Bouncing alternatives to inflation

Cargese - 06/07/10

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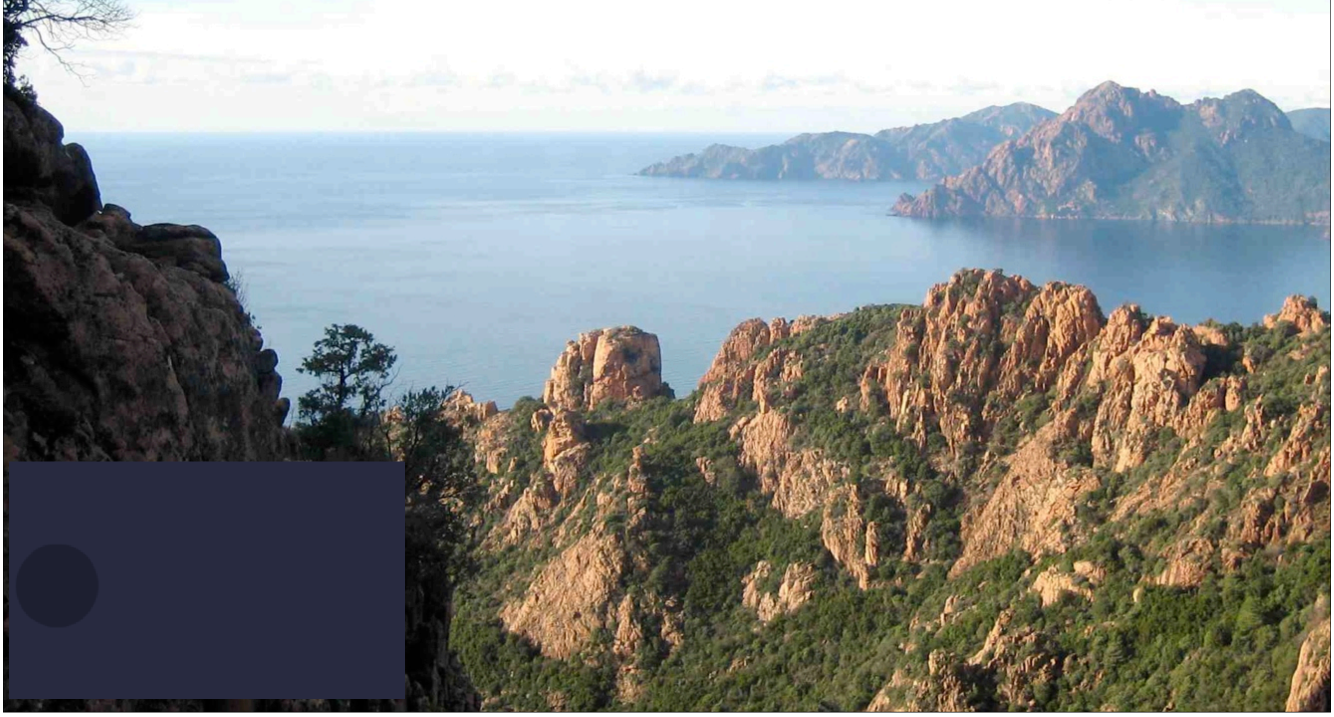
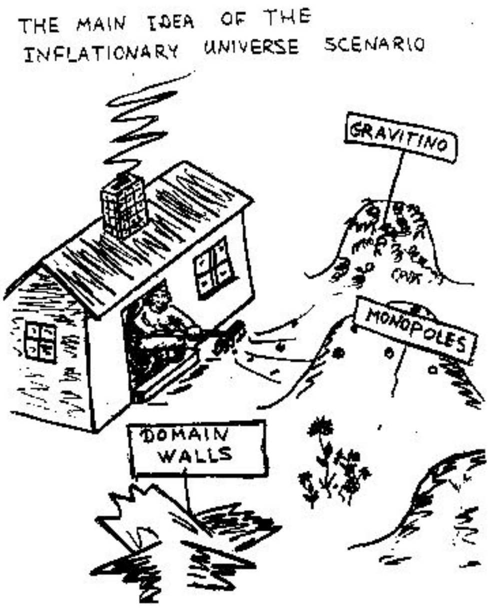


Problems with standard model:

- *Singularity*
- *Horizon*
- *Flatness*
- *Homogeneity*
- *Perturbations*
- *Dark matter*
- *Dark energy / cosmological constant*
- *Baryogenesis*
- ...

