The Chaplygin gas

*a model for dark energy*

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Plan of the talk

• Generalities
Plan of the talk

- Generalities
- Some remarkable properties of the Chaplygin gas
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- Generalities
- Some remarkable properties of the Chaplygin gas
- Chaplygin cosmology: theory and observations
Plan of the talk

- Generalities
- Some remarkable properties of the Chaplygin gas
- Chaplygin cosmology: theory and observations
- Tachyon cosmological models (depending on time)
Generalities
Dark energy: what do we know?

A new type of nonbaryonic dark "substance" totally unknown in laboratory experiments: new physics.
Dark energy: what do we know?

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The energy of the quantum vacuum?
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A modification of gravity?
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- Most of the universe energy is made of it.
- It does not cluster gravitationally.
- It has (effective?) negative pressure.

The energy of the quantum vacuum?

A modification of gravity?

The signature of extra-dimensions?
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$
General Relativistic Cosmology

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

Homogeneity and isotropy
\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]
General Relativistic Cosmology

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\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - pg_{\mu\nu} \]
General Relativistic Cosmology

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\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - pg_{\mu\nu} \]

\[
T_{\mu\nu} = \begin{pmatrix}
\rho & 0 & 0 & 0 \\
0 & p & 0 & 0 \\
0 & 0 & p & 0 \\
0 & 0 & 0 & p
\end{pmatrix}
\]
in comoving coordinates
Cosmological constant

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = 8\pi GT_{\mu\nu} \]
Cosmological constant

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]
Cosmological constant

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \]
Cosmological constant

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \tilde{T}_{\mu\nu} \]
Cosmological constant

\[ R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G \tilde{T}_{\mu\nu} \]

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \]
Cosmological constant

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \tilde{T}_{\mu\nu} \]

\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu} \quad p = -\rho \]
Cosmological constant

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \tilde{T}_{\mu\nu} \]

\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu} \]

\[ p = -\rho \]

\[ T_{\mu\nu} = \frac{1}{8\pi G} \begin{bmatrix} \Lambda & 0 & 0 & 0 \\ 0 & -\Lambda & 0 & 0 \\ 0 & 0 & -\Lambda & 0 \\ 0 & 0 & 0 & -\Lambda \end{bmatrix} \]

in comoving coordinates
Cosmology

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \]
Cosmology

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \]

Raychaudhuri eq.

\[ \frac{\ddot{a}}{a} = -\frac{4}{3} \pi G (\rho + 3p) + \frac{\Lambda}{3} \]
Cosmology

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \]

Raychaudhuri eq. \[ \frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p) + \frac{\Lambda}{3} \]

Friedmann eq. \[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3}\pi G\rho + \frac{\Lambda}{3} - \frac{K}{a^2} \]
\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \]

Raychaudhuri eq. \[ \frac{\ddot{a}}{a} = -\frac{4}{3} \pi G (\rho + 3p) + \frac{\Lambda}{3} \]

Friedmann eq. \[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi G \rho + \frac{\Lambda}{3} - \frac{K}{a^2} \]

Together imply energy conservation for each component
\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \]

Raychaudhuri eq. \[ \frac{\ddot{a}}{a} = -\frac{4}{3} \pi G (\rho + 3p) + \frac{\Lambda}{3} \]

Friedmann eq. \[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi G \rho + \frac{\Lambda}{3} - \frac{K}{a^2} \]

\[ \dot{\rho}_i = -3 \frac{\dot{a}}{a} (\rho_i + p_i) \]
Spherical universe
Spherical universe

\[ dl^2 = \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ = \frac{1}{K} \left( d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) \right) \]
Hyperbolic universe
Hyperbolic universe
Hyperbolic universe

\[ \begin{aligned}
  x_0 &= A \cosh \chi \\
  x_1 &= A \sinh \chi \sin \theta \sin \phi \\
  x_2 &= A \sinh \chi \sin \theta \cos \phi \\
  x_3 &= A \sinh \chi \cos \theta
\end{aligned} \]

\[ x_0^2 - x_1^2 - x_2^2 - x_3^2 = A^2 \]
Hyperbolic universe

\[
\begin{align*}
x_0 &= A \cosh \chi \\
x_1 &= A \sinh \chi \sin \theta \sin \phi \\
x_2 &= A \sinh \chi \sin \theta \cos \phi \\
x_3 &= A \sinh \chi \cos \theta \\
\end{align*}
\]

\[
x_0^2 - x_1^2 - x_2^2 - x_3^2 = A^2
\]

\[
dl^2 = \frac{1}{K} \left( d\chi^2 + \sinh^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) \right) \\
= \frac{dr^2}{1 + Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)
\]
Flat universe

\[ dl^2 = dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \]
Dust: $p=0$

$$a\dot{\rho} + 3\dot{a}(\rho + p) = 0$$
Dust: $p=0$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

\[ a\dot{\rho} + 3\dot{a}\rho = 0 \]
Dust: $p=0$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

\[ a\dot{\rho} + 3\dot{a}\rho = \frac{1}{a^2} \frac{d}{dt}(\rho a^3) = 0 \]
Dust: $p=0$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

\[ a\dot{\rho} + 3\dot{a}\rho = \frac{1}{a^2} \frac{d}{dt}(\rho a^3) = 0 \]

\[ \rho_{dust}(t)a^3(t) = const \]
Dust: $p=0$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

- \[ a\dot{\rho} + 3\dot{a}\rho = \frac{1}{a^2} \frac{d}{dt}(\rho a^3) = 0 \]
- \[ \rho_{dust}(t)a^3(t) = const \]
- \[ \rho_{dust}(t) = \frac{\rho_0a_0^3}{a^3(t)} \]
**Dust: p=0**

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

- \( a\dot{\rho} + 3\dot{a}\rho = \frac{1}{a^2} \frac{d}{dt}(\rho a^3) = 0 \)
- \( \rho_{dust}(t)a^3(t) = const \)
- \( \rho_{dust}(t) = \frac{\rho_0a_0^3}{a^3(t)} = \rho_0(1 + z)^3 \)
Dust: $p=0$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

\[ a\dot{\rho} + 3\dot{a}\rho = \frac{1}{a^2} \frac{d}{dt}(\rho a^3) = 0 \]

\[ \rho_{dust}(t)a^3(t) = \text{const} \]

\[ \rho_{dust}(t) = \frac{\rho_0 a_0^3}{a^3(t)} = \rho_0(1 + z)^3 \]

\[ t_0 = \text{now} \]
Radiation: $p = \frac{\rho}{3}$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]
Radiation: \( p = \frac{\rho}{3} \)

\[
a\dot{\rho} + 3\dot{a}(\rho + p) = 0
\]

\[
a\dot{\rho} + 4\dot{a}\rho = 0
\]
Radiation: \( p = \frac{\rho}{3} \)

\[
a \dot{\rho} + 3 \dot{a} (\rho + p) = 0
\]

\[
a \dot{\rho} + 4 \dot{a} \rho = \frac{1}{a^3} \frac{d}{dt} (\rho a^4) = 0
\]
Radiation: $p = \frac{\rho}{3}$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

\[ a\dot{\rho} + 4\dot{a}\rho = \frac{1}{a^3} \frac{d}{dt}(\rho a^4) = 0 \]

\[ \rho_{\text{radiation}}(t)a^4(t) = \text{const} \]
Radiation: \( p = \frac{\rho}{3} \)

\[ a \dot{\rho} + 3 \ddot{a} (\rho + p) = 0 \]

- \( a \dot{\rho} + 4 \ddot{a} \rho = \frac{1}{a^3} \frac{d}{dt}(\rho a^4) = 0 \)
- \( \rho_{\text{radiation}}(t) a^4(t) = \text{const} \)
- \( \rho_{\text{radiation}}(t) = \frac{\rho_0 a_0^4}{a^4(t)} \)
Radiation: \( p = \frac{\rho}{3} \)

\[
a\dot{\rho} + 3\dot{a}(\rho + p) = 0
\]

\[
a\dot{\rho} + 4\dot{a}\rho = \frac{1}{a^3} \frac{d}{dt}(\rho a^4) = 0
\]

\[
\rho_{\text{radiation}}(t)a^4(t) = \text{const}
\]

\[
\rho_{\text{radiation}}(t) = \frac{\rho_0 a_0^4}{a^4(t)} = \rho_0 (1 + z)^4
\]
Radiation: \( p = \frac{\rho}{3} \)

\[ a \dot{\rho} + 3 \dot{a} (\rho + p) = 0 \]

- \[ a \dot{\rho} + 4 \dot{a} \rho = \frac{1}{a^3} \frac{d}{dt} (\rho a^4) = 0 \]
- \( \rho_{\text{radiation}}(t) a^4(t) = \text{const} \)
- \( \rho_{\text{radiation}}(t) = \frac{\rho_0 a_0^4}{a^4(t)} = \rho_0 (1 + z)^4 \)
- \( t_0 = \text{now} \)
Cosmological constant: \( \rho = -\rho \)

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]
Cosmological constant: $\rho = -\rho$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

- $a\dot{\rho} = 0$
Cosmological constant: $\rho = -\dot{\rho}$

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]

- $a\dot{\rho} = 0$
- $\rho_\Lambda = const$
Cosmological constant: \( \rho = -\rho \)

\[
a\dot{\rho} + 3\dot{a}(\rho + p) = 0
\]

- \( a\dot{\rho} = 0 \)
- \( \rho_\Lambda = \text{const} \)
- for every time
The search for $\Omega$

A set of dimensionless parameters.

