Inflationary Gravitational Waves contribution

to the radiation energy-density in the early Universe

Matteo Forconi, 2^{nd} year PhD student at La Sapienza University

'Towards a reliable calculation of relic radiation from primordial gravitational waves'
W. Giarè, M. Forconi, E. Di Valentino, A. Melchiorri
arXiv 2210.14159
Future Cosmology, 23-29 April Cargèse

SAPIENZA UNIVERSITÀ DI ROMA

INFN Istituto Nazionale di Fisica Nucleare

TOR VERGATA UNIVERSITÀ DEGLI STUDI DI ROMA

Primordial Tensor Power Spectrum

A unique prediction of Inflationary Theory is the existence of PGWs

In Slow-Roll:

$$\mathcal{P}_{\mathrm{T}}(k) = A_{\mathrm{T}} \left(\frac{k}{k_\star}\right)^{n_{\mathrm{T}}}$$

$$A_{\mathrm{T}} \equiv r A_{\mathrm{S}} \quad \text{and} \quad n_{\mathrm{T}} \equiv \frac{d \ln \mathcal{P}_{\mathrm{T}}}{d \ln k}$$

Primordial Tensor Power Spectrum

A unique prediction of Inflationary Theory is the existence of PGWs

Searching Tensor modes with:

- CMB B-modes

$-0.55 < n_T < 2.54$

In Slow-Roll:

$$\mathcal{P}_{\mathrm{T}}(k) = A_{\mathrm{T}} \left(\frac{k}{k_\star}\right)^{n_{\mathrm{T}}}$$

$$A_{\mathrm{T}} \equiv r A_{\mathrm{S}} \quad \text{and} \quad n_{\mathrm{T}} \equiv \frac{d \ln \mathcal{P}_{\mathrm{T}}}{d \ln k}$$

Primordial Tensor Power Spectrum

A unique prediction of Inflationary Theory is the existence of PGWs

Searching Tensor modes with:

- CMB B-modes

  $-0.55 < n_T < 2.54$

- Stochastic Background of Gravitational Waves

  $\Omega_{GW}(k_{LV}) \leq 1.7 \times 10^{-7} \quad \rightarrow \quad n_T < 0.52$

- Big Bang Nucleosynthesis

  $\Delta N_{eff} < 0.4 \quad \rightarrow \quad n_t < 0.4$

In Slow-Roll:

$$\mathcal{P}_{\mathrm{T}}(k) = A_{\mathrm{T}} \left(\frac{k}{k_\star}\right)^{n_{\mathrm{T}}}$$

$$A_{\mathrm{T}} \equiv r A_{\mathrm{S}} \quad \text{and} \quad n_{\mathrm{T}} \equiv \frac{d \ln \mathcal{P}_{\mathrm{T}}}{d \ln k}$$

Primordial Tensor Power Spectrum

A unique prediction of Inflationary Theory is the existence of PGWs

Searching Tensor modes with:

- CMB B-modes

  $-0.55 < n_T < 2.54$

- Stochastic Background of Gravitational Waves

  $\Omega_{GW}(k_{LV}) \leq 1.7 \times 10^{-7} \quad \rightarrow \quad n_T < 0.52$

- Big Bang Nucleosynthesis

  $\Delta N_{eff} < 0.4 \quad \rightarrow \quad n_t < 0.4$

In Slow-Roll:

$$\mathcal{P}_{\rm T}(k) = A_{\rm T}\left(\frac{k}{k_\star}\right)^{n_{\rm T}}$$

$$A_{\rm T} \equiv r A_{\rm S} \quad \text{and} \quad n_{\rm T} \equiv \frac{d\ln \mathcal{P}_{\rm T}}{d\ln k}$$

But since $\quad k_{\rm LV} \gg k_\star, \quad$ and $\quad \Delta N_{eff}^{GW} \propto \int_{f_{min}}^{f_{max}} \frac{df}{f}\mathcal{P}_T(f)$

do we parameterize the Power Spectrum ($\mathcal{P}_T$) correctly?

