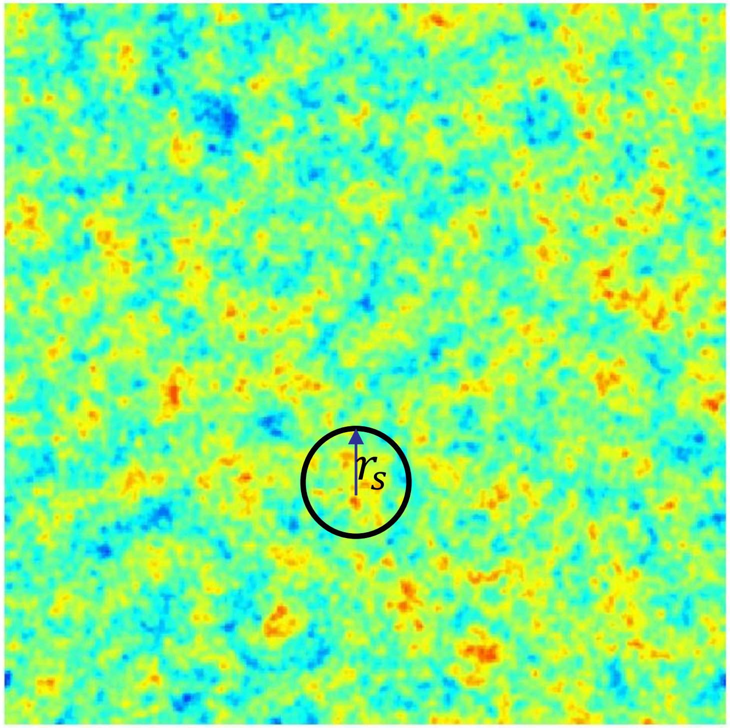
$$\Rightarrow \rho_\nu^{\text{massless}}$$

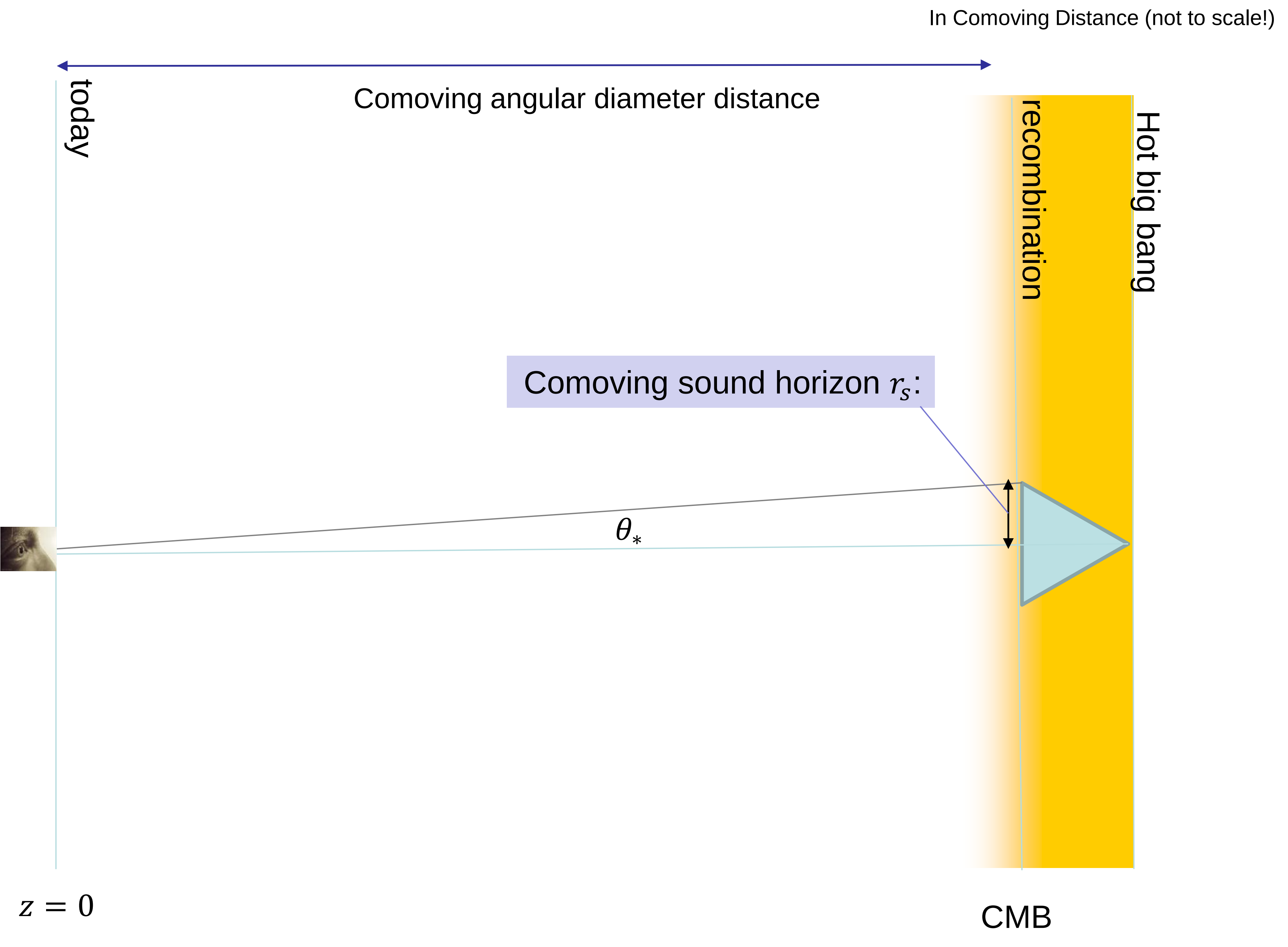
If we knew early-universe expansion rate, atomic physics $\Rightarrow$ T_*, z_*, η_* of recombination

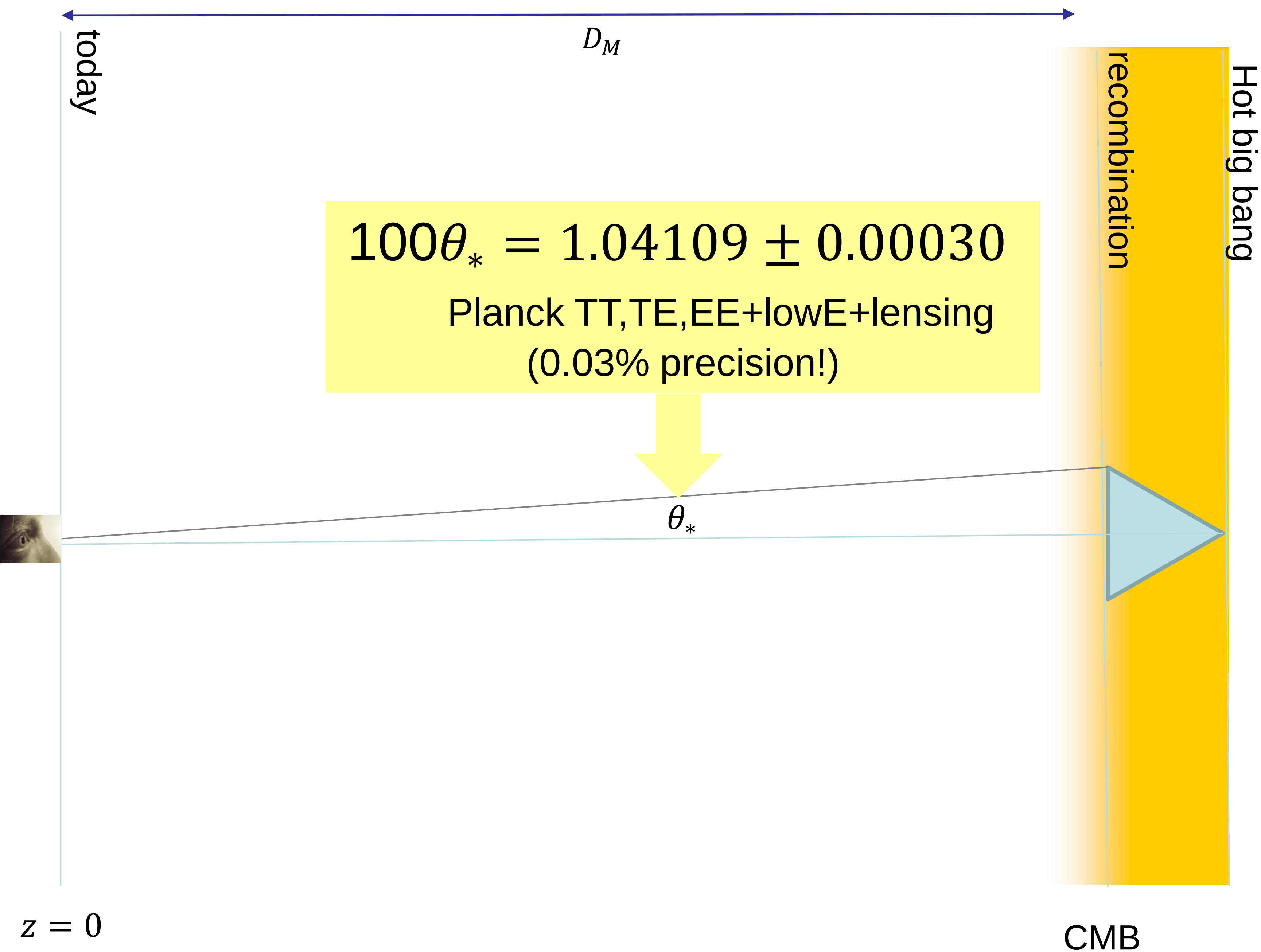
Know $\rho_\gamma(a), \rho_\nu(a)$, need to infer matter densities ρ_b, ρ_c.

CMB perturbations

CMB acoustic scale at last scattering surface



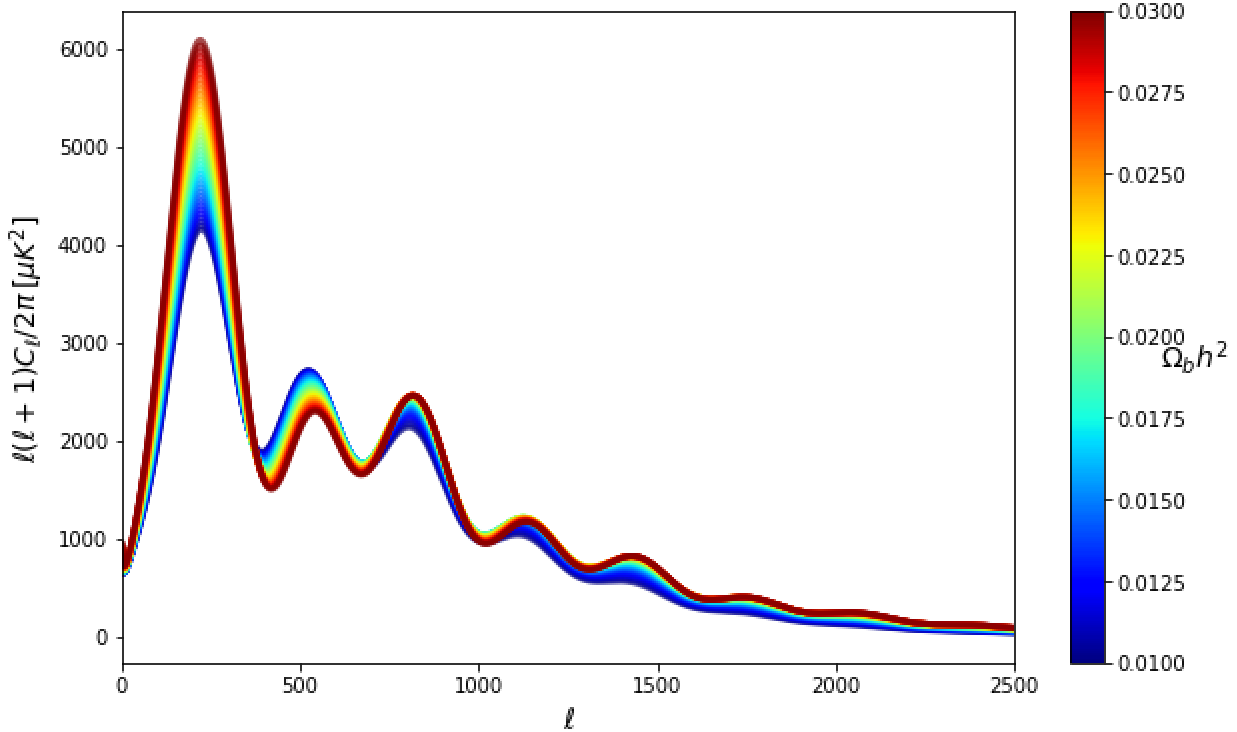

In Comoving Distance (not to scale!)
Comoving angular diameter distance
today
recombination
Hot big bang
Comoving sound horizon r_s:
θ_*
$z = 0$
CMB



D_M
today
recombination
Hot big bang
$100\theta_* = 1.04109 \pm 0.00030$
Planck TT,TE,EE+lowE+lensing
(0.03% precision!)
θ_*
$z = 0$
CMB