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi G (\rho_M + \rho_R + \rho_\Lambda) - \frac{K}{a^2}$$
The search for $\Omega$

A set of dimensionless parameters.

$$H^2 = \left( \frac{\ddot{a}}{a} \right)^2 = \frac{8}{3} \pi G (\rho_M + \rho_R + \rho_\Lambda) - \frac{K}{a^2}$$

$$= \frac{8}{3} \pi G \left( \frac{\rho_{M_0} a_0^3}{a^3(t)} + a_0 \frac{\rho_{R_0} a_0^4}{a^4(t)} + \rho_\Lambda + \frac{\rho_{K_0} a_0^2}{a^2(t)} \right)$$
The search for $\Omega$

A set of dimensionless parameters.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G (\rho_M + \rho_R + \rho_\Lambda) - \frac{K}{a^2}$$

$$= \frac{8}{3}\pi G \left(\frac{\rho_M a_0^3}{a^3(t)} + a_0 \frac{\rho_R a_0^4}{a^4(t)} + \rho_\Lambda + \frac{\rho_K a_0^2}{a^2(t)}\right)$$

$$\frac{H^2}{H_0^2} = \Omega_M (1 + z)^3 + \Omega_R (1 + z)^4 + \Omega_\Lambda + \Omega_K (1 + z)^2$$
The search for $\Omega$

A set of dimensionless parameters.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3} \pi G (\rho_M + \rho_R + \rho_\Lambda) - \frac{K}{a^2}$$

$$= \frac{8}{3} \pi G \left( \frac{\rho_{M0} a_0^3}{a^3(t)} + a_0 \frac{\rho_{R0} a_0^4}{a^4(t)} + \rho_\Lambda + \frac{\rho_{K0} a_0^2}{a^2(t)} \right)$$

$$\frac{H^2}{H_0^2} = \Omega_{M0} (1 + z)^3 + \Omega_{R0} (1 + z)^4 + \Omega_{\Lambda0} + \Omega_{K0} (1 + z)^2$$

$$\Omega_{i0} = \frac{8\pi G}{3H_0^2} \rho_{i0}$$
The search for $\Omega$

A set of dimensionless parameters.

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi G (\rho_M + \rho_R + \rho_\Lambda) - \frac{K}{a^2}$$

$$= \frac{8}{3} \pi G \left( \frac{\rho_{M0} a_0^3}{a^3(t)} + a_0 \frac{\rho_{R0} a_0^4}{a^4(t)} + \rho_\Lambda + \frac{\rho_{K0} a_0^2}{a^2(t)} \right)$$

$$\frac{H^2}{H_0^2} = \Omega_{M0} (1 + z)^3 + \Omega_{R0} (1 + z)^4 + \Omega_{\Lambda0} + \Omega_{K0} (1 + z)^2$$

$$\frac{H_0^2}{H^2} = \Omega_{M0} + \Omega_{R0} + \Omega_{\Lambda0} + \Omega_{K0} = 1$$
\[ \Omega_{i0} = \frac{8\pi G}{3H_0^2} \rho_{i0} \]
\[ \Omega_{i0} = \frac{8\pi G}{3H_0^2}\rho_{i0} \]

\[ \Omega_{\Lambda 0} = \text{Dark Energy} \approx 0.7 \]
\[ \Omega_{i0} = \frac{8\pi G}{3H_0^2}\rho_{i0} \]

\[ \Omega_{\Lambda 0} = \text{Dark Energy} \approx 0.7 \]

\[ \Omega_{M 0} = \text{baryons and NBDM} \approx 0.25 \]

\[ = \Omega_{B 0} + \Omega_{DM 0} \approx 0.05 + 0.20 \]
\[ \Omega_{i0} = \frac{8 \pi G}{3 H_0^2} \rho_{i0} \]

\[ \Omega_{\Lambda 0} = \text{Dark Energy} \quad \simeq 0.7 \]

\[ \Omega_{M 0} = \text{baryons and NBDM} \quad \simeq 0.25 \]
\[ = \Omega_{B0} + \Omega_{DM0} \quad \simeq 0.05 + 0.20 \]

\[ \Omega_{R0} = \text{CMB} \quad \simeq 0.0001 \]
\[ \Omega_{i0} = \frac{8\pi G}{3H_0^2\rho_{i0}} \]

\[ \Omega_{\Lambda 0} = \text{Dark Energy} \approx 0.7 \]

\[ \Omega_{M0} = \text{baryons and NBDM} \approx 0.25 \]
\[ = \Omega_{B0} + \Omega_{DM0} \approx 0.05 + 0.20 \]

\[ \Omega_{R0} = \text{CMB} \approx 0.0001 \]

\[ \Omega_{K0} = \text{Space Curvature} \approx 0 \]
The fate of the $\Lambda$CDM universe

$$H^2 = \frac{8}{3} \pi G \left( \frac{\rho_{M_0}}{a^3(t)} + \frac{\rho_{R_0}}{a^4(t)} + \rho_\Lambda + \frac{\rho_{K_0}}{a^2(t)} \right)$$
The fate of the $\Lambda$CDM universe

\[ H^2 = \frac{8}{3} \pi G \left( \frac{\rho_{M_0}}{a^3(t)} + \frac{\rho_{R_0}}{a^4(t)} + \rho_\Lambda + \frac{\rho_{K_0}}{a^2(t)} \right) \]

\[ H^2 \to \frac{8}{3} \pi G \rho_\Lambda \]
Only $\Lambda > 0$

$$\ddot{a} = \frac{1}{3} \Lambda a$$
Only $\Lambda > 0$

\[
\ddot{a} = \frac{1}{3} \Lambda a \quad \dot{a}^2 = \frac{1}{3} \Lambda a^2 - K
\]
Only $\Lambda > 0$

$$\ddot{a} = \frac{1}{3} \Lambda a \quad \dot{a}^2 = \frac{1}{3} \Lambda a^2 - K$$

$$K = -1 \quad a(t) = \sqrt{\frac{3}{\Lambda}} \sinh \sqrt{\frac{\Lambda}{3}} t \quad \text{hyperbolic}$$
Only $\Lambda > 0$

\[
\ddot{a} = \frac{1}{3} \Lambda a \quad \dot{a}^2 = \frac{1}{3} \Lambda a^2 - K
\]

$K = -1 \quad a(t) = \sqrt{\frac{3}{\Lambda}} \sinh \sqrt{\frac{\Lambda}{3}} t \quad \text{hyperbolic}$

$K = 0 \quad a(t) = \exp \sqrt{\frac{\Lambda}{3}} t \quad \text{flat}$
Only $\Lambda > 0$

\[ \ddot{a} = \frac{1}{3} \Lambda a \quad \dot{a}^2 = \frac{1}{3} \Lambda a^2 - K \]

\[ K = -1 \quad a(t) = \sqrt{\frac{3}{\Lambda}} \sinh \sqrt{\frac{\Lambda}{3}} t \quad \text{hyperbolic} \]

\[ K = 0 \quad a(t) = \exp \sqrt{\frac{\Lambda}{3}} t \quad \text{flat} \]

\[ K = 1 \quad a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}} t \quad \text{spherical} \]
de Sitter universe

\[ M^{d+1} \quad \eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1) \]
de Sitter universe

\[ \mathbb{M}^{d+1} \quad \eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1) \]

\[ x_0^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = -R^2 \]
de Sitter universe

\[ \mathbb{M}^{d+1} \quad \eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1) \]

\[ x_0^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = -R^2 \]
de Sitter universe

\[ M^{d+1} \quad \eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1) \]

\[ x_0^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = -R^2 \]
Spherical de Sitter

$$\begin{align*}
X_0 &= R \sinh \frac{t}{R} \\
X_i &= R \cosh \frac{t}{R} \omega_i
\end{align*}$$
Spherical de Sitter

\[
\begin{aligned}
X_0 &= R \sinh \frac{t}{R} \\
X_i &= R \cosh \frac{t}{R} \omega_i \\
|\vec{\omega}|^2 &= 1
\end{aligned}
\]
Spherical de Sitter

\[
\begin{align*}
X_0 &= R \sinh \frac{t}{R} \\
X_i &= R \cosh \frac{t}{R} \omega_i \\
|\vec{\omega}|^2 &= 1
\end{align*}
\]

\[R = \sqrt{\frac{3}{\Lambda}}\]
Spherical de Sitter

\[
\begin{aligned}
X_0 &= R \sinh \frac{t}{R} \\
X_i &= R \cosh \frac{t}{R} \omega_i \\
|\vec{\omega}|^2 &= 1
\end{aligned}
\]

\[R = \sqrt{\frac{3}{\Lambda}}\]

\[
ds^2 = dX_0^2 - dX_1^2 - \ldots dX_4^2 =
\]

\[= dt^2 - R^2 \cosh^2 \frac{t}{R} \left( d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) \right)\]
Flat de Sitter
Flat de Sitter

\[ X_0 + X_d = R \exp \frac{t}{R} \]
Flat de Sitter

\[ X_0 + X_d = R \exp \frac{t}{R} \]
\[
\begin{align*}
X_0 &= R \sinh \frac{t}{R} + \frac{1}{2R} e^{\frac{t}{R}} |\vec{x}|^2 \\
X_i &= \exp \left( \frac{t}{R} \right) x_i \\
X_d &= R \cosh \frac{t}{R} - \frac{1}{2R} e^{\frac{t}{R}} |\vec{x}|^2
\end{align*}
\]
\[ X_0 + X_d = R \exp \frac{t}{R} \]

\[ \begin{aligned}
X_0 &= R \sinh \frac{t}{R} + \frac{1}{2R} e^{\frac{t}{R}} |x|^2 \\
X_i &= \exp \left( \frac{t}{R} \right) x_i \\
X_d &= R \cosh \frac{t}{R} - \frac{1}{2R} e^{\frac{t}{R}} |x|^2
\end{aligned} \]

\[ ds^2 = dX_0^2 - dX_1^2 - \ldots - dX_4^2 = \]

\[ = dt^2 - \exp \frac{2t}{R} \left( dx_1^2 + dx_2^2 + dx_3^2 \right) \]
Open de Sitter

\[ X_0 = R \sinh \frac{t}{R} \cosh \chi \]
\[ X_1 = R \sinh \frac{t}{R} \sinh \chi \sin \theta \sin \phi \]
\[ X_2 = R \sinh \frac{t}{R} \sinh \chi \sin \theta \cos \phi \]
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X_4 &= R \cosh \frac{t}{R}
\end{align*}
\]

\[
ds^2 = dX_0^2 - dX_1^2 - \ldots dX_4^2 =
\]

\[
= dt^2 - R^2 \sinh^2 \frac{t}{R} \left(d\chi^2 + \sinh^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2)\right)
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\[
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\[
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Candidates for dark energy

