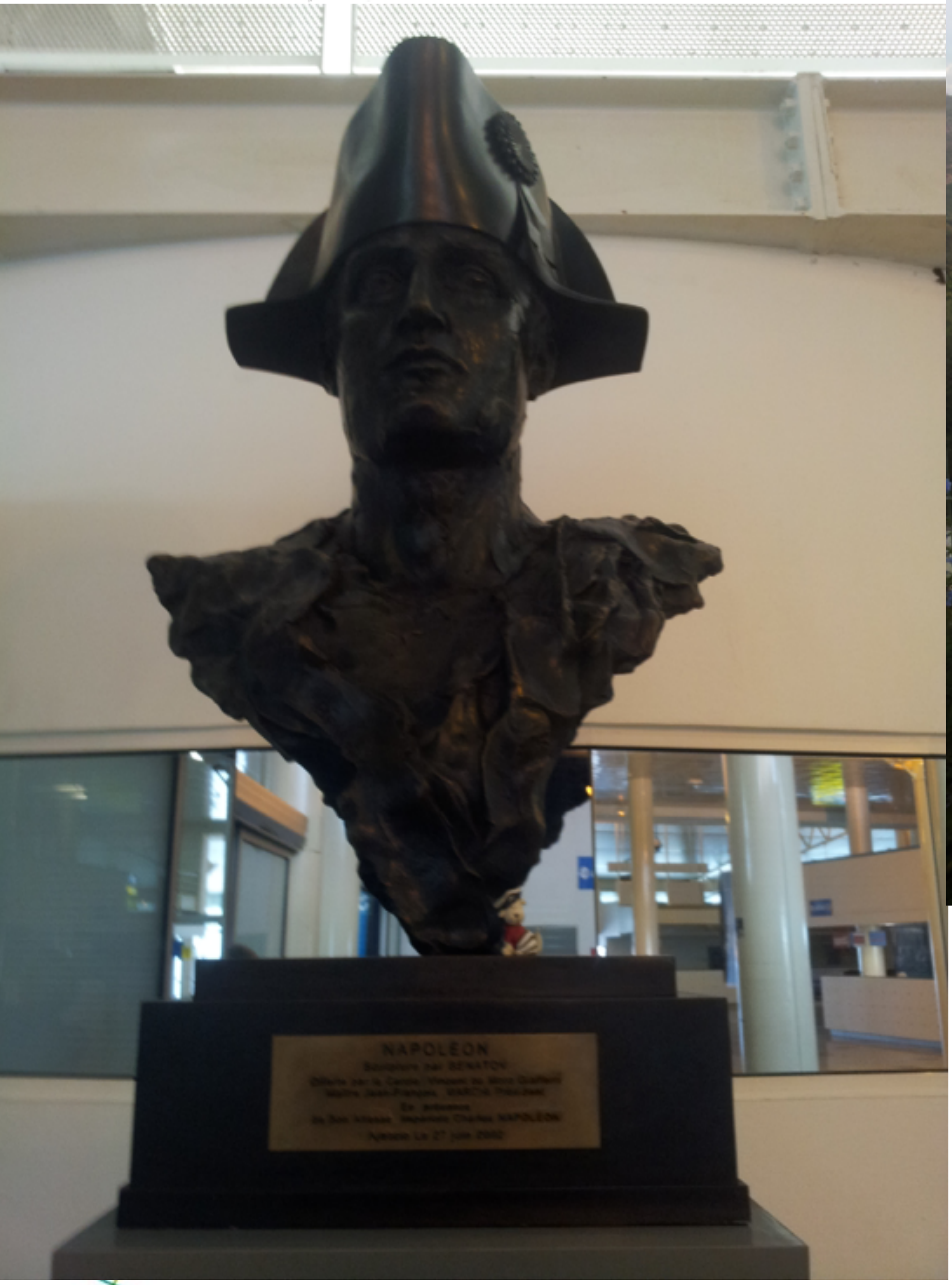
Bouncing Universe Models



Patrick Peter

Institut d'Astrophysique de Paris

GRεCO

Cargèse - 18 Sept. 2014

A brief history of bouncing cosmology

- R. C. Tolman, "*On the Theoretical Requirements for a Periodic Behaviour of the Universe*", PRD 38, 1758 (1931)
- G. Lemaître, "*L'Univers en expansion*", Ann. Soc. Sci. Bruxelles (1933)

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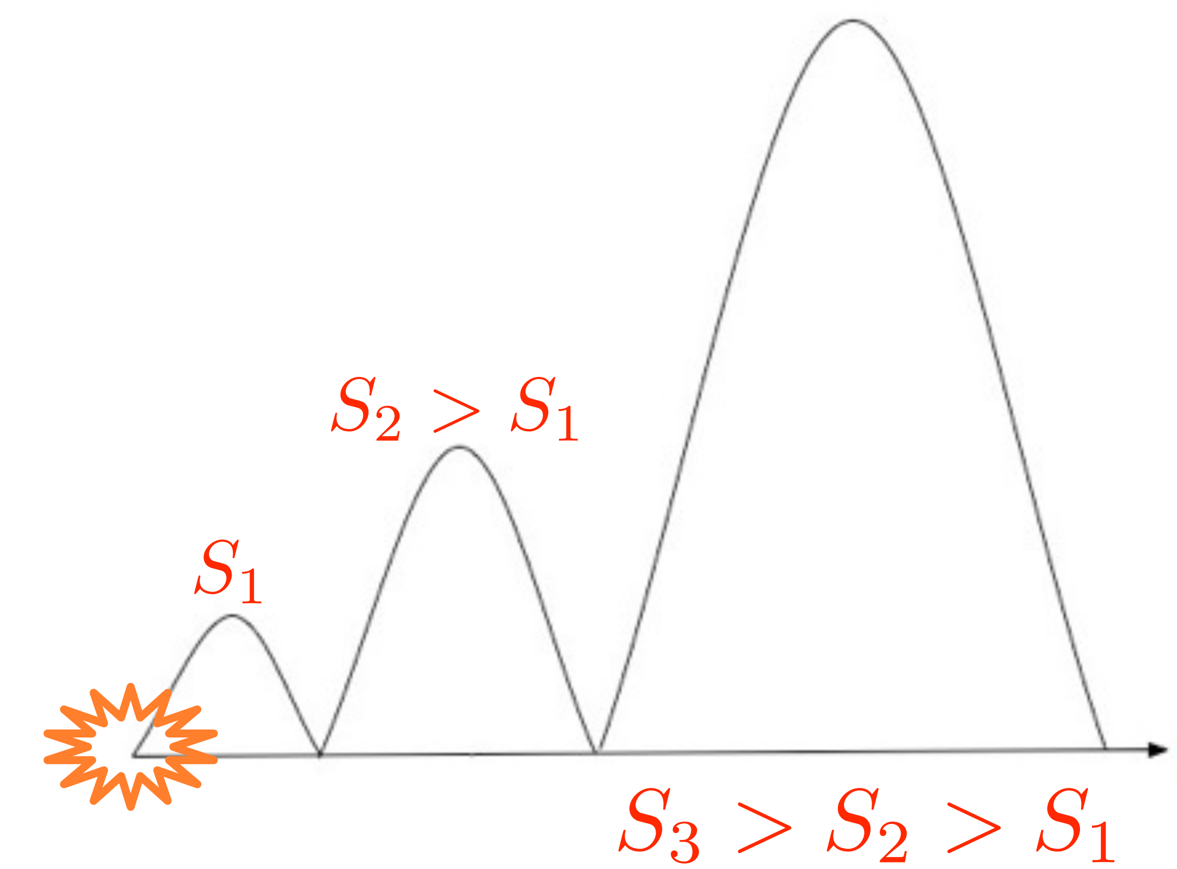
- Penrose: BH formation

Quantum nucleation?



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- M. Novello & J. M. Salim, "*Nonlinear photons in the universe*", Phys. Rev. 20, 377 (1979)
- V. N. Melnikov, S.V. Orlov, Phys. Lett. A 70, 263 (1979).
- R. Durrer & J. Laukerman, "*The oscillating Universe: an alternative to inflation*", Class. Quantum Grav. 13, 1069 (1996)
  - PBB - Ekpyrotic - Modified gravity - Quantum cosmology - Quintom - Horava-Lifshitz - Lee-Wick - ...
- M. Novello & S.E. Perez Bergliaffa, "*Bouncing cosmologies*", Phys. Rep. 463, 127 (2008)

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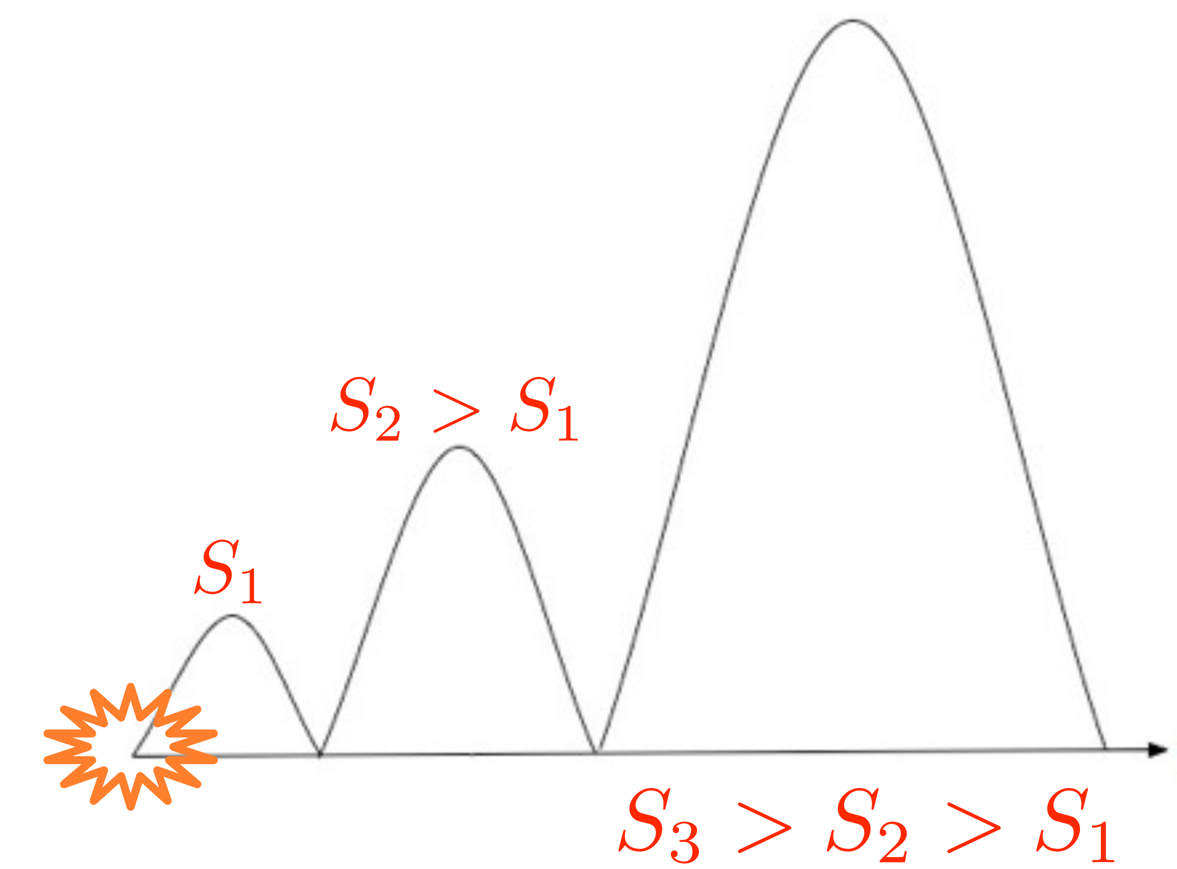

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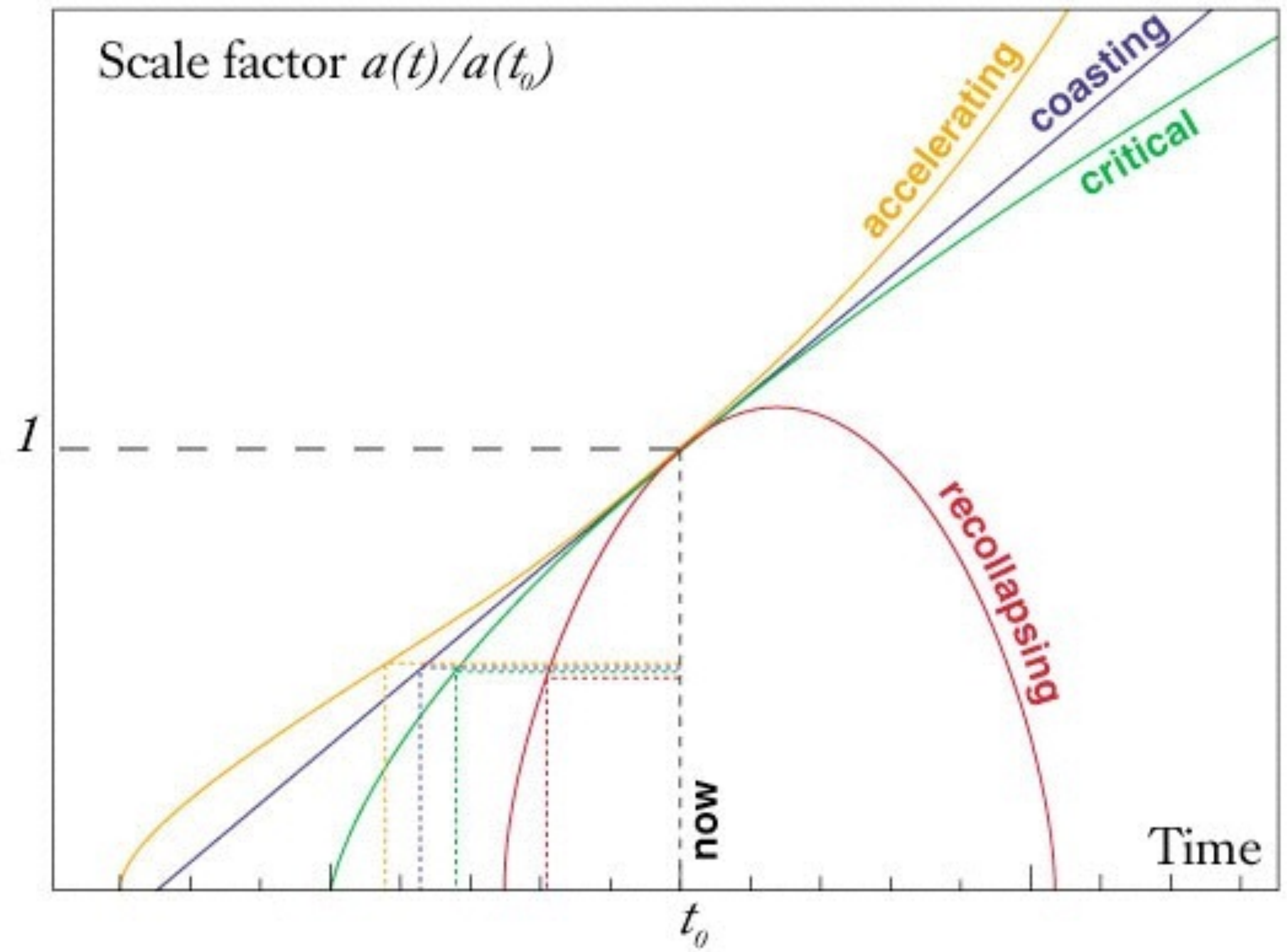
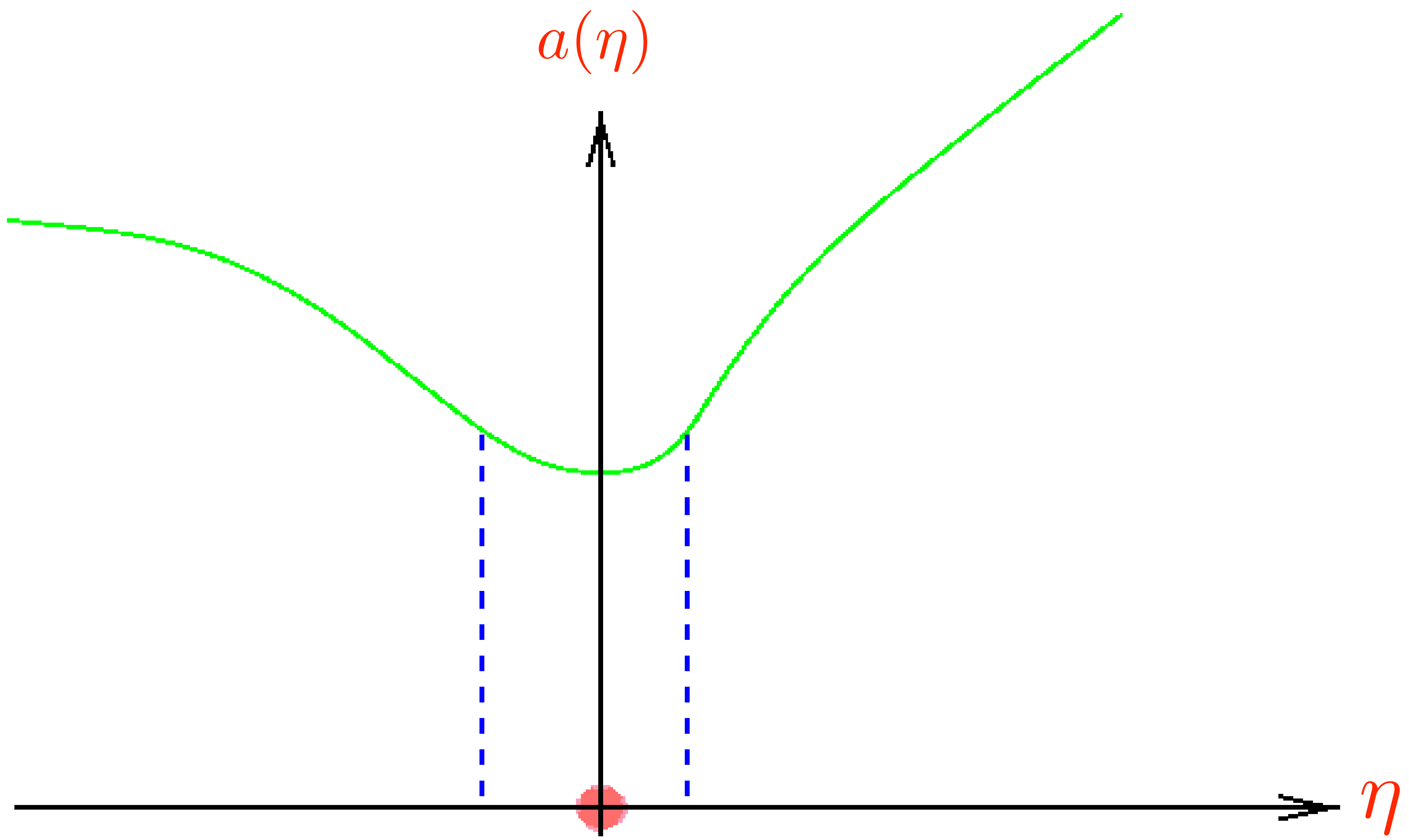
- D. Battefeld & PP, "*A Critical Review of Classical Bouncing Cosmologies*", *1406.2790*

Singularity problem




$a(\eta)$
η

Implementing a bounce = problem with GR!

Violation of Null Energy Condition (NEC) $\rho + p \geq 0$

Instabilities for perfect fluids

Implementing a bounce = problem with GR!

Violation of Null Energy Condition (NEC) $\rho + p \geq 0$

Positive spatial curvature + scalar field

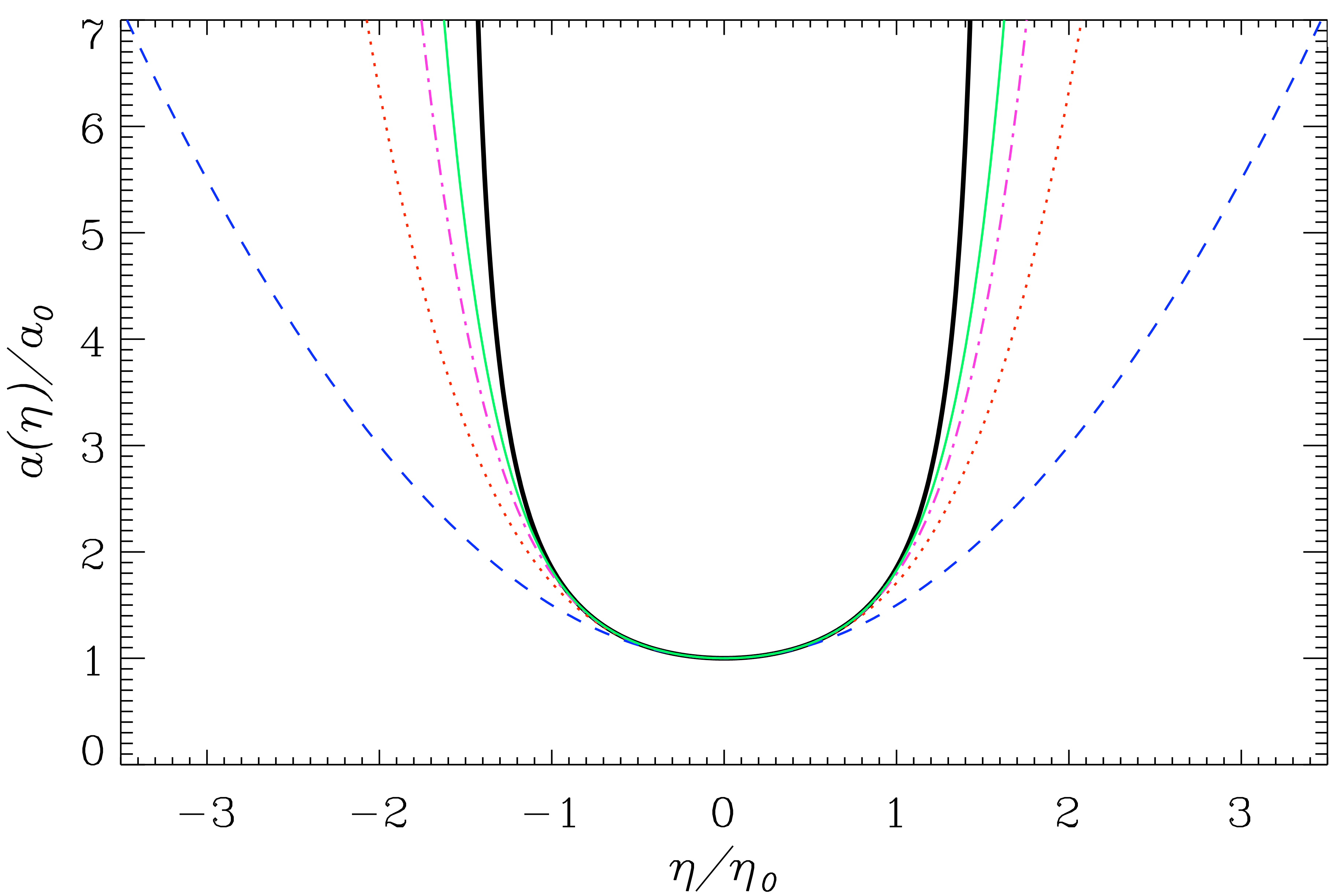
$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-\mathcal{K}r^2} + r^2 d\Omega^2\right)$$

Self consistent bounce:

→ One d.o.f. + 4 dimensions G.R.

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{6\ell_{\rm Pl}^2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right]$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) - \frac{\mathcal{K}}{a^2}$$ Positive spatial curvature



J. Martin & PP., *Phys. Rev.* **D68**, 103517 (2003)

Self consistent bounce:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\mathrm{d}\Omega^2\right)$$

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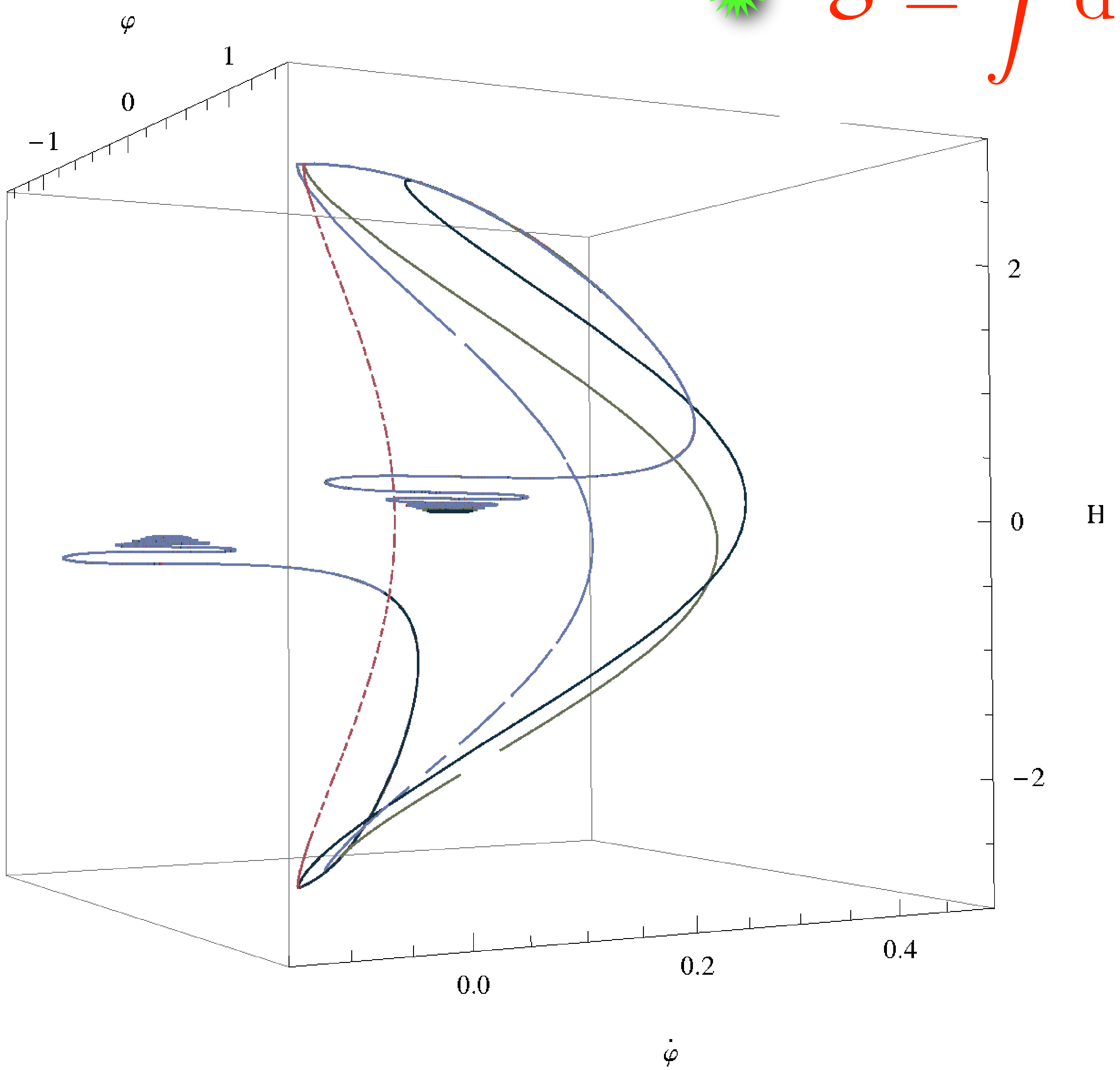
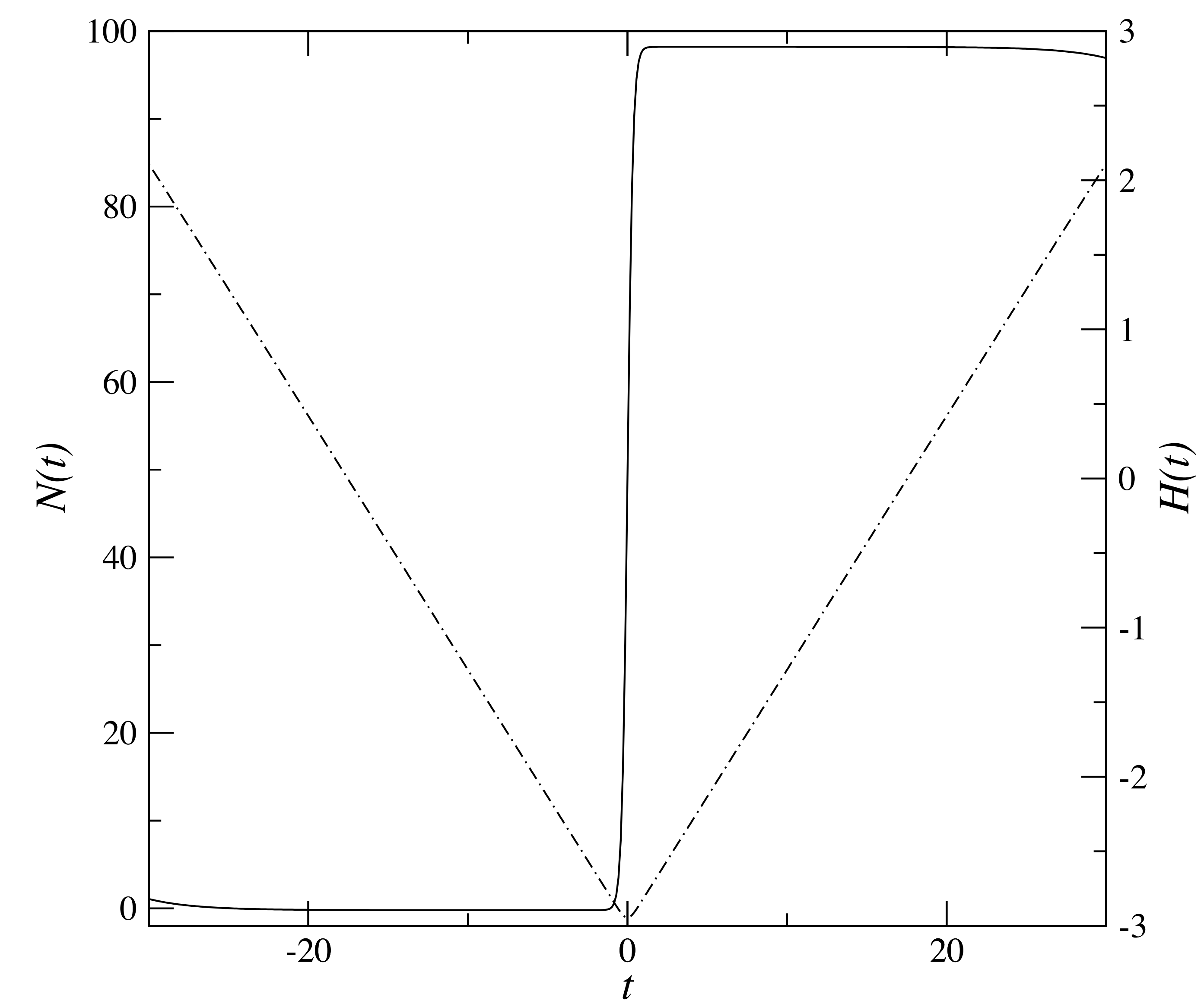




F. Falciano, M. Lilley & P. P., *Phys. Rev.* **D77**, 083513 (2008)

Implementing a bounce = problem with GR!