Accepted solution = INFLATION

Topological defects (monopoles)



(Linde's book)

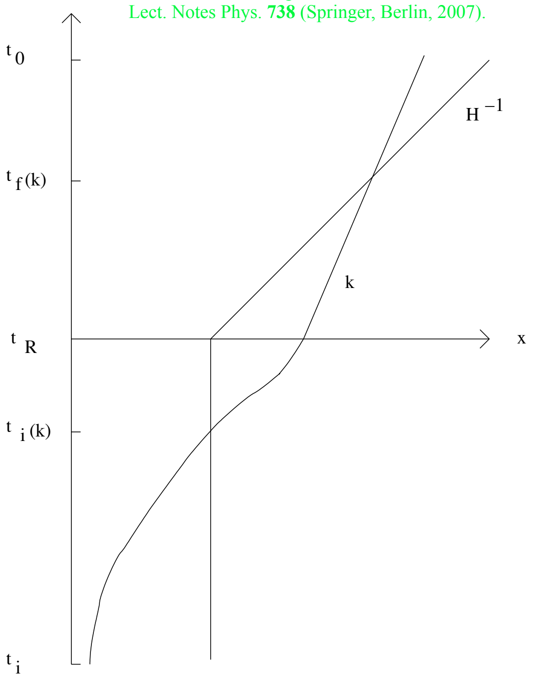
Inflation:

- solves cosmological puzzles
- uses GR + scalar fields [(semi-)classical]
- can be implemented in high energy theories
- makes falsifiable predictions ...
- ... consistent with all known observations
- string based ideas (brane inflation, ...)

Alternative model???

- string based ideas (PBB, other brane models, string gas, …)
- Quantum gravity / cosmology
- singularity, initial conditions & homogeneity
- bounces
- provide challengers!

From R. Brandenberger, *in* M. Lemoine, J. Martin & P. P. (Eds.), "Inflationary cosmology", Lect. Notes Phys. **738** (Springer, Berlin, 2007).



- Scalar field origin?
- Trans-Planckian

$$\exists t; \ell(t) = \ell_0 \frac{a(t)}{a_0} \lesssim \ell_{\rm Pl}$$

- Hierarchy (amplitude)

$$\frac{V(\varphi)}{\Delta\varphi^4} \lesssim 10^{-12}$$

- Singularity

$$\exists t_{(\pm\infty)}; a(t) \to 0$$

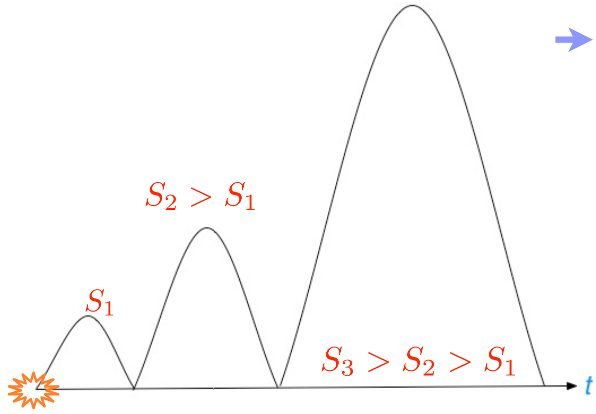
- Validity of GR?

$$E_{\rm inf} \simeq 10^{-3} M_{\rm Pl}$$

A brief history of bouncing cosmology

- R. C. Tolman, "*On the Theoretical Requirements for a Periodic Behaviour of the Universe*", PRD 38, 1758 (1931)
- G. Lemaître, "*L'Univers en expansion*", Ann. Soc. Sci. Bruxelles (1933)
- A. A. Starobinsky, "*On one non-singular isotropic cosmological model*", Sov. Astron. Lett. 4, 82 (1978)
- R. Durrer & J. Laukerman, "*The oscillating Universe: an alternative to inflation*", Class. Quantum Grav. 13, 1069 (1996)

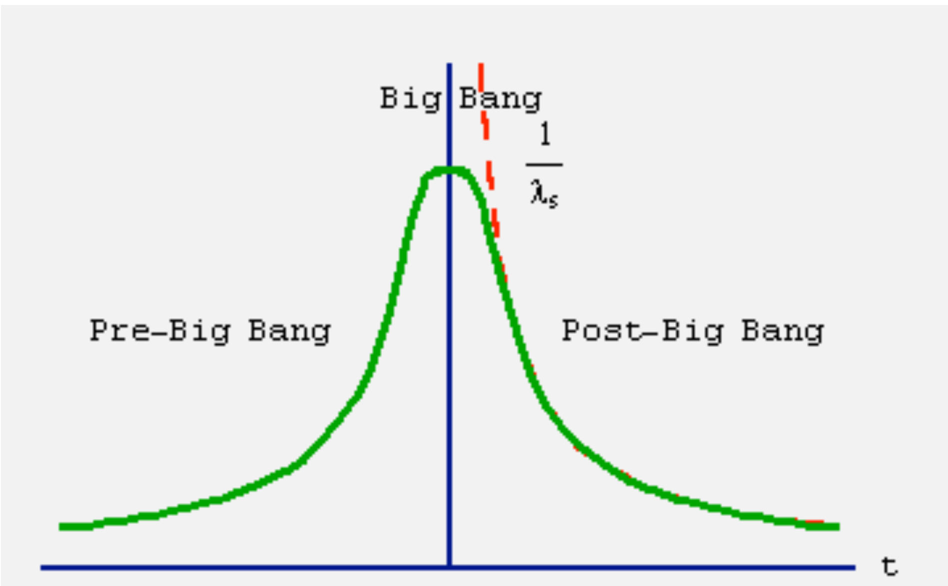
Quantum nucleation?



