Dynamics and observational constraints on Brans-Dicke cosmological model

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Published papers

- OH, M. Kamionka, M. Szydłowski, *Dynamics and cosmological constraints on Brans-Dicke cosmology*, `arXiv:1404.7112 [astro-ph.CO]`
- OH, M. Szydłowski, *Dynamical complexity of the Brans-Dicke cosmology*, JCAP12(2013)016, `arXiv:1310.1961 [gr-qc]`
- OH, M. Szydłowski, *Brans-Dicke theory and the emergence of ΛCDM model*, Phys. Rev. D88, 064018 (2013), `arXiv:1304.3300 [gr-qc]`

The Brans-Dicke cosmology

The action for the Brans-Dicke theory in so-called Jordan frame is in the following form

$$S = \int \mathrm{d}^4x\sqrt{-g}\left\{\phi R - \frac{\omega_{\text{BD}}}{\phi}\nabla^{\alpha}\phi\nabla_{\alpha}\phi - 2\,V(\phi)\right\} + 16\pi S_m \qquad (1)$$

where the barotropic matter is described by

$$S_m = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}_m\,, \qquad (2)$$

and ω_{BD} is a dimensionless parameter of the theory.

Field Equations

Variation of the total action (1) with respect to the metric tensor $\delta S/\delta g^{\mu\nu} = 0$ gives the field equations for the theory

$$\phi\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) - \frac{\omega_{\mathrm{BD}}}{\phi}\left(\nabla_{\mu}\phi\,\nabla_{\nu}\phi - \frac{1}{2}\,g_{\mu\nu}\,\nabla^{\alpha}\phi\,\nabla_{\alpha}\phi\right) + \\ + g_{\mu\nu}V(\phi) + (g_{\mu\nu}\Box\phi - \nabla_{\mu}\nabla_{\nu}\phi) = 8\pi\, T^{(m)}_{\mu\nu}\,, \tag{3}$$

where the energy momentum tensor for the matter content is

$$T^{(m)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\,\frac{\delta}{\delta g^{\mu\nu}}\left(\sqrt{-g}\mathcal{L}_m\right)\,. \tag{4}$$

FRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\Big(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\Big) \quad (5)$$

the energy conservation condition

$$3H^2 = \frac{\omega_{\mathrm{BD}}}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{V(\phi)}{\phi} - 3H\frac{\dot{\phi}}{\phi} + \frac{8\pi}{\phi}\rho_m \quad (6)$$

the acceleration equation

$$\dot{H} = -\frac{\omega_{\mathrm{BD}}}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{1}{3+2\omega_{\mathrm{BD}}}\frac{2\,V(\phi) - \phi V'(\phi)}{\phi} + 2H\frac{\dot{\phi}}{\phi} - \frac{8\pi}{\phi}\rho_m\frac{2+\omega_{\mathrm{BD}}(1+w_m)}{3+2\omega_{\mathrm{BD}}} \quad (7)$$

the dynamical equation for the BD scalar

$$\ddot{\phi} + 3H\dot{\phi} = 2\frac{2\,V(\phi) - \phi\,V'(\phi)}{3+2\omega_{\mathrm{BD}}} + 8\pi\rho_m\frac{1-3w_m}{3+2\omega_{\mathrm{BD}}} \quad (8)$$

Dynamical variables

In what follows we introduce following energy phase space variables

$$x \equiv \frac{\dot{\phi}}{H\phi}, \qquad y \equiv \sqrt{\frac{V(\phi)}{3\phi}}\frac{1}{H}, \qquad \lambda \equiv -\phi\frac{V'(\phi)}{V(\phi)}$$

the energy conservation condition (6) can be presented as

$$\Omega_m = \frac{8\pi\rho_m}{3H^2\phi} = 1 + x - \frac{\omega_{\rm BD}}{6}x^2 - y^2\,, \tag{9}$$

and the acceleration equation (7)

$$\begin{aligned}\frac{\dot{H}}{H^2} = 2x - \frac{\omega_{\rm BD}}{2}x^2 - \frac{3}{3+2\omega_{\rm BD}}y^2(2+\lambda) \\ - 3\left(1 + x - \frac{\omega_{\rm BD}}{6}x^2 - y^2\right)\frac{2+\omega_{\rm BD}(1+w_m)}{3+2\omega_{\rm BD}}\,,\end{aligned} \tag{10}$$

