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COSMOLOGICAL BACKREACTION FROM INHOMOGENEITIES/ CORRELATIONS

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Future cosmology school - 28.04.2023 - Cargèse

CRAL
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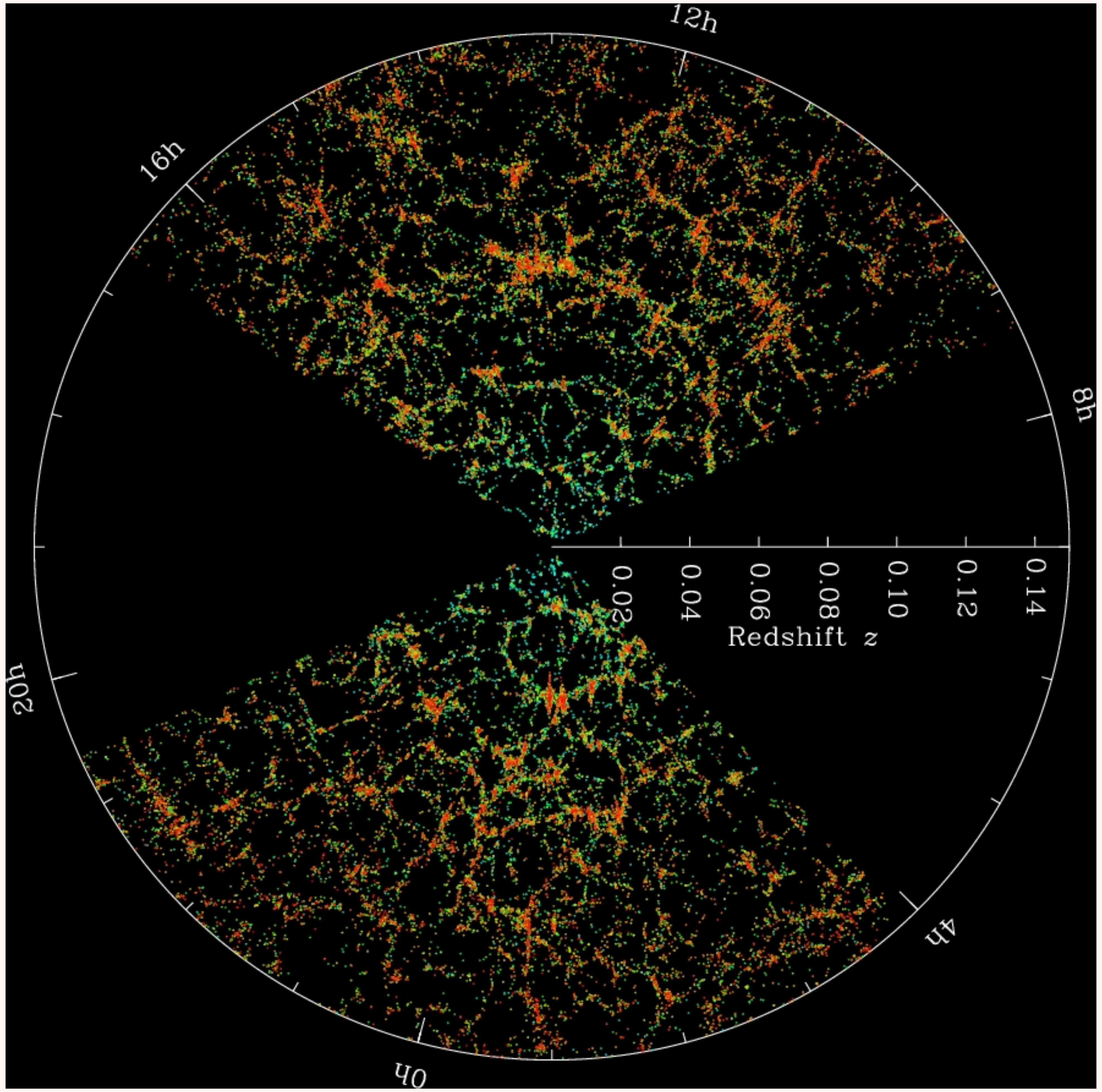
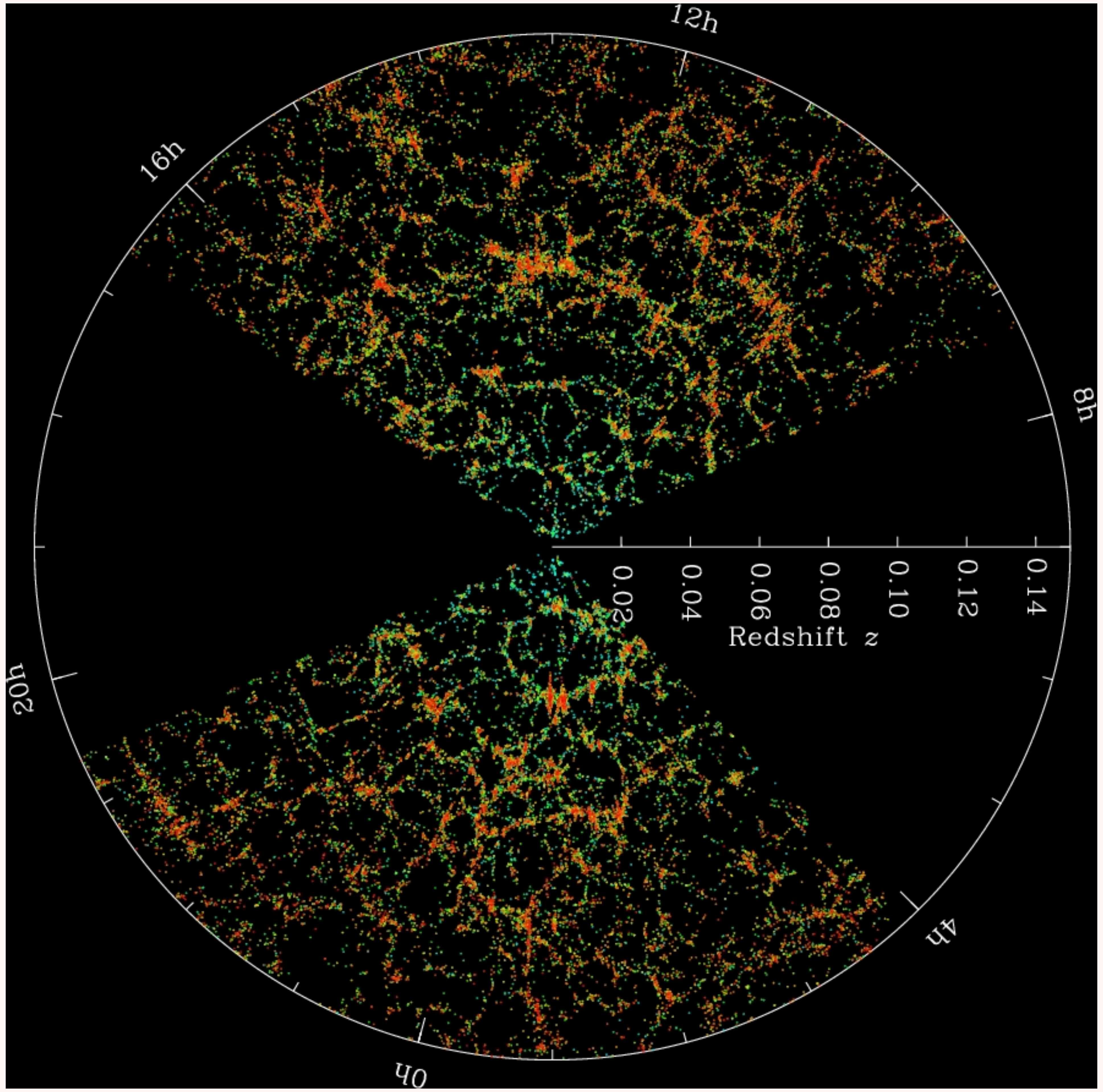
Maison de la Simulation