- Penrose: BH formation

- PBB - Ekpyrotic - Modified gravity - Quantum cosmology - Quintom - Horava-Lifshitz - Lee-Wick - ...

Pre Big Bang scenario: (cf. M.Gasperini & G. Veneziano, arXiv: hep-th/0703055)



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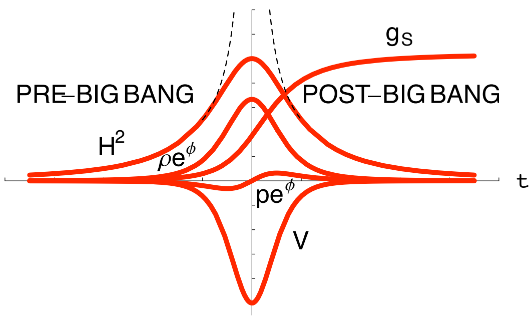
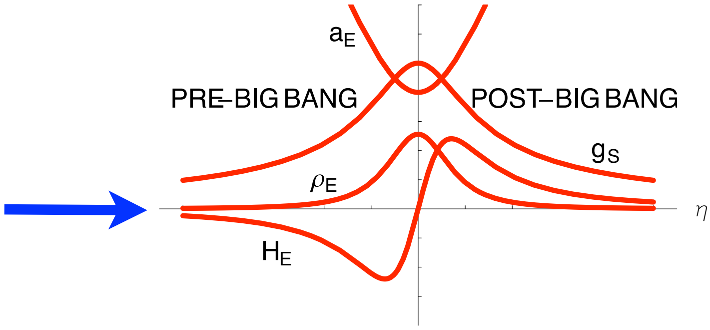


string frame



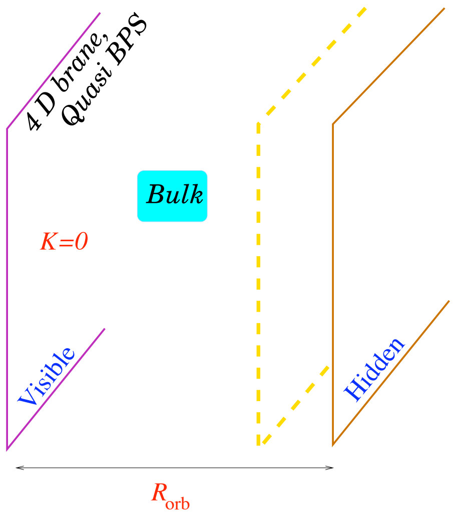
Einstein frame

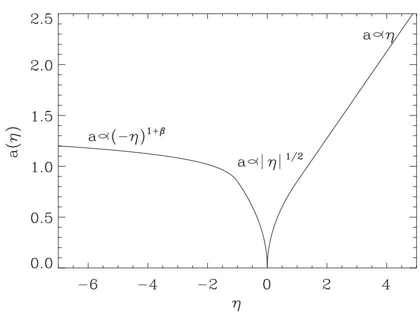
Ekpyrotic scenario:

$$\mathcal{S}_5 \propto \int_{\mathcal{M}_5} \mathrm{d}^5x\sqrt{-g_5}\left[R_{(5)} - \frac{1}{2}(\partial\varphi)^2 - \frac{3}{2}\frac{\mathrm{e}^{2\varphi}\mathcal{F}^2}{5\,!}\right],$$

$$\mathcal{S}_4 = \int_{\mathcal{M}_4} \mathrm{d}^4x\sqrt{-g_4}\left[\frac{R_{(4)}}{2\kappa} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right].$$

$$V(\varphi) = -V_{\mathrm{i}}\exp\left[-\frac{4\sqrt{\pi\gamma}}{m_{\mathrm{Pl}}}(\varphi-\varphi_{\mathrm{i}})\right].$$





Standard Failures and some solutions

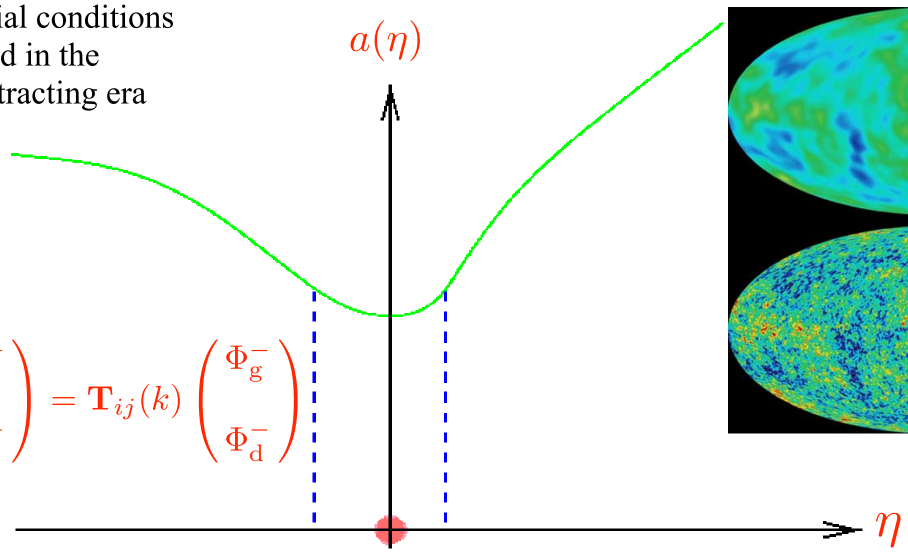
- **Singularity** — Merely a non issue in the bounce case!
- **Horizon** — $d_\mathrm{H} \equiv a(t)\int_{t_\mathrm{i}}^{t}\frac{\mathrm{d}\tau}{a(\tau)}$ can be made divergent easily if $t_\mathrm{i} \to -\infty$
- **Flatness** — $\frac{\mathrm{d}}{\mathrm{d}t}|\Omega - 1| = -2\frac{\ddot{a}}{\dot{a}^3}$ $\quad \ddot{a} < 0 \;\&\; \dot{a} < 0$