Violation of Null Energy Condition (NEC) $\rho + p \geq 0$

Positive spatial curvature + scalar field

Modify GR?

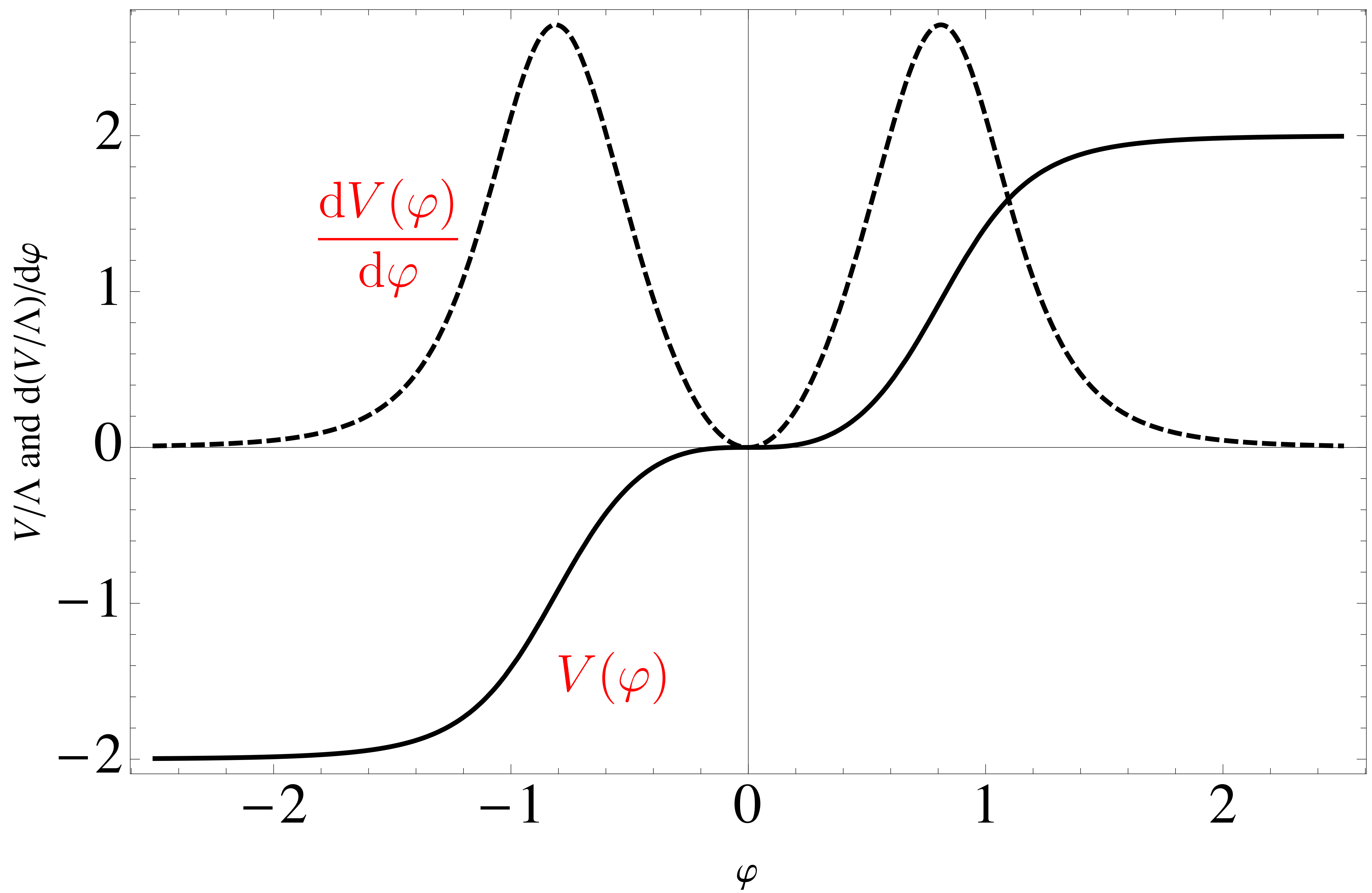
Add new terms?

K-bounce, Ghost condensates, Galileons...?

→ Modify GR to non singular theories (curvature invariants)

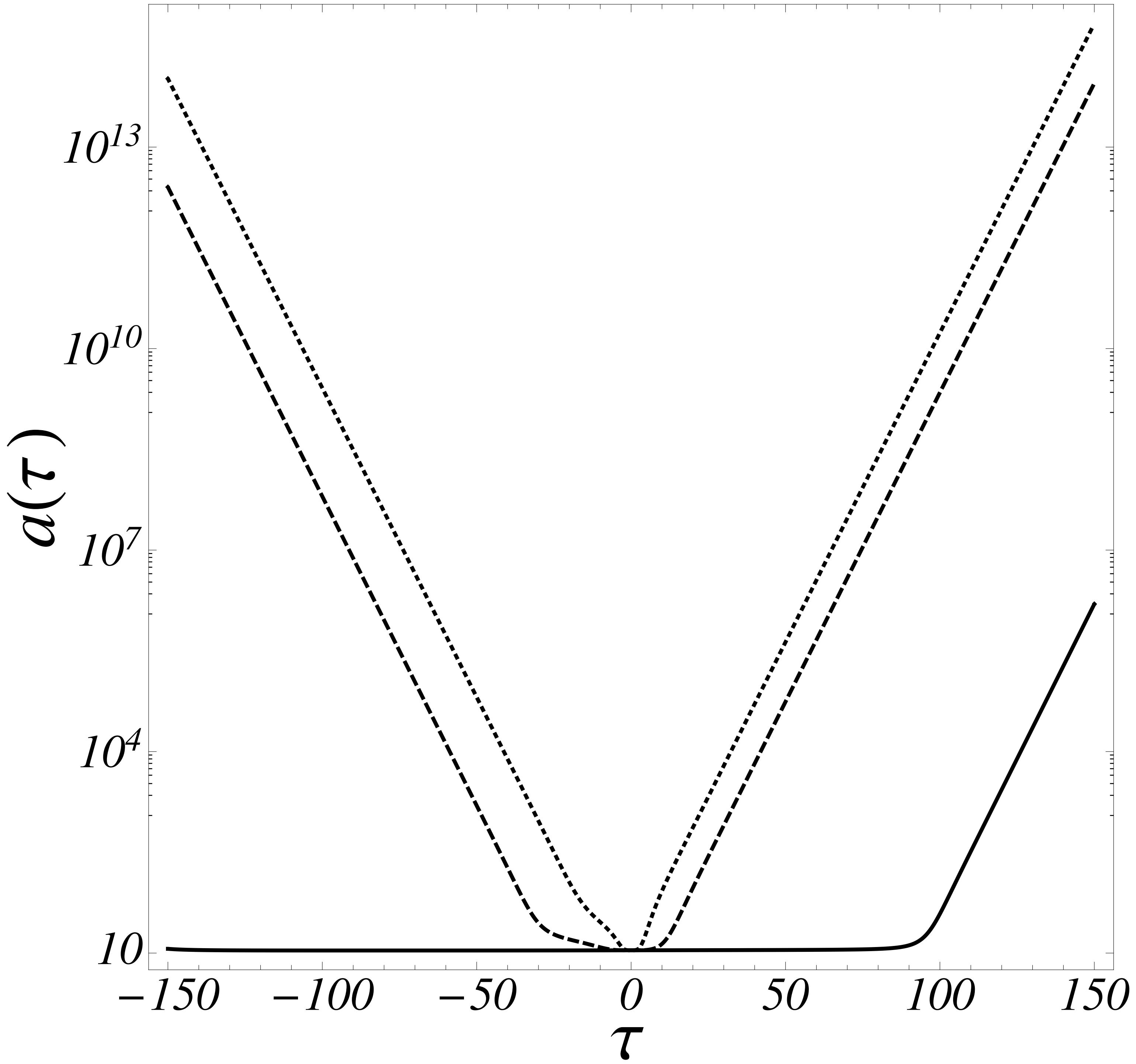
$$\mathcal{S} = \frac{1}{16\pi G_{\rm N}} \int \mathrm{d}^4x\sqrt{-g}\left[R + \sum_{i=1}^{N} \varphi_i I^{(i)} - V\left(\boldsymbol{\varphi}\right)\right] \quad \Longrightarrow \frac{\mathrm{d}V}{\mathrm{d}\varphi} = I$$

R. Brandenberger, V. F. Mukhanov and A. Sornborger, *Phys. Rev.* **D48**, 1629 (1993)

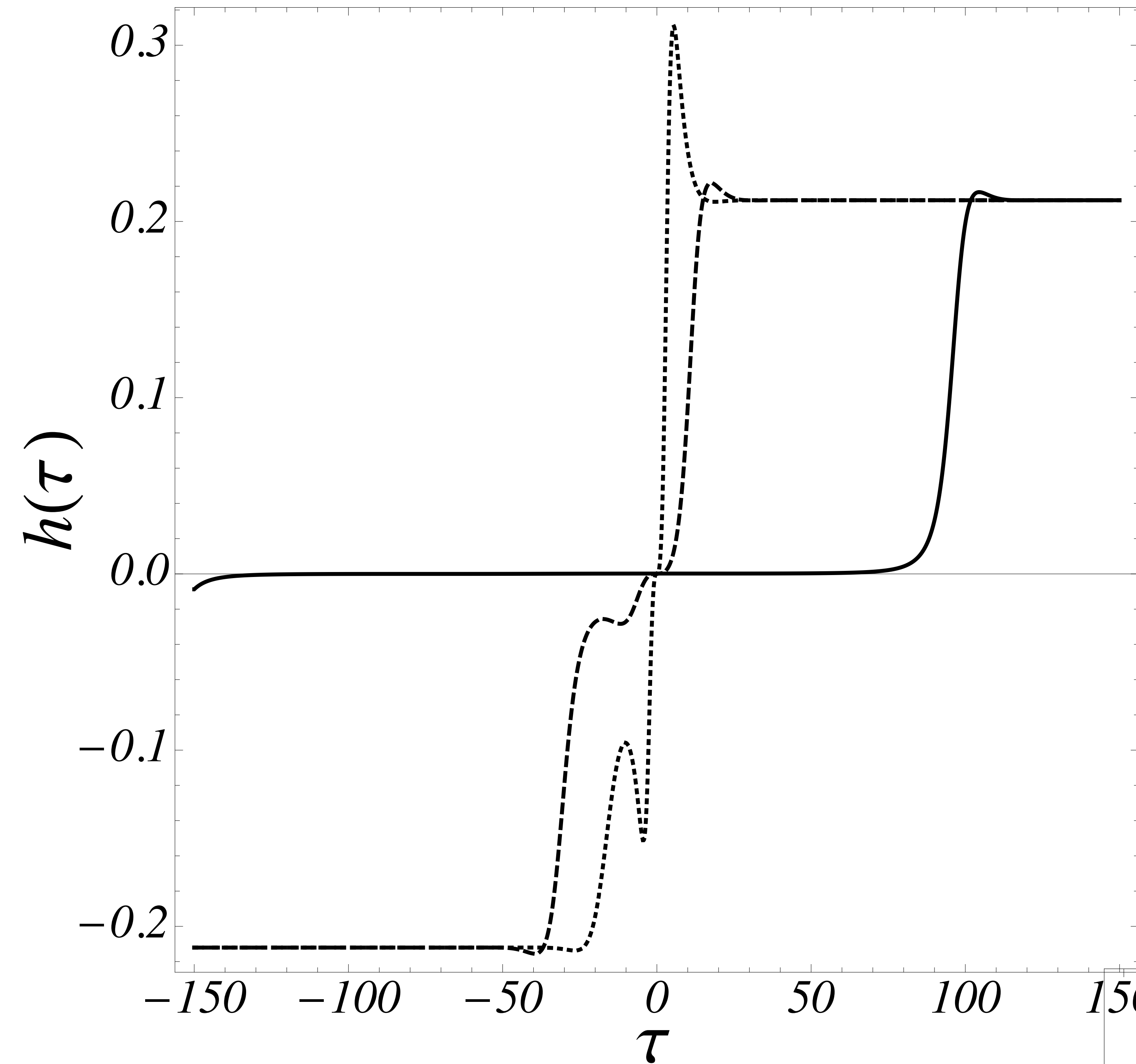


$$I = R - \sqrt{3\left(4R_{\mu\nu}R^{\mu\nu} - R^2\right)}$$

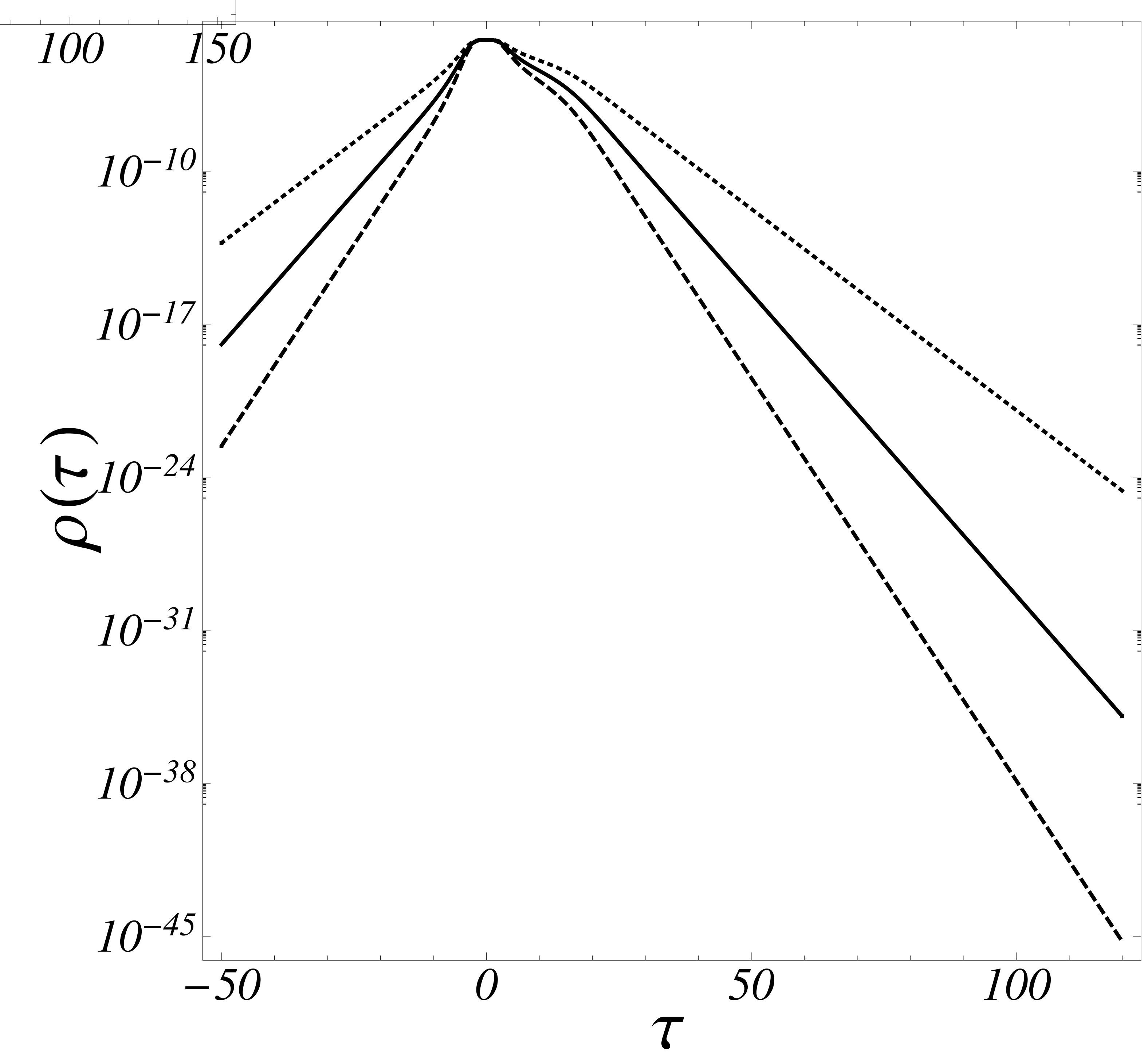
R. Abramo, P. P. & I. Yasuda, *Phys. Rev.* **D81**, 023511 (2010)



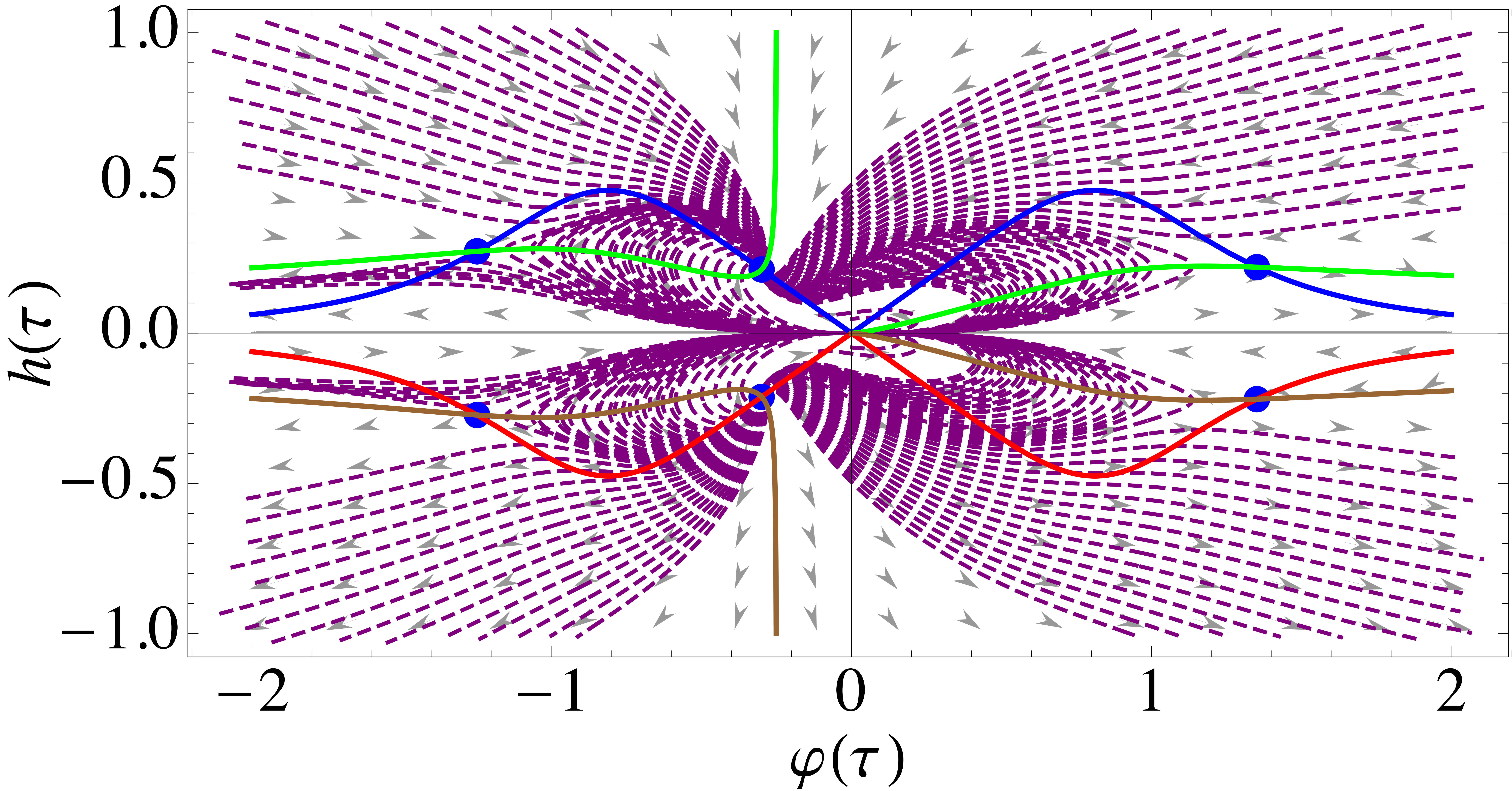
10^{13}
10^{10}
10^{7}
10^{4}
10
$a(\tau)$
−150 −100 −50 0 50 100 150
τ



0.3
0.2
0.1
0.0
−0.1
−0.2
$h(\tau)$
−150 −100 −50 0 50 100 150
τ



10^{-10}
10^{-17}
10^{-24}
10^{-31}
10^{-38}
10^{-45}
$\rho(\tau)$
−50 0 50 100
τ


1.0
0.5
0.0
−0.5
−1.0
h(τ)
−2
−1
0
1
2
φ(τ)

K-bounce: $\mathcal{L} = p(X,\varphi)$

$$X \equiv \frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

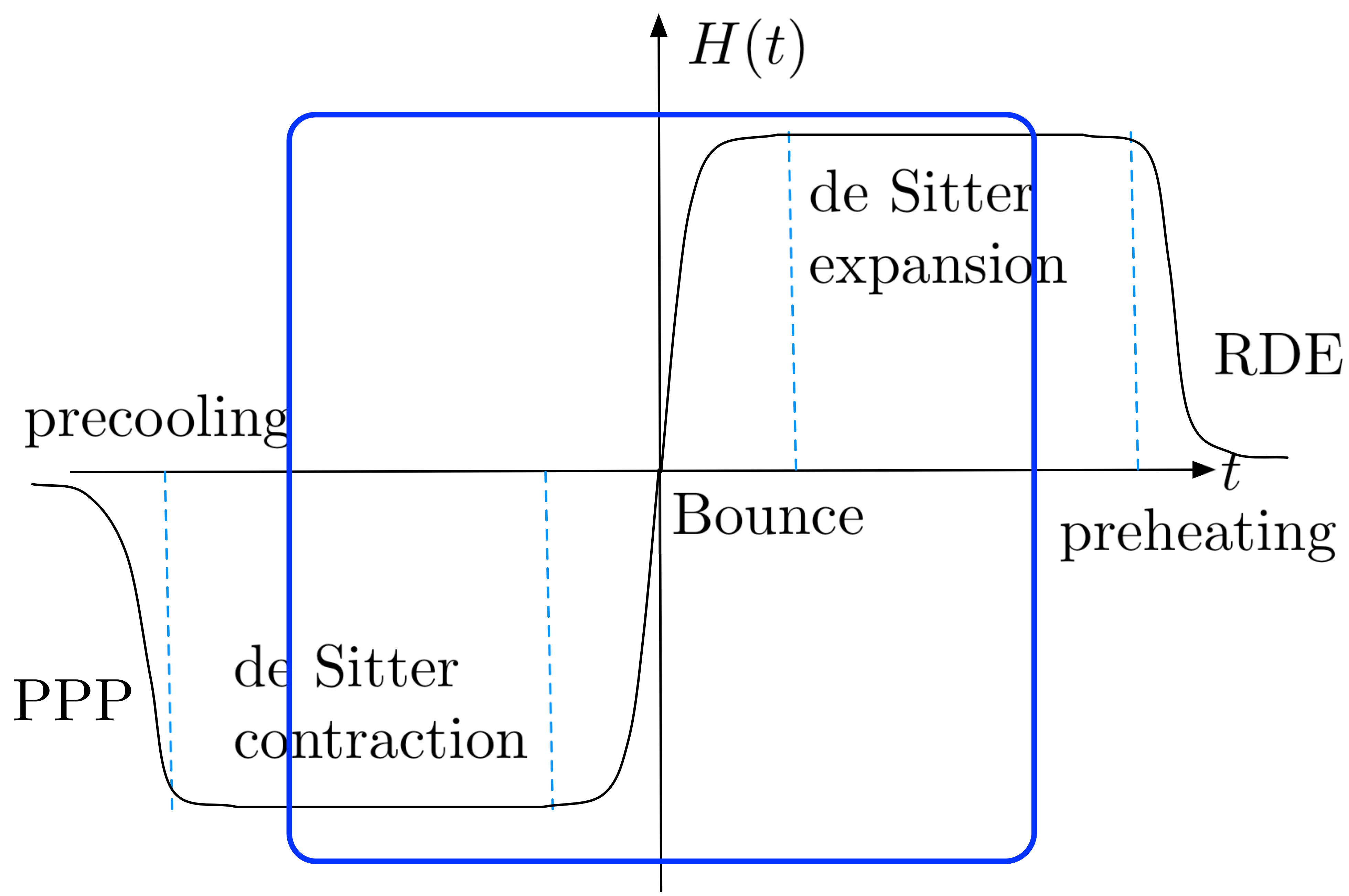
$$\Longrightarrow \quad T^{\mu\nu} = (\rho + p)\, u^\mu u^\nu - p g^{\mu\nu}$$

$$\rho \equiv 2X\frac{\partial p}{\partial X} - p$$

$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{2X}}$$

vanishing spatial curvature possible in 4 dimensions G.R.?

$$\rho\left(t_{\rm bounce}\right) = 0 \Longrightarrow p\left(t_{\rm bounce}\right) < 0$$



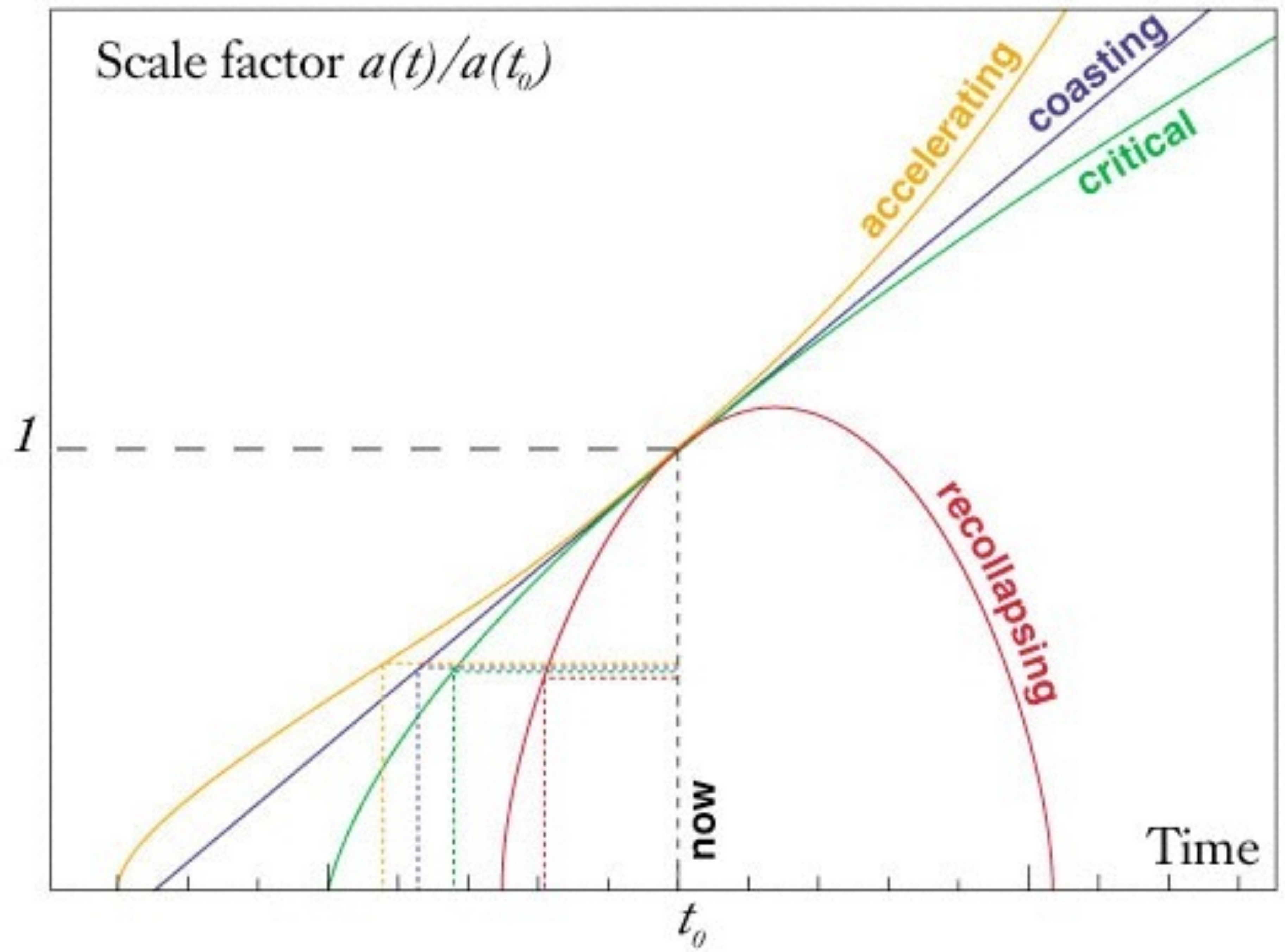
Quantized scalar field effect model:

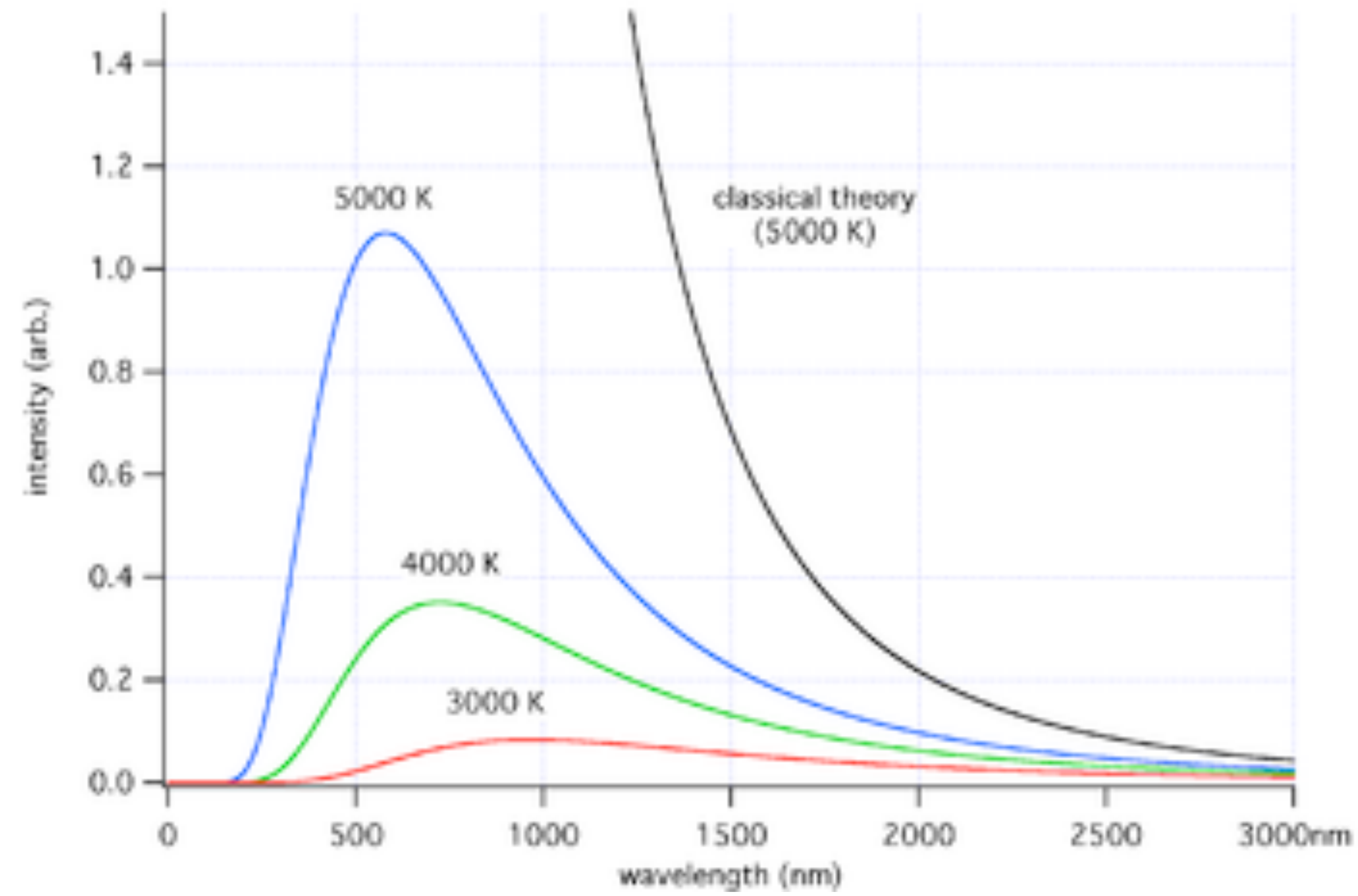
Parker & Fulling '73: massive scalar field, if $\langle a^\dagger a \rangle \gg 1$, then $\exists$ solution ($\mathcal{K} > 0$)

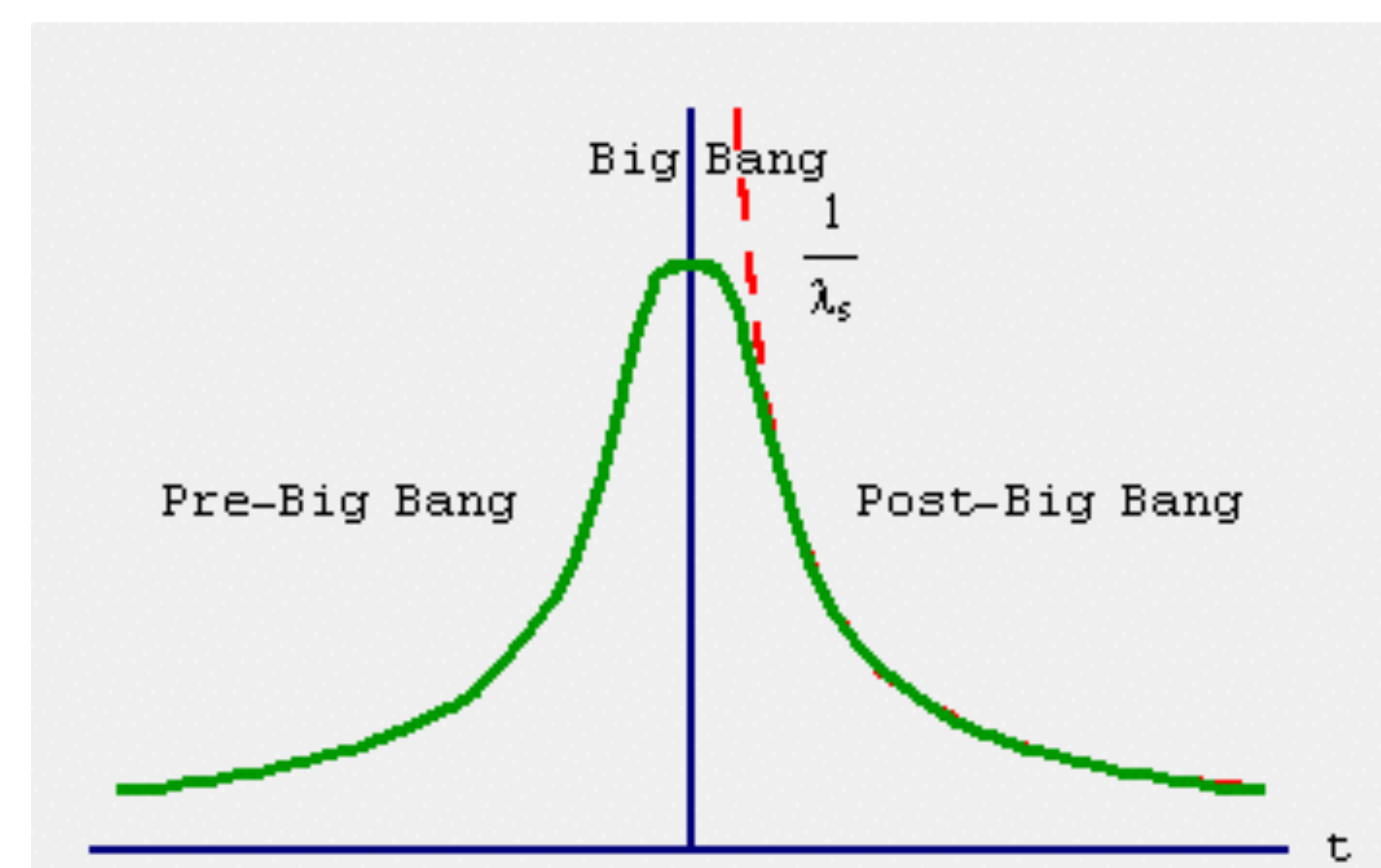
$$a(t) = \left(\frac{|B_2|^2 - |B_1|^2}{m^2 |B_2|^2} + \frac{8\pi G m^2 |B_2|^2 t^2}{3}\right)^{1/2};$$

Probability that it occurs: $\mathcal{P} \sim 10^{-43}$

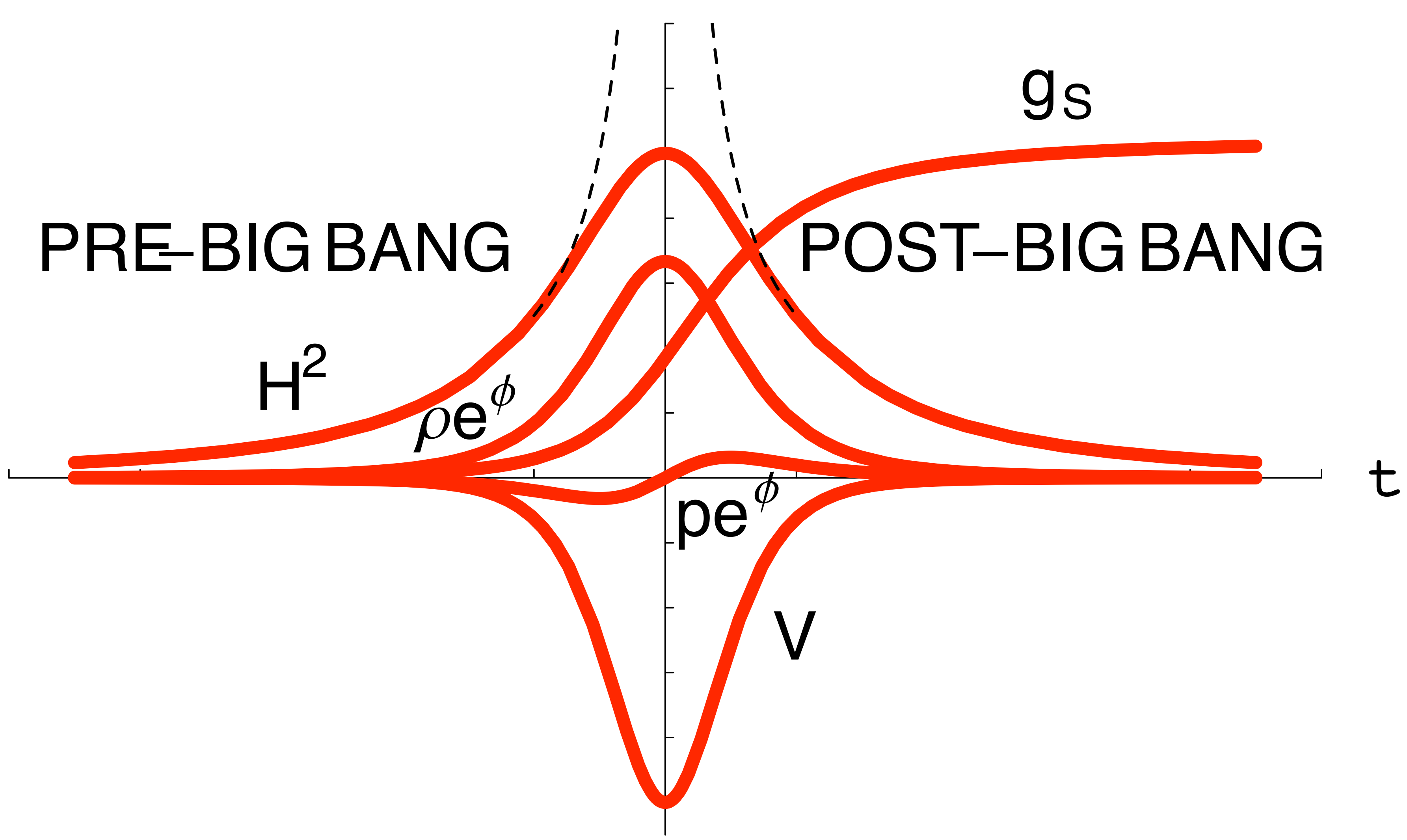
Singularity problem   Purely classical effect?



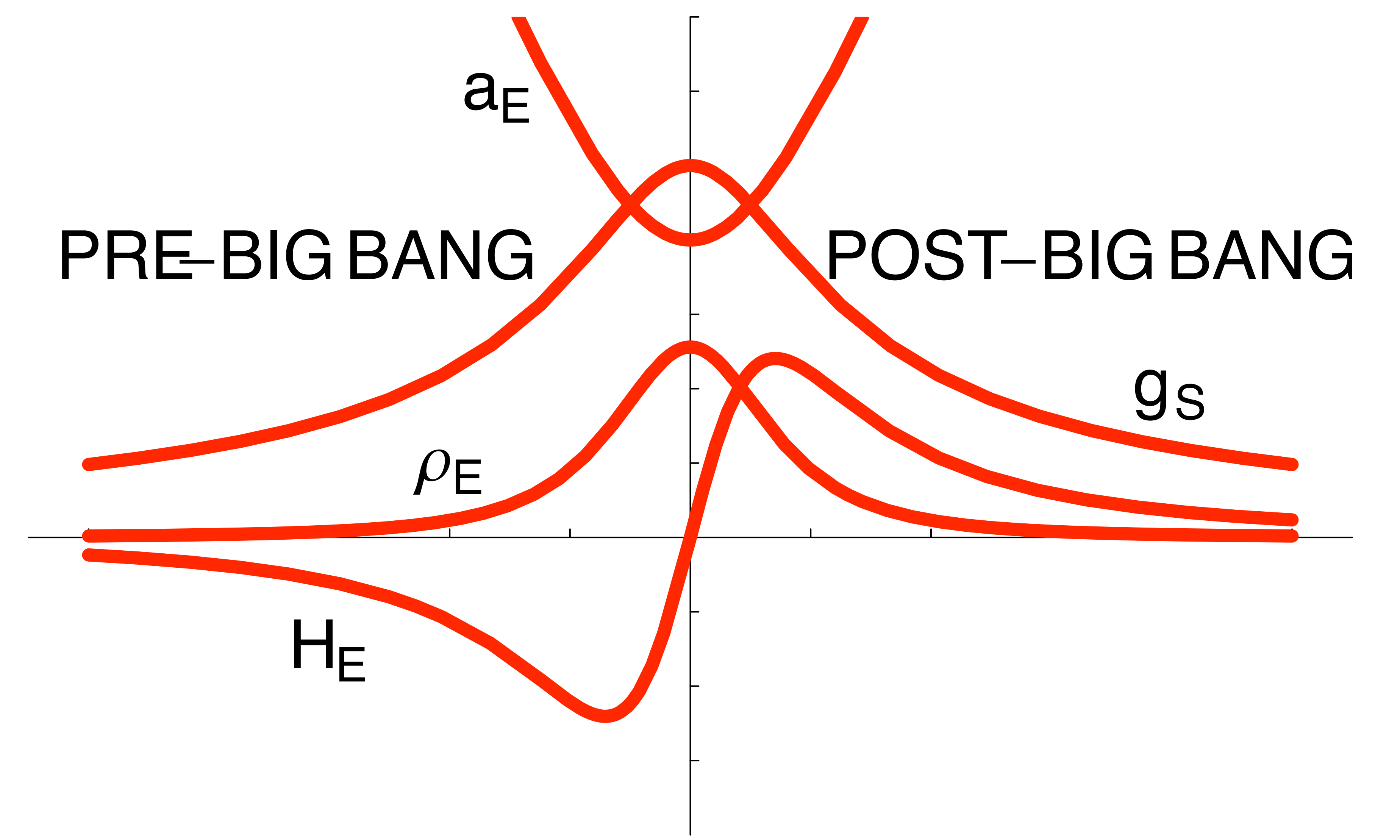




Pre Big Bang scenario: (cf. M. Gasperini & G. Veneziano, arXiv: hep-th/0703055)



string frame



Einstein frame

Quantum cosmology

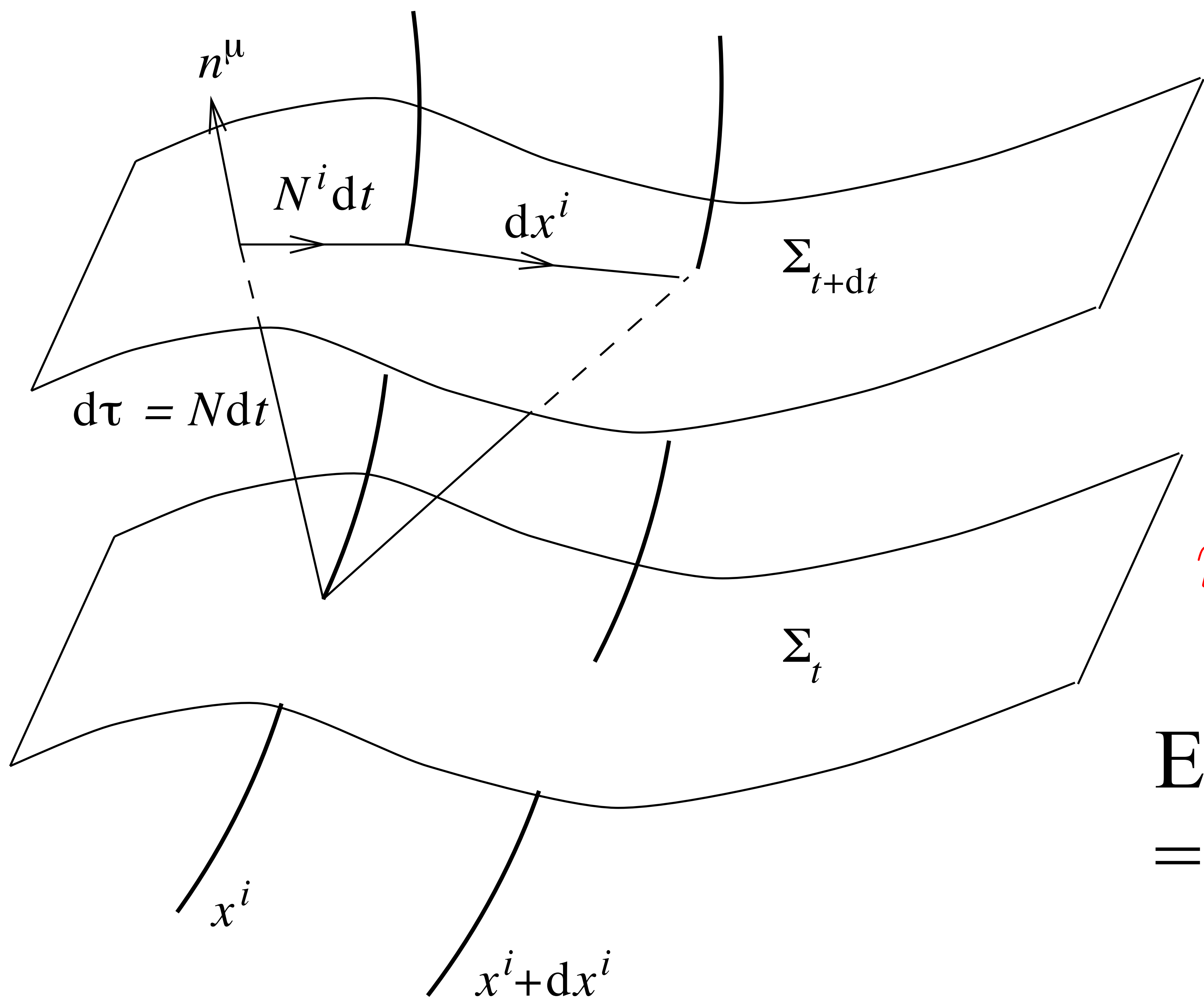
- Hamiltonian GR

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

Shift vector

Lapse function

Intrinsic metric
= first fundamental form



n^μ Normal to Σ_t Intrinsic curvature tensor ${}^3R^i{}_{jkl}(h)$

Extrinsic curvature
= second fundamental form

$$K_{ij} \equiv -\nabla_j n_i = -\Gamma^0{}_{ij} n_0 = \frac{1}{2\mathcal{N}}\left(\nabla_j\mathcal{N}_i + \nabla_i\mathcal{N}_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action: $$\mathcal{S} = \frac{1}{16\pi G_{\mathrm{N}}}\left[\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + 2\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{h}K^i{}_i\right] + \mathcal{S}_{\mathrm{matter}}$$

In 3+1 expansion: $\mathcal{S} \equiv \int \mathrm{d}t L = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}t\mathrm{d}^3x\, \mathcal{N}\sqrt{h}\left(K_{ij}K^{ij} - K^2 + {}^3R - 2\Lambda\right) + \mathcal{S}_{\mathrm{matter}}$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(K^{ij} - h^{ij}K\right)$$

$$\pi_\Phi \equiv \frac{\delta L}{\delta \dot{\Phi}} = \frac{\sqrt{h}}{\mathcal{N}}\left(\dot{\Phi} - \mathcal{N}^i \frac{\partial \Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}} = 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}_i} = 0 \end{aligned}\right\} \text{Primary constraints}$$

Hamiltonian $H \equiv \int \mathrm{d}^3x \left(\pi^0\dot{\mathcal{N}} + \pi^i\dot{\mathcal{N}}_i + \pi^{ij}\dot{h}_{ij} + \pi_\Phi\dot{\Phi}\right) - L = \int \mathrm{d}^3x \left(\pi^0\dot{\mathcal{N}} + \pi^i\dot{\mathcal{N}}_i + \mathcal{N}\mathcal{H} + \mathcal{N}_i\mathcal{H}^i\right)$

$$\left.\begin{aligned} &\text{Variation wrt lapse } \mathcal{H} = 0 \text{ Hamiltonian constraint} \\ &\text{Variation wrt shift } \mathcal{H}^i = 0 \text{ momentum constraint} \end{aligned}\right\}$$

$\Longrightarrow$ Classical description

- Superspace & canonical quantisation

Relevant configuration space?

matter fields

$$\mathrm{Riem}(\Sigma) \equiv \left\{ h_{ij}\left(x^\mu\right), \Phi\left(x^\mu\right) \ \mid x \in \Sigma \right\}$$

parameters

GR $\Longrightarrow$ invariance / diffeomorphisms $\Longrightarrow$ $\mathrm{Conf} = \dfrac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}_0(\Sigma)}$ superspace

Wave functional $\Psi\left[h_{ij}(x), \Phi(x)\right]$

Dirac canonical quantisation

$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}} \qquad \pi_\Phi \to -i\frac{\delta}{\delta \Phi} \qquad \pi^0 \to -i\frac{\delta}{\delta \mathcal{N}} \qquad \pi^i \to -i\frac{\delta}{\delta \mathcal{N}_i}$$

Primary constraints

$$\hat{\pi}\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}} = 0$$

$$\hat{\pi}^i\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}_i} = 0$$

Momentum constraint $\hat{\mathcal{N}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\frac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\mathrm{N}}\hat{T}^{0i}\Psi$

$\Longrightarrow$ Ψ is the same for configurations $\{h_{ij}(x), \Phi(x)\}$ related by a coordinate transformation

Hamiltonian constraint

$$\boxed{\hat{\mathcal{H}}\Psi = \left[-16\pi G_{\mathrm{N}}\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(-{}^3R + 2\Lambda + 16\pi G_{\mathrm{N}}\hat{T}^{00}\right)\right]\Psi = 0}$$

Wheeler - De Witt equation

$$\mathcal{G}_{ijkl} = \frac{1}{2}h^{-1/2}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

DeWitt metric...

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t),\phi(t)\right]$

Conceptual and technical problems:

Infinite number of dof $\longrightarrow$ a few: mathematical consistency?
Freeze momenta? Heisenberg uncertainties?
QM = minisuperspace of QFT

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t), \phi(t)\right]$

However, one can actually make calculations!

Exemple : Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70) $$p = p_0\left[\frac{\dot{\varphi}+\theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$$

$(\varphi,\theta,s) =$ Velocity potentials

canonical transformation: $T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega}$ ...

+ rescaling (volume…) + units… : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$a^{3\omega}$

Wheeler-De Witt $\quad H\Psi = 0$

$$\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial \Psi}{\partial T} = \frac{1}{4}\frac{\partial^2 \Psi}{\partial \chi^2}}$$

space defined by $\chi > 0 \longrightarrow$ constraint $\quad \bar{\Psi}\frac{\partial \Psi}{\partial \chi} = \Psi \frac{\partial \bar{\Psi}}{\partial \chi}$

Gaussian wave packet

$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2 + T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2 + T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $\quad S = \frac{T\chi^2}{T_0^2 + T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$

What do we do with the wave function of the Universe???

Quantum mechanics of closed systems

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

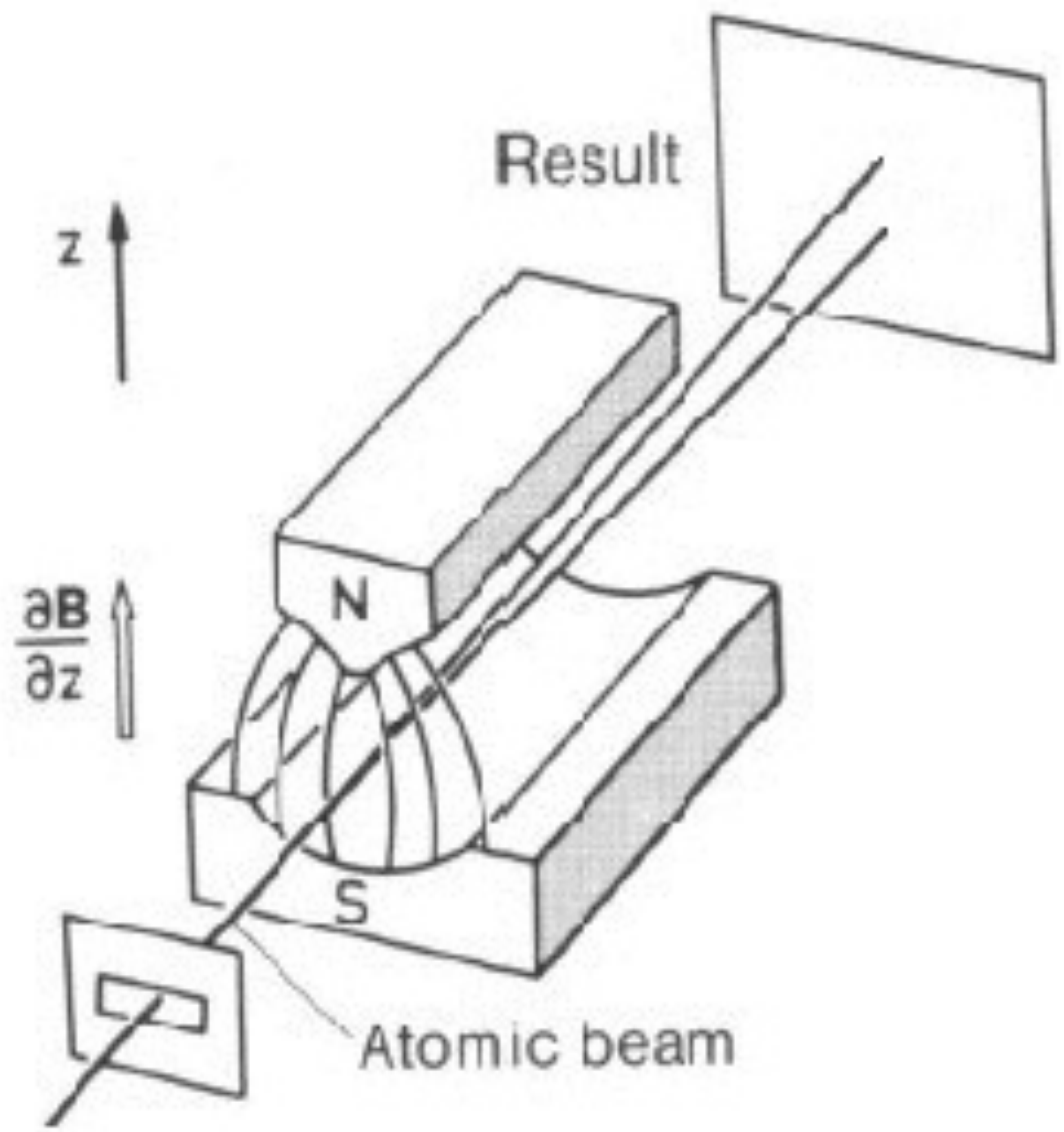
Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution
Wavepacket reduction = non linear / stochastic

Mutually incompatible

+ *External observer*

The measurement problem in quantum mechanics



Stern-Gerlach

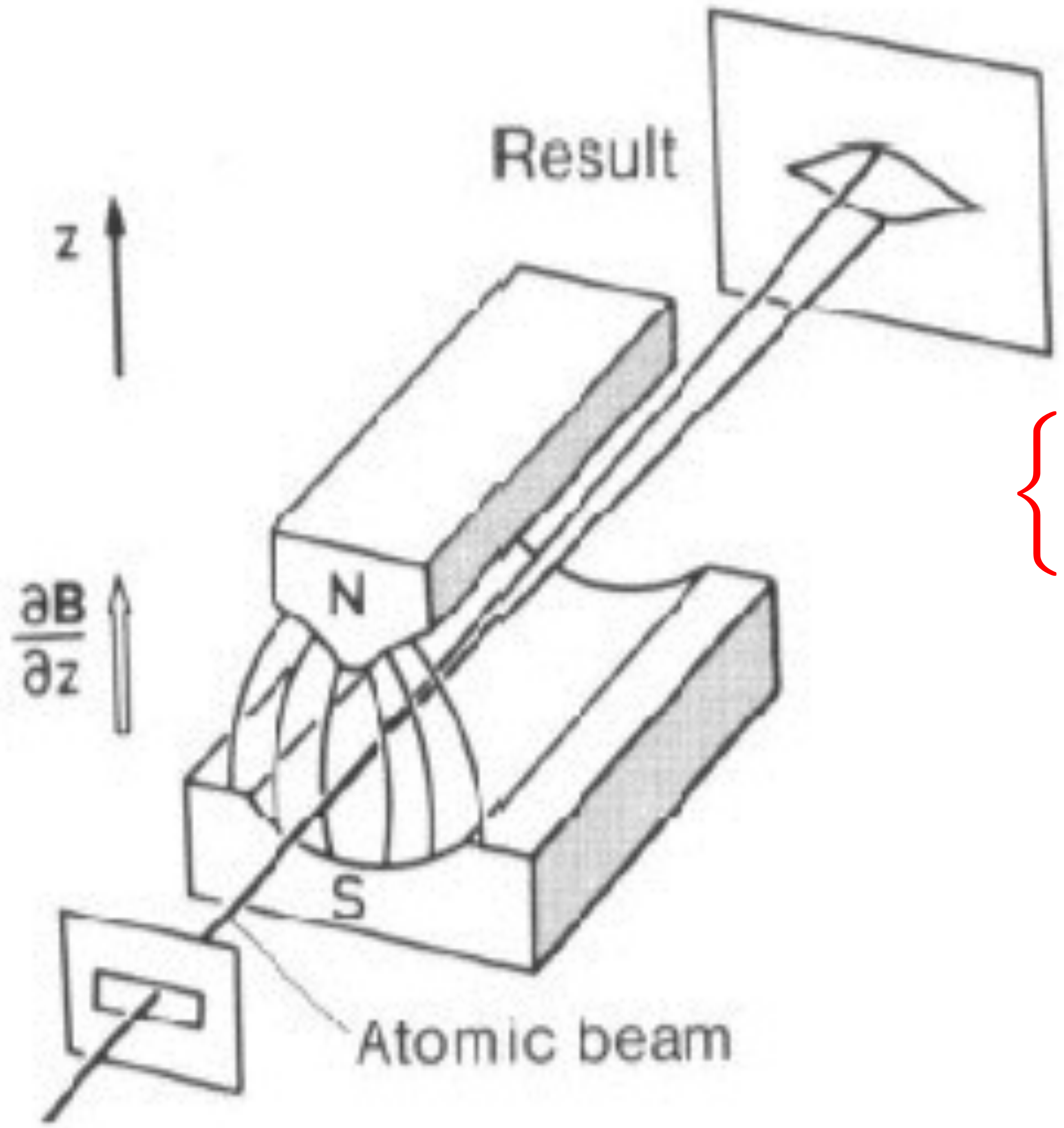
$$|\Psi_{\rm in}\rangle = \frac{1}{\sqrt{2}}\overbrace{(|\uparrow\rangle + |\downarrow\rangle)}^{\text{pure state}} \otimes |{\rm SG_{in}}\rangle$$

Unitary, deterministic
Schödinger evolution

$$\begin{aligned} |\Psi_{\rm f}\rangle &= \exp\left[\int_{t_{\rm in}}^{t_{\rm f}} \hat{H}(\tau)\, {\rm d}\tau\right] |\Psi_{\rm in}\rangle \\ &= \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |{\rm SG}_{\uparrow}\rangle + |\downarrow\rangle \otimes |{\rm SG}_{\downarrow}\rangle) \end{aligned}$$

Problem: how to reach the actual measurement $|\uparrow\rangle \otimes |{\rm SG}_{\uparrow}\rangle$ or $|\downarrow\rangle \otimes |{\rm SG}_{\downarrow}\rangle$?

