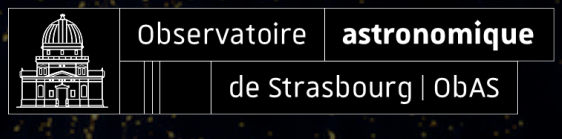
Structure formation in cosmological context 1°

Katarina Kraljic
Observatoire astronomique de Strasbourg

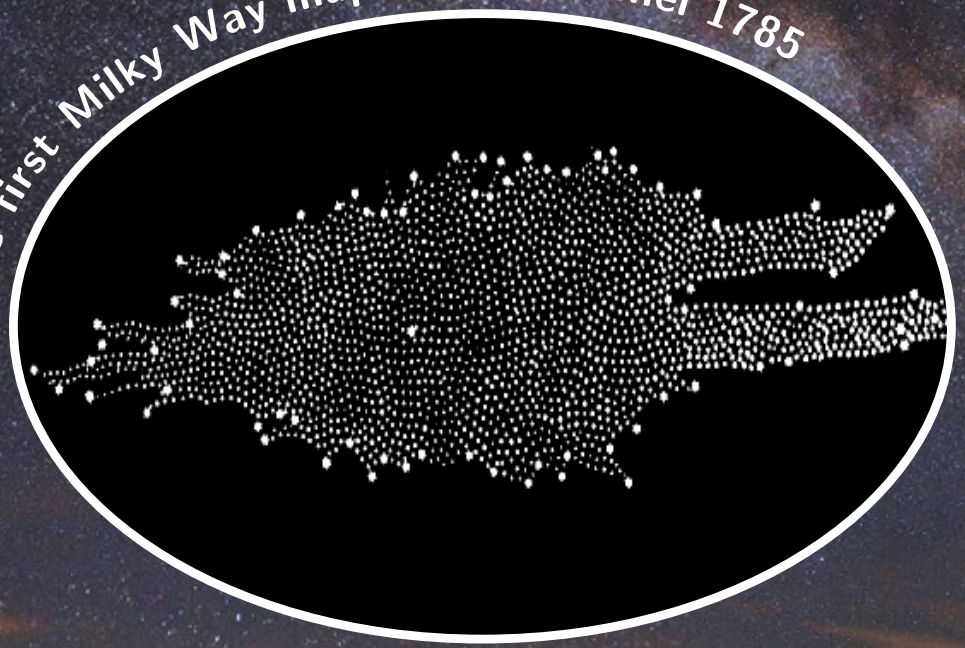


The Cosmic Dawn: First Galaxies and Cosmic Reionization, IESC Cargèse, 5-11 Oct 2025

© A. Pontzen & F. Governato; NASA/ESA/S. Beckwith (STScI) and The HUDF Team

Mapping the Universe

Mapping the Universe



William Herschel
(1738 - 1822)

Caroline Herschel
(1750 - 1848)

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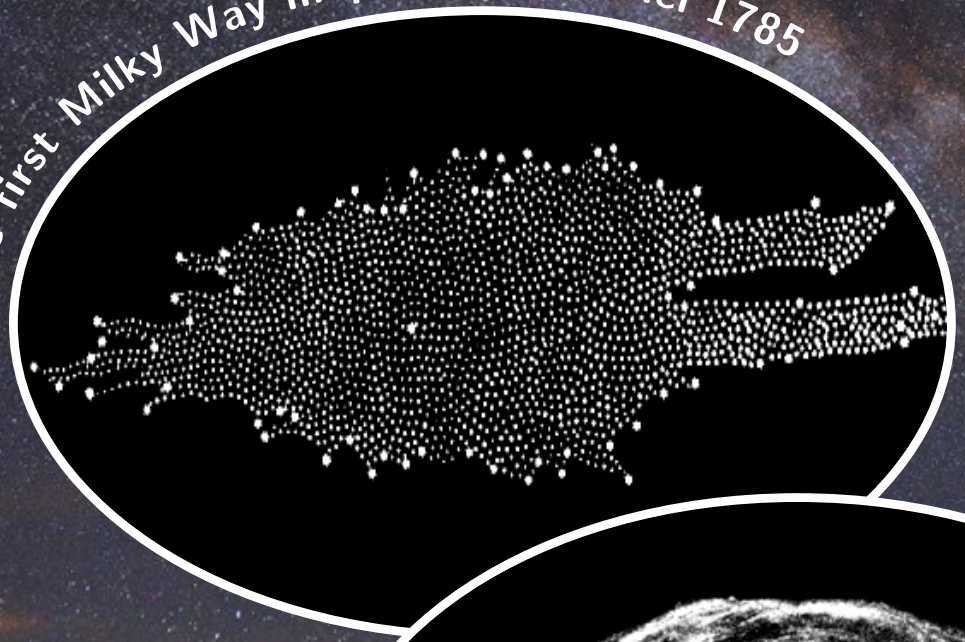
Mapping the Universe



William Parsons
(1800 - 1867)

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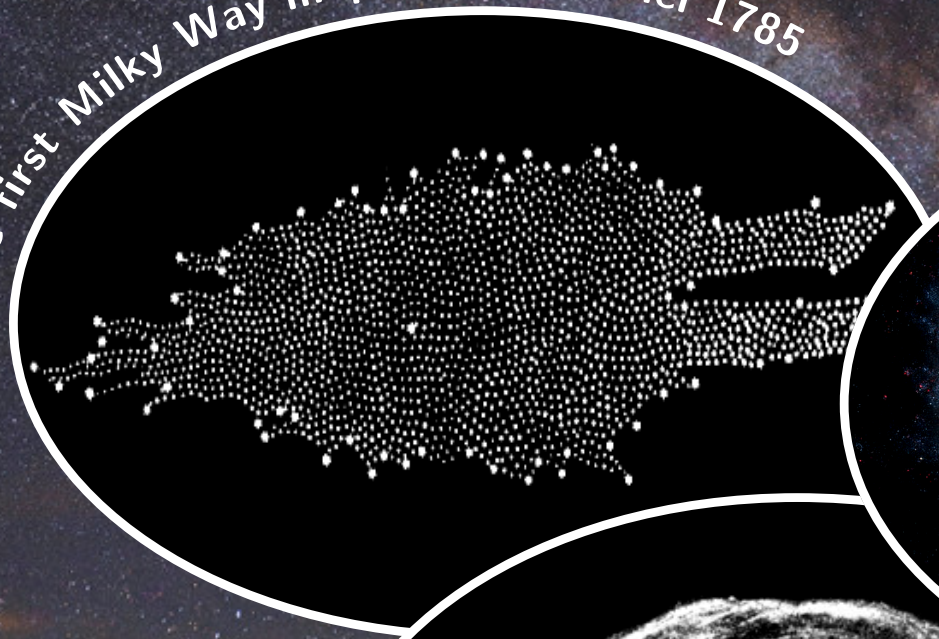
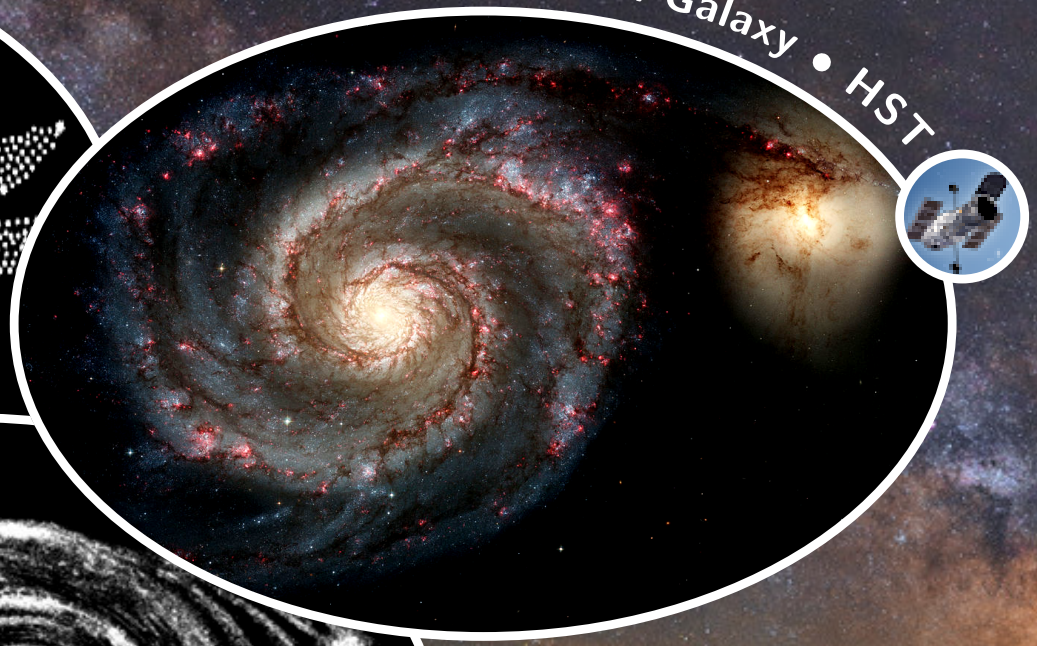
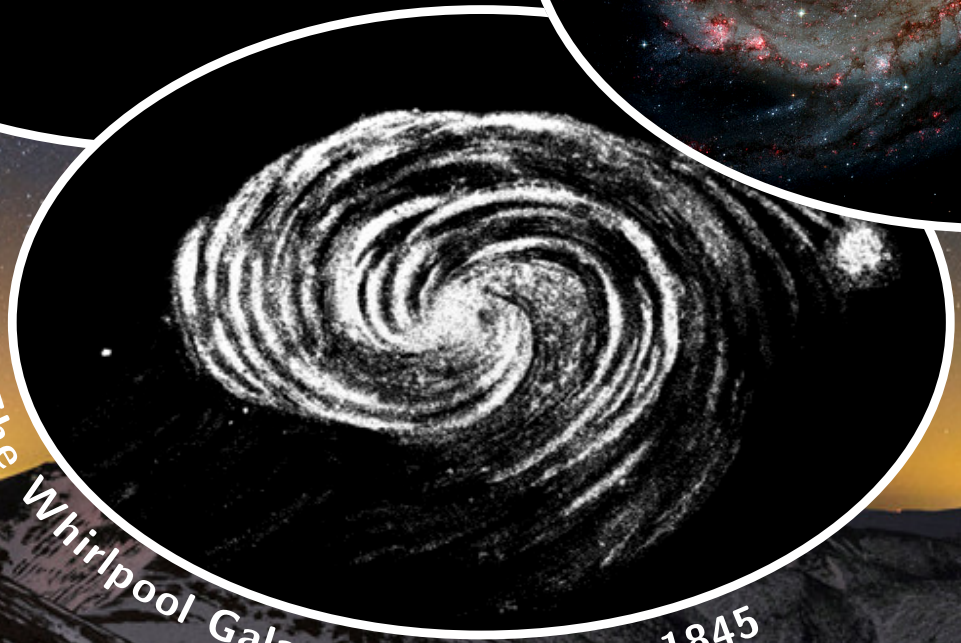
Mapping the Universe







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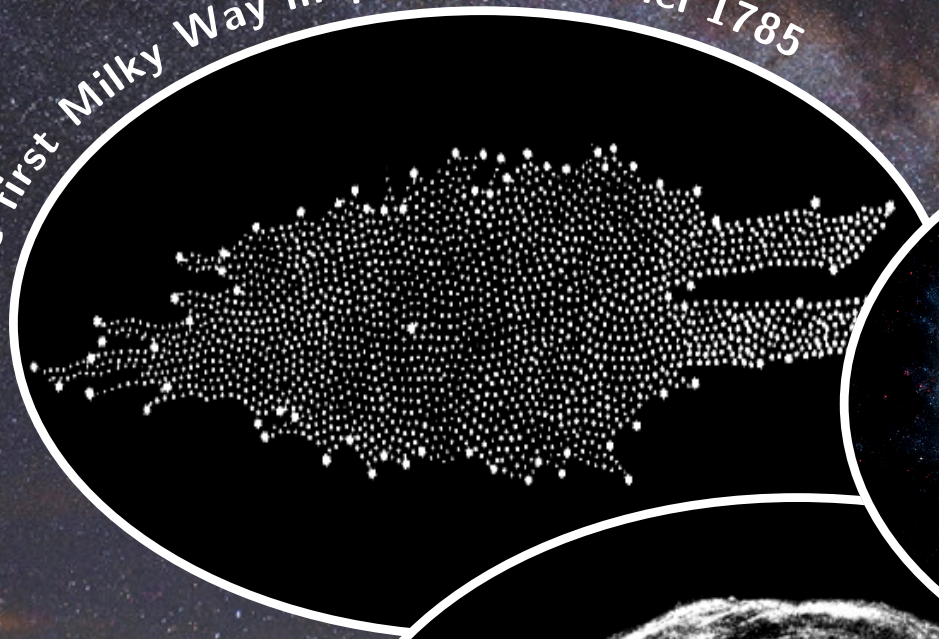
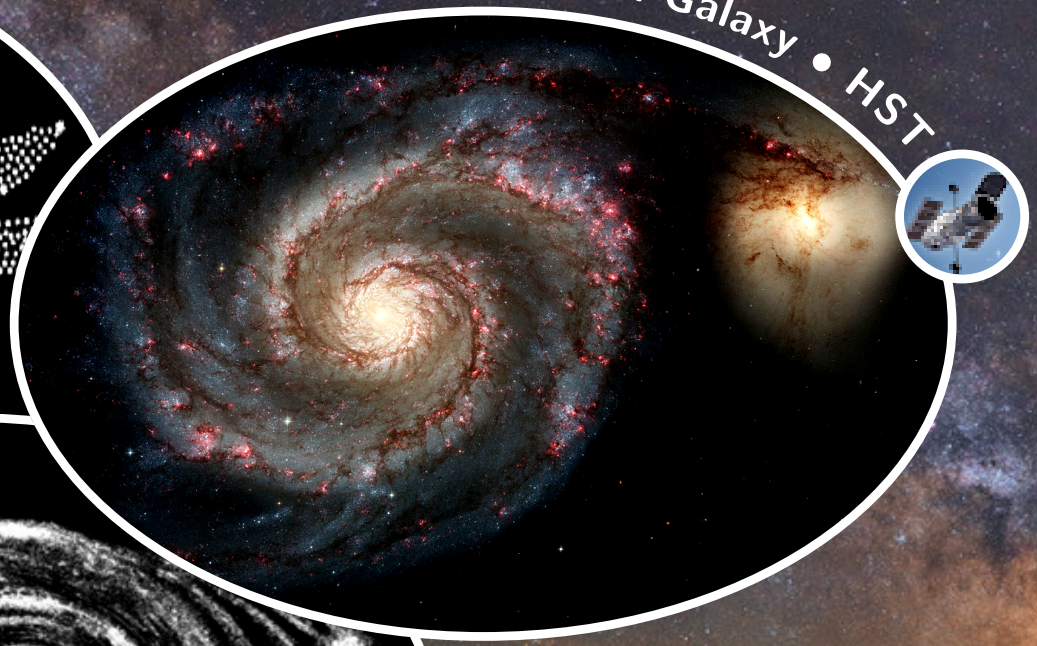
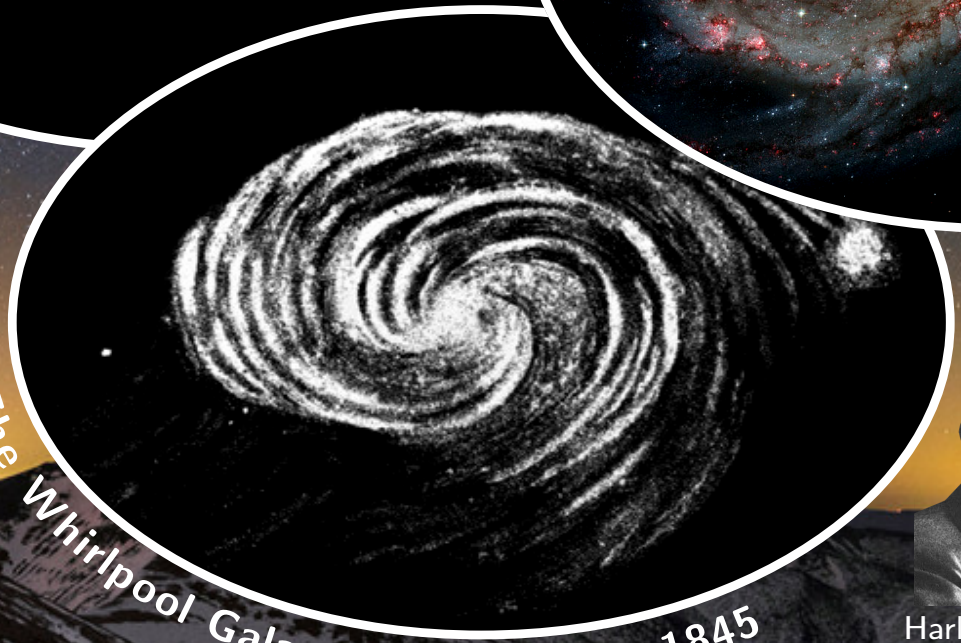
Mapping the Universe







Harlow Shapley
(1885 - 1972)

Heber Curtis
(1872 - 1942)

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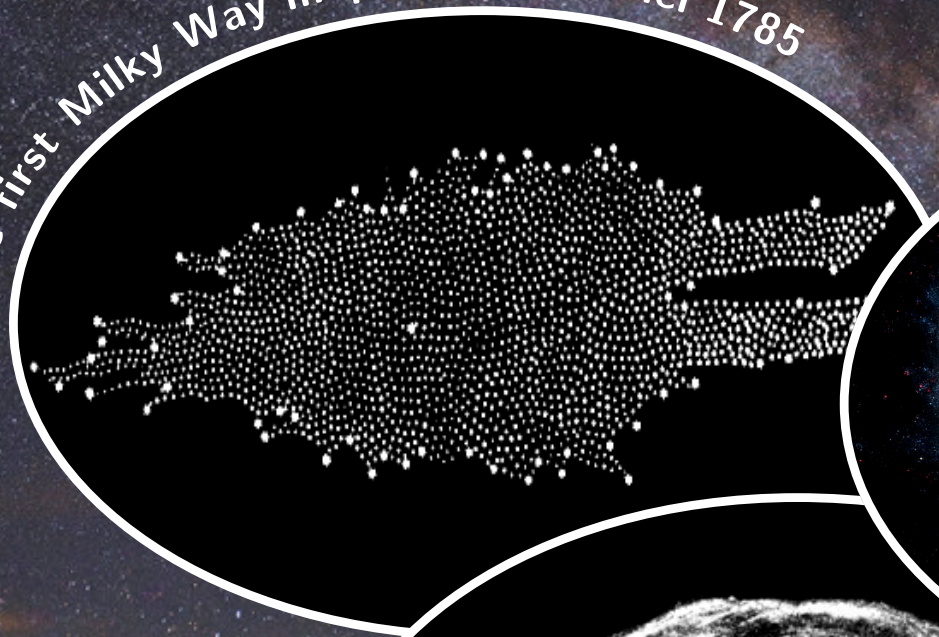
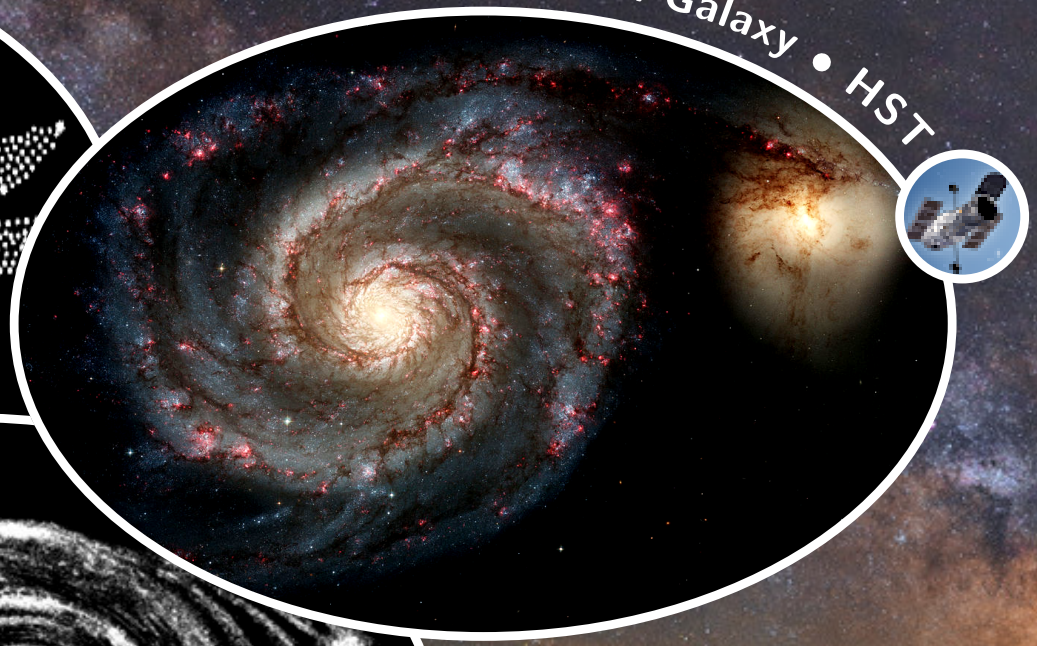
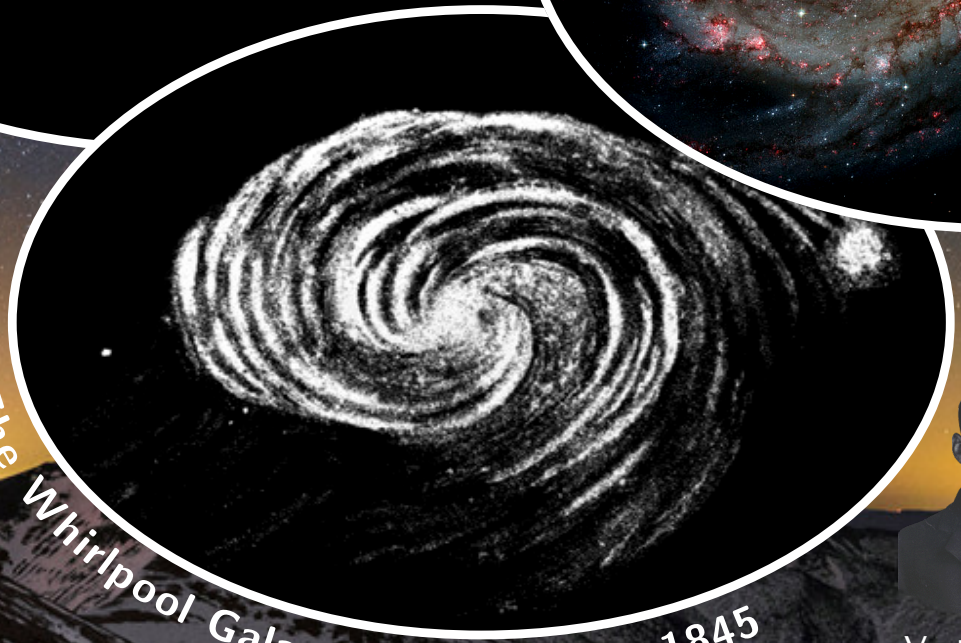
Mapping the Universe