ΛCDM baryon density at fixed T_0 , θ_*, $\Omega_m h^2$

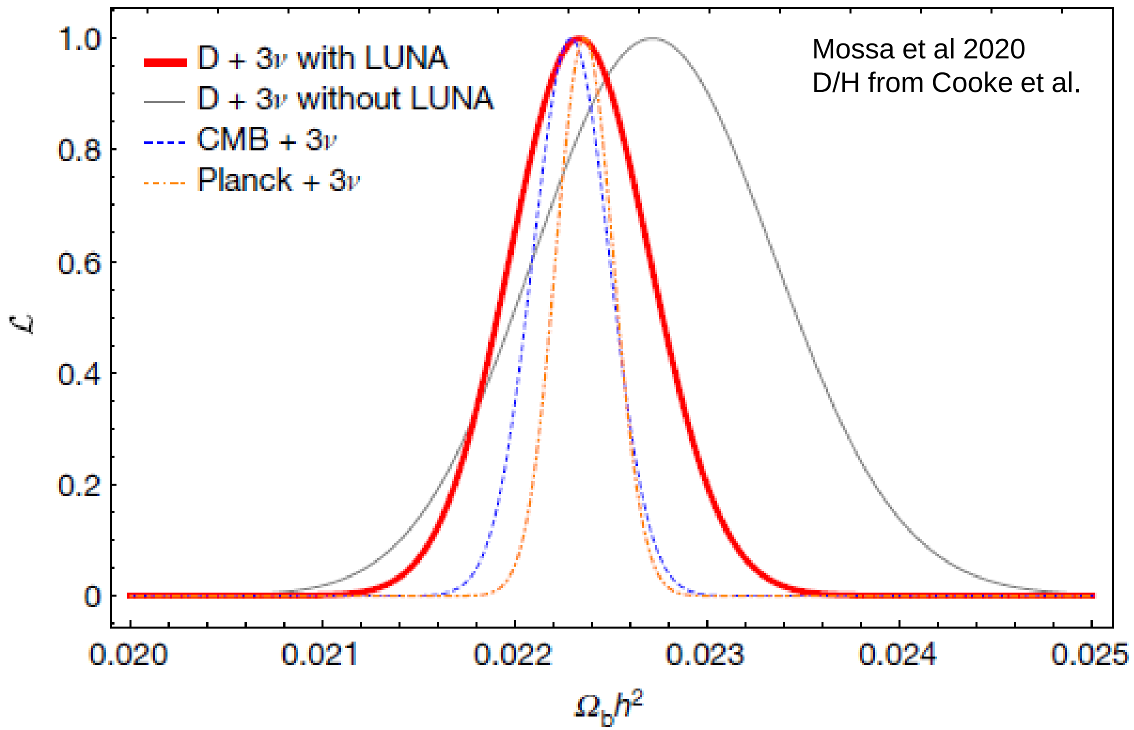
(baryons deepen overdensity compressions: enhance odd peaks of spectrum)



Odd/even height ratio distinctive and quite robust:

$$\Omega_b h^2 = 0.0224 \pm 0.0002$$

Consistency with standard Big-Bang Nucleosynthesis



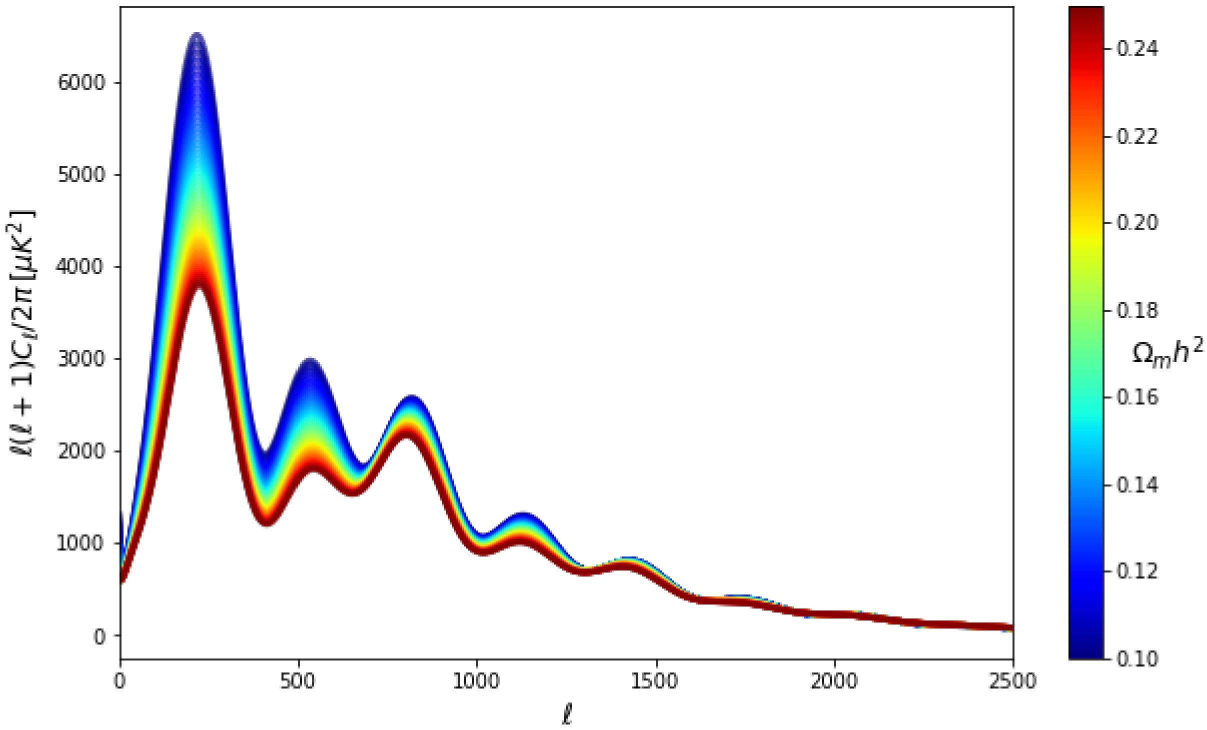
BUT: Lithium problem remains around 5σ

Measured: $^7\mathrm{Li/H} = (1.58 \pm 0.35) \times 10^{-10}$ arXiv: 1505.01076

Prediction: $^7\mathrm{Li/H} = 4.5 \times 10^{-10}$

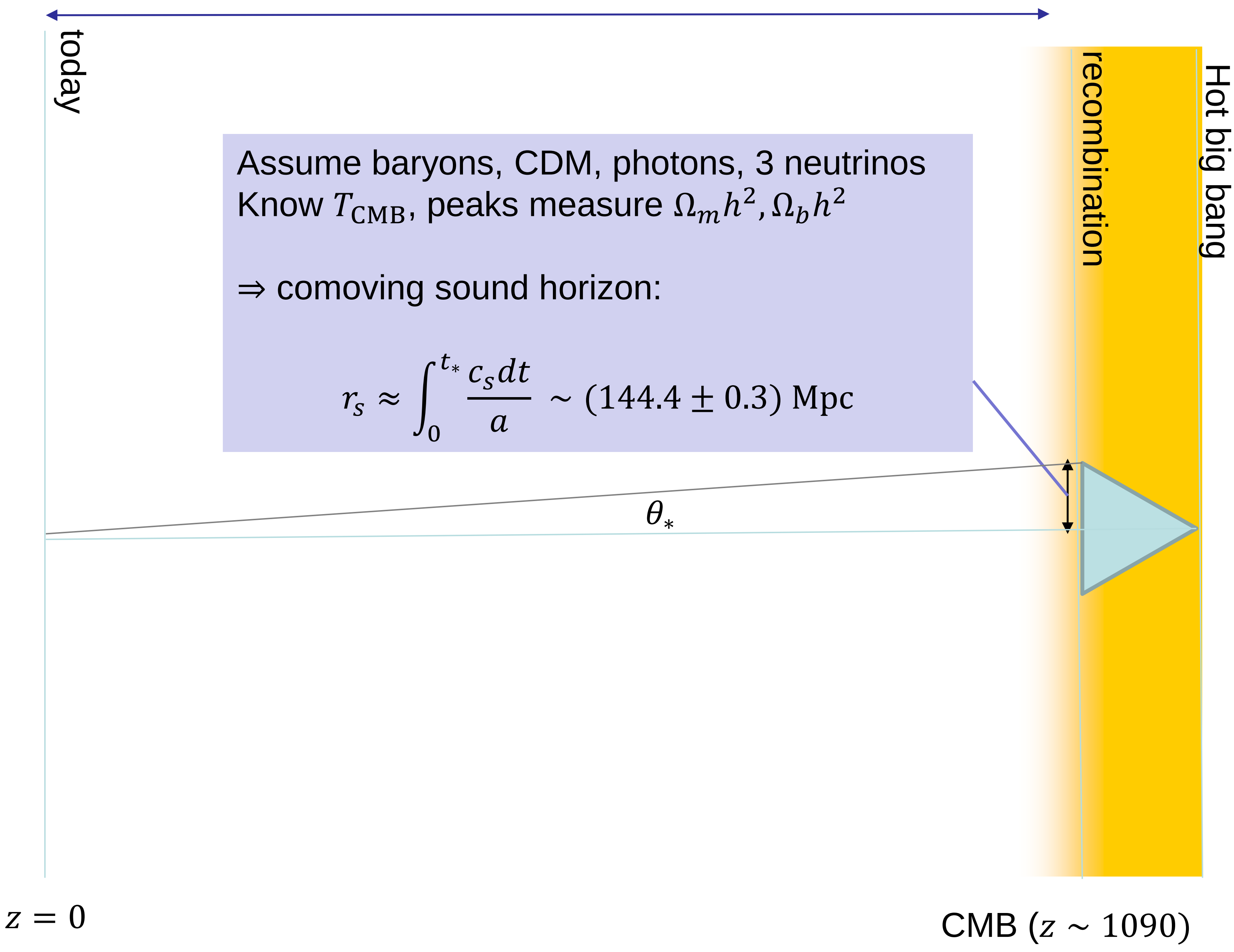
ΛCDM matter density at fixed T_0, θ_*, $\Omega_b h^2$

(more matter *lowers* amplitude for modes that enter horizon in matter domination)



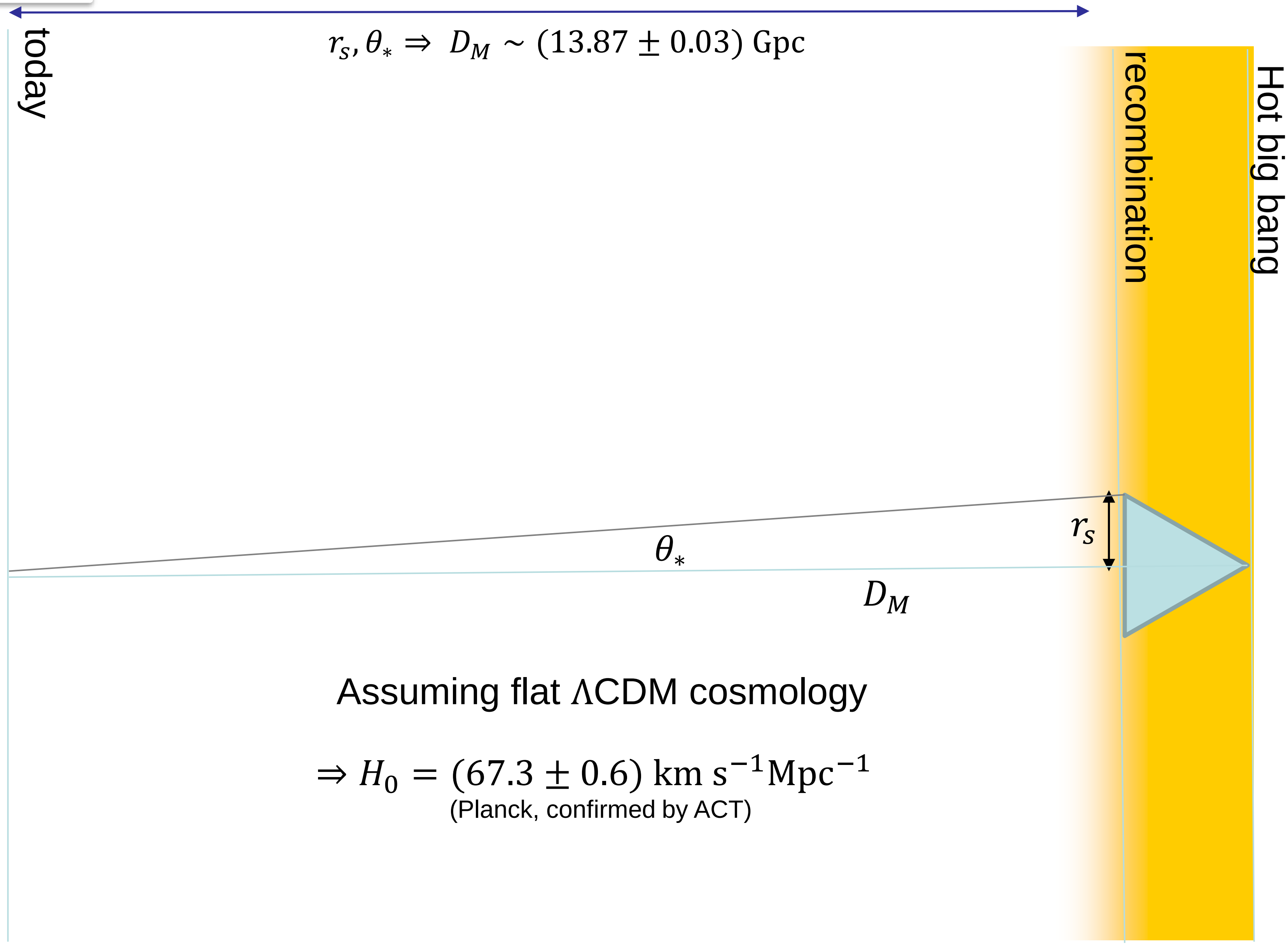
Can be partly compensated by changing initial power A_s, n_s and foregrounds.
But detailed shape is still quite distinctive and robust:

$$\Omega_m h^2 = 0.143 \pm 0.001$$


today
recombination
Hot big bang
Assume baryons, CDM, photons, 3 neutrinos
Know T_{CMB}, peaks measure $\Omega_m h^2, \Omega_b h^2$
⇒ comoving sound horizon:
$r_s \approx \int_0^{t_*} \frac{c_s dt}{a} \sim (144.4 \pm 0.3)\ \mathrm{Mpc}$
θ_*
$z = 0$
CMB ($z \sim 1090$)

