The Chaplygin gas

a model for dark energy

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Plan of the talk

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- Generalities

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- Some remarkable properties of the Chaplygin gas

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- Chaplygin cosmology: theory and observations

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- Chaplygin cosmology: theory and observations
- Tachyon cosmological models (depending on time)

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The signature of extra-dimensions?

General Relativistic Cosmology

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Homogeneity and isotropy

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$$T_{\mu\nu} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$ in comoving coordinates

Cosmological constant

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$$T_{\mu\nu} = \frac{1}{8\pi G}\begin{bmatrix} \Lambda & 0 & 0 & 0 \\ 0 & -\Lambda & 0 & 0 \\ 0 & 0 & -\Lambda & 0 \\ 0 & 0 & 0 & -\Lambda \end{bmatrix}$$

in comoving coordinates

Cosmology

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Together imply energy conservation for each component

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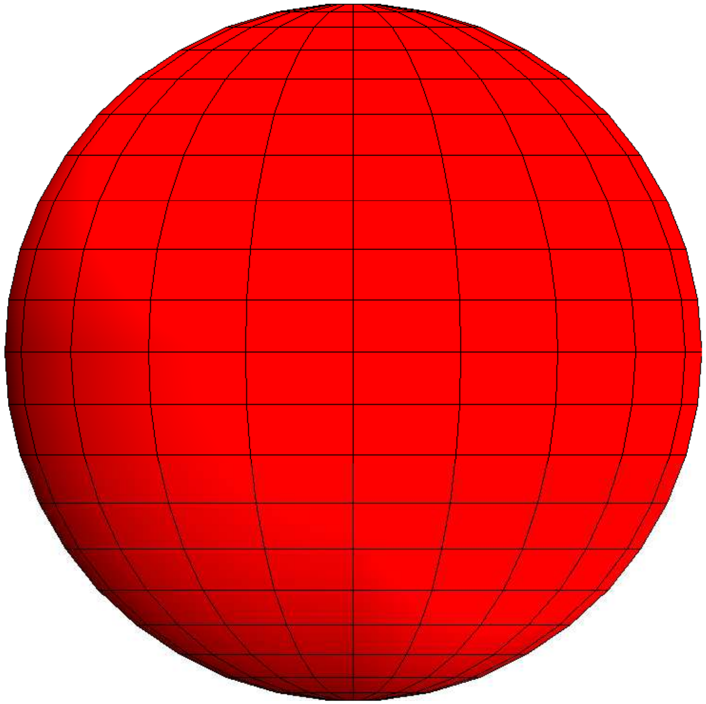
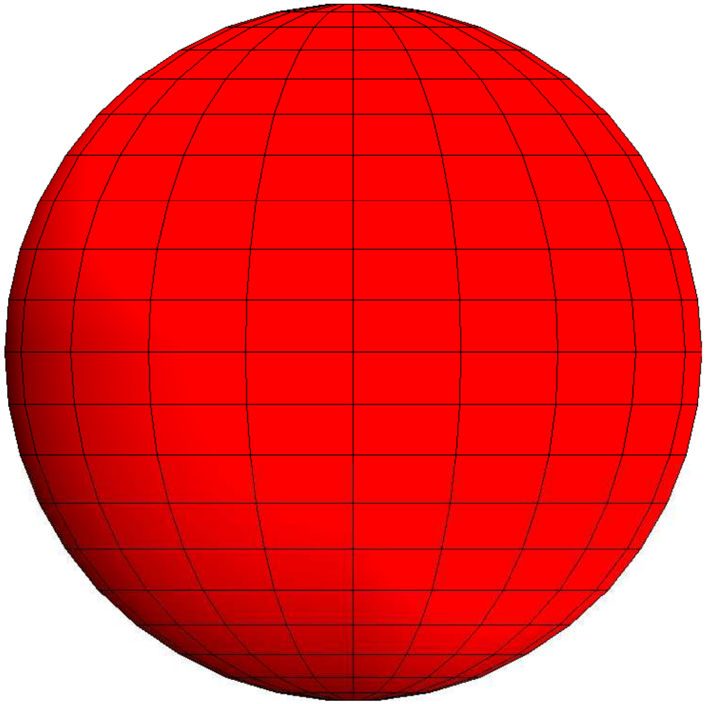
$$\dot{\rho}_i = -3\frac{\dot{a}}{a}(\rho_i + p_i)$$

Spherical universe

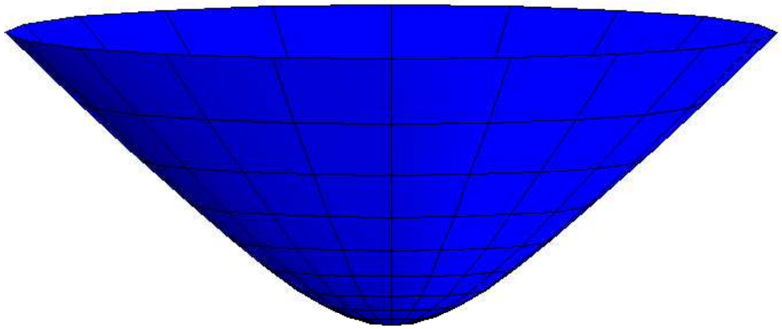


Spherical universe

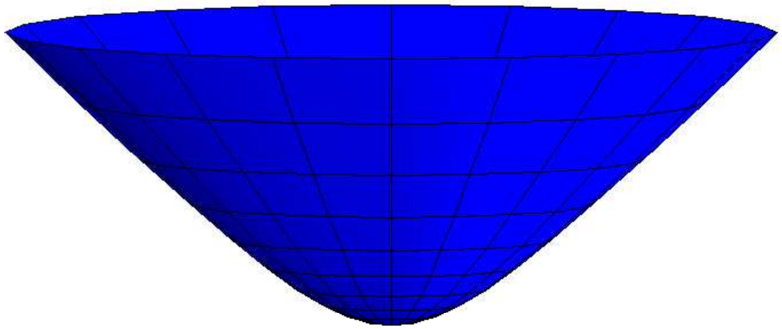


$$
\begin{aligned}
dl^2 &= \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \\
&= \frac{1}{K}\left(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)\right)
\end{aligned}
$$

Hyperbolic universe



Hyperbolic universe



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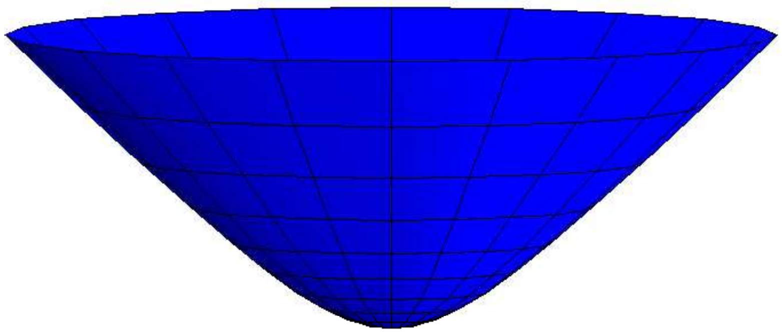
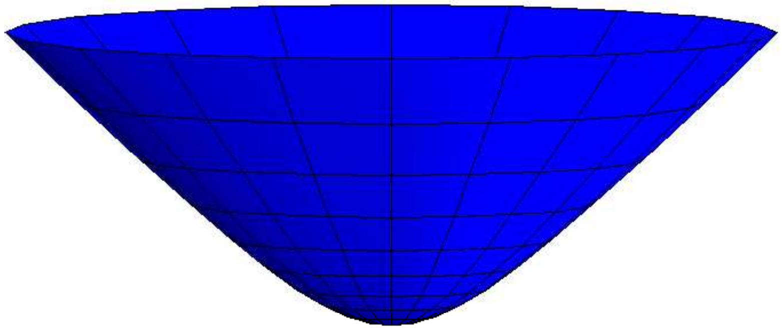


$$\begin{cases} x_0 &= A\cosh\chi \\ x_1 &= A\sinh\chi\sin\theta\sin\phi \\ x_2 &= A\sinh\chi\sin\theta\cos\phi \\ x_3 &= A\sinh\chi\cos\theta \end{cases}$$

$$x_0^2 - x_1^2 - x_2^2 - x_3^2 = A^2$$

Hyperbolic universe



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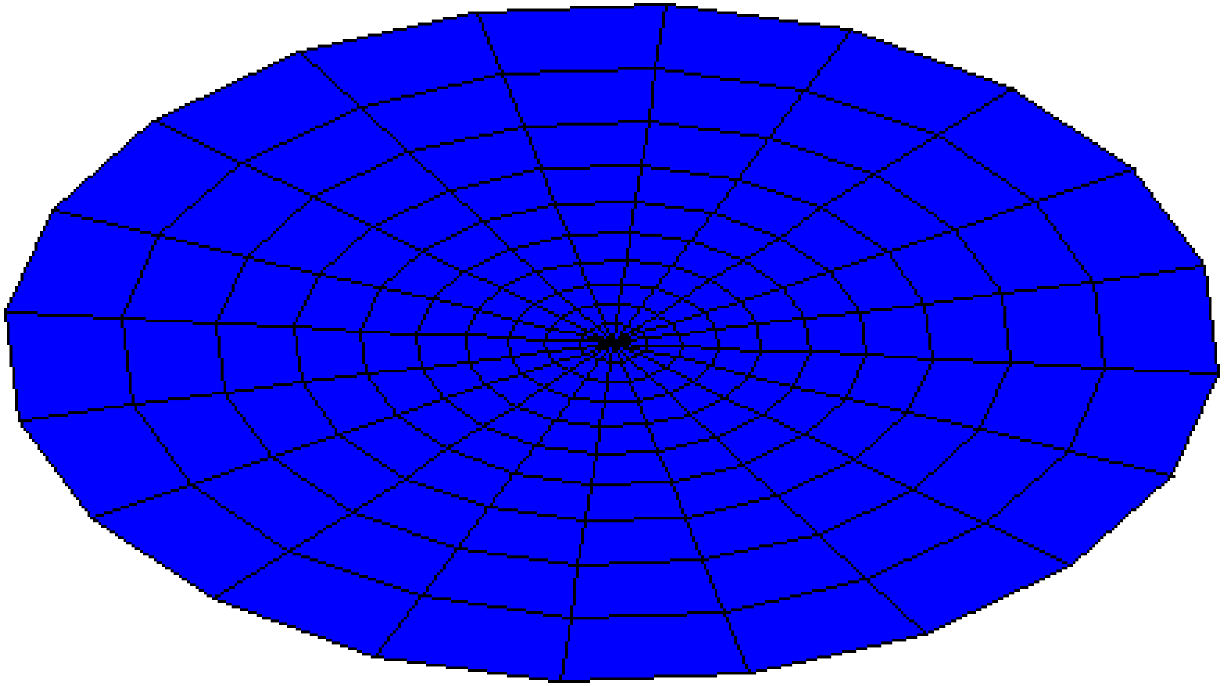
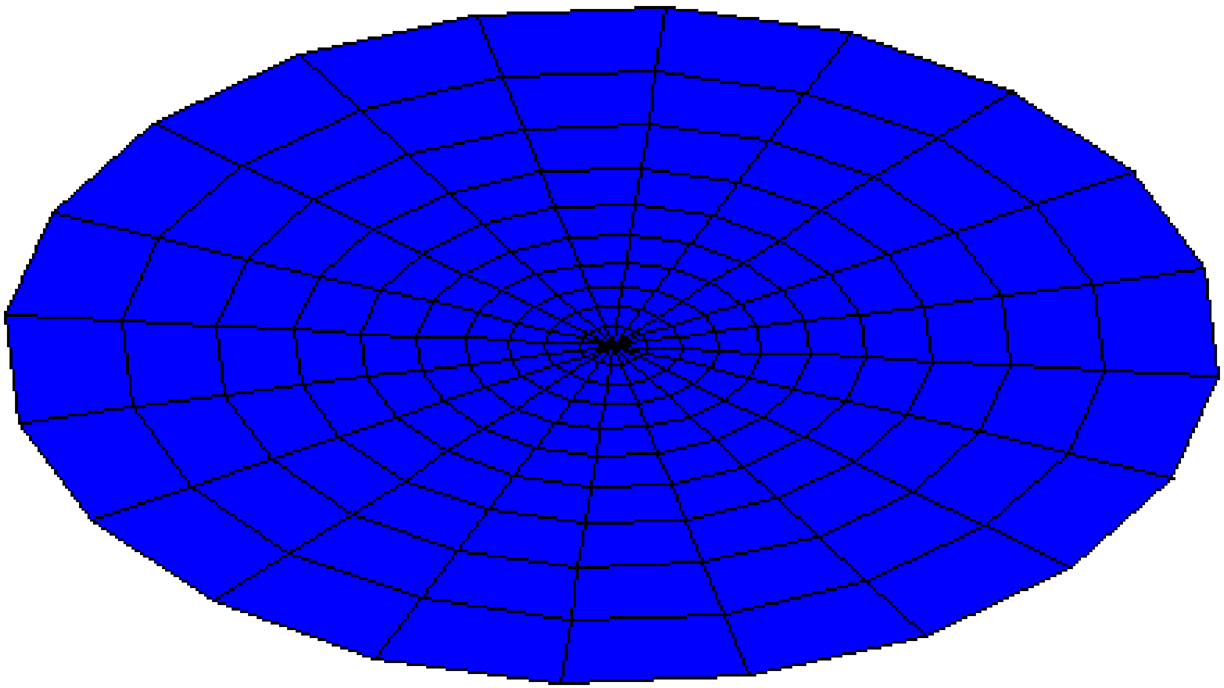
$$x_0^2 - x_1^2 - x_2^2 - x_3^2 = A^2$$

$$\begin{aligned} dl^2 &= \tfrac{1}{K}\left(d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta d\phi^2)\right) \\ &= \tfrac{dr^2}{1+Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \end{aligned}$$