Vesto Slipher
(1875 - 1969)

Edwin Hubble
(1889 - 1953)

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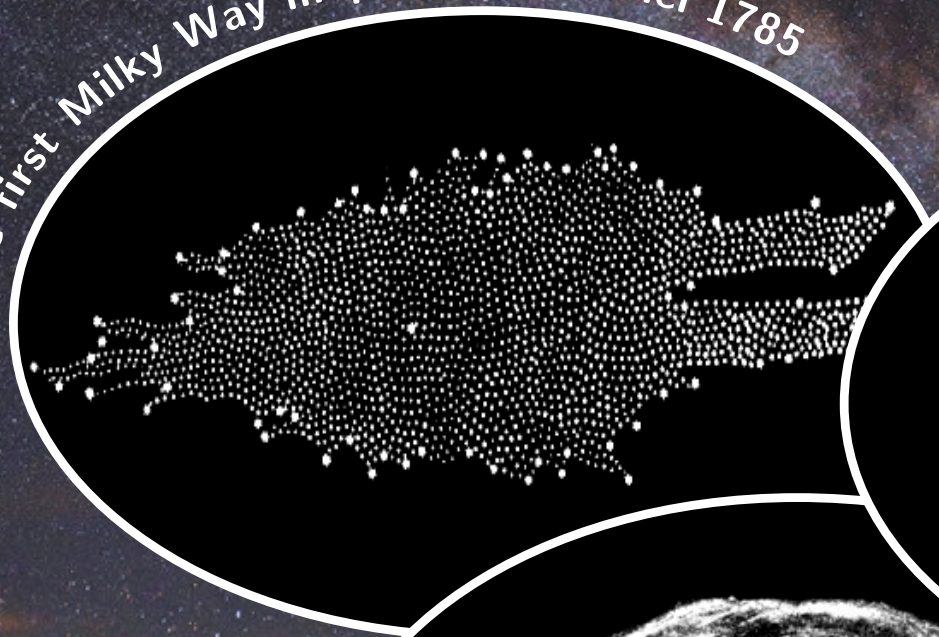
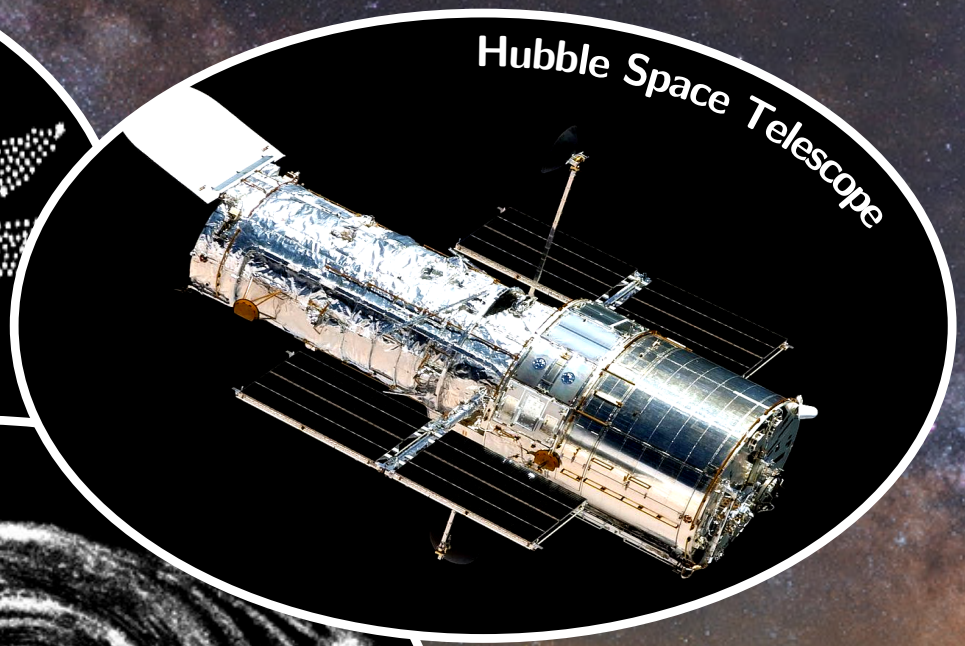
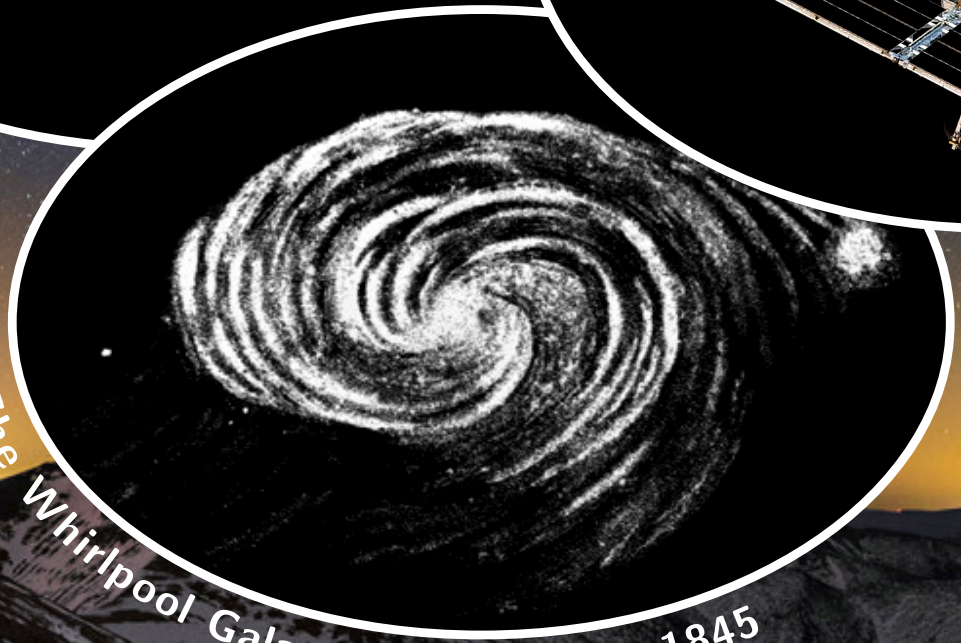
Mapping the Universe







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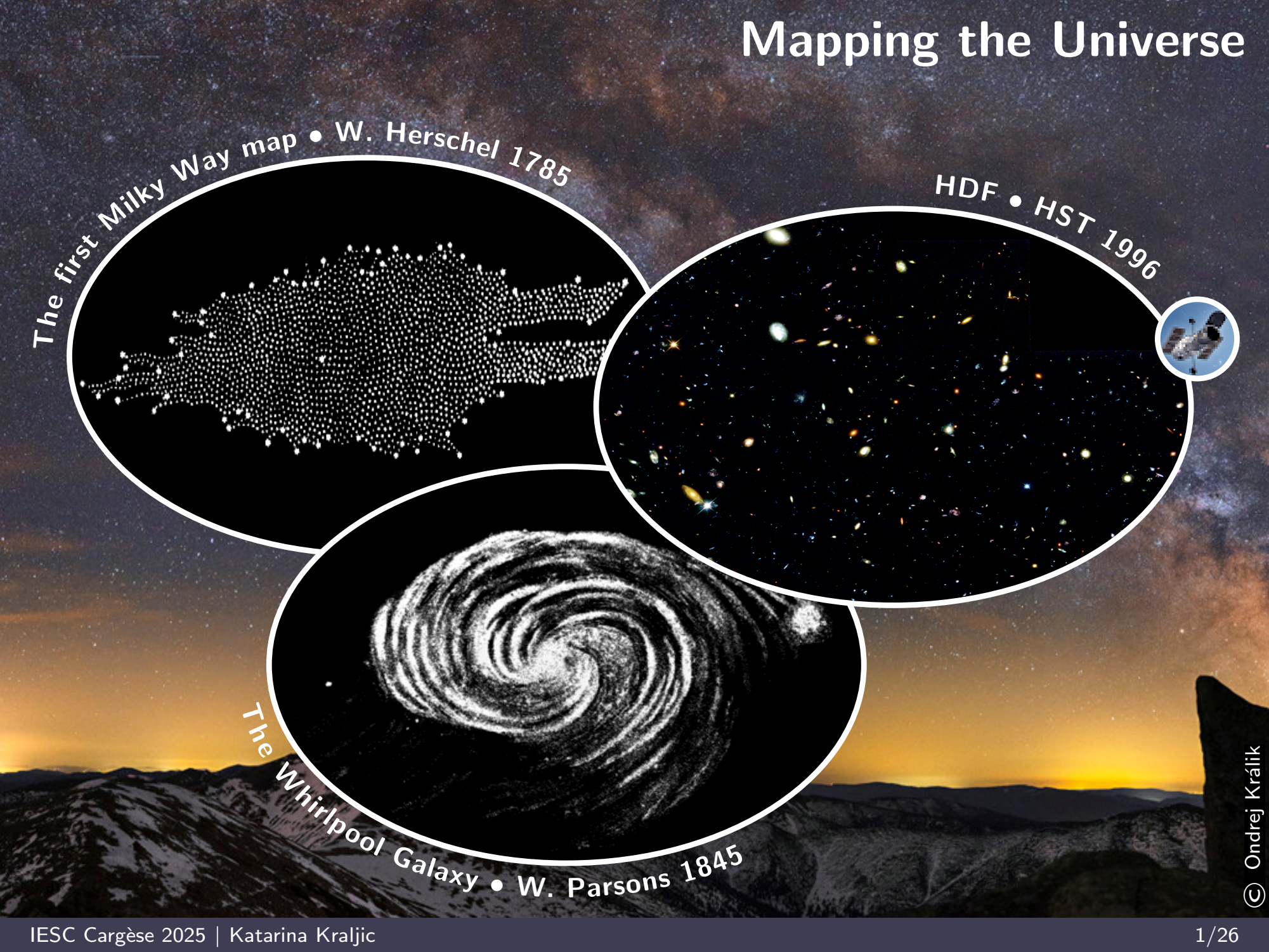
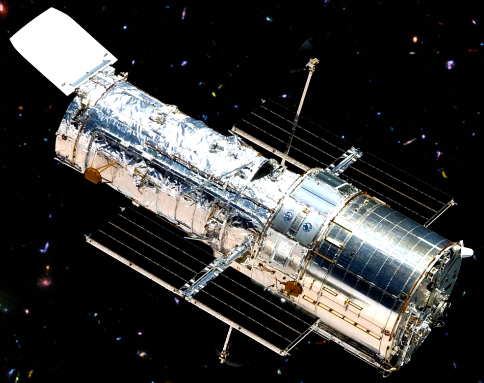
Mapping the Universe



Mapping the Universe

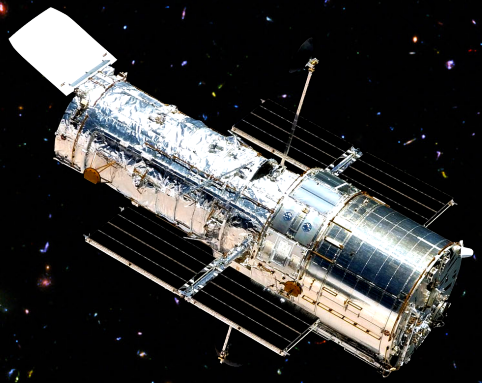
HUDF



Hubble Space Telescope

Mapping the Universe

HUDF

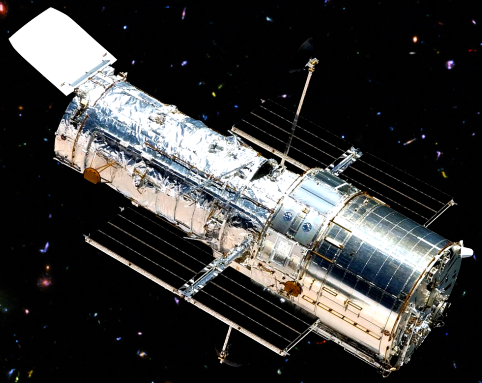


Hubble Space Telescope

collisions

Mapping the Universe

HUDF

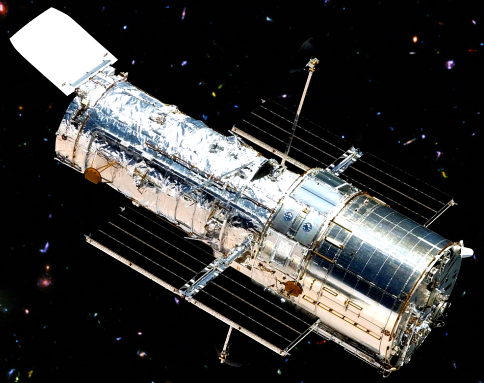


Hubble Space Telescope

spirals

Mapping the Universe

HUDF

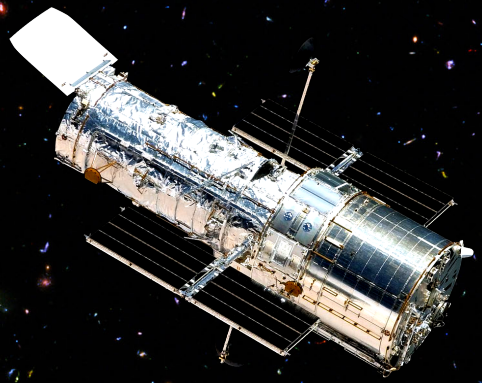


Hubble Space Telescope

ellipticals

Mapping the Universe

HUDF

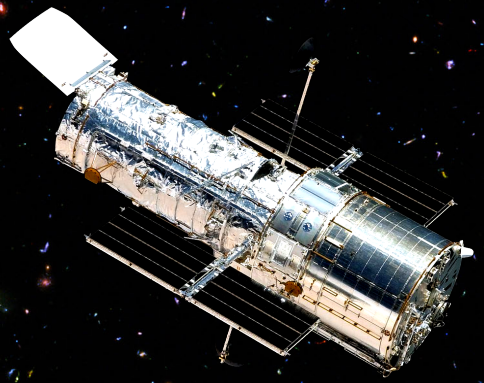


Hubble Space Telescope

bar

Mapping the Universe

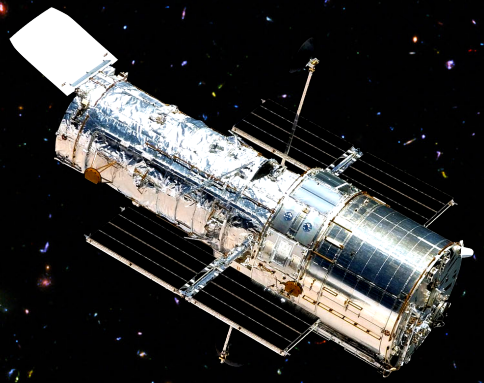
HUDF



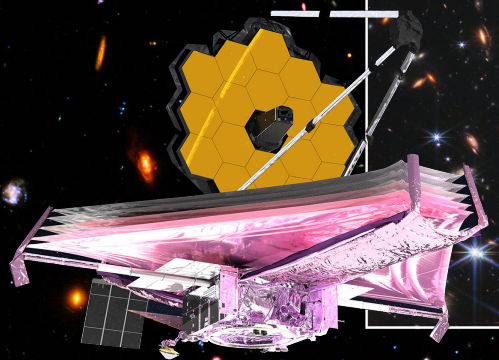
Hubble Space Telescope

Mapping the Universe

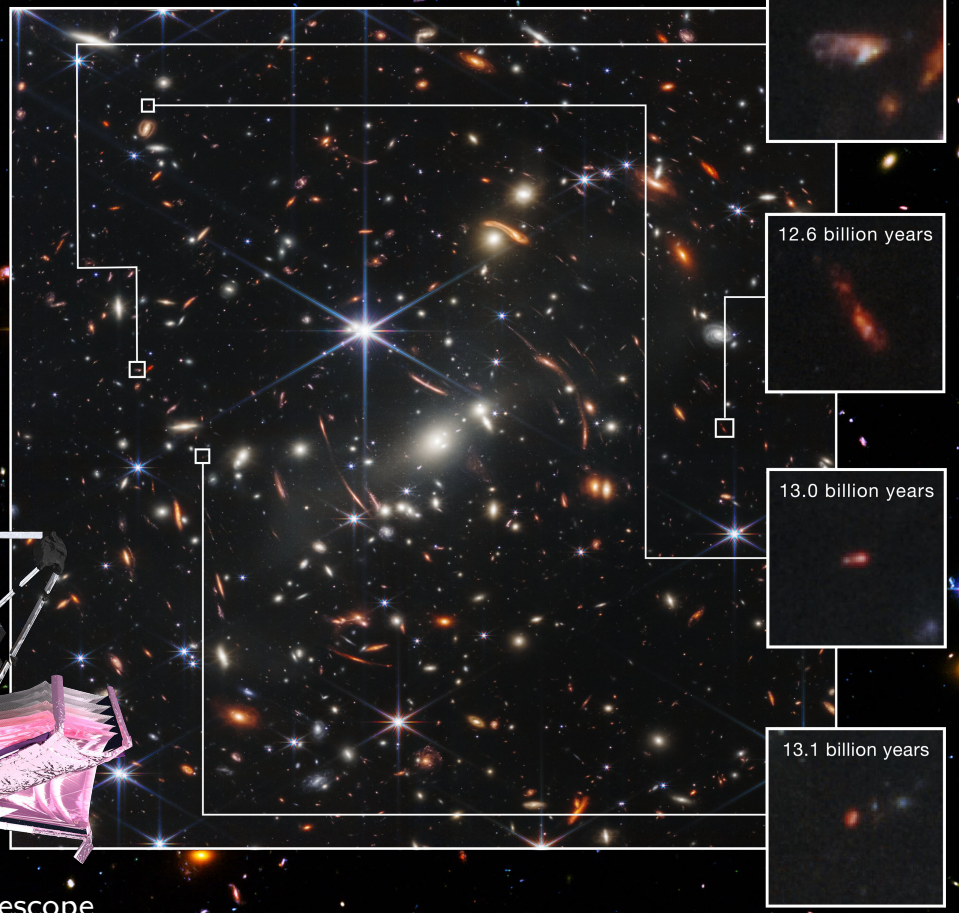
HUDF



Hubble Space Telescope



James Webb Space Telescope



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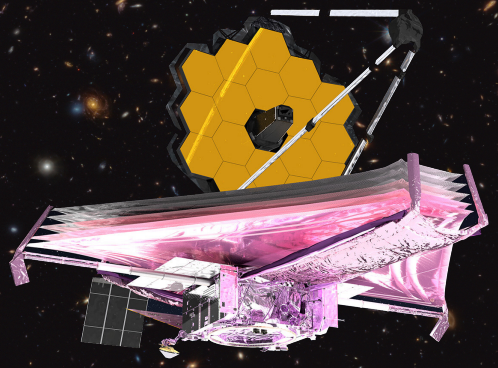
Mapping the Universe