Extra Radiation from PGWs

Primordial Gravitational Waves behaves as extra radiation $\Longrightarrow$ Contributes to the Energy budget of the Universe: $\Omega_{GW}(f) \simeq \frac{\mathcal{P}_T(f)}{24 z_{eq}}$

$$\text{standard CMB parametrization} \rightarrow \simeq \frac{1}{n_T}\left[\left(\frac{f}{f_\star}\right)^{n_T}\right]_{f_{min}}^{f_{max}}$$

Extra Radiation from PGWs

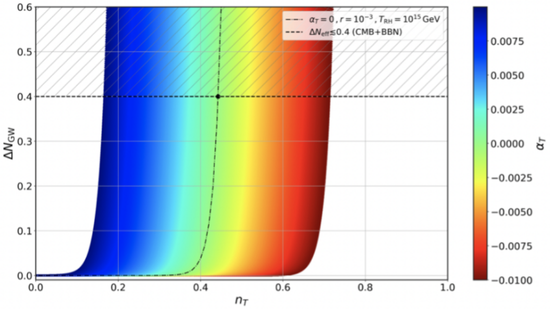
Primordial Gravitational Waves behaves as extra radiation $\Longrightarrow$ Contributes to the Energy budget of the Universe: $\Omega_{GW}(f) \simeq \frac{\mathcal{P}_T(f)}{24 z_{eq}}$

$$\text{standard CMB parametrization} \rightarrow \simeq \frac{1}{n_T}\left[\left(\frac{f}{f_\star}\right)^{n_T}\right]_{f_{min}}^{f_{max}}$$

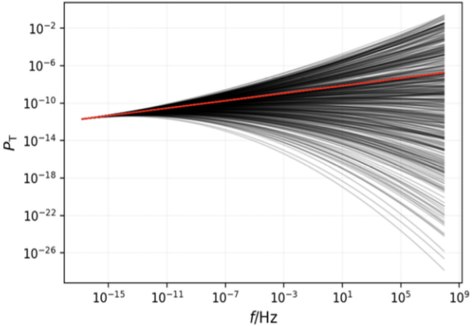
$$\text{Add naive scale-dependence } \alpha_T \equiv dn_T/d\ln k \rightarrow \propto \sqrt{\frac{2}{|\alpha_T|}}e^{-\frac{n_T^2}{2|\alpha_T|}}$$

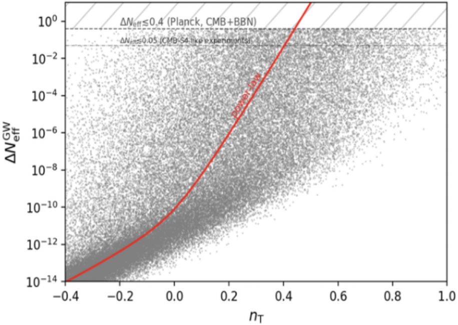
Extra Radiation from PGWs

Primordial Gravitational Waves behaves as extra radiation $\Longrightarrow$ Contributes to the Energy budget of the Universe: $\Omega_{GW}(f) \simeq \frac{\mathcal{P}_T(f)}{24 z_{eq}}$



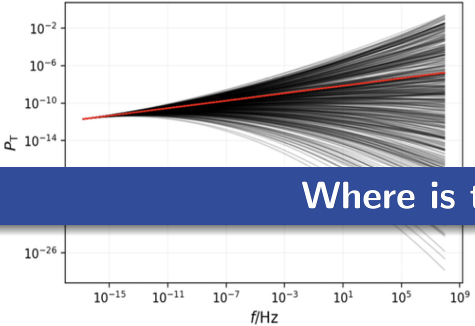
More general parametrization: $\ln \mathcal{P}_T = \sum_{j=0}^{\infty} a_j (x - x_0)^j$ with $x_0 = \ln f_{\rm CMB}$

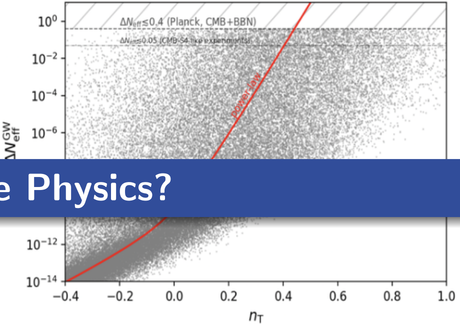




10^6 different simulations up to $j_{\rm max} = 10$

More general parametrization: $\ln \mathcal{P}_T = \sum_{j=0}^{\infty} a_j (x - x_0)^j$ with $x_0 = \ln f_{\mathrm{CMB}}$





Where is the Physics?