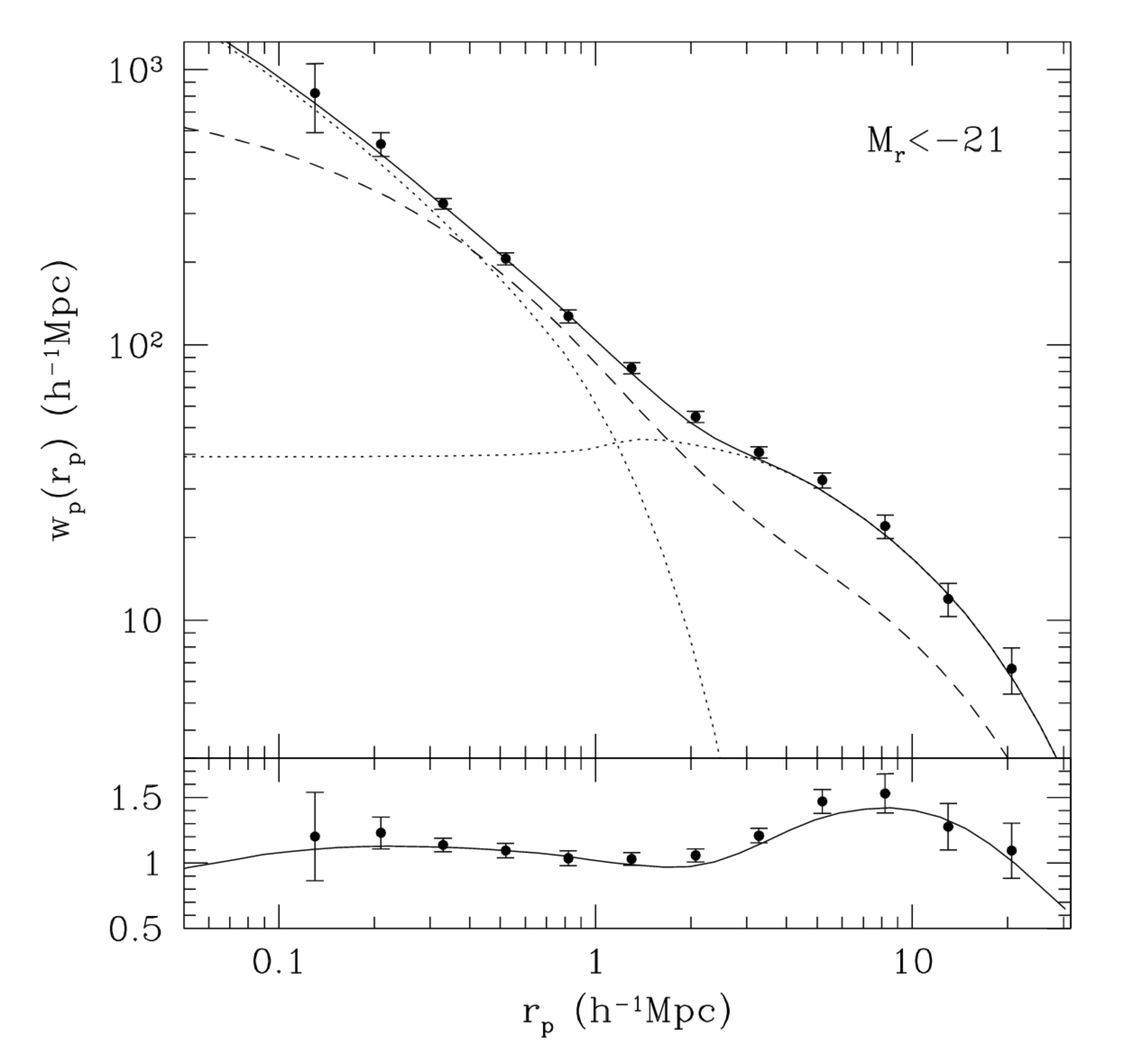


A zoomed-out view of galaxies identified by the Sloan Digital Sky Survey. Filaments and voids can be identified.
Credit : M. Blanton, SDSS

CORRELATIONS IN LARGE SCALE STRUCTURES



A zoomed-out view of galaxies identified by the Sloan Digital Sky Survey. Credit : M. Blanton, SDSS