Flat universe



Flat universe



$$dl^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta \; d\phi^2)$$

Dust: p=0

$$a\dot{\rho} + 3\dot{a}(\rho + p) = 0$$

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Cosmological constant: $p = -\rho$

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- $a\dot{\rho} = 0$
- $\rho_\Lambda = const$
- for every time

The search for Ω

A set of dimensionless parameters.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G(\rho_M + \rho_R + \rho_\Lambda) - \frac{K}{a^2}$$

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$$= \frac{8}{3}\pi G\left(\frac{\rho_{M_0} a_0^3}{a^3(t)} + a_0\frac{\rho_{R_0} a_0^4}{a^4(t)} + \rho_\Lambda + \frac{\rho_{K_0} a_0^2}{a^2(t)}\right)$$

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$$\frac{H^2}{H_0^2} = \Omega_{M0}(1+z)^3 + \Omega_{R0}(1+z)^4 + \Omega_{\Lambda 0} + \Omega_{K0}(1+z)^2$$

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$$\frac{H_0^2}{H_0^2} = \Omega_{M0} + \Omega_{R0} + \Omega_{\Lambda 0} + \Omega_{K0} = 1$$

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$$\Omega_{\Lambda 0} \;\; = \;\; Dark\ Energy \;\; \simeq 0.7$$

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$$\begin{aligned}
\Omega_{\Lambda 0} &= Dark\ Energy && \simeq 0.7 \\
\Omega_{M0} &= baryons\ and\ NBDM && \simeq 0.25 \\
&= \Omega_{B0} \quad + \quad \Omega_{DM0} && \simeq 0.05 + 0.20
\end{aligned}$$

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\Omega_{R0} &= CMB && \simeq 0.0001 \\
\Omega_{K0} &= Space\ Curvature && \simeq 0
\end{aligned}$$

The fate of the ΛCDM universe

$$H^2 = \frac{8}{3}\pi G \left(\frac{\rho_{M_0}}{a^3(t)} + \frac{\rho_{R_0}}{a^4(t)} + \rho_\Lambda + \frac{\rho_{K_0}}{a^2(t)} \right)$$

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$$H^2 \longrightarrow \frac{8}{3}\pi G \rho_\Lambda$$

Only $\Lambda > 0$

$$\ddot{a} = \frac{1}{3}\,\Lambda\,a$$

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Only $\Lambda > 0$

$$\ddot{a} = \frac{1}{3}\,\Lambda\,a \qquad \dot{a}^2 = \frac{1}{3}\,\Lambda\,a^2 - K$$

$$K = -1 \qquad a(t) = \sqrt{\frac{3}{\Lambda}}\sinh\sqrt{\frac{\Lambda}{3}}\,t \qquad \mathit{hyperbolic}$$

$$K = 0 \qquad a(t) = \exp\sqrt{\frac{\Lambda}{3}}\,t \qquad \mathit{flat}$$

$$K = 1 \qquad a(t) = \sqrt{\frac{3}{\Lambda}}\cosh\sqrt{\frac{\Lambda}{3}}\,t \qquad \mathit{spherical}$$

de Sitter universe

$$\mathbb{M}^{d+1} \quad \eta_{\mu\nu} = diag(1, -1, \ldots, -1)$$

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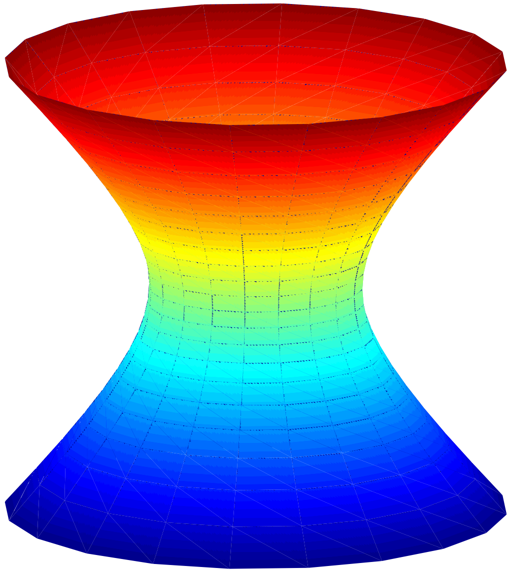

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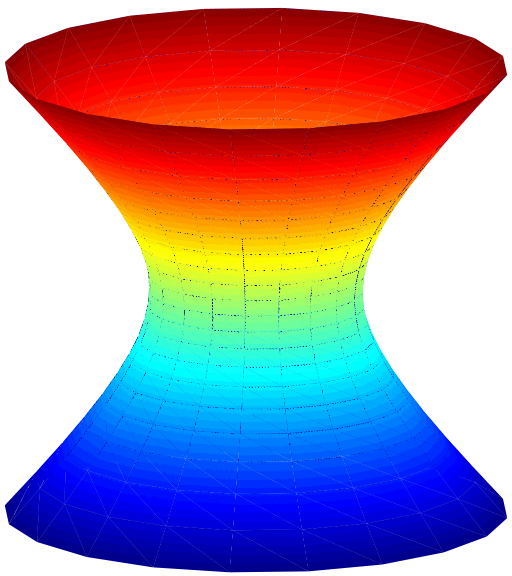
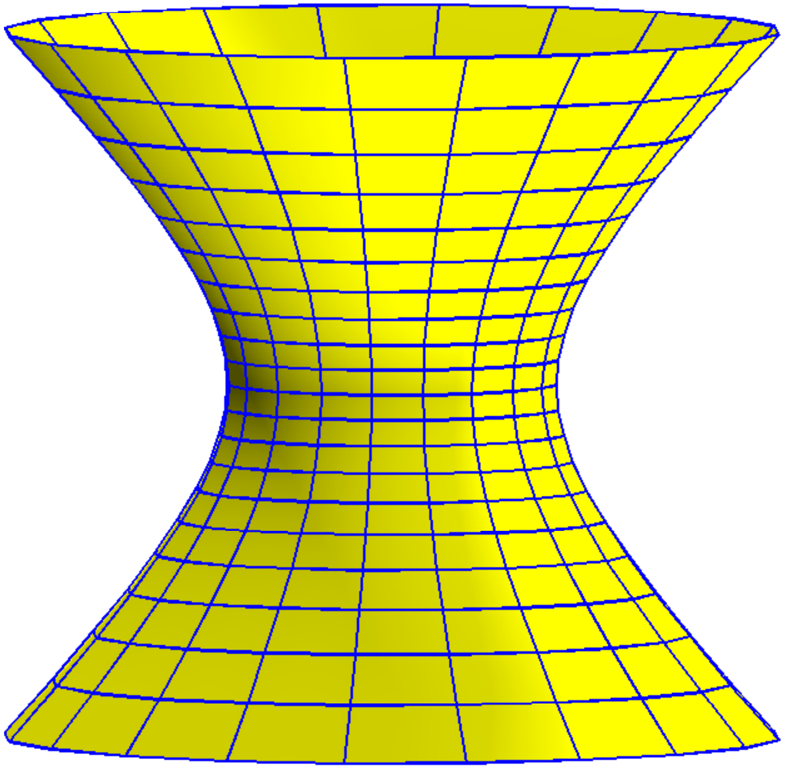


Spherical de Sitter



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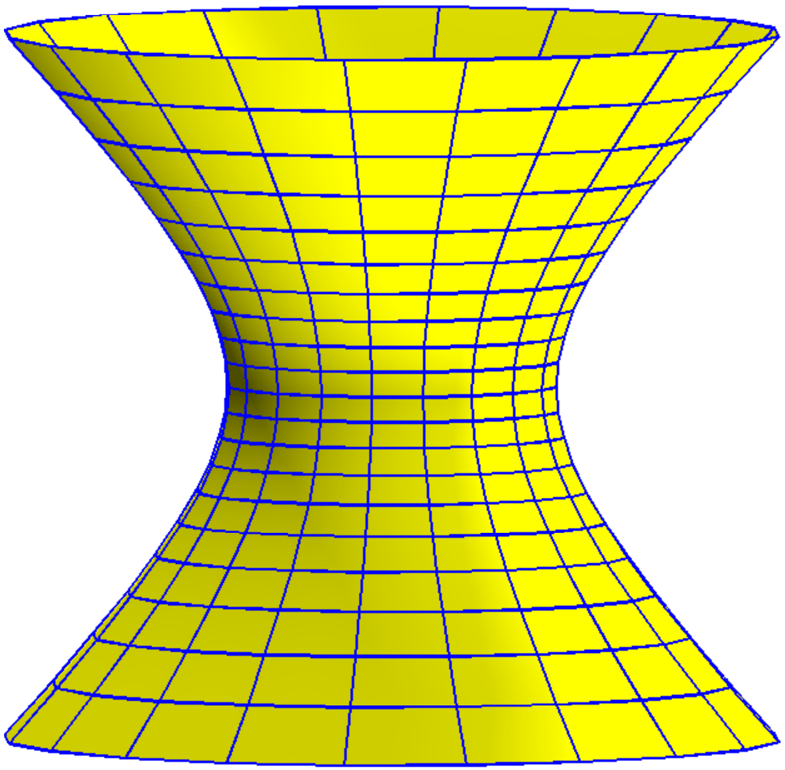

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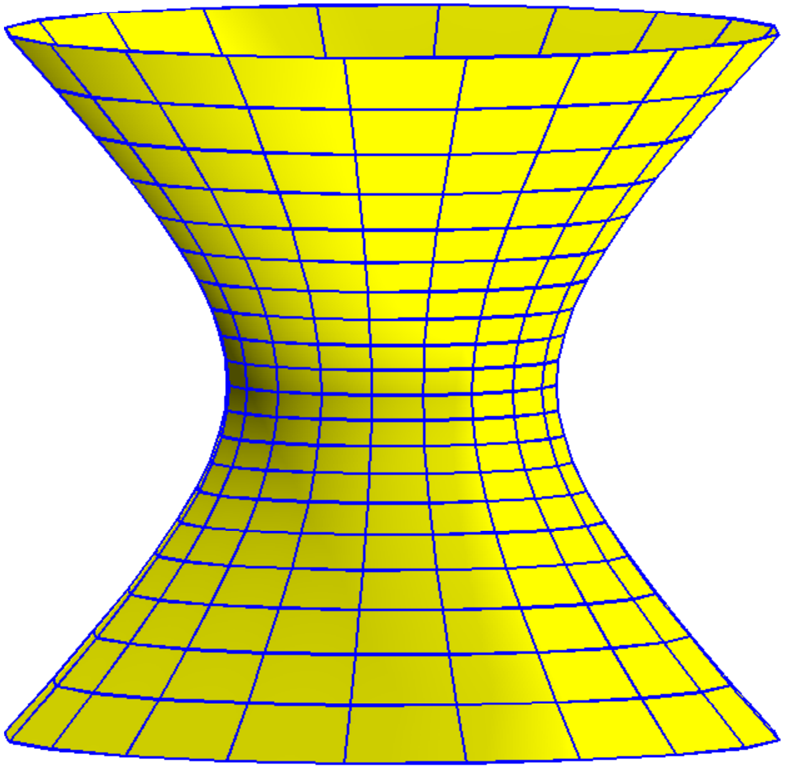

