Bimetric gravity and dipolar dark matter

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IAP

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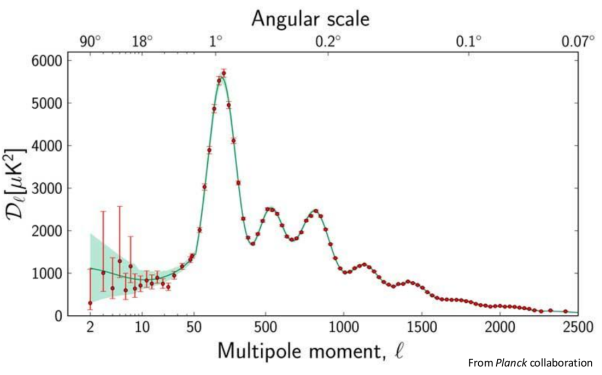
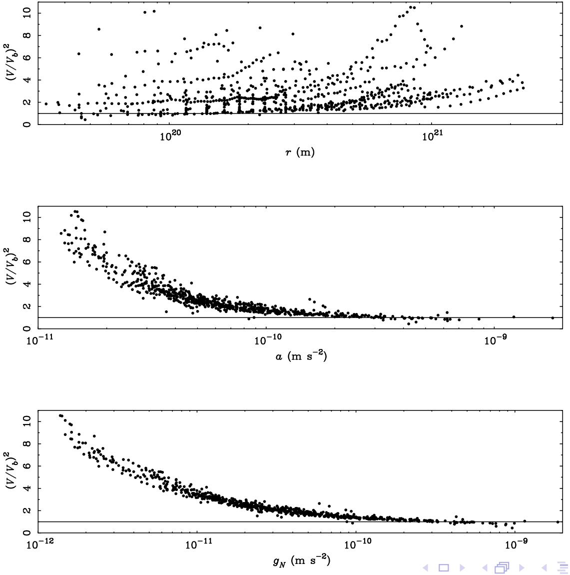
Plan

The cosmological concordance model



- Temperature fluctuations in the cosmic microwave background,
- Accelerated expansion of the universe,
- Formation and growth of large scale structure,

The mass discrepancy - acceleration relation [Famaey & McGaugh, 2012]



Milgrom's law (1983)

Modification of the Newtonian gravitational acceleration

$$\mu\left(|\mathbf{g}|/a_0\right)\mathbf{g} = \mathbf{g}_N\,,$$

- $a_0 \approx 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$ is the MOND acceleration constant,
- μ is the MOND interpolating function:

$$\begin{cases} \mu(x) \underset{x\gg 1}{\longrightarrow} 1 & \text{in the newtonian regime } g \gg a_0\,, \\ \mu(x) \underset{x\ll 1}{\longrightarrow} x & \text{in the MOND regime } g \ll a_0\,. \end{cases}$$

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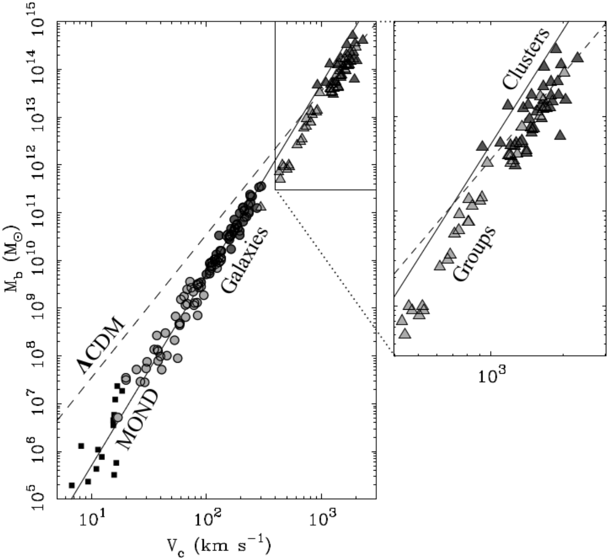
In the low acceleration limit we have

$$\frac{V_c^2}{r} = g = \sqrt{\frac{GMa_0}{r^2}} \qquad \Longrightarrow \qquad V_c^4 = GMa_0\,.$$