The measurement problem in quantum mechanics



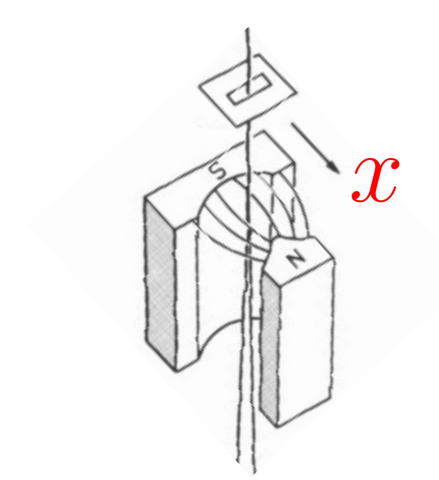
Stern-Gerlach

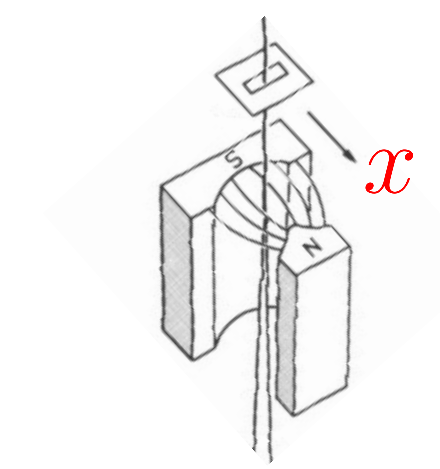
Statistical mixture

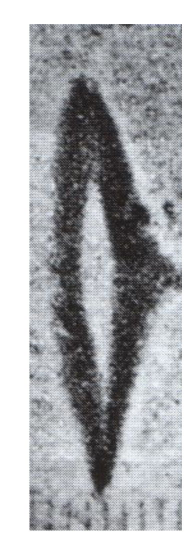
$$\Big\{|\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle\Big\} \cup \Big\{|\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle\Big\}$$

$$\Big\{|\uparrow\rangle \otimes |\mathrm{SG_{in}}\rangle\Big\} \cup \Big\{|\downarrow\rangle \otimes |\mathrm{SG_{in}}\rangle\Big\}$$

$$\Big\{(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG_{in}}\rangle\Big\}$$

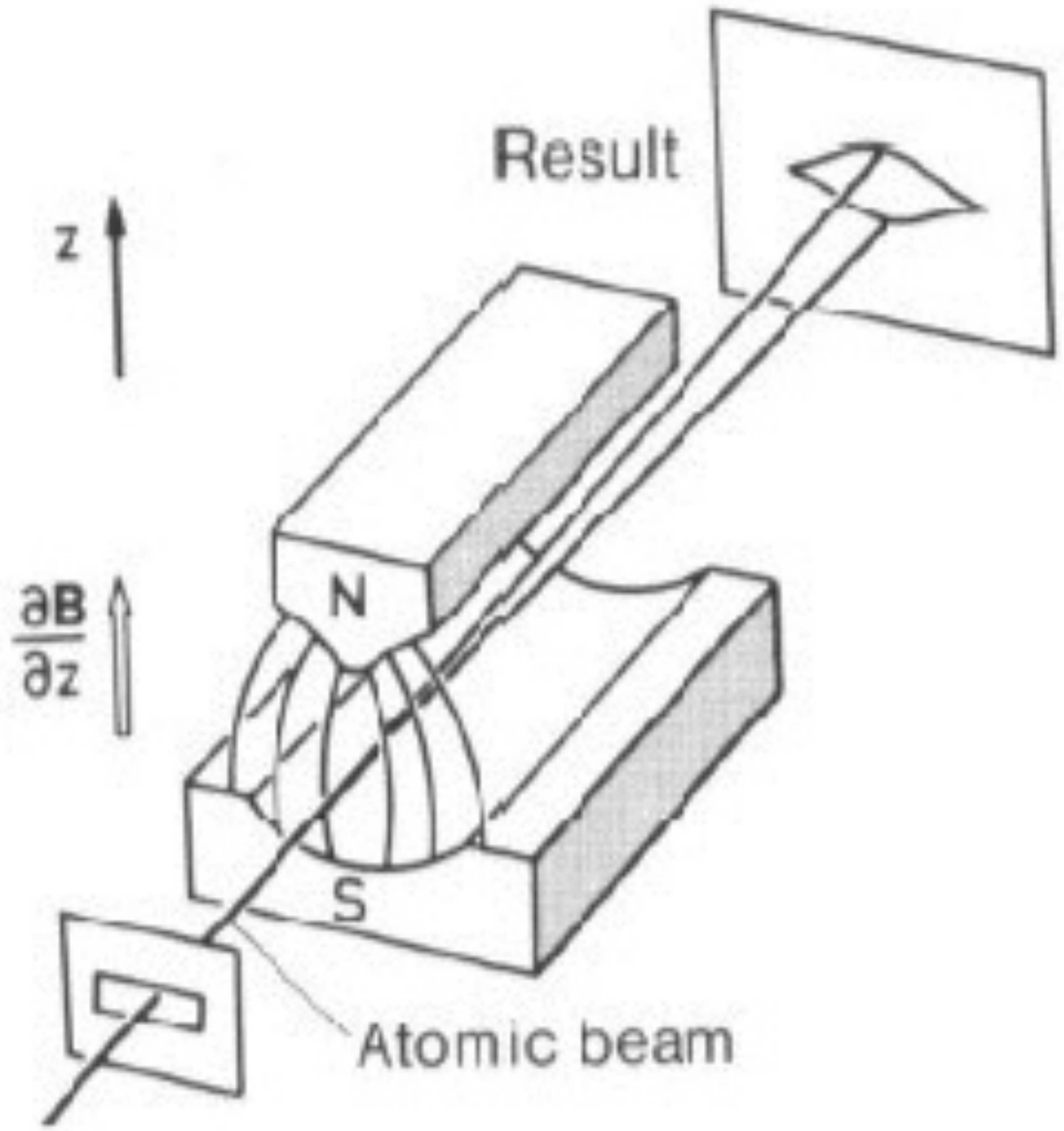






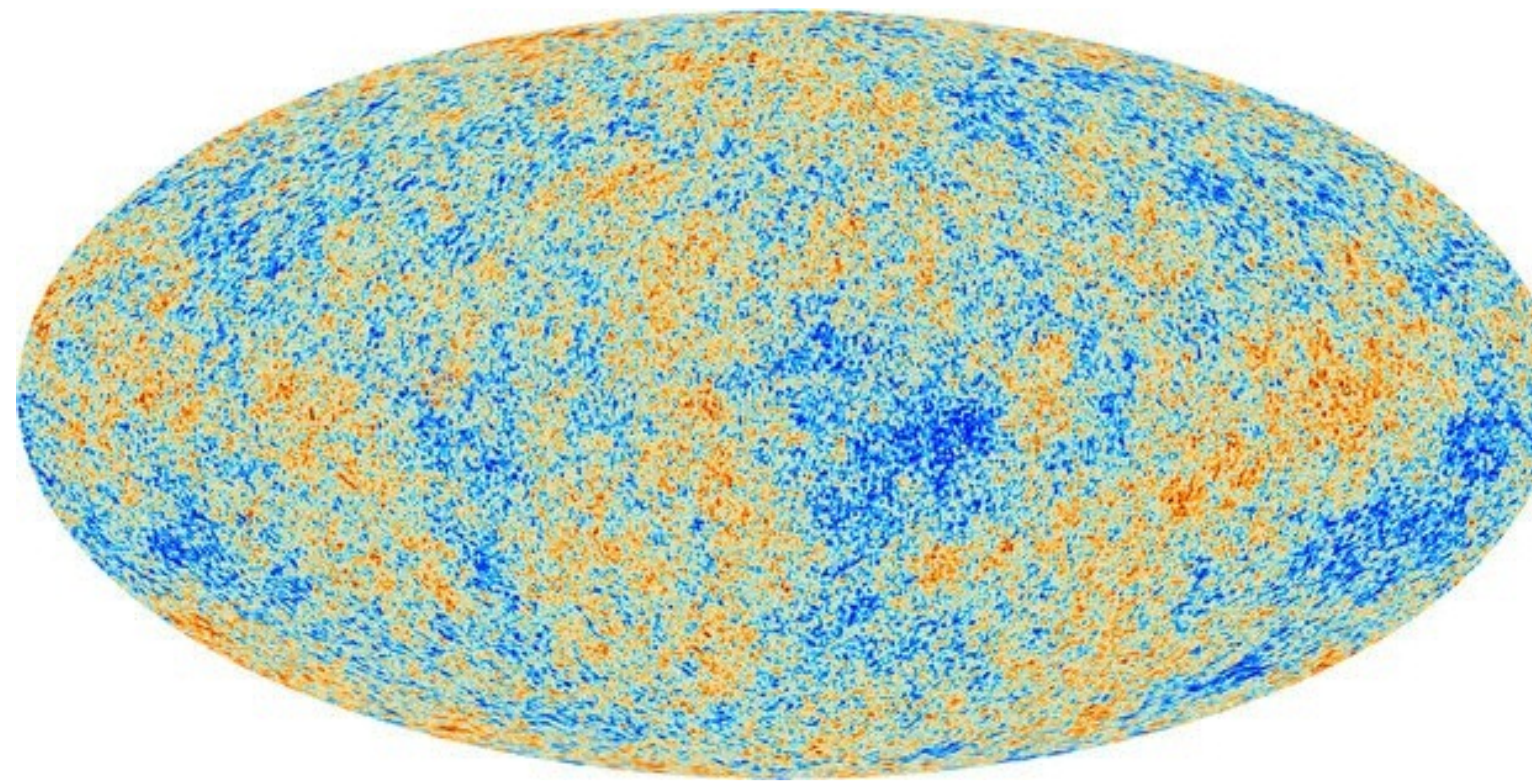
What about situations in which one has only one realization?

The measurement problem in quantum mechanics



What about the Universe itself?

Stern-Gerlach



What about situations in which one has only one realization?

Hidden Variable Theories

Schrödinger $$i\frac{\partial\Psi}{\partial t}=\left[-\frac{\nabla^2}{2m}+V\left(\boldsymbol{r}\right)\right]\Psi$$

Polar form of the wave function $$\Psi=A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS\left(\boldsymbol{r},t\right)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t}+\frac{\left(\nabla S\right)^2}{2m}+V\left(\boldsymbol{r}\right)+Q\left(\boldsymbol{r},t\right)=0$$

quantum potential

$$\equiv-\frac{1}{2m}\frac{\nabla^2 A}{A}$$

$$\Psi = A\,(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^{*}\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^{2}} = -\boldsymbol{\nabla}S$$

$$\Psi = A(\boldsymbol{r}, t)\, \mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (BdB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (Bohm)

$$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V+Q) \qquad Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = -\boldsymbol{\nabla}S$$

$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

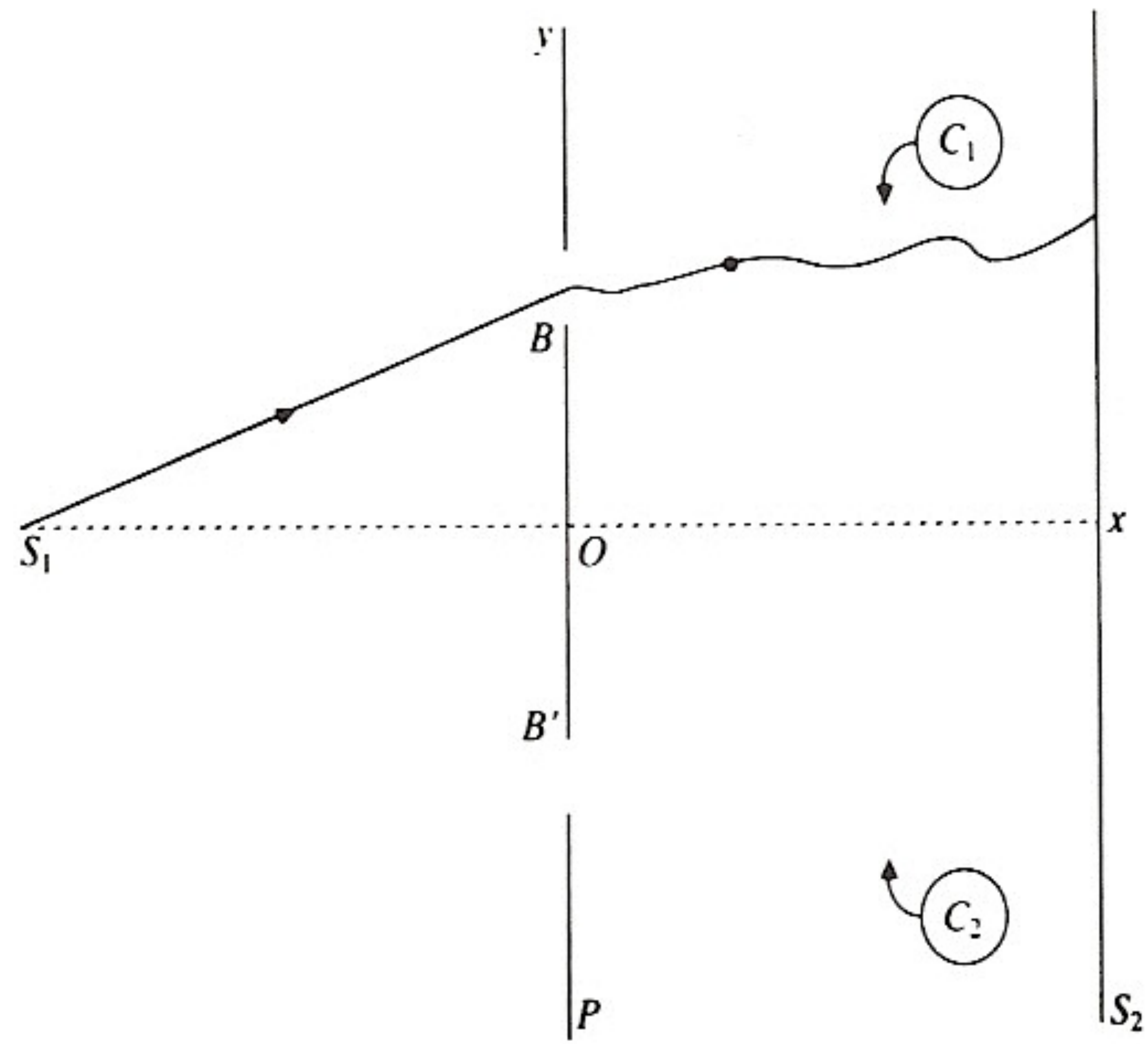
Trajectories satisfy (de Broglie) $m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^* \boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = -\boldsymbol{\nabla} S$

Properties:

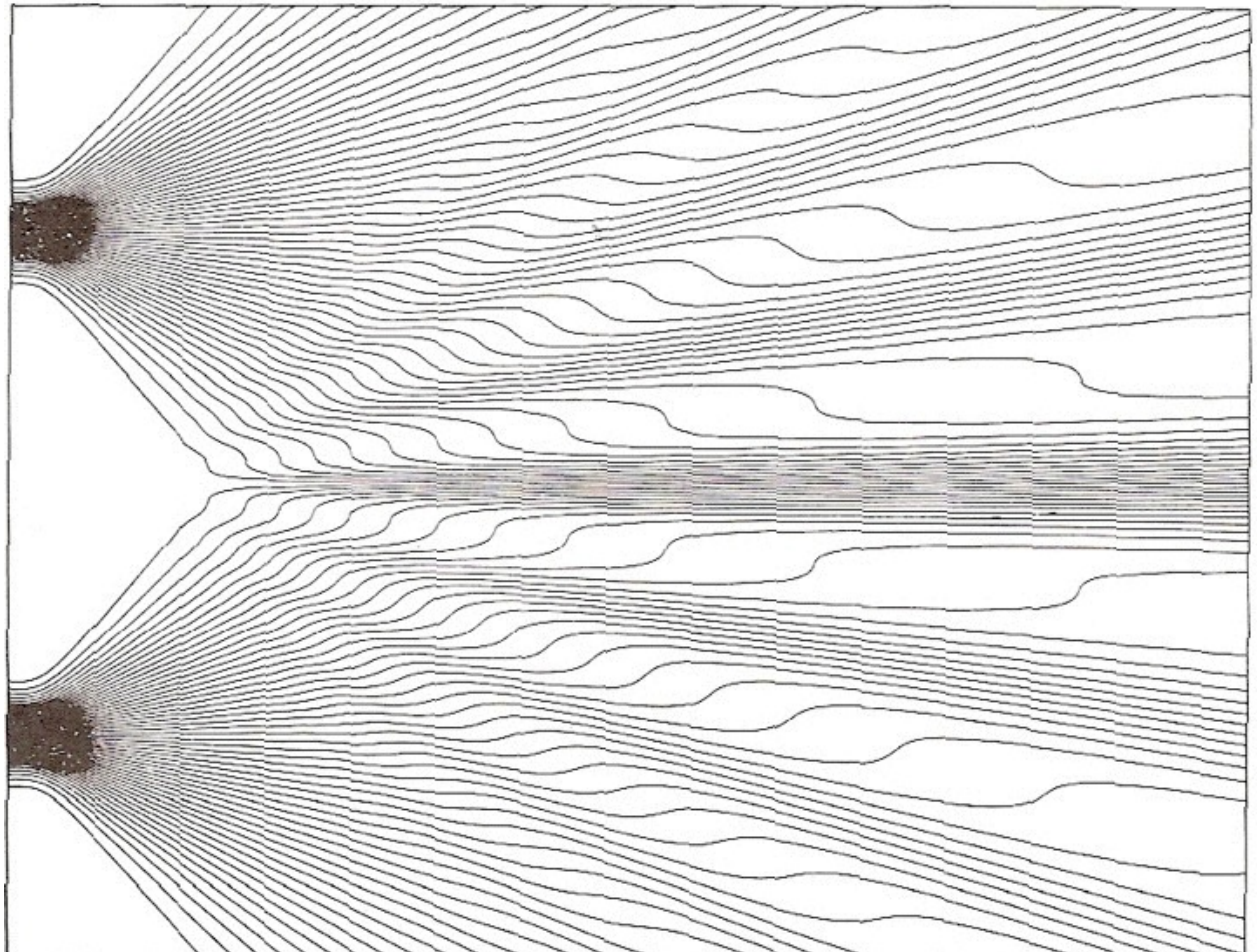
- strictly equivalent to Copenhagen QM
  - probability distribution (attractor)

    $\exists t_0 ; \rho(\boldsymbol{x}, t_0) = |\Psi(\boldsymbol{x}, t_0)|^2$
- classical limit well defined $Q \longrightarrow 0$
- state dependent
- $\nexists$ intrinsic reality
  - non local …
- no need for external classical domain/observer!

The two-slit experiment:



The two-slit experiment:



Surrealistic trajectories?

Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(\cancel{V} + Q\right)$$

... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

R. P. Feynman (1961)

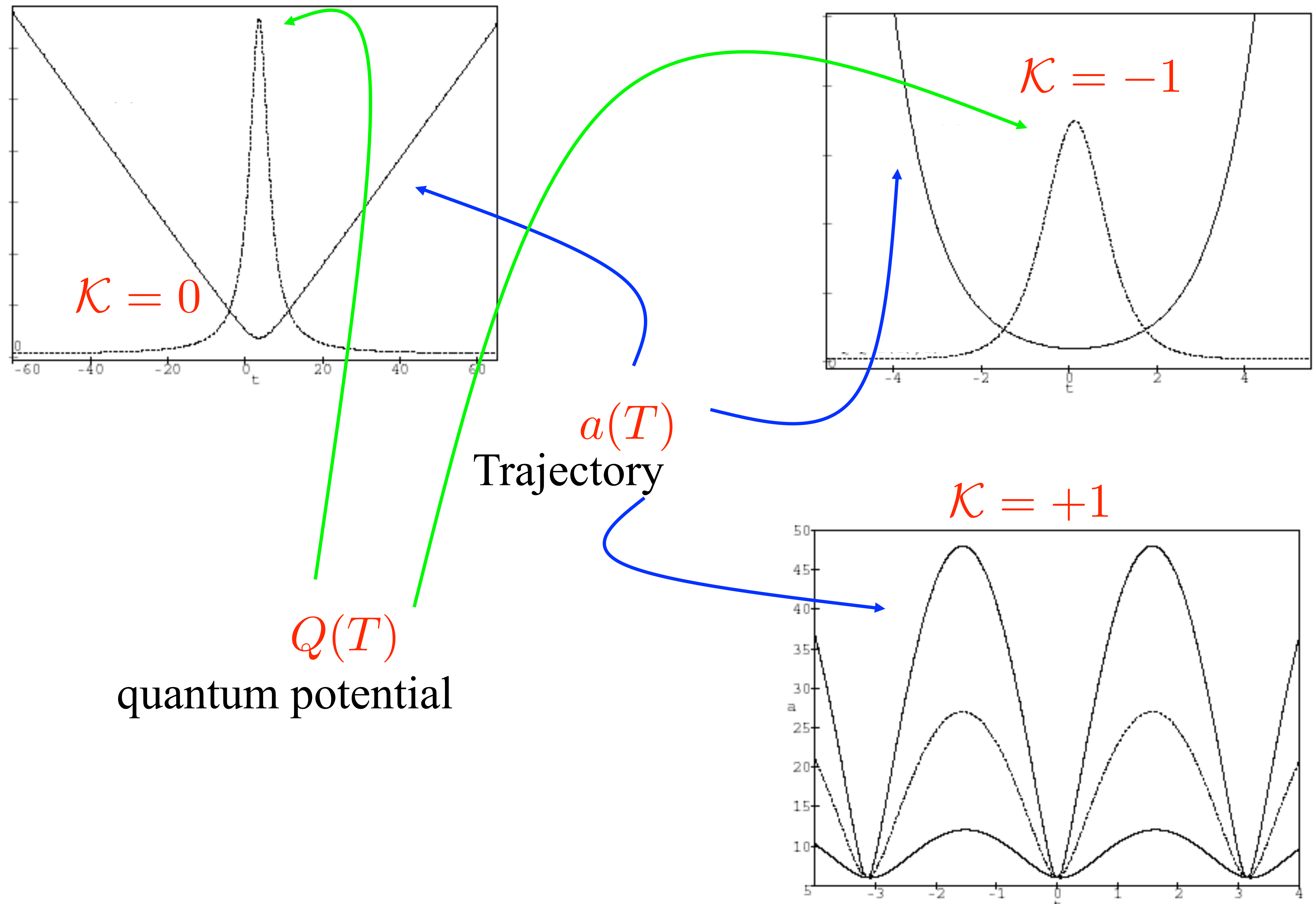
Back to the QC wave function

Gaussian wave packet

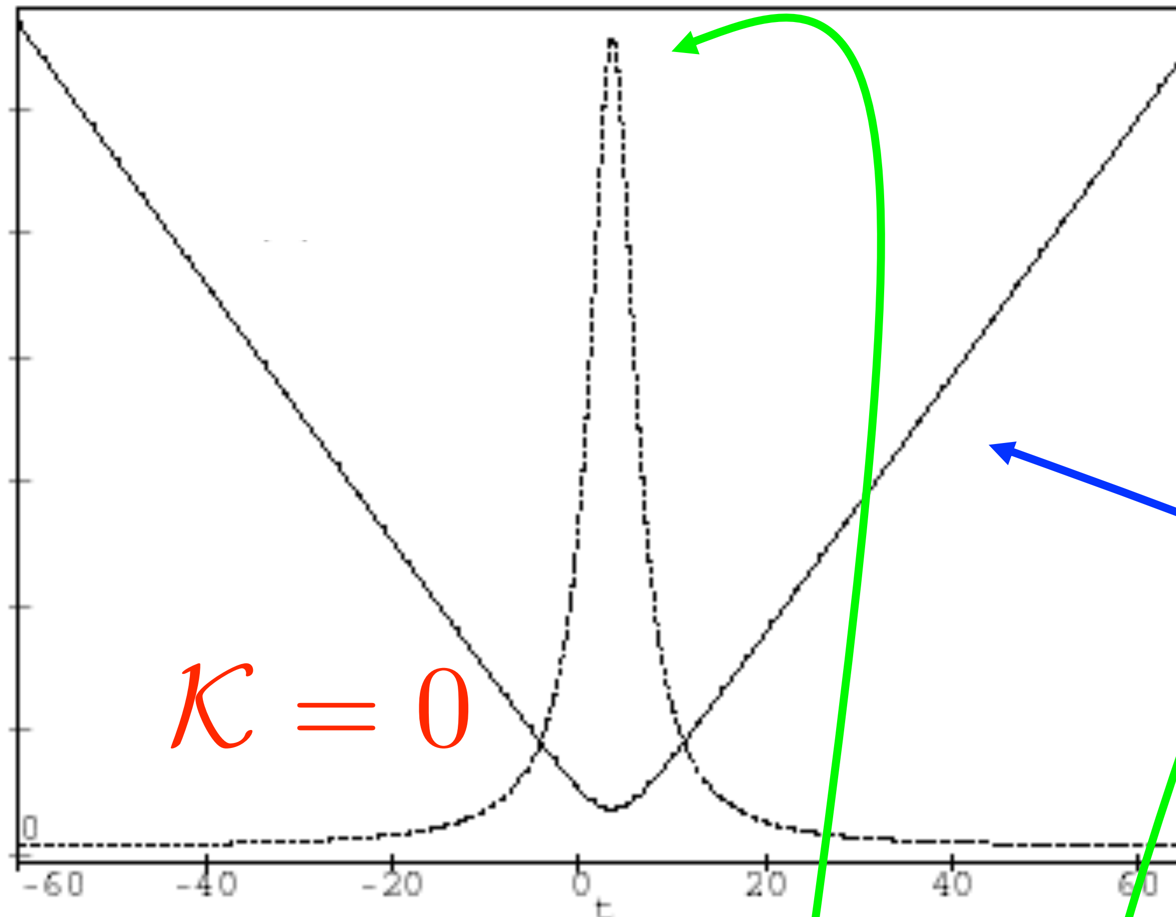
$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