Dynamical system

$$
\begin{aligned}
x' &= -3x - x^2 - x\frac{\dot{H}}{H^2} + \frac{6}{3+2\omega_{\mathrm{BD}}}y^2(2+\lambda) + \\
&+ 3\left(1 + x - \frac{\omega_{\mathrm{BD}}}{6}x^2 - y^2\right)\frac{1-3w_m}{3+2\omega_{\mathrm{BD}}}\,, \\
y' &= -y\left(\frac{1}{2}x(1+\lambda) + \frac{\dot{H}}{H^2}\right)\,, \\
\lambda' &= x\lambda\Big(1 - \lambda(\Gamma - 1)\Big)\,,
\end{aligned}
\tag{11}
$$

where $()' = \frac{\mathrm{d}}{\mathrm{d}\ln a}$ and

$$
\Gamma = \frac{V''(\phi)V(\phi)}{V'(\phi)^2}\,.
$$

From now on we will assume that $\Gamma = \Gamma(\lambda)$. The critical points of the system (11) depend on the explicit form of the $\Gamma(\lambda)$ function. One can notice that the single critical point $(x^* = 0\,, y^* = \pm 1\,, \lambda^* = -2)$ do not depend on the assumed $\Gamma(\lambda)$. Additionally, the acceleration equation (10) calculated at this point vanishes, giving rise to the deSitter expansion.

Invariant manifolds

For an arbitrary power-law potential functions of the type $V(\phi) = V_0\,\phi^n$ system reduces to a 2-dimensional one, i.e. the power-law potentials are invariant submanifolds of the system.

$$\lambda = -n = \text{const.}\,, \quad \Gamma(\lambda) = 1 - \frac{1}{n} \tag{12}$$

Simple inspection of the acceleration equation (10) and the system (11) for the power-law potential $\lambda = -n$ confirms that the de Sitter expansion is possible only for the quadratic $n = 2$ or the linear $n = 1$ potential function. In the Einstein frame one has

$$\widetilde{m}^2 = \frac{32\pi}{(3+2\omega_{\rm BD})G} V_0 (n-2)^2\,\phi^{n-2}\,, \tag{13}$$

while in the Jordan frame

$$m^2 = \frac{2}{3+2\omega_{\rm BD}} V_0\,n(n-2)\,\phi^{n-1}\,. \tag{14}$$

For the linear potential function the scalar field ϕ has a finite range in the Einstein frame and is tachyonic in the Jordan frame. Only the quadratic potential function with $n = 2$ leads to the BD field ϕ which has an infinite range in both frames.

A quadratic potential function

We present a detailed dynamical analysis of the system (11) with a quadratic potential function $V(\phi) \propto \phi^2$. Within this assumption we have $\lambda = -2$ and dynamics reduces to a $2D$ dynamical system. The acceleration equation is :

$$\frac{\dot{H}}{H^2} = -\frac{\omega_{\text{BD}}}{2}x^2 + 2x - 3\left(1 + x - \frac{\omega_{\text{BD}}}{6}x^2 - y^2\right)\frac{2 + \omega_{\text{BD}}(1 + w_m)}{3 + 2\omega_{\text{BD}}}, \quad (15)$$

and the effective equation of state parameter is :

$$w_{\text{eff}} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}\,. \quad (16)$$

Dynamical system for $V(\phi) \propto \phi^2$

$$
\begin{aligned}
x' &= -3x\Bigg\{1 + x - \frac{\omega_{\mathrm{BD}}}{6}x^2 - \\
&\qquad - \left(1 + x - \frac{\omega_{\mathrm{BD}}}{6}x^2 - y^2\right)\frac{2 + \omega_{\mathrm{BD}}(1 + w_m)}{3 + 2\omega_{\mathrm{BD}}}\Bigg\} \\
&\quad +3\left(1 + x - \frac{\omega_{\mathrm{BD}}}{6}x^2 - y^2\right)\frac{1 - 3w_m}{3 + 2\omega_{\mathrm{BD}}}, \qquad (17) \\
y' &= 3y\Bigg\{-\frac{1}{2}x + \frac{\omega_{\mathrm{BD}}}{6}x^2 + \\
&\qquad + \left(1 + x - \frac{\omega_{\mathrm{BD}}}{6}x^2 - y^2\right)\frac{2 + \omega_{\mathrm{BD}}(1 + w_m)}{3 + 2\omega_{\mathrm{BD}}}\Bigg\}.
\end{aligned}
$$