10^6 different simulations up to $j_{\max} = 10$

EFT Approach

The Effective Field Theory of inflation is a general framework for describing fluctuations around a quasi-de Sitter background

$$S = \int d^4x\sqrt{-g}\left[\overbrace{\frac{1}{2}M_{pl}^2R - c(t)g^{00} - \Lambda(t)}^{background} + \overbrace{\sum_{n\geq 2}\frac{1}{n!}M_n(t)^4(g^{00}+1)^n - \frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu_\nu + ...}^{corrections}\right]$$

EFT Approach

The Effective Field Theory of inflation is a general framework for describing fluctuations around a quasi-de Sitter background

$$S = \int d^4x\sqrt{-g}\left[\overbrace{\frac{1}{2}M_{pl}^2 R - c(t)g^{00} - \Lambda(t)}^{background} + \overbrace{\sum_{n\geq 2}\frac{1}{n!}M_n(t)^4(g^{00}+1)^n - \frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^{\mu}_{\nu} + \dots}^{corrections}\right]$$

The system of the Hubble flow equations gives the evolution of any generic quantity $Q(\phi)$

EFT Approach

The Effective Field Theory of inflation is a general framework for describing fluctuations around a quasi-de Sitter background

$$\begin{cases} \frac{d\epsilon_Q}{dN} = \epsilon_Q(\theta - \epsilon - \eta - \epsilon_Q) + \epsilon\rho_Q \\ \frac{d\rho_Q}{dN} = \rho_Q(\theta - 2\eta - \epsilon_Q) +^2 \chi_Q \\ \dots \\ \frac{d^l\chi_Q}{dN} =^l \chi_Q[l(l\theta - (l-1)\epsilon - (l+1)\eta - \epsilon_Q] +^{l+1} \chi_Q \end{cases}$$

SR Parameters for $Q(\phi)$

$$^l\chi_Q(\phi) = \left(\frac{c(\phi)}{M_{\rm pl}^2}\right)^l \left(\frac{1}{H(\phi}\right)^{l-1} \left(\frac{1}{H'(\phi)}\right)^{l+1} \frac{1}{Q}\frac{d^{l+1}Q}{d\phi^{l+1}}$$

EFT Approach

The Effective Field Theory of inflation is a general framework for describing fluctuations around a quasi-de Sitter background

$$\begin{cases} \frac{d\epsilon_Q}{dN} = \epsilon_Q(\theta - \epsilon - \eta - \epsilon_Q) + \epsilon\rho_Q \\ \frac{d\rho_Q}{dN} = \rho_Q(\theta - 2\eta - \epsilon_Q) +^2 \chi_Q \\ \dots \\ \frac{d^l\chi_Q}{dN} =^l \chi_Q[l(l\theta - (l-1)\epsilon - (l+1)\eta - \epsilon_Q] +^{l+1} \chi_Q \end{cases}$$

Many coefficients. Many systems?

Fortunately not.

SR Parameters for $Q(\phi)$

$$^l\chi_Q(\phi) = \left(\frac{c(\phi)}{M_{\rm pl}^2}\right)^l \left(\frac{1}{H(\phi)}\right)^{l-1} \left(\frac{1}{H'(\phi)}\right)^{l+1} \frac{1}{Q}\frac{d^{l+1}Q}{d\phi^{l+1}}$$