CMB

$r_s, \theta_* \Rightarrow \; D_M \sim (13.87 \pm 0.03)\ \mathrm{Gpc}$



Assuming flat ΛCDM cosmology

$$\Rightarrow H_0 = (67.3 \pm 0.6)\ \mathrm{km\ s^{-1} Mpc^{-1}}$$

(Planck, confirmed by ACT)

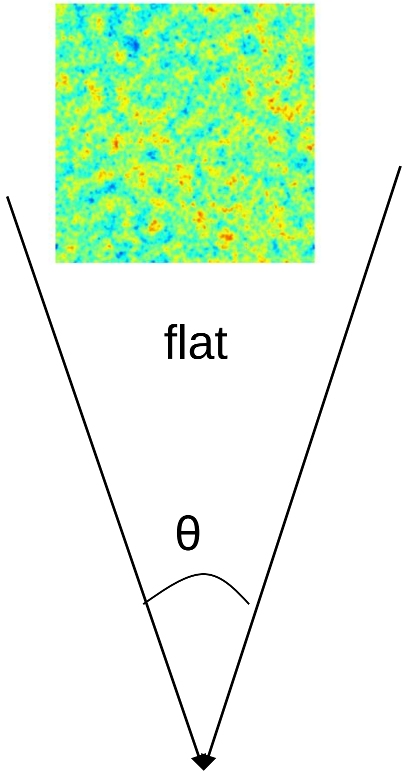
HUBBLE PARAMETER RESULTS

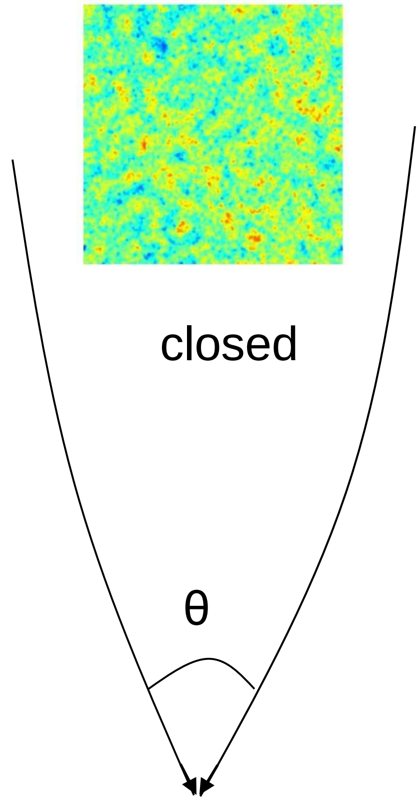
Recent measurement from standard candle distance ladder:
Parallax+cepheids+SN (+ megamaser)

$$H_0 = (73 \pm 1)\ \mathrm{km\ s^{-1} Mpc^{-1}}$$

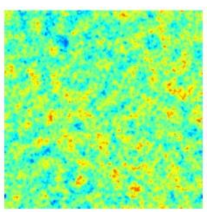
Riess et al. 2022
https://arxiv.org/abs/2112.04510

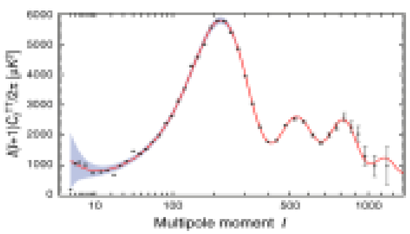
Inconsistent with CMB inverse distance ladder ***in ΛCDM*** e.g. Geometry: curvature

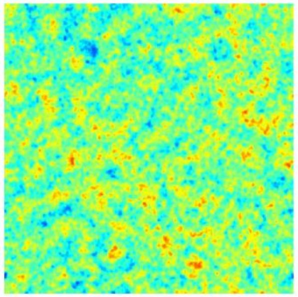


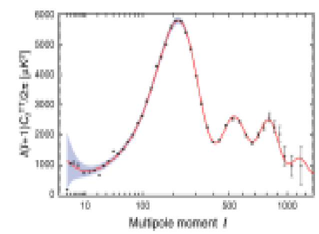


We see:

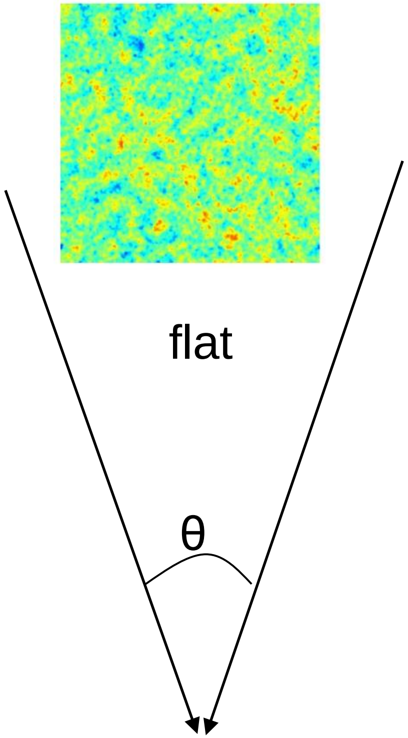


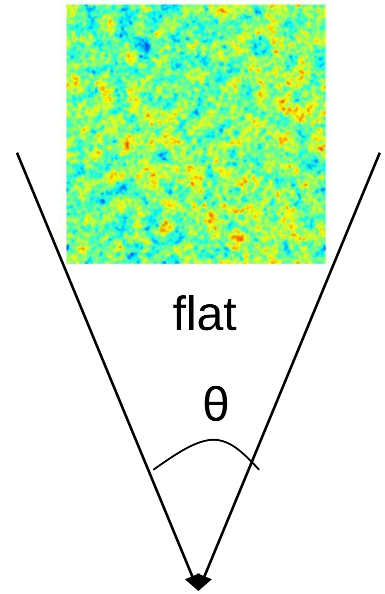




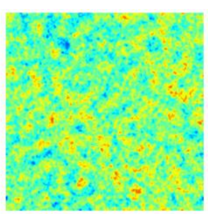


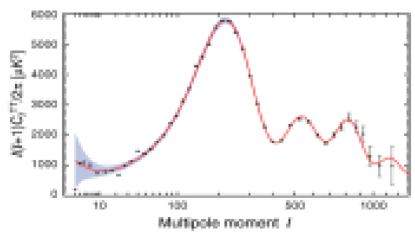
or is it just closer??

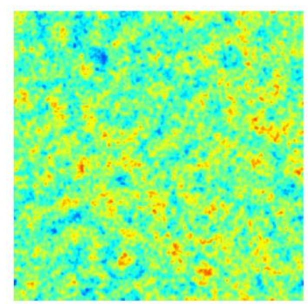


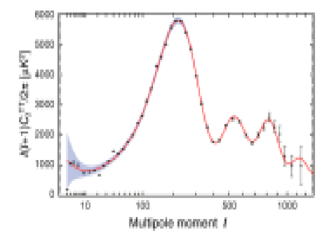


We see:

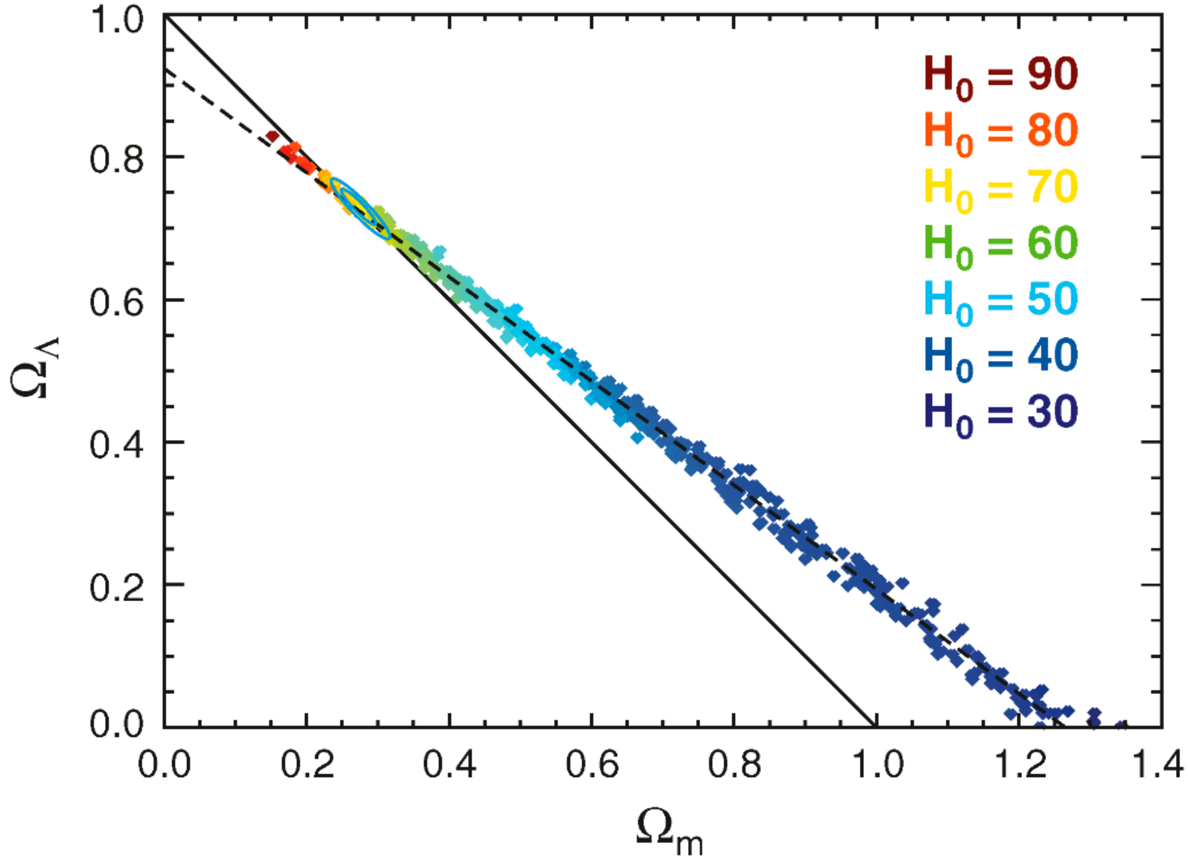








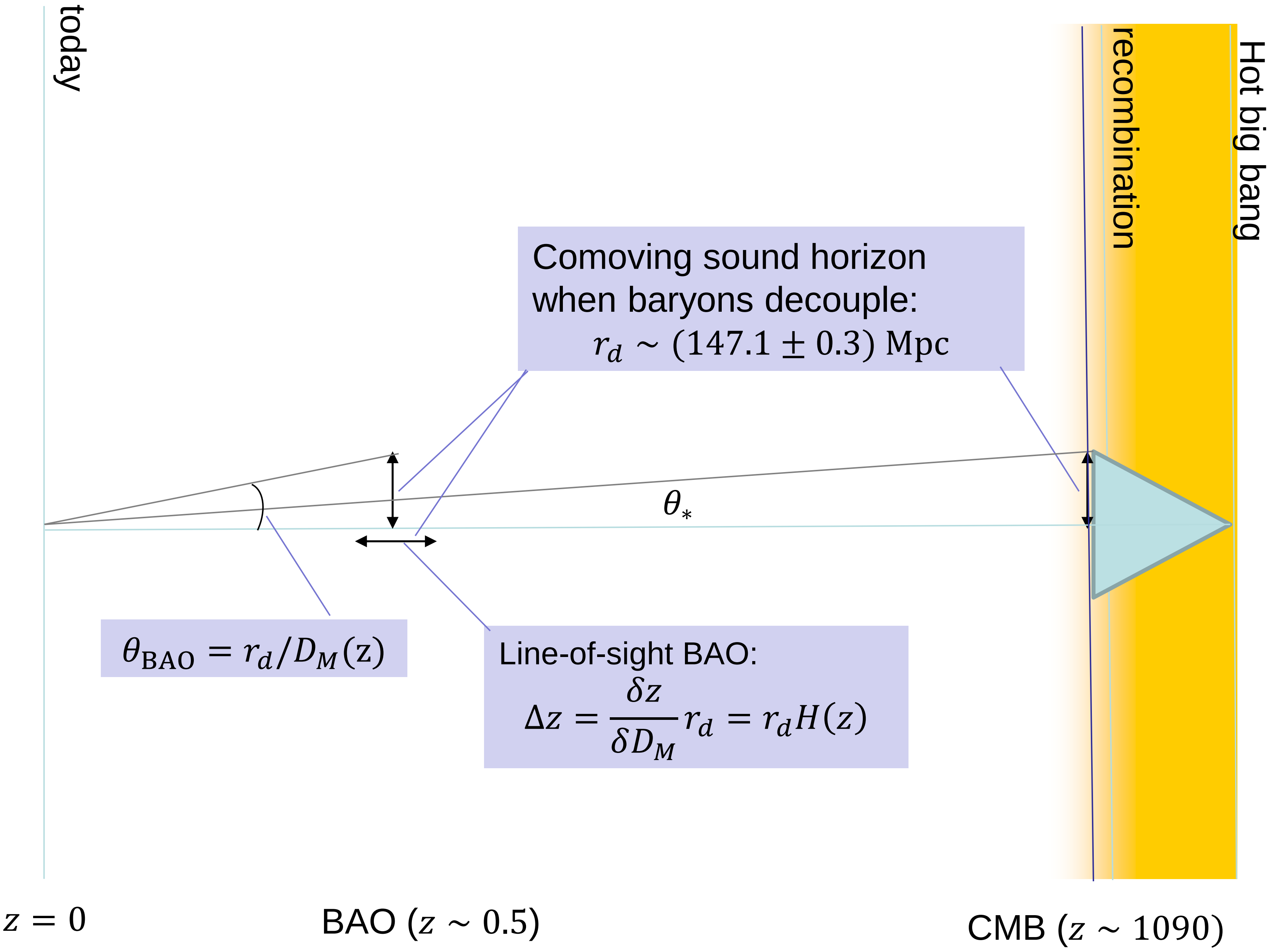
Angular diameter distance degeneracy between parameters
(limitation of small-scale CMB being from a single source plane)