Critical point $x_2^* = \frac{1-3w_m}{1+\omega_{\rm BD}(1-w_m)}$, $y_2^* = 0$ with effective equation of state parameter

$$w_{\rm eff}\Big|_2^* = w_m + \frac{1}{3}\frac{1-3w_m}{1+\omega_{\rm BD}(1-w_m)} .$$

Using the linearized solutions in the vicinity of this critical point

$$x_2(a) = x_2^* + \Delta x \left(\frac{a}{a_2^{(i)}}\right)^{\lambda_1} , \qquad (18a)$$

$$y_2(a) = \Delta y \left(\frac{a}{a_2^{(i)}}\right)^{\lambda_2} , \qquad (18b)$$

where the eigenvalues of the linearization matrix are

$$\lambda_1 = -\frac{3}{2}(1-w_m) - \frac{1}{2}x_2^* , \qquad \lambda_2 = \frac{3}{2}(1+w_m) + x_2^*$$

and $\Delta x = x_2^{(i)} - x_2^*$, $\Delta y = y_2^{(i)} - y_2^*$ are the initial conditions, and $a_2^{(i)}$ is the initial value of the scale factor near the critical point.

One can easy obtain corresponding formula for the Hubble function. The equation (15) up to linear terms in initial conditions reduces to

$$\frac{\mathrm{d}\ln H^2}{\mathrm{d}\tau} \approx -3(1+w_m) - x_2^*$$

and after integration and up to linear terms in initial conditions we obtain

$$\left(\frac{H(a)}{H(a_2^{(i)})}\right) \approx \left(\frac{a}{a_2^{(i)}}\right)^{-3(1+w_m)} \left(\frac{a}{a_2^{(i)}}\right)^{-x_2^*} \tag{19}$$

where $x_2^* = \frac{1-3w_m}{1+\omega_{\mathrm{BD}}(1-w_m)}$ is the coordinate of the critical point.

Critical point: $x_3^* = 0$, $y_3^* = \pm 1$ with effective equation of state parameter

$$w_{\text{eff}}\Big|_3^* = -1$$

linearized solutions are

$$\begin{aligned}
x_3(a) &= \frac{1}{w_m}\frac{1+2\omega_{\text{BD}}w_m}{3+2\omega_{\text{BD}}}\left[\Delta x - 2y_3^*\frac{1-3w_m}{1+2\omega_{\text{BD}}w_m}\Delta y\right]\left(\frac{a}{a_3^{(i)}}\right)^{\lambda_1} - \\
&\quad -\frac{1}{w_m}\frac{1-3w_m}{3+2\omega_{\text{BD}}}\left[\Delta x - 2y^*\Delta y\right]\left(\frac{a}{a_3^{(i)}}\right)^{\lambda_2}, \\
y_3(a) &= y_3^* + \frac{1}{2y_3^* w_m}\frac{1+2\omega_{\text{BD}}w_m}{3+2\omega_{\text{BD}}}\left\{\left[\Delta x - 2y_3^*\frac{1-3w_m}{1+2\omega_{\text{BD}}w_m}\Delta y\right]\left(\frac{a}{a_3^{(i)}}\right)^{\lambda_1} - \right. \\
&\quad \left. -\left[\Delta x - 2y^*\Delta y\right]\left(\frac{a}{a_3^{(i)}}\right)^{\lambda_2}\right\}
\end{aligned}$$

where $\lambda_1 = -3$ and $\lambda_2 = -3(1+w_m)$ are the eigenvalues of the linearization matrix and $\Delta x = x_3^{(i)} - x_3^*$, $\Delta y = y_3^{(i)} - y_3^*$ are the initial conditions.