  accelerated expansion (inflation) or decelerated contraction (bounce)
- **Homogeneity** — Large & flat Universe + low initial density + diffusion

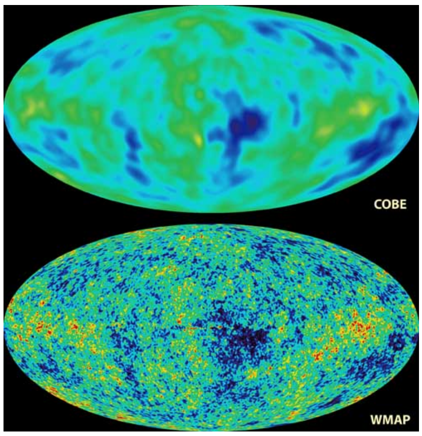
  $$\frac{t_\mathrm{dissipation}}{t_\mathrm{Hubble}} \propto \frac{\lambda}{R_\mathrm{H}^{1/3}}\left(1 + \frac{\lambda}{AR_\mathrm{H}^2}\right)$$ enough time to dissipate any wavelength

  $\Longrightarrow$ vacuum state!
- **Perturbations** — see coming slides
- **Others** — dark matter/energy, baryogenesis, ...

Initial conditions
fixed in the
contracting era



$$\begin{pmatrix}\Phi_\mathrm{g}^+\\ \Phi_\mathrm{d}^+\end{pmatrix} = \mathbf{T}_{ij}(k)\begin{pmatrix}\Phi_\mathrm{g}^-\\ \Phi_\mathrm{d}^-\end{pmatrix}$$



???????

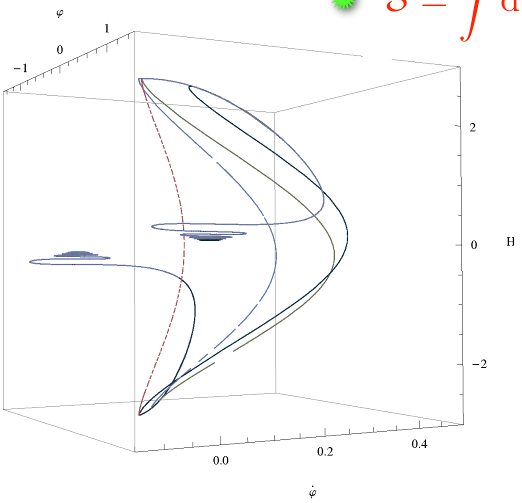
Self consistent bounce:

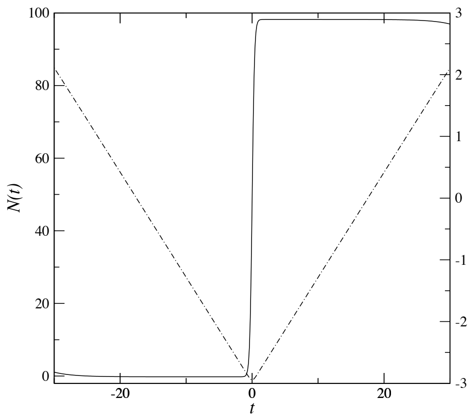
$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-\mathcal{K}r^2} + r^2 d\Omega^2\right)$$

→ One d.o.f. + 4 dimensions G.R.

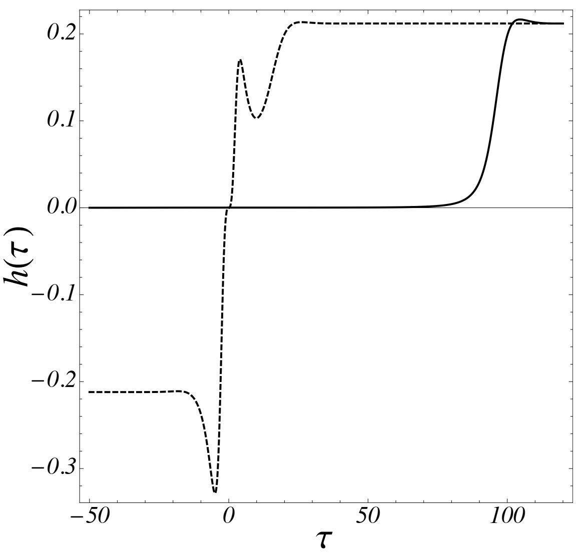
$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{6\ell_{\rm Pl}^2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right]$$