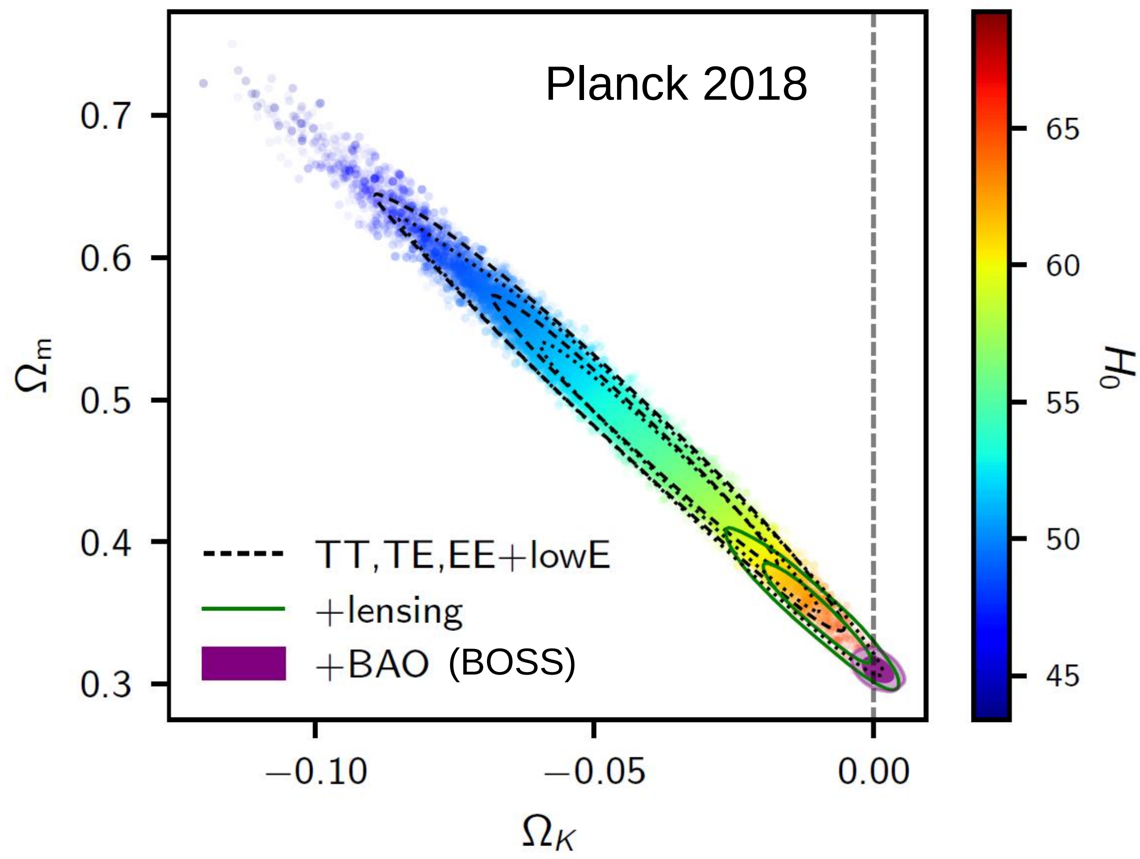


WMAP 7

Need other information to break remaining degeneracies

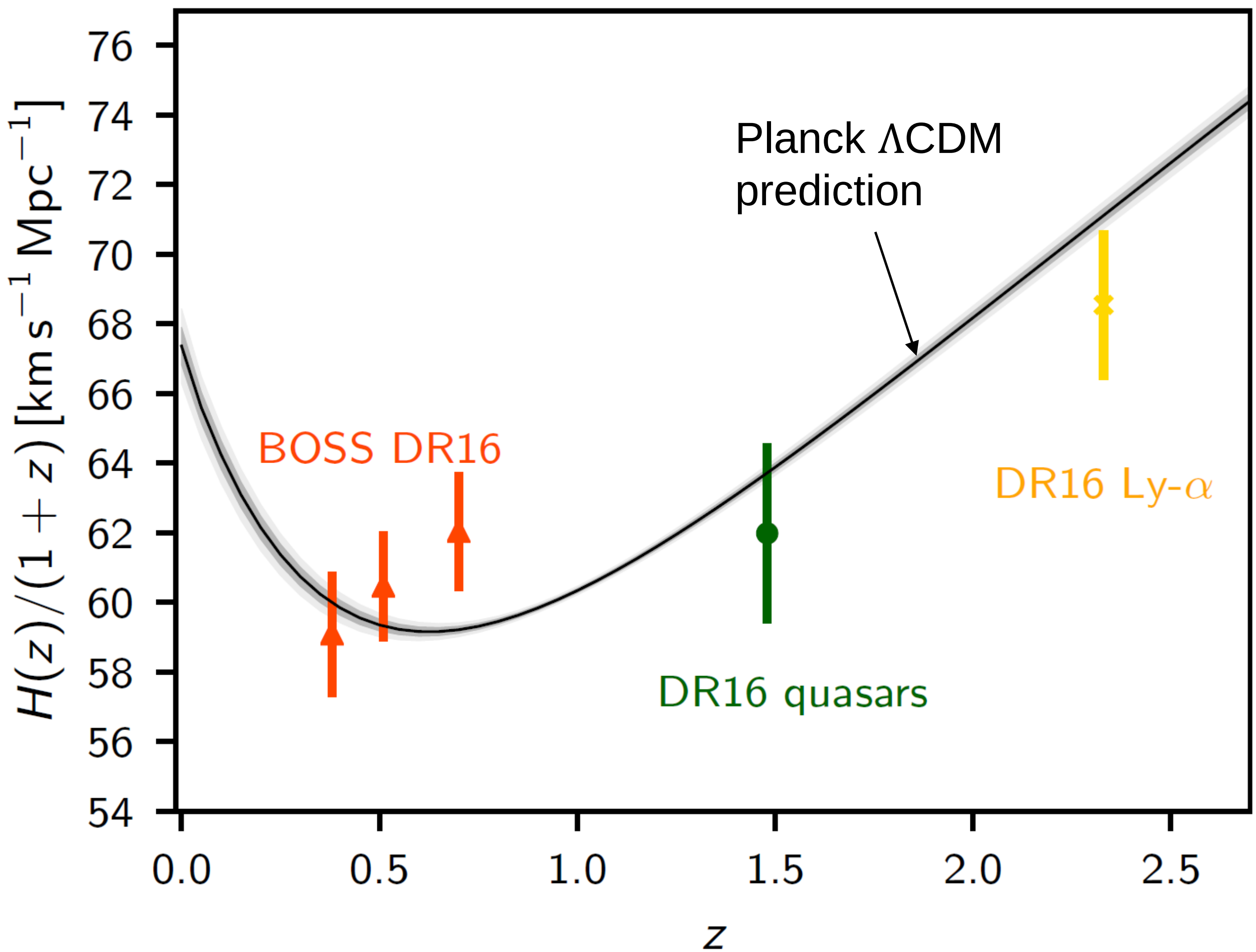
CMB and BAO consistency in ΛCDM





Planck 2018
0.7
0.6
0.5
0.4
0.3
Ω_m
−0.10
−0.05
0.00
Ω_K
TT,TE,EE+lowE
+lensing
+BAO (BOSS)
65
60
55
50
45
H_0

Assuming ΛCDM and Planck sound horizon r_d



Boss DR16 from
arXiv: 2007.08991

CMB Lensing Reconstruction – Quadratic Estimators

- Fixed lenses introduce statistically-anisotropic correlations:

$$\Delta\langle X_{l_1m_1}Y_{l_2m_2}\rangle_{\mathrm{CMB}} = \sum_{LM}(-1)^M \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix} \mathcal{W}^{XY}_{l_1l_2L}\phi_{LM}$$

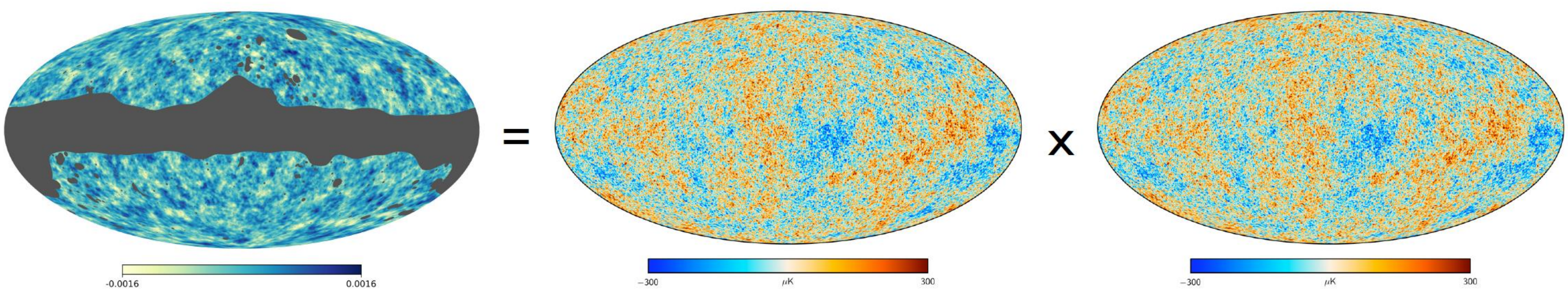
- Noisy lensing estimates from quadratic CMB combinations:

$$\hat{\phi}_{LM} = \frac{(-1)^M}{2}\frac{1}{\mathcal{R}^{XY}_L}\sum_{l_1m_1,l_2m_2}\begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix}\left[\mathcal{W}^{XY}_{l_1l_2L}\right]^*\bar{X}_{l_1m_1}\bar{Y}_{l_2m_2}$$