- The cosmological constant $\Lambda \rightarrow p = -\rho$
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- A perfect fluid $\rightarrow p = w\rho$; $-1 < w < 0$;
  $w = -1/3$ cosmic strings, $w = -2/3$ domain walls.
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  Quintessence scalar field: $= \frac{1}{2}(\nabla_\mu \Phi)^2 - V(\Phi)$
  “$k$ - essence” field (tachyons?) $L \sim K(\nabla_\mu \Phi)$
  Braneworld models. DGP model
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- UDM-quartessence
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- The Chaplygin gas
Some remarkable properties of the Chaplygin gas
The Chaplygin gas

\[ p = -\frac{A}{\rho} \quad (Chaplygin, \ 1904) \]
The Chaplygin gas

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- Integrable - large symmetry group
- Obtainable from Nambu-Goto action for \( d \) - branes moving in a \((d + 2)\) - dimensional spacetime
- Admits a supersymmetric generalization (only known fluid)
Chaplygin gas from branes

Example: string in 3-dimensional spacetime.
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\[ H = \frac{1}{2} \int \left[ \Pi^2 + (\partial_\sigma x)^2 \right] d\sigma \]
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Hamilton equations: \( \partial_\tau x = \Pi, \)
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Hamilton equations: \( \partial_\tau x = \Pi, \quad \partial_\tau^2 x - \partial_\sigma^2 x = 0 \)
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Example: string in 3-dimensional spacetime. Light cone gauge Hamiltonian density:

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Hamilton equations:
\[ \partial_\tau x = \Pi, \quad \partial^2_\tau x - \partial^2_\sigma x = 0 \]

- Introduce the density \( \rho(x) = (\partial_\sigma x)^{-1} \)
- Introduce the velocity \( v = \Pi \)
Hodograph transformation

\[ \partial_\tau x = v = \Pi, \quad \partial_\sigma x = \frac{1}{\rho}, \quad \partial_\tau^2 x - \partial_\sigma^2 x = 0 \]
Hodograph transformation:

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Hodograph transformation:

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Hodograph transformation:

$$\tau, \sigma \rightarrow t, x$$
Hodograph transformation

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Hodograph transformation:

\[ \tau, \sigma \rightarrow t, x \]

\[
\begin{align*}
   t &= \tau \\
   x &= x(\sigma, \tau)
\end{align*}
\]
Hodograph transformation

\[ \partial_\tau x = v = \Pi, \quad \partial_\sigma x = \frac{1}{\rho}, \quad \partial_\tau^2 x - \partial_\sigma^2 x = 0 \]

Hodograph transformation:

\[ \tau, \sigma \rightarrow t, x \]

\[ \begin{align*}
  dt &= d\tau \\
  dx &= \partial_\tau x \, d\tau + \partial_\sigma x \, d\sigma
\end{align*} \]
Hodograph transformation:

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Hodograph transformation:

\[ \tau, \sigma \rightarrow t, x \]

\[ \begin{cases} 
    dt = d\tau \\
    dx = v \, d\tau + \frac{1}{\rho} \, d\sigma 
\end{cases} \]
Hodograph transformation

\[ \partial_\tau x = v = \Pi, \quad \partial_\sigma x = \frac{1}{\rho}, \quad \partial_x^2 x - \partial_\tau^2 x = 0 \]

Hodograph transformation:

\[ \tau, \sigma \rightarrow t, x \]

\[ \begin{align*}
  dt &= d\tau \\
  dx &= v \, d\tau + \frac{1}{\rho} \, d\sigma
\end{align*} \quad \rightarrow \quad \begin{align*}
  \partial_\tau &= \partial_t + v \partial_x \\
  \partial_\sigma &= \frac{1}{\rho} \partial_x
\end{align*} \]
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First Hamilton’s equation: \( \partial_\tau x = v \)
Hodograph transformation

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First Hamilton’s equation: \[ \partial_\tau x = v \quad \rightarrow \quad \partial_\sigma \partial_\tau x = \partial_\sigma v \]
Hodograph transformation

\[ \partial_\tau x = v = \Pi, \quad \partial_\sigma x = \frac{1}{\rho}, \quad \partial_\tau^2 x - \partial_\sigma^2 x = 0 \]

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First Hamilton’s equation: \( \partial_\tau x = v \quad \rightarrow \quad \partial_\sigma \partial_\tau x = \partial_\sigma v \)

\[ \rightarrow \quad (\partial_t + v \partial_x) \frac{1}{\rho} = \frac{1}{\rho} \partial_x v \]
Hodograph transformation:

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Hodograph transformation:

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\begin{aligned}
& dt = d\tau \\
& dx = v \, d\tau + \frac{1}{\rho} \, d\sigma
\end{aligned}
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\end{aligned}
\]

First Hamilton’s equation: \( \partial_\tau x = v \rightarrow \partial_\sigma \partial_\tau x = \partial_\sigma v \)

\[
\rightarrow (\partial_t + v \partial_x) \frac{1}{\rho} = \frac{1}{\rho} \partial_x v \rightarrow \partial_t \rho + \partial_x (\rho v) = 0
\]
Hodograph transformation:

\[ \partial_{\tau} x = v = \Pi, \quad \partial_{\sigma} x = \frac{1}{\rho}, \quad \partial_{\tau}^{2} x - \partial_{\sigma}^{2} x = 0 \]

Hodograph transformation:

\[ \tau, \sigma \rightarrow t, x \]

\[ \begin{cases} 
  dt = d\tau \\
  dx = v \ d\tau + \frac{1}{\rho} \ d\sigma 
\end{cases} \rightarrow \begin{cases} 
  \partial_{\tau} = \partial_{t} + v \partial_{x} \\
  \partial_{\sigma} = \frac{1}{\rho} \partial_{x} 
\end{cases} \]

First Hamilton’s equation:

\[ \partial_{\tau} x = v \rightarrow \partial_{\sigma} \partial_{\tau} x = \partial_{\sigma} v \]

\[ \rightarrow (\partial_{t} + v \partial_{x}) \frac{1}{\rho} = \frac{1}{\rho} \partial_{x} v \rightarrow \partial_{t} \rho + \partial_{x} (\rho v) = 0 \]

Continuity equation
Hodograph transformation

\[ \partial_{\tau} x = \Pi = v, \quad \partial_{\tau}^2 x - \partial_{\sigma}^2 x = 0, \quad \rho(x) = (\partial_{\sigma} x)^{-1} \]

\[
\begin{cases}
    dt = d\tau \\
    dx = v \, d\tau + \frac{1}{\rho} \, d\sigma
\end{cases}
\quad \rightarrow \quad \begin{cases}
    \partial_{\tau} = \partial_t + v \partial_x \\
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\end{cases}
\]
String (Hamilton's) equation: \( \partial^2_\tau x - \partial^2_\sigma x = 0 \)
Hodograph transformation

\[ \partial_{\tau} x = \Pi = v, \quad \partial_{\tau}^2 x - \partial_{\sigma}^2 x = 0, \quad \rho(x) = (\partial_{\sigma} x)^{-1} \]

\[
\begin{align*}
\left\{ \begin{array}{l}
dt = d\tau \\
\ dx = v \, d\tau + \frac{1}{\rho} \, d\sigma
\end{array} \right. \quad \rightarrow \quad \left\{ \begin{array}{l}
\partial_{\tau} = \partial_t + v \partial_x \\
\partial_{\sigma} = \frac{1}{\rho} \partial_x
\end{array} \right.
\end{align*}
\]

String (Hamilton’s) equation: \[ \partial_{\tau}^2 x - \partial_{\sigma}^2 x = 0 \]

\[
\rho(\partial_t + v \partial_x) v = \partial_x (1/\rho)
\]
Hodograph transformation

\[ \partial_\tau x = \Pi = v, \quad \partial^2_\tau x - \partial^2_\sigma x = 0, \quad \rho(x) = (\partial_\sigma x)^{-1} \]

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String (Hamilton’s) equation: \( \partial^2_\tau x - \partial^2_\sigma x = 0 \)

\[ \rho(\partial_t + v \partial_x) v = \partial_x (1/\rho) \]

Euler’s equation
Hodograph transformation

\[ \partial_\tau x = \Pi = v, \quad \partial_\tau^2 x - \partial_\sigma^2 x = 0, \quad \rho(x) = (\partial_\sigma x)^{-1} \]

\[
\begin{align*}
\begin{cases}
\quad dt = d\tau \\
\quad dx = v\, d\tau + \frac{1}{\rho} \, d\sigma
\end{cases}
\rightarrow
\begin{cases}
\quad \partial_\tau = \partial_t + v \partial_x \\
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\end{cases}
\end{align*}
\]

String (Hamilton’s) equation: \( \partial_\tau^2 x - \partial_\sigma^2 x = 0 \)

\[ \rho(\partial_t + v \partial_x) v = \partial_x (1/\rho) \]

Euler’s equation provided \( p = -\frac{1}{\rho} \)
The multidimensional scenario

L. Randall, R. Sundrum, 1999
The multidimensional scenario

Idea: a warped universe
The multidimensional scenario

Idea: a warped universe

\[ ds^2 = e^{-2|y|/l}(dt^2 - dx_1^2 - dx_2^2 - dx_3^2) - dy^2 \]
The multidimensional scenario

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AdS geometry
The multidimensional scenario

Idea: a warped universe

\[ ds^2 = e^{-2|y|/l} (dt^2 - dx_1^2 - dx_2^2 - dx_3^2) - dy^2 \]

\[ l = \text{AdS radius}; \, \Lambda = -\frac{6}{l^2} \]
The multidimensional scenario

Idea: a warped universe

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- \( l = \text{AdS radius} \);
- \( \Lambda = -\frac{6}{l^2} \)
- \( y \) - fifth additional non-compact coordinate
The multidimensional scenario

Idea: a warped universe

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- \( l = \text{AdS radius}; \ \Lambda = -\frac{6}{l^2} \)
- \( y \) - fifth additional non-compact coordinate
- At \( y = 0 \): orbifold conditions
The brane

- Christoffel symbols have jumps at \( y = 0 \)
The brane

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- The curvature tensor has $\delta$ terms
Christoffel symbols have jumps at $y = 0$

The curvature tensor has $\delta$ terms

Must introduce a brane at $y = 0$
Christoffel symbols have jumps at $y = 0$

The curvature tensor has $\delta$ terms

Must introduce a brane at $y = 0$

Brane tension: $\lambda = \frac{6}{l}$
Brane in adS-BH background

Brane in adS-BH background

Another type of foliation
Brane in adS-BH background

Another type of foliation

Instead of a Minkowski brane take a brane with $R \times S^d$ topology in a $(d+2)$-dimensional adS bulk
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Tension not enough: need matter on the brane. A fluid?
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Equation of state  

$$p = -\frac{(d - 1)\rho}{d} - \frac{4d}{\rho l^2}$$
Another type of foliation

Instead of a Minkowski brane take a brane with $R \times S^d$ topology in a $(d + 2)$-dimensional adS bulk

Tension not enough: need matter on the brane. A fluid?