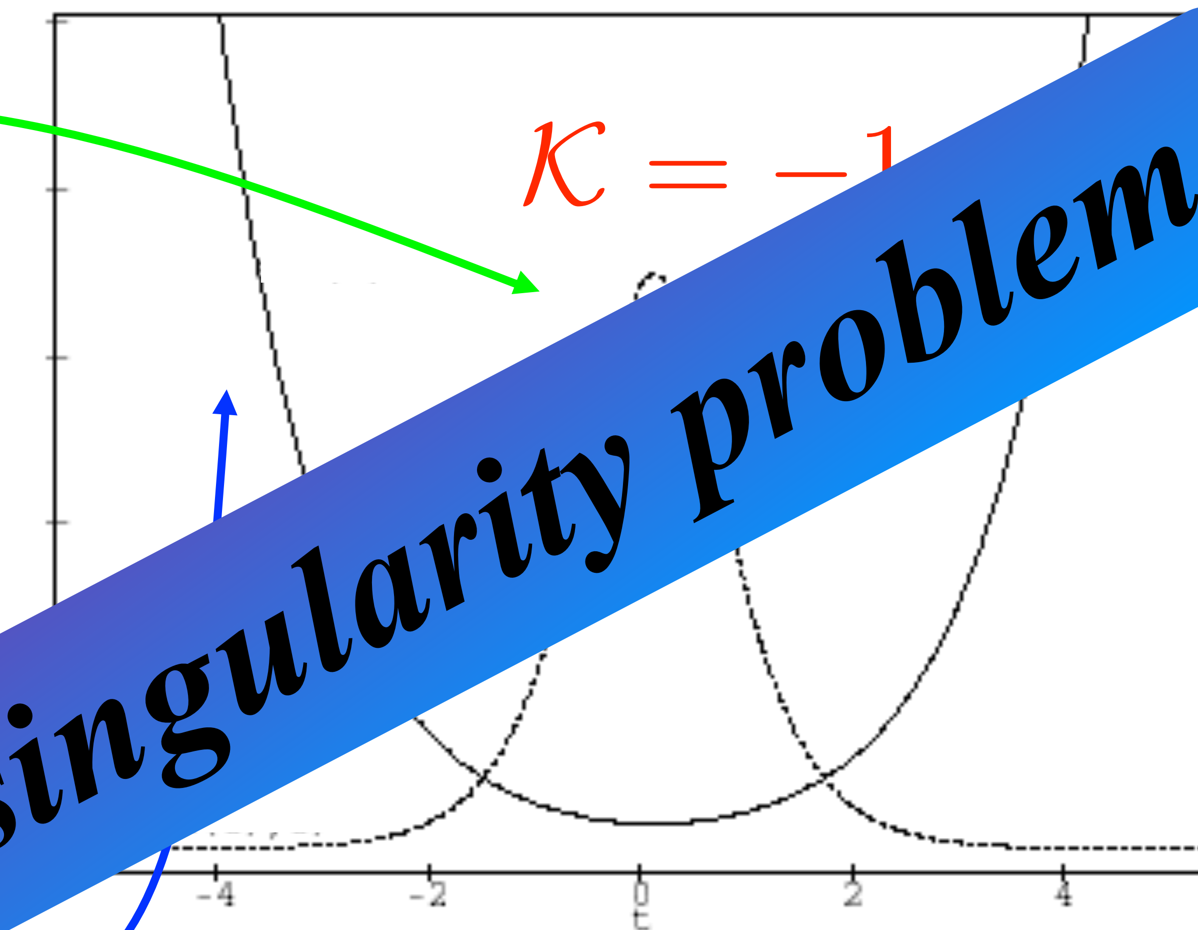
phase $$S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

Hidden trajectory $$a = a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$



J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,
Phys. Lett. **A241**, 229 (1998)

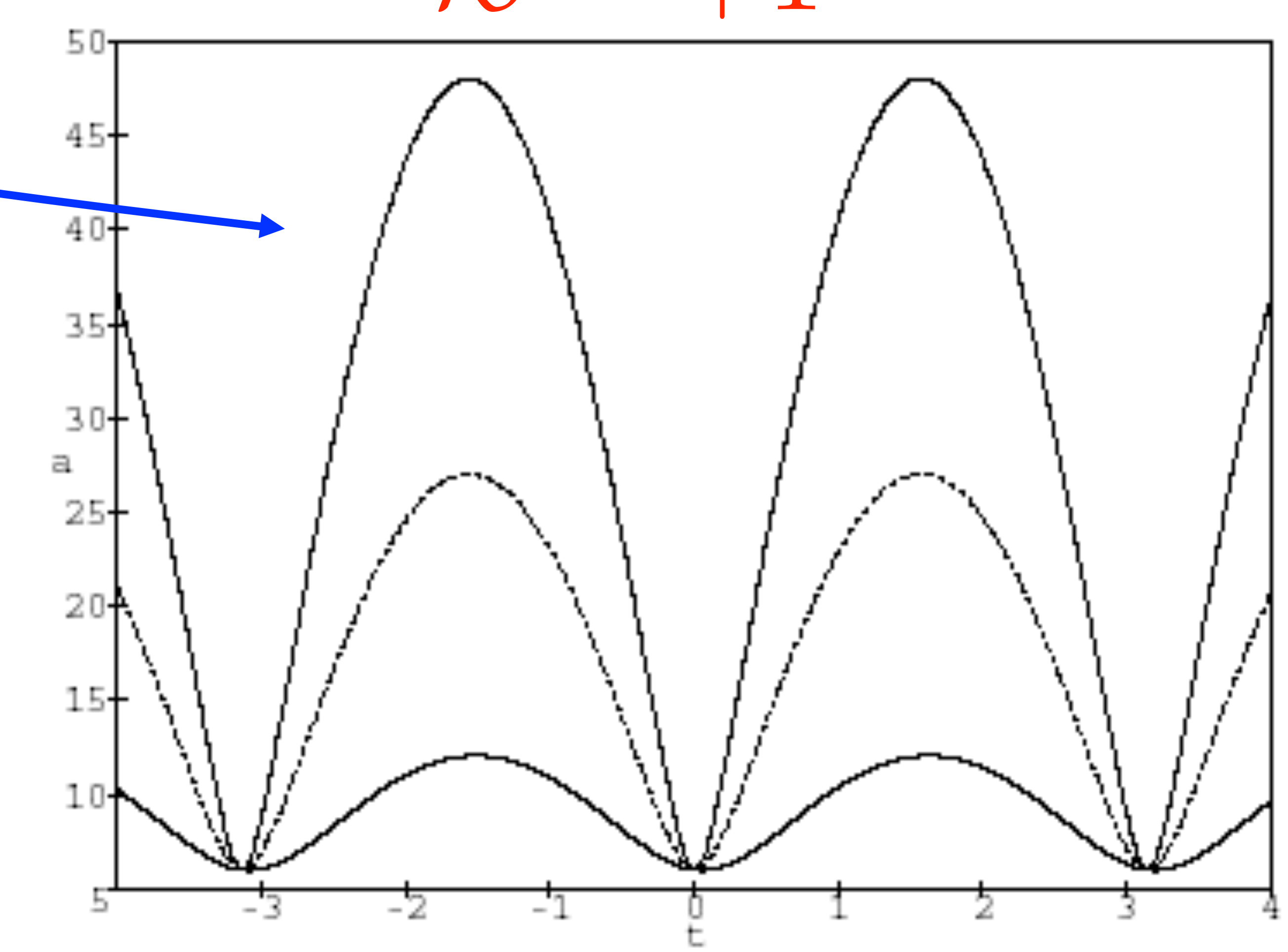




Natural quantum solution to the singularity problem!

um potential

$\mathcal{K} = +1$



J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal, *Phys. Lett.* **A241**, 229 (1998)

Implementing a bounce = problem with GR!

Violation of Null Energy Condition (NEC) $\rho + p \geq 0$

Positive spatial curvature + scalar field

Modify GR?

Add new terms?

K-bounce, Ghost condensates, Galileons...?

Various instabilities may arise!
(e.g. radiation for matter bounce or curvature perturbations)

The problem with contraction: BKL/shear instability

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\sum_i \mathrm{e}^{2\theta_i(t)}\sigma^i\sigma^i$$

Ricci flat:

$$\sigma^i = \mathrm{d}x^i$$

$$\sum_i \theta_i = 0$$

Average scale factor

$$\frac{\dot{a}}{a}$$ Mean Hubble parameter

$$H_i \equiv \frac{1}{a\mathrm{e}^{\theta_i}}\frac{\mathrm{d}}{\mathrm{d}t}\left(a\mathrm{e}^{\theta_i}\right) = H + \dot{\theta}_i$$

Friedmann equations

$$\left.\begin{aligned} H^2 &= \frac{\rho_{\mathrm{T}}}{3M_{\mathrm{Pl}}^2} + \frac{1}{6}\sum_i \dot{\theta}_i^2 \\ \dot{H} &= -\frac{\rho_{\mathrm{T}} + p_{\mathrm{T}}}{2M_{\mathrm{Pl}}^2} - \frac{1}{2}\sum_i \dot{\theta}_i^2 \end{aligned}\right\} \ddot{\theta}_i + 3H\dot{\theta}_i = 0$$

The problem with contraction: BKL/shear instability

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t) \sum_i \mathrm{e}^{2\theta_i(t)} \sigma^i \sigma^i$$

Ricci flat:

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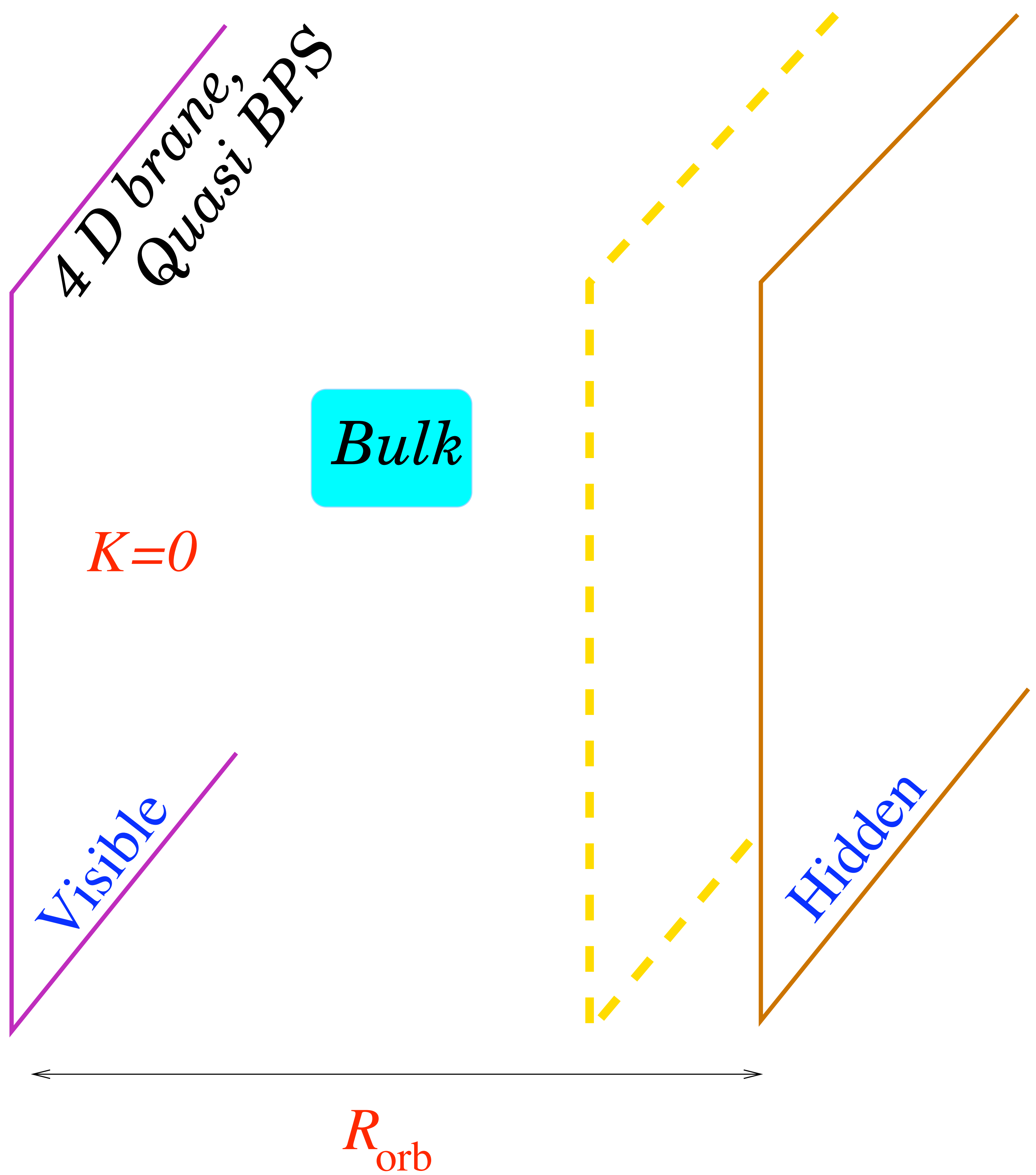
$$\rho_{\mathrm{shear}} \propto a^{-6}$$

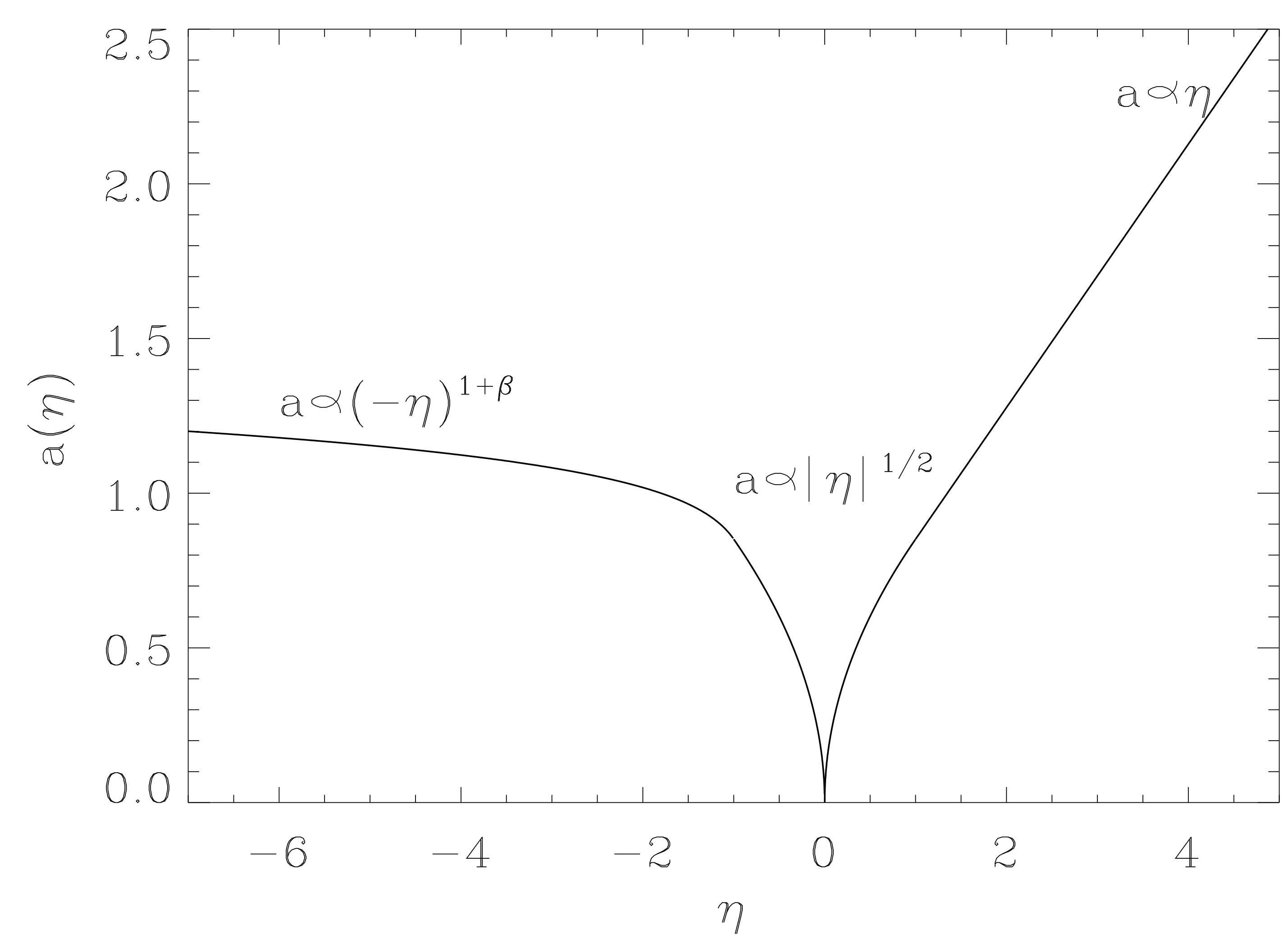
Ekpyrotic/cyclic scenario:

$$\mathcal{S}_5 \propto \int_{\mathcal{M}_5} \mathrm{d}^5x \sqrt{-g_5}\left[R_{(5)} - \frac{1}{2}\left(\partial\varphi\right)^2 - \frac{3}{2}\frac{\mathrm{e}^{2\varphi}\mathcal{F}^2}{5\,!}\right],$$



$$\mathcal{S}_4 = \int_{\mathcal{M}_4} \mathrm{d}^4x \sqrt{-g_4}\left[\frac{R_{(4)}}{2\kappa} - \frac{1}{2}\left(\partial\phi\right)^2 - V(\phi)\right],$$

$$V(\varphi) = -V_\mathrm{i} \exp\left[-\frac{4\sqrt{\pi\gamma}}{m_{\mathrm{Pl}}}(\varphi - \varphi_\mathrm{i})\right],$$



Singular ...

Singular ...

... the Universe contracts towards a "big crunch" until the scale factor $a(t)$ is so small that quantum gravity effects become important. The presumption is that these quantum gravity effects introduce deviations from conventional general relativity and produce a bounce that preserves the smooth, flat conditions achieved during the ultraslow contraction phase.

PRL **105,** 261301 (2010)

Non singular bounce

... where the Universe stops contraction and reverses to expansion at a finite value of $a(t)$ where classical general relativity is still valid. A significant advantage of this scenario is that the entire cosmological history can be described by 4D effective field theory and classical general relativity, without invoking extra dimensions or quantum gravity effects.

PRL **105,** 261301 (2010)

Ekpyrotic solution:

$$w_{\mathrm{ekp}} \gg 1 \quad \Longrightarrow \quad \rho_{\mathrm{ekp}} \propto a^{-3(1+w_{\mathrm{ekp}})} \ll a^{-6} \quad \text{when} \quad a \to 0$$

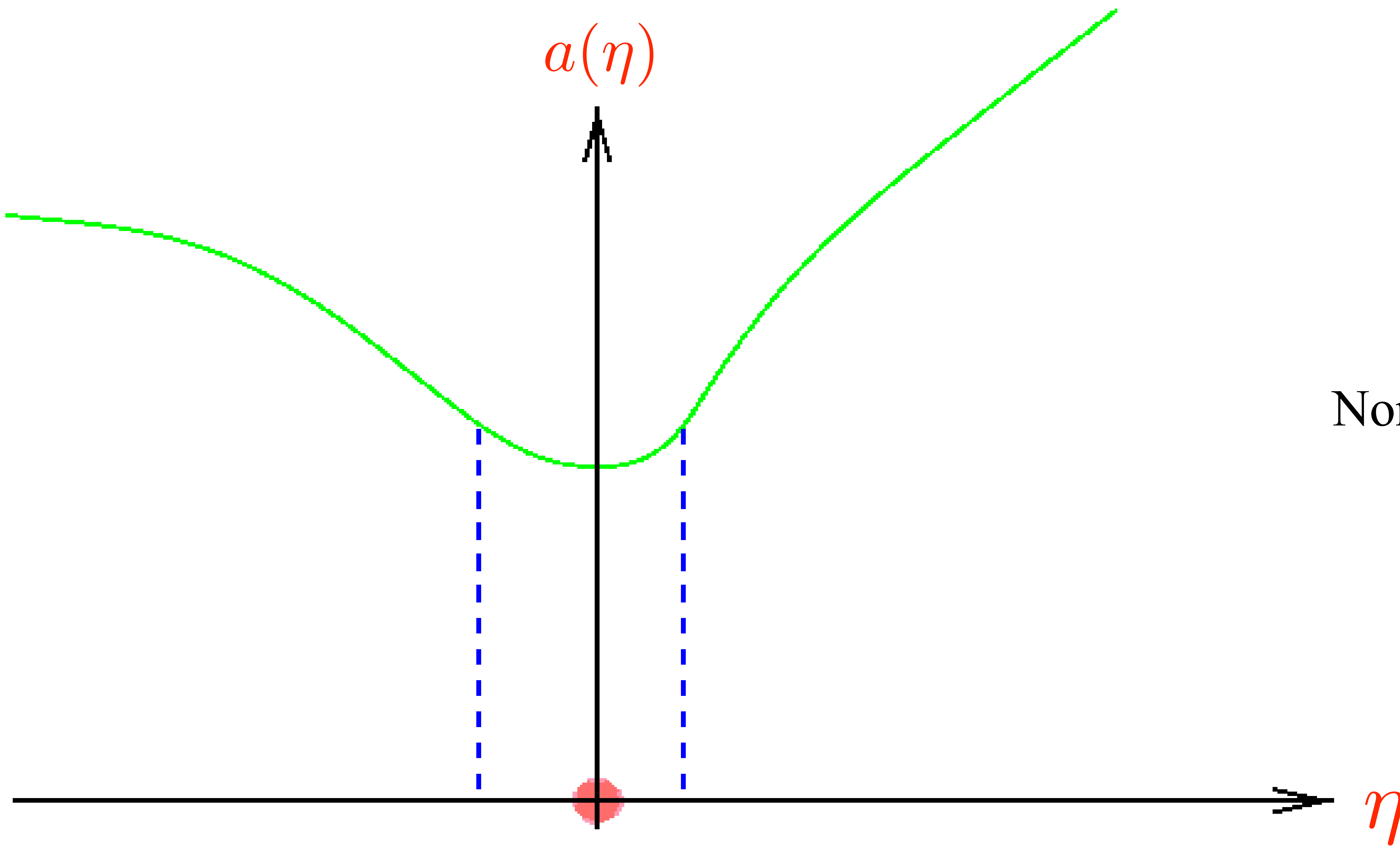
Hence a singular bounce!

Problem: regular bounce $\exists$ phase with $w_{\mathrm{bounce}} < -1$

So finally...

$$\rho_{\mathrm{Shear}} \equiv \frac{M_{\mathrm{Pl}}^2}{2} \sum_i \dot{\theta}_i^2 \propto a^{-6} \gg \rho_{\mathrm{Fluid}}$$

Singularity!



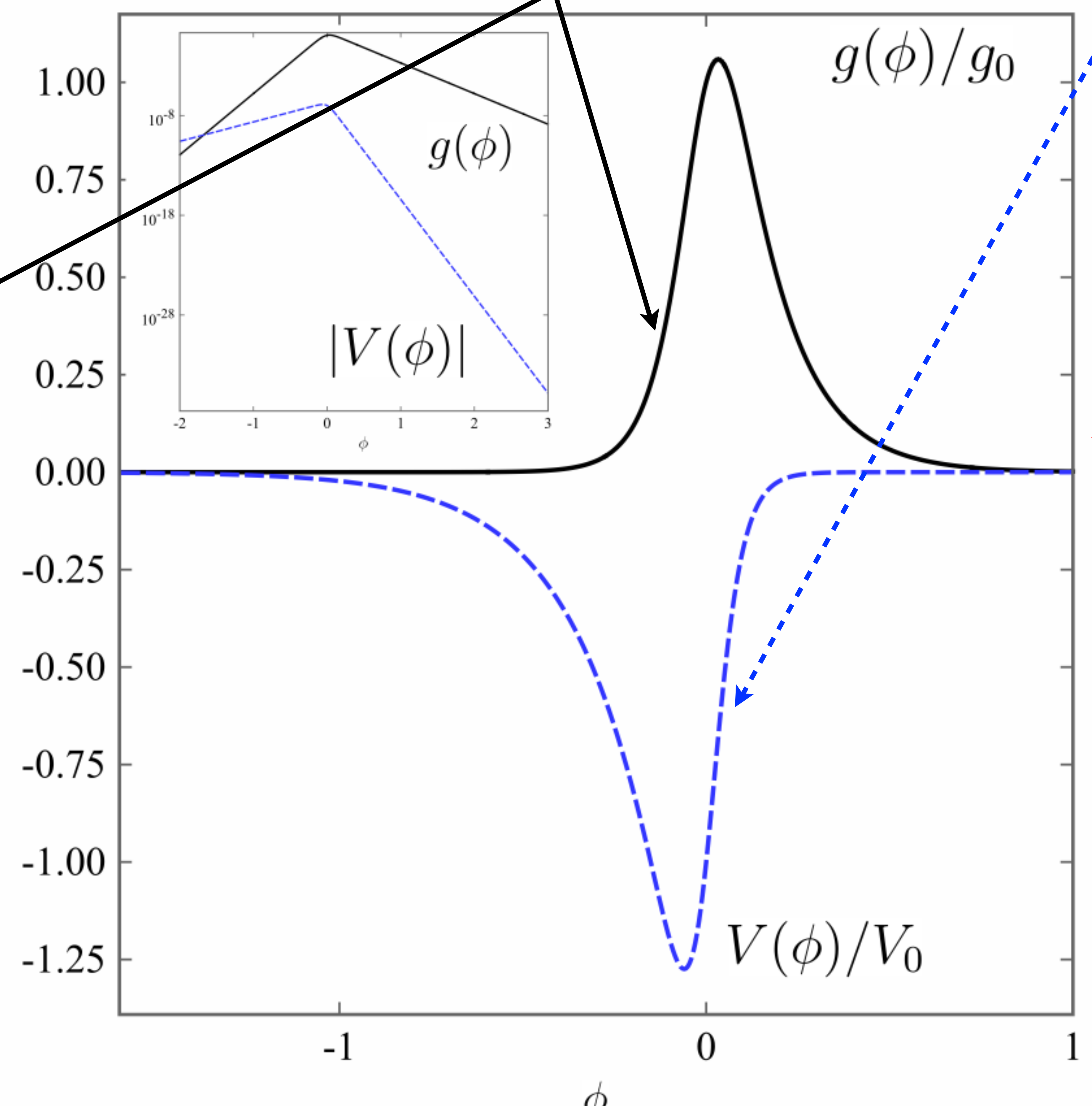
Non singular bounce

Y. Cai, R. Brandenberger & PP, Class. Quantum Grav. 30 (2013) 075019 [arXiv:1301.4703]

A nonsingular bounce model

$$\mathcal{L}\left[\phi\left(x\right)\right] = K(\phi, X) + G(\phi, X)\Box\phi$$ with kinetic term $X \equiv \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ + Fluid

Specific choices: $K(\phi, X) = M_{\rm Pl}^2\left[1-g(\phi)\right]X + \beta X^2 - V(\phi)$ $G(X) = \gamma X$



$$g(\phi) = \frac{2g_0}{\mathrm{e}^{-\sqrt{\frac{2}{p}}\phi} + \mathrm{e}^{b_g\sqrt{\frac{2}{p}}\phi}}$$

$$V(\phi) = -\frac{2V_0}{\mathrm{e}^{-\sqrt{\frac{2}{q}}\phi} + \mathrm{e}^{b_V\sqrt{\frac{2}{q}}\phi}}$$

Stress-energy tensor

$$T^{\phi}_{\mu\nu} = \left(-K + 2XG_{,\phi} + G_{,X}\nabla_\sigma X\nabla^\sigma\phi\right)g_{\mu\nu} + \left(K_{,X} + G_{,X}\Box\phi - 2G_{,\phi}\right)\nabla_\mu\phi\nabla_\nu\phi - G_{,X}\left(\nabla_\mu X\nabla_\nu\phi + \nabla_\nu X\nabla_\mu\phi\right)$$

$\Rightarrow$ Energy density & Pressure

$$\rho_\phi = \frac{1}{2}M^2_{\rm Pl}(1-g)\dot\phi^2 + \frac{3}{4}\beta\dot\phi^4 + 3\gamma H\dot\phi^3 + V(\phi)$$

$$p_\phi = \frac{1}{2}M^2_{\rm Pl}(1-g)\dot\phi^2 + \frac{1}{4}\beta\dot\phi^4 - \gamma\dot\phi^2\ddot\phi - V(\phi)$$

+ Fluid $p = w\rho$

Einstein equation + $\nabla_\mu T^{\mu\nu}_{\rm Fluid} = 0$

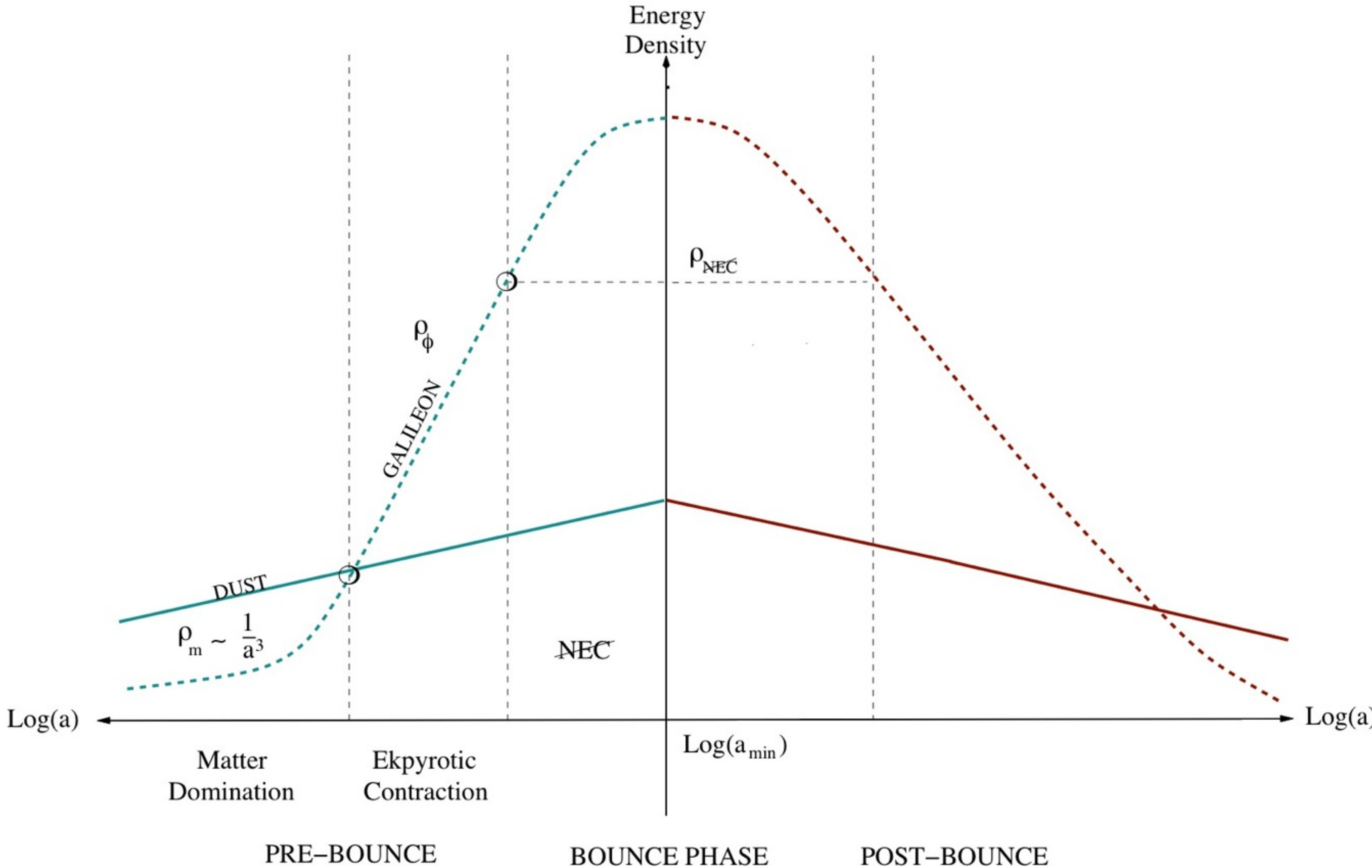
+ modified Klein-Gordon $\mathcal{P}\ddot\phi + \mathcal{D}\dot\phi + V_{,\phi} = 0$

with...