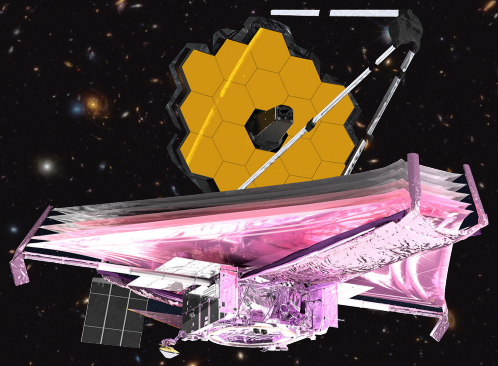
JADES



James Webb Space Telescope

Mapping the Universe

JADES

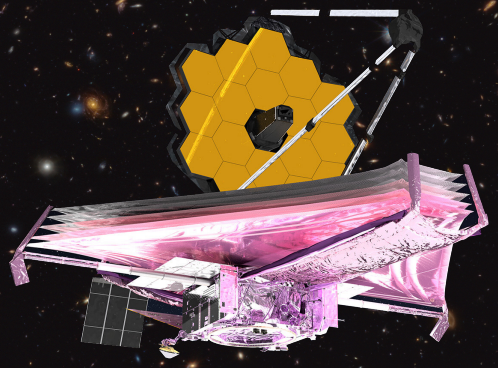


James Webb Space Telescope

spirals

Mapping the Universe
JADES

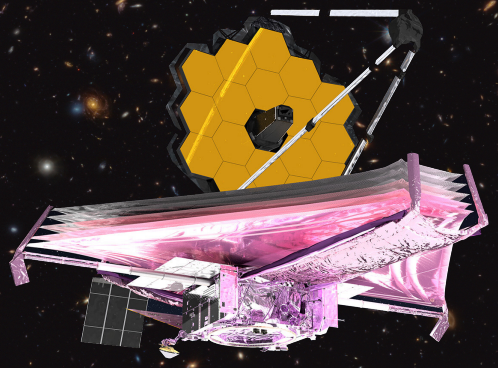
James Webb Space Telescope



bars

Mapping the Universe
JADES

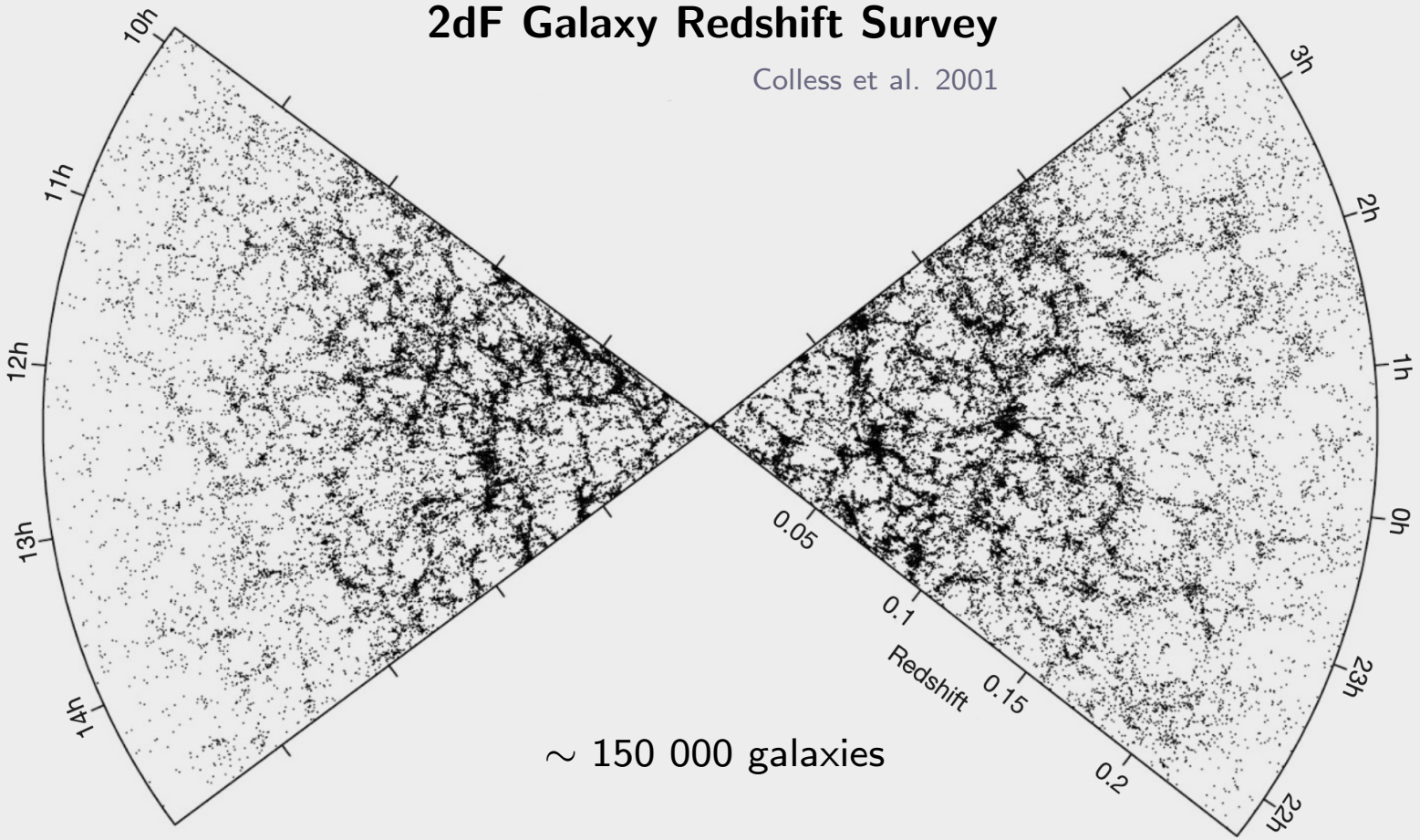
James Webb Space Telescope



bars

Mapping the Universe

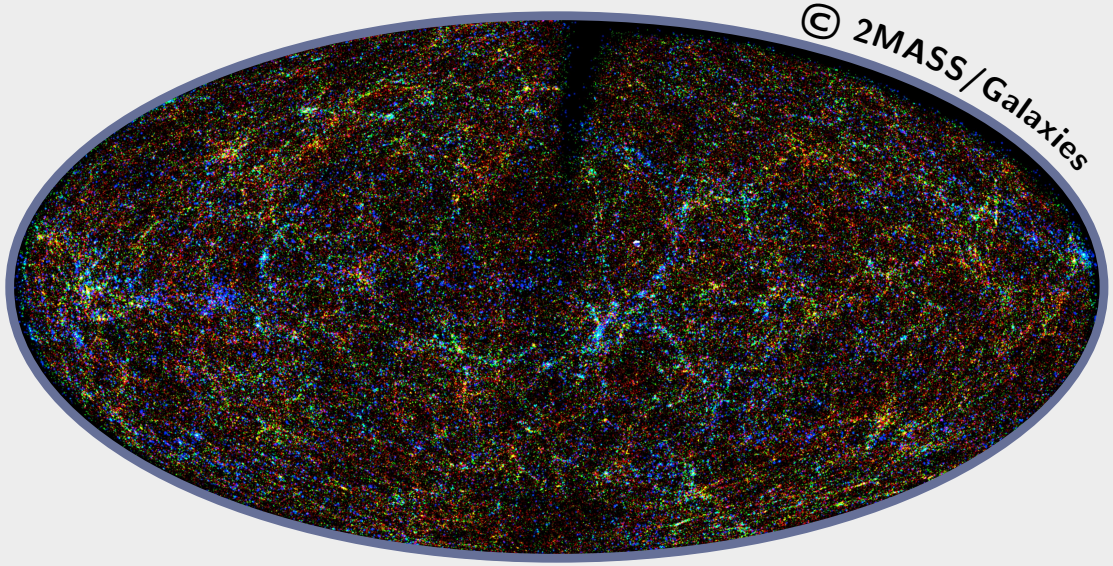
Large-scale galaxy distribution



Peacock et al. 2001

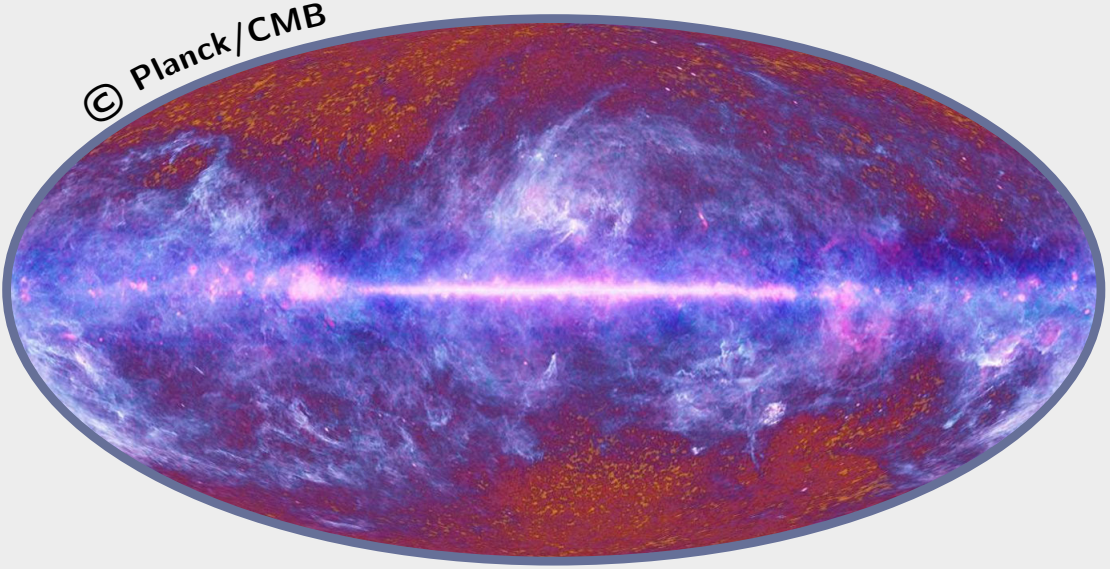
Mapping the Universe

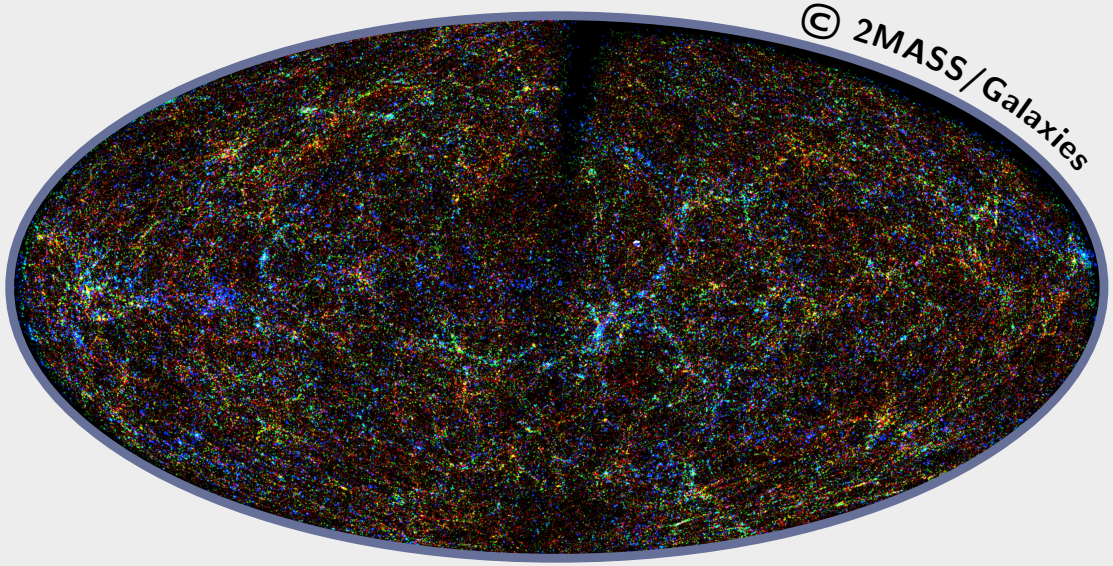
Connecting cosmic epochs



Mapping the Universe

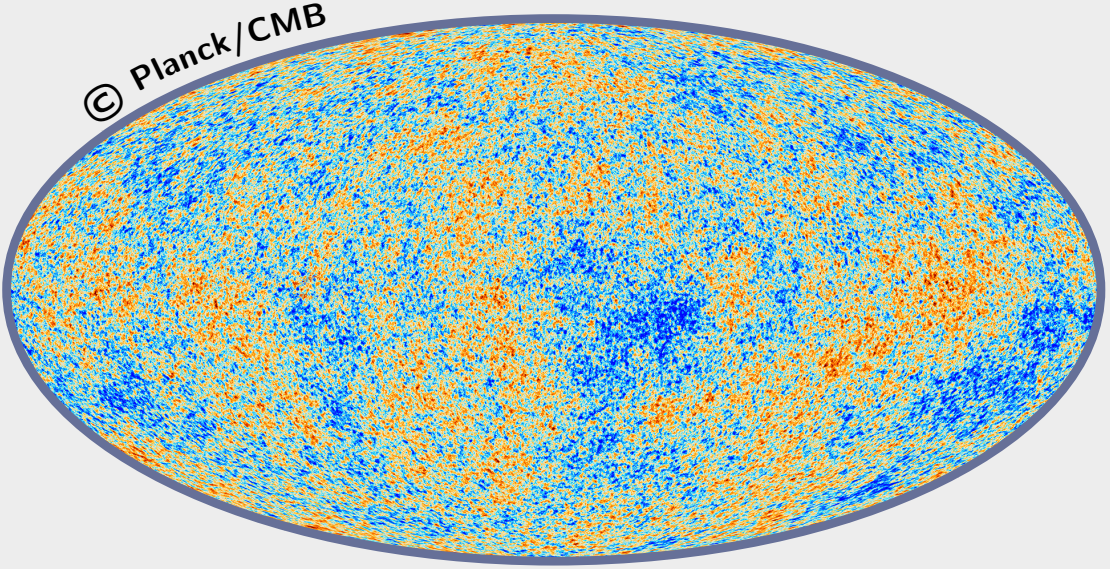
Connecting cosmic epochs

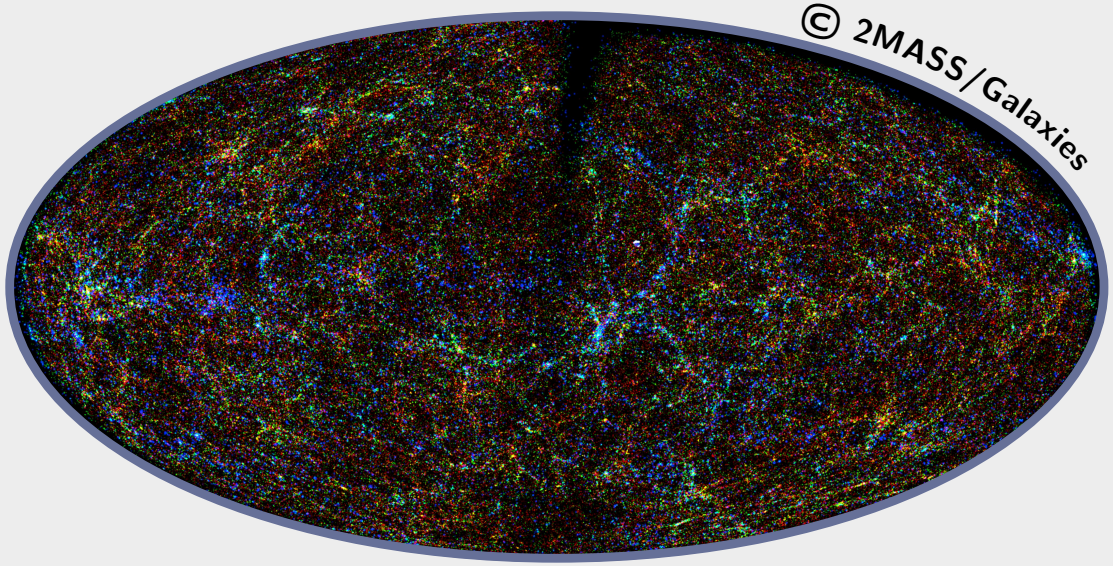




Mapping the Universe

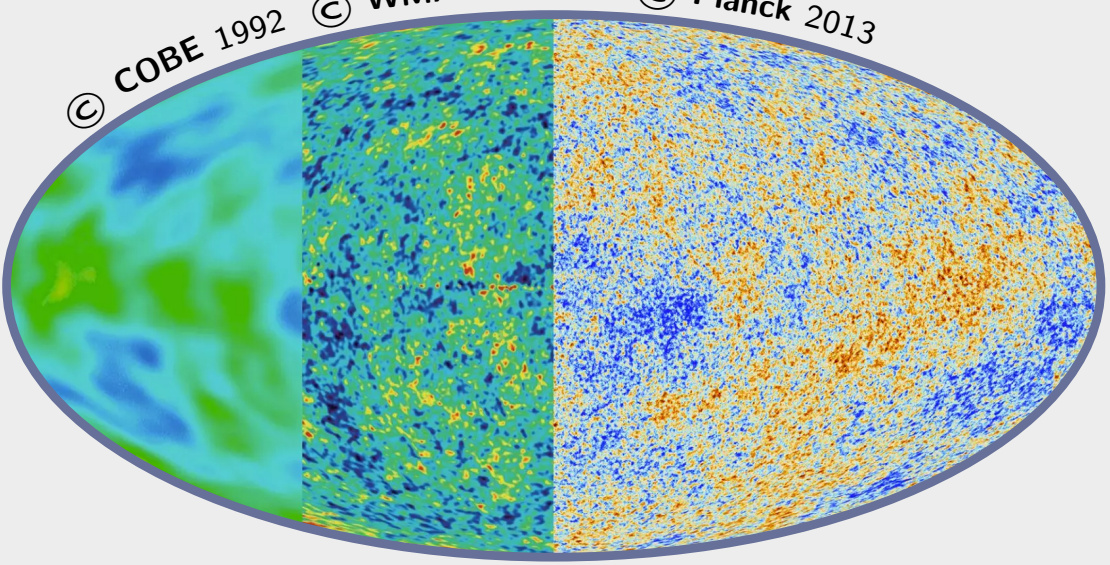
Connecting cosmic epochs

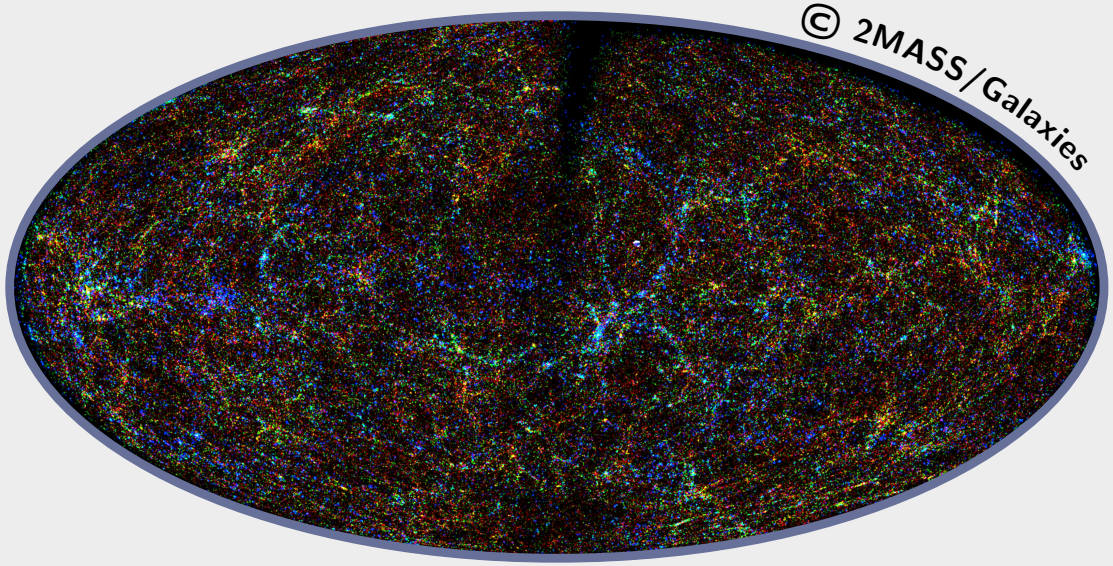




Mapping the Universe

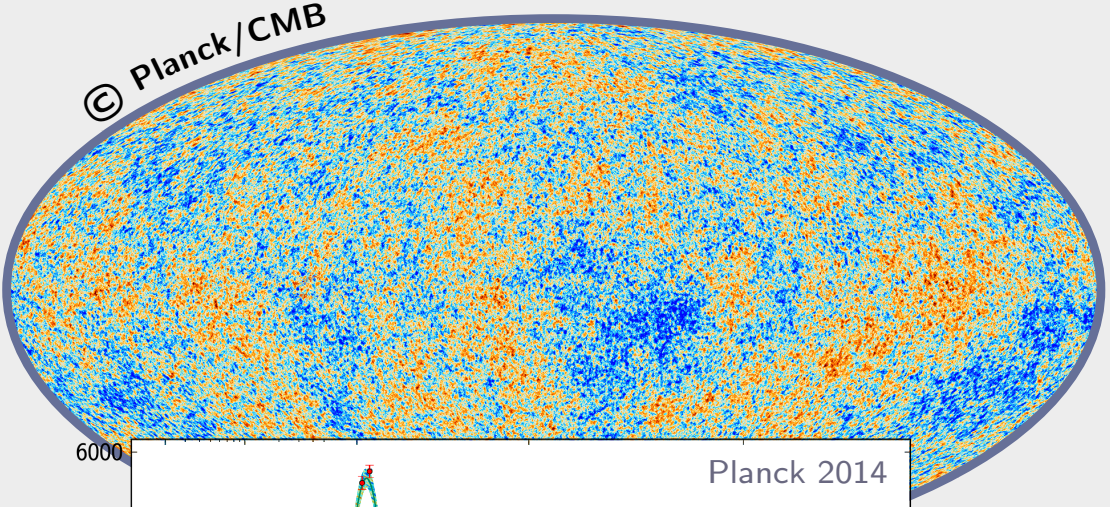
Connecting cosmic epochs

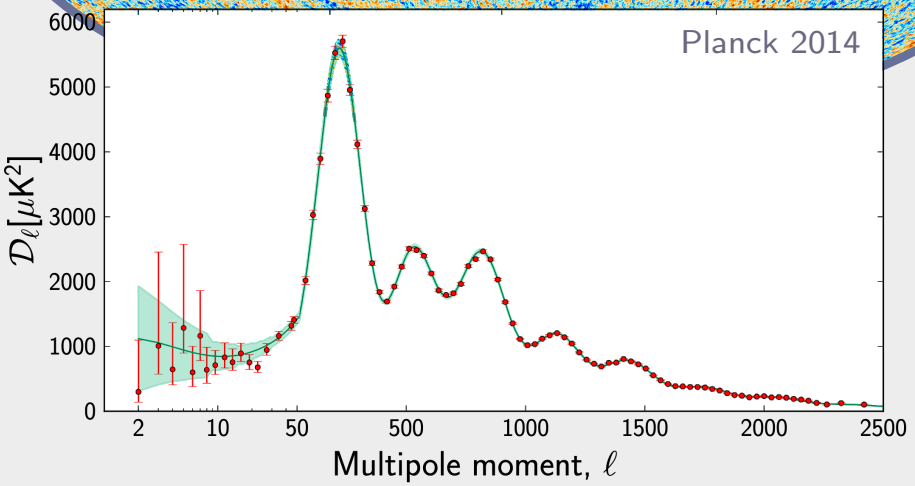


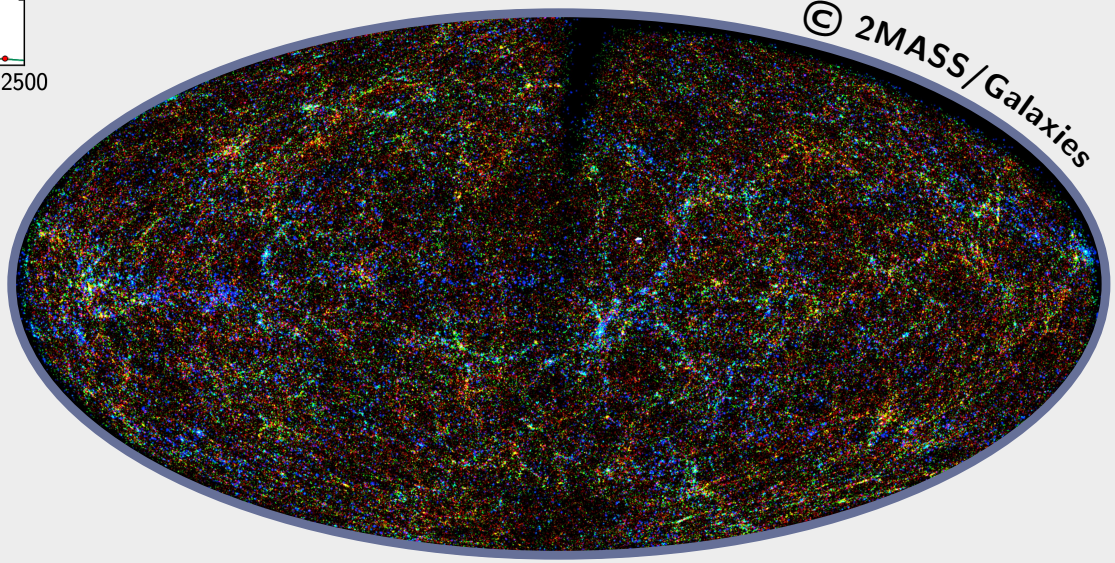


Mapping the Universe

Connecting cosmic epochs

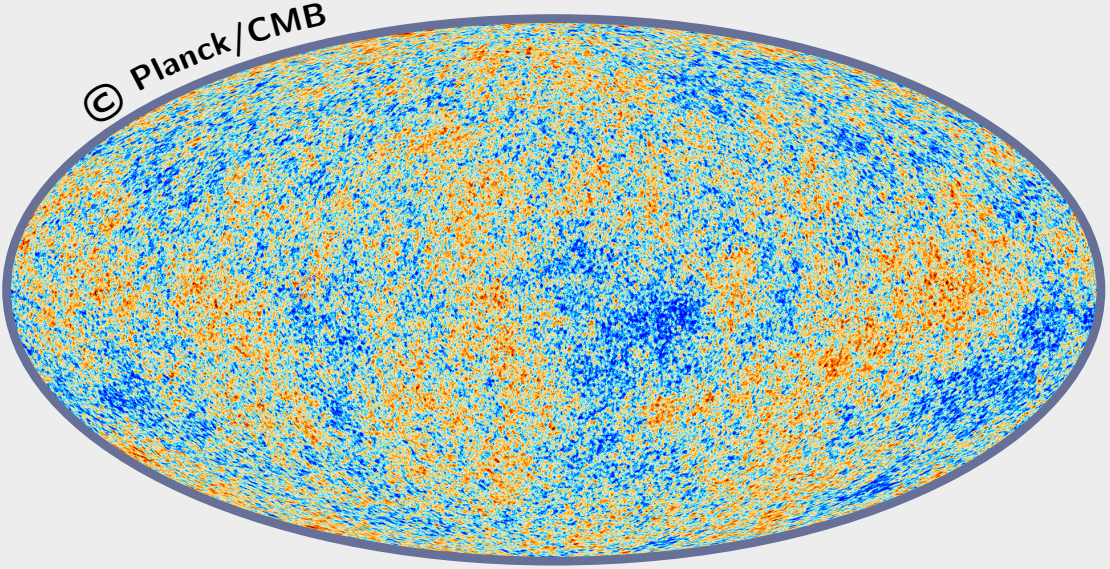




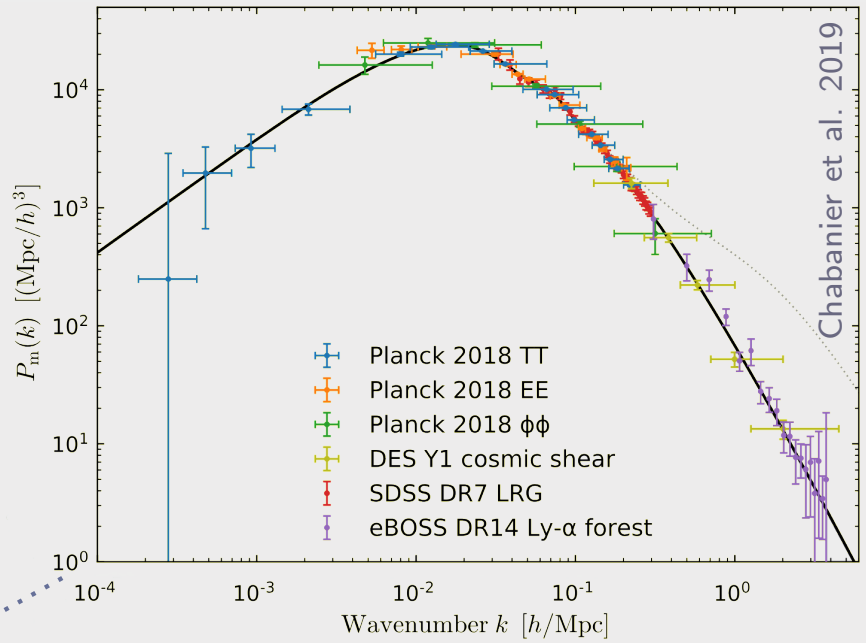


Mapping the Universe

Connecting cosmic epochs



Early Universe

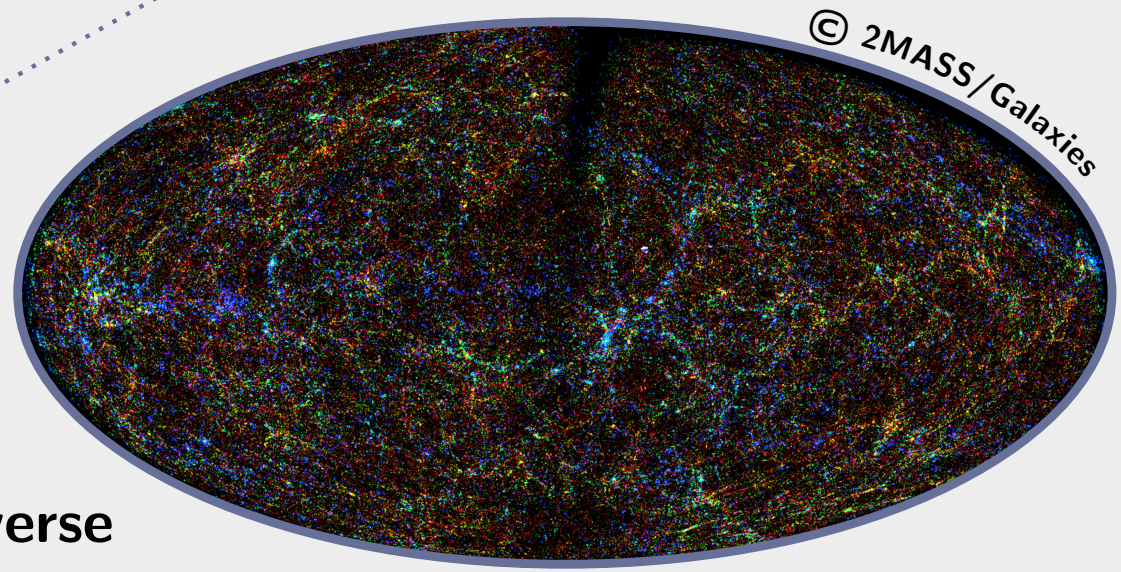


Concordance ΛCDM^{++} model