Equation of state

$$p = -\frac{(d-1)\rho}{d} - \frac{4d}{\rho l^2}$$

For $d = 1$

$$p = -\frac{4}{l^2} \frac{1}{\rho}$$
Embedding of a $3 + 1$ brane in a $4 + 1$ bulk is described by coordinates $x^M = (x^\mu, x^4)$
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Bulk metric is $g_{MN}$.
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The induced metric on the $3 + 1$ brane is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \theta_{,\mu} \theta_{,\nu}$$
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Bulk metric is $g_{MN}$

The induced metric on the $3+1$ brane is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \theta_{,\mu}\theta_{,\nu}$$

$\theta(x^\mu)$ is a scalar field describing the embedding of the brane into the bulk.
Embedding of a $3 + 1$ brane in a $4 + 1$ bulk is described by coordinates $x^M = (x^\mu, x^4)$

Bulk metric is $g_{MN}$

The induced metric on the $3 + 1$ brane is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \theta,\mu \theta,\nu$$

The action for the brane ($f$ is the brane tension):

$$S_{brane} = \int d^4x \sqrt{-\tilde{g}}(-f + \ldots)$$
Embedding of a $3 + 1$ brane in a $4 + 1$ bulk is described by coordinates $x^M = (x^\mu, x^4)$

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The induced metric on the $3 + 1$ brane is

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The action for the brane ($f$ is the brane tension):

$$S_{brane} = \int d^4x \sqrt{-\tilde{g}}(-f + \ldots)$$

$$= \int d^4x \sqrt{-g} \sqrt{1 - g^{\mu\nu} \theta,\mu \theta,\nu}(-f + \ldots)$$
\[ T_{\mu\nu} = f \left( \frac{\theta_{,\mu} \theta_{,\nu}}{\sqrt{1 - g_{\mu\nu} \theta_{,\mu} \theta_{,\nu}}} + g_{\mu\nu} \sqrt{1 - g_{\mu\nu} \theta_{,\mu} \theta_{,\nu}} \right) \]

\[ = (\rho + p) u_\mu u_\nu - p g_{\mu\nu} \]
\[
T_{\mu\nu} = f \left( \frac{\theta,_{\mu}\theta,_{\nu}}{\sqrt{1 - g_{\mu\nu}\theta,_{\mu}\theta,_{\nu}}} + g_{\mu\nu}\sqrt{1 - g_{\mu\nu}\theta,_{\mu}\theta,_{\nu}} \right)
\]
\[
= (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu}
\]
with the following identifications:
\[ T_{\mu\nu} = f \left( \frac{\theta,_{\mu}\theta,_{\nu}}{\sqrt{1 - g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}}} + g_{\mu\nu}\sqrt{1 - g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}} \right) \]

\[ = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu} \]

**Four-velocity** \( u_{\mu} = \frac{\theta,_{\mu}}{\sqrt{g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}}} \)
\[ T_{\mu\nu} = f \left( \frac{\theta_{,\mu}\theta_{,\nu}}{\sqrt{1 - g_{\mu\nu}\theta_{,\mu}\theta_{,\nu}}} + g_{\mu\nu}\sqrt{1 - g_{\mu\nu}\theta_{,\mu}\theta_{,\nu}} \right) \]

\[ = (\rho + p)u_\mu u_\nu - p g_{\mu\nu} \]

**Four-velocity** \( u_\mu = \frac{\theta_{,\mu}}{\sqrt{g_{\mu\nu}\theta_{,\mu}\theta_{,\nu}}} \)

**Pressure** \( p = -f \sqrt{1 - g_{\mu\nu}\theta_{,\mu}\theta_{,\nu}} \)
\[ T_{\mu\nu} = f \left( \frac{\theta,_{\mu}\theta,_{\nu}}{\sqrt{1 - g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}}} + g_{\mu\nu}\sqrt{1 - g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}} \right) \]

\[ = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu} \]

Four-velocity \( u_{\mu} = \frac{\theta,_{\mu}}{\sqrt{g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}}} \)

Pressure \( p = -f \sqrt{1 - g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}} \)

Energy density: \( \rho = f \frac{1}{\sqrt{1 - g^{\mu\nu}\theta,_{\mu}\theta,_{\nu}}} \)
\[ T_{\mu\nu} = f \left( \frac{\theta,\mu\theta,\nu}{\sqrt{1 - g^{\mu\nu}\theta,\mu\theta,\nu}} + g_{\mu\nu}\sqrt{1 - g^{\mu\nu}\theta,\mu\theta,\nu} \right) \]

\[ = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \]

**Four-velocity** \( u_\mu = \frac{\theta,\mu}{\sqrt{g^{\mu\nu}\theta,\mu\theta,\nu}} \)

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Chaplygin’s gas with \( A = f^2 \).
Chaplygin cosmology: theory and observations
FRW-Chaplygin Cosmology

The Chaplygin gas

\[ a\dot{\rho} + 3\dot{a}(\rho + p) = 0 \]
\[ a \dot{\rho} + 3 \dot{a} \left( \rho - \frac{A}{\rho} \right) = 0 \]
The equation for the Chaplygin gas is given by:

\[ a \dot{\rho} + 3 \ddot{a} \left( \rho - \frac{A}{\rho} \right) = 0 \]

and

\[ \rho = \sqrt{A + \frac{B}{a^6}} \]
$$a \dot{\rho} + 3 \dot{a} \left( \rho - \frac{A}{\rho} \right) = 0 \quad \rho = \sqrt{A + \frac{B}{a^6}}$$

B is an integration constant chosen positive
\[ a \dot{\rho} + 3 \dot{a} \left( \rho - \frac{A}{\rho} \right) = 0 \]

\[ \rho = \sqrt{A + \frac{B}{a^6}} \]

For small \( a(t) \) (early times): dust-like behavior

\[ \rho \sim \frac{\sqrt{B}}{a^3} \]
\[ a \dot{\rho} + 3 \dot{a} \left( \rho - \frac{A}{\rho} \right) = 0 \]

\[ \rho = \sqrt{A + \frac{B}{a^6}} \]

For small \( a(t) \) (early times): dust-like behavior

\[ \rho \sim \frac{\sqrt{B}}{a^3} \]

For large \( a(t) \) (late times): cosmological constant-like behavior

\[ \rho \sim \sqrt{A}, \quad p \sim -\sqrt{A} \]
\[
a \dot{\rho} + 3 \dot{a} \left( \rho - \frac{A}{\rho} \right) = 0 \quad \rho = \sqrt{A + \frac{B}{a^6}}
\]

For small \(a(t)\) (early times): dust-like behavior

\[
\rho \sim \frac{\sqrt{B}}{a^3}
\]

For large \(a(t)\) (late times): cosmological constant-like behavior

\[
\rho \sim \sqrt{A}, \quad p \sim -\sqrt{A}
\]

\(\Lambda = \sqrt{A}\) is the asymptotic cosmological constant
The subleading terms at large values of $a$ give

$$\rho \approx \sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6} \quad p \approx -\sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6}$$
The subleading terms at large values of $a$ give

$$\rho \approx \sqrt{A} + \sqrt{\frac{B}{4A}} \ a^{-6} \ \ \ p \approx -\sqrt{A} + \sqrt{\frac{B}{4A}} \ a^{-6}$$

Mixture of c.c. $\Lambda = \sqrt{A}$ and stiff matter $p = \rho$
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Mixture of c.c. $\Lambda = \sqrt{A}$ and stiff matter $p = \rho$

Chaplygin’s cosmic evolution:
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Mixture of c.c. $\Lambda = \sqrt{A}$ and stiff matter $p = \rho$

Chaplygin’s cosmic evolution:

dust – like

matter
The subleading terms at large values of $a$ give

$$\rho \approx \sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6} \quad p \approx -\sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6}$$

Mixture of c.c. $\Lambda = \sqrt{A}$ and stiff matter $p = \rho$

Chaplygin’s cosmic evolution:

dust-like matter $\rightarrow$ constant

cosmological +

stiff matter
The subleading terms at large values of \( a \) give

\[
\rho \approx \sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6} \quad p \approx -\sqrt{A} + \sqrt{\frac{B}{4A}} a^{-6}
\]

Mixture of c.c. \( \Lambda = \sqrt{A} \) and stiff matter \( p = \rho \)

Chaplygin’s cosmic evolution:

- Dust-like matter \( \rightarrow \) Cosmological constant
- + Stiff matter \( \rightarrow \) deSitter universe
Now:

\[
\begin{align*}
p &= p_{\Lambda_0} + p_M = -\frac{3}{4\pi G}\Lambda_0 \\
\rho &= \rho_{\Lambda_0} + \rho_M = \frac{3}{4\pi G}\Lambda_0 + \rho_M \\
\frac{\rho_M}{\rho_{\Lambda_0}} &\sim \frac{3}{7}
\end{align*}
\]
\[ \Lambda \text{ will increase} \]