EFT Approach

The Effective Field Theory of inflation is a general framework for describing fluctuations around a quasi-de Sitter background

$$\begin{cases} \frac{d\epsilon_Q}{dN} = \epsilon_Q(\theta - \epsilon - \eta - \epsilon_Q) + \epsilon\rho_Q \\ \frac{d\rho_Q}{dN} = \rho_Q(\theta - 2\eta - \epsilon_Q) +^2 \chi_Q \\ \dots \\ \frac{d^l\chi_Q}{dN} =^l \chi_Q[l(l\theta - (l-1)\epsilon - (l+1)\eta - \epsilon_Q] +^{l+1} \chi_Q \end{cases}$$

SR Parameters for $Q(\phi)$

$$^l\chi_Q(\phi) = \left(\frac{c(\phi)}{M_{\rm pl}^2}\right)^l \left(\frac{1}{H(\phi}\right)^{l-1} \left(\frac{1}{H'(\phi)}\right)^{l+1} \frac{1}{Q}\frac{d^{l+1}Q}{d\phi^{l+1}}$$

Many coefficients. Many systems?

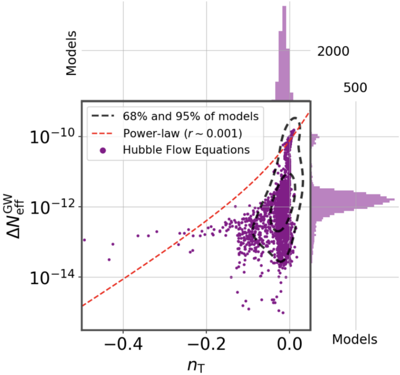
Fortunately not.

$$\mathcal{P}_T = \frac{1}{c_T}\left(\frac{H^2}{\pi^2 M_{pl}^2}\right)$$

Theoretical Monte Carlo

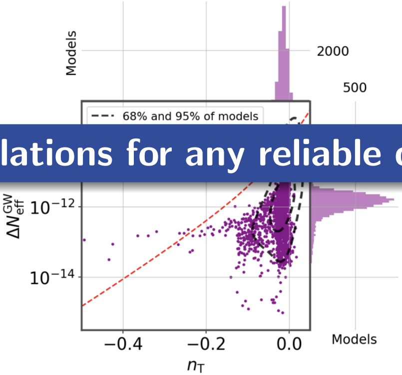
At high frequencies the GW production becomes strongly model-dependent

$$\Delta N_{eff} < 0.4 \Longrightarrow \Delta N_{eff} \simeq 10^{-10} - 10^{-14}$$



At high frequencies the GW production becomes strongly model-dependent

$$\Delta N_{eff} < 0.4 \Longrightarrow \Delta N_{eff} \simeq 10^{-10} - 10^{-14}$$

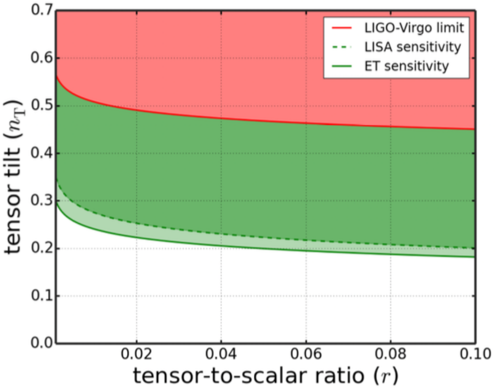


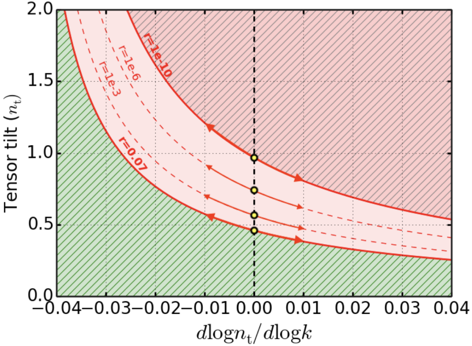
More accurate calculations for any reliable conclusion on inflation!

Thank You!