Using the linearized solutions we obtain the following form of the Hubble function in the vicinity of the critical point under considerations

$$\left(\frac{H(a)}{H(a_0)}\right)^2 \approx 1-\Omega_{DM,0}-\Omega_{M,0}+\Omega_{DM,0}\left(\frac{a}{a_0}\right)^{-3}+\Omega_{M,0}\left(\frac{a}{a_0}\right)^{-3(1+w_m)}, \tag{21}$$

where

$$\Omega_{DM,0} = -\frac{4}{3w_m}\frac{1+2\omega_{\text{BD}}w_m}{3+2\omega_{\text{BD}}}\left\{\Delta x - 2y_3^*\frac{1-3w_m}{1+2\omega_{\text{BD}}w_m}\Delta y\right\}\left(\frac{a_0}{a_3^{(i)}}\right)^{-3},$$

$$\Omega_{M,0} = \frac{2}{3w_m(1+w_m)}\frac{2+3\omega_{\text{BD}}w_m(1+w_m)}{3+2\omega_{\text{BD}}}\left\{\Delta x - 2y_3^*\Delta y\right\}\left(\frac{a_0}{a_3^{(i)}}\right)^{-3(1+w_m)}.$$

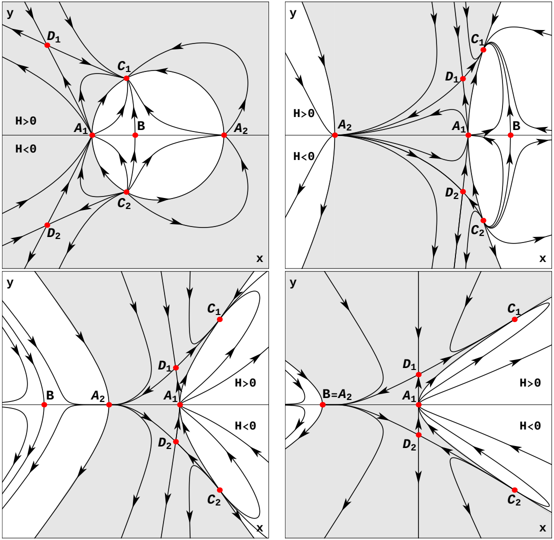


Figure : The phase plane portrait for system under considerations filled with the dust matter $w_m = 0$ and : $\omega_{\rm BD} > 0$ ($\omega_{\rm BD} = 5$), $-1 < \omega_{\rm BD} < 0$ ($\omega_{\rm BD} = -1/2$), $-4/3 < \omega_{\rm BD} < -1$ ($\omega_{\rm BD} = -7/6$), $\omega_{\rm BD} = -4/3$.

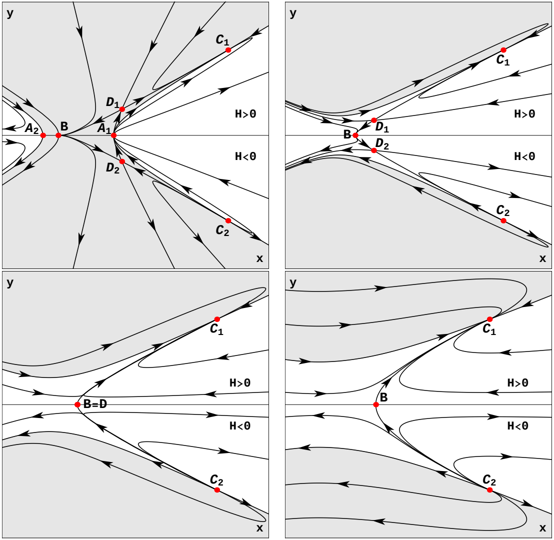


Figure : $w_m = 0$: $-3/2 < \omega_{\rm BD} < -4/3$, $-5/3 < \omega_{\rm BD} < -3/2$, $\omega_{\rm BD} = -5/3$, $\omega_{\rm BD} < -5/3$