Now:

\[
\begin{aligned}
    p &= p_{\Lambda_0} + p_M = -\frac{3}{4\pi G} \Lambda_0 \\
    \rho &= \rho_{\Lambda_0} + \rho_M = \frac{3}{4\pi G} \Lambda_0 + \rho_M \\
    \frac{\rho_M}{\rho_{\Lambda_0}} &\sim \frac{3}{7}
\end{aligned}
\]

Comparing these data with the Chaplygin gas model one has:

\[ \Lambda_\infty \sim 1.2 \Lambda_0 \]
Now:

\[
\begin{align*}
p &= p_{\Lambda_0} + p_M = -\frac{3}{4\pi G}\Lambda_0 \\
\rho &= \rho_{\Lambda_0} + \rho_M = \frac{3}{4\pi G}\Lambda_0 + \rho_M \\
\frac{\rho_M}{\rho_{\Lambda_0}} &\sim \frac{3}{7}
\end{align*}
\]

Comparing these data with the Chaplygin gas model one has:

\[\Lambda_\infty \sim 1.2\Lambda_0\]

Comological constant may be increasing
Time evolution

\[ (\frac{\dot{a}}{a})^2 = \frac{8}{3} \pi G \rho + \frac{\Lambda}{3} - \frac{K}{a^2} \]
Friedmann eq. \( (\frac{\dot{a}}{a})^2 = \frac{8}{3} \pi G \sqrt{A + \frac{B}{a^6} - \frac{K}{a^2}} \)
Friedmann eq. \((\frac{\dot{a}}{a})^2 = \frac{8}{3} \pi G \sqrt{A + \frac{B}{a^6}} - \frac{K}{a^2}\)

for the flat case \(K=0\) it can be solved
Time evolution

\[ \text{Friedmann eq.} \quad \left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi G \sqrt{A + \frac{B}{a^6} - \frac{K}{a^2}} \]

\[ t = \frac{1}{6 A^{\frac{1}{4}}} \left( \ln \left( \frac{1 + \frac{B}{A a^6}}{(1 + \frac{B}{A a^6})^{\frac{1}{4}}} + 1 \right) - 2 \arctan\left( 1 + \frac{B}{A a^6} \right)^{\frac{1}{4}} + \pi \right) \]
For open or flat space ($K = -1, 0$) the universe always evolves from deceleration to acceleration.
For open or flat space \((K = -1, 0)\) the universe always evolves from deceleration to acceleration.

For the closed models \((K = 1)\), there exists a static Einstein universe solution

\[
a_0 = (3A)^{-\frac{1}{4}}, \quad B = \frac{2}{3\sqrt{3A}}
\]
For open or flat space \((K = -1, 0)\) the universe always evolves from deceleration to acceleration.

For the closed models \((K = 1)\), there exists a static Einstein universe solution:

\[ a_0 = (3A)^{-\frac{1}{4}}, \quad B = \frac{2}{3\sqrt{3A}} \]

When \(B > \frac{2}{3\sqrt{3A}}\) \(a(t)\) can take any value.
For open or flat space \((K = -1, 0)\) the universe always evolves from deceleration to acceleration.

For the closed models \((K = 1)\), there exists a static Einstein universe solution

\[
a_0 = (3A)^{-\frac{1}{4}}, \quad B = \frac{2}{3\sqrt{3}A}
\]

When \(B > \frac{2}{3\sqrt{3}A}\) \(a(t)\) can take any value.

When \(B < \frac{2}{3\sqrt{3}A}\) there are forbidden radii; either

\[
a < a_1 = \frac{1}{\sqrt{3}A} \left(\sqrt{3} \sin \frac{\varphi}{3} - \cos \frac{\varphi}{3}\right) \text{ or } a > a_2 = \frac{2}{\sqrt{3}A} \cos \frac{\varphi}{3},
\]

\[\varphi = \pi - \arccos \frac{3\sqrt{3}AB}{2}\]
Mixed component of the energy-momentum

\[ T_{\mu}^{\nu} = (\rho + p)u_{\mu}u^{\nu} - \delta_{\mu}^{\nu}p \]
Mixed component of the energy-momentum

\[ T_{\mu}^{\nu} = (\rho + p)u_{\mu}u^{\nu} - \delta_{\mu}^{\nu}p \]

In comoving coordinates

\[ u^0 = 1/\sqrt{g_{00}} \quad u^i = 0 \]
Mixed component of the energy-momentum
\[ T_{\mu}^{\nu} = (\rho + p) u_{\mu} u^{\nu} - \delta_{\mu}^{\nu} p \]

In comoving coordinates \( u^0 = 1/\sqrt{g_{00}} \) \( u^i = 0 \)

\[ T_{0}^{0} = \rho, \quad T_{i}^{j} = -\delta_{i}^{j} p, \quad T_{0}^{i} = 0 \]
Mixed component of the energy-momentum

\[ T^\nu_\mu = (\rho + p)u_\mu u^\nu - \delta^\nu_\mu p \]

In comoving coordinates \( u^0 = 1/\sqrt{g_{00}} \)  \( u^i = 0 \)

\[ T^0_0 = \rho, \quad T^j_i = -\delta^j_i p, \quad T^i_0 = 0 \]

\[ T^0_i = (\rho + p)u_i u^0 = (\rho + p)g_{i0}(u^0)^2 = (\rho + p)g_{i0}/g_{00} \]
The inhomogeneous C. G.

Mixed component of the energy-momentum
\[ T^\nu_{\mu} = (\rho + p)u_\mu u^\nu - \delta^\nu_{\mu}p \]

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Energy conservation
\[ T^\mu_0 ;\mu = 0 \] reads
\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho + p) \]
The inhomogeneous C. G.

Mixed component of the energy-momentum

\[ T^\nu_\mu = (\rho + p)u_\mu u^\nu - \delta^\nu_\mu p \]

In comoving coordinates \( u^0 = 1/\sqrt{g_{00}} \quad u^i = 0 \)

\[ T^0_0 = \rho, \quad T^j_i = -\delta^j_i p, \quad T^i_0 = 0 \]

\[ T^i_0 = (\rho + p)u_i u^0 = (\rho + p)g_{i0}(u^0)^2 = (\rho + p)g_{i0}/g_{00} \]

Energy conservation \( T_0^\mu ;_\mu = 0 \) reads

\[ \dot{\rho} = -\frac{1}{2} \dot{\gamma} (\rho + p) \]

\[ \gamma_{ij} = \frac{g_{i0}g_{j0}}{g_{00}} - g_{ij}, \quad \gamma^{ij} = -g^{ij} \]
\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho + p) \]
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Chaplygin gas \( p = -\frac{A}{\rho} \)
\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho + p) \]

\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho - \frac{A}{\rho}) \]
\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho + p) \]

\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho - \frac{A}{\rho}) \]

This equation can be integrated

\[ \rho(\vec{x}, t) = \sqrt{A + \frac{B(\vec{x})}{\gamma(\vec{x}, t)}} \]
\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho + p) \]

\[ \dot{\rho} = -\frac{1}{2} \frac{\dot{\gamma}}{\gamma} (\rho - \frac{A}{\rho}) \]

This equation can be integrated

\[ \rho(\vec{x}, t) = \sqrt{A + \frac{B(\vec{x})}{\gamma(\vec{x}, t)}} \]

\(B(\vec{x})\) is an arbitrary function of the spatial coordinates
The "statefinder" diagnostic 1

Proliferation of models explaining cosmic acceleration. How to discriminate between them?
Proliferation of models explaining cosmic acceleration.
How to discriminate between them?

Statefinder parameters
The "statefinder" diagnostic

Proliferation of models explaining cosmic acceleration. How to discriminate between them?

Statefinder parameters

Sahni, Saini, Starobinsky, Alam 2002
The "statefinder" diagnostic 1

Proliferation of models explaining cosmic acceleration. How to discriminate between them?

Statefinder parameters

\[ r \equiv \frac{\dddot{a}}{aH^3}, \quad s \equiv \frac{r - 1}{3(q - 1/2)} \]

\[ q \equiv -\frac{\ddot{a}}{aH^2} \quad \text{the deceleration parameter} \]
The "statefinder" diagnostic 1

Proliferation of models explaining cosmic acceleration. How to discriminate between them?

Statefinder parameters

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\[ q \equiv -\frac{\ddot{a}}{aH^2} \quad \text{the deceleration parameter} \]

Involve the \textit{third} time derivative of the cosmological radius \(a\)
\[ \dot{\rho} = \frac{\partial p}{\partial \rho} \dot{\rho} = -3\sqrt{\rho} (\rho + p) \frac{\partial p}{\partial \rho} \]
\dot{p} = \frac{\partial p}{\partial \rho} \dot{\rho} = -3\sqrt{\rho} (\rho + p) \frac{\partial p}{\partial \rho}

r = 1 + \frac{9}{2} \left(1 + \frac{p}{\rho}\right) \frac{\partial p}{\partial \rho}, \quad s = \left(1 + \frac{\rho}{p}\right) \frac{\partial p}{\partial \rho}
The "statefinder" diagnostic 2

\[
\dot{\rho} = \frac{\partial p}{\partial \rho} \dot{\rho} = -3\sqrt{\rho} (\rho + p) \frac{\partial p}{\partial \rho}
\]

\[
r = 1 + \frac{9}{2} \left(1 + \frac{p}{\rho}\right) \frac{\partial p}{\partial \rho}, \quad s = \left(1 + \frac{\rho}{p}\right) \frac{\partial p}{\partial \rho}
\]

For the Chaplygin gas one has:

\[
v_s^2 = \frac{\partial p}{\partial \rho} = \frac{A}{\rho^2} = -\frac{p}{\rho} = 1 + s
\]
\[ \dot{p} = \frac{\partial p}{\partial \rho} \dot{\rho} = -3\sqrt{\rho} (\rho + p) \frac{\partial p}{\partial \rho} \]

\[ r = 1 + \frac{9}{2} \left( 1 + \frac{p}{\rho} \right) \frac{\partial p}{\partial \rho}, \quad s = \left( 1 + \frac{\rho}{p} \right) \frac{\partial p}{\partial \rho} \]

For the Chaplygin gas one has:

\[ v_s^2 = \frac{\partial p}{\partial \rho} = \frac{A}{\rho^2} = -\frac{p}{\rho} = 1 + s \]

\[ r = 1 - \frac{9}{2} s (1 + s) \]
The "statefinder" diagnostic

![Graph showing the statefinder diagnostic with a fixed point labeled \( \Lambda CDM \) and a point labeled "today".](image)
The Chaplygin gas statefinder $s$ takes negative values (in contrast with quintessence).
For $q \approx -0.5$ the current values of the statefinder (within our model) are $s \approx -0.3$, $r \approx 1.9$
The Chaplygin gas statefinder $s$ takes negative values (in contrast with quintessence). For $q \approx -0.5$ the current values of the statefinder (within our model) are $s \approx -0.3$, $r \approx 1.9$.