Backup slides

Observing Tensor Modes





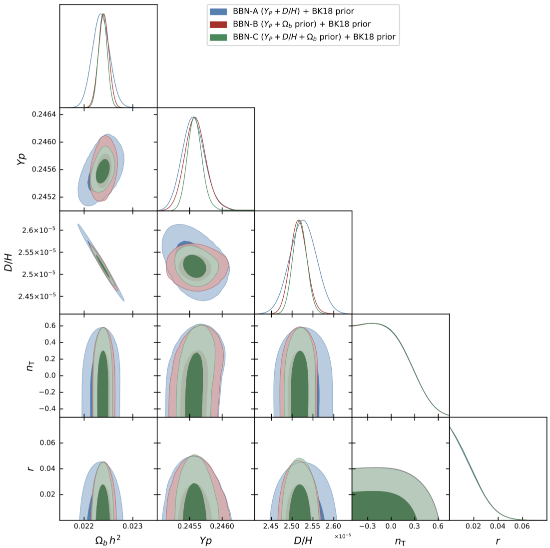
Extra Radiation from PGWs

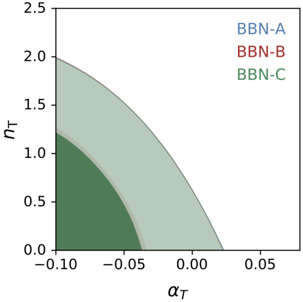
Primordial Gravitational Waves behaves as extra radiation $\Longrightarrow$ Contributes to the Energy budget of the Universe: $\Omega_{GW}(f) \simeq \frac{\mathcal{P}_T(f)}{24 z_{eq}}$

The total amount of radiation is:

$$\rho_{rad} = \frac{\pi^2}{30}\left[2\,T_\gamma^4 + \frac{7}{4}\,T_{e^\pm}^4 + \frac{7}{4}N_{eff}\,T_\nu^4 + 2\,T_{GW}^4\right] \Longrightarrow \boxed{\Delta N_{eff}^{GW} = \frac{8}{7}\frac{\rho_{GW}}{\rho_\gamma}} \text{ at } T \gtrsim \mathcal{O}(1MeV)$$

$$\text{and today: } \Delta N_{eff}^{GW} \simeq \frac{h_0^2}{5.6\times 10^{-6}}\left(\frac{1}{z_{eq}}\right)\int_{f_{min}}^{f_{max}} \frac{df}{f}\mathcal{P}_T(f)$$


BBN-A ($Y_P + D/H$) + BK18 prior
BBN-B ($Y_P + \Omega_b$ prior) + BK18 prior
BBN-C ($Y_P + D/H + \Omega_b$ prior) + BK18 prior
Yp
D/H
n_T
r
$\Omega_b h^2$
0.2464
0.2460
0.2456
0.2452
2.6×10^{-5}
2.55×10^{-5}
2.5×10^{-5}
2.45×10^{-5}
0.6
0.4
0.2
0.0
−0.2
−0.4
0.06
0.04
0.02
0.022
0.023
0.2455
0.2460
2.45
2.50
2.55
2.60
$\times10^{-5}$
−0.3
0.0
0.3
0.6
0.02
0.04
0.06


BBN-A
BBN-B
BBN-C
n_T
α_T
2.5
2.0
1.5
1.0
0.5
0.0
−0.10
−0.05
0.00
0.05

Stochastic Approach

More general parametrization: $\ln \mathcal{P}_T = \sum_{j=0}^{\infty} a_j (x - x_0)^j$ with $x_0 = \ln f_{\mathrm{CMB}}$

1. $a_0 = \ln (rA_s)$
2. $a_1 = n_T \in [-0.5; 1]$
3. $a_\mathrm{j} = \frac{1}{j!} \frac{d^j \ln \mathcal{P}_T}{d \ln^j f}$
4. $a_1 \gg a_\mathrm{j} \gg a_{\mathrm{j}+1}$

EFT Approach

The Effective Field Theory of inflation is a general framework for describing fluctuations around a quasi-de Sitter background

$$S = \int d^4x\sqrt{-g}\left[\overbrace{\frac{1}{2}M_{pl}^2 R - c(t)g^{00} - \Lambda(t)}^{background} + \overbrace{\sum_{n\geq 2}\frac{1}{n!}M_n(t)^4(g^{00}+1)^n - \frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu_\nu + \ldots}^{different\ models}\right]$$