$$
\begin{aligned}
\mathcal{P} &= (1-g)M_{\rm Pl}^2 + 6\gamma H\dot\phi + 3\beta\dot\phi^2 + \frac{3\gamma^2}{2M_{\rm Pl}^2}\dot\phi^4 \\
\mathcal{D} &= 3(1-g)M_{\rm Pl}^2 H + \left(9\gamma H^2 - \frac{1}{2}M_{\rm Pl}^2 g_{,\phi}\right)\dot\phi + 3\beta H\dot\phi^2 \\
&\quad -\frac{3}{2}(1-g)\gamma\dot\phi^3 - \frac{9\gamma^2 H\dot\phi^4}{2M_{\rm Pl}^2} - \frac{3\beta\gamma\dot\phi^5}{2M_{\rm Pl}^2} \\
&\quad -\frac{3}{2}G_{,X}\sum_i \dot\theta_i^2\dot\phi - \frac{3G_{,X}}{2M_{\rm Pl}^2}(\rho_{\rm m}+p_{\rm m})\dot\phi
\end{aligned}
$$

5 phases:

A.	➡	Matter contraction	Produces scale invariant perturbations
B.	➡	Ekpyrotic contraction	Removes anisotropies
C.	➡	The bounce itself	Leads to expansion
D.	➡	Fast-roll expansion	Connects to standard model!!
E.	➡	Radiation + Matter + ...	BB cosmology



Energy
Density
ρ$_{\cancel{NEC}}$
ρ$_\phi$
GALILEON
DUST
$\rho_m \sim \frac{1}{a^3}$
$\cancel{NEC}$
Log(a)
Log(a)
Log(a_{min})
Matter
Domination
Ekpyrotic
Contraction
PRE–BOUNCE
BOUNCE PHASE
POST–BOUNCE

→ Anisotropies can remain small all throughout!!!!

explicit example...

$$V_0 = 10^{-7}, \quad g_0 = 1.1, \quad \beta = 5, \quad \gamma = 10^{-3}$$

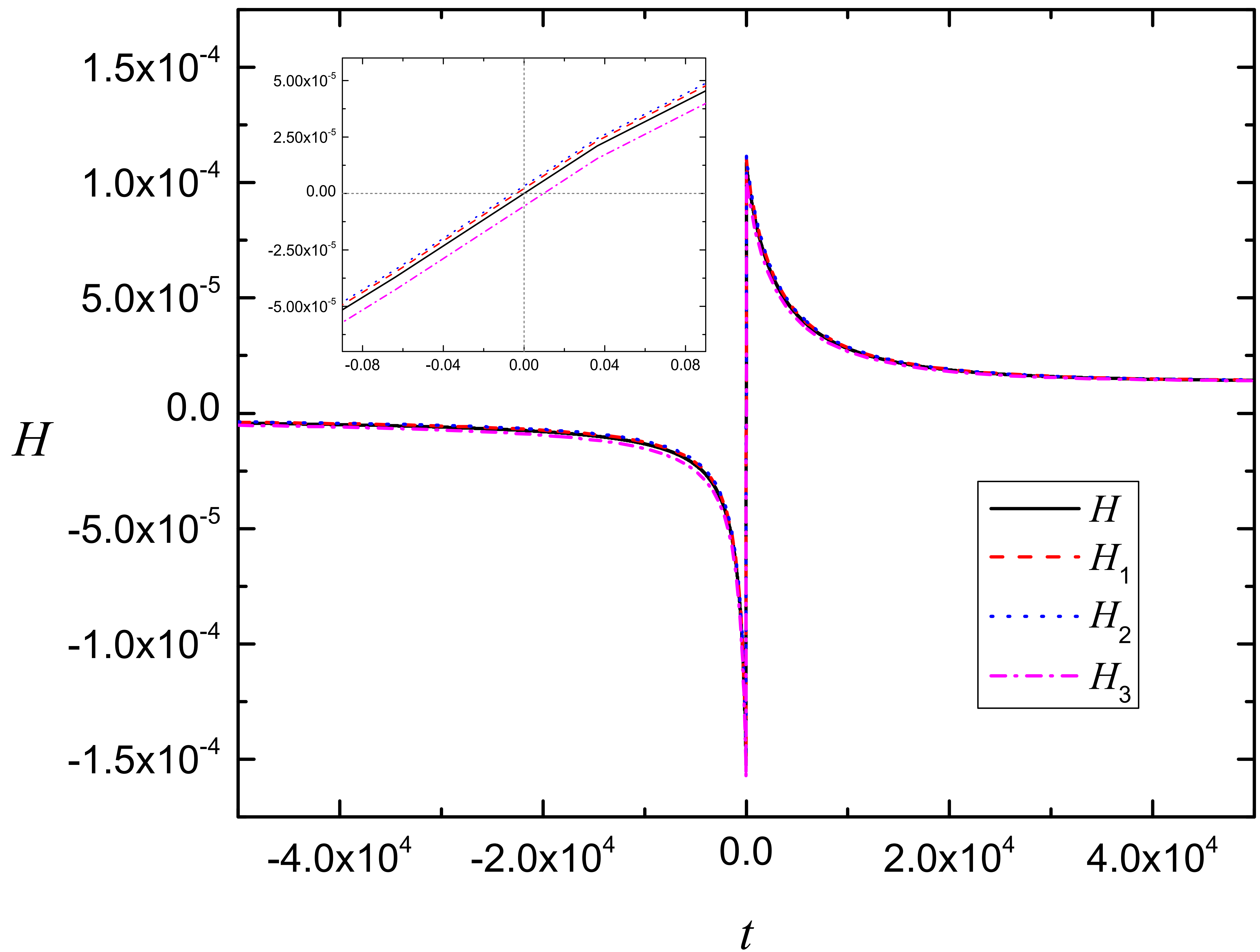
$$b_V = 5, \quad b_g = 0.5, \quad p = 0.01, \quad q = 0.1$$

$$\rho_{\mathrm{m,B}} = 2.8 \times 10^{-10}, \quad M_{\theta,1} = 2.2 \times 10^{-6}$$

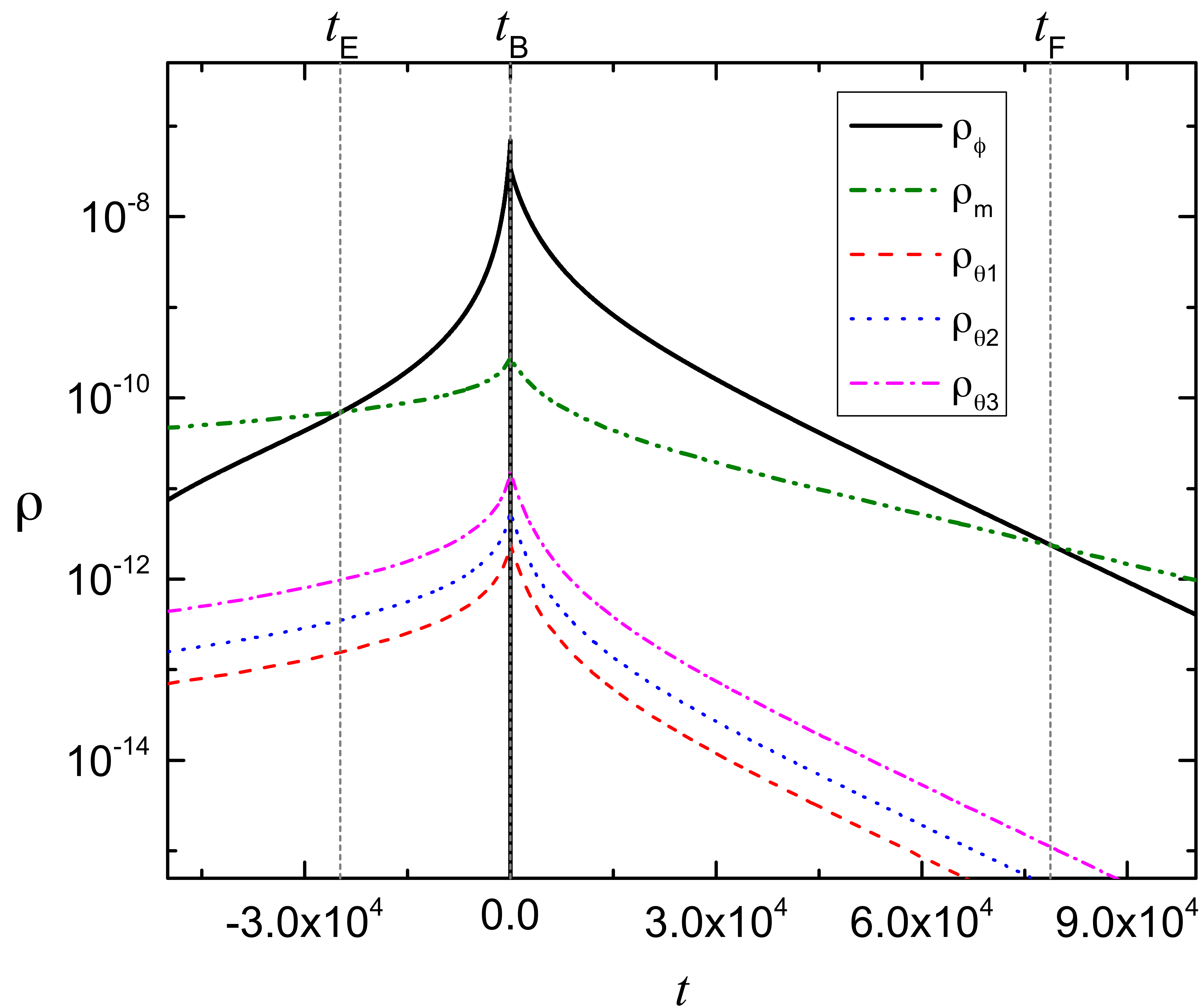
$$M_{\theta,2} = 3.4 \times 10^{-6}, \quad M_{\theta,3} = -5.6 \times 10^{-6}$$

$$\phi_{\mathrm{ini}} = -2, \quad \dot{\phi}_{\mathrm{ini}} = 7.8 \times 10^{-6}.$$

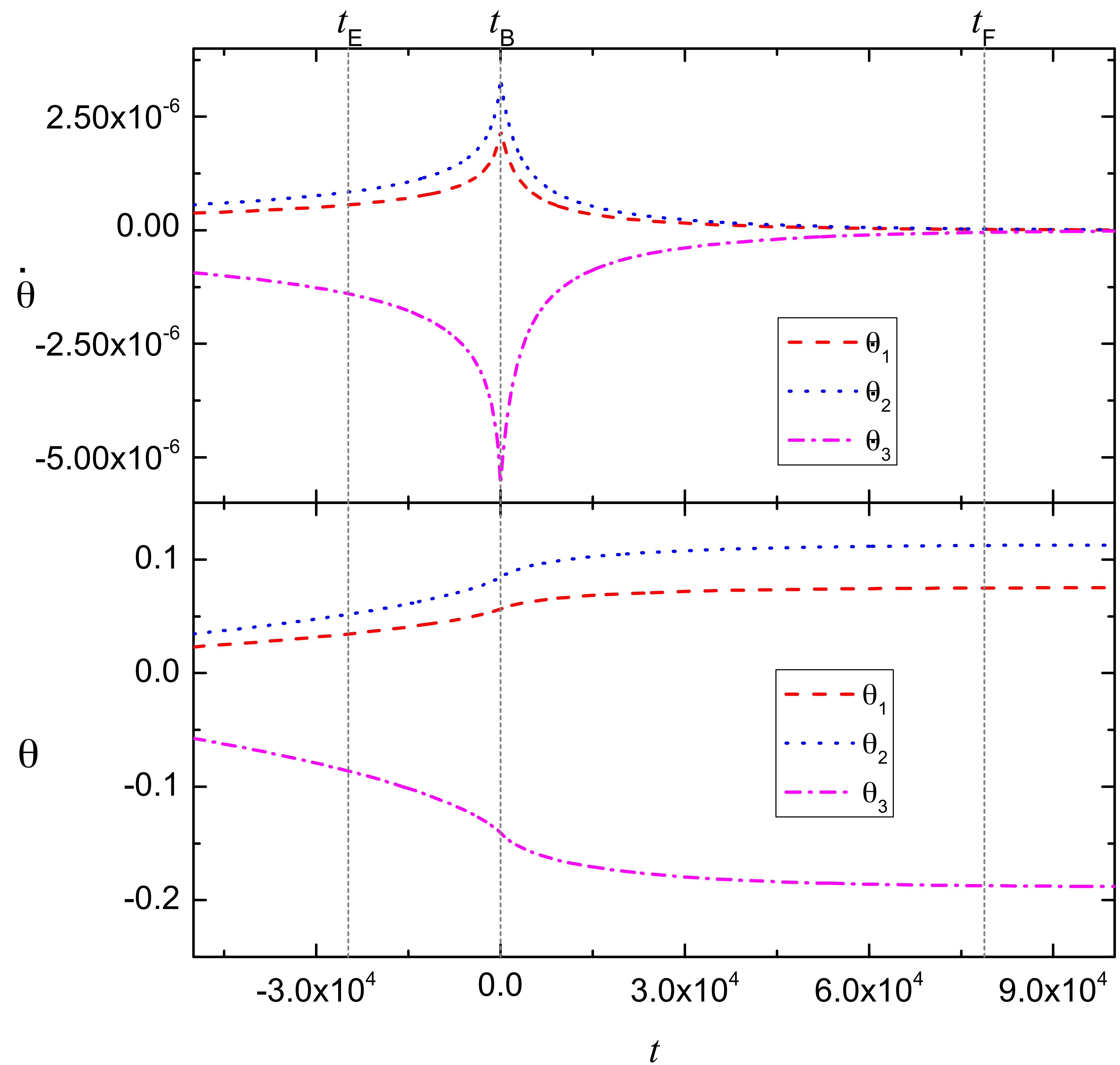
Hubble parameters



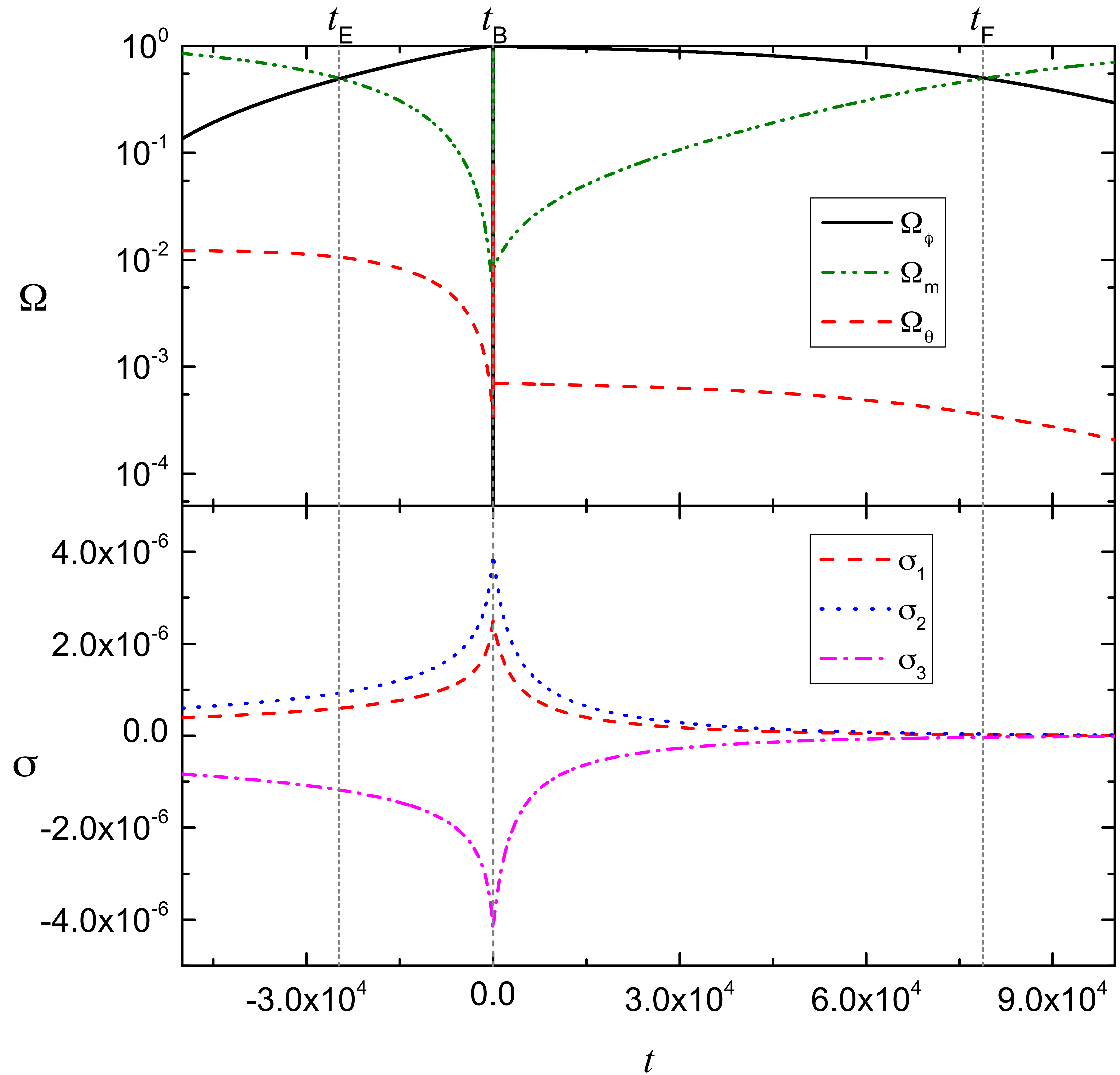
Energy densities



Anisotropies

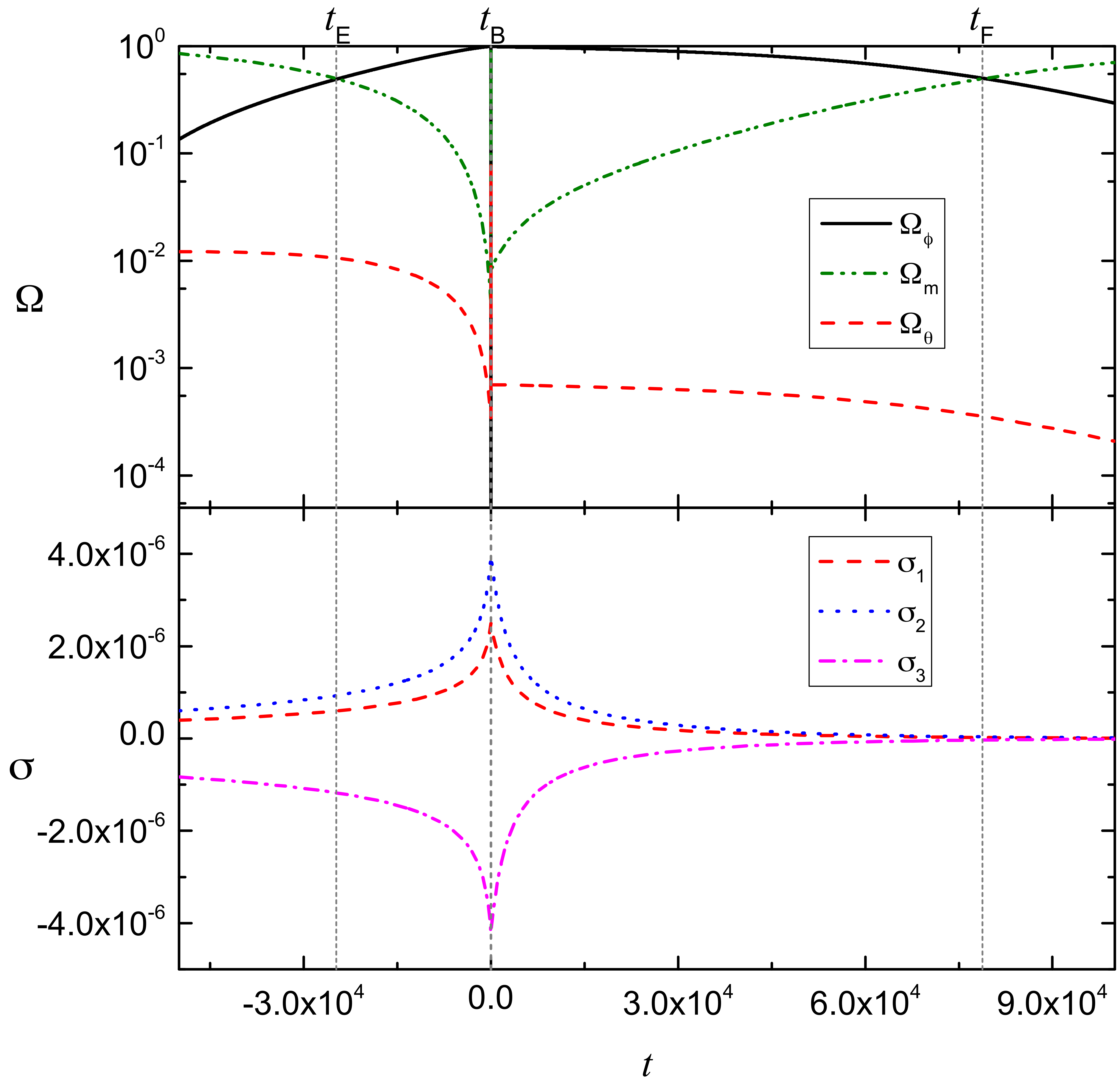


Density parameters and shears



Density parameters and shears

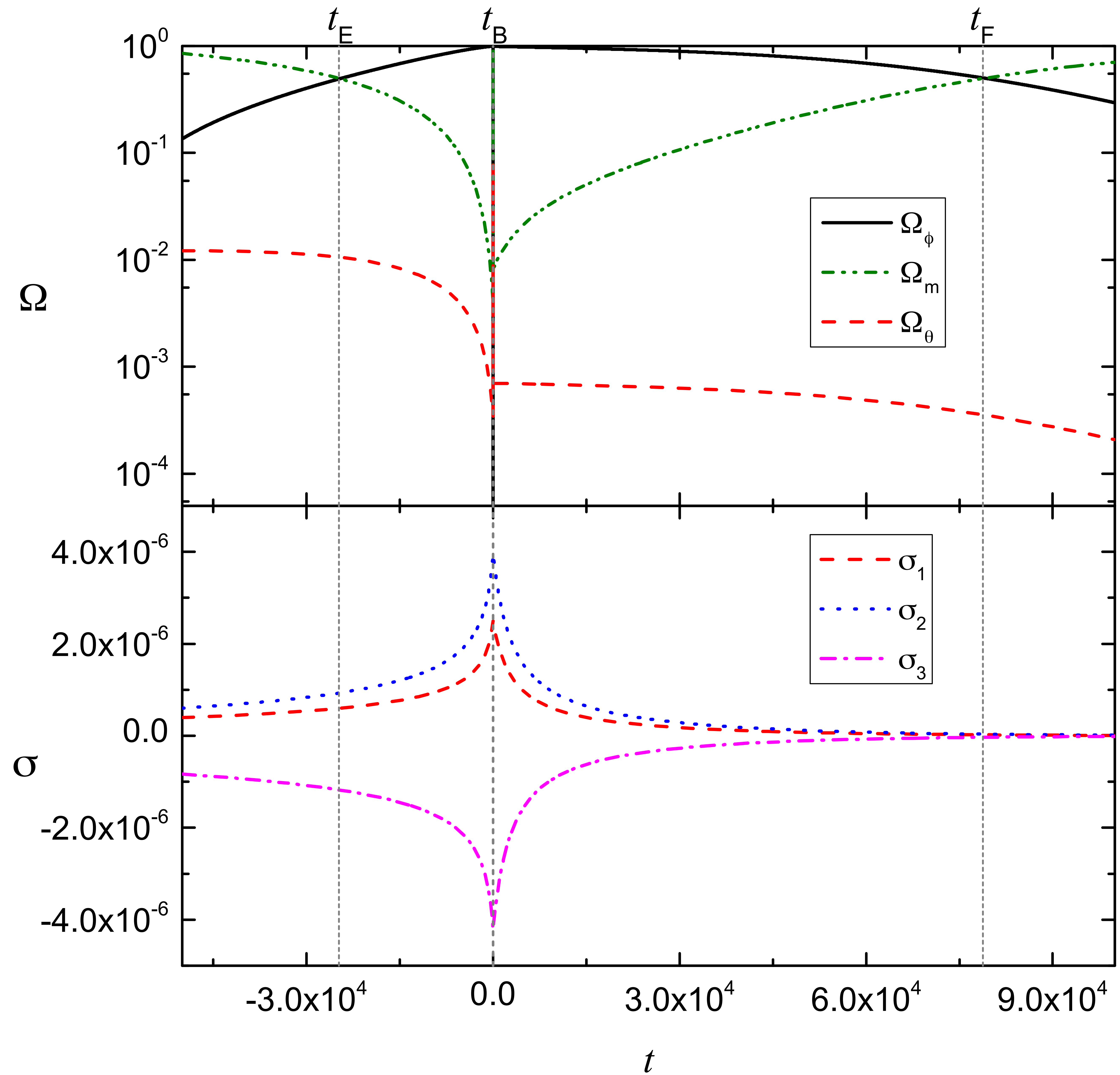
$$\Omega_{\mathrm{I}} \equiv \frac{\rho_{\mathrm{I}}}{\sum_{\mathrm{I}} \rho_{\mathrm{I}}}$$



Density parameters and shears

$$\Omega_{\mathrm{I}} \equiv \frac{\rho_{\mathrm{I}}}{\sum_{\mathrm{I}} \rho_{\mathrm{I}}}$$

$$\sigma_i \equiv \dot{\theta}_i \mathrm{e}^{2\theta_i}$$



Standard Failures and inflationary solutions

Singularity

Not solved... actually not addressed!

Horizon

$$d_{\mathrm{H}} \equiv a(t)\int_{t_{\mathrm{i}}}^{t}\frac{\mathrm{d}\tau}{a(\tau)}$$

can be made as big as one wishes

Flatness

$$\frac{\mathrm{d}}{\mathrm{d}t}|\Omega - 1| = -2\frac{\ddot{a}}{\dot{a}^3}$$

$$\ddot{a} > 0 \quad \& \quad \dot{a} > 0$$

accelerated expansion (inflation)

Homogeneity & Isotropy

Initial Universe = very small patch
Accelerated expansion drives the shear to zero...

+ attractor

$\Longrightarrow$ vacuum state!

Perturbations

Bonus of the theory: superb predictions!!!

Others

dark matter/energy, baryogenesis, ...

P. P. & N. Pinto-Neto, *Phys. Rev.* **D78**, 063506 (2008)

Standard Failures and bouncing solutions

Singularity Merely a non issue in the bounce case!

Horizon $d_\mathrm{H} \equiv a(t) \int_{t_\mathrm{i}}^{t} \frac{\mathrm{d}\tau}{a(\tau)}$ can be made divergent easily if $t_\mathrm{i} \to -\infty$

Flatness $\frac{\mathrm{d}}{\mathrm{d}t} |\Omega - 1| = -2 \frac{\ddot{a}}{\dot{a}^3}$ $\ddot{a} < 0 \;\&\; \dot{a} < 0$

accelerated expansion (inflation) or decelerated contraction (bounce)

Homogeneity Large & flat Universe + low initial density + diffusion

$$\frac{t_\mathrm{dissipation}}{t_\mathrm{Hubble}} \propto \frac{\lambda}{R_\mathrm{H}^{1/3}} \left(1 + \frac{\lambda}{A R_\mathrm{H}^2}\right)$$

enough time to dissipate any wavelength

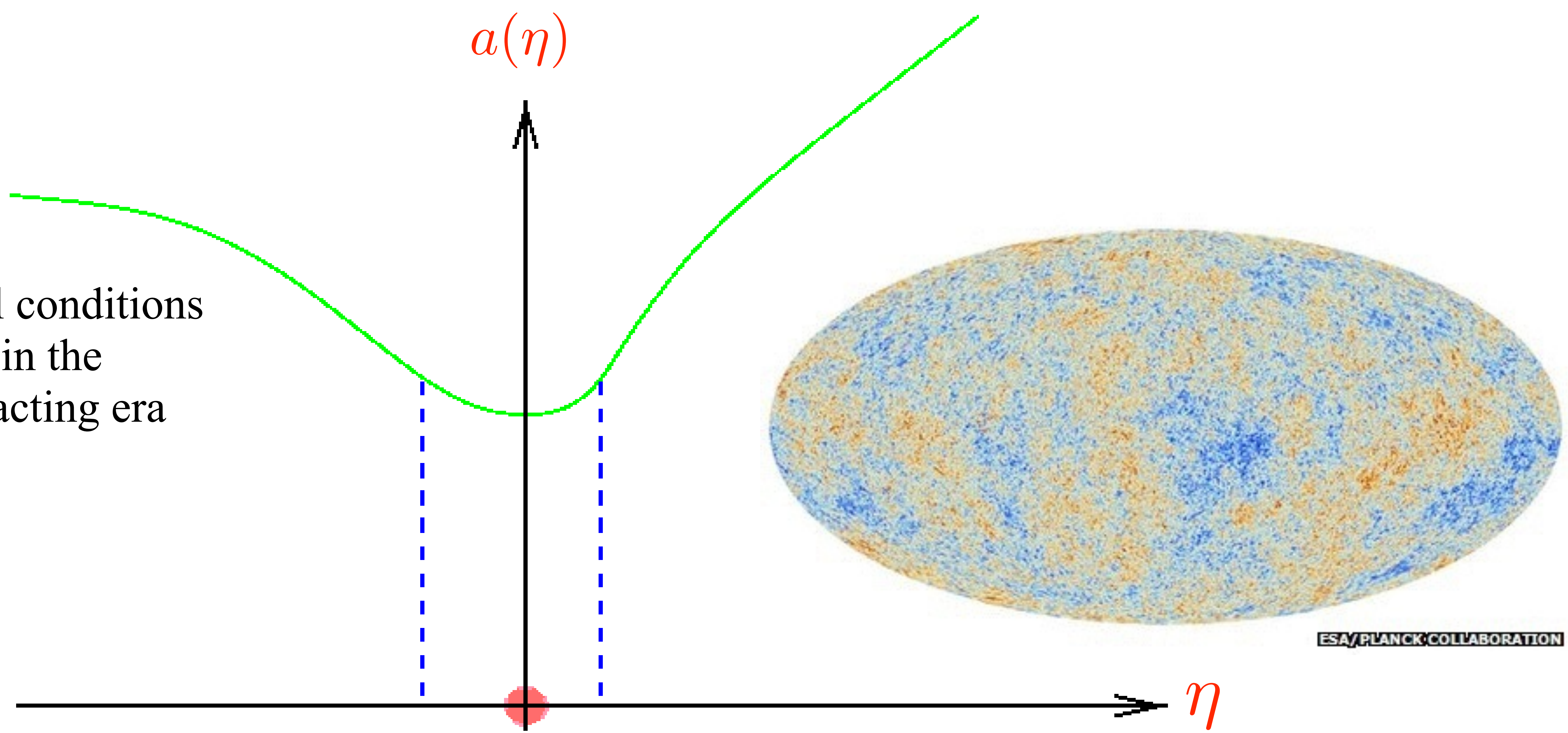
$\Longrightarrow$ vacuum state! ... debatable though

Isotropy Potentially problematic: model dependent

Others dark matter/energy, baryogenesis, ...