Baryonic mass vs rotation velocity [McGaugh, 2014]



The MOND equation (Bekenstein & Milgrom, 1984)

Modified Poisson equation for the gravitational field $\mathbf{g} = \nabla U$

- Field theoretic version of MOND:

$$\boldsymbol{\nabla} \cdot \left(\mu \left(\frac{g}{a_0} \right) \mathbf{g} \right) = -4\pi G \rho_b \ ,$$

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$$\boldsymbol{\nabla} \cdot \left(\mu \left(\frac{g}{a_0} \right) \mathbf{g} \right) = -4\pi G \rho_b \ ,$$

- Writing $\mu = 1 + \chi$ where χ is the gravitational susceptibility, the analogy with a dielectric medium is apparent,

$$\Delta U = -4\pi G \left(\rho_b + \rho_{\text{pol}} \right) \ ,$$

where $\rho_{\text{pol}} = -\boldsymbol{\nabla} \cdot \mathbf{P}$ and $\mathbf{P} = -\dfrac{\chi}{4\pi G} \mathbf{g}$.

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Some Relativistic MOND theories

Modified gravity theories

- Tensor-Vector-Scalar theory (TeVeS) [Bekenstein 2004, Sanders 2005]
- Non canonical Einstein-aether theories [Zlosnik et *al.* 2007, Halle et *al.* 2008]
- A bimetric theory of gravity (BIMOND) [Milgrom 2009]
- Non local theories [Deffayet et *al.* 2011]

Modified dark matter theories

- Dipolar Dark Matter [Blanchet & Le Tiec 2008;2009]

TeVeS [Bekenstein 2004, Sanders 2005]

- Einstein metric $\tilde{g}_{\mu\nu}$, scalar field ϕ, dynamical unit time-like vector field U^μ,

$$S_U = -\frac{c^4}{16\pi G}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[K^{\alpha\beta\mu\nu}U_{[\alpha,\mu]}U_{[\beta,\nu]} - \lambda\left(\tilde{g}^{\mu\nu}U_\mu U_\nu + 1\right)\right],$$

$$S_\phi = -\frac{c^4}{2k^2l^2G}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\, f\left(kl^2\left(\tilde{g}^{\mu\nu} - U^\mu U^\nu\right)\phi_{,\mu}\phi_{,\nu}\right),$$

$$K^{\alpha\beta\mu\nu} = c_1\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu} + c_2\tilde{g}^{\alpha\beta}\tilde{g}^{\mu\nu} + c_3\tilde{g}^{\alpha\nu}\tilde{g}^{\beta\mu} + c_4U^\alpha U^\mu\tilde{g}^{\beta\nu}.$$

- $a_0 = \frac{\sqrt{3k}}{4\pi l}$ and $\mu \approx \left(1 + \frac{k}{4\pi f'}\right)^{-1}$.

Non-canonical Einstein-aether theories [Zlosnik et *al.* 2007, Halle et *al.* 2008]

- Tensor-vector theory in the physical frame,
- Violates Lorentz invariance as it selected a preferred frame at each point in space-time

$$S_U = -\frac{c^4}{16\pi G l^2}\int \mathrm{d}^4x\sqrt{-g}\left[f\left(l^2 K^{\alpha\beta\mu\nu}U_{[\alpha,\mu]}U_{[\beta,\nu]}\right)\right.$$
$$\left. - l^2\lambda\left(g^{\mu\nu}U_\mu U_\nu + 1\right)\right],$$

- $\mu = f' + (1-f')/(1-C/2)$ and $l = \frac{(2-C)c^2}{3/2C^{3/2}a_0}$ to recover MOND.