Normalisation

Known lensing-induced correlations

Inverse-variance-weighted CMB fields

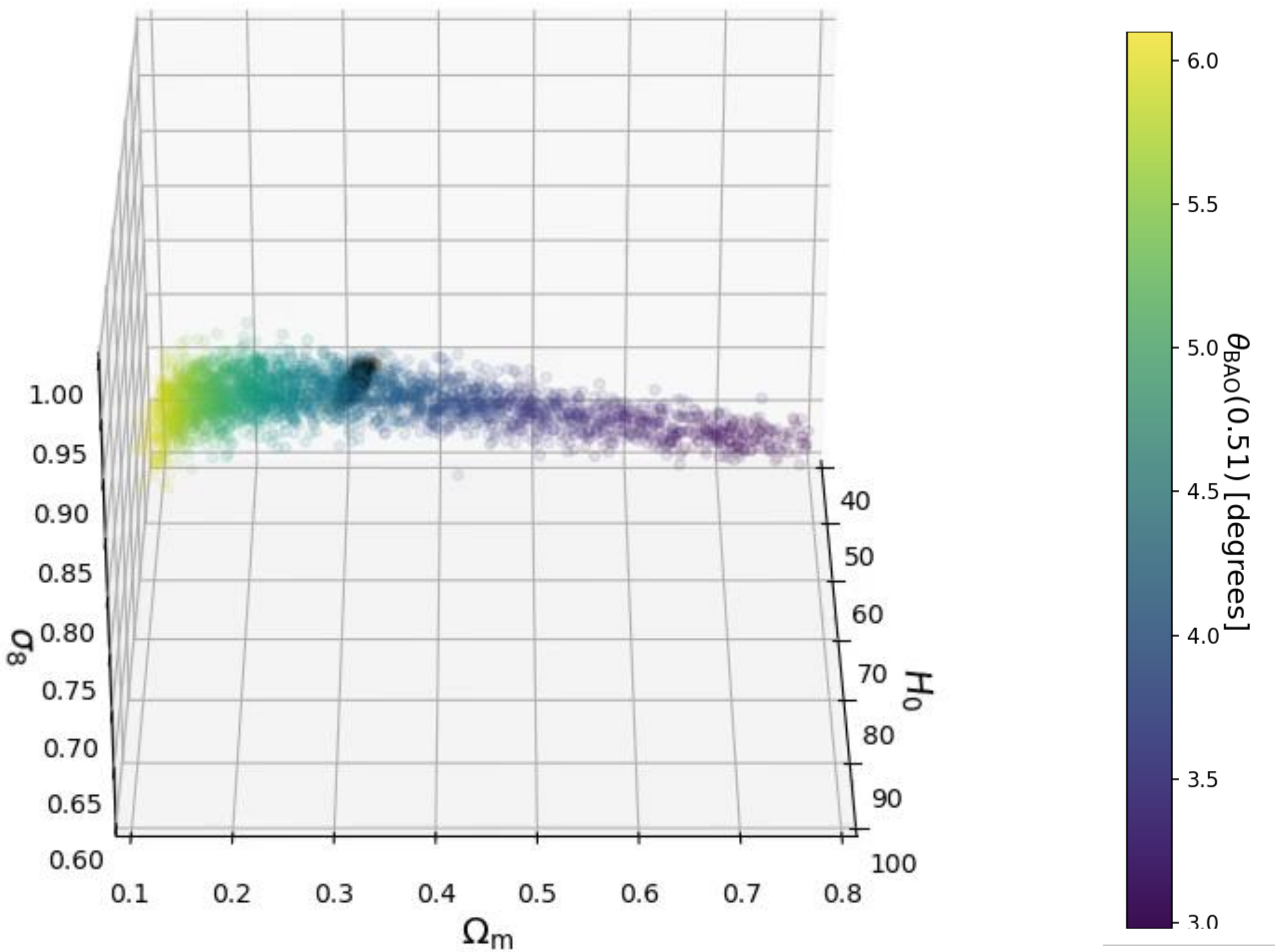


Credit: Anthony Challinor

Planck 2018 CMB lensing ΛCDM parameters

2018 lensing only

2018 TTTEEE+lowE

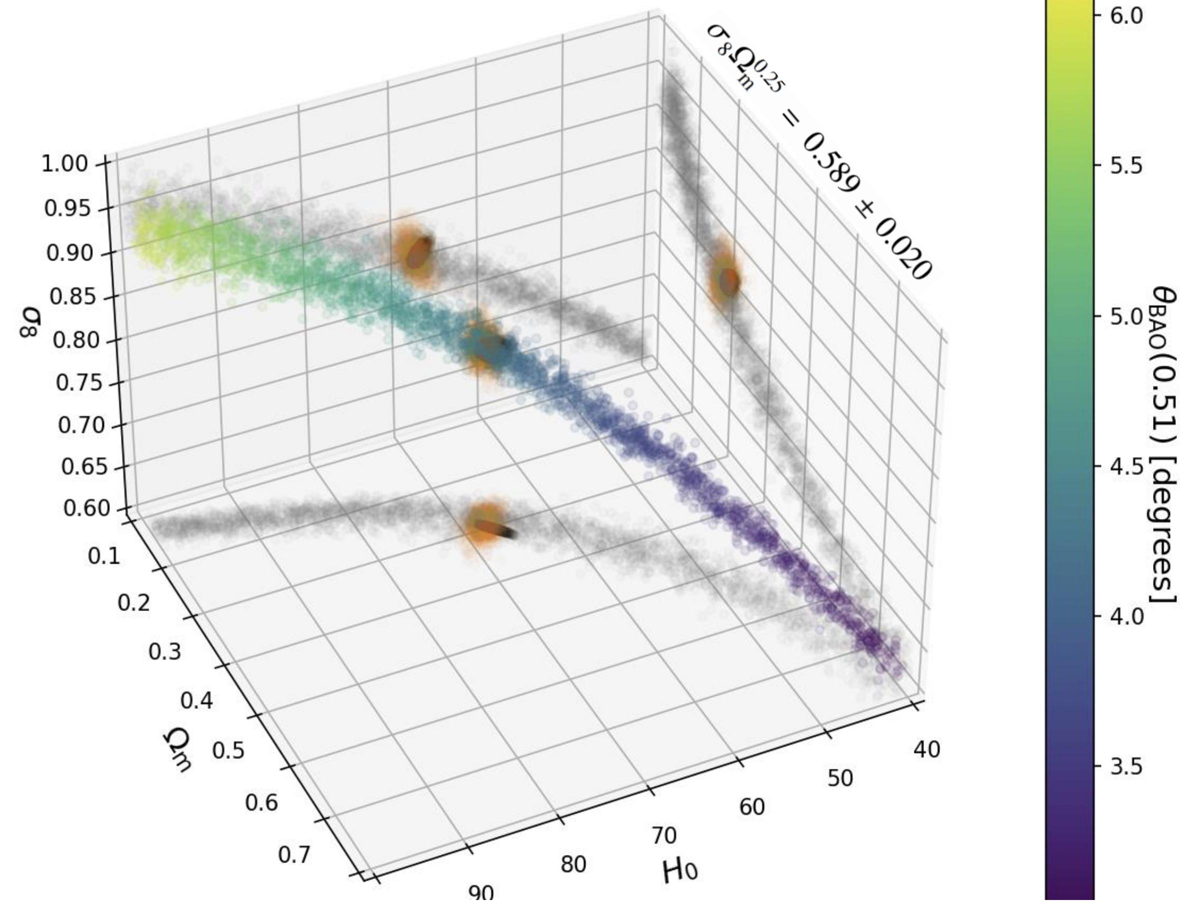


$$\theta_\mathrm{BAO}(0.51) \equiv r_s/D_\mathrm{M}(z=0.51)$$

Planck 2018 CMB lensing ΛCDM parameters

("Lensing-only" priors: $\Omega_b h^2 = 0.0222 \pm 0.0005, n_s = 0.96 \pm 0.02, 0.4 < h < 1$)

Planck lensing 2018
+ BOSS BAO ($+\Omega_b h^2$ BBN)
Planck 2018 TTTEEE



CMB lensing + BAO inverse distance ladder (with $\Omega_b h^2$ prior from abundance measurements)

$$\left.\begin{aligned} H_0 &= 67.9^{+1.2}_{-1.3}\ \mathrm{km\ s^{-1}Mpc^{-1}}, \\ \sigma_8 &= 0.811 \pm 0.019, \\ \Omega_m &= 0.303^{+0.016}_{-0.018}, \end{aligned}\right\} \quad 68\,\%, \text{ lensing+BAO}$$

arXiv:1807.06210

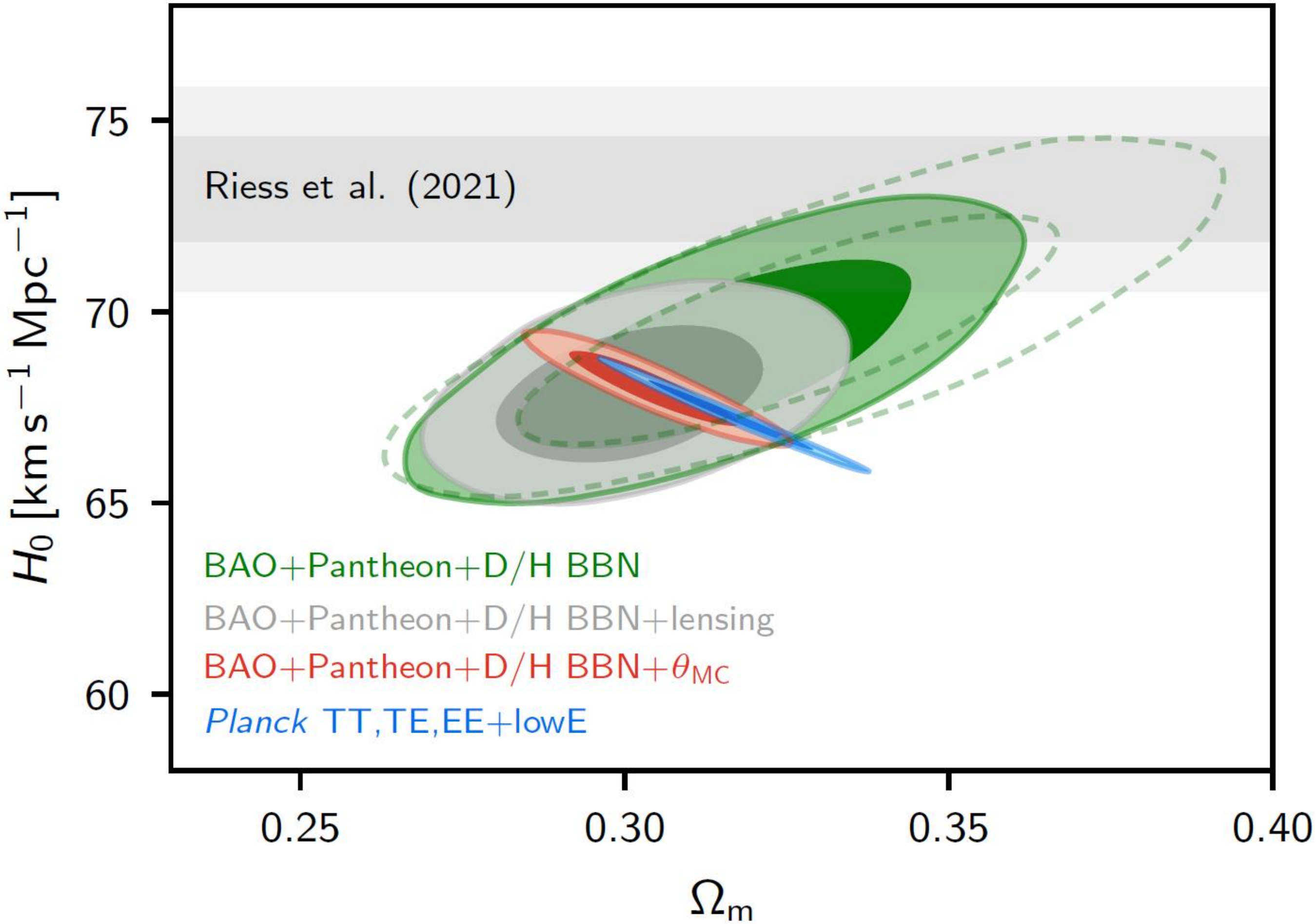
ACT+Planck CMB lensing+BAO

$$H_0 = 68.1 \pm 1.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}.$$

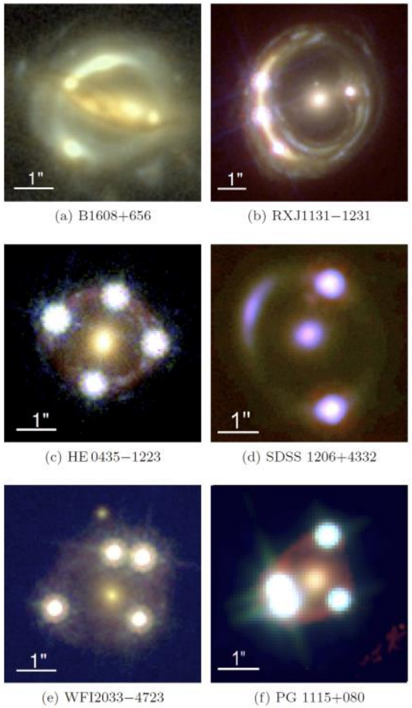
arXiv:2304.05203

Independent ΛCDM inverse distance ladder is also consistent with Planck

Element abundance (D/H) measurements + BBN ⇒ $\Omega_b h^2$
Supernovae (or other data) ⇒ Ω_m
$\Omega_b h^2 + \Omega_m \Rightarrow r_s$ comoving standard ruler assuming ΛCDM

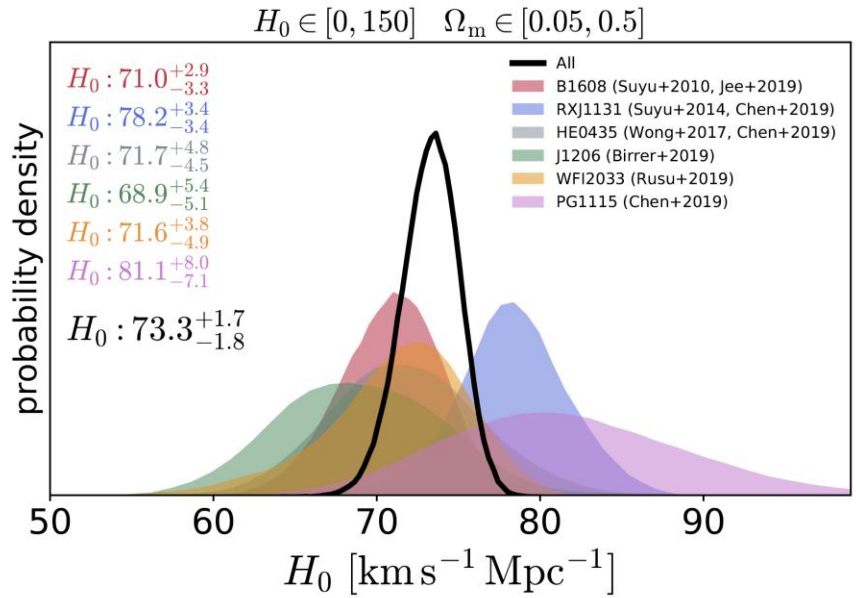


Strong Lensing



Lens modelling etc..

$$D_{\Delta t} \equiv (1+z_{\rm d})\frac{D_{\rm d}D_{\rm s}}{D_{\rm ds}}$$



H0LiCOW: $H_0 = 73.3^{+1.7}_{-1.8}\,\mathrm{km\ s^{-1}Mpc^{-1}}$ Wong et al. arXiv:1907.04869

(some cosmology dependence)