Future experiments will discriminate the pure Chaplygin gas model from $\Lambda$CDM model.
Chaplygin gas + Dust

\[ r = 1 + \frac{9(\rho + p)}{2(\rho + \rho_d)} \frac{\partial p}{\partial \rho}, \quad s = \frac{\rho + p}{p} \frac{\partial p}{\partial \rho} \]
Chaplygin gas + Dust

\[ r = 1 + \frac{9(\rho + p)}{2(\rho + \rho_d)} \frac{\partial p}{\partial \rho}, \quad s = \frac{\rho + p}{p} \frac{\partial p}{\partial \rho} \]

If the second fluid is the Chaplygin gas

\[ r = 1 - \frac{9}{2} \frac{s(s + 1)}{1 + \frac{\rho_d}{\rho}} \]
Chaplygin gas + Dust

\[ r = 1 + \frac{9(\rho + p)}{2(\rho + \rho_d)} \frac{\partial p}{\partial \rho}, \quad s = \frac{\rho + p}{p} \frac{\partial p}{\partial \rho} \]

If the second fluid is the Chaplygin gas

\[ r = 1 - \frac{9}{2} \frac{s(s + 1)}{1 + \frac{\rho_d}{\rho}} \]

Dust: \( \rho_1 = \frac{C}{a^3} \)
Chaplygin gas + Dust

\[ r = 1 + \frac{9(\rho + p)}{2(\rho + \rho_d)} \frac{\partial p}{\partial \rho}, \quad s = \frac{\rho + p}{p} \frac{\partial p}{\partial \rho} \]

If the second fluid is the Chaplygin gas

\[ r = 1 - \frac{9 s(s + 1)}{2 \left( 1 + \frac{\rho_d}{\rho} \right)} \]

Dust: \( \rho_1 = \frac{C}{a^3} \)  \hspace{1cm} \text{Chaplygin gas:} \ \rho_2 = \sqrt{A + \frac{B}{a^6}} \]
If the second fluid is the Chaplygin gas

\[ r = 1 - \frac{9 s(s + 1)}{2 \left( 1 + \frac{\rho_d}{\rho} \right)} \]

\[ \frac{\rho_d}{\rho} = \sqrt{-s \kappa} \]
Chaplygin gas + Dust

\[ r = 1 + \frac{9(\rho + p)}{2(\rho + \rho_d)} \frac{\partial p}{\partial \rho}, \quad s = \frac{\rho + p}{p} \frac{\partial p}{\partial \rho} \]

If the second fluid is the Chaplygin gas

\[ r = 1 - \frac{9s(s + 1)}{2} \frac{\rho_d}{1 + \frac{\rho_d}{\rho}} \]

\[ \frac{\rho_d}{\rho} = \sqrt{-s\kappa} \]

\[ \kappa \equiv \frac{C}{\sqrt{B}} \] is the ratio between the energy densities of dust and of the Chaplygin gas at the beginning of the cosmological evolution
$r = 1 - \frac{9}{2} \frac{s(s+1)}{1+\kappa \sqrt{-s}}$
\[ r = 1 - \frac{9}{2} \frac{s(s+1)}{1+\kappa \sqrt{-s}} \]
Possible solution of the cosmic coincidence co-nundrum. Here initial values of $\rho_d$ and $\rho$ can have the same order of magnitude.
The generalized Chaplygin gas


\[ p = -\frac{A}{\rho^\alpha} \quad 0 \leq \alpha \leq 1 \]
The generalized Chaplygin gas

\[ p = -\frac{A}{\rho^\alpha} \quad \rho = \left( A + \frac{B}{a^3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} \]
The generalized Chaplygin gas

\[
p = -\frac{A}{\rho^\alpha} \quad \rho = \left( A + \frac{B}{a^3(1+\alpha)} \right)^{\frac{1}{1+\alpha}}
\]

\[
w_\alpha = - \left[ 1 + \frac{B}{A} \frac{1}{a^{3(\alpha+1)}} \right]^{-1}
\]
The generalized Chaplygin gas

\[ p = -\frac{A}{\rho^\alpha} \quad \rho = \left( A + \frac{B}{a^3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} \]

\[ w_\alpha = -\left[ 1 + \frac{B}{A a^3(\alpha+1)} \right]^{-1}, \quad c_\alpha^2 = \frac{dp}{d\rho} = -\alpha w_\alpha. \]
The generalized Chaplygin gas

\[ p = -\frac{A}{\rho^\alpha} \quad \rho = \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \]

\[ w_\alpha = -\left[ 1 + \frac{B}{A a^{3(\alpha+1)}} \right]^{-1}, \quad c_\alpha^2 = \frac{dp}{d\rho} = -\alpha w_\alpha. \]

Comes from a (rather artificial) Born-Infeld action:

\[ L = -A^{\frac{1}{1+\alpha}} \left[ 1 - (g^{\mu\nu} \theta,_{\mu}\theta,_{\nu})^{\frac{1+\alpha}{2\alpha}} \right]^{\frac{\alpha}{1+\alpha}} \]

Bento, Bertolami, Sen 2002
The generalized Chaplygin gas

\[ p = - \frac{A}{\rho^\alpha} \quad \rho = \left( A + \frac{B}{a^3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} \]

\[ w_\alpha = - \left[ 1 + \frac{B}{A a^3(\alpha+1)} \right]^{-1} \]

\[ c_\alpha^2 = \frac{dp}{d\rho} = -\alpha w_\alpha. \]

dust-like matter \rightarrow \text{a perfect fluid} \rightarrow \text{deSitter universe}

\[ p = \alpha \rho \]
The generalized Chaplygin gas

\[ p = -\frac{A}{\rho^\alpha} \quad \rho = \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \]

\[ w_\alpha = -\left[ 1 + \frac{B}{A a^{3(\alpha+1)}} \right]^{-1}, \quad c_\alpha^2 = \frac{dp}{d\rho} = -\alpha w_\alpha. \]

cosm. const.

dust – like matter → + a perfect fluid → deSitter universe

\[ p = \alpha \rho \]
The gCg: superluminal case

\[ p = -\frac{A}{\rho^\alpha} \quad \alpha \geq 1 \]
The gCg: superluminal case

\[ p = -\frac{A}{\rho^\alpha} \quad \alpha \geq 1 \quad \frac{\mathcal{H}^2}{\mathcal{H}_0^2} = \left( \tilde{A} + \frac{1 - \tilde{A}}{a^3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} a^2, \]

\[ \tilde{A} = A/(A + B) \] and \( \mathcal{H}_0 \) is the Hubble constant.
The gCg: superluminal case

\[ p = -\frac{A}{\rho^\alpha} \quad \alpha \geq 1 \quad \frac{\mathcal{H}^2}{\mathcal{H}_0^2} = \left( \bar{A} + \frac{1 - \bar{A}}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} a^2, \]

\[ \bar{A} = \frac{A}{A + B} \] and \( \mathcal{H}_0 \) is the Hubble constant.

Redshift of the transition to the superluminal gCg:

\[ z_{sl} = \left[ \frac{\bar{A} (\alpha - 1)}{1 - \bar{A}} \right]^{\frac{1}{3(\alpha+1)}} - 1. \]
The gCg: superluminal case

\[ p = -\frac{A}{\rho^\alpha} \quad \alpha \geq 1 \quad \frac{\mathcal{H}^2}{\mathcal{H}_0^2} = \left( \tilde{A} + \frac{1 - \tilde{A}}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} a^2, \]

\[ \tilde{A} = \frac{A}{A + B} \text{ and } \mathcal{H}_0 \text{ is the Hubble constant.} \]
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\[ z_{sl} = \left[ \frac{\tilde{A} (\alpha - 1)}{1 - \tilde{A}} \right]^{\frac{1}{3(\alpha+1)}} - 1. \]

\( \tilde{A} \) and \( \alpha \) are not independent
\[ p = -\frac{A}{\rho^\alpha} \quad \alpha \geq 1 \quad \frac{\mathcal{H}^2}{\mathcal{H}_0^2} = \left( \tilde{A} + \frac{1 - \tilde{A}}{a^3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} a^2, \]

\( \tilde{A} = \frac{A}{A + B} \) and \( \mathcal{H}_0 \) is the Hubble constant.

