Relativistic Intrinsic Lagrangian Perturbation Theory

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Motivations

Zel'dovich idea for perturbation theory :

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→ Formulate the equations in terms of a single dynamical field $\boldsymbol{\eta}^a$.

→ Perturb it and try to find the separable non propagating and propagating parts.

→ Inject the solution **without truncation** into the functionals of the metric to keep non-linear terms.

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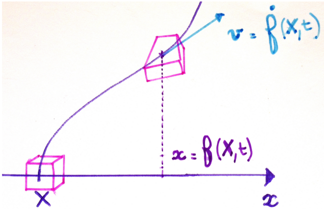

Euler-Newton System and Lagrange-Newton System

$$\frac{d}{dt}\mathbf{v} = \mathbf{g} \quad ; \quad \frac{d}{dt}\varrho + \mathbf{v}\cdot\boldsymbol{\nabla}\varrho = 0 \ , \tag{1}$$

$$\boldsymbol{\nabla}\times\mathbf{g} = \mathbf{0} \quad ; \quad \boldsymbol{\nabla}\cdot\mathbf{g} = \Lambda - 4\pi G\varrho \ , \tag{2}$$

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$$\ddot{\mathbf{f}} = \mathbf{g} \quad ; \varrho = \frac{\varrho_{\mathbf{i}}}{J} \;/\; J = \frac{1}{6}\epsilon_{ijk}\epsilon^{lmn}\; f^i_{\;|l} f^j_{\;|m} f^k_{\;|n} \;, \tag{7}$$

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Conclusions :

- Newton's equations expressed in a single dynamical field $\mathbf{f}$.

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- This description breaks down when two trajectories cross each other...

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Lagrangian perturbation theory

- Only the trajectory field is perturbed : $\mathbf{f} = \mathbf{f}^H + \mathbf{P}$
- Large density contrasts are allowed.

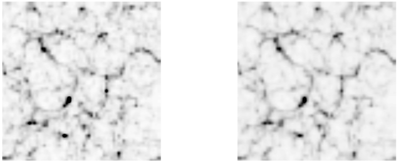

Figure : Density from **Eulerian** numerical simulation (*left*) and **Lagrangian** analytical solution to order 2 (*right*). *A. Melott. Size of the box :* $200h^{-1}Mpc$.

Extend to Relativity the Lagrangian perturbation theory !

Standard Perturbation Theory

Two manifolds :

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SPT perturbs the metric :

$$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{Minkowski} + \underbrace{h_{\mu\nu}}_{\ll 1}$$

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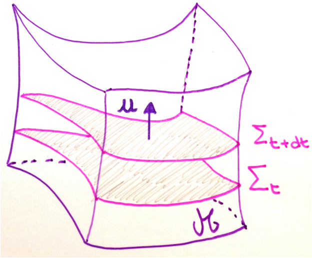
⇒ Spacetime can be foliated into spatial hypersurfaces of constant time : Σ_t.

- The normal vector to Σ_t is the 4-velocity $\mathbf{u}$.



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Lagrangian coordinates : $\mathbf{X}$; $\{\mathbf{d}X^i\}$ local coordinate basis.

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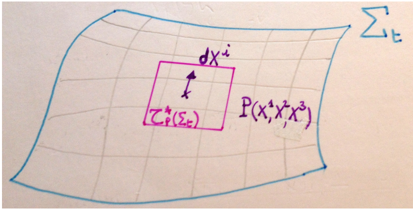

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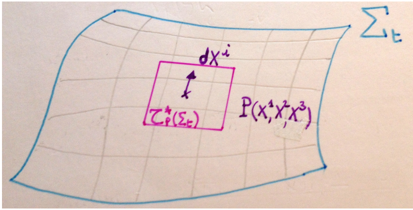

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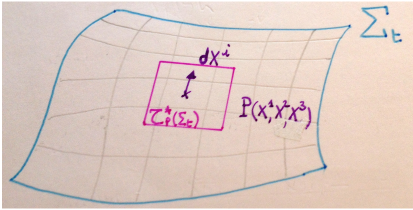

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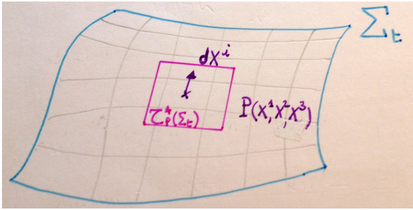

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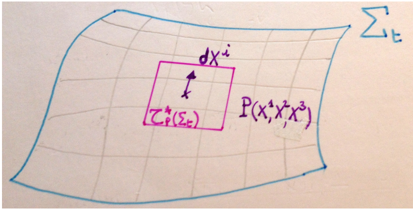

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Non–integrable Cartan's coframe fields : $\boldsymbol{\eta}^a = \eta^a{}_i\,\mathbf{d}X^i$

η^a encode the geometry of spacetime & the dynamics of the fluid :

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- Thus i is a coordinate index.