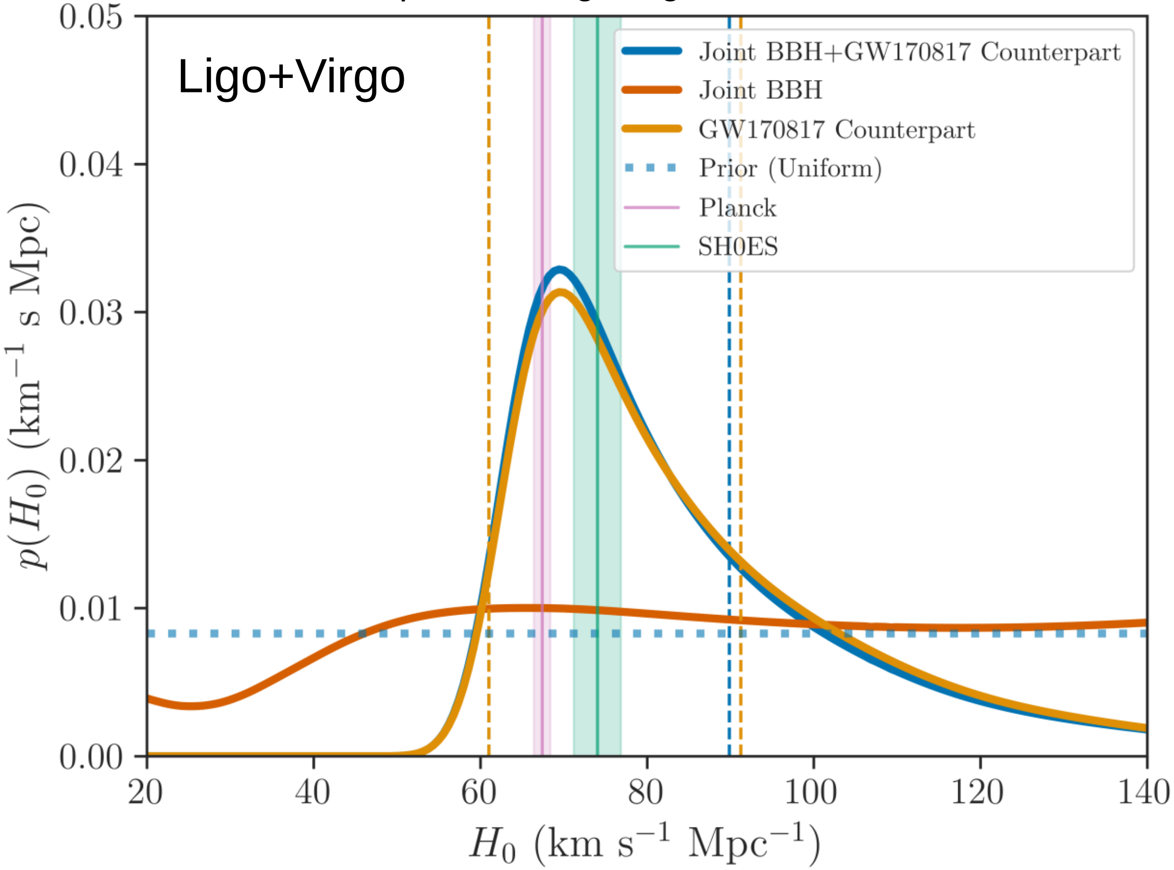
Independent of CMB and local distance ladder and mostly redshift $z >\sim 0.1$

⇒ tension with CMB independent of very local environment

TDCOSMO+SLACS: $H_0 = 67.4^{+4.1}_{-3.2}\,\mathrm{km\ s^{-1}Mpc^{-1}}$ Birrer et al. arXiv: 2007.02941

Gravitational Waves

https://www.ligo.org/science/Publication-O2H0/



Need much larger sample

Possible solutions to the H_0 tension in $\Lambda\mathbf{CDM}$

Biases in data or underestimated error bars
- inverse distance ladder: BAO and CMB consistent $\Rightarrow$ *both* CMB and BAO being wrong?
- Local H_0 and strong lensing independent; multiple local distance ladders agree
  *T*RGB results and strong lensing modelling dependent

New physics prior to recombination:
- decrease sound horizon r_d: BAO and Planck H_0 both shift proportionately
- other changes that affect relevant inferred parameters (e.g. $\Omega_m h^2$)

New physics at lower redshift/dark energy/modified gravity
- fitting BAO and $H(z)/H_0$ from supernovae leaves little wiggle room
  (or find problem with supernovae)

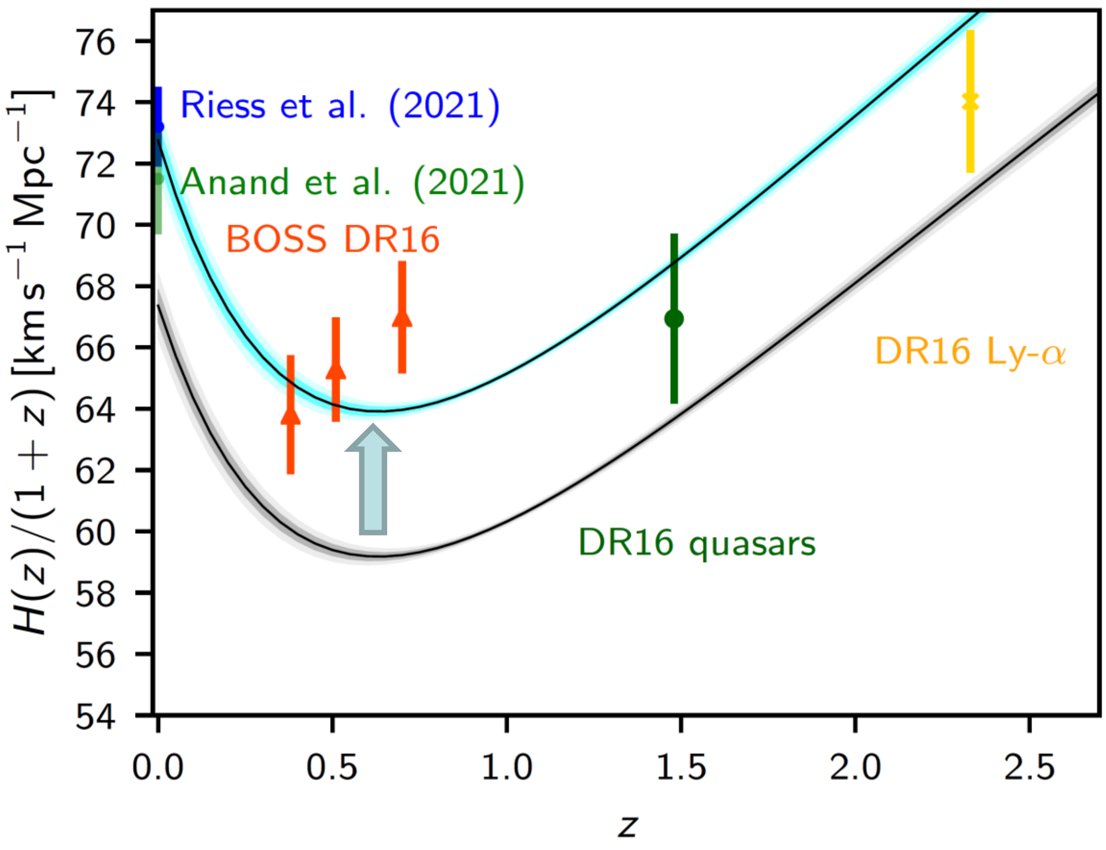
New physics/very unusual conditions in our local neighbourhood
- strong lensing results then in tension?

Largish statistical fluctuation

Some combination of the above

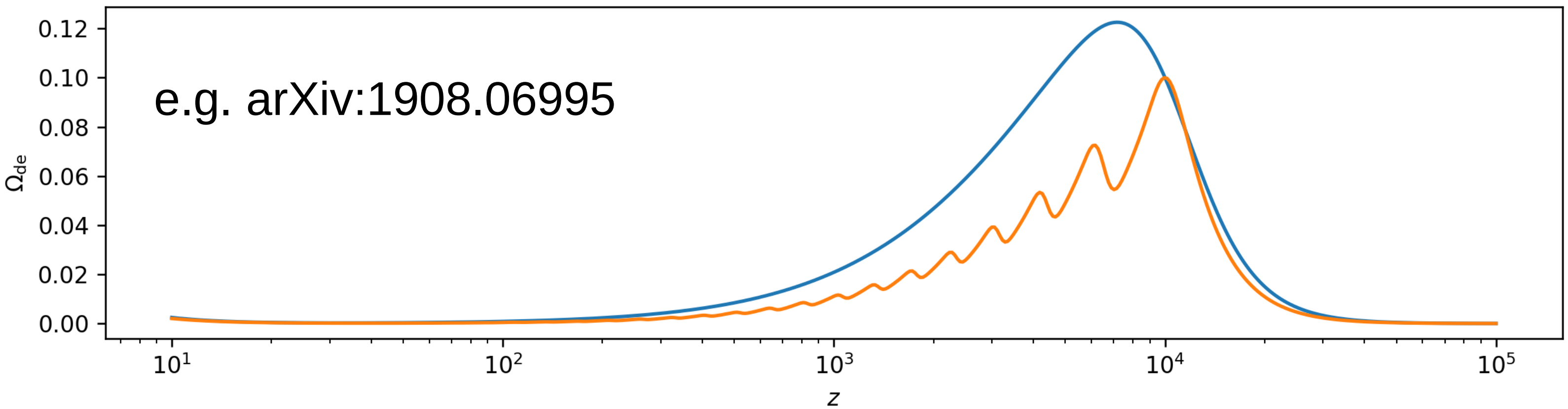
New early universe physics – decrease sound horizon r_d by $\mathcal{O}(10\%)$

e.g. increase expansion rate, decrease sound speed, shift recombination, ..

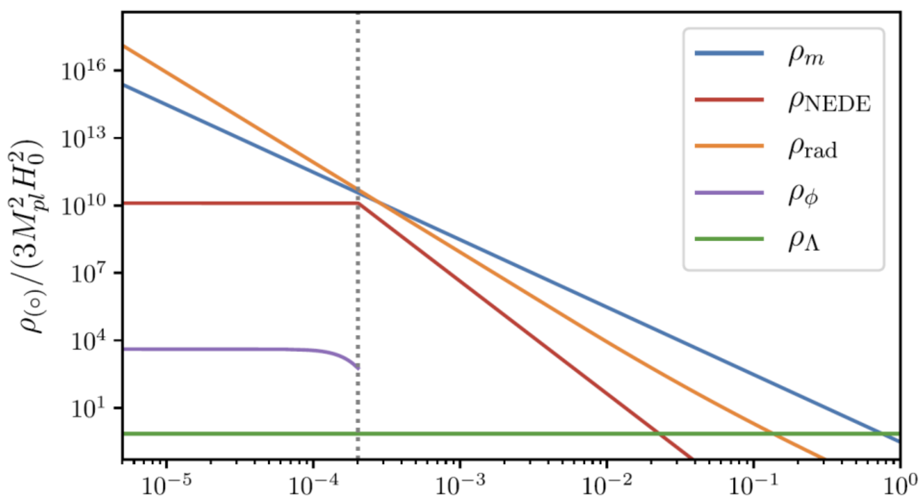


But, simple models e.g. extra relativistic degrees of freedom ($N_{\mathrm{eff}} \neq 3.044$) not favoured by Planck spectra (and disfavoured by BBN D/H)

More complex models possible, e.g. Poulin et al. early dark energy



Or New Early Dark Energy



[arXiv:2006.06686]

Has to peak around 10%, decay rapidly not to mess up C_l

But fine tuned and makes fit to large-scale structure data worse…

22°57'31" SOUTH, 67°47'15" WEST / ELEVATION: 5190 m

SIMONS OBSERVATORY

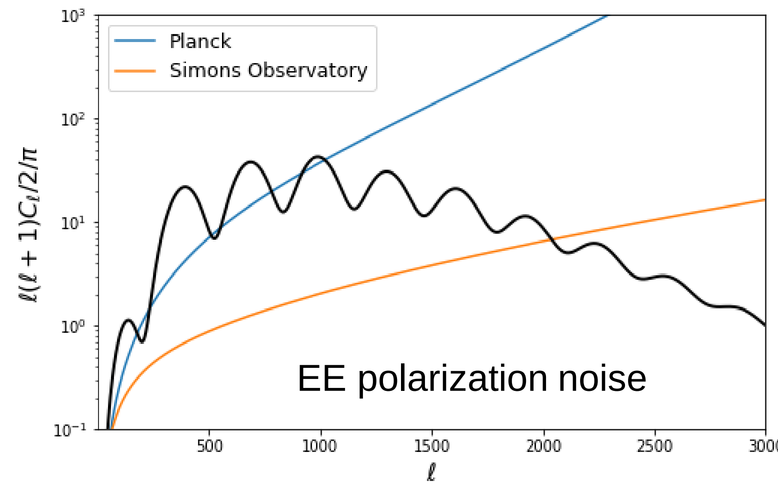
PARQUE ASTRONÓMICO ATACAMA, CHILE

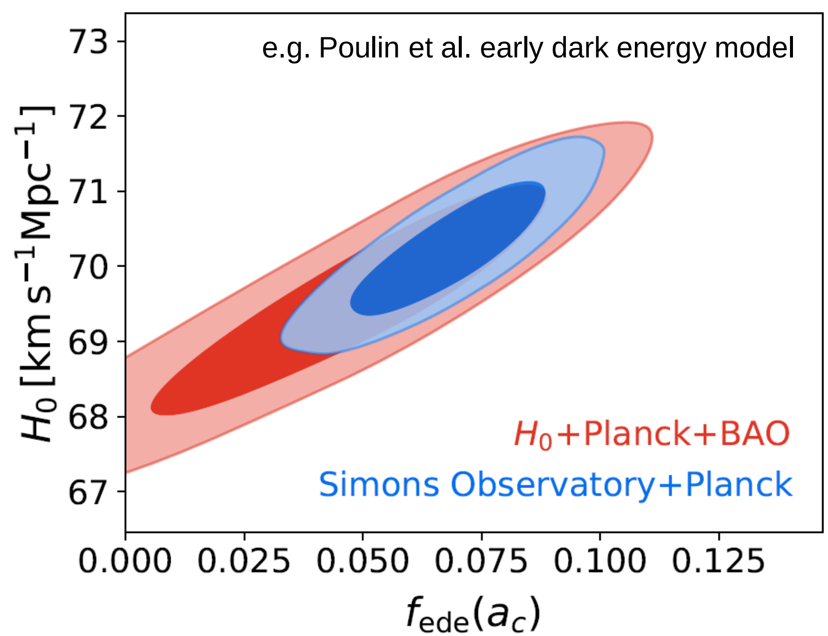
+ ActPol, SPTpol (now)
+ CMB-S4 (beyond)