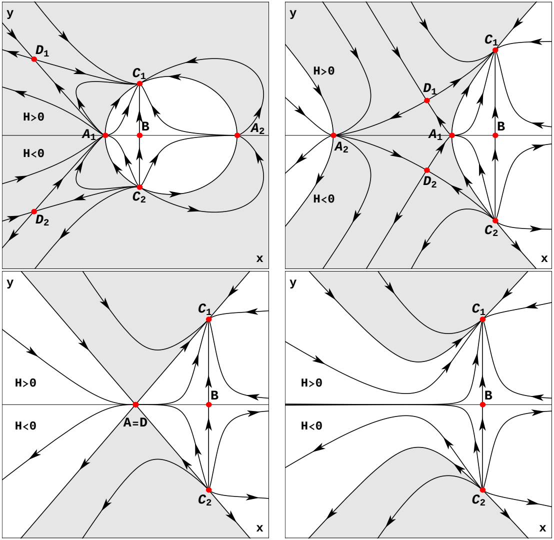


Figure : The phase plane portraits for the model filled with the relativistic matter ($w_m = 1/3$) and $\omega_{\mathrm{BD}} > 0$, $-3/2 < \omega_{\mathrm{BD}} < 0$, $\omega_{\mathrm{BD}} = -3/2$, $\omega_{\mathrm{BD}} < -3/2$.

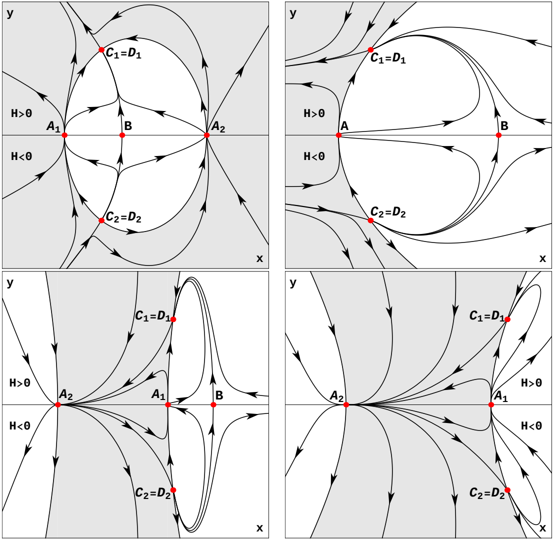


Figure : The phase plane diagrams for system (17) filled with the cosmological constant $w_m = -1$ and $\omega_{\mathrm{BD}} > 0$ (top left), $\omega_{\mathrm{BD}} = 0$ (top right), $0 > \omega_{\mathrm{BD}} > -1/2$ (bottom left), $\omega_{\mathrm{BD}} = -1/2$ (bottom right).

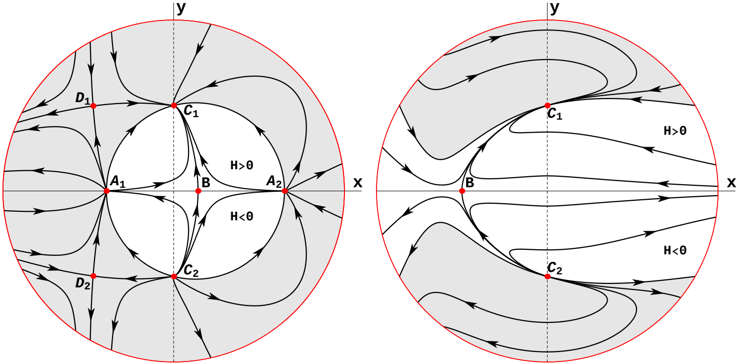


Figure : Diagrams of the evolutional paths in phase space compactified with circle at infinity for the model filled with dust matter $w_m = 0$ and : $\omega_{\text{BD}} > 0$ (left) , $\omega_{\text{BD}} < -5/3$ (right). Diagrams plotted for fixed values ($\omega_{\text{BD}} = 5$ and $\omega_{\text{BD}} = -2$), all phase space diagram in given range are topologically equivalent. The circle at infinity consists of bounces during the evolution of the universe.