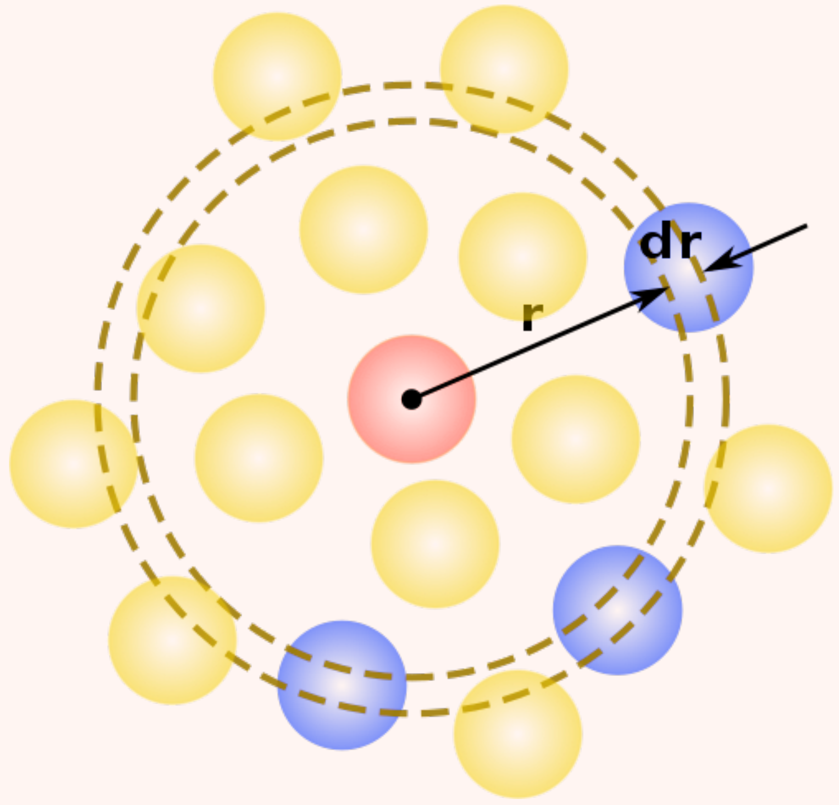
The projected correlation function for SDSS galaxies (data points with error bars). Taken from Zehavi et al. (2004)

The large-scale galaxy matter distribution of the Universe is highly correlated at small scales.