Ω_m, Ω_Λ, Ω_b, n_s, A_s, Σm_ν, w_0, w_a

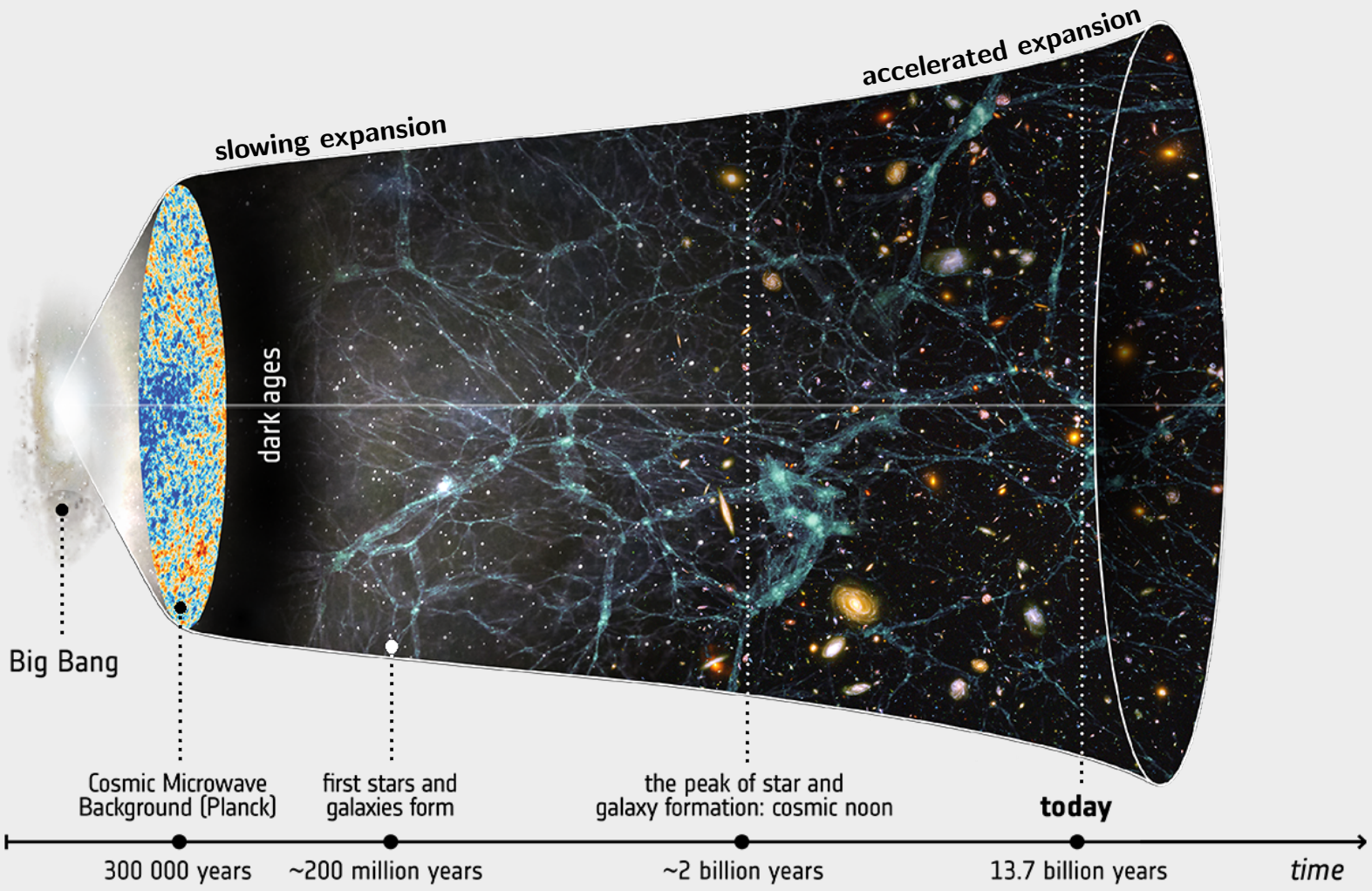
energy content | inflation | neutrinos | DE EoS

+ General relativity
+ Nature of Dark Matter
+ Astrophysics



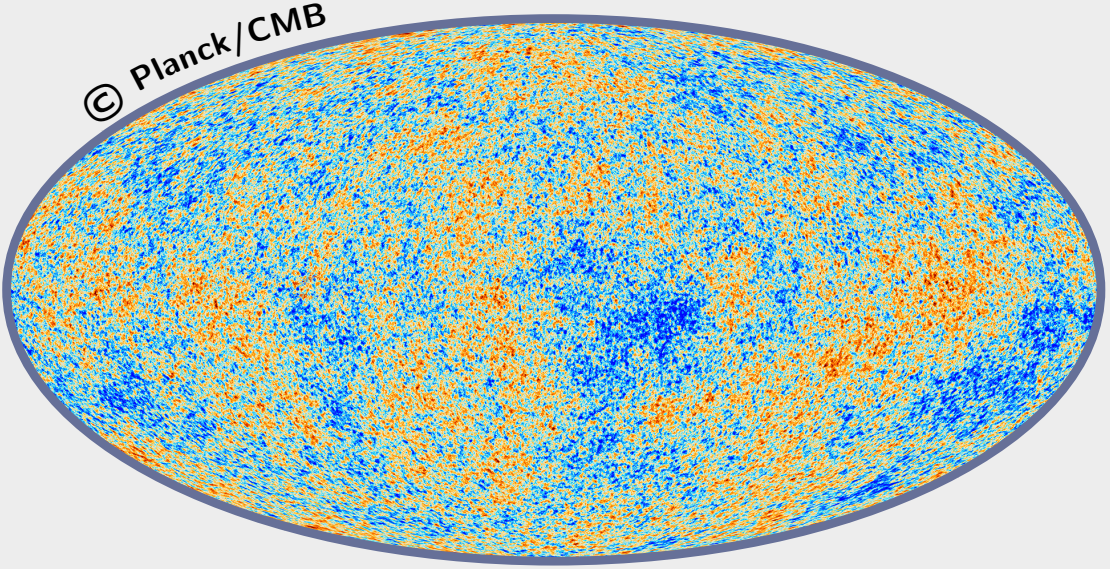
Today's Universe

The Universe across space and time



Mapping the Universe

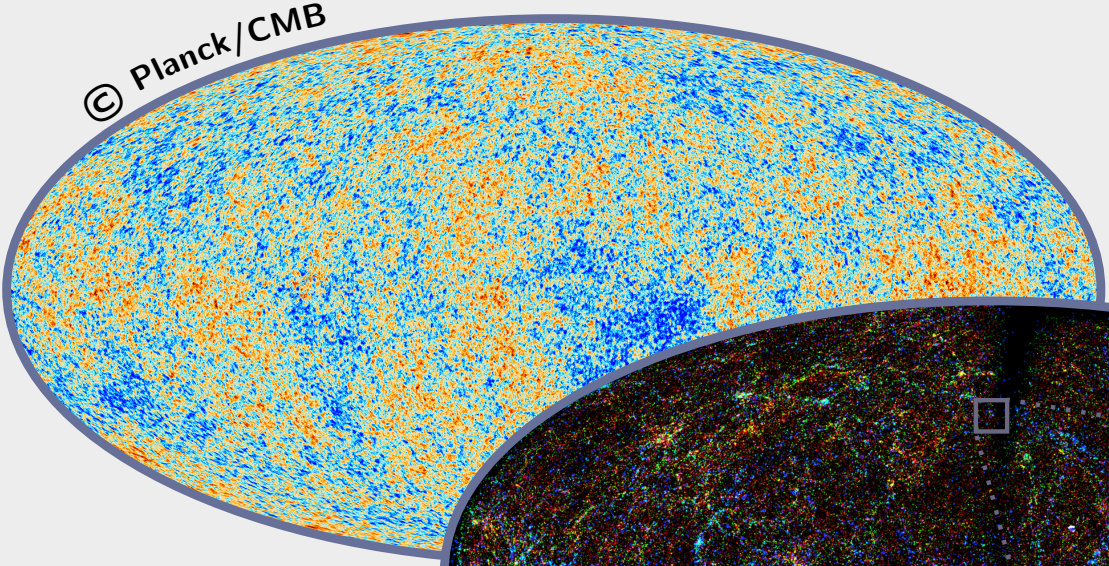
Non-linear challenge

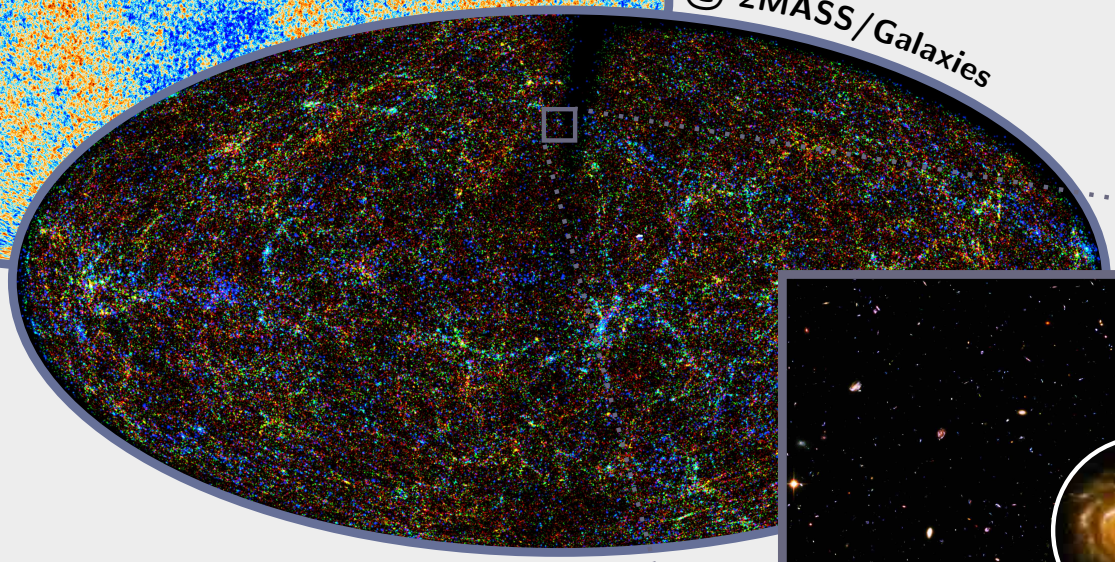


linear

Mapping the Universe

Non-linear challenge

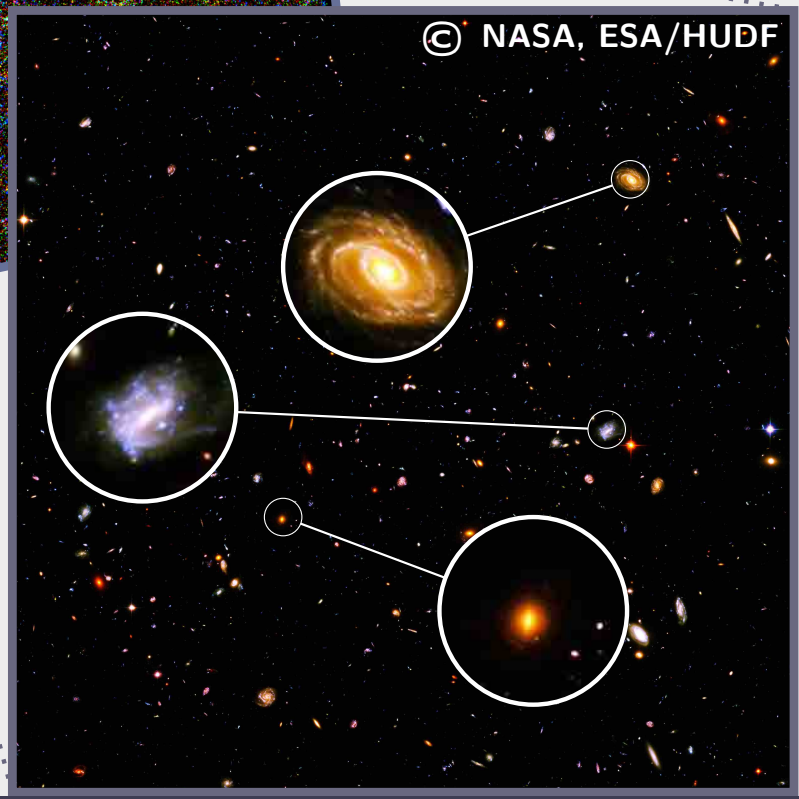




linear

mildly non-linear

highly non-linear



Structure formation

. . . Understanding galaxy formation

Structure formation

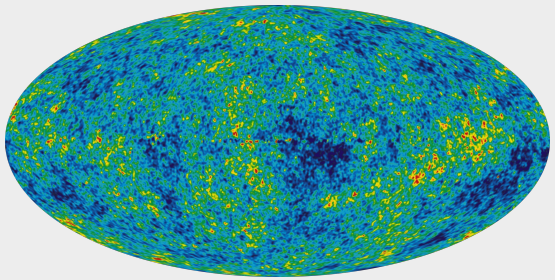
. . . Understanding galaxy formation

① initial conditions

Structure formation

. . . Understanding galaxy formation

① initial conditions



ΛCDM

- CDM paradigm: success on large scales
- simplest (6 parameters only)
- sufficiently accurate

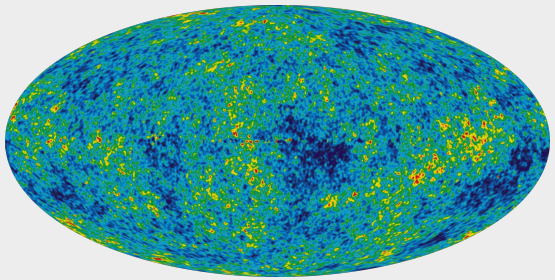
Structure formation

. . . Understanding galaxy formation

① initial conditions

⊕

② ingredients



ΛCDM

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Structure formation

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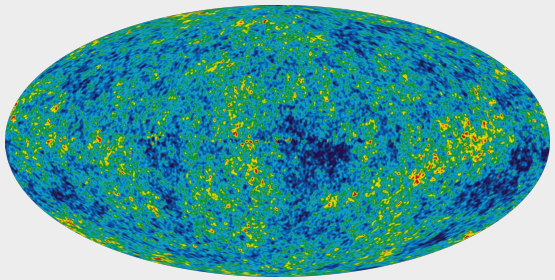
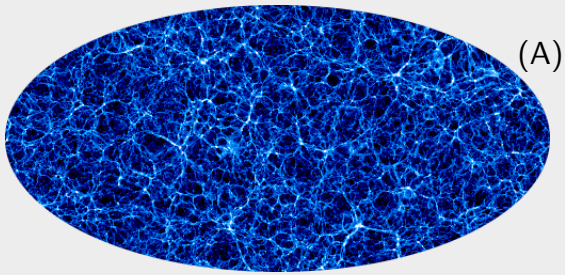




(A) DM only

ΛCDM

- CDM paradigm: success on large scales
- simplest (6 parameters only)
- sufficiently accurate