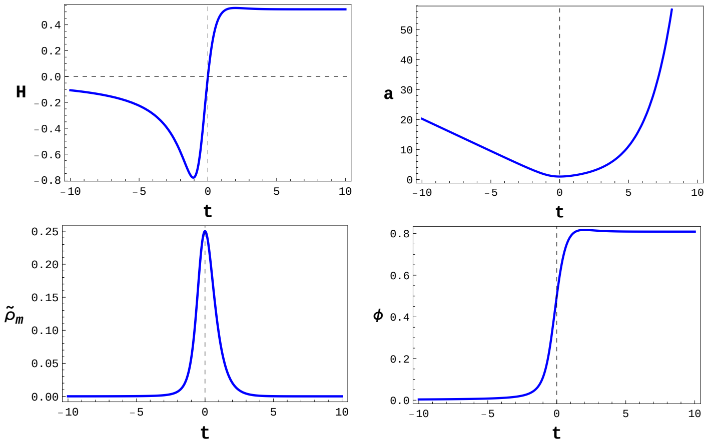


Figure : Cosmological time evolution of the Hubble function H, the scale factor a, the barotropic matter density $\widetilde{\rho}_m = 8\pi\rho_m$ and the scalar field ϕ for sample evolutional trajectory with a bounce and the model parameters $\omega_{\rm BD} = -2$ and $w_m = 0$. The initial conditions taken at the bounce are : $\phi_{(i)} = 1/2$, $a_{(i)} = 1$, $H_{(i)} = 0$ and $\dot{\phi}_{(i)} = 1/2$.

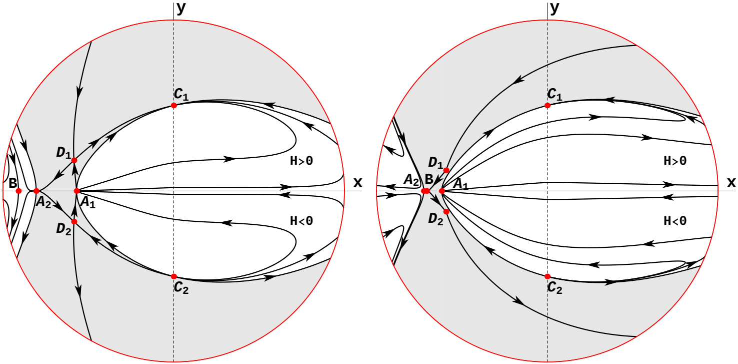


Figure : Diagrams of the evolutional paths in phase space compactified with circle at infinity for the model filled with dust matter $w_m = 0$ and : $-4/3 < \omega_{\rm BD} < -1$ (left) , $-3/2 < \omega_{\rm BD} < -4/3$ (right). The circle at infinity consists of bounces during the evolution of the universe. For clarity of the presentation we omitted trajectories in the nonphysical region.

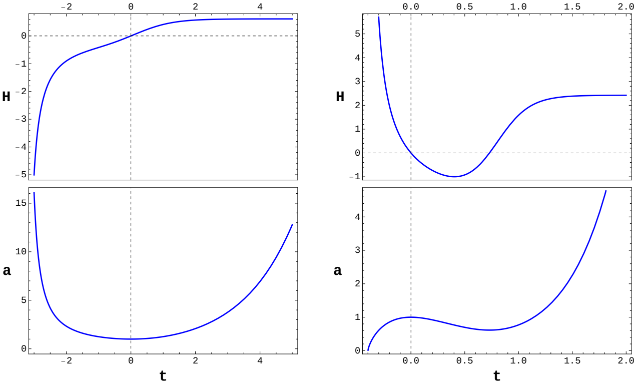


Figure : Cosmological time evolution of the Hubble function H, the scale factor a for sample evolutional trajectories with one bounce (left panel $-4/3 < \omega_{\rm BD} < -1$) and two bounces (right panel $-3/2 < \omega_{\rm BD} < -4/3$) in the model with dust matter $w_m = 0$. The initial conditions taken at the bounce are : $H_{(i)} = 0$, $\phi_{(i)} = 1/2$ and $\dot{\phi}_{(i)} = 1/2$ (left panel) and $\dot{\phi}_{(i)} = 1$ (right panel). Note the coexistence of the bounces and the singularity in the second case.