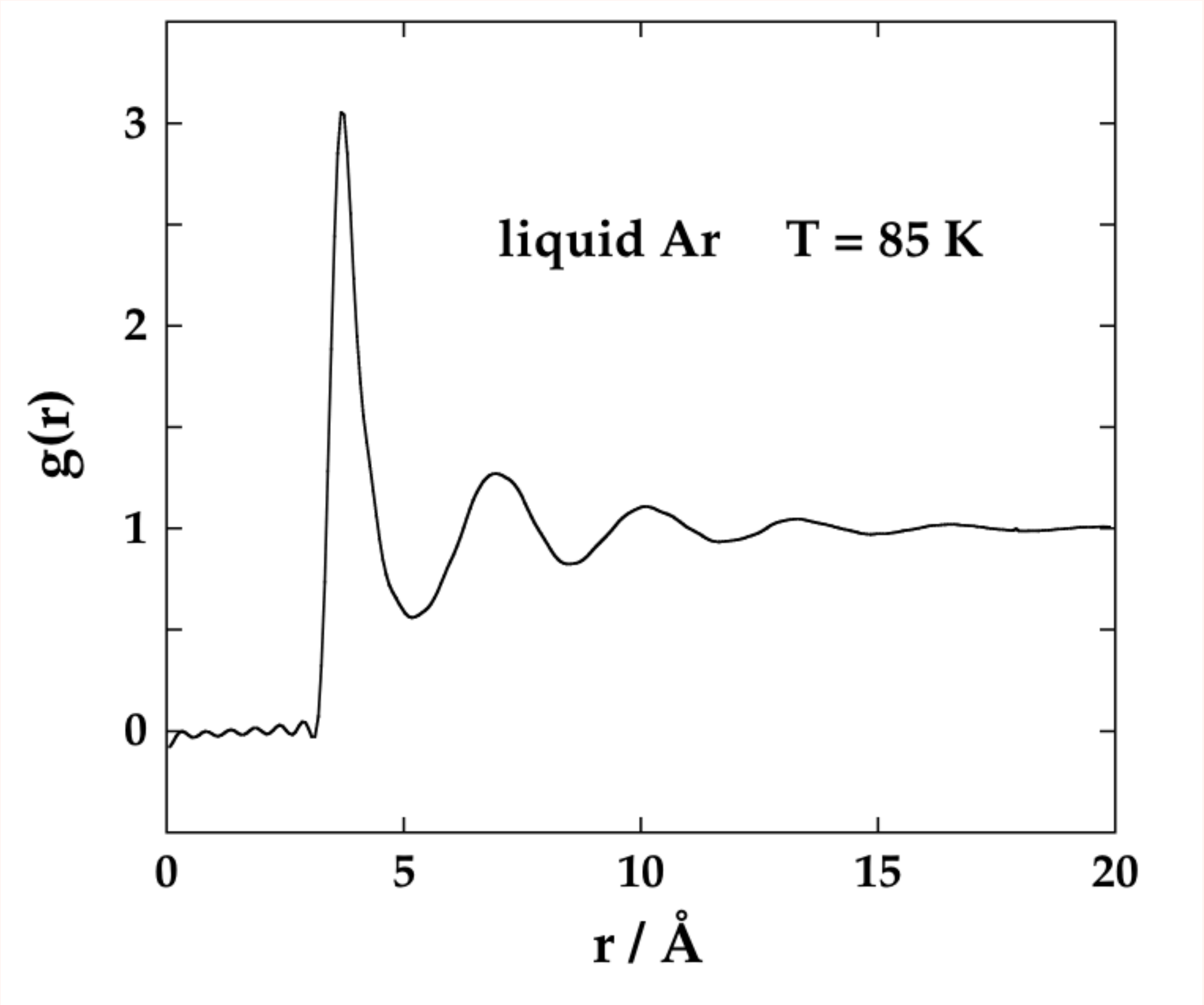
CORRELATIONS IN LIQUIDS

Radial distribution function

$$g(r) = \frac{1}{\rho}\left\langle \frac{1}{N} \sum_{1 \le i \ne j \le N} \delta\left(\mathbf{r} - \mathbf{r}_j + \mathbf{r}_i\right) \right\rangle$$



$4\pi r^2 \rho g(r) dr$ is the mean number of particles within the range r to $r + dr$, given that there is a particle at $r = 0$



Radial distribution function of liquid argon near the triple point, after Yarnell et al.

Thermodynamic properties

Energy

$$E = \frac{N k_B T}{\gamma - 1} + \frac{2\pi N^2}{V} \int_0^\infty g(r)\phi(r) r^2 dr$$

Equation of state

$$PV = N k_B T - \frac{2\pi N^2}{3V} \int_0^\infty g(r) \frac{d\phi(r)}{dr} r^3 dr$$

Transport coefficients

- shear viscosity η
- bulk viscosity χ

Liquids are correlated due to short-range interactions.

Correlations determine their physical properties.

BACKREACTION AND AVERAGING PROBLEMS

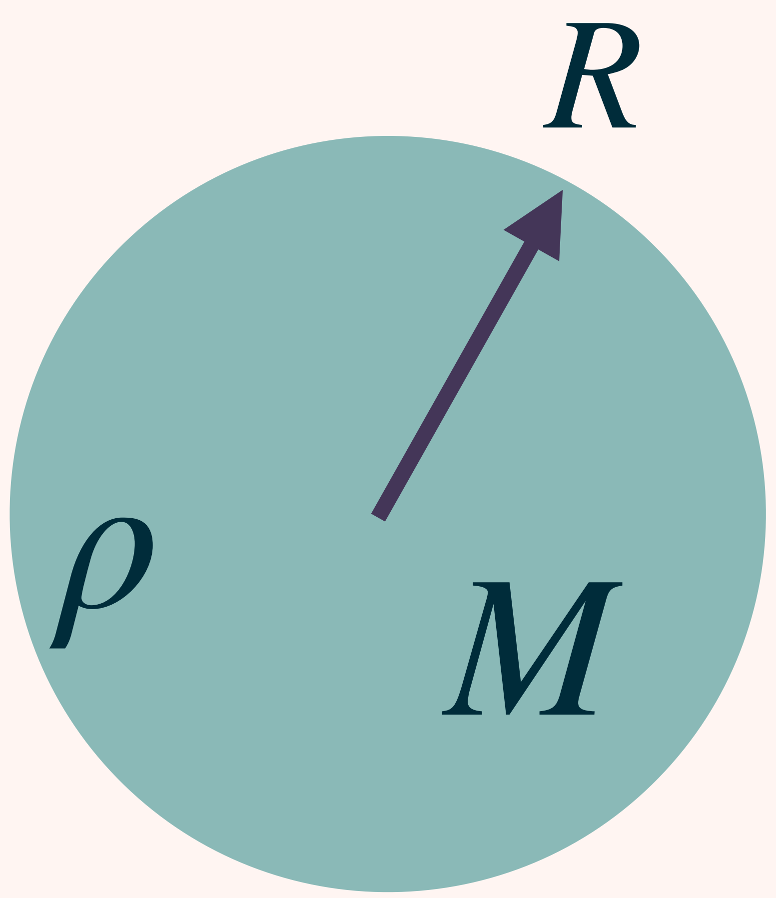
Question : Do correlations have an impact on the dynamics of the Universe (e.g. its expansion) ?