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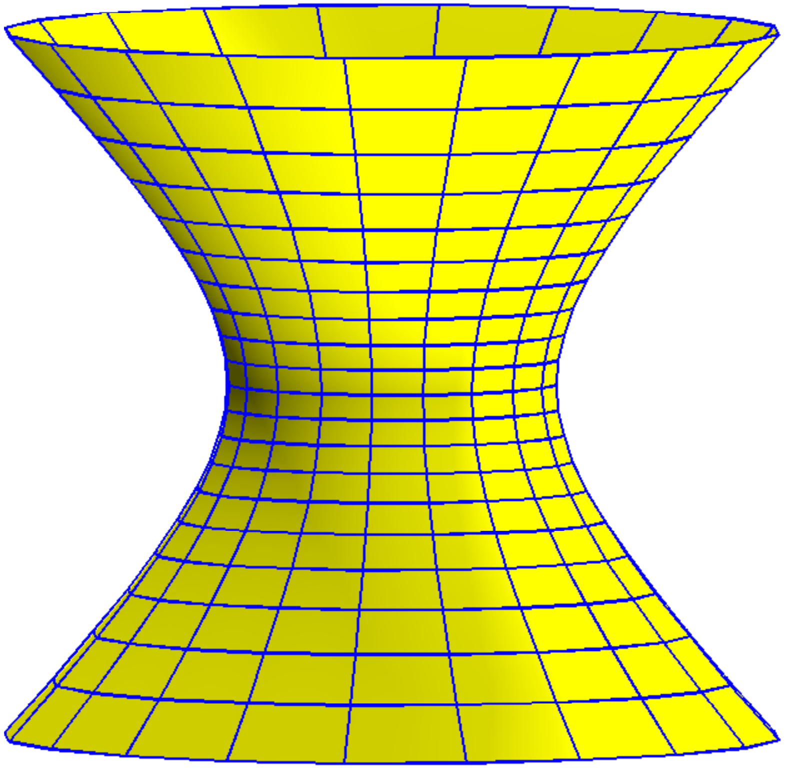

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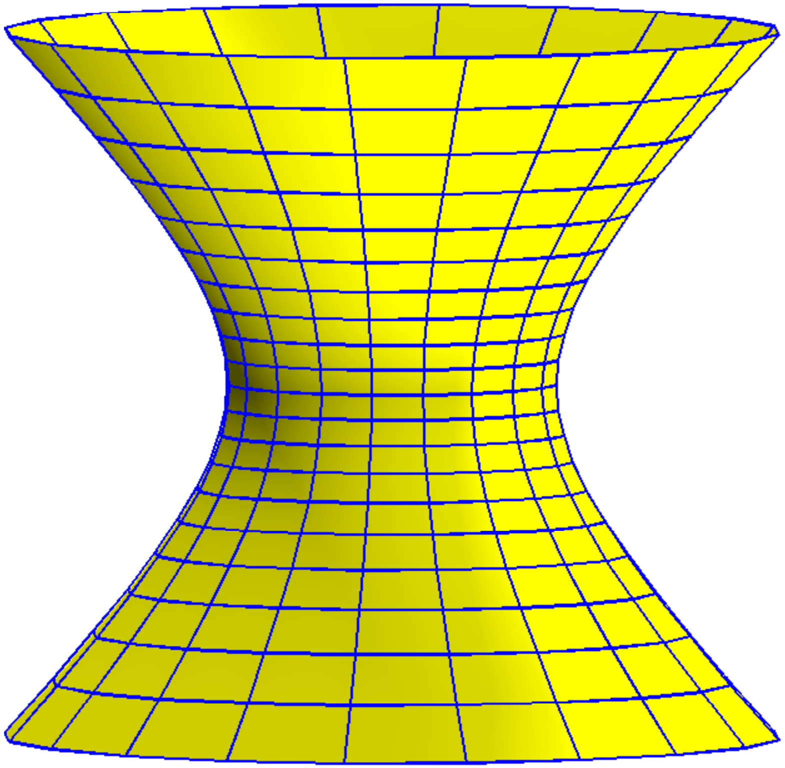

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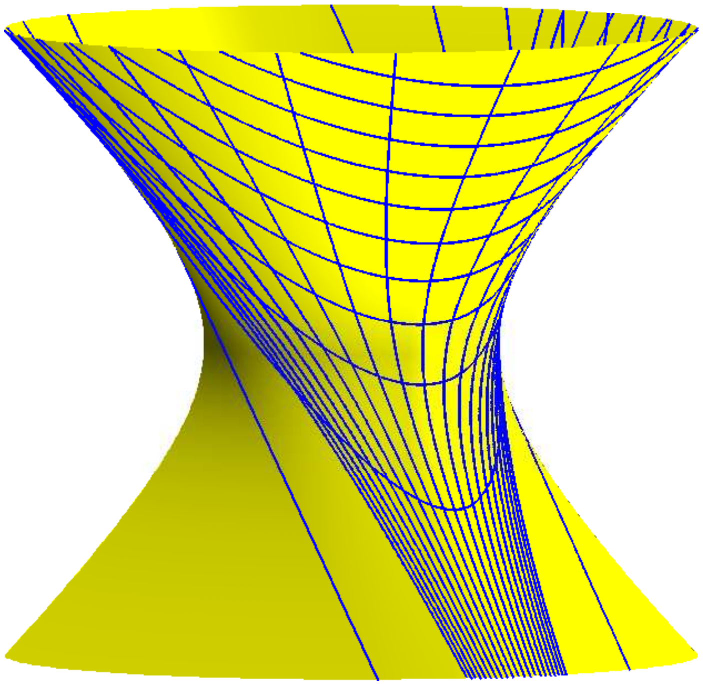

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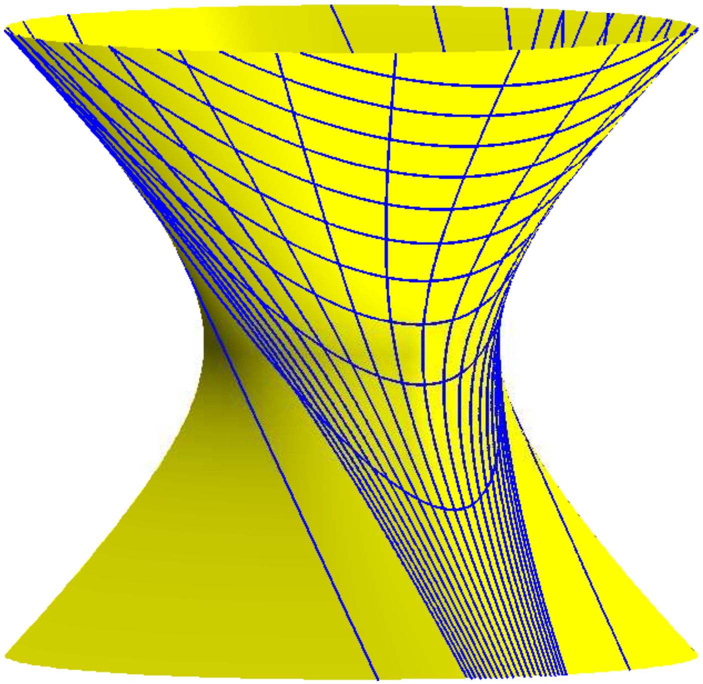

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$$
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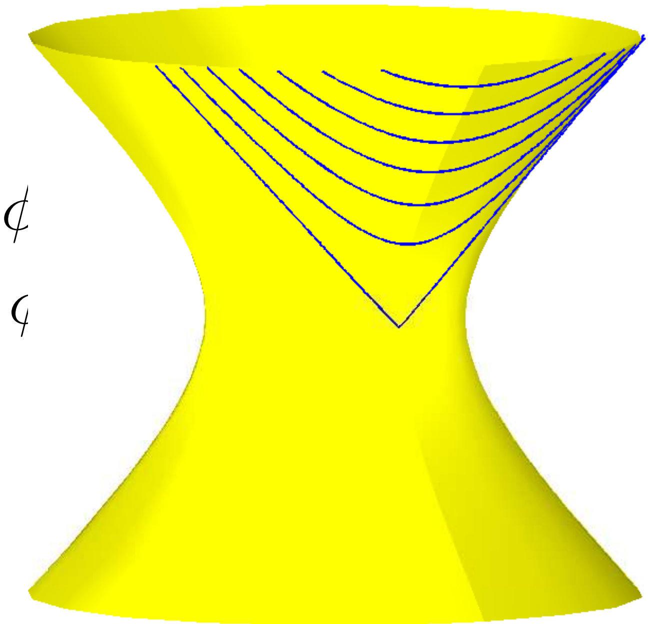
$$



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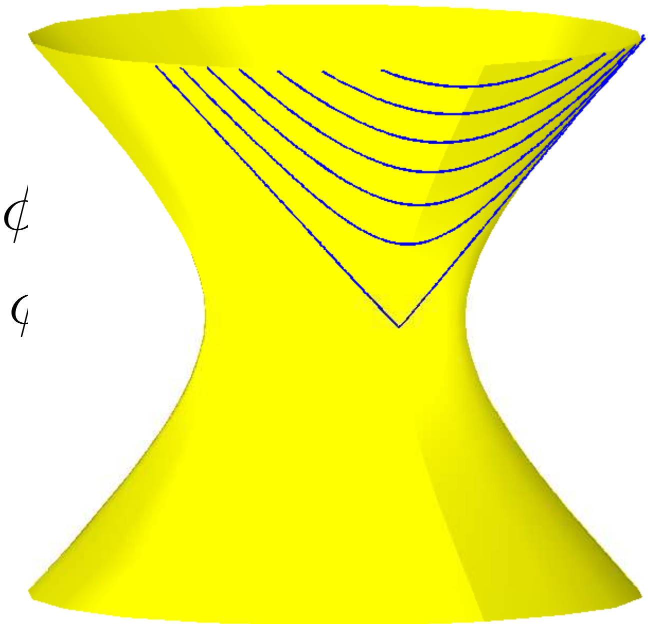

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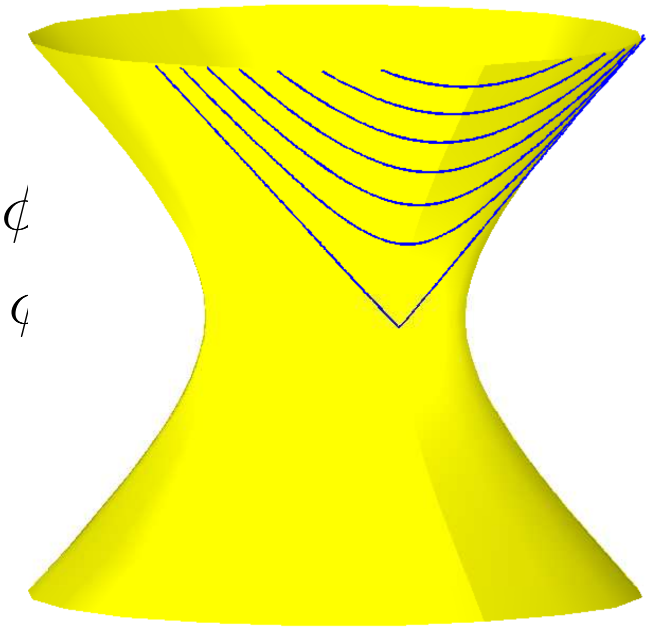

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Some remarkable properties of the Chaplygin gas

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- Admits a supersymmetric generalization (only known fluid)

Chaplygin gas from branes

J. Goldstone, M. Bordeman, J. Hoppe

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Example: string in 3-dimensional spacetime.