Dipolar Dark Matter [Blanchet & Le Tiec 2008;2009]

- Dark matter fluid endowed with a dipole moment vector ξ^μ.

$$S_{\mathrm{DDM}} = \int \mathrm{d}^4x \sqrt{-g} \left[-\rho + J^\mu \dot{\xi}_\mu - \mathcal{W}(P_\perp) \right] ,$$

with $P_\perp = \rho \, \xi_\perp$ the polarization field.

$$\mathcal{W}(P_\perp) = \frac{\Lambda}{8\pi} + 2\pi \, P_\perp^2 + \frac{16\pi^2}{3a_0} \, P_\perp^3 + \mathcal{O}(P_\perp^4) \, .$$

- Recovers the first order cosmological perturbations.
- But requires a weak clustering hypothesis of DDM to recover MOND.

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Motivations

Some Relativistic MOND theories

Modified Dark Matter and bimetric gravity

A new kind of modified Dark Matter and bimetric gravity

The model

- Two metrics $g_{\mu\nu}$ and $\underline{g}_{\mu\nu}$ interacting with each other through $f_{\mu\nu}$ defined by the implicit relation

$$f_{\mu\nu} = f^{\rho\sigma} g_{\rho\mu} \underline{g}_{\nu\sigma} = f^{\rho\sigma} g_{\rho\nu} \underline{g}_{\mu\sigma} \,,$$

- Two kinds of dark matter ρ and $\underline{\rho}$, with mass currents $J^\mu = \rho\, u^\mu$ and $\underline{J}^\mu = \underline{\rho}\, \underline{u}^\mu$, and respectively coupled to $g_{\mu\nu}$ and $\underline{g}_{\mu\nu}$,
- Ordinary baryonic matter ρ_b living in the sector $g_{\mu\nu}$,
- A vector field K_μ living in the interacting sector $f_{\mu\nu}$ and with a non-canonical kinetic term.

The action

$$S = \int \mathrm{d}^4x \left\{ \sqrt{-g}\left(\frac{R-2\Lambda}{32\pi} - \rho_{\mathrm{b}} - \rho\right) + \sqrt{-\underline{g}}\left(\frac{\underline{R}-2\Lambda}{32\pi} - \underline{\rho}\right) \right.$$
$$\left. + \sqrt{-f}\left[\frac{\mathcal{R}-2\Lambda}{16\pi\varepsilon} + (j^{\mu} - \underline{j}^{\mu})K_{\mu} + \frac{a_0^2}{2\alpha}W(-\frac{H^{\mu\nu}H_{\mu\nu}}{2a_0^2})\right]\right\}$$

The action

$$S = \int \mathrm{d}^4x \left\{ \sqrt{-g}\left(\frac{R-2\Lambda}{32\pi} - \rho_\mathrm{b} - \rho\right) + \sqrt{-\underline{g}}\left(\frac{\underline{R}-2\Lambda}{32\pi} - \underline{\rho}\right) \right.$$
$$\left. + \sqrt{-f}\left[\frac{\mathcal{R}-2\Lambda}{16\pi\varepsilon} + (j^\mu - \underline{j}^\mu)K_\mu + \frac{a_0^2}{8\pi} W(-\frac{H^{\mu\nu}H_{\mu\nu}}{2a_0^2})\right]\right\}.$$

- The function W is determined phenomenologically to recover
  - MOND in the weak field limit,

$$W(X) = X - \frac{2}{3}X^{3/2} + \mathcal{O}(X^2)\,, \quad \text{when} \quad X \to 0\,,$$

  - and GR for strong fields

$$W(X) = A + \frac{B}{X^\alpha} + \mathcal{O}(X^{-\alpha-1})\,, \quad \alpha > 0\,.$$

- There is the same cosmological constant in all sectors in order to be in agreement with Λ-CDM.
- ε measures the strength of the interaction between the two sectors, will be assume very small in the (post-)Newtonian limit.