Redshift of the transition to the superluminal gCg:

\[ z_{sl} = \left[ \frac{\tilde{A} (\alpha - 1)}{1 - \tilde{A}} \right]^{\frac{1}{3(\alpha+1)}} - 1. \]

\( \tilde{A} \) and \( \alpha \) are not independent

Redshift the transition to the accelerated phase

\[ z_{tr} = \left[ \frac{2\tilde{A}}{1 - \tilde{A}} \right]^{\frac{1}{3(\alpha+1)}} - 1 \simeq 0.4 \]
\[ p = -\frac{A}{\rho^\alpha} \quad \alpha \geq 1 \quad \frac{H^2}{H_0^2} = \left( \frac{\bar{A}}{a^3(1+\alpha)} + \frac{1 - \bar{A}}{\alpha^3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} a^2, \]

\[ \bar{A} = \frac{A}{A + B} \text{ and } H_0 \text{ is the Hubble constant.} \]

Redshift of the transition to the superluminal gCg:

\[ z_{sl} = \left[ \frac{\bar{A}(\alpha - 1)}{1 - \bar{A}} \right]^{\frac{1}{3(\alpha + 1)}} - 1. \]

\[ \bar{A} \text{ and } \alpha \text{ are not independent} \]

Redshift the transition to the accelerated phase

\[ \bar{A} = \frac{(1 + z_{tr})^{3(1+\alpha)}}{2 + (1 + z_{tr})^{3(1+\alpha)}} \approx \frac{(1.45)^{3(1+\alpha)}}{2 + (1.45)^{3(1+\alpha)}}. \]
Comparison with observations

Observations seem to favor the generalised Chaplygin gas over other models
Comparison with observations

Observations seem to favor the generalised Chaplygin gas over other models

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<td>220.4 / 191</td>
<td>7.1</td>
<td>28</td>
<td>30</td>
</tr>
<tr>
<td>Flat Chaplygin</td>
<td>301.0 / 192</td>
<td>0.0</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

**Note.** — From Davis et al. astro-ph:0701510

The flat cosmological constant (flat $\Lambda$) model is preferred by both the AIC and the BIC. The $\Delta$AIC and $\Delta$BIC values for all other models in the table are then measured with respect to these lowest values. The goodness of fit (GoF) approximates the probability of finding a worse fit to the data. The models are given in order of increasing $\Delta$AIC.
Perturbations

The pure gCg has passed many tests of standard cosmology. However the behaviour of the gCg under perturbations is still problematic [Tegmark et al., Bean et al.]
Perturbations

Power spectrum of large scale structures seems to indicate [Tegmark et al] that the best fit value of $\alpha$ is very close to zero, rendering the gCg indistinguishable from $\Lambda$CDM (but see previous remarks on the ccp).
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- This behaviour has to be attributed to the gCg sound velocity which, during the cosmic evolution, grows from 0 to $\sqrt{\alpha}$ driving inhomogeneities to oscillate (if $\alpha > 0$) or to blow-up (if $\alpha < 0$).
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This behaviour has to be attributed to the gCg sound velocity which, during the cosmic evolution, grows from 0 to $\sqrt{\alpha}$ driving inhomogeneities to oscillate (if $\alpha > 0$) or to blow–up (if $\alpha < 0$).

This feature seems to be common to all UDM models [Tegmark et al.] that might appear to be ruled out.
Gauge invariant perturbations

V. Gorini, A. Kamenshick, U.M., O. Piattella, A. Starobinsky work in progress
\begin{equation}
\begin{aligned}
-k^2 \Phi - 3a \mathcal{H}^2 \dot{\Phi} - 3 \mathcal{H}^2 \dot{\Phi} &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
\mathcal{H} \dot{\Phi} + a \mathcal{H} \ddot{\Phi} &= a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(a \mathcal{H})^2 \dddot{\Phi} + \left(4a \mathcal{H}^2 + a^2 \mathcal{H} \dot{\mathcal{H}}\right) \dot{\Phi} + \left(2a \mathcal{H} \dot{\mathcal{H}} + \mathcal{H}^2\right) \Phi &= \\
&= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i,
\end{aligned}
\end{equation}
\[
-k^2 \Phi - 3aH^2 \dot{\Phi} - 3\dot{H}^2 \Phi = a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
H \Phi + aH \dot{\Phi} = a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(aH)^2 \ddot{\Phi} + \left(4aH^2 + a^2 H \dot{H} \right) \dot{\Phi} + \left(2aH \dot{H} + H^2 \right) \Phi = \\
= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i .
\]

\(a(\eta)\) is the scale factor as a function of the conformal time \(\eta\). Its present epoch value is normalized to unity.
Gauge invariant perturbations

\[
\begin{aligned}
-k^2 \Phi - 3a \mathcal{H}^2 \dot{\Phi} - 3 \mathcal{H}^2 \Phi &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
\mathcal{H} \Phi + a \mathcal{H} \dot{\Phi} &= a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(a\mathcal{H})^2 \ddot{\Phi} + \left(4a \mathcal{H}^2 + a^2 \mathcal{H} \mathcal{H} \right) \dot{\Phi} + \left(2a \mathcal{H} \mathcal{H} + \mathcal{H}^2 \right) \Phi &= \\
= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i,
\end{aligned}
\]

\(\mathcal{H}(\eta) = a' / a\), where the prime denotes derivation with respect to the conformal time.
Gauge invariant perturbations

\[
\begin{align*}
-k^2 \Phi - 3a\mathcal{H}^2 \dot{\Phi} - 3\mathcal{H}^2 \Phi &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
\mathcal{H} \dot{\Phi} + a\mathcal{H} \dot{\Phi} &= a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(a\mathcal{H})^2 \ddot{\Phi} + \left(4a\mathcal{H}^2 + a^2 \mathcal{H}\dot{\mathcal{H}}\right) \dot{\Phi} + \left(2a\mathcal{H}\dot{\mathcal{H}} + \mathcal{H}^2\right) \Phi &= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i,
\end{align*}
\]

Φ is the Bardeen gauge–invariant potentials (no shear)
\begin{equation*}
\begin{aligned}
-k^2 \Phi - 3a\mathcal{H}^2 \dot{\Phi} - 3\mathcal{H}^2 \Phi &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
\mathcal{H} \Phi + a\mathcal{H} \dot{\Phi} &= a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(a\mathcal{H})^2 \ddot{\Phi} + \left(4a\mathcal{H}^2 + a^2 \mathcal{H} \dot{\mathcal{H}}\right) \dot{\Phi} + \left(2a\mathcal{H} \dot{\mathcal{H}} + \mathcal{H}^2\right) \Phi &= \\
&= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i,
\end{aligned}
\end{equation*}

$V$ is the gauge–invariant expression of the scalar potential of the velocity field.
Gauge invariant perturbations

\[
\begin{aligned}
-k^2 \Phi - 3aH^2 \dot{\Phi} - 3H^2 \Phi &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
\mathcal{H} \Phi + a\mathcal{H} \dot{\Phi} &= a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(aH)^2 \ddot{\Phi} + \left(4aH^2 + a^2 \mathcal{H} \mathcal{H}\right) \dot{\Phi} + \left(2a\mathcal{H} \mathcal{H} + H^2\right) \Phi &= \\
&= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i, \\
\end{aligned}
\]

\(\rho_i\) and \(p_i\) are the background energy and pressure of the component \(i\).
Gauge invariant perturbations

\[
\begin{aligned}
-k^2 \Phi - 3aH^2 \dot{\Phi} - 3\dot{H}^2 \Phi &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
\mathcal{H} \Phi + a \mathcal{H} \dot{\Phi} &= a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(\mathcal{H})^2 \ddot{\Phi} + \left(4aH^2 + a^2 \mathcal{H} \dot{\mathcal{H}}\right) \dot{\Phi} + \left(2a\mathcal{H} \dot{\mathcal{H}} + \mathcal{H}^2\right) \Phi &= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i,
\end{aligned}
\]

\(\delta \rho_i\) and \(\delta p_i\) are the gauge–invariant expressions of the perturbations of the energy density and pressure.
Gauge invariant perturbations

\[
\begin{align*}
-k^2 \Phi - 3a\mathcal{H}^2 \dot{\Phi} - 3\mathcal{H}^2 \Phi &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
\mathcal{H} \Phi + a\mathcal{H} \dot{\Phi} &= a \sum_{i=1}^{N} (\rho_i + p_i) V_i \\
(a\mathcal{H})^2 \ddot{\Phi} + \left(4a\mathcal{H}^2 + a^2 \mathcal{H} \dot{\mathcal{H}}\right) \dot{\Phi} + \left(2a\mathcal{H} \dot{\mathcal{H}} + \mathcal{H}^2\right) \Phi &= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i,
\end{align*}
\]

\[
\delta p_i = c_{si}^2 \delta \rho_i, \quad c_{si}^2 \equiv \frac{\partial p_i}{\partial \rho_i}, \quad \delta_i \equiv \frac{\delta \rho_i}{\rho_i},
\]
Gauge invariant perturbations

\[
\begin{align*}
-k^2 \Phi - 3a\mathcal{H}^2 \dot{\Phi} - 3\mathcal{H}^2 \Phi &= a^2 \sum_{i=1}^{N} \rho_i \delta_i \\
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(a\mathcal{H})^2 \ddot{\Phi} + \left( 4a\mathcal{H}^2 + a^2 \mathcal{H} \dot{\mathcal{H}} \right) \dot{\Phi} + \left( 2a\mathcal{H} \dot{\mathcal{H}} + \mathcal{H}^2 \right) \Phi &= a^2 \sum_{i=1}^{N} c_{si}^2 \rho_i \delta_i,
\end{align*}
\]

\[\delta p_i = c_{si}^2 \delta \rho_i, \quad c_{si}^2 \equiv \frac{\partial p_i}{\partial \rho_i}, \quad \delta_i \equiv \frac{\delta \rho_i}{\rho_i},\]

The spatial Fourier transform allows us to treat each mode independently in the linear approximation.
The system can be solved by eliminating $\delta$ and by extracting a second order equation for $\Phi$:

$$\ddot{\Phi} + \left( \frac{\dot{\mathcal{H}}}{\mathcal{H}} + \frac{4 + 3c_s^2}{a} \right) \dot{\Phi} + \left( \frac{2\dot{\mathcal{H}}}{a\mathcal{H}} + \frac{1 + 3c_s^2}{a^2} + \frac{k^2c_s^2}{a^2\mathcal{H}^2} \right) \Phi = 0.$$
\( N = 1 \) - Density contrast

Evolution of the density contrast in the gCg \( k = 100 \; h \) \( Mpc^{-1} \) scale of a protogalaxy. Oscillations take place too early for \( \alpha \gtrsim 10^{-5} \), thus preventing structure formation.
$N = 1$ - Density contrast