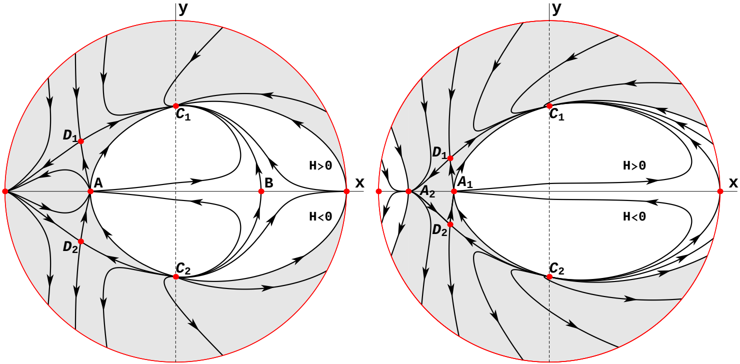


Figure : Diagrams of the evolutional paths in the phase space compactified with circle at infinity for the model filled with dust matter $w_m = 0$ and : $\omega_{\rm BD} = 0$ (left) , $\omega_{\rm BD} = -1$ (right). In both cases we have two critical points at infinity $x \to \pm\infty$, $y \to 0$. In the first case, one is stable during expansion and the second one is unstable during expansion. In the second case the critical points are of a saddle type.

What is the value of ω_{BD} ?

- $f(R)$ theories of gravity

$$S = \int \mathrm{d}^4x\sqrt{-g}f(R) \quad \rightarrow \quad S = \int \mathrm{d}^4x\Big\{\phi R - 2V(\phi)\Big\} \qquad \omega_{\mathrm{BD}} = 0$$

- low-energy limit of the bosonic string theory

$$S = \int \mathrm{d}^4x\sqrt{-g}e^{-2\Phi}\Big\{R + 4\nabla_\alpha\Phi\nabla^\alpha\Phi - \Lambda\Big\} \quad \rightarrow$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big\{\phi R + \frac{1}{\phi}\nabla_\alpha\phi\nabla^\alpha\phi - \Lambda\phi\Big\} \qquad \omega_{\mathrm{BD}} = -1$$

An arbitrary potential function

The critical point ($x^* = 0$, $y^* = 1$, $\lambda^* = -2$) corresponds to the de Sitter expansion.
The eigenvalues of the linearization matrix are

$$l_1 = -3(1 + w_m)\,,$$
$$l_{2,3} = -\frac{3}{2}\left(1 \pm \sqrt{\frac{3 + 2\omega_{\mathrm{BD}} + \delta}{3 + 2\omega_{\mathrm{BD}}}}\right)\,, \tag{22}$$

where δ parameter is defined as

$$\delta = \frac{8}{3}\lambda^*\big(1 - \lambda^*(\Gamma(\lambda^*) - 1)\big) = \frac{16}{3}\big(1 - 2\,\Gamma^*\big)\,, \tag{23}$$

and depends on the second derivative of the potential function at the de Sitter state.

Model 1

The first case, characterized by the purely real eigenvalues, we make the following substitution

$$\frac{\delta}{3+2\omega_{\text{BD}}} = \frac{4}{9}n(n-3)\,. \tag{24}$$

The Hubble function is

$$\left(\frac{H(a)}{H(a_0)}\right)^2 = \Omega_{\Lambda,0} + \Omega_{M,0}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{n,0}\left(\frac{a}{a_0}\right)^{-n} + \Omega_{3n,0}\left(\frac{a}{a_0}\right)^{-3+n}, \tag{25}$$

where

$$\Omega_{M,0} = \left(1-\frac{16}{3\delta}\right)\Omega_{bm,0}\,,$$

and $\Omega_{n,0}$, $\Omega_{3n,0}$ are functions of the initial conditions Δx, Δy, $\Delta\lambda$ and

$$\Omega_{\Lambda,0} = 1 - \Omega_{M,0} - \Omega_{n,0} - \Omega_{3n,0}\,. \tag{26}$$