If $H_0 > 71\ \mathrm{km\ s^{-1} Mpc^{-1}}$,
new pre-recombination physics likely detectable at $> 5\sigma$ soon

Distinct physical models give different precision predictions

High resolution/sensitivity polarization:
precision small-scale EE, TE, TT power spectrum





Summary

- **Forward distance (calibration) ladder**
  calibrate intrinsic luminosity of Sn1A standard candles
  Gaia (parallax) → Cepheids/TRGB/etc → Sn1A

  Relies on standardizing SN and relating
  - properties in our galaxy to other galaxies
  - SN in nearby galaxies to SN in Hubble flow
  + …

  Does *not* rely on distance to intermediate calibrators as long as SN1A and calibrator at the same distance (e.g. same external galaxy)

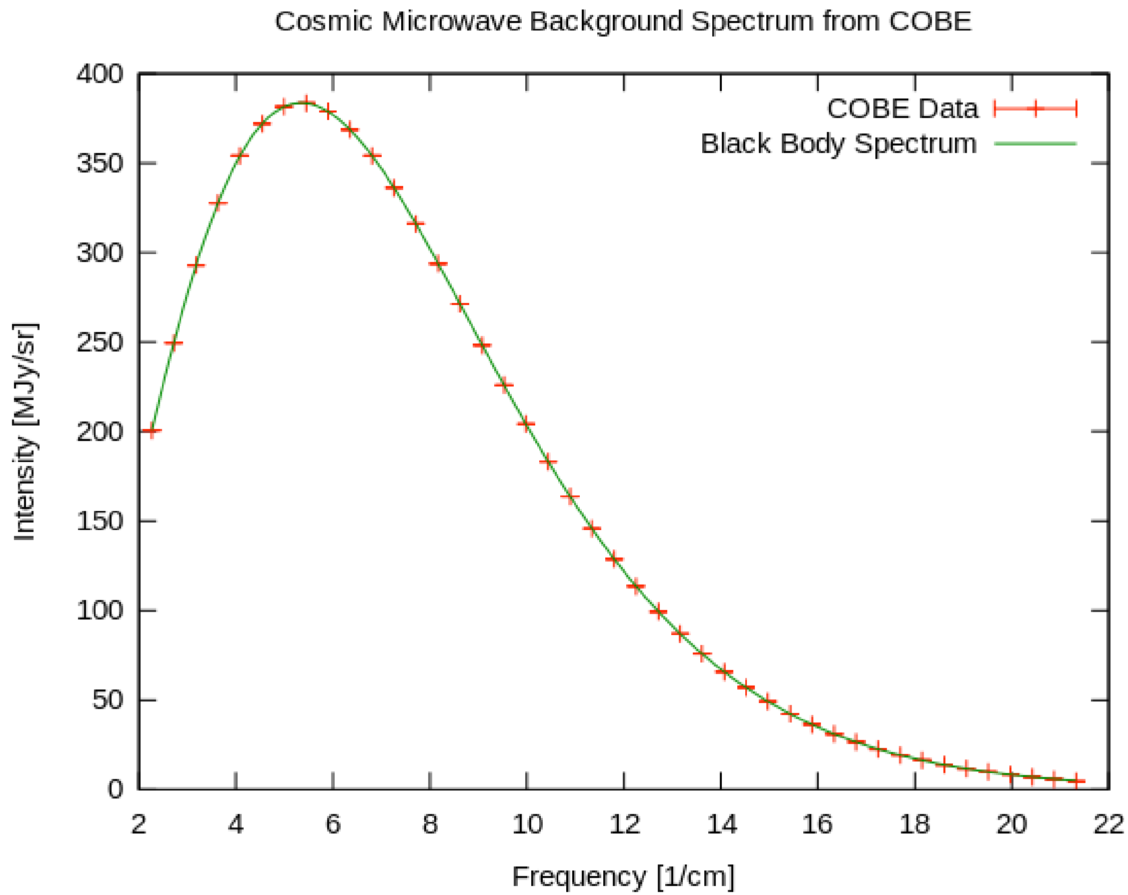
- **Inverse distance ladder**
  CMB acoustic peaks or D/H+BBN $\Rightarrow \Omega_\mathrm{b}\mathrm{h}^2$
  + CMB peak amplitudes $\Rightarrow \Omega_m h^2$
  + COBE $T_\mathrm{CMB} \Rightarrow \Omega_r h^2$
  Early $\Lambda\mathrm{CDM} \Rightarrow z_*,\ r_s$ prediction of comoving standard ruler

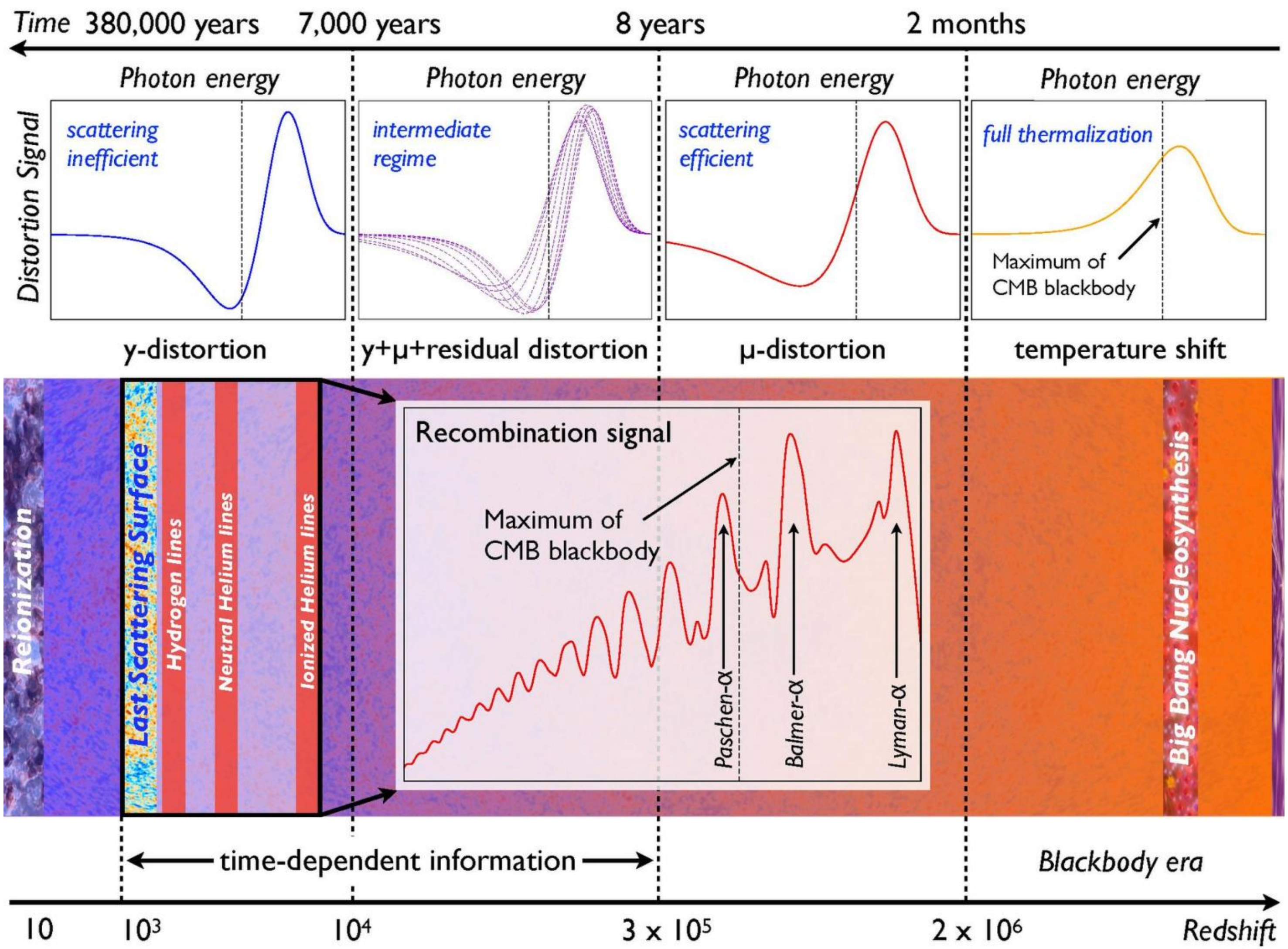
  Measure r_s sky angle θ_* to high precision (or θ_BAO)
  $\Rightarrow D_M$ comoving angular diameter distance. $D_M(z_*) + \Lambda\mathrm{CDM} \Rightarrow H_0$

Homework: resolve tension

CMB Spectral Distortions?

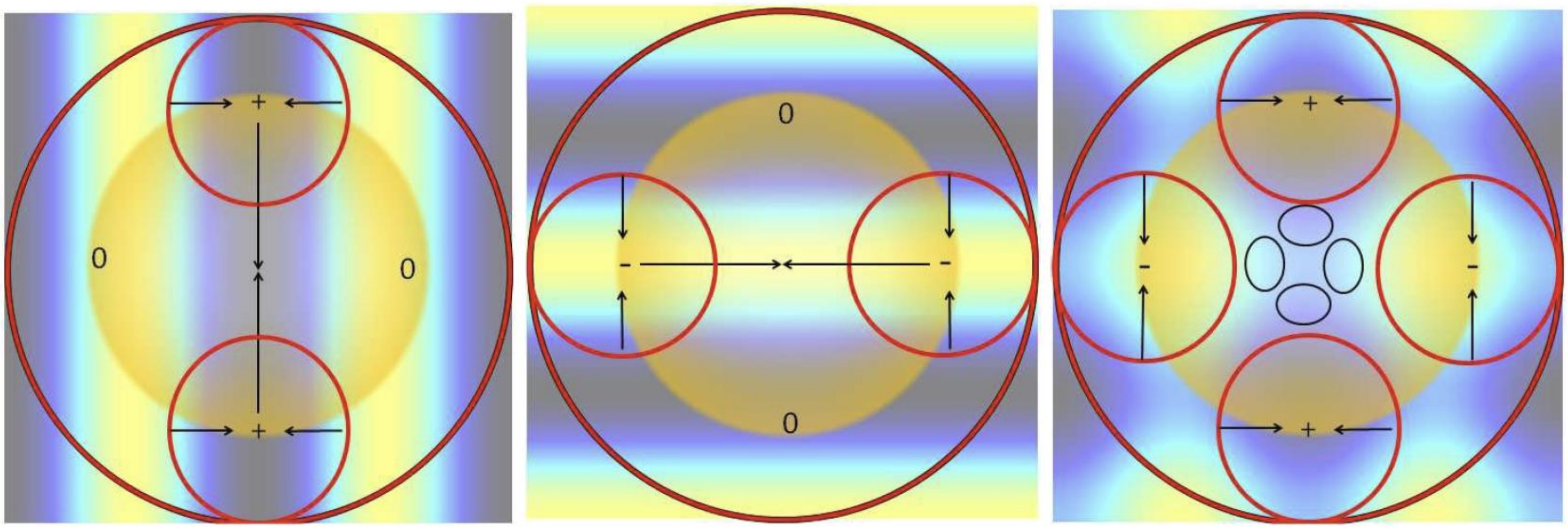
Deviations from blackbody in monopole (+ perturbations?)