Einstein equations for irrotational dust (ADM)

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Einstein's equations fully expressed in terms of $\boldsymbol{\eta}^a$.

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- $X^i \longrightarrow x^m = f^m(X^i, t)$
- $ds^2 = g_{ij}\, dX^i\, dX^j = \delta_{mn} f^m_{|i} dX^i\, f^n_{|j} dX^j = \delta_{mn} dx^m\, dx^n.$

Electric part of the Lagrange-Einstein system :

$$\delta_{ab}\ddot{\eta}^a{}_{[i}\eta^b{}_{j]} = 0 \tag{17}$$

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MR of this system of equations

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We recover the Lagrange-Newton system for $\ddot{\mathbf{f}} = \mathbf{g}$.

First–order intrinsic Lagrangian perturbation theory

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- a part that deviates to flatness and contains GW !

Perturbated coframe :

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- Metric $\quad g_{ij} = a^2(t)\left(\delta_{ij} + G^{(1)}_{ij} + 2P_{(ij)} + P^a{}_i\, P_{a\,j}\right)$
- Perturbation decomposition $P_{ij} = \frac{1}{3}P\delta_{ij} + \Pi_{ij} + \mathfrak{P}_{ij}$

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$$P_{ij} = \frac{1}{3} P \delta_{ij} + {}^{E}\Pi_{ij} + {}^{H}\Pi_{ij}$$

Trace equation :

We combine the **Hamilton constraint**

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We choose :

$${}^E W_{ij}^{tl} + H_{\mathbf{i}}{}^E U_{ij}^{tl} = \mathcal{D}_{ij}(H_{\mathbf{i}} S - \phi) \quad ; \quad {}^H W_{ij}^{tl} + H_{\mathbf{i}}{}^H U_{ij}^{tl} = \tilde{W}_{ij}^{tl} + H_{\mathbf{i}}{}^H \tilde{U}_{ij}^{tl}$$

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For an Einstein-de Sitter background ($\Lambda = 0$, $k = 0$)

$$\Psi(\mathbf{X}, t) = {}^{\Psi}C^{-1}(\mathbf{X}) \left(\frac{t}{t_i}\right)^{-1} + {}^{\Psi}C^{2/3}(\mathbf{X}) \left(\frac{t}{t_i}\right)^{2/3} + {}^{\Psi}C^{0}(\mathbf{X})$$

$${}^{\Psi}C^{-1} = -\frac{3}{5}(S t_{\mathbf{i}} + \phi {t_{\mathbf{i}}}^2) \ ; \ {}^{\Psi}C^{2/3} = \frac{3}{5} S t_{\mathbf{i}} - \frac{9}{10} \phi {t_{\mathbf{i}}}^2 \ ; \ {}^{\Psi}C^{0} = \frac{\phi}{4\pi G \rho_{H_{\mathbf{i}}}}$$

The non–integrable dynamics

We solve :

$$ {}^{H}\ddot{\Pi}_{ij} + 3H\,{}^{H}\dot{\Pi}_{ij} - a^{-2}\ {}^{H}\Pi_{ij}{}^{|k}{}_{|k} = a^{-2}\left({}^{H}W^{tl}_{ij} + H_{\mathbf{i}}\,{}^{H}U^{tl}_{ij}\right)\ . \qquad (27) $$

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- Homogeneous solution ${}^{H}\Pi^{hom}_{ij}(\mathbf{X},t)$ contains propagation : **GW**.

Monochromatic solution of pulsation ω :

$$ {}^{H}\Pi^{\omega}{}_{ij}(\mathbf{X},t) = \left(C^{\omega}_{1}\ \mathcal{J}_0(3\omega t_i^{2/3}t^{1/3}) + C^{\omega}_{2}\ \mathcal{Y}_0(3\omega t_i^{2/3}t^{1/3})\right) \left\{C^{K+}_{ij}\ e^{i\mathbf{K}\cdot\mathbf{X}} + C^{K-}_{ij}\ e^{-i\mathbf{K}\cdot\mathbf{X}}\right\} $$

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$P(\mathbf{X}, t), {}^E\Pi_{ij}(\mathbf{X}, t), {}^H\Pi_{ij}(\mathbf{X}, t)$: $\mathbf{X} =$ local coordinate on curved space

Conclusion and Outlook

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The difference is that our coordinates are defined on locally curved space sections.

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- What does this theory gives at further orders ?

References

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