In connection with
- backreaction problem: to what extent do inhomogeneities impact the cosmological expansion ?
- averaging problem: how to derive a smooth spacetime from an inhomogeneous spacetime, in a covariant way? e.g. Macroscopic gravity (Zalaletdinov 1993), volume average of scalars (Buchert 2000)

However, many results pointing to backreaction being a small effect of order $(v/c)^2 \sim 10^{-5}$
- Effective field theory (Baumann 2012)
- Relativistic simulations (Adamek+ 2019)
- And others (see references in Peebles 2010)

NEWTONIAN DERIVATION OF FRIEDMANN EQUATIONS

Homogeneous (uncorrelated) sphere of fluid



Conservation of mechanical energy (assumed zero here)

$$E_m = E_{kin} + E_{int} = 0$$

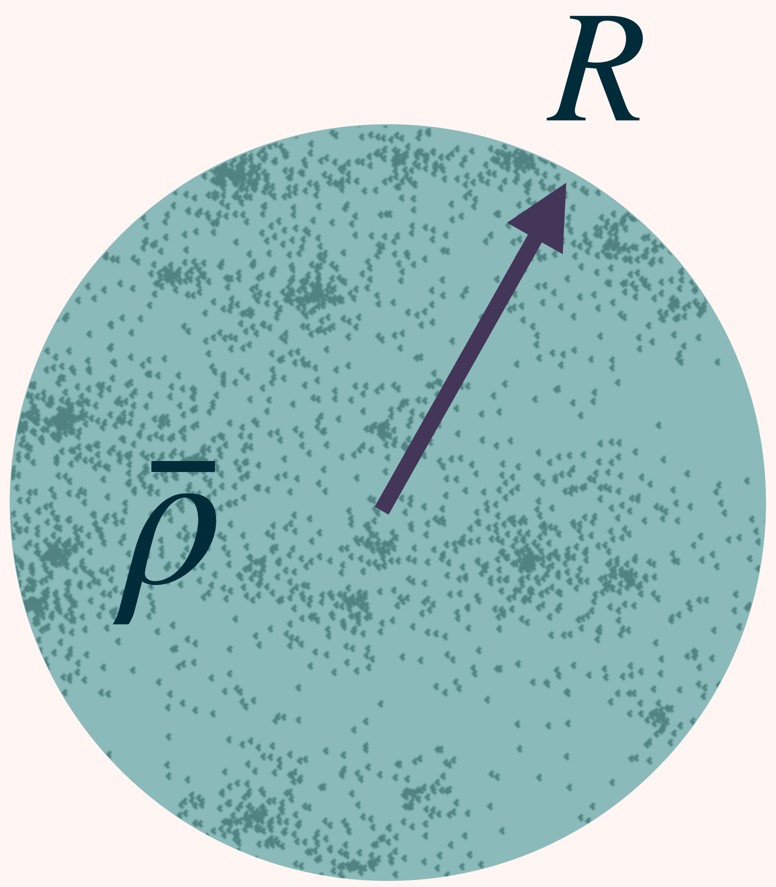
$$E_m = \frac{1}{2} m \dot{R}^2 - \frac{GmM}{R} = 0$$

Leads to the Friedmann equation

$$H^2 = \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3} \rho$$

HEURISTIC NEWTONIAN DERIVATION IN TREMBLIN ET AL. (2022)

Sphere composed of a correlated fluid $P_2(r, r')$



Energy conservation : $E_{kin} + E_{int} = 0$

$$\frac{1}{2}M\dot{R}^2 \approx \frac{1}{2}M\langle v^2\rangle = \alpha_{ni}\frac{3GM^2}{5R}$$

with $$\alpha_{ni} = \frac{\iint_{V,V} P_2(\boldsymbol{r},\boldsymbol{r}')/\left|\boldsymbol{r}-\boldsymbol{r}'\right| dVdV'}{\iint_{V,V} P_1(\boldsymbol{r})P_1(\boldsymbol{r}')/\left|\boldsymbol{r}-\boldsymbol{r}'\right| dVdV'}$$