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Continuity equation

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Euler's equation provided $p = -\frac{1}{\rho}$

The multidimensional scenario

L.Randall, R. Sundrum, 1999

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Idea: a warped universe

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AdS geometry

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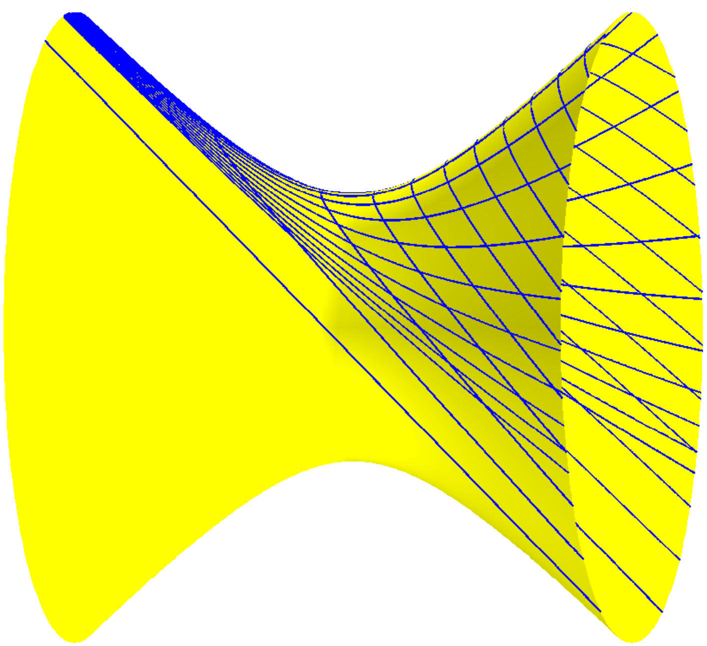

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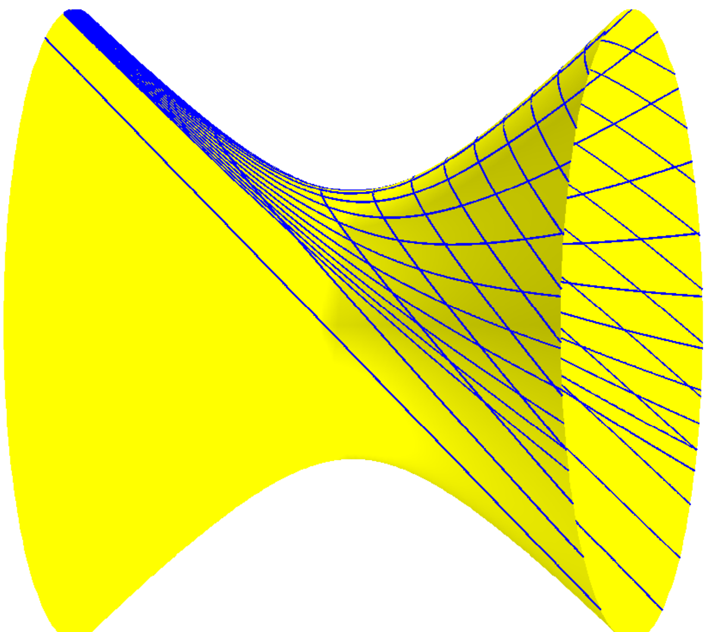

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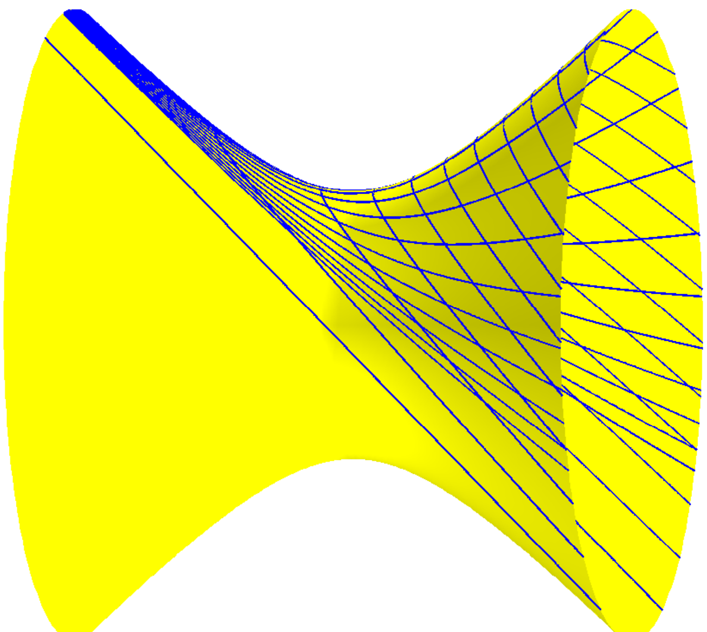

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- At $y = 0$: orbifold conditions

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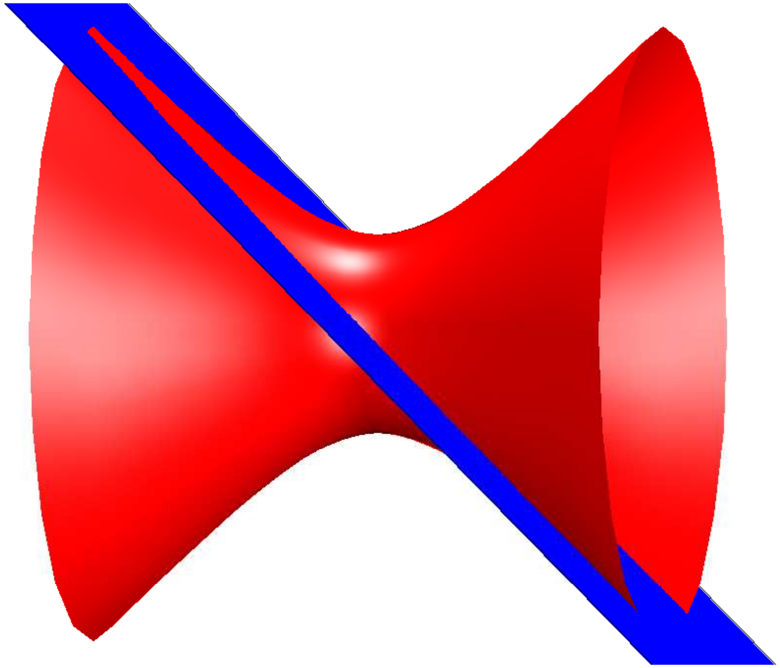

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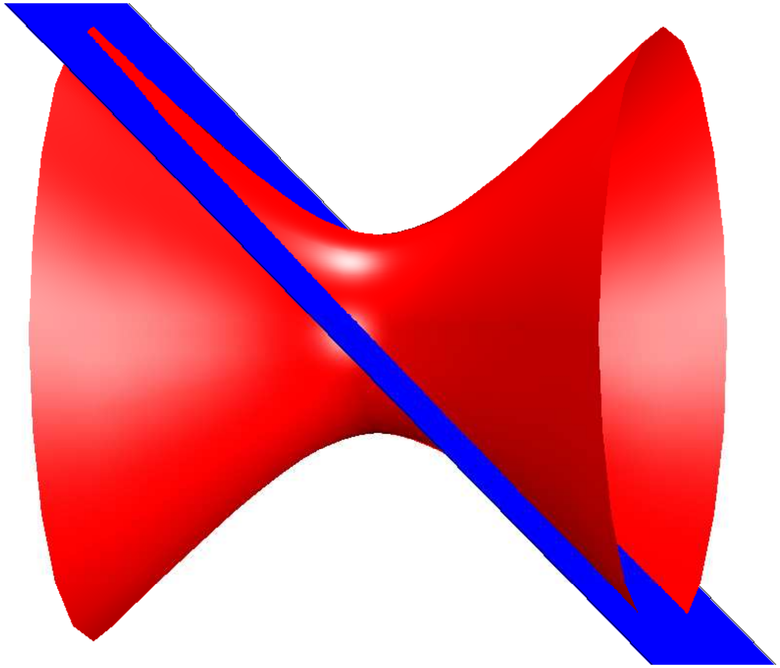

- Brane tension: $\lambda = \frac{6}{l}$

Brane in adS-BH background

A. Kamenshchik, U. M., V. Pasquier, Phys. Lett. B487 (2000)

Brane in adS-BH background

Another type of foliation

Brane in adS-BH background

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Tension not enough: need matter on the brane. A fluid?

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$$\text{For} \quad d = 1 \qquad p = -\frac{4}{l^2}\frac{1}{\rho}$$

Chaplygin gas from branes II

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$\theta(x^\mu)$ is a scalar field describing the embedding of the brane into the bulk.

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$$= \int d^4x \sqrt{-g}\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}(-f + \ldots)$$

$$T_{\mu\nu} = f\left(\frac{\theta_{,\mu}\theta_{,\nu}}{\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}} + g_{\mu\nu}\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}\right)$$

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Chaplygin's gas with $A = f^2$.

Chaplygin cosmology: theory and observations

FRW-Chaplygin Cosmology

A. Kamenshchik, U. M., V. Pasquier, Phys. Lett. B (2001)

FRW-Chaplygin Cosmology

$$a\dot{\rho} + 3\dot{a}(\rho + p) = 0$$

FRW-Chaplygin Cosmology

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B is an integration constant chosen positive

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$\Lambda = \sqrt{A}$ is the asymptotic cosmological constant

FRW-Chaplygin Cosmology

The subleading terms at large values of a give

$$\rho \approx \sqrt{A} + \sqrt{\frac{B}{4A}}\, a^{-6} \quad p \approx -\sqrt{A} + \sqrt{\frac{B}{4A}}\, a^{-6}$$

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$$\begin{matrix} \mathrm{dust-like} \\ \mathrm{matter} \end{matrix} \longrightarrow \begin{matrix} \mathrm{cosmological} \\ \mathrm{constant} \\ + \\ \mathrm{stiff\ matter} \end{matrix}$$

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Chaplygin's cosmic evolution:

$$\begin{array}{ccccc} & & \text{cosmological} & & \\ \text{dust}-\text{like} & & \text{constant} & & \text{deSitter} \\ & \rightarrow & & \rightarrow & \\ \text{matter} & & + & & \text{universe} \\ & & \text{stiff matter} & & \end{array}$$

Λ will increase

Now:

$$\begin{cases} p = p_{\Lambda_0} + p_M = -\frac{3}{4\pi G}\Lambda_0 \\ \rho = \rho_{\Lambda_0} + \rho_M = \frac{3}{4\pi G}\Lambda_0 + \rho_M \\ \frac{\rho_M}{\rho_{\Lambda_0}} \sim \frac{3}{7} \end{cases}$$

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Comparing these data with the Chaplygin gas model one has:

$$\Lambda_\infty \sim 1.2\Lambda_0$$

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Comological constant may be increasing

Time evolution

$$Friedmann\ eq.\quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho + \frac{\Lambda}{3} - \frac{K}{a^2}$$

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for the flat case K=0 it can be solved

Time evolution

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$$t = \frac{1}{6\ A^{\frac{1}{4}}}\left(\ln\frac{(1+\frac{B}{Aa^6})^{\frac{1}{4}}+1}{(1+\frac{B}{Aa^6})^{\frac{1}{4}}-1} - 2\arctan\left(1+\frac{B}{Aa^6}\right)^{\frac{1}{4}} + \pi\right)$$

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- When $B > \frac{2}{3\sqrt{3A}}$ $a(t)$ can take any value
- When $B < \frac{2}{3\sqrt{3A}}$ there are forbidden radii; either $a < a_1 = \frac{1}{\sqrt{3A}}\left(\sqrt{3}\sin\frac{\varphi}{3} - \cos\frac{\varphi}{3}\right)$ or $a > a_2 = \frac{2}{\sqrt{3A}}\cos\frac{\varphi}{3}$, $\varphi = \pi - \arccos 3\sqrt{3A}B/2$.

The inhomogeneous C. G.

Mixed component of the energy-momentum
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Energy conservation $T_{0\ ;\mu}^{\mu} = 0$ reads

$$\dot\rho = -\frac{1}{2}\frac{\dot\gamma}{\gamma}(\rho + p)$$

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Chaplygin gas $p = -\frac{A}{\rho}$

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$B(\vec{x})$ is an arbitrary function of the spatial coordinates

The "statefinder" diagnostic 1

V. Gorini, A. Kamenshchik, U. M. (2003)

The "statefinder" diagnostic 1

Proliferation of models explaining cosmic acceleration.
How to discriminate between them?