Equation of motion

Einstein field equations

$$\sqrt{-g}\,(G^{\mu\nu}+\Lambda g^{\mu\nu})+\frac{\sqrt{-f}}{\varepsilon}\,\mathcal{A}^{\mu\nu}_{\rho\sigma}\Big(\mathcal{G}^{\rho\sigma}+\Lambda f^{\rho\sigma}\Big)=16\pi\Big[\sqrt{-g}\,(T^{\mu\nu}_{\mathrm{b}}+T^{\mu\nu})$$
$$+\sqrt{-f}\,\mathcal{A}^{\mu\nu}_{\rho\sigma}\,\tau^{\rho\sigma}\Big]\,,$$

$$\sqrt{-\underline{g}}\,(\underline{G}^{\mu\nu}+\Lambda\underline{g}^{\mu\nu})+\frac{\sqrt{-f}}{\epsilon}\,\underline{\mathcal{A}}^{\mu\nu}_{\rho\sigma}\Big(\mathcal{G}^{\rho\sigma}+\Lambda f^{\rho\sigma}\Big)=16\pi\Big[\sqrt{-\underline{g}}\,\underline{T}^{\mu\nu}+\sqrt{-f}\,\underline{\mathcal{A}}^{\mu\nu}_{\rho\sigma}\,\tau^{\rho\sigma}\Big]$$

Equations of motion

$$\begin{aligned} a_b^\mu &= 0\,,\\ a^\mu &= u^\nu\,H_{\mu\nu}\,,\\ \underline{a}^\mu &= -\underline{u}^\nu\,H_{\mu\nu}\,. \end{aligned}$$

$$\mathcal{D}_\nu\,(W'\,H^{\mu\nu})=4\pi\,\big(j^\mu-\underline{j}^\mu\big)\;.$$

Linear order

Hypothesis: plasma-like solutions

The two fluid of DM particles slightly differ from an equilibrium configuration by small displacement vectors y^μ and $\underline{y}^\mu$,

$$\begin{aligned} j^\mu &= j_0^\mu + \mathcal{D}_\nu \left(j_0^\nu y_\perp^\mu - j_0^\mu y_\perp^\nu \right) + \mathcal{O}\left(y^2\right) , \\ \underline{j}^\mu &= j_0^\mu + \mathcal{D}_\nu \left(j_0^\nu \underline{y}_\perp^\mu - j_0^\mu \underline{y}_\perp^\nu \right) + \mathcal{O}\left(y^2\right) . \end{aligned}$$

- We work at linear order around $f_{\mu\nu}$:

$$\begin{aligned} g_{\mu\nu} &= f_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2) , \\ \underline{g}_{\mu\nu} &= f_{\mu\nu} - h_{\mu\nu} + \mathcal{O}(h^2) , \end{aligned}$$

- All perturbation variables are of the same order of magnitude

$$\nabla y \sim \nabla \underline{y} \sim h .$$

Linearised matter and gravitational fields

- Defining the relative displacement $\xi^\mu = y^\mu - \underline{y}^\mu$,

$$j^\mu - \underline{j}^\mu = \mathcal{D}_\nu \left(j_0^\nu \xi_\perp^\mu - j_0^\mu \xi_\perp^\nu \right)$$

- We insert it in the equation of motion for the vector field $\mathcal{D}_\nu \left(W' \, H^{\mu\nu} \right) = 4\pi \left(j^\mu - \underline{j}^\mu \right)$ and integrate it

$$W' H^{\mu\nu} = \alpha \left(j_0^\nu \xi_\perp^\mu - j_0^\mu \xi_\perp^\nu \right) .$$

$\Longrightarrow \tau^{\mu\nu} = \mathcal{O}(h^2)$ for weak fields.

- Taking the difference of the gravitational field equations we get

$$T_{\mathrm{b}}^{\mu\nu} = \mathcal{O}(h) .$$

Linearised equations of motion

$$\begin{aligned} a^\mu &= -4\pi \rho_0 \xi_\perp^\mu \, , \\ \underline{a}^\mu &= 4\pi \rho_0 \xi_\perp^\mu \, . \end{aligned}$$

Cosmological perturbations

We expand the two metrics around the FLRW background metric

$$\overset{\circ}{\mathrm{d}s}^{\,2} = a^2(\eta)\left(-\mathrm{d}\eta^2 + \gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right)\,,$$

Background equations

$$3\left(\mathcal{H}^2 + K\right) = \frac{\varepsilon\kappa}{\kappa+\varepsilon}\,\overset{\circ}{\rho}\,a^2 + \Lambda a^2\,,$$
$$\mathcal{H}^2 + 2\mathcal{H}' + K = \Lambda a^2\,.$$

- No baryons in the background,
- The two kinds of dark matter overlap in the backround,
- We recover the standard term for the cosmological constant.