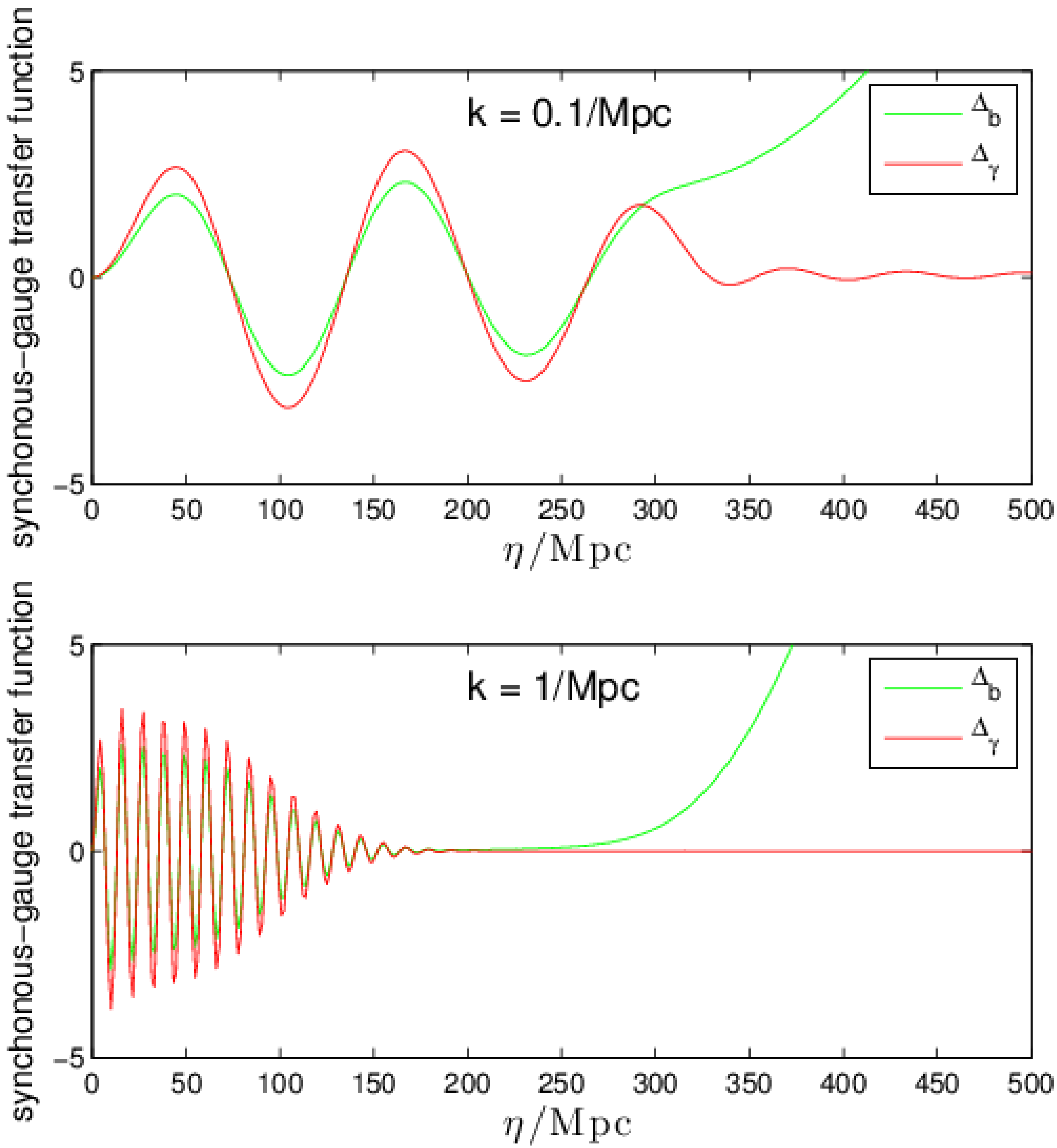


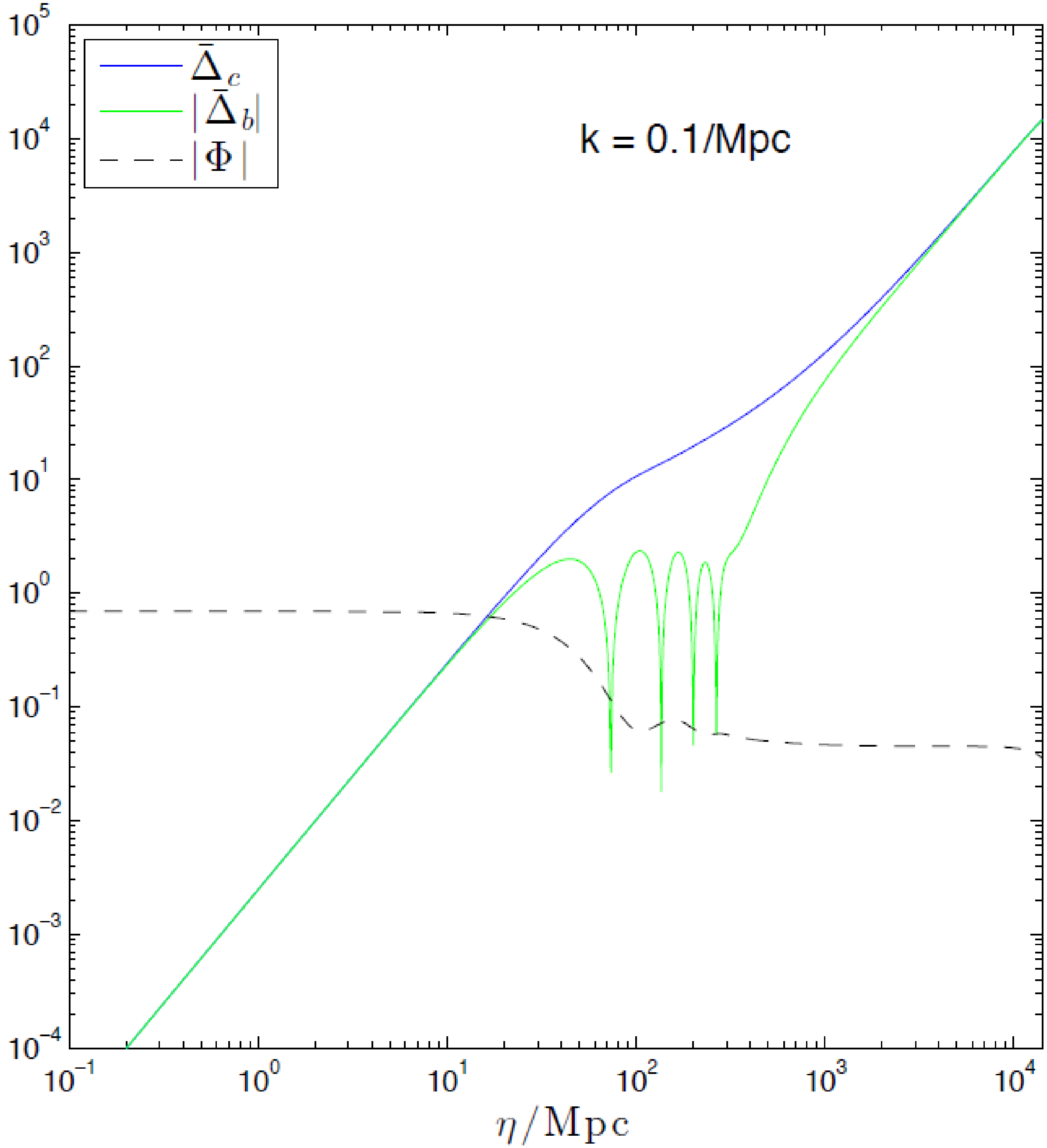


Chluba et al. arXiv:1909.01593

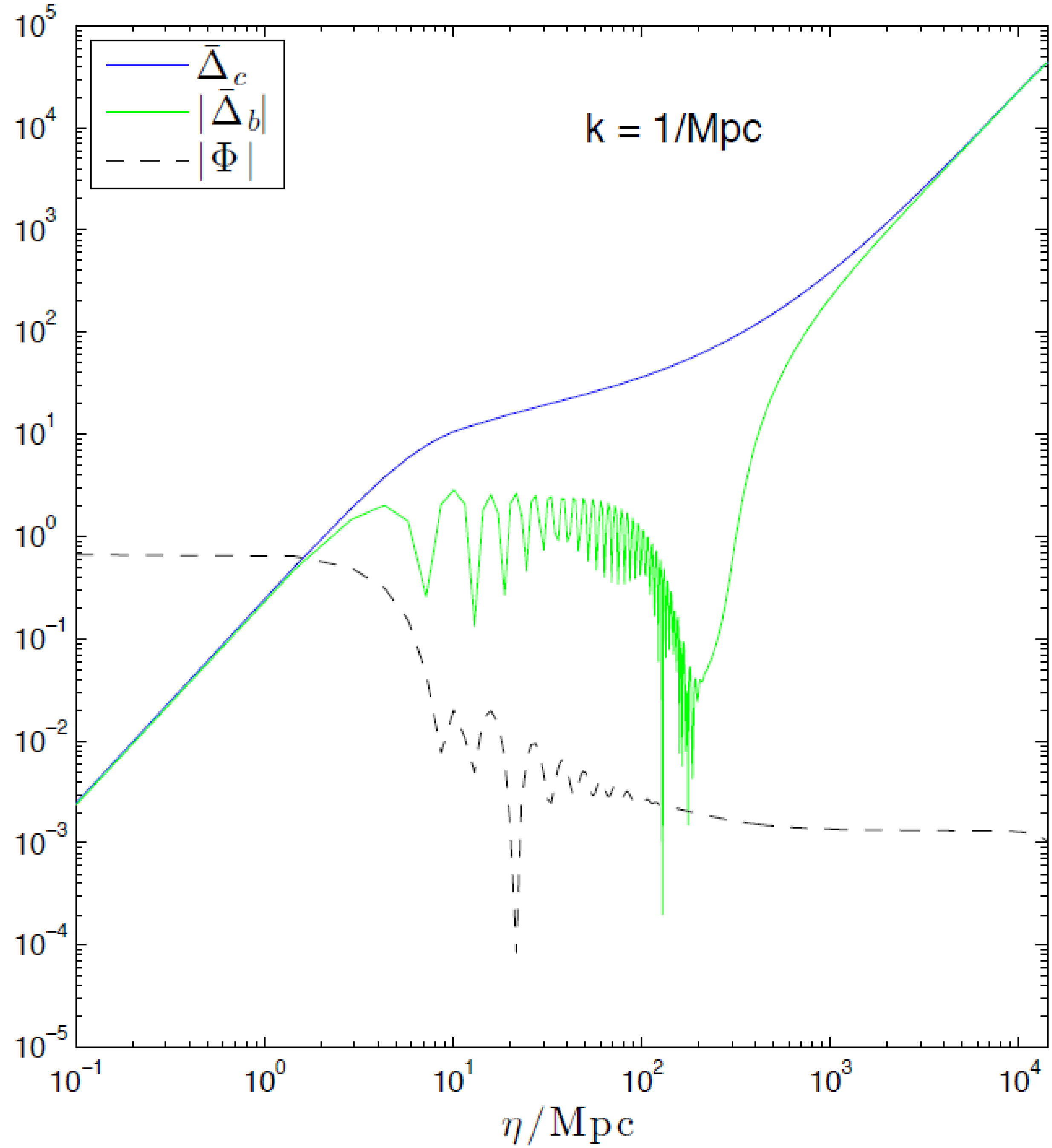
Large-scale polarization from reionization



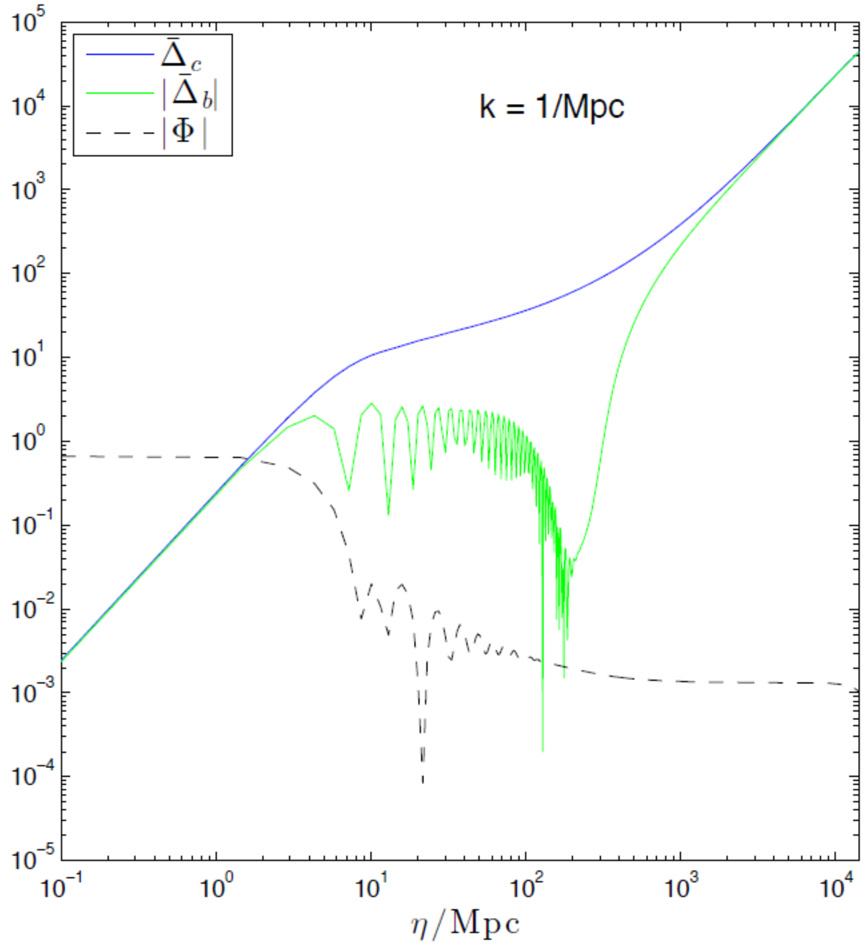

k = 0.1/Mpc
Δ_b
Δ_γ
synchonous–gauge transfer function
η/Mpc
k = 1/Mpc
Δ_b
Δ_γ
synchonous–gauge transfer function
η/Mpc



$\bar{\Delta}_c$
$|\bar{\Delta}_b|$
$|\Phi|$
k = 0.1/Mpc
$\eta\,/\,\mathrm{Mpc}$



$\bar{\Delta}_c$
$|\bar{\Delta}_b|$
$|\Phi|$
k = 1/Mpc
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-1}
10^0
10^1
10^2
10^3
10^4
η/Mpc



Outside horizon
No causal contact

Acoustic oscillations
in radiation fluid

Growth of structure

Transfer functions
for each perturbation:

e.g.
$\Delta_c = \delta\rho_c/\rho_c$
$\Delta_b = \delta\rho_b/\rho_b$
Φ = gravitational potential

Note: CAMB integrates transfer functions for variables defined in the CDM frame/synchronous gauge.

General regular linear primordial perturbation