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Sahni, Saini, Starobinsky, Alam 2002

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$$r \equiv \frac{\dddot{a}}{aH^3}, \quad s \equiv \frac{r-1}{3(q-1/2)}$$

$$q \equiv -\frac{\ddot{a}}{aH^2} - \text{the deceleration parameter}$$

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Involve the *third* time derivative of the cosmological radius a

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$$v_s^2 = \frac{\partial p}{\partial \rho} = \frac{A}{\rho^2} = -\frac{p}{\rho} = 1 + s$$

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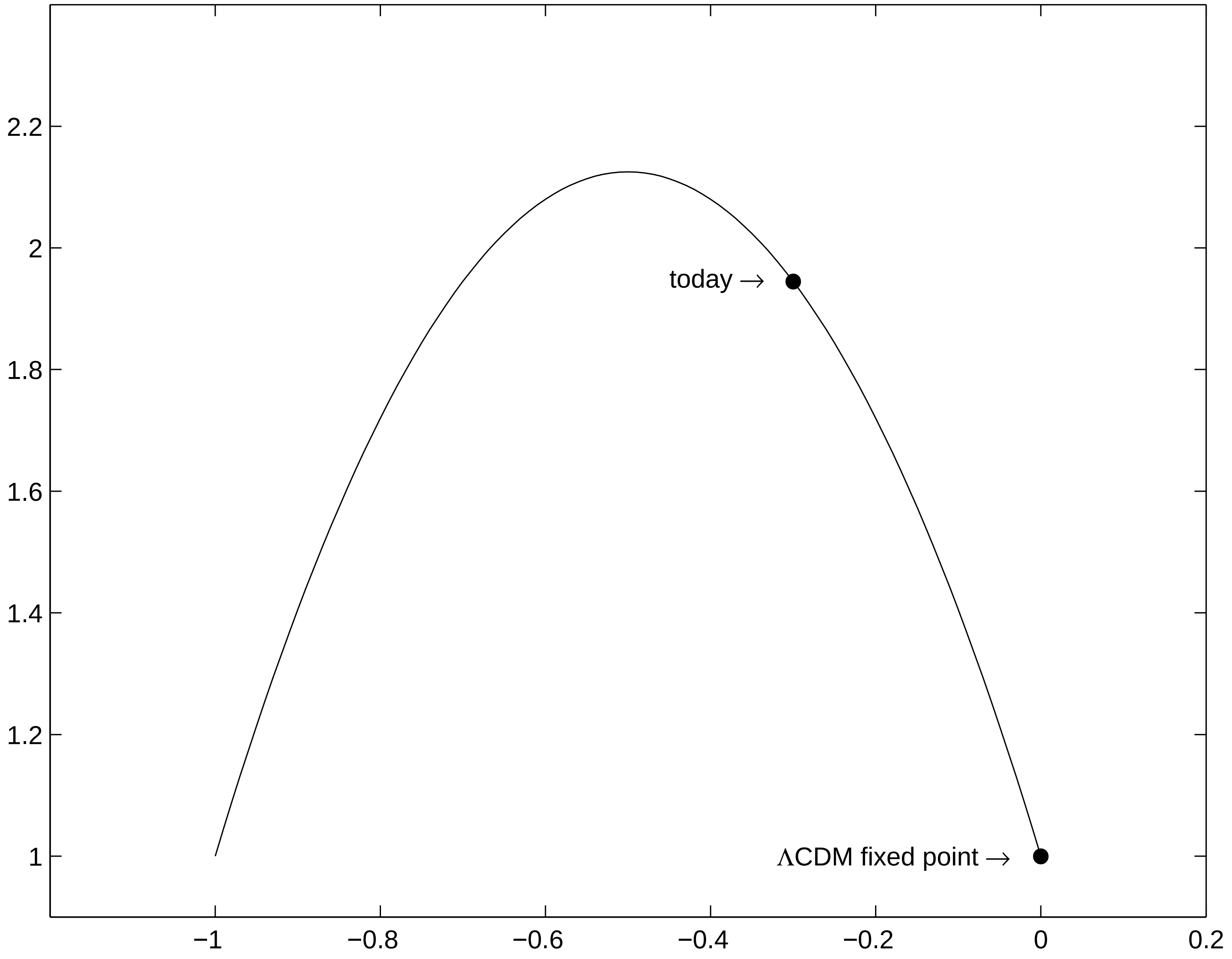
$$r = 1 - \frac{9}{2}\,s\,(1 + s)$$

The "statefinder" diagnostic 3



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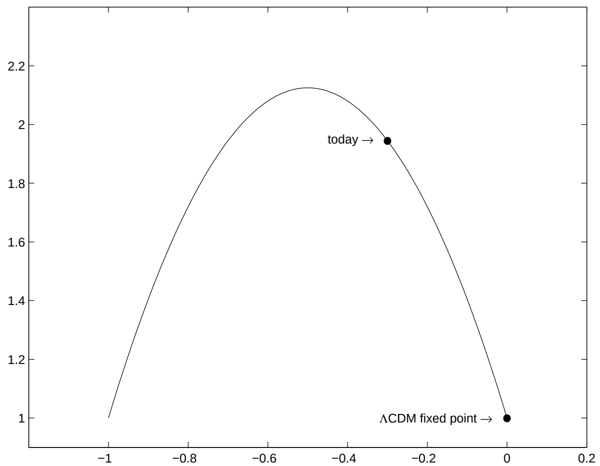

The Chaplygin gas statefinder s takes negative values (in contrast with quintessence).
For $q \approx -0.5$ the current values of the statefinder (within our model) are $s \approx -0.3, \quad r \approx 1.9$

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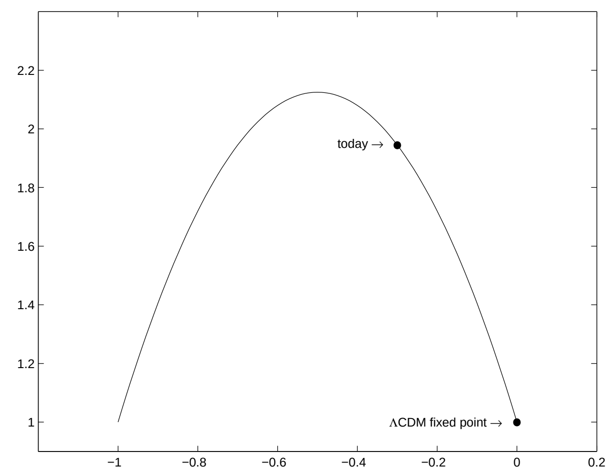

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Future experiments will discriminate the pure Chaplygin gas model from ΛCDM model

Chaplygin gas + Dust

$$r = 1 + \frac{9(\rho + p)}{2(\rho + \rho_d)} \frac{\partial p}{\partial \rho}, \qquad s = \frac{\rho + p}{p} \frac{\partial p}{\partial \rho}$$

Chaplygin gas + Dust

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If the second fluid is the Chaplygin gas

$$r = 1 - \frac{9}{2} \frac{s(s+1)}{1 + \frac{\rho_d}{\rho}}$$

Chaplygin gas + Dust

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Chaplygin gas + Dust

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Chaplygin gas + Dust

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If the second fluid is the Chaplygin gas

$$r = 1 - \frac{9}{2} \frac{s(s+1)}{1 + \frac{\rho_d}{\rho}}$$

$$\frac{\rho_d}{\rho} = \sqrt{-s}\kappa$$

Chaplygin gas + Dust

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$$\frac{\rho_d}{\rho} = \sqrt{-s}\kappa$$

$\kappa \equiv \frac{C}{\sqrt{B}}$ is the ratio between the energy densities of dust and of the Chaplygin gas at the beginning of the cosmological evolution

Chaplygin gas + Dust

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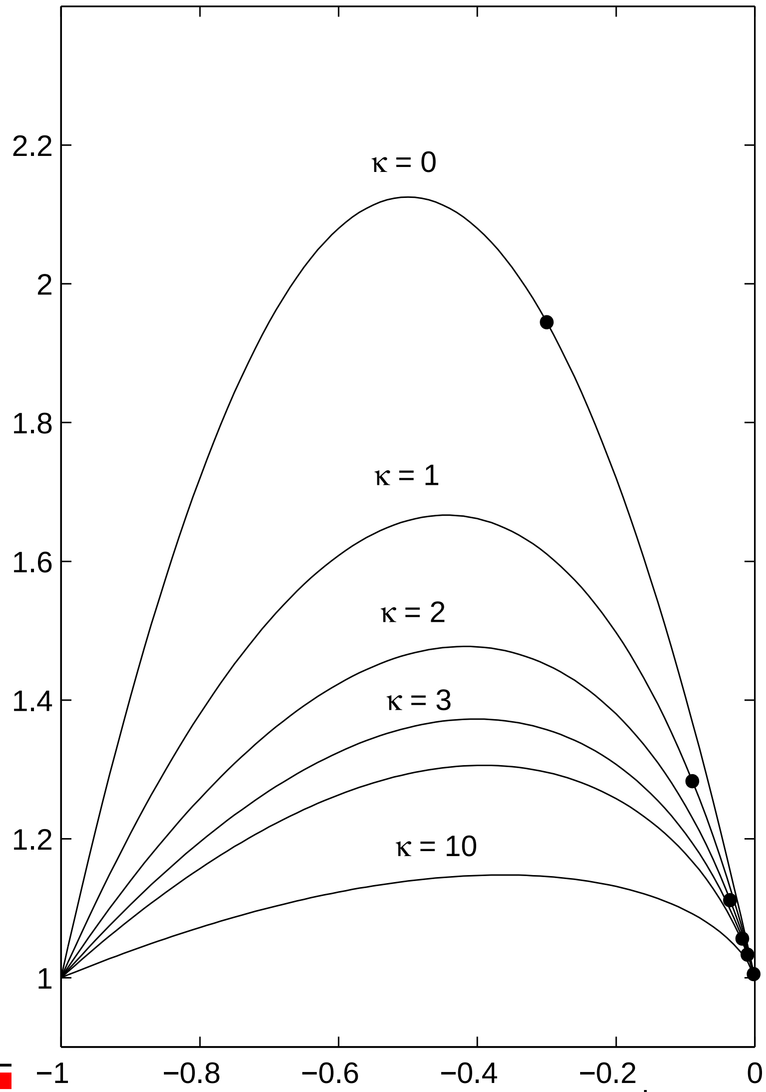
Possible solution of the cosmic coincidence conundrum. Here initial values of ρ_d and ρ can havs same order of magnitude.



The generalized Chaplygin gas

A. Kamenshick, U.M., V. Pasquier (2001)

$$p = -\frac{A}{\rho^{\alpha}} \qquad 0 \leq \alpha \leq 1$$

The generalized Chaplygin gas

$$p = -\frac{A}{\rho^{\alpha}} \qquad \rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}$$

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$$w_{\alpha} = -\left[1 + \frac{B}{A}\frac{1}{a^{3(\alpha+1)}}\right]^{-1},$$

The generalized Chaplygin gas

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$$w_\alpha = -\left[1 + \frac{B}{A}\frac{1}{a^{3(\alpha+1)}}\right]^{-1}, \qquad c_\alpha^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} = -\alpha w_\alpha.$$

The generalized Chaplygin gas

$$p = -\frac{A}{\rho^\alpha} \qquad \rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}$$

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Comes from a (rather artificial) Born-Infeld action:

$$L = -A^{\frac{1}{1+\alpha}}\left[1 - (g^{\mu\nu}\theta_{,\mu}\theta_{,\nu})^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}}$$

Bento, Bertolami, Sen 2002

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$$\begin{matrix} & & \mathrm{cosm.const.} & & \\ \mathrm{dust-like} & & + & & \mathrm{deSitter} \\ & \longrightarrow & & \longrightarrow & \\ \mathrm{matter} & & \mathrm{a\ perfectfluid} & & \mathrm{universe} \\ & & p = \alpha\rho & & \end{matrix}$$

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$$z_{\rm tr} = \left[\frac{2\bar{A}}{1-\bar{A}}\right]^{\frac{1}{3(\alpha+1)}} - 1 \simeq 0.4$$

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Redshift the transition to the accelerated phase

$$\bar{A} = \frac{(1+z_{\rm tr})^{3(1+\alpha)}}{2+(1+z_{\rm tr})^{3(1+\alpha)}} \approx \frac{(1.45)^{3(1+\alpha)}}{2+(1.45)^{3(1+\alpha)}}.$$

Comparison with observations

Observations seem to favor the generalised Chaplygin gas over other models

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TABLE 1
SUMMARY OF THE INFORMATION CRITERIA RESULTS

	χ^2/ dof	GoF (%)	ΔAIC	ΔBIC
Flat cosmo. const.	194.5 / 192	43.7	0	0
Flat Gen. Chaplygin	193.9 / 191	42.7	1	5
Cosmological const.	194.3 / 191	42.0	2	5
Flat constant w	194.5 / 191	41.7	2	5
Flat w(a)	193.8 / 190	41.0	3	10
Constant w	193.9 / 190	40.8	3	10
Gen. Chaplygin	193.9 / 190	40.7	3	10
Cardassian	194.1 / 190	40.4	4	10
DGP	207.4 / 191	19.8	15	18
Flat DGP	210.1 / 192	17.6	16	16
Chaplygin	220.4 / 191	7.1	28	30
Flat Chaplygin	301.0 / 192	0.0	30	30

NOTE. — From Davis et al. astro-ph:0701510
The flat cosmological constant (flat Λ) model is preferred by both the AIC and the BIC. The ΔAIC and ΔBIC values for all other models in the table are then measured with respect to these lowest values. The goodness of fit (GoF) approximates the probability of finding a worse fit to the data. The models are given in order of increasing ΔAIC.

Perturbations

The pure gCg has passed many tests of standard cosmology. However the behaviour of the gCg under perturbations is still problematic [*Tegmark et al.* , *Bean et al.*]

Perturbations

- Power spectrum of large scale structures seems to indicate [*Tegmark et al*] that the best fit value of α is very close to zero, rendering the gCg indistinguishable from ΛCDM (but see previous remarks on the ccp).

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- This feature seems to be common to all UDM models [*Tegmark et al.*] that might appear to be ruled out.