The diagram shows the gCg density contrast $\delta_{\text{Ch}}$ as a function of the scale factor $a$, for different values of $\alpha$. The curves represent different values of $\alpha$: $\alpha = 0$, $\alpha = 10^{-7}$, $\alpha = 10^{-5}$, $\alpha = 0.1$, $\alpha = 1$, $\alpha = 2$, and $\alpha = 3$. The density contrast is plotted on the y-axis, with values ranging from $-5 \times 10^9$ to $20 \times 10^9$. The scale factor $a$ is shown on the x-axis, ranging from 0 to 0.1.
For $\alpha \sim 0.1$ the sound velocity becomes non negligible much earlier than at other values of $\alpha$. The range $10^{-7} \lesssim \alpha \lesssim 3$ appears thus to be ruled out since structure formation is prevented.
The plots for $\alpha = 0$ and $\alpha = 10^{-7}$ are superposed. At larger $\alpha$ the power spectrum tends to a limiting behaviour which is systematically below that of the $\Lambda$CDM one.
In the matter dominated–regime we obtain the system

\[
\begin{align*}
\ddot{\delta}_b + \left( \frac{\dot{H}}{H} + \frac{2}{a} \right) \dot{\delta}_b &= \frac{1}{H^2} \left( \rho_b \delta_b + \rho_{Ch} \delta_{Ch} \right), \\
\ddot{\delta}_{Ch} + \left( \frac{\dot{H}}{H} + \frac{2}{a} \right) \dot{\delta}_{Ch} + \frac{k^2}{a^2 H^2} c_s^2 \dot{\delta}_{Ch} &= \frac{1}{H^2} \left( \rho_b \delta_b + \rho_{Ch} \delta_{Ch} \right),
\end{align*}
\]

where \(c_s^2\) is the gCg square sound velocity.
\[ \dot{N} = 2 \ - \ \text{gCg + Baryons} \]

In the matter dominated–regime we obtain the system

\[
\begin{cases}
\ddot{\delta}_b + \left( \frac{\dot{H}}{H} + \frac{2}{a} \right) \dot{\delta}_b = \frac{1}{H^2} \left( \rho_b \delta_b + \rho_{\text{Ch}} \delta_{\text{Ch}} \right) \\
\ddot{\delta}_{\text{Ch}} + \left( \frac{\dot{H}}{H} + \frac{2}{a} \right) \dot{\delta}_{\text{Ch}} + \frac{k^2}{a^2 H^2} c_s^2 \dot{\delta}_{\text{Ch}} = \frac{1}{H^2} \left( \rho_b \delta_b + \rho_{\text{Ch}} \delta_{\text{Ch}} \right),
\end{cases}
\]

where \( c_s^2 \) is the gCg square sound velocity.
Following Solov’eva and Starobinsky we introduce the variables

\[ x = k \gamma^{-2} a^{-\frac{3}{2} \gamma}, \quad \gamma = -2 \alpha - \frac{8}{3} \]

The system is reduced to a fourth-order equation
In the matter dominated–regime we obtain the system

\[
\begin{align*}
\ddot{\delta}_b + \left( \frac{\dot{H}}{H} + \frac{2}{a} \right) \dot{\delta}_b &= \frac{1}{H^2} \left( \rho_b \delta_b + \rho_{Ch} \delta_{Ch} \right) \\
\ddot{\delta}_{Ch} + \left( \frac{\dot{H}}{H} + \frac{2}{a} \right) \dot{\delta}_{Ch} + \frac{k^2}{a^2 H^2} c_s^2 \dot{\delta}_{Ch} &= \frac{1}{H^2} \left( \rho_b \delta_b + \rho_{Ch} \delta_{Ch} \right),
\end{align*}
\]

where \( c_s^2 \) is the gCg square sound velocity.

\[
\left[ \left( \Delta + \frac{2}{3\gamma} \right) \Delta \left( \Delta - \frac{1}{3\gamma} \right) \left( \Delta - \frac{1}{\gamma} \right) + \right. \\
\left. x \left( \frac{2\gamma - 1/3}{\gamma} \Delta + \frac{1}{\gamma^2} \left( \gamma \left( \gamma - \frac{1}{3} \right) - \frac{2}{3} \Omega_{b0} \right) \right) \right] \delta_{Ch} = 0
\]

\( \Delta = x \cdot d/dx \).
\( N = 2 \) - Power spectrum
\( N = 2 \) - Power spectrum

Power spectra of baryons. For all \( \alpha \) good agreement with the observed spectrum. Better for small \( \alpha = 0, 10^{-5} \) (\( \simeq \Lambda \)CDM) and for \( \alpha = 3, 5 \) (superluminal).
Conclusions

The generalized Chaplygin gas cosmological model, with no additional fluid components, is compatible with structure formation and large scale structure only for $\alpha$ sufficiently small ($\alpha < 10^{-5}$), in which case it is indistinguishable from the $\Lambda$CDM model.
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The generalized Chaplygin gas cosmological model, with no additional fluid components, is compatible with structure formation only for $\alpha$ sufficiently small ($\alpha < 10^{-5}$), in which case it is indistinguishable from the $\Lambda$CDM model.

Adding to the generalized Chaplygin gas model a baryon component large scale structures are compatible with observations for all values of $\alpha$. However very small values of $\alpha$ and $\alpha \gtrsim 3$ are favoured.
Conclusions

- The generalized Chaplygin gas cosmological model, with no additional fluid components, is compatible with structure formation only for $\alpha$ sufficiently small ($\alpha < 10^{-5}$), in which case it is indistinguishable from the $\Lambda$CDM model.

- Adding to the generalized Chaplygin gas model a baryon component large scale structures are compatible with observations for all values of $\alpha$. However very small values of $\alpha$ and $\alpha \gtrsim 3$ are favoured.

- A coincidence(?): the transition from the subluminal to the superluminal regime and the transition to the accelerated expansion of the universe may have the same redshift; $z_{sl} = z_{tr}$ for $\alpha \simeq 3$. 
Tachyonic models
"Equivalent" scalar field

Want a homogeneous scalar field with same cosmic evolution as the Chaplygin gas
"Equivalent" scalar field

\[ L(\phi) = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]
"Equivalent" scalar field

\[
L(\phi) = \frac{1}{2} \dot{\phi}^2 - V(\phi)
\]

\[
\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \sqrt{A + \frac{B}{a^6}}
\]
"Equivalent" scalar field

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\[ \dot{\phi}^2 = \frac{B}{a^6 \sqrt{A + \frac{B}{a^6}}} \quad V(\phi) = \frac{2a^6 \left(A + \frac{B}{a^6}\right) - B}{2a^6 \sqrt{A + \frac{B}{a^6}}} \]
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\[
V(\phi) = \frac{2a^6 \left(A + \frac{B}{a^6}\right) - B}{2a^6 \sqrt{A + \frac{B}{a^6}}}
\]

\[
\frac{d\phi}{dt}
\]
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\[ \frac{d\phi}{dt} = \frac{d\phi}{da} \frac{\dot{a}}{a} a \]
"Equivalent" scalar field

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\[ \frac{d\phi}{da} = \frac{\sqrt{B}}{a(Aa^6 + B)^{1/2}} \]
"Equivalent" scalar field

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\[ \frac{d\phi}{da} = \frac{\sqrt{B}}{a(Aa^6 + B)^{1/2}} \]

\[ a^6 = \frac{4B \exp(6\phi)}{A(1 - \exp(6\phi))^2} \]
"Equivalent" scalar field

\[ L(\phi) = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]

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\[ \frac{d\phi}{da} = \frac{\sqrt{B}}{a(Aa^6 + B)^{1/2}} \]

\[ V(\phi) = \frac{1}{2} \sqrt{A} \left( \cosh 3\phi + \frac{1}{\cosh 3\phi} \right) \]
The cosmological evolution of the model with a scalar field with this potential coincides with that of the Chaplygin gas model provided the initial values $\phi(t_0)$ and $\dot{\phi}(t_0)$ satisfy the relation

$$\dot{\phi}^4(t_0) = 4(V^2(\phi(t_0)) - A)$$
Chapygin = "Free" Tachyons

Sen’s action:

\[ S = - \int d^4x \sqrt{-g} V(T) \sqrt{1 - g^{\mu \nu} T_{\mu T, \nu}} \]
Chapygin = "Free" Tachyons

For a spatially homogeneous model

\[ L = -V(T)\sqrt{1 - \dot{T}^2} \]
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If the tachyon potential is constant \( V(T) = V_0 \)
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*Frolov, Kofman, Starobinsky 2002*
A simple example

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One can reconstruct a potential:

\[ V(T) = \frac{4\sqrt{-k}}{9(1 + w)T^2} \]
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An exact solution that gives back the above cosmic evolution \( T = \sqrt{1 + wt} \)
A simple example

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Feinstein, Padhmanhaban 2002
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Field equation:

\[ \ddot{T} + 3(1 - \dot{T}^2) \frac{\dot{a}}{a} \dot{T} + (1 - \dot{T}^2) \frac{V,_{T}}{V} = 0 \]

An exact solution that gives back the above cosmic evolution:

\[ T = \sqrt{1 + wt} \]

Feinstein, Padhmanhaban 2002
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The potential has the same form: \( V(T) = \frac{4\sqrt{k}}{9(1+k)T^2} \)
A more complicated example

A two-fluid cosmological model
V. Gorini, A. Kamenshchik, U. Moschella, V. Pasquier
hep-th/0311111 PRD 2004
A more complicated example

A two-fluid cosmological model

\[ p_1 = -\rho_1 = -\Lambda \quad p_2 = k \rho_2, \quad -1 < k < 0 \]

\[ a(t) = a_0 \left( \sinh \frac{3\sqrt{\Lambda(1+k)t}}{2} \right)^\frac{2}{3(1+k)} \]
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An exact field solution that gives \( a(t) \):

\[ T(t) = \frac{2}{3\sqrt{\Lambda(1+k)}} \arctan \sinh \frac{3\sqrt{\Lambda(1+k)t}}{2} \]
Tachyon Potential $k < 0$
Time evolution $k < 0$
Tachyon Potential $k > 0$
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There are two types of trajectories: infinitely expanding universes

universes, hitting a cosmological singularity of a special type that we have called *Big Brake*

\[
\ddot{a}(t_B) = -\infty, \quad \dot{a}(t_B) = 0, \\
0 < a(t_B) < \infty
\]
The future of the universe

- An infinite expansion
The future of the universe

- An infinite expansion
- Big Crunch
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