Perturbations: $\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$

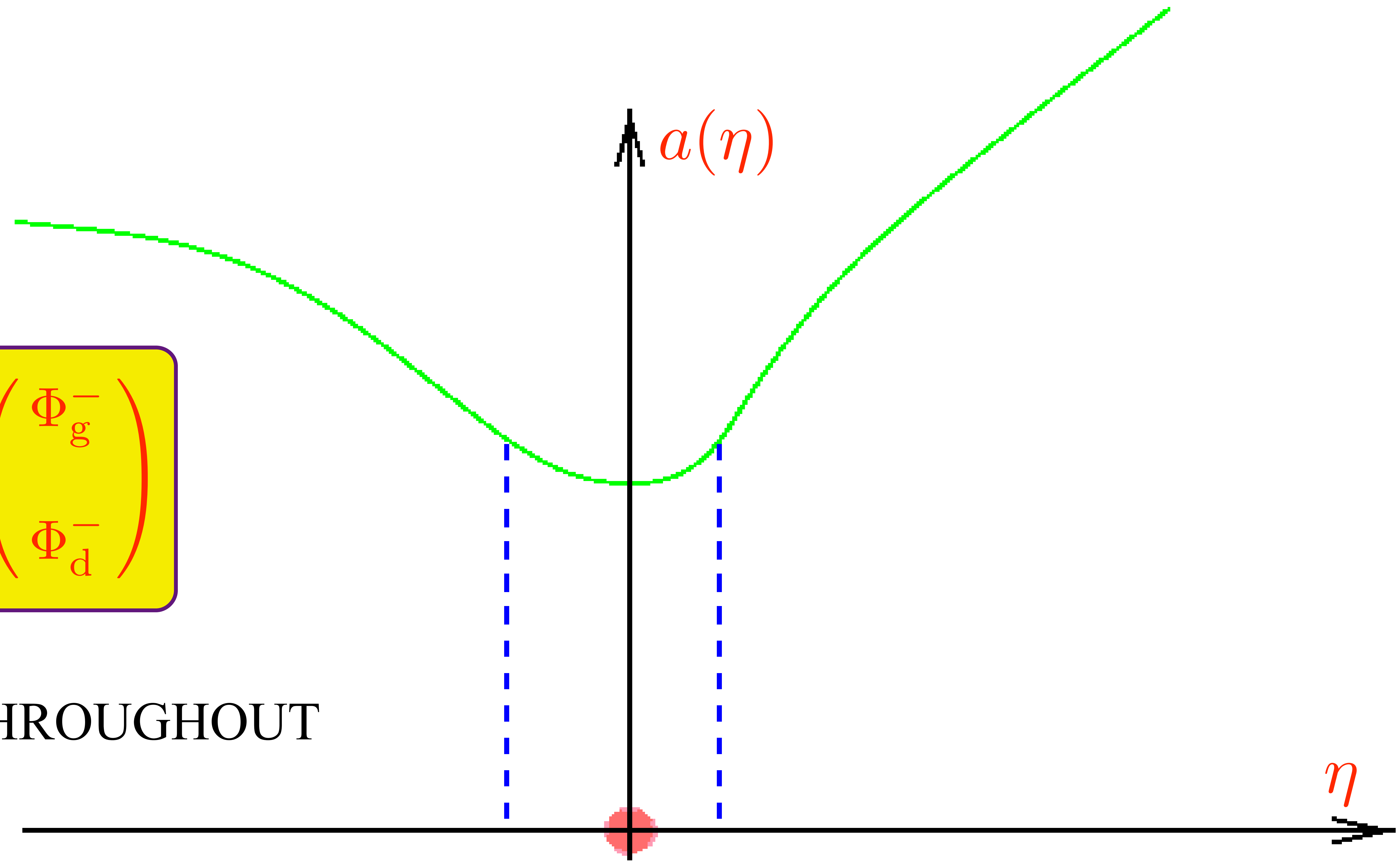
Initial conditions
fixed in the
contracting era



???????

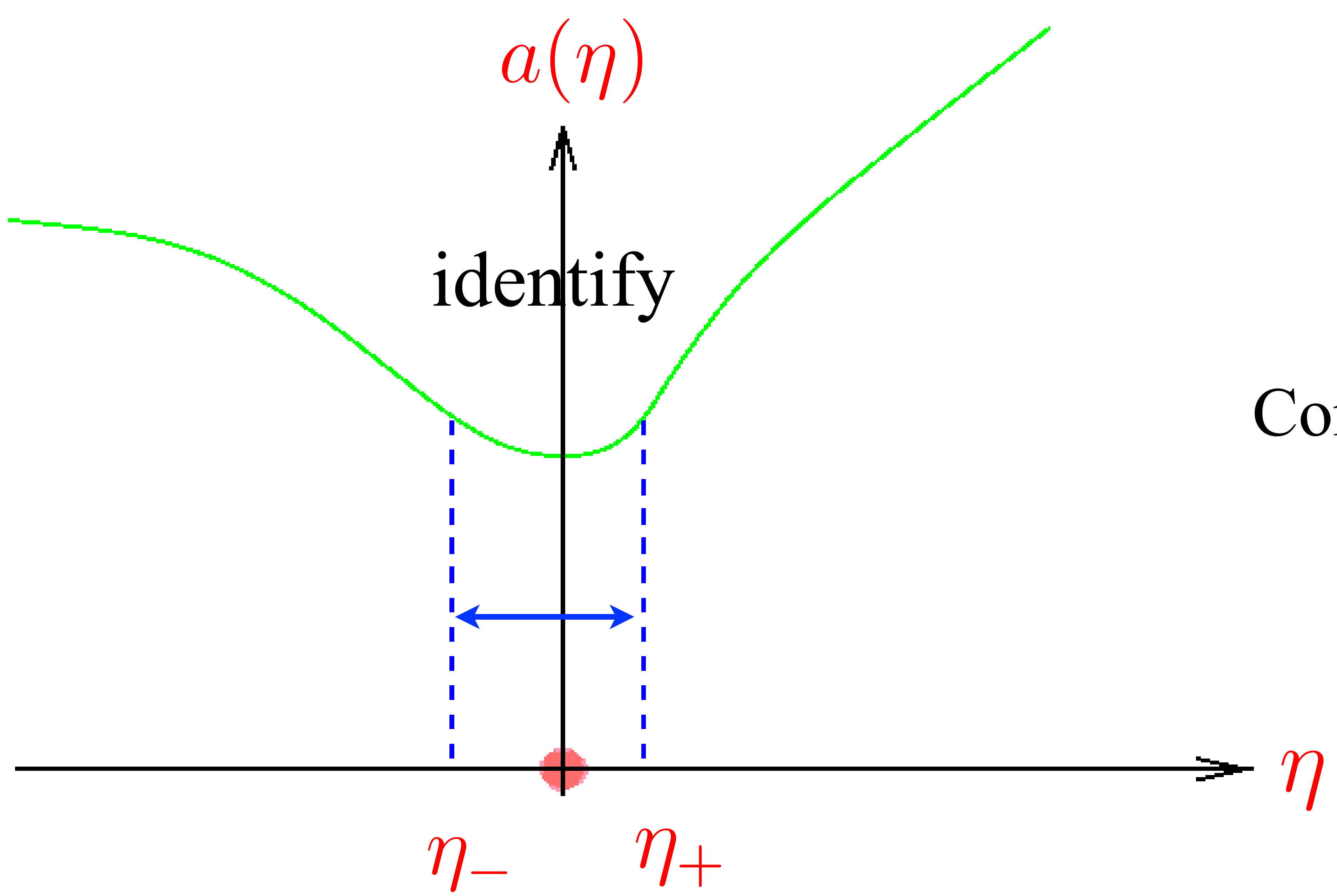
Perturbations: $\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$

$$\boxed{\begin{pmatrix} \Phi_{\mathrm{g}}^{+} \\ \\ \Phi_{\mathrm{d}}^{+} \end{pmatrix} = \mathbf{T}_{ij}(k) \begin{pmatrix} \Phi_{\mathrm{g}}^{-} \\ \\ \Phi_{\mathrm{d}}^{-} \end{pmatrix}}$$



ASSUME LINEARITY THROUGHOUT

A generic model-independent treatment of the bounce phase?



Geometric matching conditions?

Continuity of metric	$[a]_\pm = 0$	OK
Continuity of extrinsic curvature	$[H]_\pm = 0$	???
Perturbations?	$[\zeta]_\pm = 0$	???

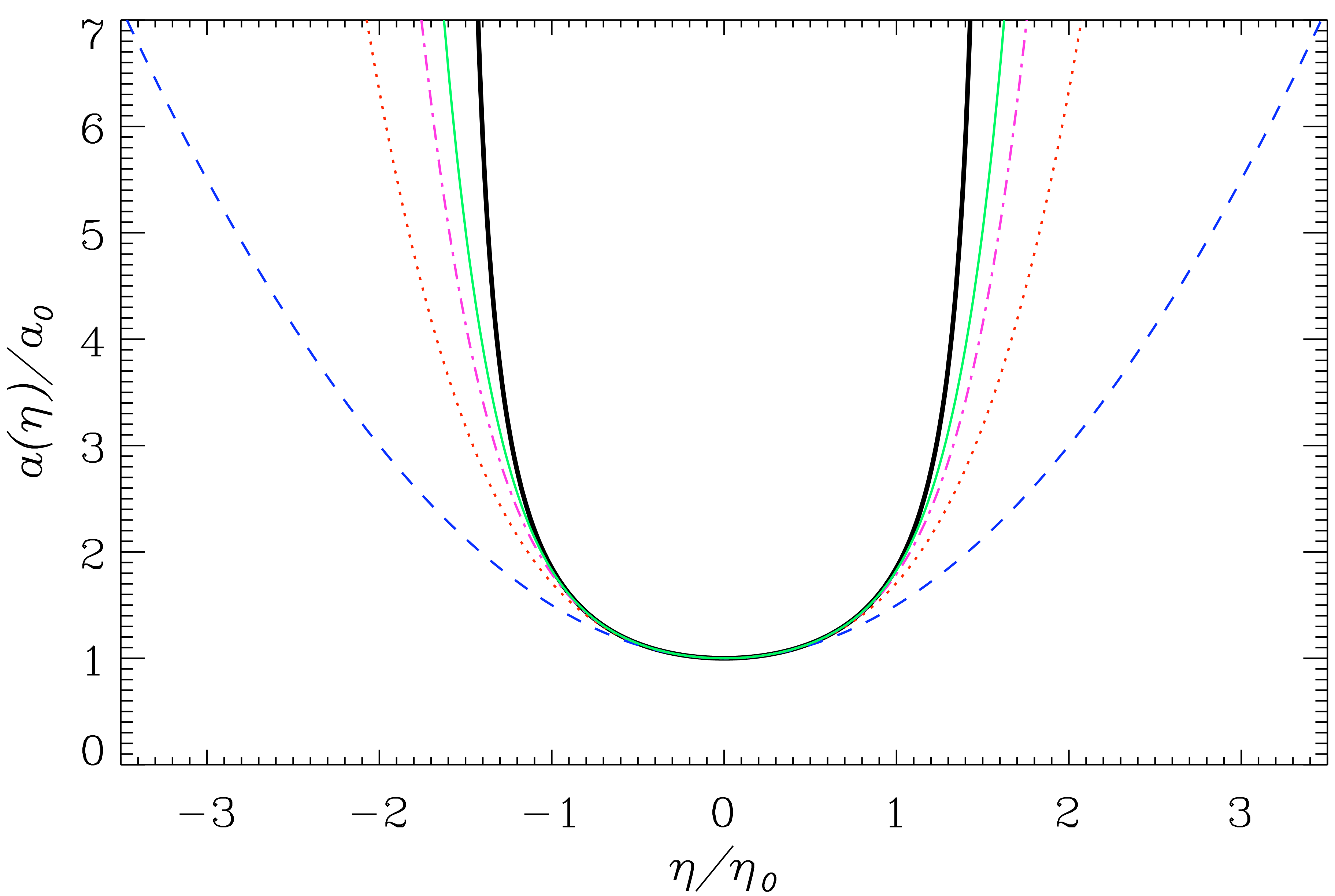
Self consistent bounce:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\mathrm{d}\Omega^2\right)$$

→ One d.o.f. + 4 dimensions G.R.

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{6\ell_{\mathrm{Pl}}^2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right]$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) - \frac{\mathcal{K}}{a^2}$$ Positive spatial curvature



J. Martin & PP., *Phys. Rev.* **D68**, 103517 (2003)

Perturbations: $$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

$$\Longleftrightarrow \qquad \Phi = \frac{3\mathcal{H}u}{2a^2\theta} \qquad \theta \equiv \frac{1}{a}\sqrt{\frac{\rho_\varphi}{\rho_\varphi + p_\varphi}\left(1 - \frac{3\mathcal{K}}{\rho_\varphi a^2}\right)}$$

$$\boxed{u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K}\left(1 - c_{\mathrm{S}}^2\right)\right]u = 0}$$

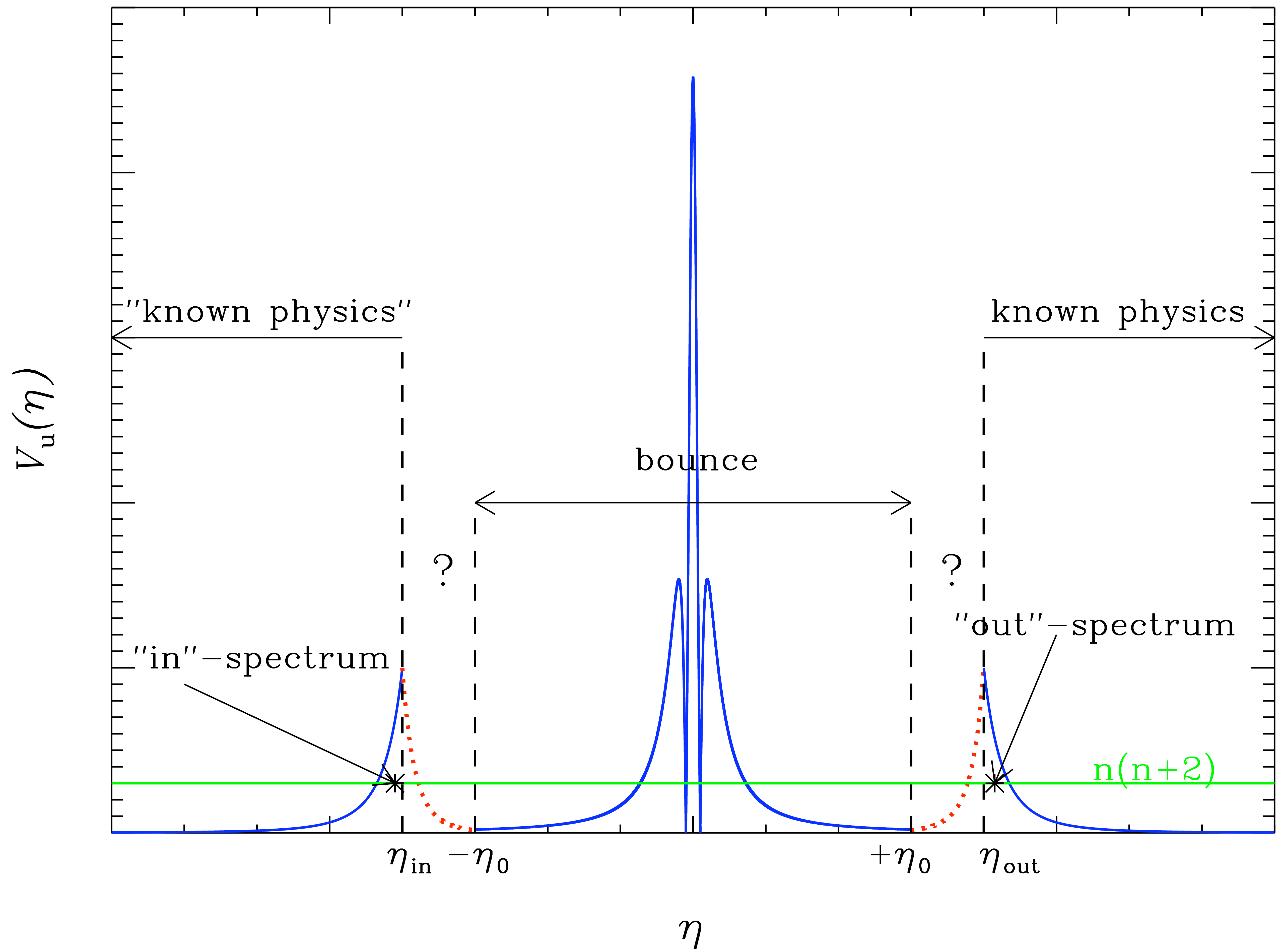
$$V_u(\eta) \equiv \frac{\theta''}{\theta} + 3\mathcal{K}(1 - c_{\mathrm{S}}^2) = \frac{P_{24}(\eta)}{Q_{24}(\eta)},$$

Non trivial transfer matrix

$$\boldsymbol{T}_{ij}(k) = \begin{bmatrix} A(k) & B(k) \\ C(k) & D(k) \end{bmatrix}$$

"Causality" argument...

J. Martin & PP, *Phys. Rev. Lett.* **92,** 061301 (2004)



Actual shape depends on the microscopic parameters

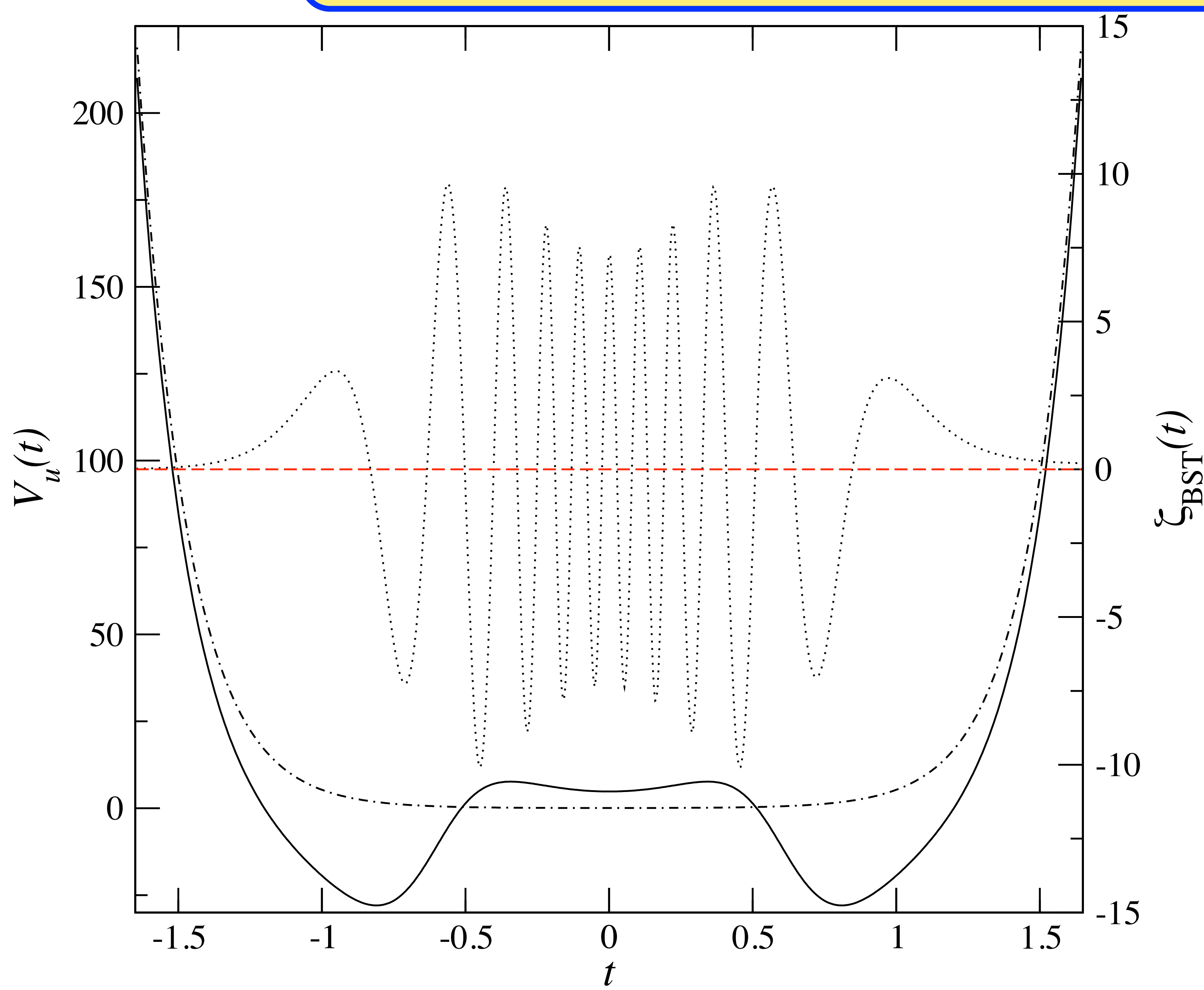
J. Martin & PP, *Phys*. *Rev*. **D68**, 103517 (2003)

Perturbations: $\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$

$$\Longleftrightarrow \qquad \Phi = \frac{3\mathcal{H}u}{2a^2\theta} \qquad \theta \equiv \frac{1}{a}\sqrt{\frac{\rho_\varphi}{\rho_\varphi + p_\varphi}\left(1 - \frac{3\mathcal{K}}{\rho_\varphi a^2}\right)}$$

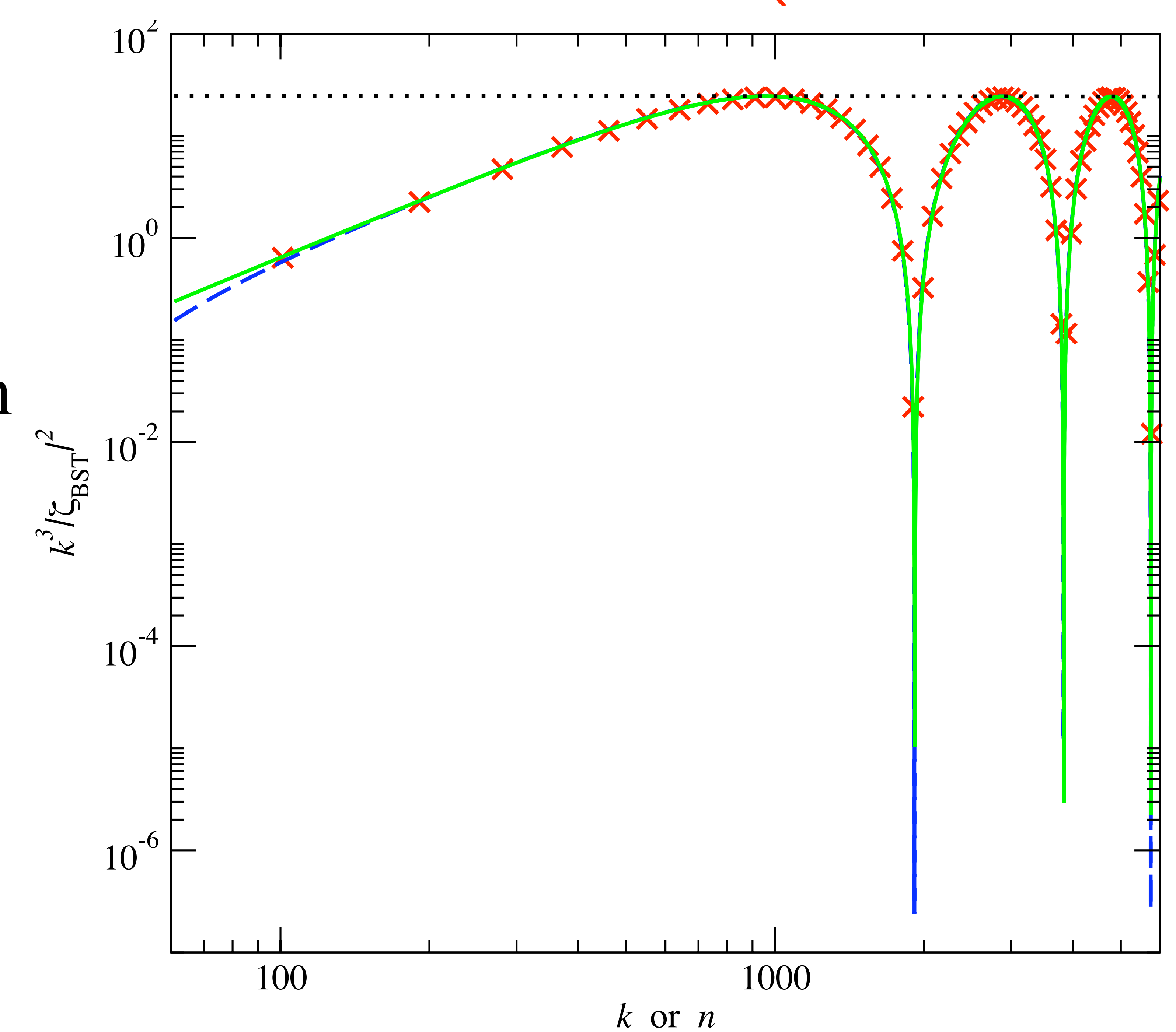
$$\boxed{u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K}\left(1 - c_{\mathrm{S}}^2\right)\right]u = 0}$$

$$\mathcal{P}_\zeta = \mathcal{A}k^{n_{\mathrm{S}}-1}\cos^2\left(\omega\frac{k_{\mathrm{ph}}}{k_\star} + \psi\right)$$



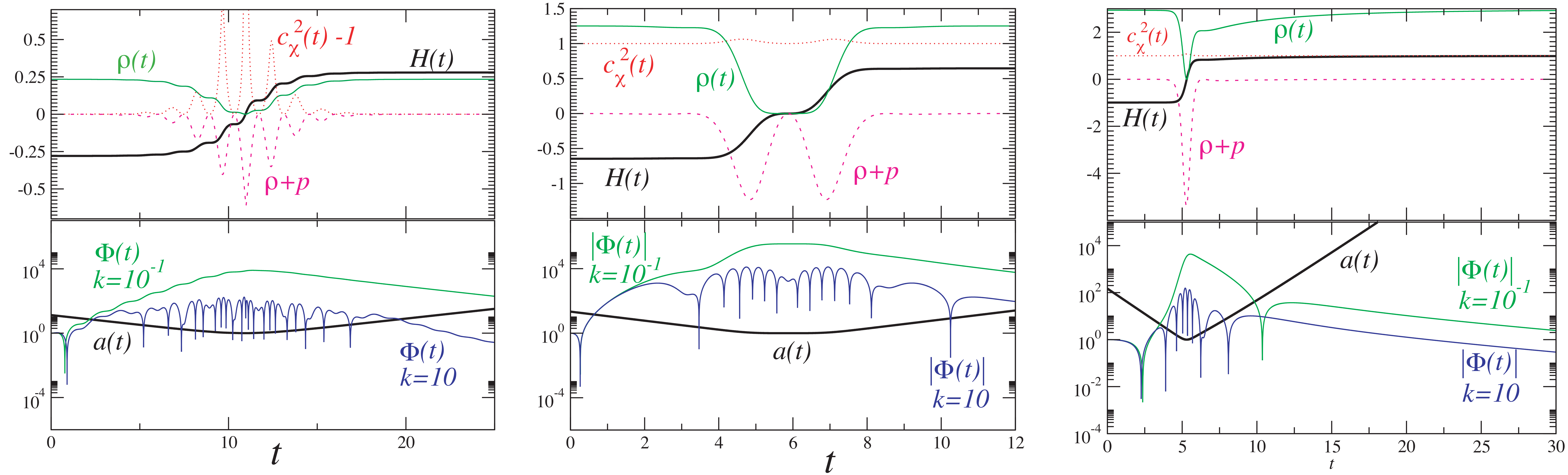
Different parameters

primordial spectrum



Model for the bounce phase only:

$$p = p_0 + p_X(X - X_0) + p_\varphi\varphi + p_{X\varphi}\varphi(X - X0) + \frac{1}{2}p_{XX}(X - X_0)^2 + \frac{1}{2}p_{\varphi\varphi}\varphi^2 + \cdots$$

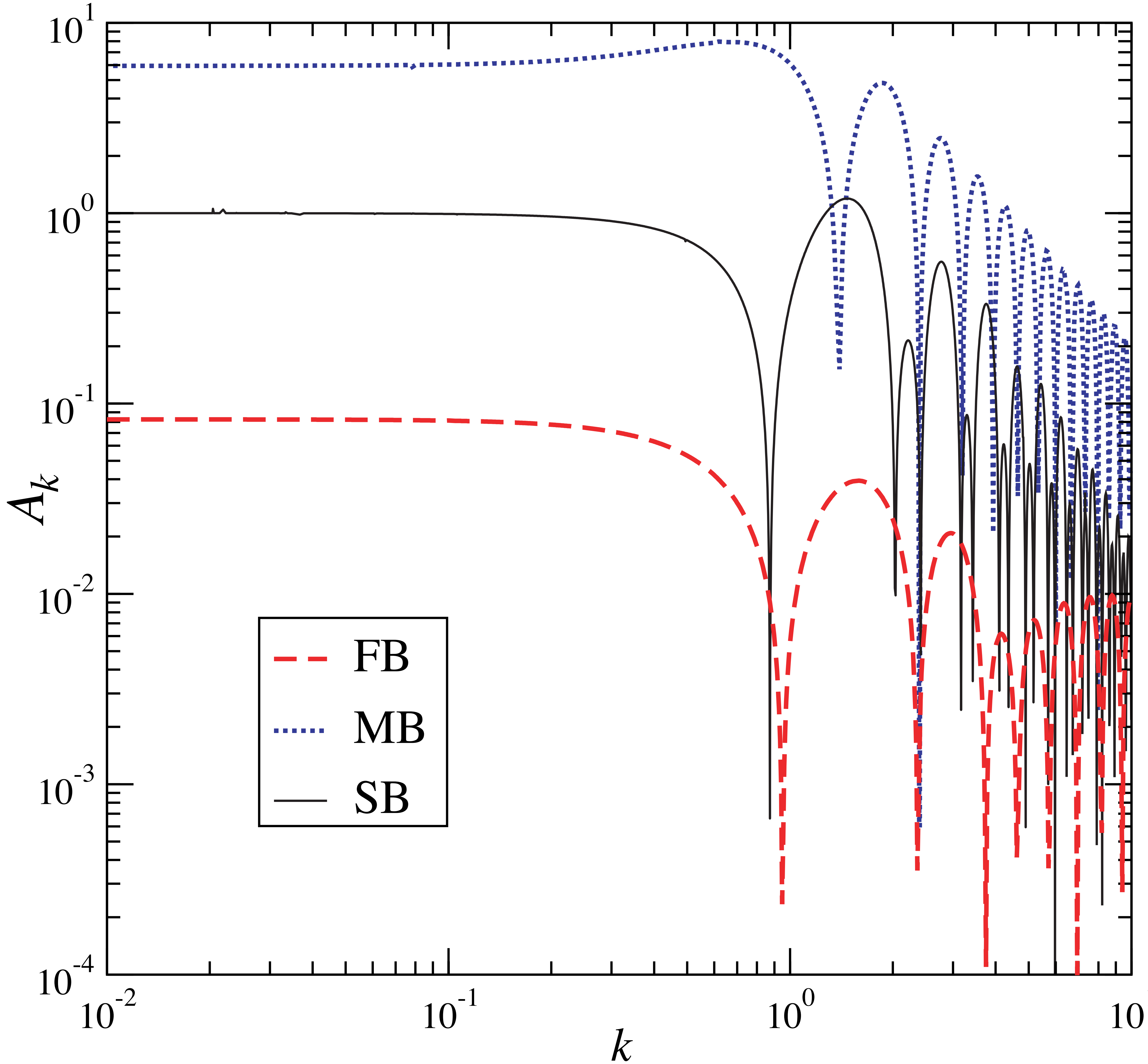


Slow

Fast

Oscillations + ζ conserved

R. Abramo & P. P., *JCAP* **09**, 001 (2007)



k-mode mixing ...