Gauge invariant perturbations

V. Gorini, A. Kamenshick, U.M., O. Piattella, A. Starobinsky work in progress

Gauge invariant perturbations

$$\begin{cases} -k^2\Phi - 3a\mathcal{H}^2\dot{\Phi} - 3\mathcal{H}^2\Phi = a^2\sum_{i=1}^{N}\rho_i\delta_i \\ \mathcal{H}\Phi + a\mathcal{H}\dot{\Phi} = a\sum_{i=1}^{N}\left(\rho_i + p_i\right)V_i \\ \left(a\mathcal{H}\right)^2\ddot{\Phi} + \left(4a\mathcal{H}^2 + a^2\mathcal{H}\dot{\mathcal{H}}\right)\dot{\Phi} + \left(2a\mathcal{H}\dot{\mathcal{H}} + \mathcal{H}^2\right)\Phi = \\ = a^2\sum_{i=1}^{N}c_{\mathrm{s}i}^2\rho_i\delta_i, \end{cases}$$

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$a(\eta)$ is the scale factor as a function of the conformal time η. Its present epoch value is normalized to unity

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$\mathcal{H}(\eta) = a'/a$, where the prime denotes derivation with respect to the conformal time

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Φ is the Bardeen gauge–invariant potentials (no shear)

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V is the gauge–invariant expression of the scalar potential of the velocity field

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ρ_i and p_i are the background energy and pressure of the component i.

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$\delta\rho_i$ and δp_i are the gauge–invariant expressions of the perturbations of the energy density and pressure

Gauge invariant perturbations

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$$\delta p_i = c_{\mathrm{s}i}^2\delta\rho_i, \quad c_{\mathrm{s}i}^2 \equiv \frac{\partial p_i}{\partial\rho_i}, \quad \delta_i \equiv \frac{\delta\rho_i}{\rho_i},$$

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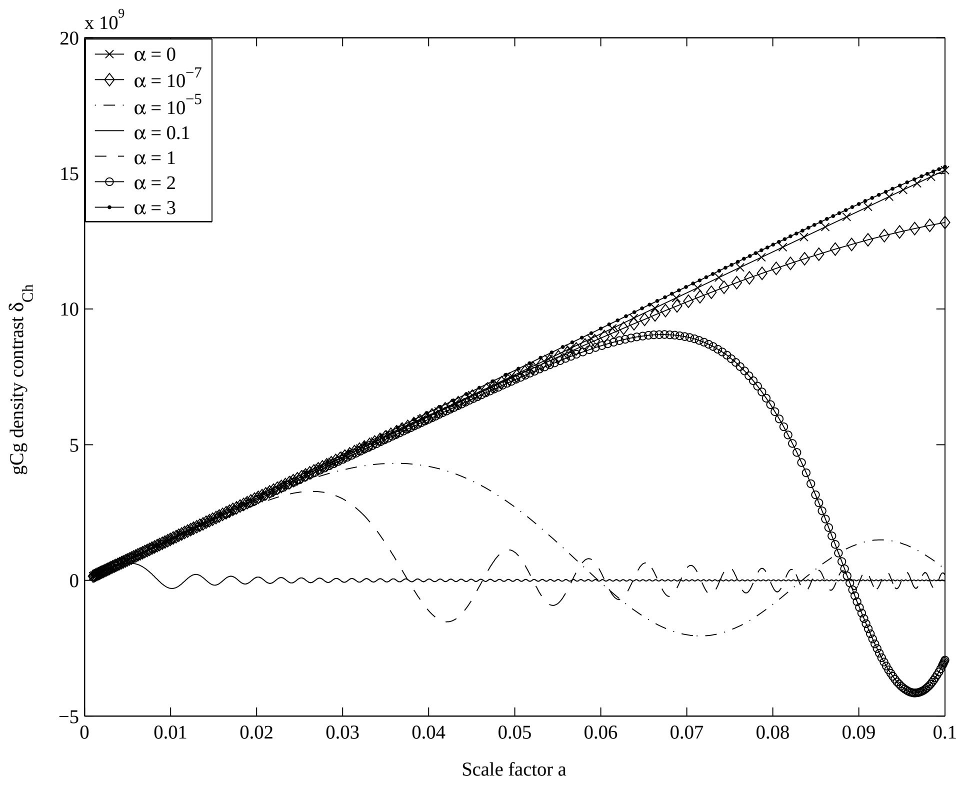
The spatial Fourier transform allows us to treat each mode independently in the linear approximation.

$N = 1$ - Density contrast

The system can be solved by eliminating δ and by extracting a second order equation for Φ:

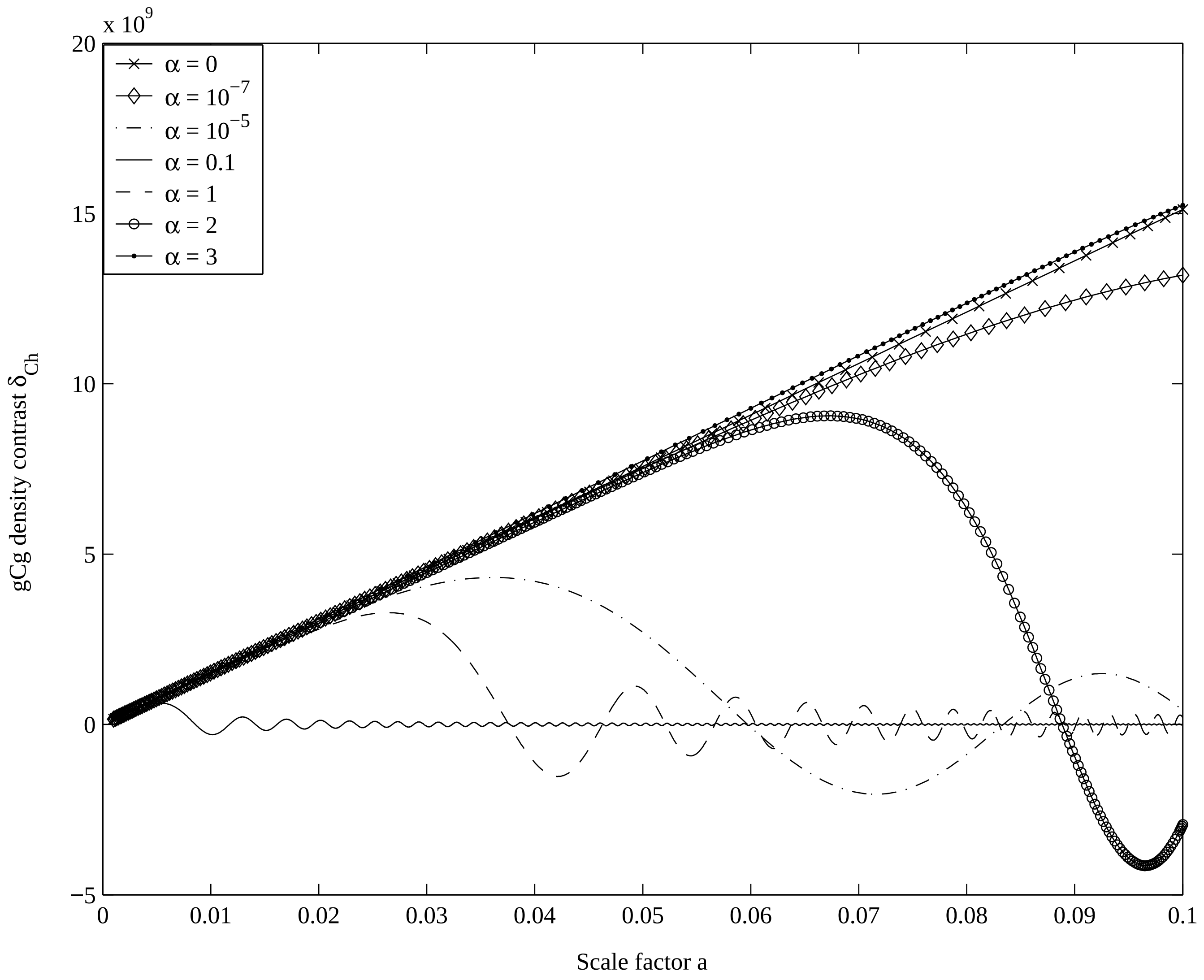
$$\ddot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}} + \frac{4 + 3c_{\mathrm{s}}^2}{a} \right) \dot{\Phi} + \left(\frac{2\dot{\mathcal{H}}}{a\mathcal{H}} + \frac{1 + 3c_{\mathrm{s}}^2}{a^2} + \frac{k^2 c_{\mathrm{s}}^2}{a^2 \mathcal{H}^2} \right) \Phi = 0.$$

$N = 1$ - Density contrast

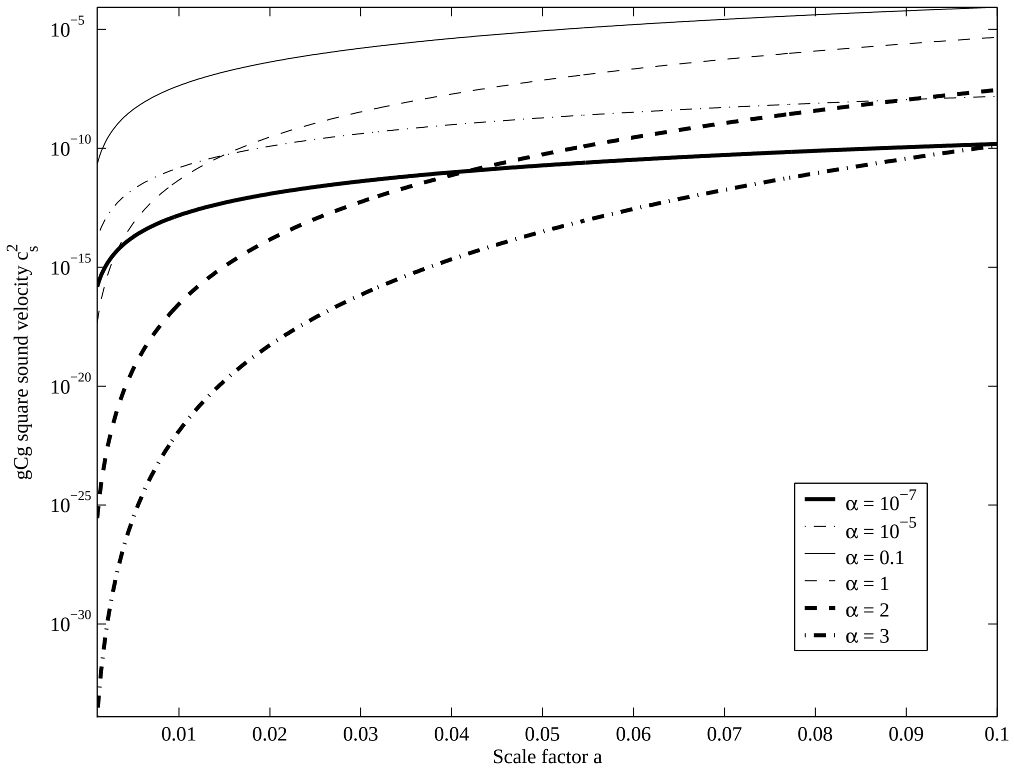


Evolution of the density contrast in the gCg $k = 100\ h\ Mpc^{-1}$ scale of a protogalaxy. Oscillations take place too early for $\alpha \gtrsim 10^{-5}$, thus preventing structure formation.

$N = 1$ - Density contrast

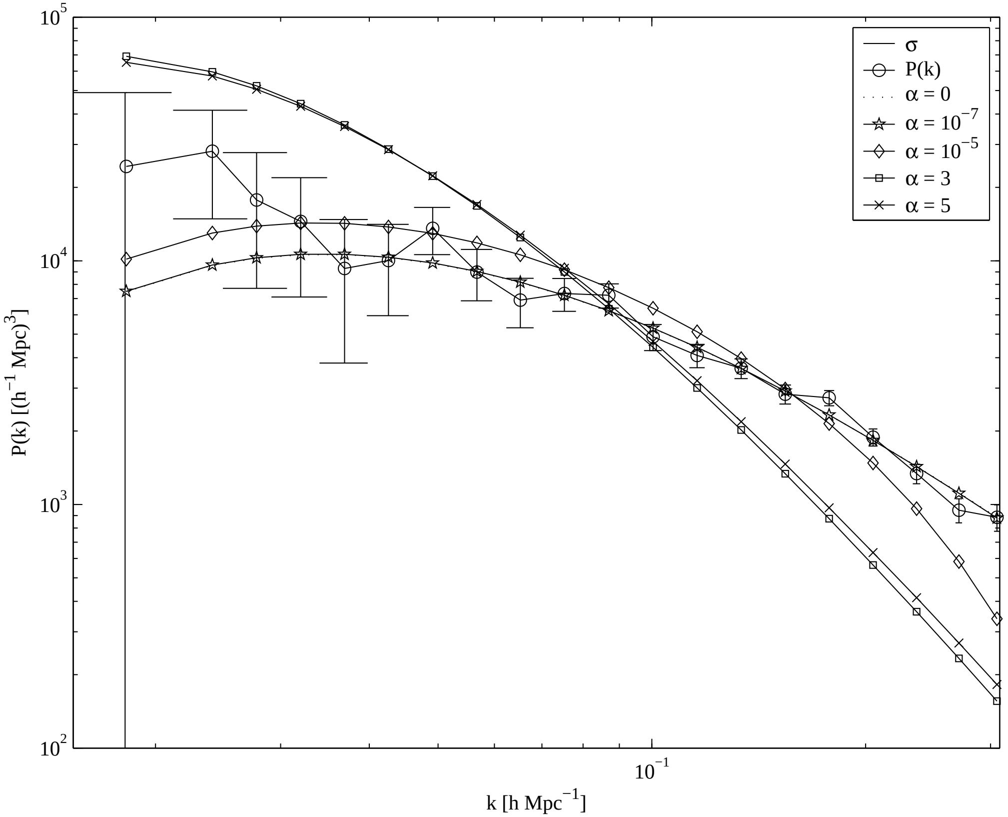


$N = 1$ - Sound velocity



For $\alpha \sim 0.1$ the sound velocity becomes non negligible much earlier than at other values of α. The range $10^{-7} \lesssim \alpha \lesssim 3$ appears thus to be ruled out since structure formation is prevented.