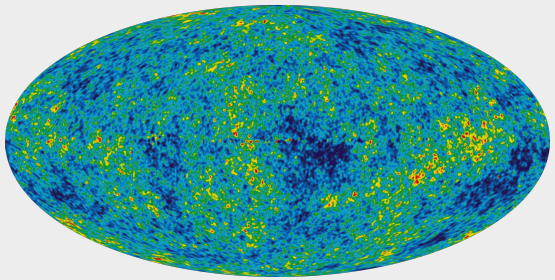
Structure formation

. . . Understanding galaxy formation

① initial conditions

⊕

② ingredients



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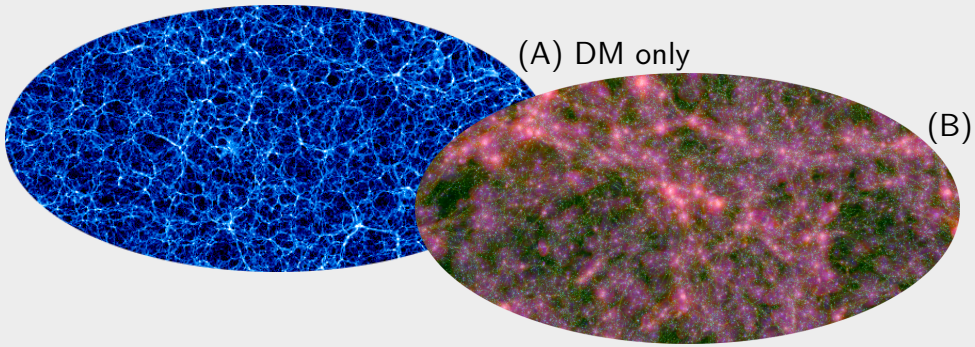

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use

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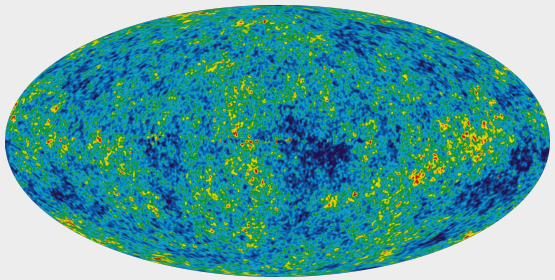

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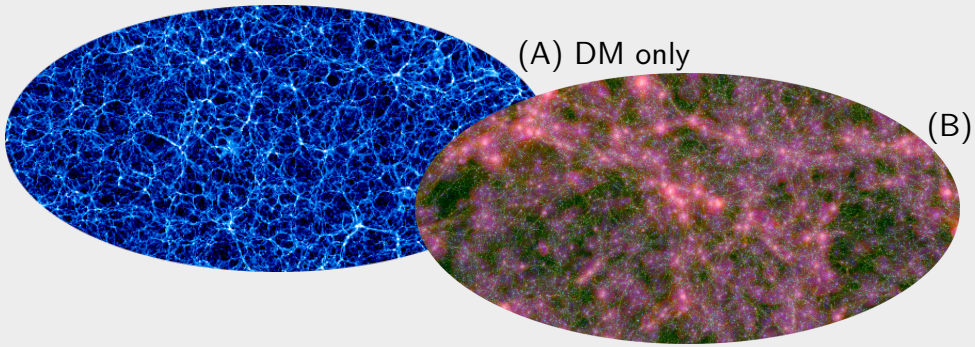

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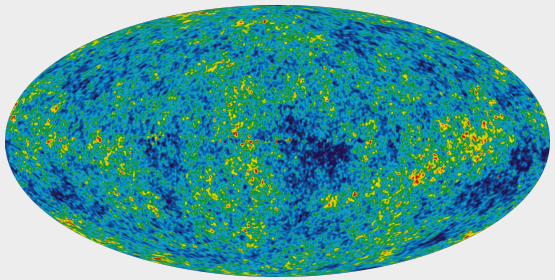

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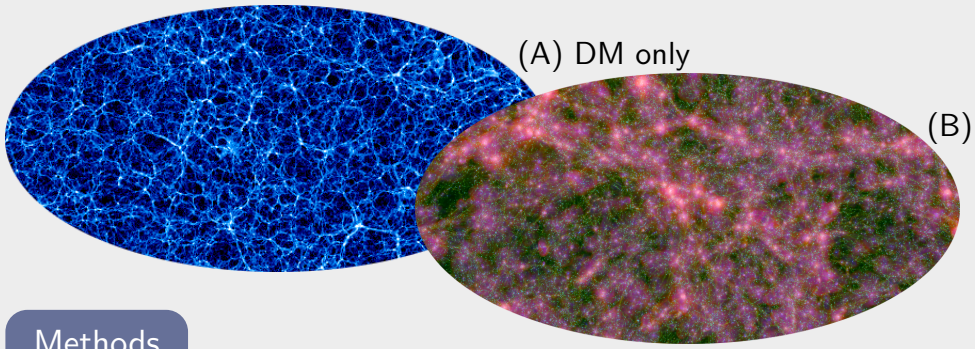

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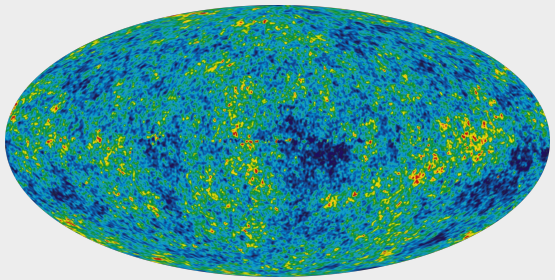

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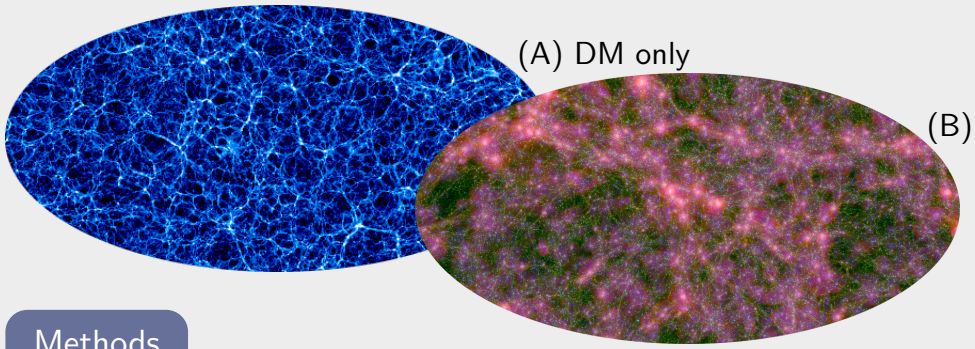

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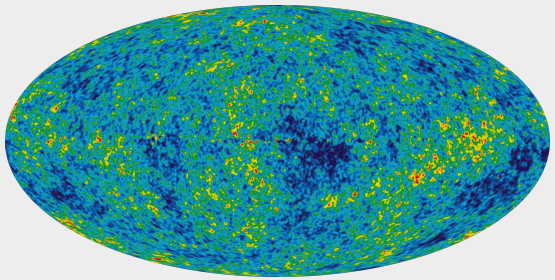

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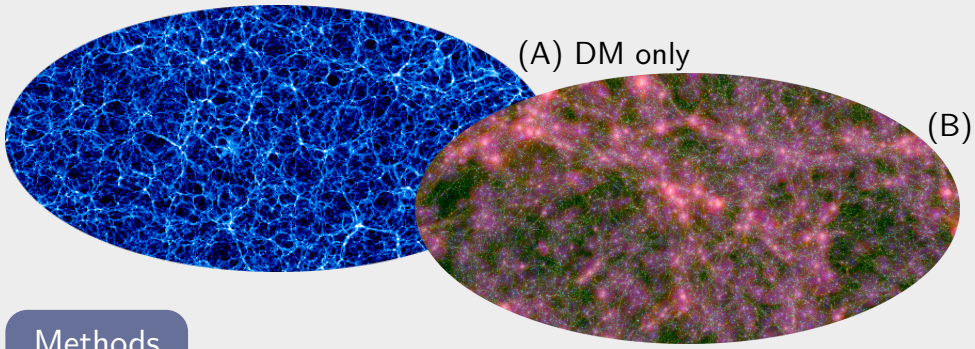

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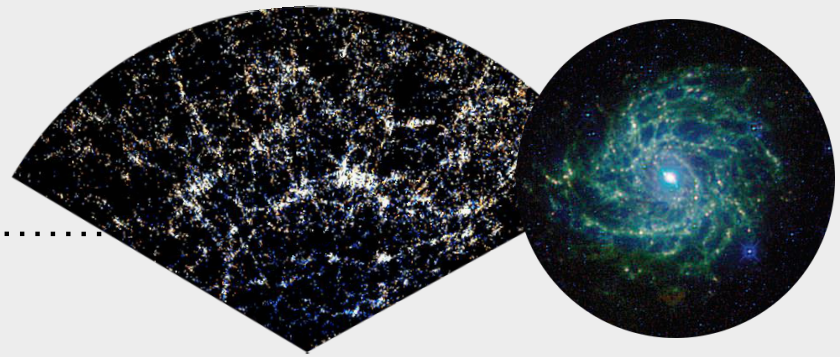

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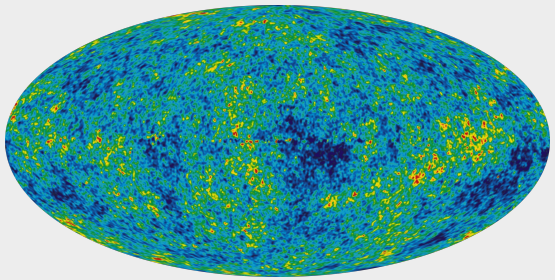

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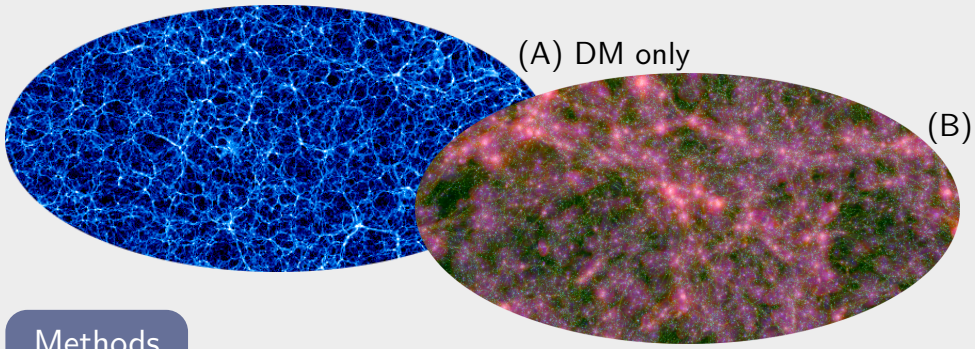

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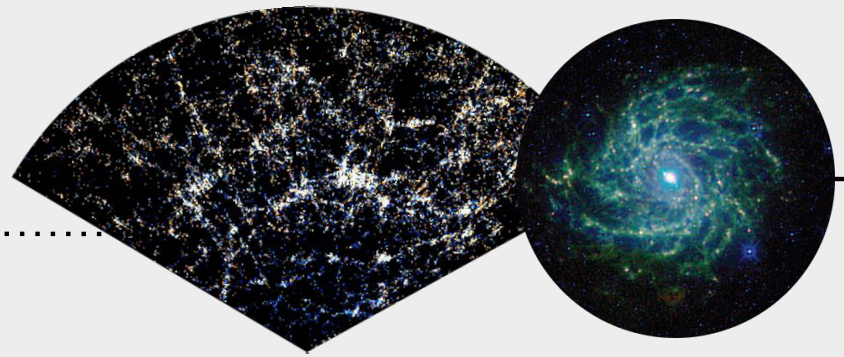

→ Test predictions

Structure formation

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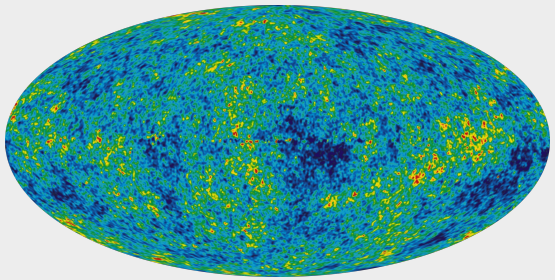

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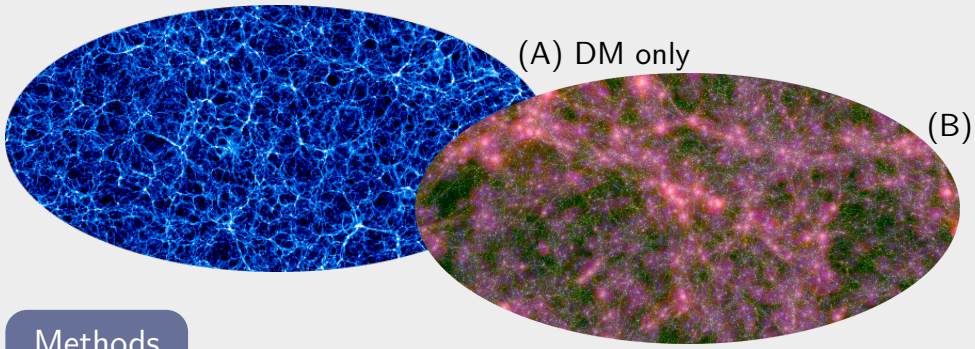

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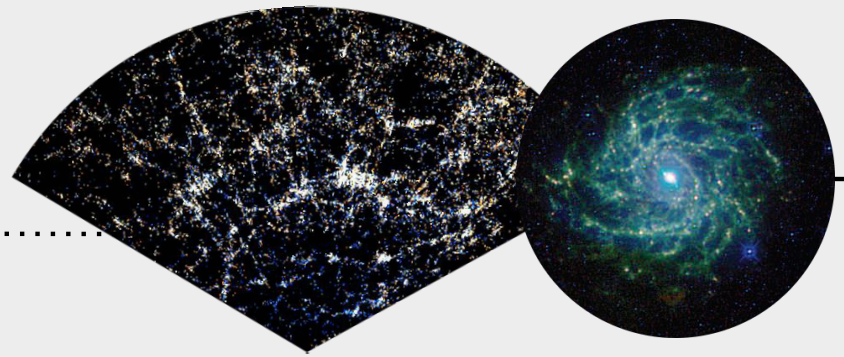
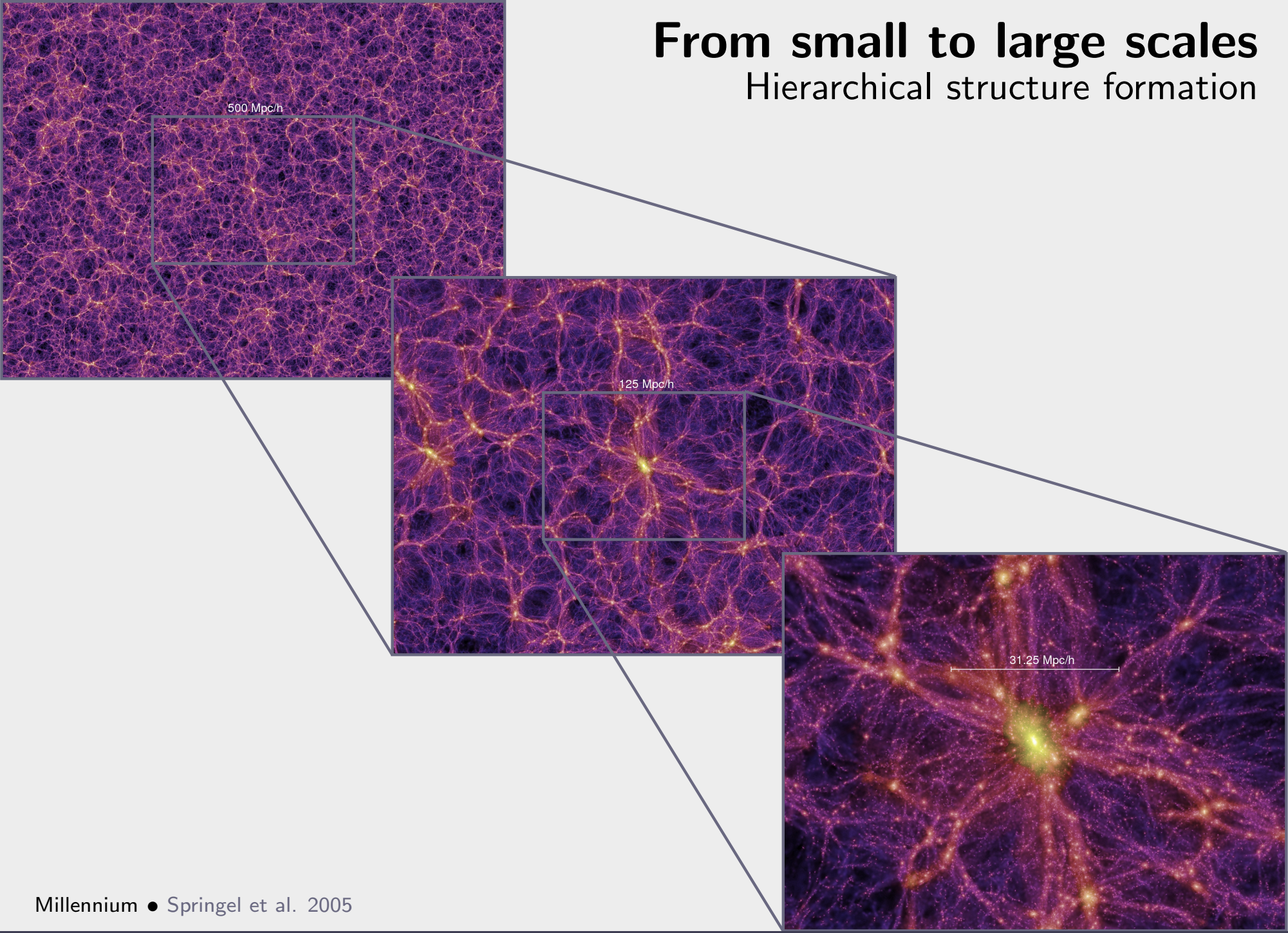


④ observations

Test predictions

From small to large scales

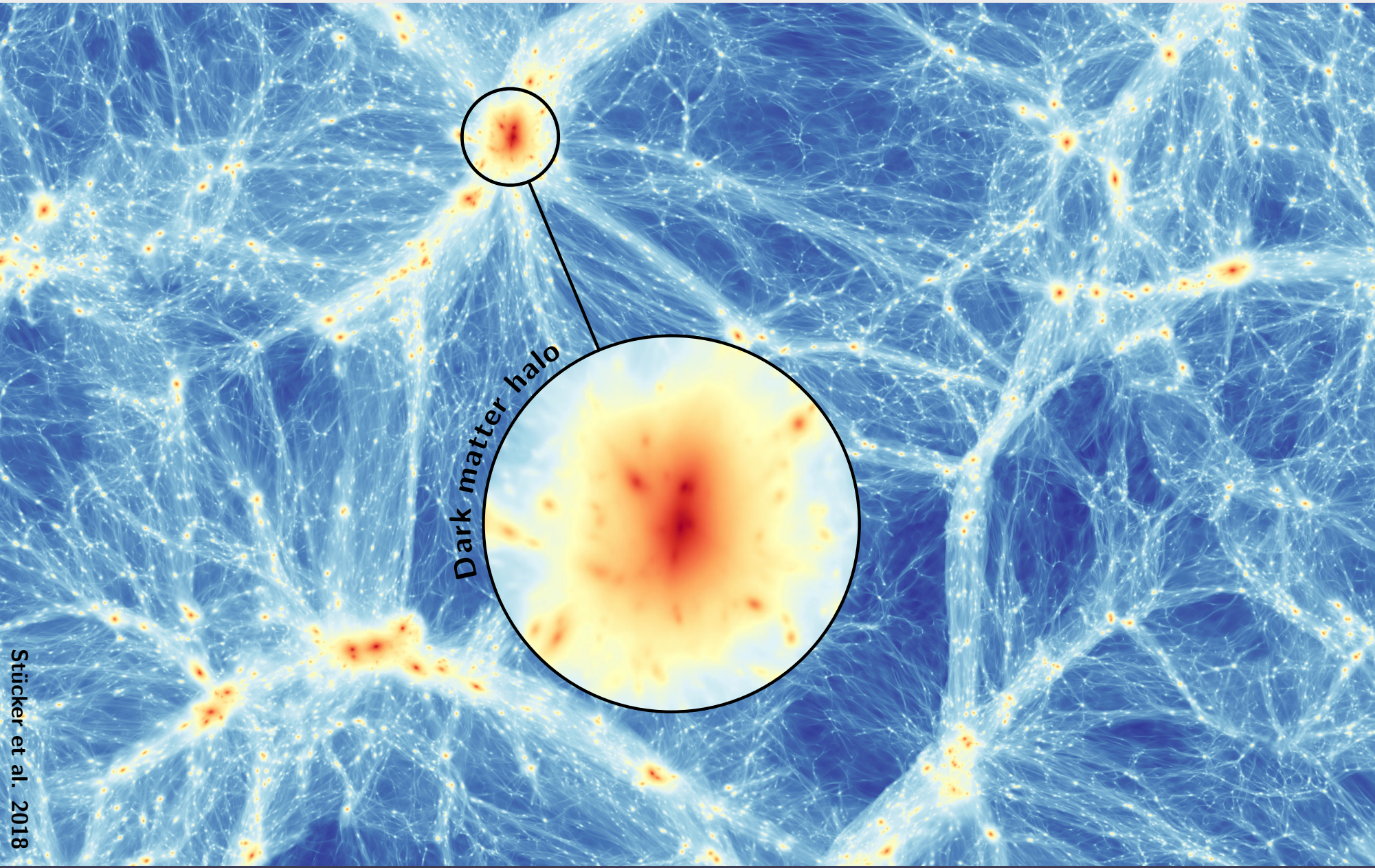
Hierarchical structure formation



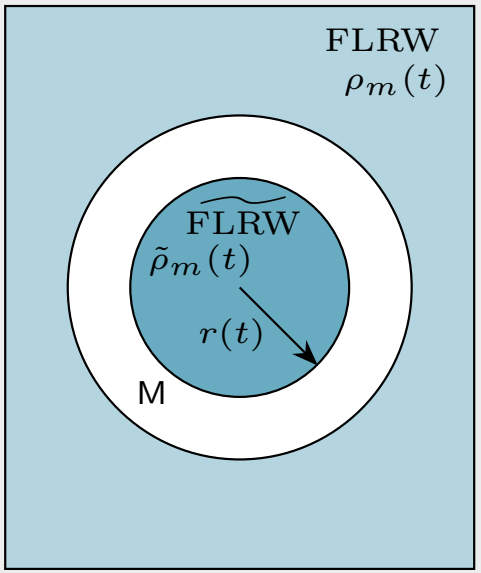
Millennium • Springel et al. 2005

Dark matter halos

Basic building blocks of non-linear structure



Spherical collapse



isolated, uniform, spherically symmetric overdense region

$$\tilde{\rho}_m(t) = \frac{M}{4/3\pi r^3(t)}$$

$r(t)$ - physical radius

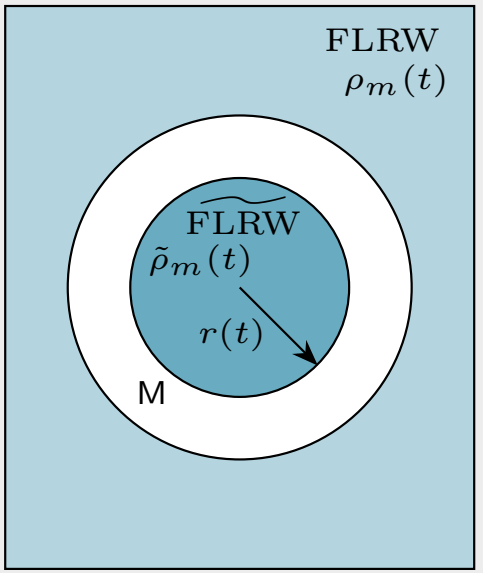
Spherical collapse

- 2nd Friedmann equation of the region:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3p)$$

$$\Rightarrow \quad \ddot{r}(t) = -\frac{GM}{r^2(t)} + \frac{8\pi G}{3}\rho_\Lambda r(t)$$