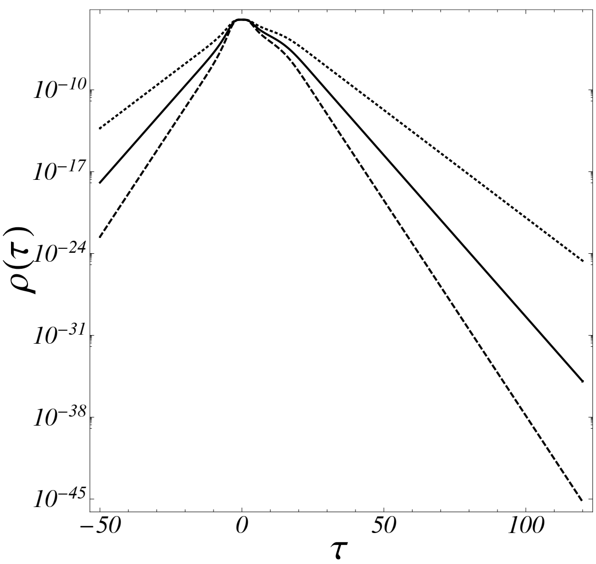
$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) - \frac{\mathcal{K}}{a^2}$$ Positive spatial curvature





F. Falciano, M. Lilley & P. P., *Phys. Rev.* **D77**, 083513 (2008)



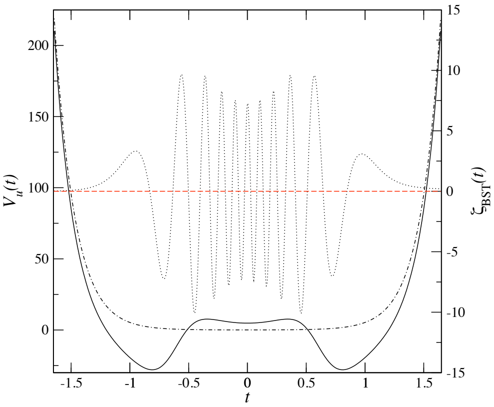


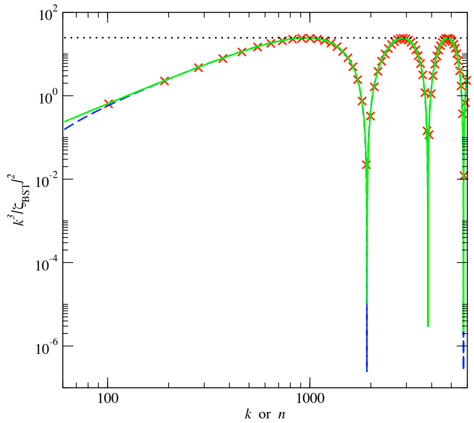
Perturbations: $$ds^2 = a^2(\eta)\left\{(1+2\Phi)\,d\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]dx^i dx^j\right\}$$

$$\Longleftrightarrow \quad \Phi = \frac{3\mathcal{H}u}{2a^2\theta} \qquad \theta \equiv \frac{1}{a}\sqrt{\frac{\rho_\varphi}{\rho_\varphi + p_\varphi}}\left(1 - \frac{3\mathcal{K}}{\rho_\varphi a^2}\right)$$

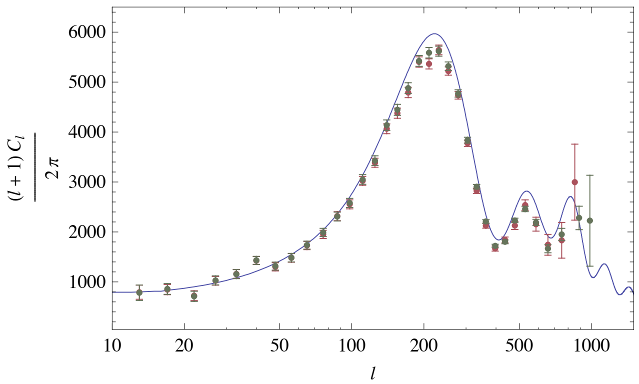
$$u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K}\left(1 - c_{\rm S}^2\right)\right]u = 0$$

$$\mathcal{P}_\zeta = \mathcal{A}k^{n_{\rm S}-1}\cos^2\left(\omega\frac{k_{\rm ph}}{k_\star} + \psi\right)$$