First order cosmological perturbations

Method

- We write the first order cosmological perturbations for the two metrics and the matter variables:
  - in the g-sector : $\{\Psi,\ \Phi,\ \Phi^i,\ E^{ij}\}$, $\{\delta^F,\ V,\ V^i\}$ and $\{\rho_b,\ u_b^\mu\}$,
  - in the $\underline{g}$-sector : $\{\underline{\Psi},\ \underline{\Phi},\ \underline{\Phi}^i,\ \underline{E}^{ij}\}$ and $\{\underline{\delta}^F,\ \underline{V},\ \underline{V}^i\}$,
  - in the f-sector : $\xi_\perp^\mu = \big(0,\ D^i z + z^i\big)$.

- Then we compare the ordinary sector $g_{\mu\nu}$ on which ordinary matter moves with Λ-CDM scenario $\longrightarrow$ identify the observed dark matter variables in the sector $g_{\mu\nu}$.

First order cosmological perturbations

We introduce new effective dark matter variables:

$$\overset{\circ}{\rho}_{\mathrm{DM}} = \frac{2\varepsilon}{1+\varepsilon}\,\overset{\circ}{\rho}\,, \qquad \delta^F_{\mathrm{DM}} = \delta^F - \frac{1}{2\varepsilon}\left(\Delta z - (A - \underline{A})\right)\,,$$

$$V_{\mathrm{DM}} = V + \frac{1}{2\varepsilon}\left(z' + \tfrac{1}{2}(B - \underline{B})\right)\,, \qquad V^i_{\mathrm{DM}} = V^i + \frac{1}{2\varepsilon}\left({z'}^i + \frac{1}{2}(B^i - \underline{B}^i)\right)\,,$$

and recover the standard continuity and Euler equations for the effective dark matter,

$$\begin{aligned}
{\delta'}^{\mathrm{F}}_{\mathrm{DM}} + \Delta V_{\mathrm{DM}} &= 0\,,\\
V'_{\mathrm{DM}} + \mathcal{H} V_{\mathrm{DM}} + \Psi &= 0\,,\\
{V'}^i_{\mathrm{DM}} + \mathcal{H} V^i_{\mathrm{DM}} &= 0\,,
\end{aligned}$$

and $\rho'_{\mathrm{B}} + 3\mathcal{H}\rho_{\mathrm{B}} = 0$ for the baryons.

First order cosmological perturbations

Gravitational perturbation equations

$$
\begin{aligned}
&\Delta\Psi - 3\mathcal{H}^2 X = 4\pi\, a^2 \Big(\rho_{\mathrm{B}} + \overset{\circ}{\rho}_{\mathrm{DM}}\, \delta^{\mathrm{F}}_{\mathrm{DM}}\Big)\,,\\
&\Psi - \Phi = 0\,,\\
&\Psi' + \mathcal{H}\Phi = -4\pi\, a^2 \Big(\overset{\circ}{\rho}_{\mathrm{DM}}\, V_{\mathrm{DM}}\Big)\,,\\
&\mathcal{H}X' + (\mathcal{H}^2 + 2\mathcal{H}')X = 0\,,\\
&(\Delta + 2K)\Phi^i = -16\pi\, a^2 \Big(\overset{\circ}{\rho}_{\mathrm{DM}}\, V^i_{\mathrm{DM}}\Big)\,,\\
&{\Phi'}^i + 2\mathcal{H}\Phi^i = 0\,,\\
&{E''}^{ij} + 2\mathcal{H}{E'}^{ij} + (2K - \Delta)E^{ij} = 0\,,
\end{aligned}
$$

and similar equations in the $\underline{g}$-sector.

- This system of equations is well defined and each variable can be determined.