A few problems…

spectral index $n_s < 1$

Non gaussianities:

phenomenological description $S = -\int \mathrm{d}^4x\sqrt{-g}\left[R + (\partial\phi)^2 + V(\phi)\right]$

$$a(\eta) = a_0\left[1 + \frac{1}{2}\left(\frac{\eta}{\eta_c}\right)^2 + \lambda_3\left(\frac{\eta}{\eta_c}\right)^3 + \frac{5}{24}(1+\lambda_4)\left(\frac{\eta}{\eta_c}\right)^4\right]$$ + *scalar field*

$$\begin{cases} \dfrac{\phi'^2}{a^2} = \dfrac{2}{a^2}\left(\mathcal{H}^2 - \mathcal{H}' + \mathcal{K}\right) \\ -\dfrac{6}{a^2}\mathcal{H}' = -2V(\phi)\left[1 - \dfrac{\phi'^2}{a^2V(\phi)}\right] \end{cases}$$

$$\phi'' + 2\mathcal{H}\phi' + a^2V_{,\phi} = 0$$

$$\varepsilon_V = \frac{V_0'}{V_0}$$ *"slow-roll"*

$$\eta_V = \frac{V_0''}{V_0}$$

$$\Upsilon \equiv \phi_0'^2/2 \longrightarrow \eta_c^2 = \frac{1}{1-\Upsilon}$$

complete set of parameters

perturbed metric $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = a^2\left(-\mathrm{e}^{2\Phi}\mathrm{d}\eta^2 + \mathrm{e}^{-2\Psi}\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\right)$

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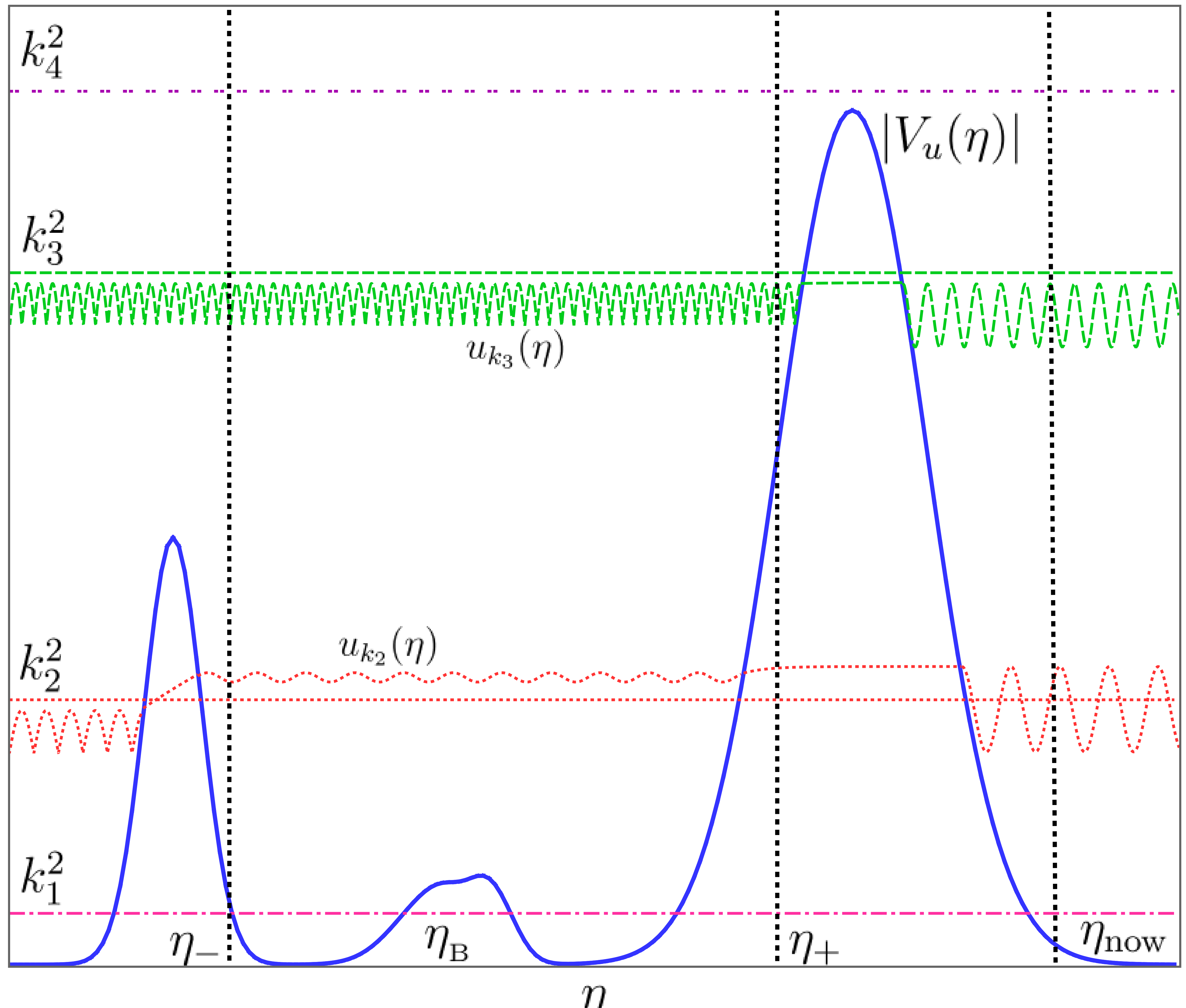
$u \propto a\Psi_{(1)}/\phi'$ → $u_{\boldsymbol{k}}'' + \left[k^2 - V_u(\eta)\right] u_{\boldsymbol{k}} = 0$



perturbations up to 2nd order $X\left(\boldsymbol{x},\eta\right)=X_{(1)}\left(\boldsymbol{x},\eta\right)+\frac{1}{2}X_{(2)}\left(\boldsymbol{x},\eta\right)+\cdots$

$$\mathcal{D}\,\Psi_{(i)}=\mathcal{S}\left[\Psi_{(i-1)}\right]$$

first order

$$\Psi''_{(1)}+F\left(\eta\right)\Psi'_{(1)}-\bar{\nabla}^2\Psi_{(1)}+W\left(\eta\right)\Psi_{(1)}=0$$

$$2\left(\mathcal{H}-\frac{\bar{\phi}''}{\bar{\phi}'}\right)\qquad\qquad 2\left(\mathcal{H}'-\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}-2\mathcal{K}\right)$$

positive spatial curvature: decomposition on the 3-sphere

$$\Psi_{(1)}(\boldsymbol{x},\eta)=\sum_{\ell mn}\Psi_{\ell mn}(\eta)Q_{\ell mn}(\chi,\theta,\varphi)$$

$Q_{\ell mn}(\chi,\theta,\varphi)=R_{\ell n}(\chi)Y_{\ell m}(\theta,\varphi)$ *hyperspherical harmonics*

Legendre

$$R_{n\ell}\left(\chi\right)=\sqrt{\frac{\left(n+1\right)\left(n+\ell+1\right)!}{\left(n-\ell\right)!}}\sqrt{\frac{\mathcal{K}}{f_{\mathcal{K}}\left(\chi\right)}}P^{-\ell-\frac{1}{2}}_{n+\frac{1}{2}}\left[\cos\left(\sqrt{\mathcal{K}}\chi\right)\right]$$

effect of the bounce itself: initial conditions = classical gaussian fields

$$\begin{bmatrix}\Psi_{(1)}\left(\boldsymbol{k},\eta_-\right)\\ \Psi'_{(1)}\left(\boldsymbol{k},\eta_-\right)\end{bmatrix}\equiv\begin{bmatrix}\hat{x}_1\left(\boldsymbol{k}\right)\\ \hat{x}_2\left(\boldsymbol{k}\right)\end{bmatrix}$$

spectra

$$\left\langle\hat{x}_i\left(\boldsymbol{k}\right)\hat{x}_j\left(\boldsymbol{k}'\right)\right\rangle\equiv\delta_{\boldsymbol{k},\boldsymbol{k}'}P_{ij}\left(k\right)$$

$$\delta_{nn'}\delta_{\ell\ell'}\delta_{mm'}$$

2nd order $$\Psi''_{(2)} + 2\left(\mathcal{H} - \frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi'_{(2)} - \bar{\nabla}^2\Psi_{(2)} + 2\left(\mathcal{H}' - 2\mathcal{K} - \mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(2)} = \mathcal{S}_{(2)}$$

$$\begin{aligned}\mathcal{S}_{(2)} = {} & 4\left(2\mathcal{H}^2 - \mathcal{H}' + 2\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'} + 6\mathcal{K}\right)\Psi^2_{(1)} + 8\Psi'^2_{(1)} + 8\left(2\mathcal{H} + \frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(1)}\Psi'_{(1)} + 8\Psi_{(1)}\bar{\nabla}^2\Psi_{(1)} - \frac{4}{3}\left(\bar{\nabla}_i\Psi_{(1)}\right)^2 \\ & - \left[2\left(2\mathcal{H}^2 - \mathcal{H}'\right) - \frac{\bar{\phi}'''}{\bar{\phi}'}\right]\phi^2_{(1)} - \frac{2}{3}\left(\bar{\nabla}_i\phi_{(1)}\right)^2 - 2\left(\frac{\bar{\phi}''}{\bar{\phi}'} + 2\mathcal{H}\right)\bar{\nabla}^{-2}\bar{\nabla}^i\left(2\Psi'_{(1)}\bar{\nabla}_i\Psi_{(1)} + \phi'_{(1)}\bar{\nabla}_i\phi_{(1)}\right) \\ & + \left[2\left(\mathcal{H}' - \mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right) + \frac{1}{3}\bar{\nabla}^2\right]\left[2F\left(\Psi_{(1)}\right) + F\left(\phi_{(1)}\right)\right] + \mathcal{H}\left[2F\left(\Psi_{(1)}\right) + F\left(\phi_{(1)}\right)\right]'\end{aligned}$$

$$F(X) = \left(\bar{\nabla}^2\bar{\nabla}^2 + 3\mathcal{K}\bar{\nabla}^2\right)^{-1}\left[\bar{\nabla}_i\bar{\nabla}^j\left(3\bar{\nabla}^i X\bar{\nabla}_j X - \delta^i_j\left(\bar{\nabla}_k X\right)^2\right)\right]$$

general solution $$\mathcal{S}_{(2)}(\boldsymbol{k},\eta) = \sum_{\boldsymbol{p}_1,\boldsymbol{p}_2}\mathcal{G}_{\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2}\tilde{\Sigma}_{ij}(k,p_1,p_2;\eta)\,\hat{a}_i(\boldsymbol{p}_1)\,\hat{a}_j(\boldsymbol{p}_2)$$

$$\Psi_{(2)}(\boldsymbol{k},\eta) = \Psi^{(0)}_{(2)}(\boldsymbol{k},\eta) + \sum_{\boldsymbol{p}_1,\boldsymbol{p}_2}\mathcal{G}_{\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2}\Pi_{ij}(k,p_1,p_2;\eta)\,\hat{x}_i(\boldsymbol{p}_1)\,\hat{x}_j(\boldsymbol{p}_2)$$

$$\Pi_{ij}(k,p_1,p_2;\eta) \equiv \int_{\eta_-}^{\eta}\mathrm{d}\eta' G\left(k,\eta,\eta'\right)\Sigma_{ij}\left(k,p_1,p_2;\eta'\right)$$

Green

Bispectrum $\langle \Psi(\boldsymbol{k}_1,\eta)\,\Psi(\boldsymbol{k}_2,\eta)\,\Psi(\boldsymbol{k}_3,\eta)\rangle \equiv \frac{1}{2}\mathcal{G}_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3}\mathcal{B}_\Psi(k_1,k_2,k_3;\eta)$

$\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$

$$\mathcal{B}_\Psi(k_1,k_2,k_3) = \frac{6}{5}f_{\rm NL}\left[P_{\Psi\Psi}(k_1)P_{\Psi\Psi}(k_2)+P_{\Psi\Psi}(k_2)P_{\Psi\Psi}(k_3)+P_{\Psi\Psi}(k_3)P_{\Psi\Psi}(k_1)\right]$$

$7P_{\Psi\Psi}(k)+11P_{\Psi\Psi'}(k)+4P_{\Psi'\Psi'}(k)$

$$f_{\rm NL} = -\frac{5(k_1+k_2+k_3)}{3\Upsilon K_3(k_1,k_2,k_3)}\left(\left[\prod_{\sigma(i,j,\ell)}(k_i+k_j-k_\ell)\right]\left\{\sum_{\sigma(i,j,\ell)}\frac{K_1(k_i)K_1(k_j)}{k_\ell^2}-4\left[\frac{K_1(k_i)K_2(k_j)}{k_j^2k_\ell^2}+\frac{K_1(k_j)K_2(k_i)}{k_i^2k_\ell^2}\right]\right\}\right.$$
$$\left.-\sum_{\sigma(i,j,\ell)}\left[\frac{7}{3}+\frac{2}{3}\left(\frac{k_i^2+k_j^2}{k_\ell^2}\right)-3\left(\frac{k_i^2-k_j^2}{k_\ell^2}\right)^2\right]K_1(k_i)K_1(k_j)\right)+\cdots,$$

$6P_{\Psi\Psi}(k)+7P_{\Psi\Psi'}(k)+2P_{\Psi'\Psi'}(k)$

$$81\sum_{\sigma(i,j)}P_{\Psi\Psi}(k_i)P_{\Psi\Psi}(k_j)+108\sum_{\sigma(i,j)}P_{\Psi\Psi}(k_i)P_{\Psi\Psi'}(k_j)+36\sum_{\sigma(i,j)}P_{\Psi\Psi}(k_i)P_{\Psi'\Psi'}(k_j)+$$
$$144\sum_{\sigma(i,j)}P_{\Psi\Psi'}(k_i)P_{\Psi\Psi'}(k_j)+48\sum_{\sigma(i,j)}P_{\Psi\Psi'}(k_i)P_{\Psi'\Psi'}(k_j)+16\sum_{\sigma(i,j)}P_{\Psi'\Psi'}(k_i)P_{\Psi'\Psi'}(k_j)$$

equilateral $k_1 = k_2 = k_3 = k$

$$f_{\rm NL}^{\rm equi} = -\frac{15k^2}{\Upsilon}\frac{K_1^2(k)}{K_3(k,k,k)}$$

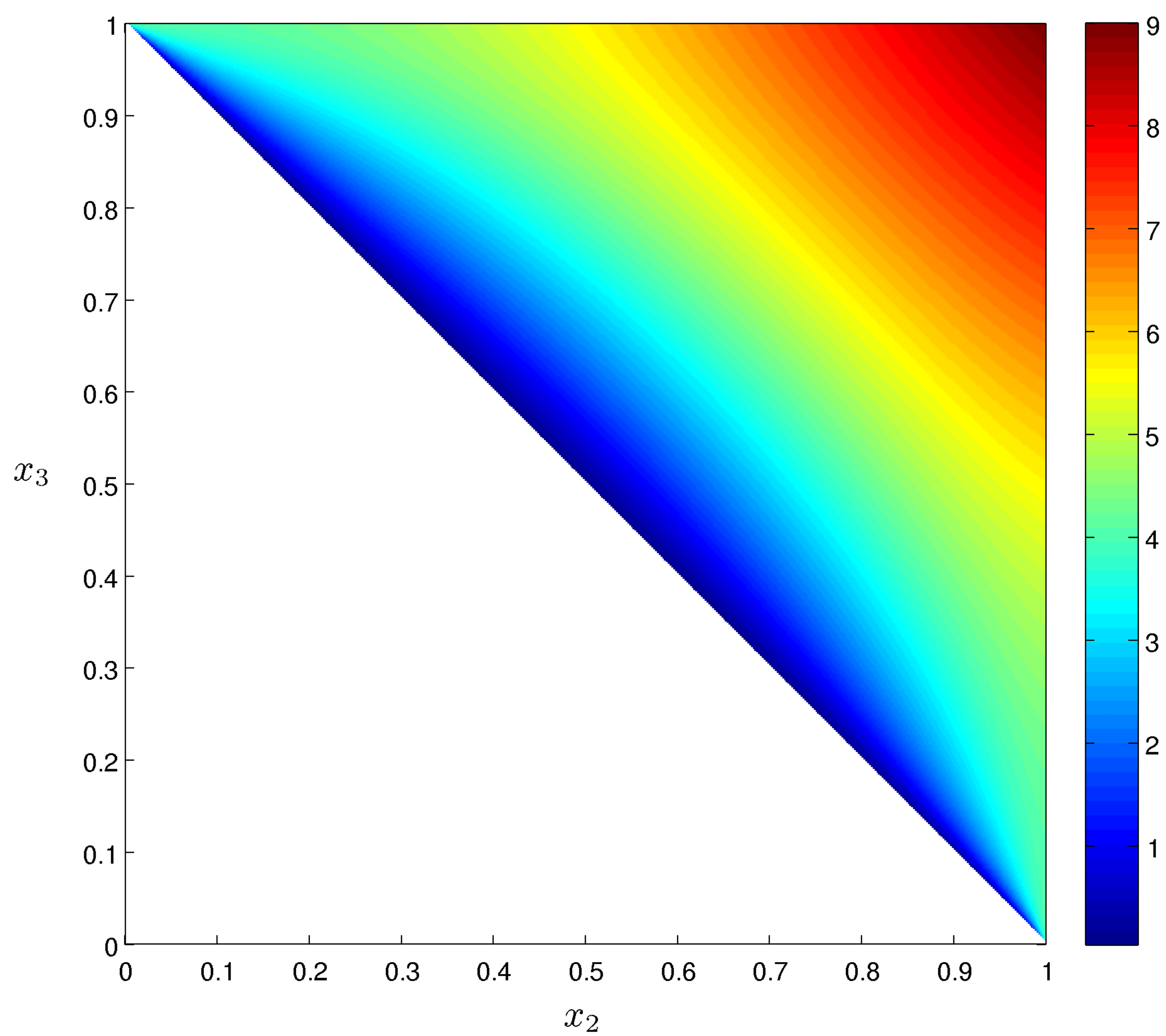
squeezed $k_i = k_j = k$ & $k_\ell = p \ll k$

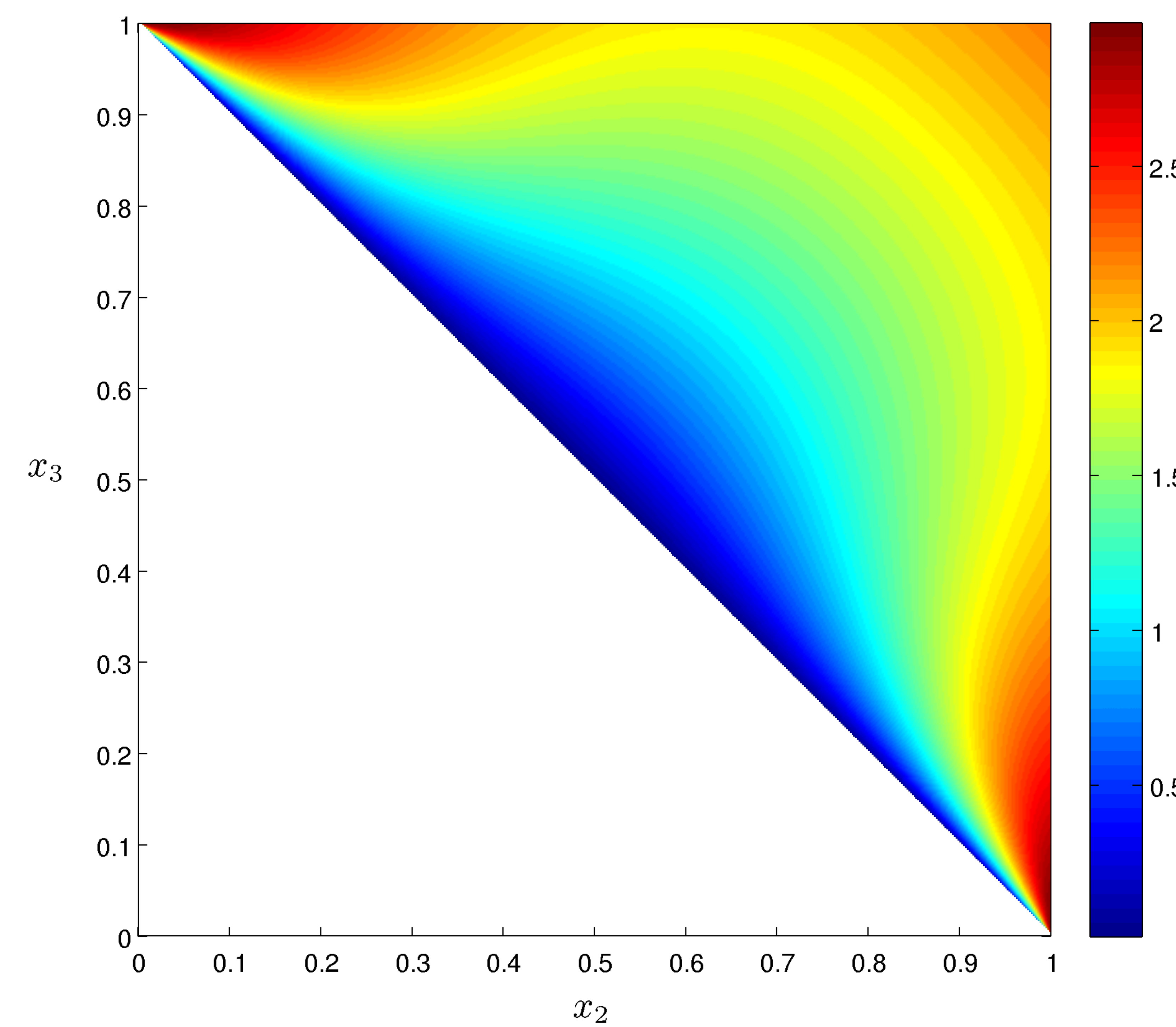
$$f_{\rm NL}^{\rm sq} = -\frac{20k^2}{3\Upsilon}\frac{K_1^2(k)+K_1(k)K_1(p)}{K_3(k,k,p)}$$

folded $k_2 = k_3 = \frac{1}{2}k_1$

$$f_{\rm NL}^{\rm fold} = \frac{40}{9\Upsilon}\frac{K_1(k)\left[K_1(k)-16K_1(2k)\right]}{K_3(k,k,2k)}$$

$$f_{\rm NL} \propto (k_1^2/\Upsilon) \times \mathcal{S}(x_2,x_3)$$





$10^3 h\,{\rm Mpc}^{-1} \lesssim k_{\rm phys} \lesssim 10^3 h\,{\rm Mpc}^{-1}$ → $10^2 \lesssim k \lesssim 10^8$ *with* $\Omega_\mathcal{K} \leq 10^{-2}$

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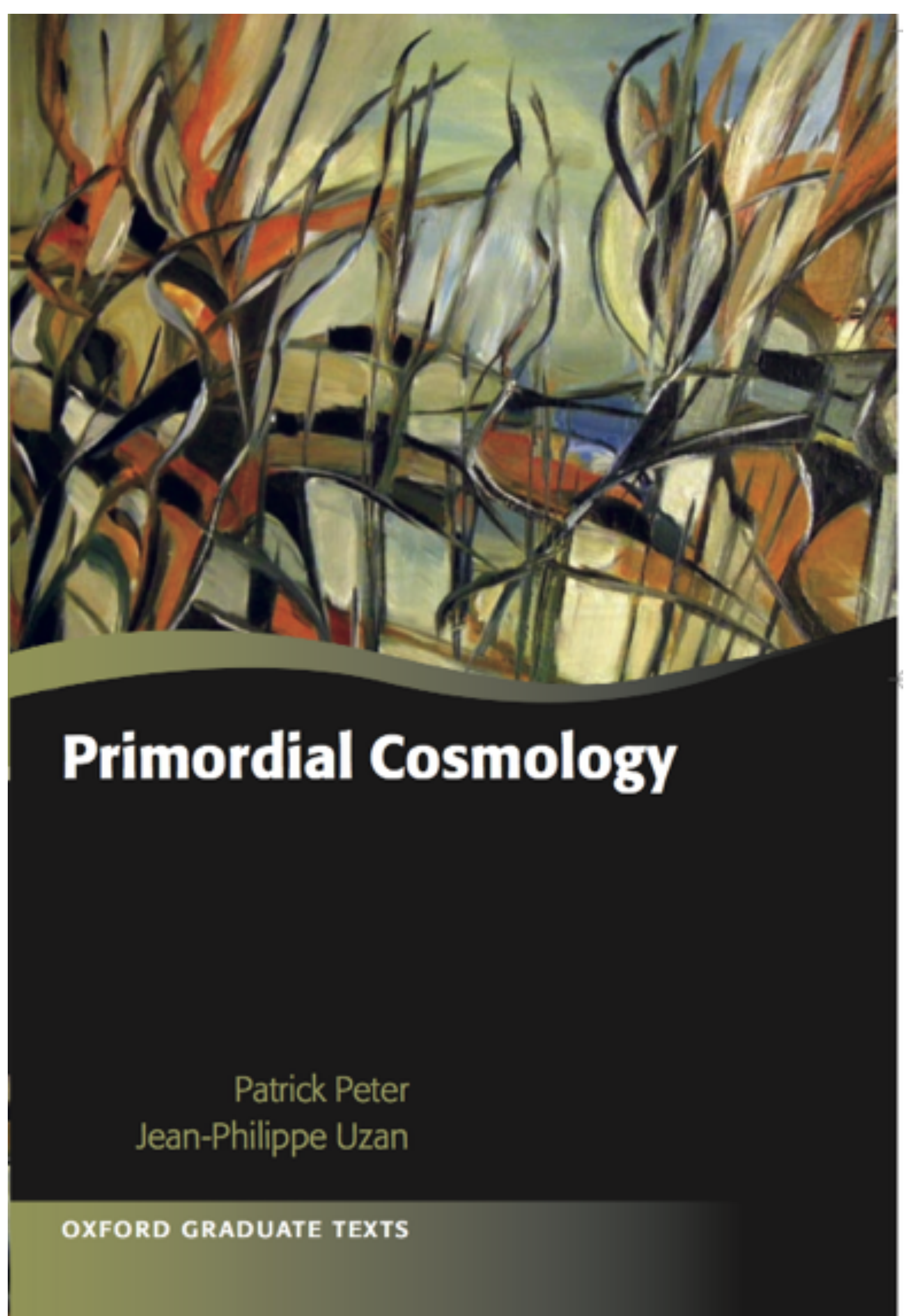
Conclusion

Bouncing cosmology = testbed for new ideas, interesting, potentially useful…

not yet an alternative to inflation

Refs.



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