Newtonian **Λ term**



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Newtonian **Λ term**

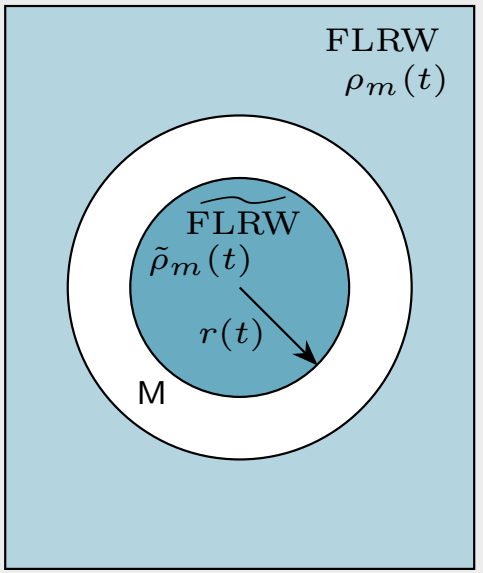
- parametric solution for the case without Λ:

$$r(t) = A(1 - \cos\theta)$$
$$t = B(\theta - \sin\theta)$$

$\rightarrow$ shell expands from $r = 0$ at $\theta = 0$ $(t = 0)$

$\rightarrow$ shell reaches $r_{\max}$ at $\theta = \pi$ $(t_{\max} = \pi B$; turn-around time)

$\rightarrow$ shell collapses back to $r = 0$ at $\theta = 2\pi$ $(t_{\rm coll} = 2t_{\max})$



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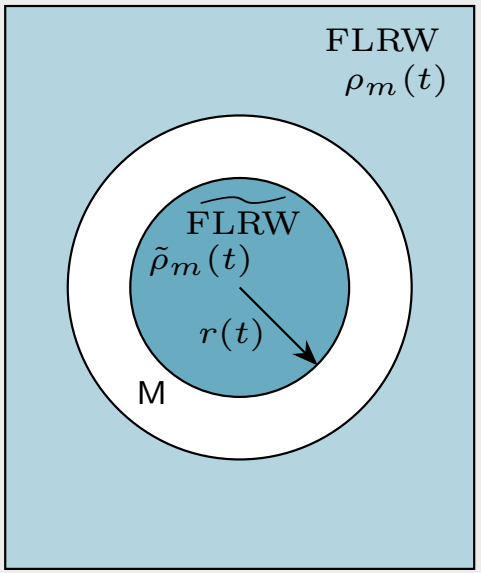

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- for Einstein-de Sitter cosmology:

$$1 + \delta = \frac{\rho}{\overline{\rho}} = \frac{9}{2}\frac{(\theta - \sin\theta)^2}{(1 - \cos\theta)^3}$$ - overdensity for spherical top-hat

$$\delta_{\rm lin} = \frac{3}{20}(6\pi)^{2/3}\left(\frac{t}{t_{\rm max}}\right)^{2/3}$$ - linearly extrapolated δ

$$\delta_{\rm lin}(t_{\rm coll}) = \frac{3}{20}(12\pi)^{2/3} \simeq 1.686$$ - linearly extrapolated critical δ_c for collapse

Shell crossing & Virialisation

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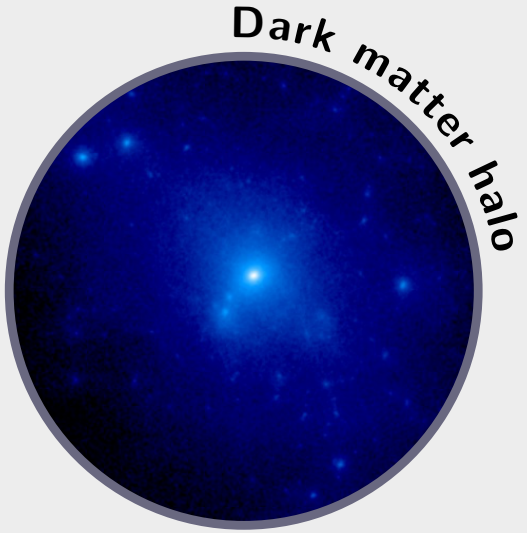
- top-hat spherical collapse only valid up to point of collapse, $\delta(t_{\rm coll}) = \infty$
- collapse is never perfectly spherical
- oscillating shells interact gravitationally $\rightarrow$ virialized halo



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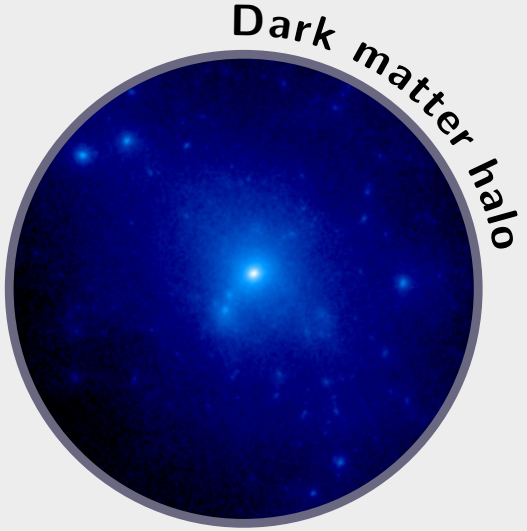
  virial equilibrium: $2K_{\rm f} + W_{\rm f} = 0$

  energy conservation: $E_{\rm f} = K_{\rm f} + W_{\rm f} = E_{\rm ta}$

$$\left.\begin{aligned} E_{\rm ta} = W_{\rm ta} &= -\frac{GM}{r_{\rm ta}} \\ E_{\rm f} = \frac{W_{\rm f}}{2} &= -\frac{GM}{2r_{vir}} \end{aligned}\right\} \Rightarrow \quad r_{\rm vir} = \frac{r_{\rm ta}}{2}$$

$$\rightarrow \theta_{\rm vir} = \frac{3\pi}{2}$$

$\rightarrow$ after virialization: $\rho_{\rm vir} = 8\rho_{\rm ta}$



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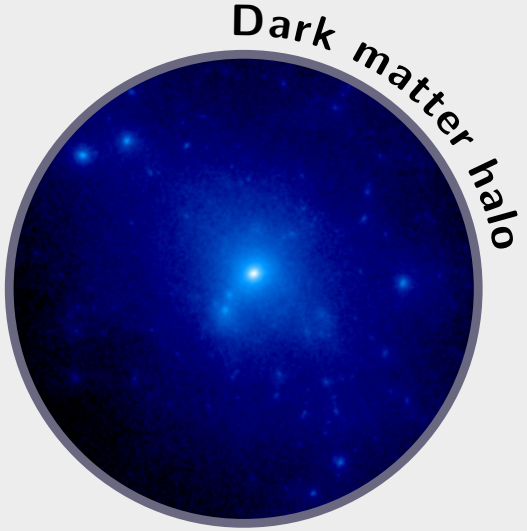

- average overdensity of a virialized DM halo at $t_{\rm coll} = 2t_{\rm ta}$:

$$\Delta_{\rm vir} \equiv 1 + \delta(t_{\rm coll}) = \frac{\rho(t_{\rm coll})}{\overline{\rho}(t_{\rm coll})} = \frac{8\rho_{\rm ta}}{\overline{\rho}(t_{\rm ta})/4} = 32(1+\delta_{\rm ta}) = 18\pi^2 \simeq 178$$

$$\Delta_{\rm vir} \simeq 178$$

Press-Schechter (PS) formalism

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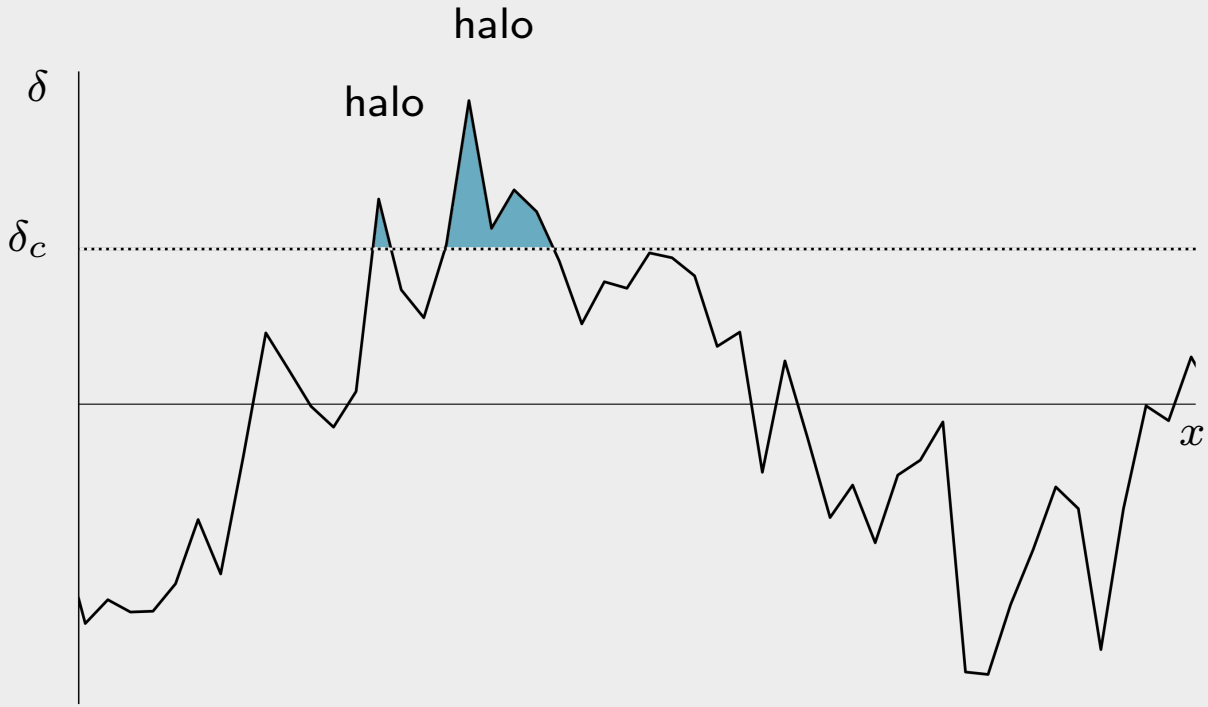
- spherical collapse model: regions with $\delta(\mathbf{x}, t) > \delta_c \simeq 1.686$ will have collapsed to to produce dark matter haloes by time t



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"Solution": $F(> M, t) = 2P(\delta_M > \delta_c)$

"fudge factor"

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- Definition: PS halo mass function $n(M, t)dM$ is the number of haloes with masses in the range $[M, M + dM]$ per volume: $n(M, t) = \frac{dn}{dM}$

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- **ansatz:** probability that $\delta_M > \delta_c(t)$ is the same as the mass fraction that at time t is contained in halos with mass greater than M Press & Schechter 1974
- for Gaussian random field:

$$F(>M,t) = 2P(\delta_M > \delta_c) = \frac{2}{\sqrt{2\pi}\sigma_M}\int_{\delta_c}^{\infty} \exp\left[-\frac{\delta_M^2}{2\sigma_M^2}\right] d\delta_M = \mathrm{erfc}\left[\frac{\delta_M}{\sqrt{2}\sigma_M}\right]$$

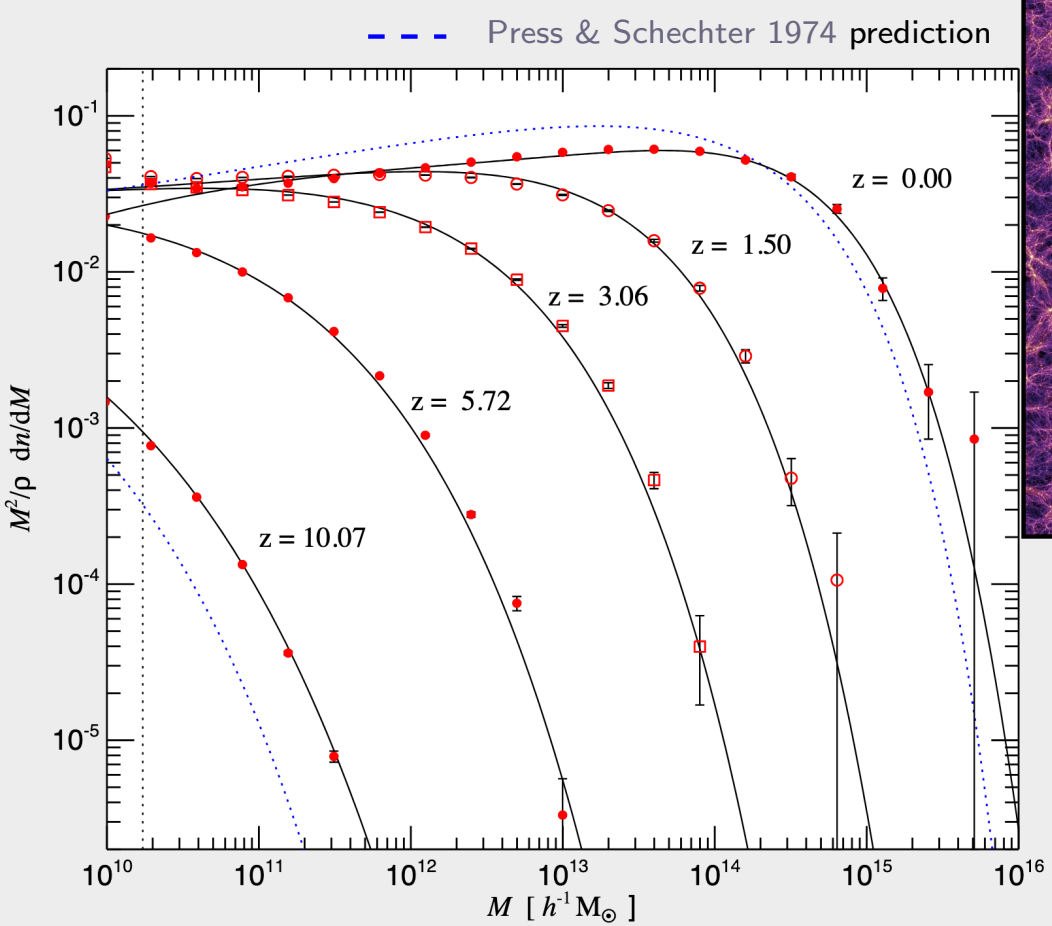
- Definition: PS halo mass function $n(M,t)dM$ is the number of haloes with masses in the range $[M, M+dM]$ per volume: $n(M,t) = \frac{dn}{dM}$

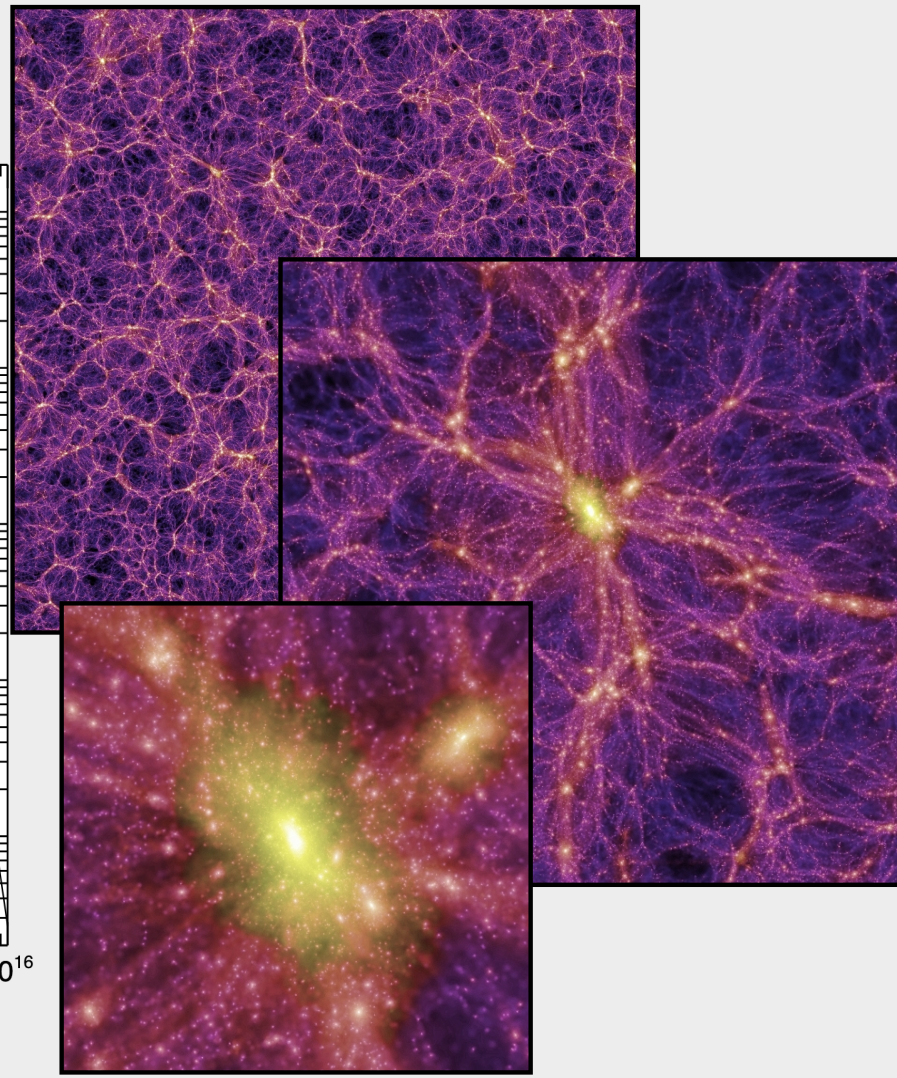
$$n(M,t)dM = \frac{\overline{\rho}}{M}\frac{\partial F(>M)}{\partial M}dM = 2\frac{\overline{\rho}}{M}\frac{\partial P(>\delta_c)}{\partial M}dM$$

$$n(M,t)dM = \sqrt{\frac{2}{\pi}}\frac{\overline{\rho}}{M^2}\frac{\delta_c}{\sigma_M}\exp\left[-\frac{\delta_c^2}{2\sigma_M^2}\right]\left|\frac{d\ln\sigma_M}{d\ln M}\right| dM$$

Dark matter halos
Halo mass function

Comparison with simulations





The Millenium simulation: Springel et al. 2005
$\sim$ 10 billion particles, box length of 500 Mpc/h

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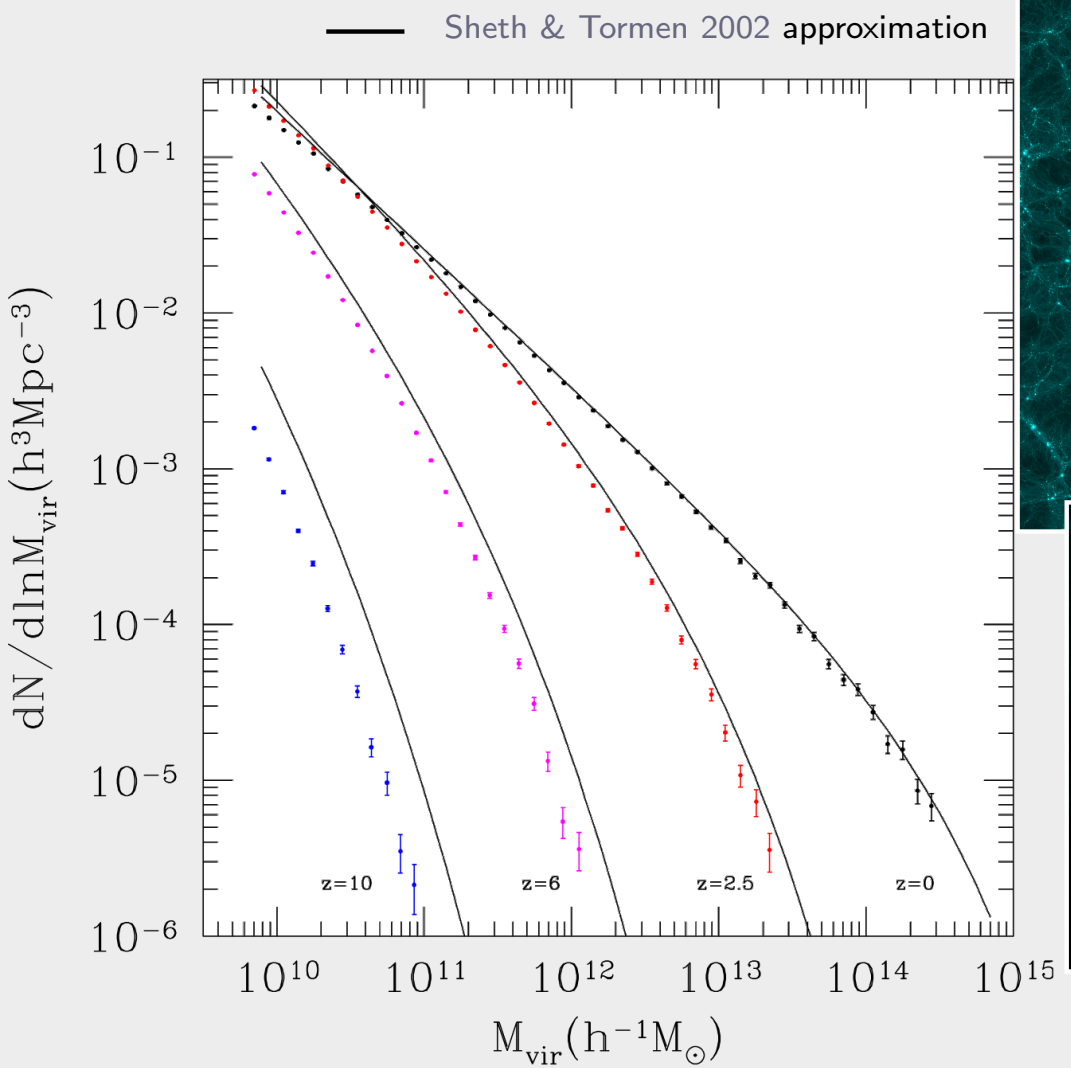