Non-relativistic limit when $\varepsilon \ll 1$

$$g_{00} = -1 + \frac{2U}{c^2} + \mathcal{O}(c^{-4})\,, \qquad g_{0i} = \mathcal{O}(c^{-3})\,, \qquad g_{ij} = \delta_{ij} + \mathcal{O}(c^{-2})\,,$$
$$\underline{g}_{00} = -1 + \frac{2\underline{U}}{c^2} + \mathcal{O}(c^{-4})\,, \qquad \underline{g}_{0i} = \mathcal{O}(c^{-3})\,, \qquad \underline{g}_{ij} = \delta_{ij} + \mathcal{O}(c^{-2})\,,$$
$$A^\mu = (\frac{\phi}{c^2}, \mathbf{0})\,, \qquad \xi_\perp^\mu = (0, \lambda^i)\,.$$

Poisson equation

$$U + \underline{U} = 0\,,$$
$$\Delta U = -4\pi\left(\rho_b^* + \rho^* - \underline{\rho}^*\right)\,.$$

Non-relativistic equations of motion

$$\frac{\mathrm{d}\mathbf{v}_b}{\mathrm{d}t} = \boldsymbol{\nabla} U\,, \qquad \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \boldsymbol{\nabla}(U + \phi)\,, \qquad \frac{\mathrm{d}\underline{\mathbf{v}}}{\mathrm{d}t} = -\boldsymbol{\nabla}(U + \phi)\,.$$

Non-relativistic limit in the case $\varepsilon \ll 1$

- Plasma-like solutions $\rho^* = \rho_0^* - \frac{1}{2}\boldsymbol{\nabla}\cdot\mathbf{P}$ and $\underline{\rho}^* = \rho_0^* + \frac{1}{2}\boldsymbol{\nabla}\cdot\mathbf{P}$, where P is the gravitational polarization field,

$$\mathrm{P} = \rho_0^*\,\boldsymbol{\lambda} = \frac{W'}{4\pi}\,\boldsymbol{\nabla}U\,.$$

- From the Poisson equation we recover the MOND formula

$$\boldsymbol{\nabla}\cdot\left[\mu\left(\frac{|\boldsymbol{\nabla}U|}{a_0}\right)\boldsymbol{\nabla}U\right] = -4\pi G\rho_b\,,$$

with $\mu = 1 - W' = \dfrac{|\boldsymbol{\nabla}U|}{a_0}$ in the weak field regime.

- We recover a dipolar dark matter medium which oscillates at the plasma frequency $\omega = \sqrt{\frac{8\pi\rho_0^*}{W'}}$,

$$\frac{\mathrm{d}^2\boldsymbol{\lambda}}{\mathrm{d}t^2} + \omega^2\boldsymbol{\lambda} = 2\boldsymbol{\nabla}U\,.$$

Solar system tests

Strong field regime

- To recover GR in the strong field regime ($X \to \infty$), we impose

$$W(X) = A + \frac{B}{X^\alpha} + \mathcal{O}\Big(\frac{1}{X^{\alpha+1}}\Big)\,, \qquad \alpha < 0\,.$$

- and expand both metrics to 2^{nd} order in h:
  $g_{\mu\nu} = f_{\mu\nu} + h_{\mu\nu} + \frac{1}{2} h_{\mu\rho} h^\rho{}_\nu$ and $\underline{g}_{\mu\nu} = f_{\mu\nu} - h_{\mu\nu} + \frac{1}{2} h_{\mu\rho} h^\rho{}_\nu$,

Post-Newtonian limit

- We obtain the same parametrized post-Newtonian parameters as in GR, $\beta = 1$, $\gamma = 1$, all others are zero.

Conclusion

Results

- It correctly reproduces the phenomenology of MOND in the non-relativistic limit provided that $\varepsilon \ll 1$,
- It passes solar system tests (same ppN parameters as GR),
- The model agrees with the standard Λ-CDM paradigm at cosmological scales, provided that the baryons are treated perturbatively.

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Remarks and perspectives

- Investigate the case where we expand the metrics around two different FLRW background,
- The arbitrary function W should be derived from a more fundamental theory.