Modified Friedmann equation

$$H^2 = \frac{\dot{R}^2}{R^2} = \alpha_{ni}\frac{8\pi G}{3}\bar{\rho}$$

Result

The Hubble rate is modified by an amplification coefficient α_{ni} related to substructures → backreaction effect

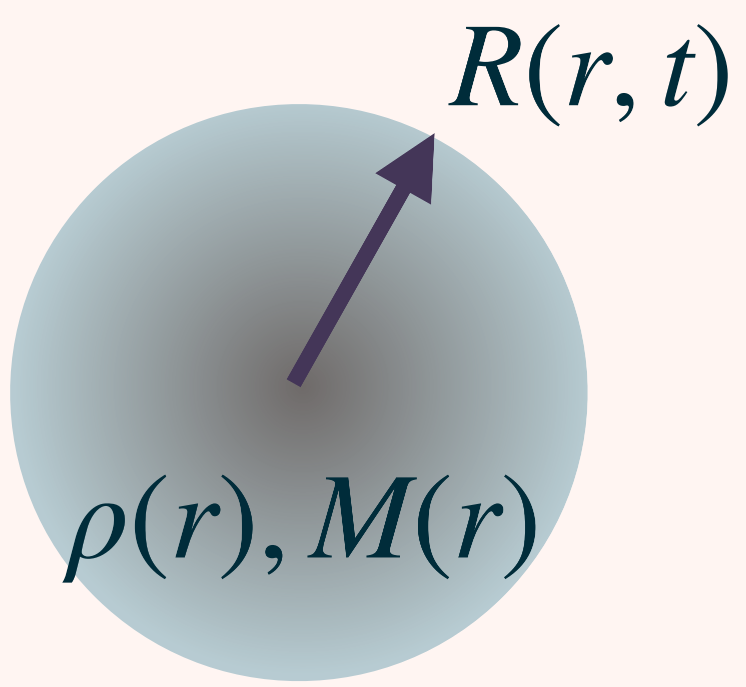
How does this Newtonian heuristic hold in a general relativistic setting?

AVERAGING INHOMOGENEITIES IN THE LTB MODEL

Idea : derive equations similar to the modified Friedmann equation of Tremblin et al. 2022, in a rigorous and general relativistic setting [1]

Lemaître-Tolman-Bondi (LTB) model

Spherically symmetric dust (pressureless) inhomogeneous model



Local expansion equation

$$\left(\frac{\partial R}{\partial t}\right)^2 = \frac{2GM(r)}{R} + 2E(r)$$

With line element

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \frac{(\partial R/\partial r)^2}{1+2E}\,\mathrm{d}r^2 + R^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

[1] PW, Tremblin, Chabrier, submitted

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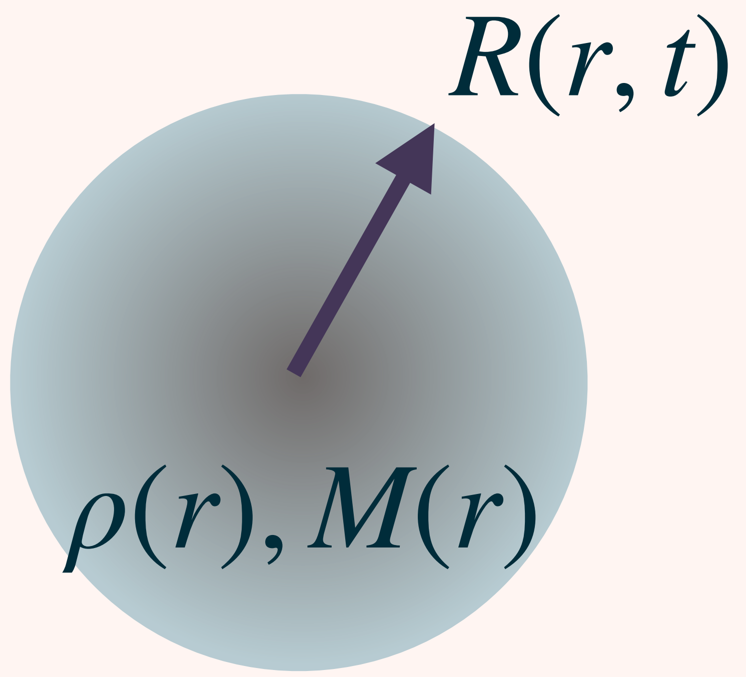