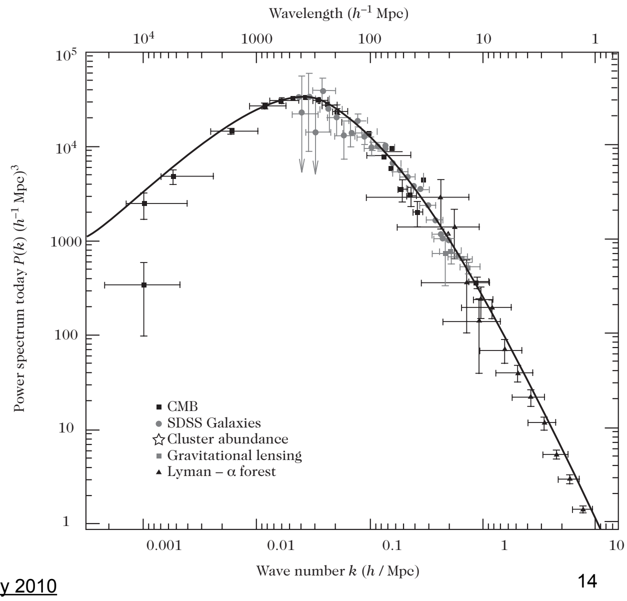


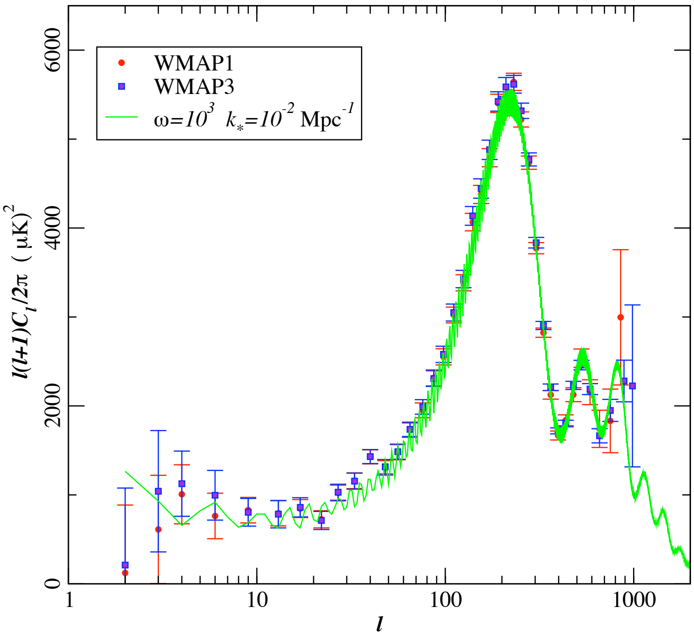


Data!



No obvious oscillations ...





A specific model: 4D Quantum cosmology

$$\mathcal{S} = \int \sqrt{-g}\left(-\frac{R}{6\ell_{\rm P}^2} + p\right) \mathrm{d}^4 x$$

Perfect fluid: $p = \omega\rho$ → **bounce**

no horizon problem if $\omega > -\frac{1}{3}$

Results:

$$n_{\rm T} = n_{\rm S} - 1 = \frac{12\omega}{1+3\omega}$$

$$\frac{T}{S} \simeq 4\times 10^{-2}\sqrt{n_{\rm S}-1}$$

Digression: about QM

Schrödinger $$i\frac{\partial\Psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V(\boldsymbol{r})\right]\Psi$$

Polar form of the wave function $$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\boldsymbol{r}) + Q(\boldsymbol{r},t) = 0$$

quantum potential $$Q \equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

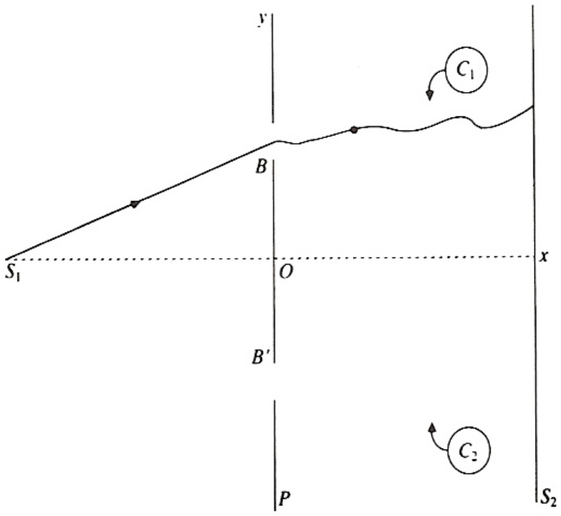
Ontological interpretation (BdB) $\exists\ x(t)$

Trajectories satisfy $$m\frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = -\nabla(V+Q)$$

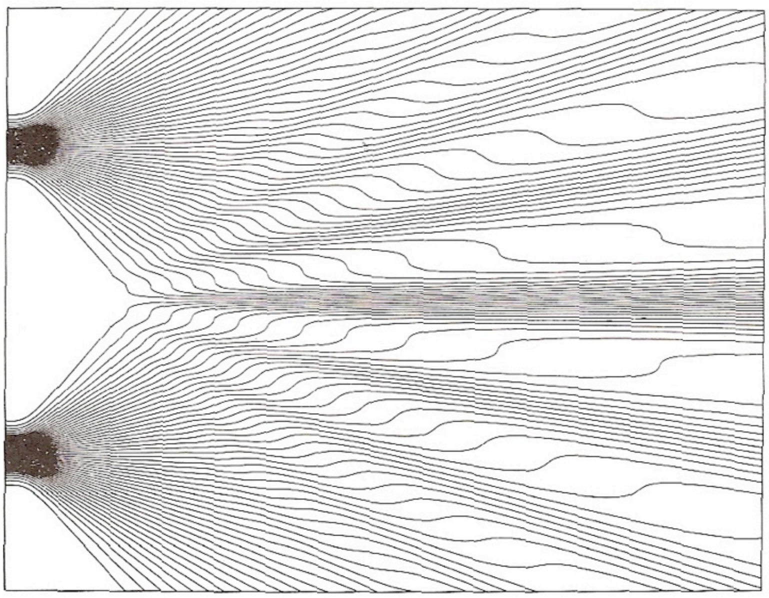
Properties:

- **strictly equivalent to Copenhagen QM**
  - **probability distribution (attractor)** $\exists t_0;\ \rho(\boldsymbol{x},t_0) = |\Psi(\boldsymbol{x},t_0)|^2$
- **classical limit well defined** $Q \longrightarrow 0$
- **state dependent**
- $\exists$ **intrinsic reality**
  - **non local …**
- **no need for external classical domain!**

The two-slit experiment:



Trajectories in the two-slit experiment



Quantum cosmology $\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$

+ canonical transformation

+ rescaling (volume …) = a simple Hamiltonian:

+ units

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$N \to a^{3\omega}$

Wheeler-De Witt $H\Psi = 0$

+ Technical trick: $\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}$

space defined by $\chi > 0$ —— constraint $\bar{\Psi}\frac{\partial\Psi}{\partial\chi} = \Psi\frac{\partial\bar{\Psi}}{\partial\chi}$

alternative way of getting the solution:

WKB exact superposition: $\Psi = \int \mathrm{e}^{iET}\rho(E)\psi_E(T)\mathrm{d}E$

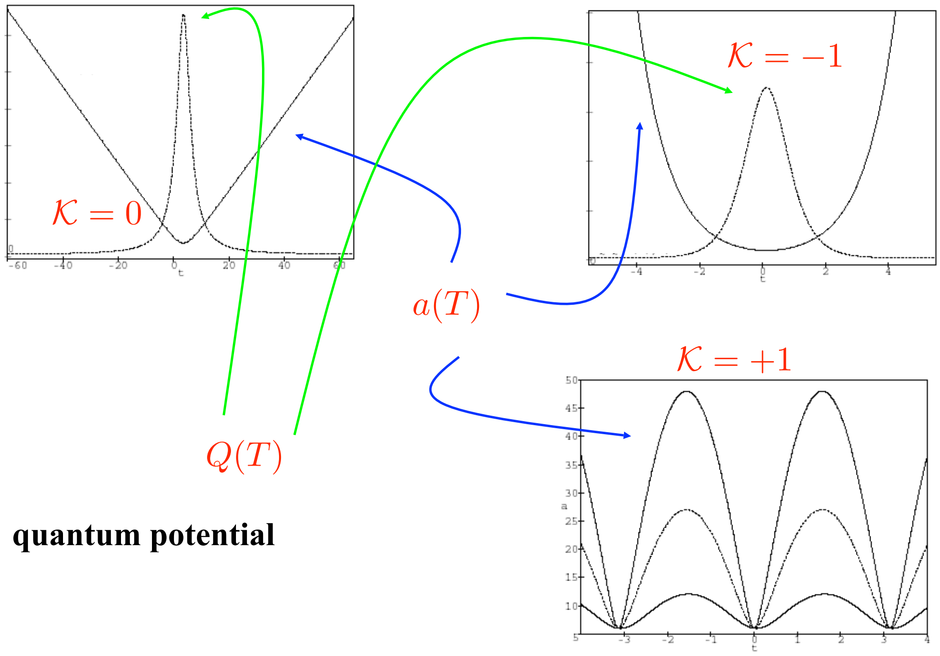
Gaussian wave packet $\longleftarrow \rho(E) \propto \mathrm{e}^{-(ET_0)^2}$

$$\Rightarrow \Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right)\mathrm{e}^{-iS(\chi,T)}$$

phase $S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$

Bohmian trajectory $\dot{a} = \{a, H\}$

$$a = a_0\left[1 + \left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$



quantum potential

What about the perturbations?

Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \cdots$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a,T\right)\Psi_{(2)}\left[v,T;a\left(T\right)\right]$$

comes from 0th order

$$\Delta\Phi = -\frac{3\ell_{\rm Pl}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

conformal time $\mathrm{d}\eta = a^{3\omega-1}\mathrm{d}T$

+ canonical transformations:

$$i\frac{\partial \Psi_{(2)}}{\partial \eta} = \int \mathrm{d}^3x \left(-\frac{1}{2}\frac{\delta^2}{\delta v^2} + \frac{\omega}{2} v_{,i}v^{,i} - \frac{a''}{a}\right)\Psi_{(2)}$$