Evolution of the Hubble parameter using Friedmann equations

$$H^2 = \frac{1}{3M_{\rm pl}^2}[c(t)+\Lambda(t)] \qquad \frac{\ddot{a}}{a} = -\frac{1}{3M_{\rm pl}^2}[2c(t)-\Lambda(t)]$$

Only two independent functions to fully characterize the background evolution: $c(\phi)\,, H(\phi)$

Hubble Flow Equations

The system of the Hubble flow equations gives the evolution of any generic quantity $Q(\phi)$

$$\begin{cases} \frac{d\epsilon}{dN} = \epsilon(\theta - 2\epsilon) \\ \frac{d\eta}{dN} = \eta(\theta - 2\eta - \epsilon) +^2 \lambda \\ \dots \\ \frac{d^l\lambda}{dN} =^l \lambda[l(\theta - \epsilon) - (l+1)\eta] +^{l+1} \lambda \\ \frac{d\theta}{dN} = \kappa\epsilon - \theta(\epsilon + \eta) \\ \frac{d\kappa}{dN} = -2\kappa\eta +^2 \xi \\ \dots \\ \frac{d^l\xi}{dN} =^l \xi[(l-1)(\theta - \epsilon) - (l+1)\eta] +^{l+1} \xi \end{cases}$$

SR Parameters for $H(\phi)$

$$^l\lambda(\phi) = \left(\frac{c(\phi)}{M_{\rm pl}^2}\right)^l \left(\frac{1}{H(\phi}\right)^l \left(\frac{1}{H'(\phi)}\right)^{l+1} \frac{d^{l+1}H(\phi)}{d\phi^{l+1}}$$

SR Parameters for $c(\phi)$

$$^l\xi(\phi) = \left(\frac{c(\phi)}{M_{\rm pl}^2}\right)^l \left(\frac{1}{H(\phi})\right)^{l-1} \left(\frac{1}{H'(\phi)}\right)^{l+1} \frac{1}{c(\phi)} \frac{d^{l+1}c(\phi)}{d\phi^{l+1}}$$

Theoretical Monte Carlo

This block is here for consistency with the other slides. But it is kind of useless...

Taking into account all the operators that induce tensor perturbations, at leading order

$$\mathcal{P}_T = \frac{1}{c_T}\left(\frac{H^2}{\pi^2 M_{pl}^2}\right) \quad \text{with} \quad c_T^{-2} = 1 - \frac{\bar{M}_3^2}{M_{pl}^2} \Longrightarrow n_T = -2\varepsilon + \varepsilon_T$$

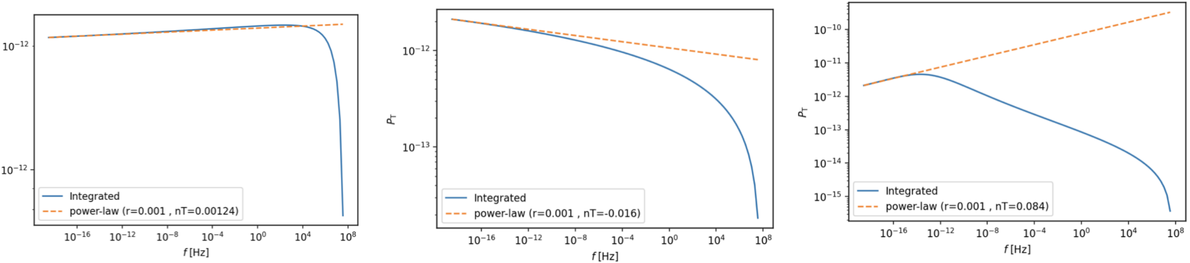
thus

$$\text{Background: } c(\phi), H(\phi). \text{ Tensor contribution: } c_T(\phi) \quad \left(\epsilon_T = -\frac{\dot{c}_T}{H c_T}, \ldots\right)$$

Theoretical Monte Carlo

This block is here for consistency with the other slides. But it is kind of useless...

Taking into account all the operators that induce tensor perturbations, at leading order



At high frequencies the GW production becomes strongly model-dependent!