$N = 1$ - Power spectrum



The plots for $\alpha = 0$ and $\alpha = 10^{-7}$ are superposed At larger α the power spectrum tends to a limiting behaviour which is systematically below that of the ΛCDM one.

$N = 2$ - gCg + Baryons

In the matter dominated–regime we obtain the system

$$\begin{cases} \ddot{\delta}_{\rm b} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}} + \frac{2}{a}\right)\dot{\delta}_{\rm b} = \frac{1}{\mathcal{H}^2}\left(\rho_{\rm b}\delta_{\rm b} + \rho_{\rm Ch}\delta_{\rm Ch}\right) \\ \ddot{\delta}_{\rm Ch} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}} + \frac{2}{a}\right)\dot{\delta}_{\rm Ch} + \frac{k^2}{a^2\mathcal{H}^2}c_{\rm s}^2\dot{\delta}_{\rm Ch} = \frac{1}{\mathcal{H}^2}\left(\rho_{\rm b}\delta_{\rm b} + \rho_{\rm Ch}\delta_{\rm Ch}\right), \end{cases}$$

where $c_{\rm s}^2$ is the gCg square sound velocity.

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where $c_{\rm s}^2$ is the gCg square sound velocity.
Following Solov'eva and Starobinsky we introduce the variables

$$x = k\gamma^{-2}a^{-\frac{3}{2}\gamma}, \quad \gamma = -2\alpha - \frac{8}{3}$$

The system is reduced to a fourth-order equation

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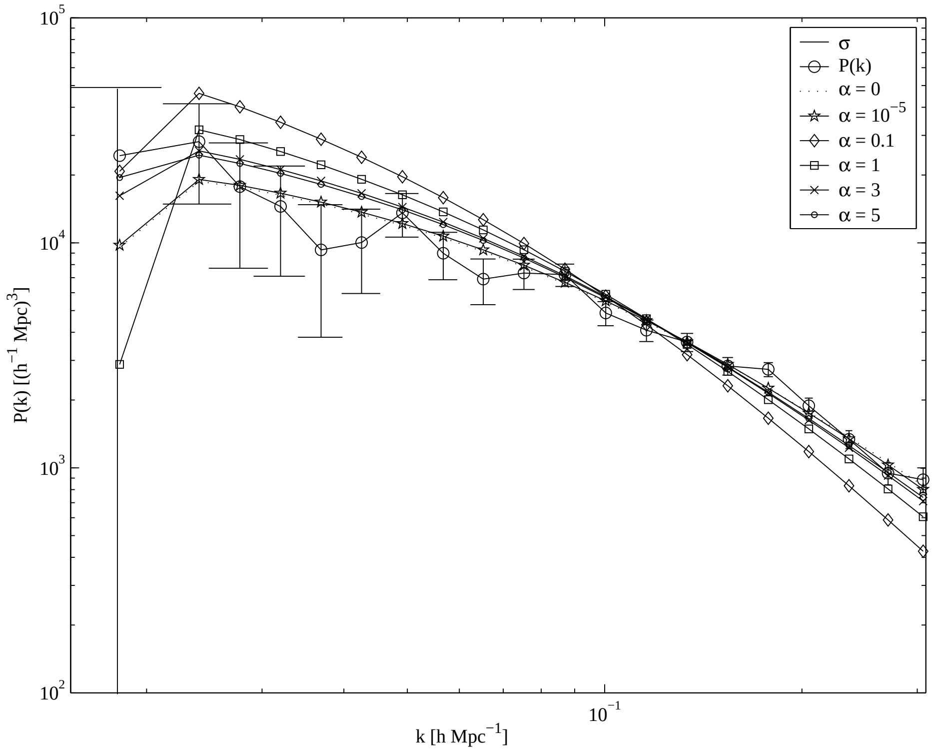
$$\left[\left(\Delta + \frac{2}{3\gamma}\right)\Delta\left(\Delta - \frac{1}{3\gamma}\right)\left(\Delta - \frac{1}{\gamma}\right) + x\left(\Delta^2 + \frac{2\gamma - 1/3}{\gamma}\Delta + \frac{1}{\gamma^2}\left(\gamma\left(\gamma - \frac{1}{3}\right) - \frac{2}{3}\Omega_{\rm b0}\right)\right)\right]\delta_{\rm Ch} = 0$$

$\Delta = x \cdot {\rm d}/{\rm d}x$.

$N = 2$ - Power spectrum



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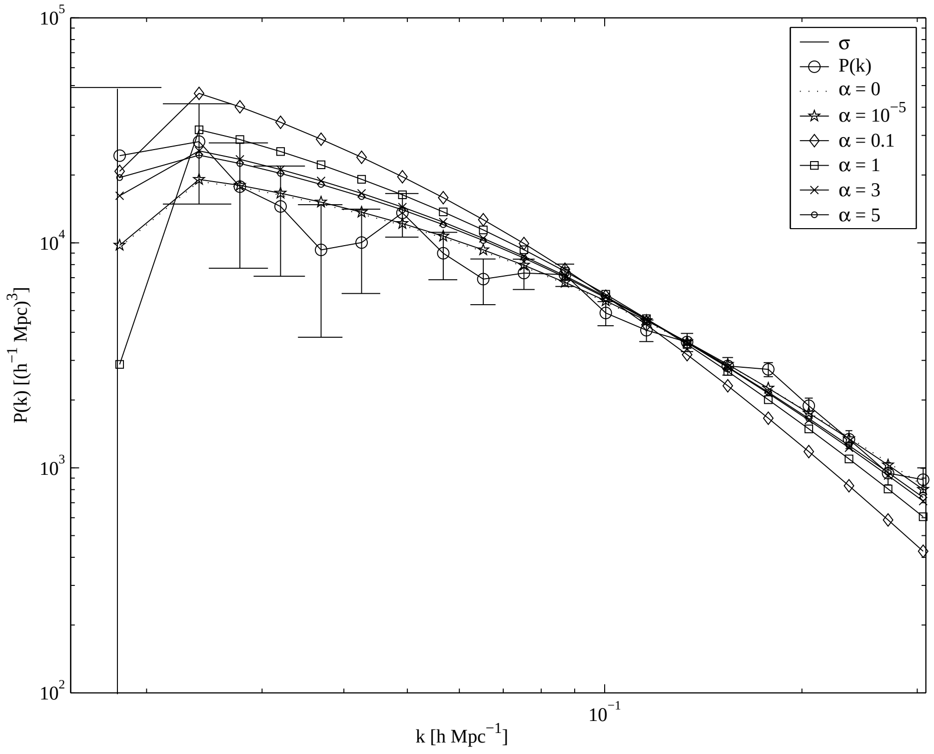

Power spectra of baryons. For all α good agreement with the observed spectrum. Better for small $\alpha = 0, 10^{-5}$ ($\simeq \Lambda$CDM) and for $\alpha = 3, 5$ (superluminal).

Conclusions

- The generalized Chaplygin gas cosmological model, with no additional fluid components, is compatible with structure formation and large scale structure only for α sufficiently small ($\alpha < 10^{-5}$), in which case it is indistinguishable from the ΛCDM model.

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- Adding to the generalized Chaplygin gas model a baryon component large scale structures are compatible with observations for all values of α. However very small values of α and $\alpha \gtrsim 3$ are favoured.
- A coincidence(?): the transition from the subluminal to the superluminal regime and the transition to the accelerated expansion of the universe may have the same redshift; $z_{\mathrm{sl}} = z_{\mathrm{tr}}$ for $\alpha \simeq 3$.

Tachyonic models

"Equivalent" scalar field

Want a homogeneous scalar field with same cosmic evolution as the Chaplygin gas

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$$L(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

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$$\frac{d\phi}{dt}$$

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"Equivalent" scalar field

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The cosmological evolution of the model with a scalar field with this potential coincides with that of the Chaplygin gas model provided the initial values $\phi(t_0)$ and $\dot{\phi}(t_0)$ satisfy the relation

$$\dot{\phi}^4(t_0) = 4(V^2(\phi(t_0)) - A)$$

Chapygin = "Free" Tachyons

Sen's action:

$$S = -\int d^4x \sqrt{-g} V(T) \sqrt{1 - g^{\mu\nu} T_{,\mu} T_{,\nu}}$$

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For a spatially homogeneous model

$$L = -V(T)\sqrt{1 - \dot{T}^2}$$

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Frolov, Kofman, Starobinsky 2002

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A more complicated example

A two-fluid cosmological model
V. Gorini, A. Kamenshchik, U. Moschella, V. Pasquier
hep-th/0311111 PRD 2004

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$$p_1 = -\rho_1 = -\Lambda \qquad p_2 = k\rho_2, \quad -1 < k < 0$$

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An exact field solution that gives $a(t)$:

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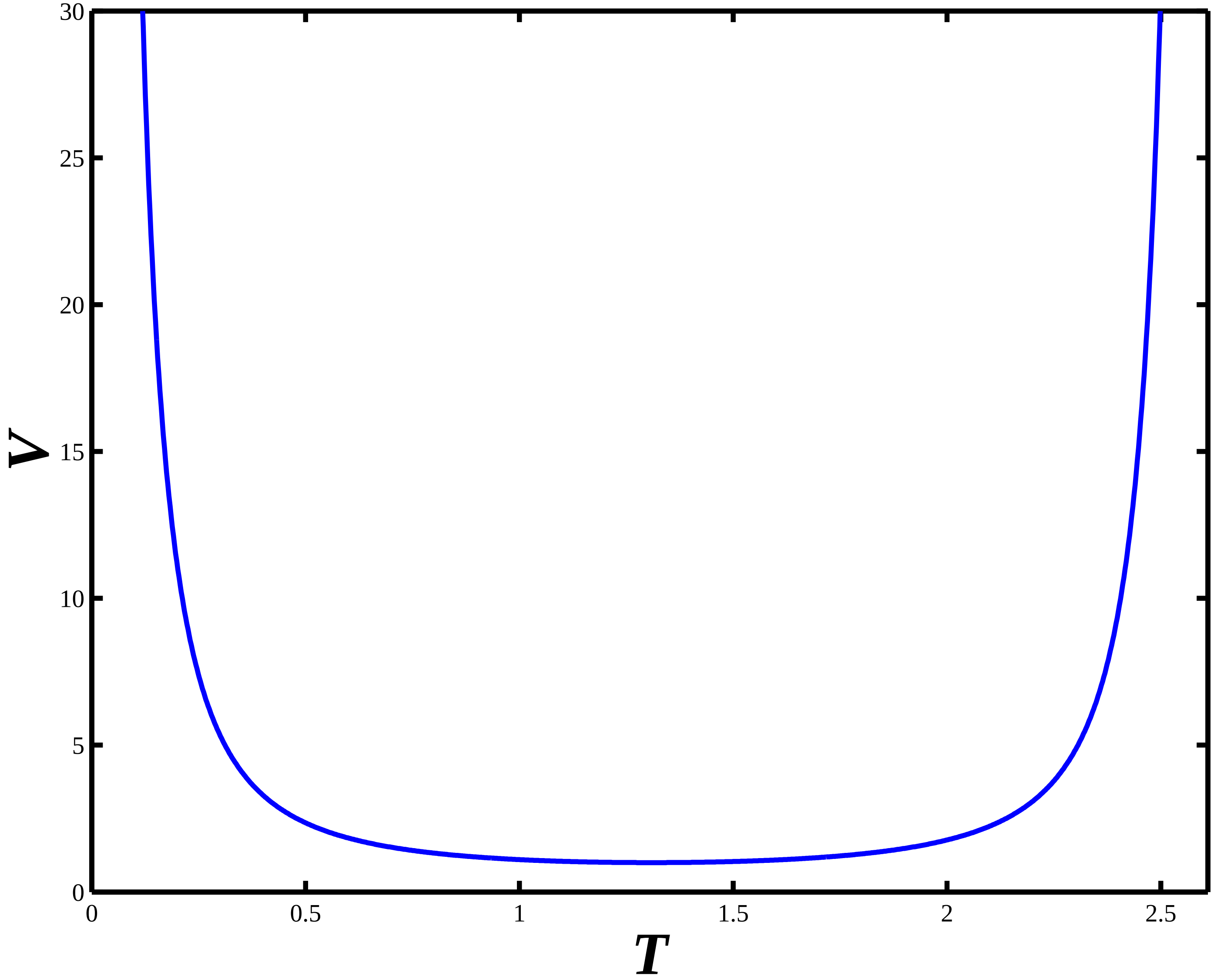
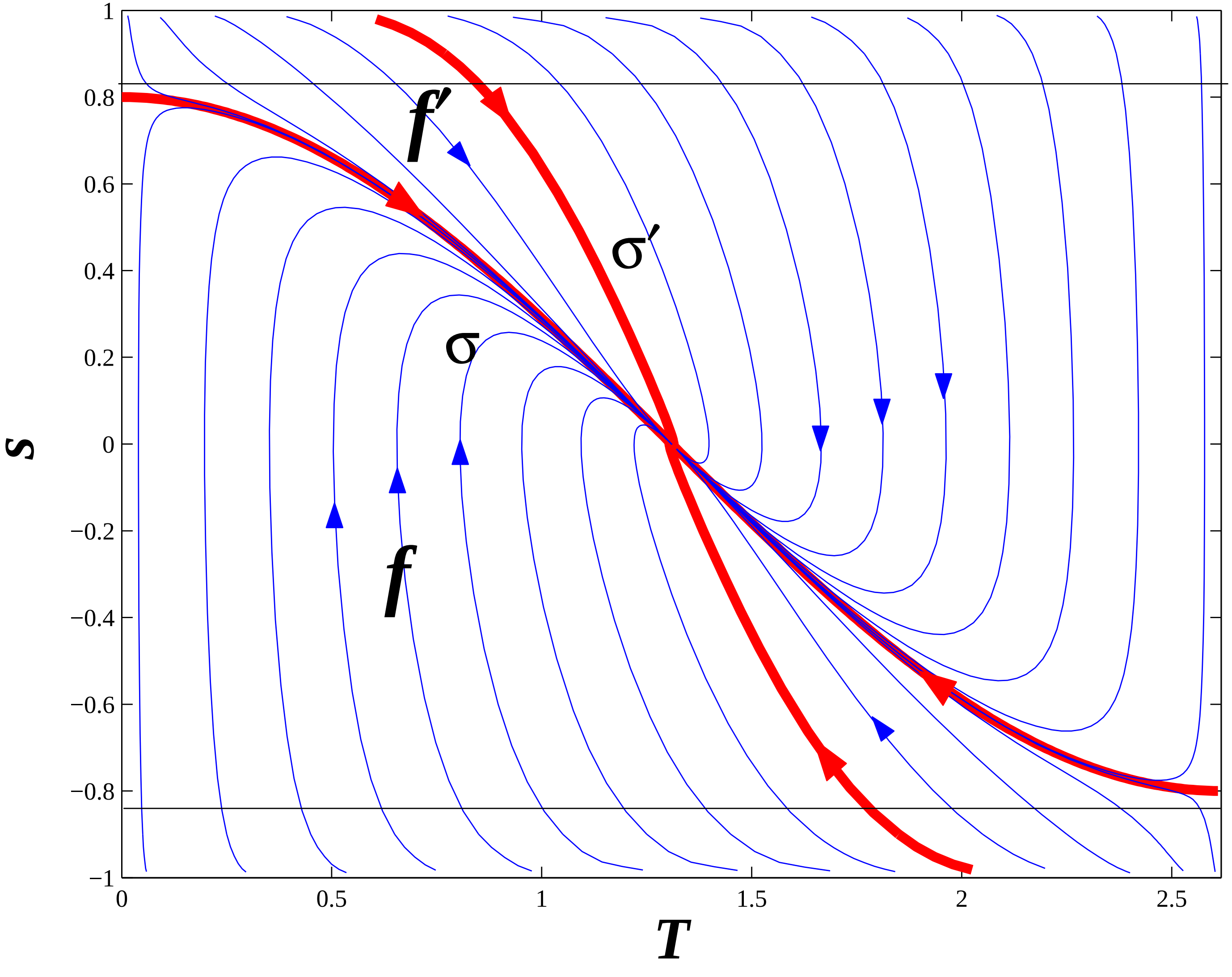
Tachyon Potential $k < 0$



Time evolution $k < 0$



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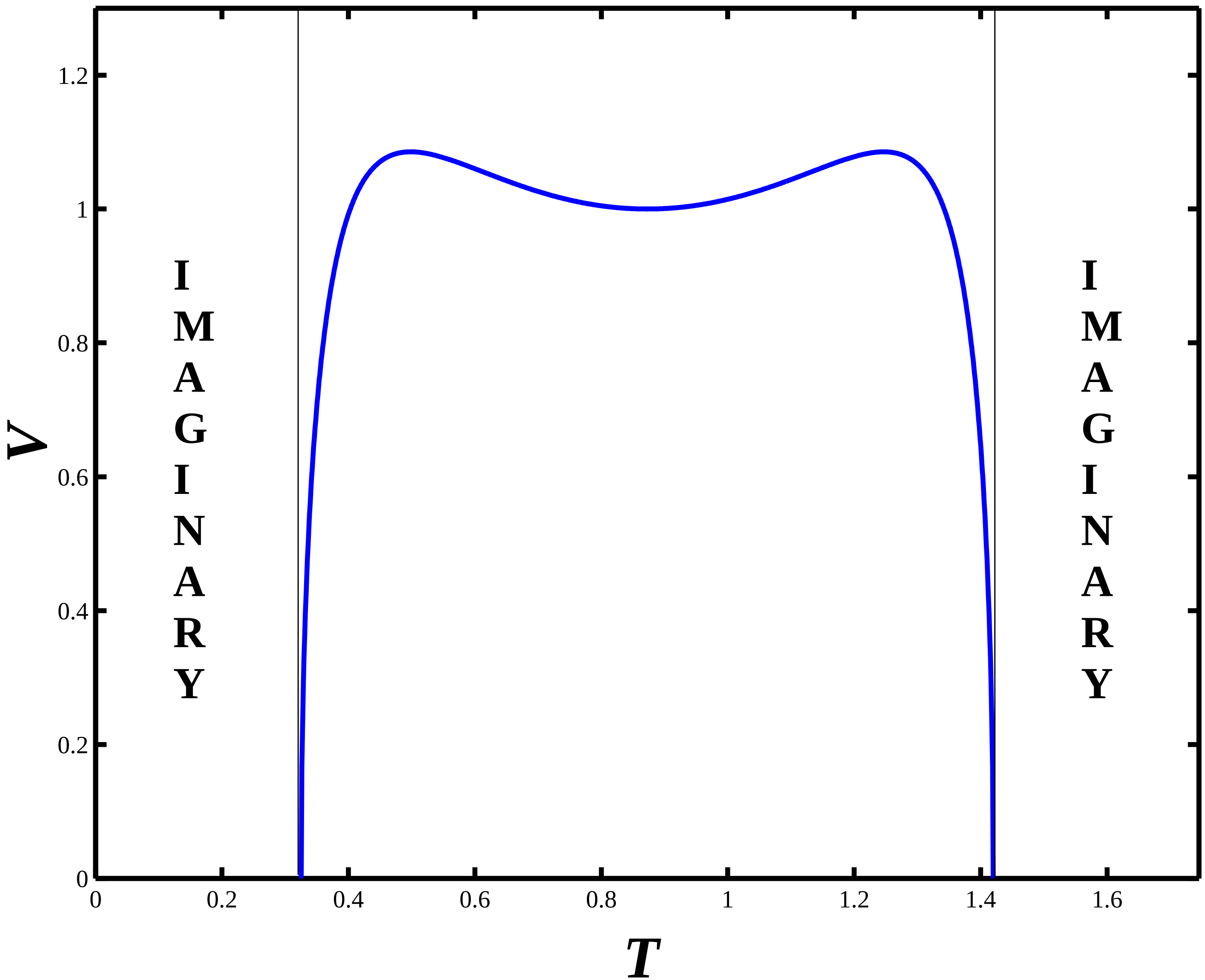

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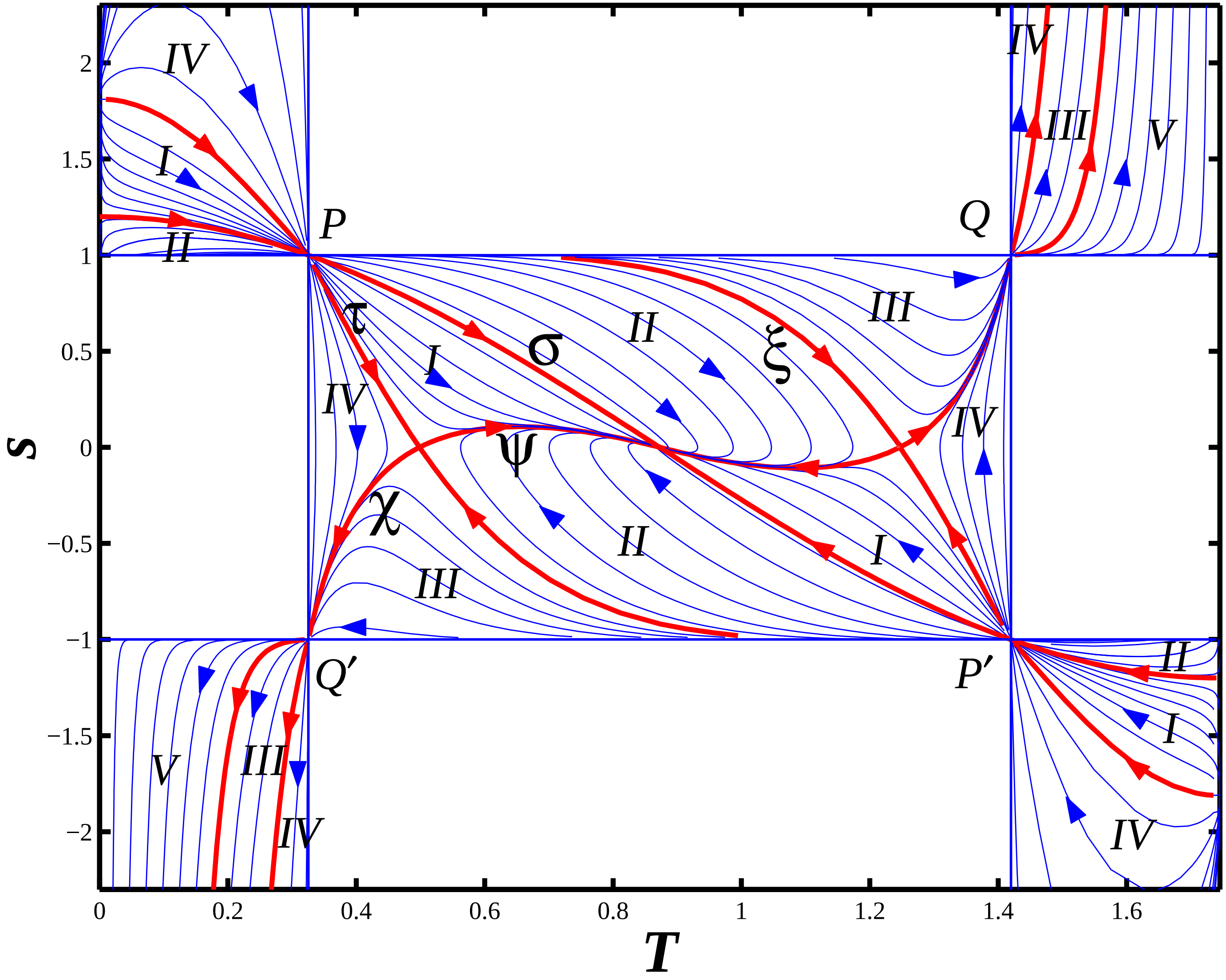

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- universes, hitting a cosmological singularity of a special type that we have called *Big Brake*

$$\ddot{a}(t_B) = -\infty,\ \dot{a}(t_B) = 0,$$

$$0 < a(t_B) < \infty$$

The future of the universe

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