Fourier mode

$$v_k'' + \left(c_\mathrm{S}^2 k^2 - \frac{a''}{a}\right) v_k = 0$$

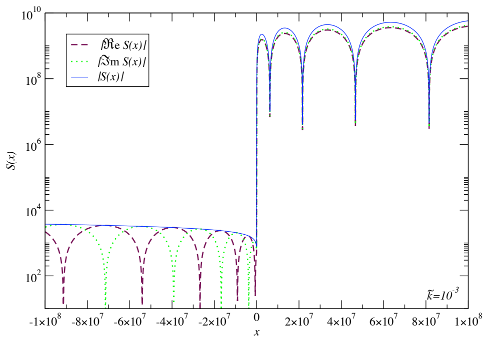
$$c_\mathrm{S}^2 = \sqrt{\omega} \neq 0$$

vacuum initial conditions

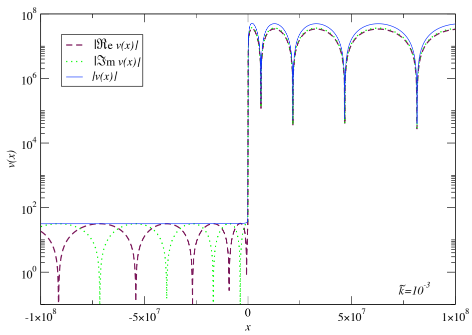
$$v_k \propto \frac{\mathrm{e}^{-ic_\mathrm{S} k\eta}}{\sqrt{2c_\mathrm{S} k}}$$

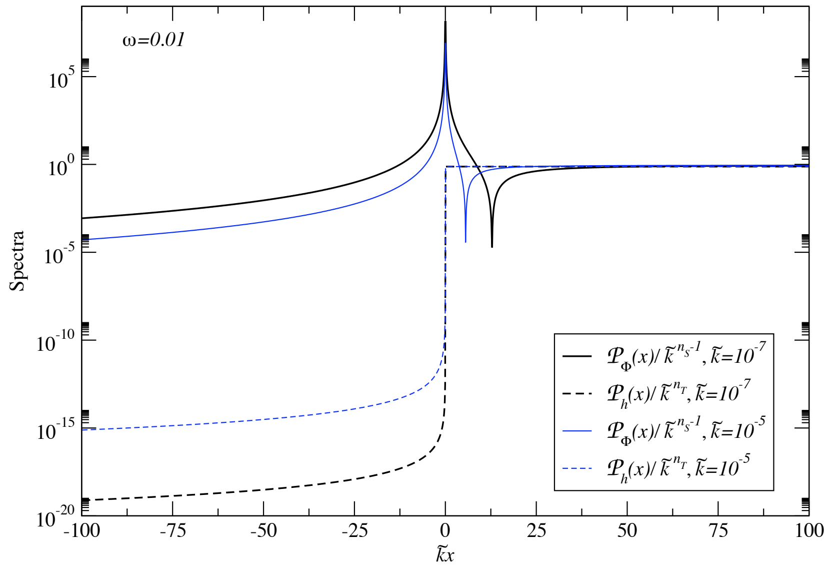
+ evolution **(matchings and/or numerics)**

$$s(x) = a^{\frac{1}{2}(1-3\omega)}\frac{v(x)}{\sqrt{T_0}}$$



$$x \equiv \frac{T}{T_0}$$





spectrum $\mathcal{P}_\Phi \propto k^3 \left|\Phi_k\right|^2 \propto A_{\mathrm{S}}^2 k^{n_{\mathrm{S}}-1}$

id. grav. waves: $\mu'' + \left(k^2 - \frac{a''}{a}\right)\mu = 0$ $\quad \mu \equiv \frac{h}{a}$

$\mu_{\mathrm{ini}} \propto \frac{\exp(-ik\eta)}{\sqrt{k\eta}}$ $\quad \mathcal{P}_h \propto k^3 \left|h_k\right|^2 \propto A_{\mathrm{T}}^2 k^{n_{\mathrm{T}}}$

same dynamics + initial conditions ⟹ **same spectrum**

$$n_{\mathrm{T}} = n_{\mathrm{S}} - 1 = \frac{12\omega}{1+3\omega}$$

CMB normalisation $A_{\mathrm{S}}^2 = 2.08 \times 10^{-10}$

⟹ **bounce curvature** $T_0 a_0^{3\omega} \simeq 1500\ell_{\mathrm{Pl}}$

WMAP constraint

$$n_{\rm s} = 0.96 \pm 0.02 \Longrightarrow w \lesssim 8 \times 10^{-4}$$

predictions

- **spectrum slightly blue**

power-law + concordance

$$\frac{T}{S} = \frac{C_{10}^{(\rm T)}}{C_{10}^{(\rm S)}} = \mathcal{F}\left(\Omega, \cdots\right) \frac{A_{\rm T}^2}{A_{\rm S}^2} \propto \sqrt{w}$$

$\mathcal{F}\left(\Omega, \cdots\right) \simeq 0.62$

$$\boxed{\frac{T}{S} \simeq 4 \times 10^{-2} \sqrt{n_{\rm s} - 1}}$$

P. P. & N. Pinto-Neto, *Phys. Rev.* **D78**, 063506 (2008)

Cosmology without inflation?

monopoles = ???

Dark energy ...

Model dependence

Cosmology without inflation!

- New solutions to old puzzles
- No singularity
- G.R. ...

New predictions (oscillations, *T/S* ...)

Future

String implementation

Non gaussianities