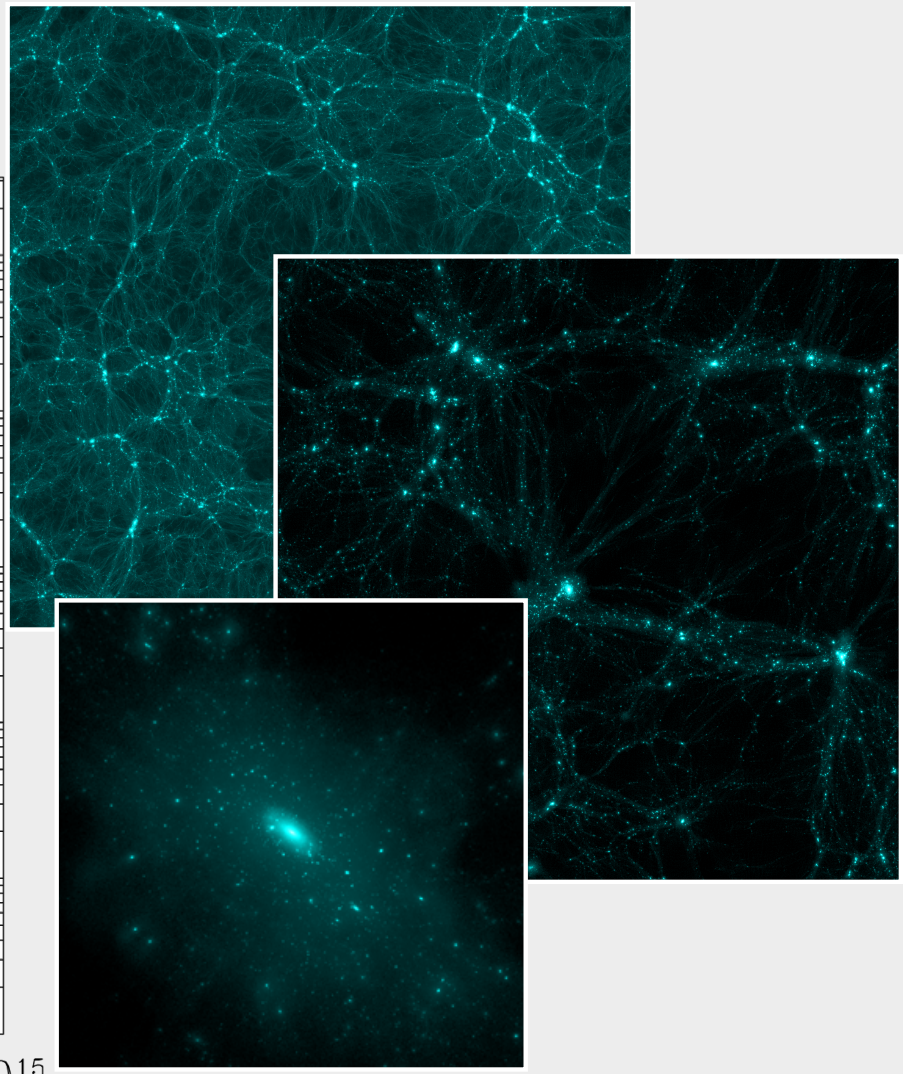

The Bolshoi simulation: Klypin et al. 2011
$\sim$ 10 billion particles, box length of 250 Mpc/h

Angular momentum

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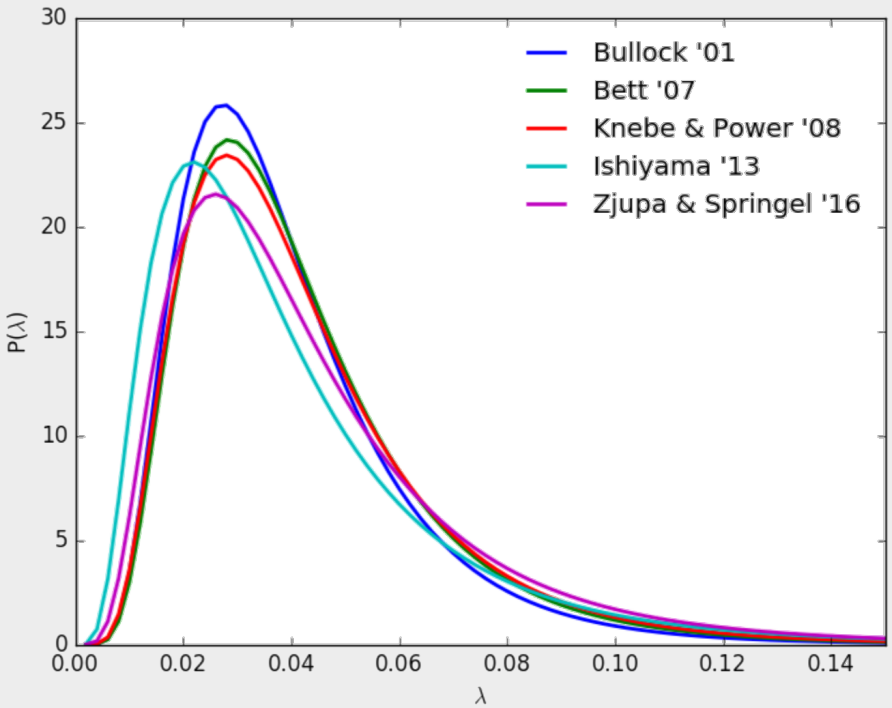
- halo's spin parameter:

$$\lambda = \frac{J|E|^{1/2}}{GM^{5/2}}$$ Peebles 1969

E, M - energy and mass

$\rightarrow$ simulations: log-normal PDF
with $\bar{\lambda} \simeq 0.03$, $\sigma \simeq 0.5$
no mass or cosmology dependence
$\Rightarrow$ dispersion supported



Stewart 2017

NFW profile

NFW profile

- CDM halos in N-body simulations broadly follow a universal density profile, well fit by a double power-law

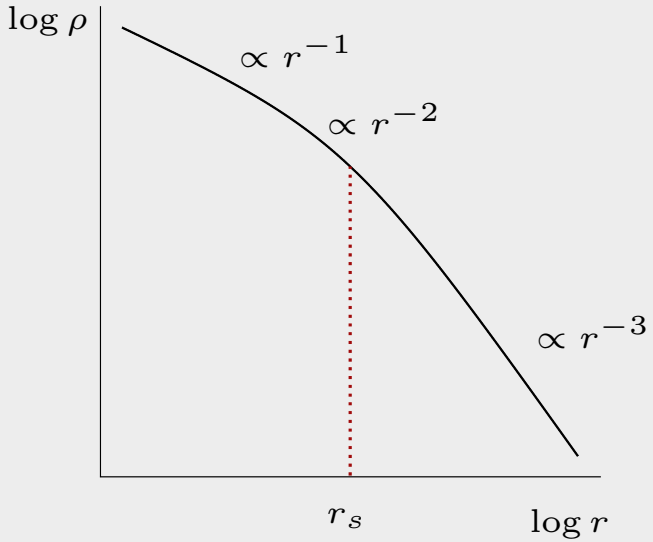
$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}$$

Navarro, Frenk & White 1997

completely characterised by the virial mass $M_{\rm vir}$ and the concentration parameter $c = r_{\rm vir}/r_s$

$$M_{\rm vir} = \frac{4}{3}\pi r_{\rm vir}^3 \Delta_{\rm vir} \rho_{\rm c}(z)$$

$$\rho_{\rm c(z)} = \frac{3H(z)^2}{8\pi G}$$



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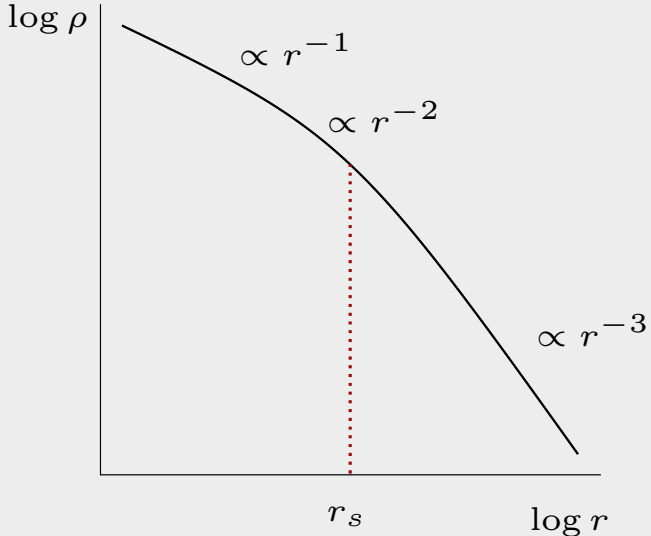
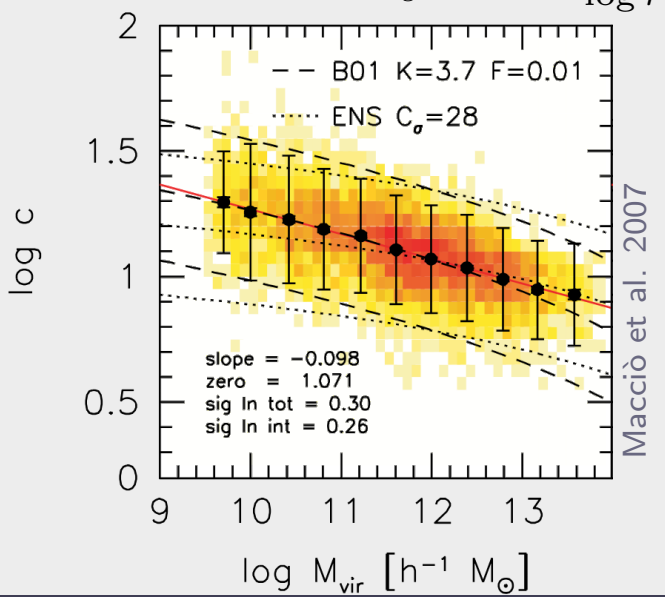


- halos that form earlier are more concentrated & more massive halos form on average later
  ➡ inverted concentration-mass relation

- simulations have shown: even at fixed $M_{\rm vir}$, c is correlated with formation time

e.g. Wechsler et al. 2002, Zhao et al. 2003



NFW profile

- models developped predicting $c(M, z)$ - concentration-mass-redshift relation - of CDM halos and its dependence on redshift z and cosmology:
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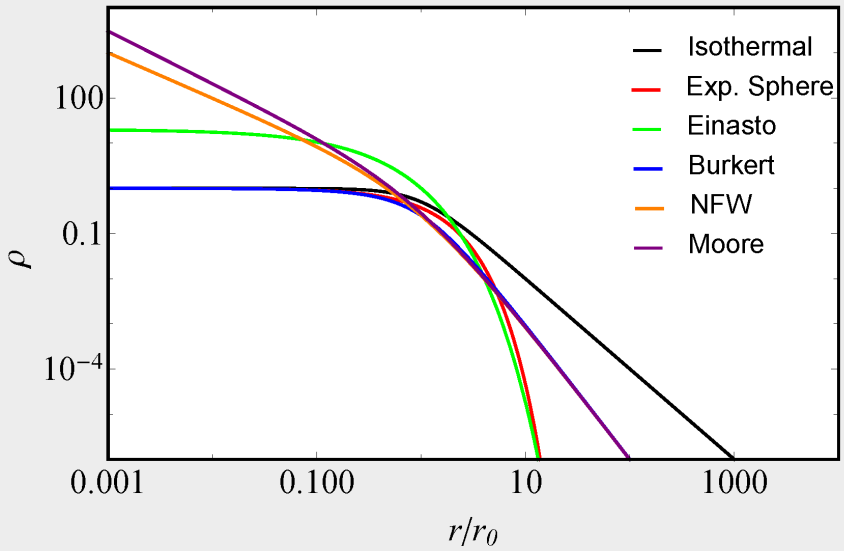
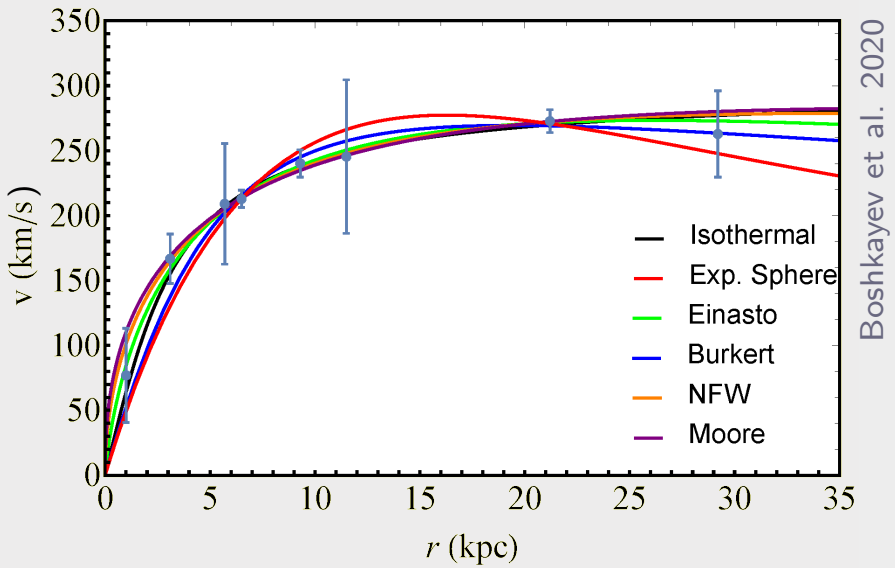
- inner slope of halo density profile $\rightarrow$ controversial





Shock heating

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  $\rightarrow$ assume: shock thermalises all the kinetic energy of the infalling gas & internal energy of the infalling gas can be ignored $\Rightarrow$ internal energy of the shocked gas = kinetic energy of the gas at infall:

$$E_{\rm int,sh} = \frac{3}{2} N k_{\rm B} T_{\rm sh} = \frac{1}{2} M_{\rm gas} v_{\rm in}^2 \qquad \text{(mono-atomic gas)}$$

$$\Rightarrow \quad T_{\rm sh} = \frac{\mu m_{\rm p}}{3 k_{\rm B}} v_{\rm in}^2 = \frac{\xi \mu m_{\rm p}}{3 k_{\rm B}} \frac{G M_{\rm vir}}{r_{\rm vir}} \qquad \xi \text{ - detailed density profile}$$

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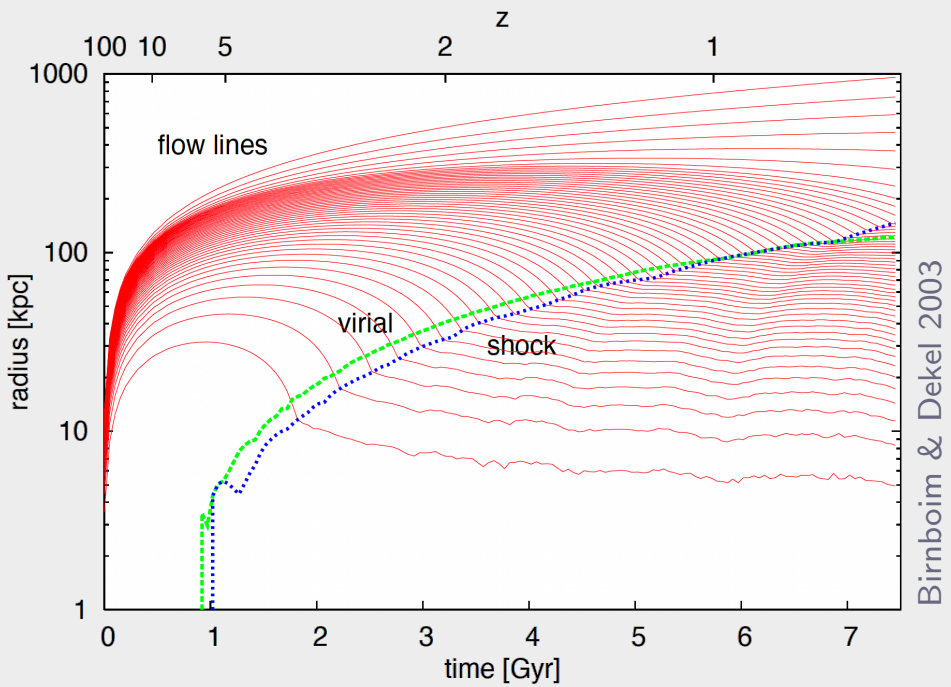
Build-up of a shock near the virial radius in a collapsing structure (1D calculation)



Hydrostatic equilibrium

- assume: non-radiative gas (no cooling/heating)
  $\rightarrow$ shocked gas heats until thermal pressure balances gravity
  $\rightarrow$ gas settles into hydrostatic equilibrium:

$$\nabla P(r) = -\rho_{\rm gas} \nabla \Phi(r)$$

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} \qquad \text{(spherical symmetry)}$$

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Virial temperature

- virial equilibrium ($2K + W = 0$):

$$3\frac{M_{\rm gas}}{\mu m_{\rm p}} k_{\rm B} T_{\rm vir} - \xi \frac{GM_{\rm gas} M_{\rm vir}}{r_{\rm vir}} = 0 \qquad \text{(no external } P\text{)}$$

$$T_{\rm vir} = \frac{\xi \mu m_{\rm p}}{3k_{\rm B}} \frac{GM_{\rm vir}}{r_{\rm vir}}$$

Caution: useful for order of magnitude estimates, in reality, gas inside virialised halos has temperature profile ...

Radiative cooling: cooling time

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- dynamical time (or 'free-fall' time):

  $$t_{\rm ff} = \left(\frac{3\pi}{32 G \bar{\rho}_{\rm sys}}\right)^{1/2} \propto (\bar{\rho}_{\rm sys})^{-1/2} \qquad \text{with } \bar{\rho}_{\rm sys} = \bar{\rho}_{\rm gas} + \bar{\rho}_{\rm DM}$$

  Note: $\bar{\rho}_{\rm sys} \sim 200\bar{\rho}$ ➡ $t_{\rm ff} \sim t_{\rm H}/10$

Radiative cooling: cooling time

- three regimes:

Note: $t_{\rm cool} \propto \rho^{-1} \propto (1+z)^{-3}$

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➡ cooling generally more efficient at high z

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$t_{\rm cool} < t_{\rm ff}$
- catastrophic cooling: cooling proceeds faster and faster
- gas cannot respond to loss of P fast enough and falls to the center of dynamic system on $t_{\rm ff}$

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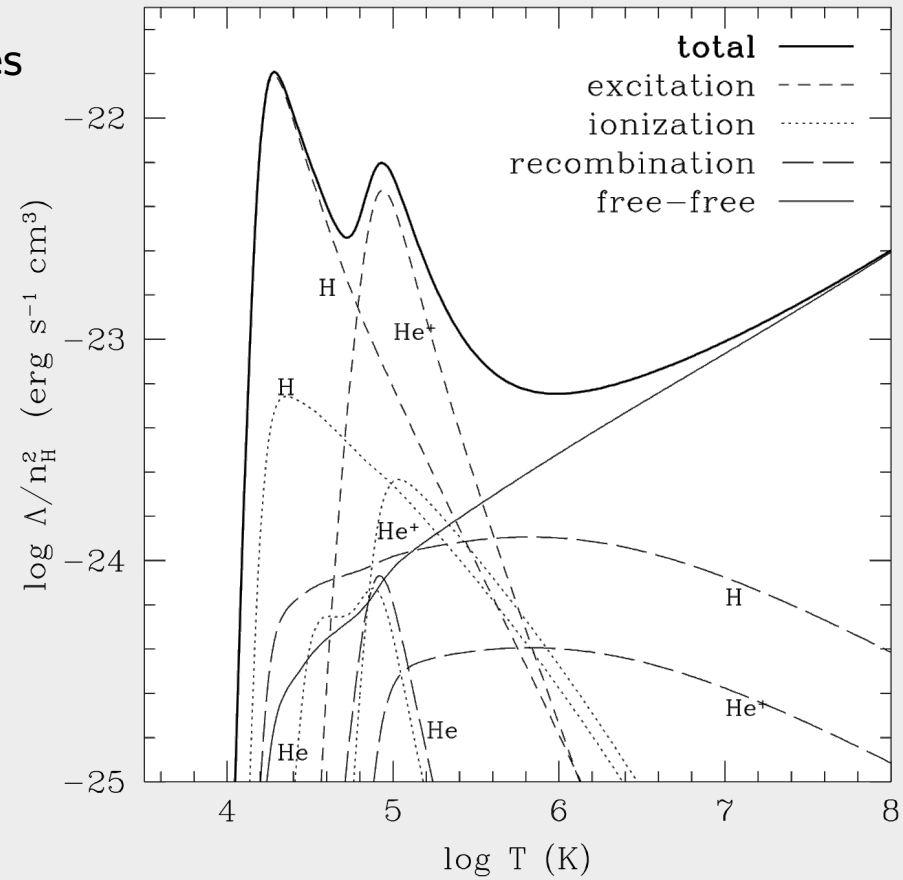
4. bound-bound (collisional excitation)
   $\mathrm{e}^- + \mathrm{X} \longrightarrow \mathrm{e}^- + \mathrm{X}' \longrightarrow \mathrm{e}^- + \mathrm{X} + \gamma$
   - impact of the free e^- knocks bound e^- to excited state
   - emits photon as it decays