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$$ds^2 = -\,dt^2 + \frac{(\partial R/\partial r)^2}{1+2E}\,dr^2 + R^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

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Mass-weighted averaging operator

$$\langle A \rangle_M \equiv \frac{1}{M(r_0)} \int_{V_{r_0}} dV \rho A$$

$$\dot{R}_M \equiv \left\langle \left(\frac{\partial R}{\partial t}\right)^2 \right\rangle_M^{1/2}$$

Mass-weighted effective scale factor

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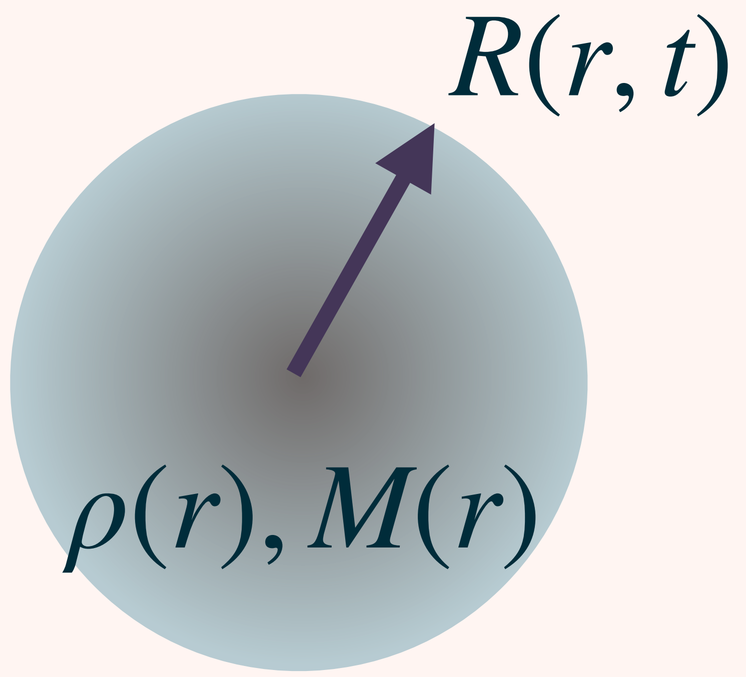
Spherically symmetric dust (pressureless) **inhomogeneous** model



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Mass-weighted effective scale factor

Effective, averaged, expansion equation

$$H_M^2 = \alpha \frac{8\pi G}{3} \bar{\rho}(R_0) + 2\frac{\langle E\rangle_M}{R_M^2}$$

$$\alpha = \frac{\int_{V_{R_0}} \int_{V_R} d\tilde{V}_R dV_{R'} P_2(R,R')/R}{\int_{V_{R_0}} \int_{V_R} d\tilde{V}_R dV_{R'} \bar{\rho}(R_0)^2/R} \cdot \frac{\langle R^2\rangle_V}{R_M^2}$$

Results

- **Two-point function** $P_2(R,R') = \rho(R)\rho(R')$ contribution to the amplification coefficient α, similar to Newtonian heuristics
- **Amplification of matter contribution** compared to FLRW spacetime

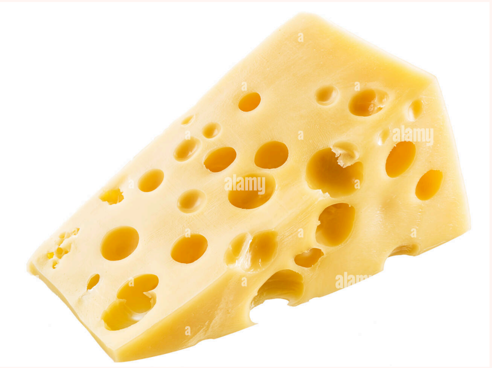
[1] PW, Tremblin, Chabrier, submitted

(PW, Tremblin, Chabrier, under review)

DISCUSSION

We showed how the Tremblin et al. 22 Newtonian heuristic can be derived from a GR model using a mass average.

We illustrated and compared the influence of the averaging procedure (e.g. with Buchert's formalism) on the form and magnitude of backreaction terms.



Limitations

- The LTB model contains only 1 spherical overdensity → extension to Swiss-cheese models?
- Lack of justification for the averaging formalism (why pick this one over others?) and of connection with observables related to light propagation