Radiative cooling: cooling function

- primordial gas* collisional ionization equilibrium** cooling:
  - → at $T < 10^4$ K: all gas is neutral, no ions
  - → at low T: collisional excitation dominates
  - → at high T: gas is fully ionised, free-free (bremsstrahlung) dominates
  - → if $T > \text{few} \times 10^4$ K: all H is ionized, no longer contributes to cooling
  - → He responsible for a second peak at $T \sim 10^5$ K



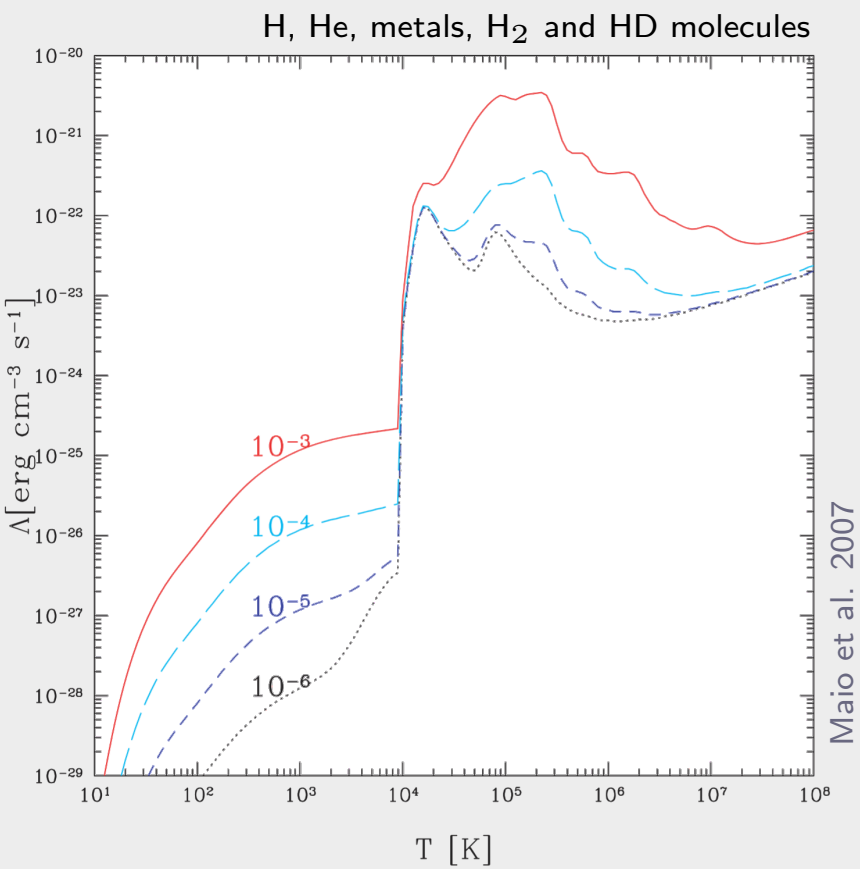
* $X = 0.76$, $Y = 0.24$ - H and He mass abundances

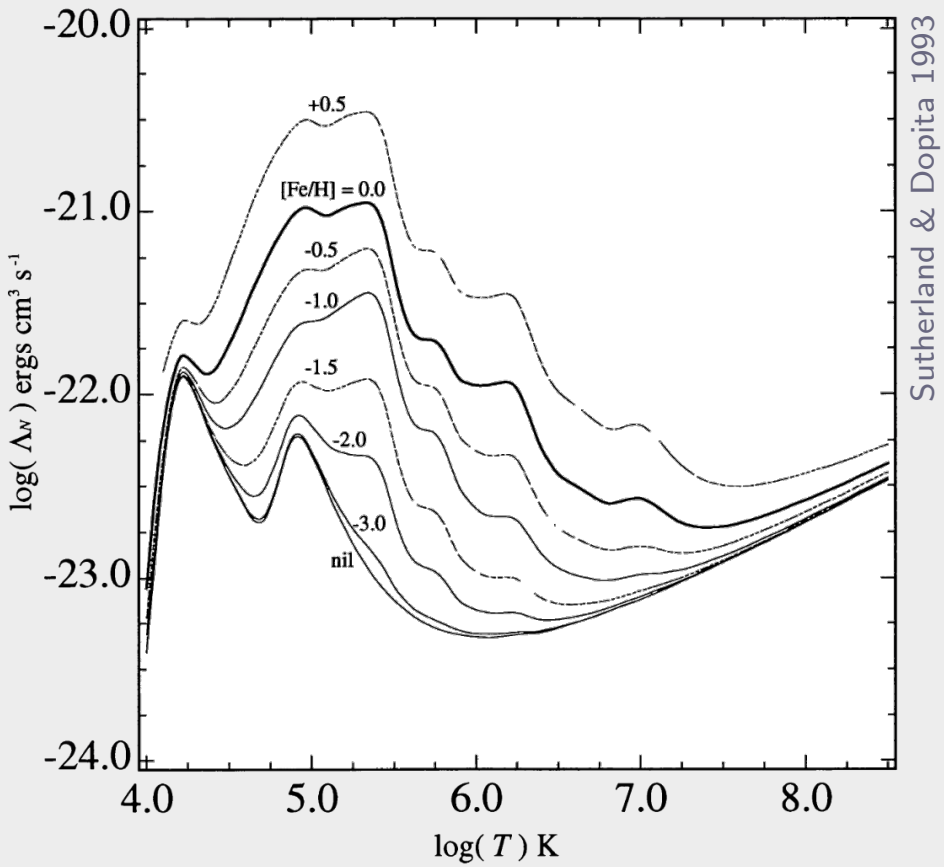
** absence of ionizing radiation and the system has equilibrated destruction and creation rates (rate of ionization from collisions with free electrons equals the rate of recombination)

Radiative cooling: cooling function

- presence of metals and molecules → new cooling channels available:

  → mainly between $10^4 - 10^7$ K, greatly increasing $\Lambda(T)$: for $Z_\odot$ by a factor of 100 compared to the primordial gas



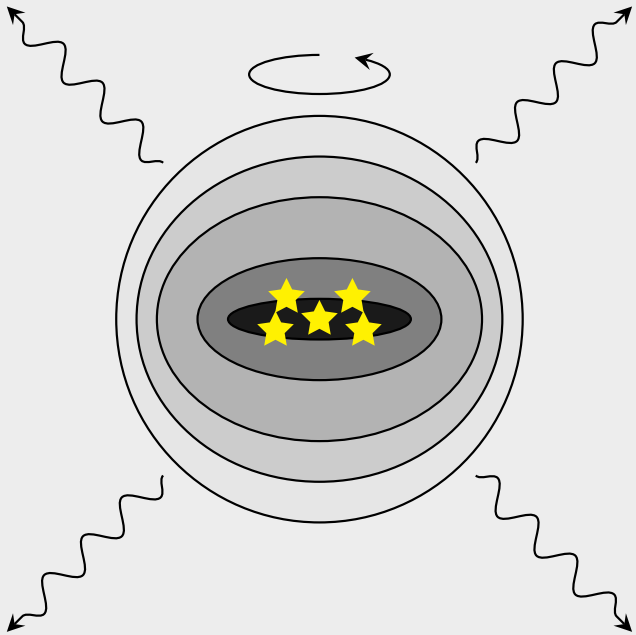


Radiative cooling: disk galaxies

- Ingredients: i) **radiative cooling of baryons**: unlike the dark matter, baryons are able to cool radiatevely

  ii) **rotation of halo**: dark matter halo has a rotation

- **Disk formation**:
  - Hot shock-heated gas inside DM halo cools down
  - Gas cools $\rightarrow$ pressure decreases $\rightarrow$ gas contracts
  - Isotropic emission of photons $\rightarrow$ angular momentum of gas is conserved
  - Gas sphere contracts $\rightarrow$ spins up and flattens
  - Disk density increases $\rightarrow$ star formation, ...
  - Disk galaxy forms



Hot vs Cold mode halo accretion

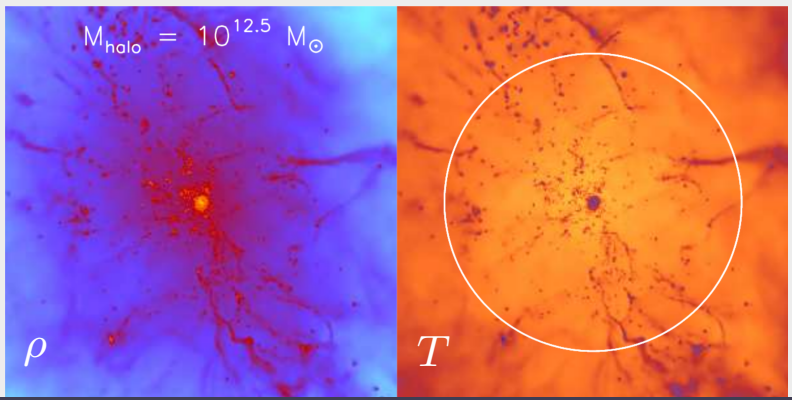
- **hot mode accretion**: slow cooling regime ($r_{\rm cool} \ll r_{\rm vir}$):

  $\rightarrow$ only the inner gas can cool $\rightarrow$ hot halo and accretion shock close to $r_{\rm vir}$ form

  $\rightarrow$ newly accreted gas $\rightarrow$ shock-heated to $T_{\rm vir}$, cools slowly in cooling flows, in quasi-hydrostatic equilibrium

  $\rightarrow$ occurs in halos $M \gtrsim 10^{12} {\rm M}_\odot$

Hot mode



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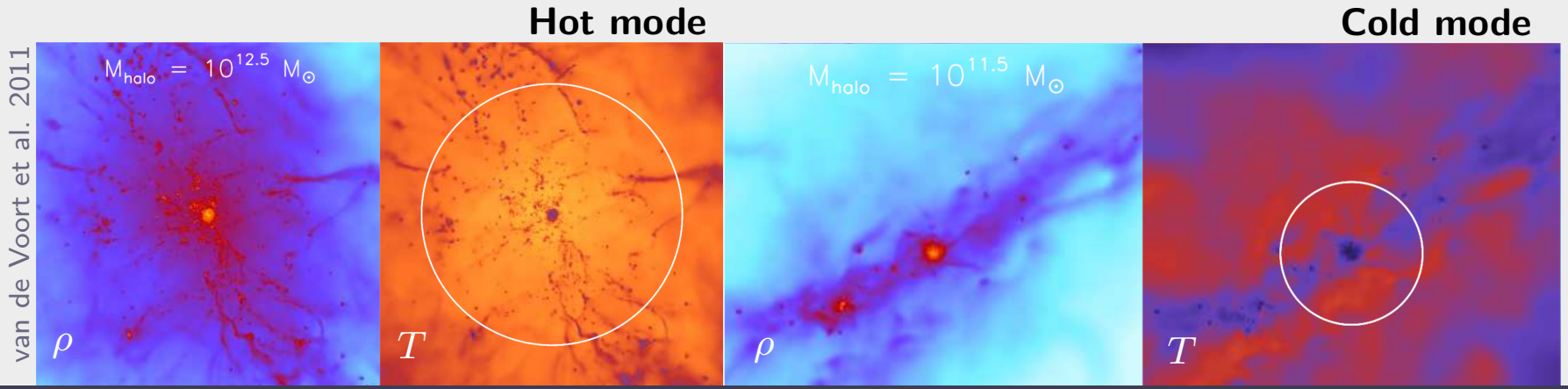
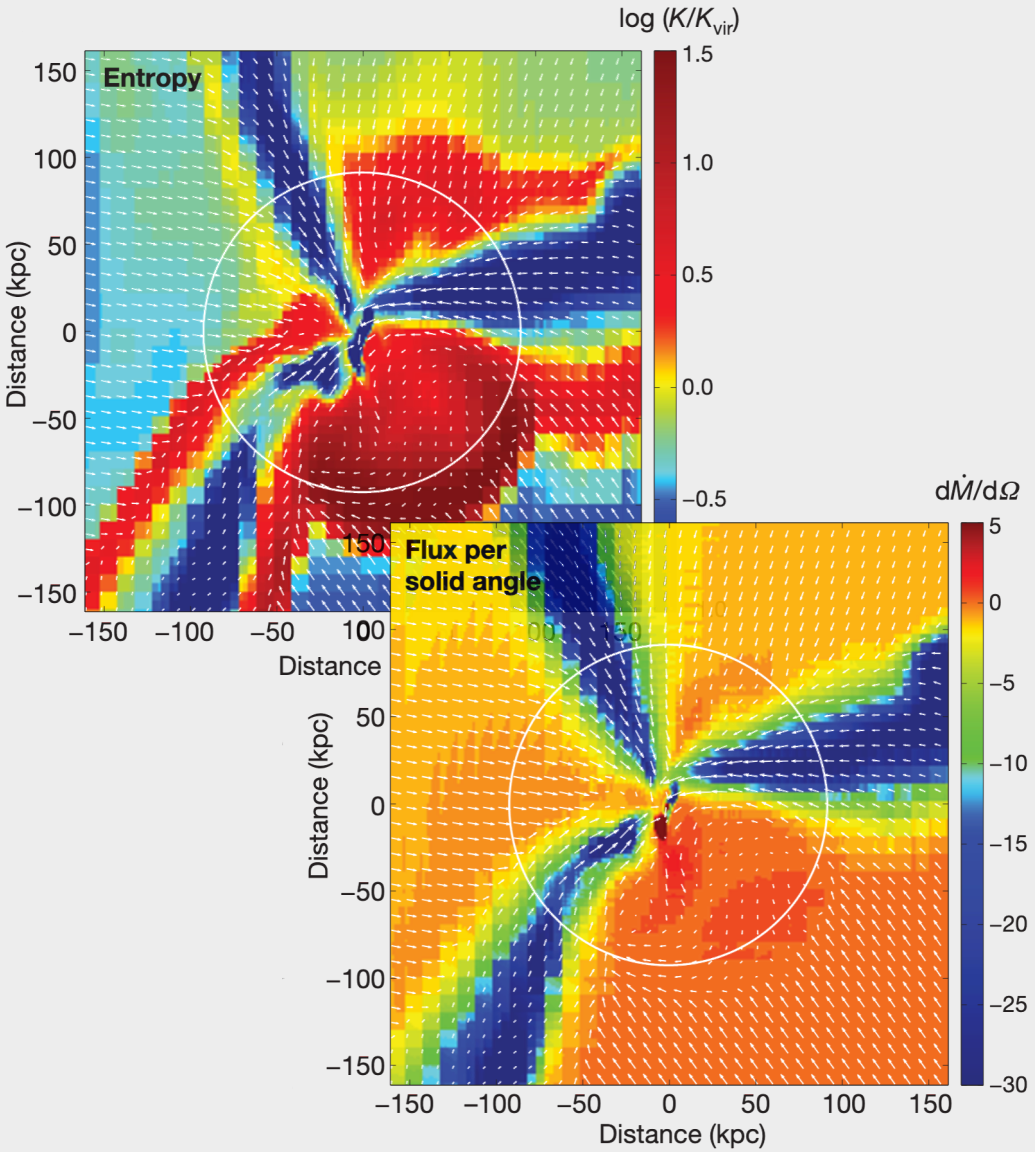
- **cold mode accretion**: fast cooling regime ($r_{\rm cool} \gg r_{\rm vir}$):
  - $\rightarrow$ 'catastrophic cooling': all the gas within the halo is expected to have cooled
  - $\rightarrow$ no hot gas $\rightarrow$ no accretion shock near $r_{\rm vir}$
  - $\rightarrow$ newly accreted gas $\rightarrow$ not shock-heated, falls to the center cold
  - $\rightarrow$ occurs in halos $M \lesssim 10^{11} - 10^{12} {\rm M}_\odot$ (primordial vs +metal cooling, filaments)

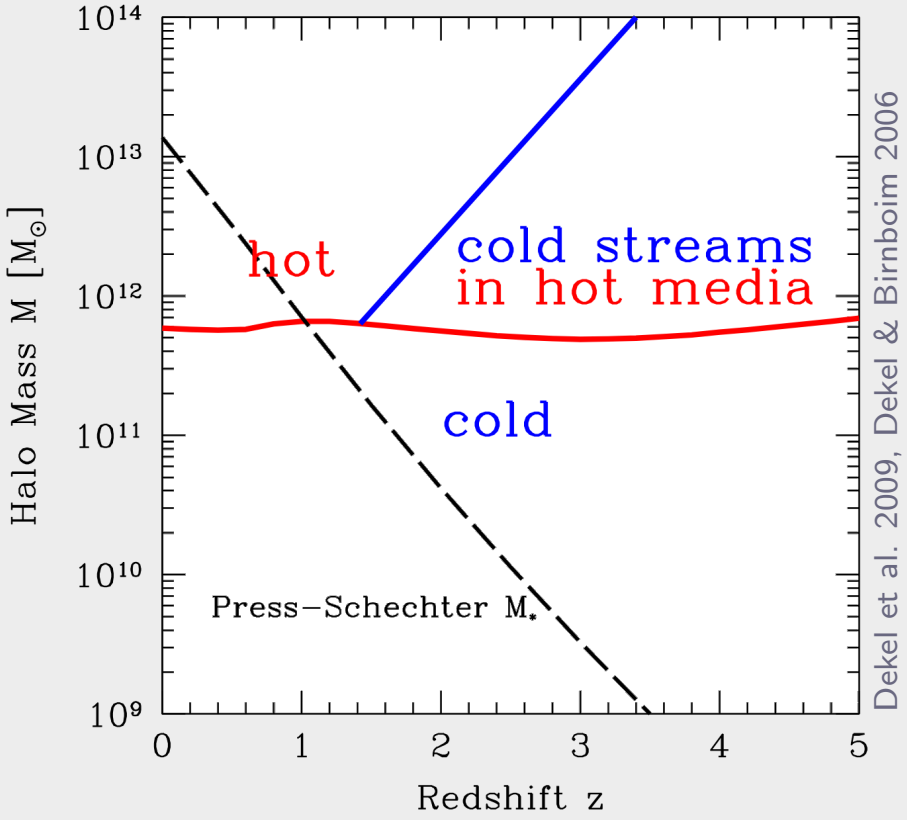


From halos to galaxies

Gas inside dark matter haloes

Hot vs Cold mode halo accretion



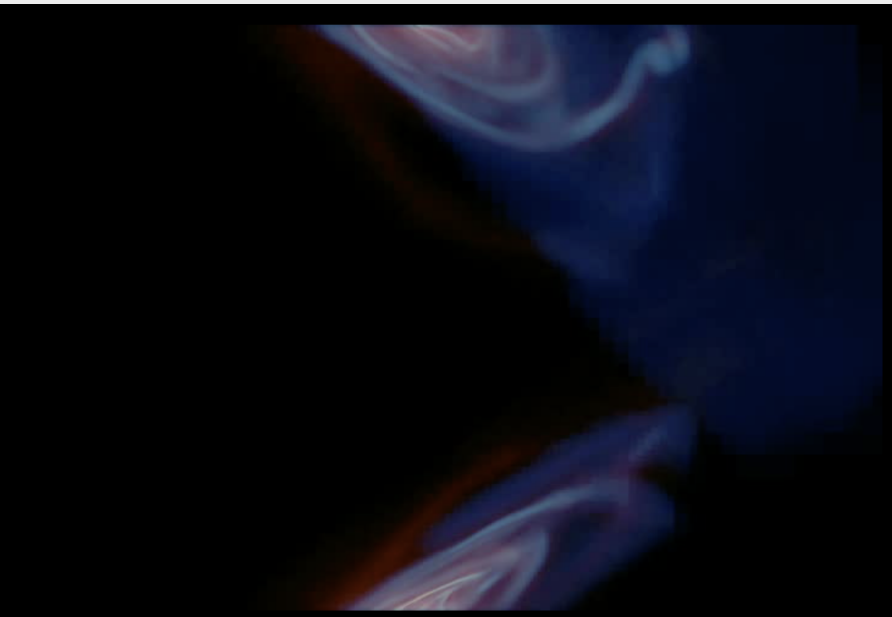


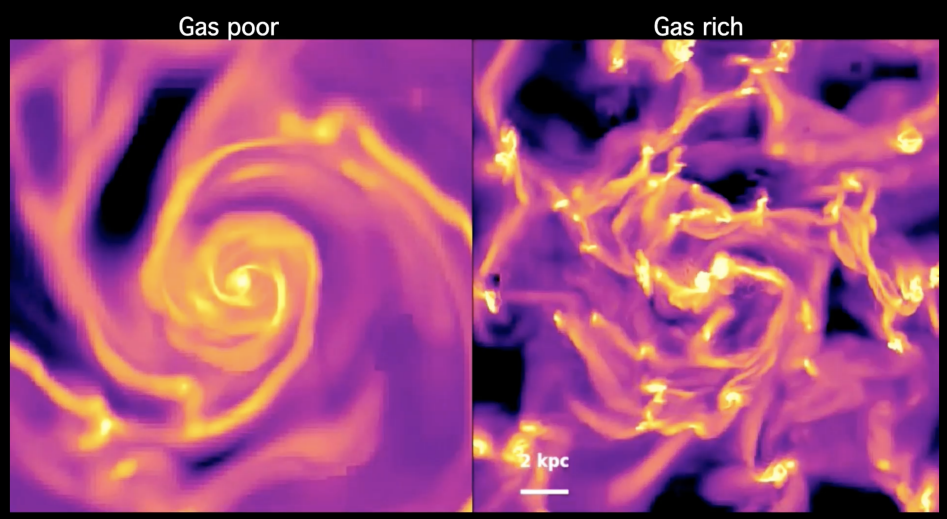
From halos to galaxies

Gas inside dark matter haloes

Mergers: elliptical galaxies

- **major merger** $\rightarrow$ mass ratio of galaxies $> 1:3$
  - $\rightarrow$ structure/morphology disruption/transformation
  - $\rightarrow$ two merging disks can form an elliptical . . . if no/not enough gas available, otherwise disk can re-grow (with a bulge)
- **minor merger** $\rightarrow$ mass ratio of galaxies $> 1:10$
  - $\rightarrow$ global structure retained, internal structure impacted
  - $\rightarrow$ many subsequent minor mergers $\rightarrow$ elliptical





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