Model 2

For the second type of behavior in the vicinity of the de Sitter state we make the following substitution

$$\frac{\delta}{3+2\omega_{\text{BD}}} = -\frac{1}{9}(9+4n^2)\,, \tag{27}$$

The Hubble function is

$$\left(\frac{H(a)}{H(a_0)}\right)^2 = \Omega_{\Lambda,0} + \Omega_{M,0}\left(\frac{a}{a_0}\right)^{-3} + \left(\frac{a}{a_0}\right)^{-3/2}\left(\Omega_{cos,0}\cos\left(n\ln\left(\frac{a}{a_0}\right)\right) + \Omega_{sin,0}\sin\left(n\ln\left(\frac{a}{a_0}\right)\right)\right), \tag{28}$$

where

$$\Omega_{M,0} = \left(1-\frac{16}{3\delta}\right)\Omega_{bm,0}\,,$$

and $\Omega_{cos,0}$, $\Omega_{sin,0}$ are functions of the initial conditions Δx, Δy, $\Delta\lambda$ and

$$\Omega_{\Lambda,0} = 1 - \Omega_{M,0} - \Omega_{cos,0} \tag{29}$$

ΛCDM nested within

Carefully choosing the initial conditions for the linearized solutions

$$\Delta x = \frac{4}{\delta}\Omega_{bm,i}\,, \qquad \Delta\lambda = -\frac{1}{2}\Omega_{bm,i}\,, \tag{30}$$

where up to linear terms in initial conditions $\Omega_{bm,i} = \Delta x - 2\Delta y$, then in (25) we have $\Omega_{n,0} = \Omega_{3n,0} = 0$ and in (28) we have $\Omega_{cos,0} = \Omega_{sin,0} = 0$ and the resulting form of the Hubble function is

$$\left(\frac{H(a)}{H(a_0)}\right)^2 \approx 1 - \Omega_{M,0} + \Omega_{M,0}\left(\frac{a}{a_0}\right)^{-3}\,, \tag{31}$$

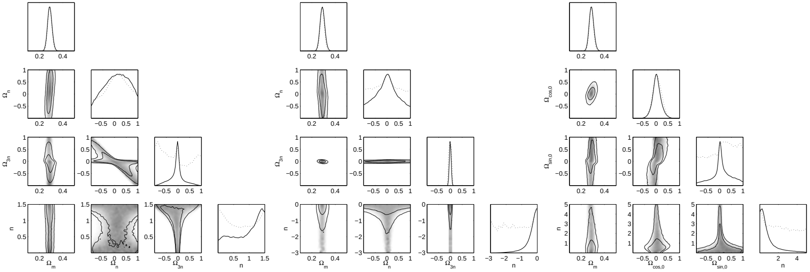
where

$$\Omega_{M,0} = \left(1 - \frac{16}{3\delta}\right)\Omega_{bm,0}\,. \tag{32}$$

This Hubble function describes the ΛCDM model with direct interpretation of the second term in the brackets as proportional to density parameter of the dark matter in the model

$$\Omega_{dm,0} = -\frac{16}{3\delta}\Omega_{bm,0}\,. \tag{33}$$

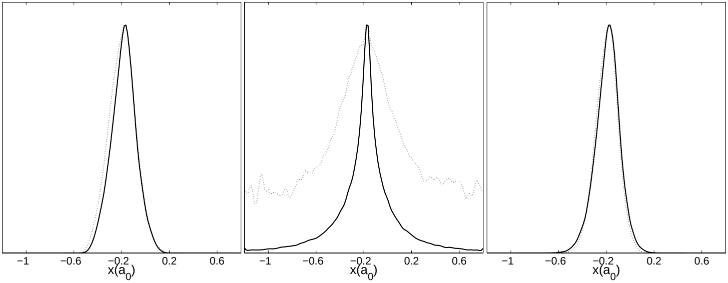
Confidence levels



Derived quantities

$$x(a_0) = \frac{3}{4}\left(\Omega_{bm,0} - \Omega_{M,0}\right) - \frac{n}{n+1}\Omega_{n,0} - \frac{n-3}{n-4}\Omega_{3n,0}\,,$$

$$x(a_0) = \frac{3}{4}\left(\Omega_{bm,0} - \Omega_{M,0}\right) + \frac{2}{4n^2+25}\left(5\,\Omega_{cos,0} + 2n\,\Omega_{sin,0}\right) - \Omega_{cos,0}\,.$$



Derived quantities

$$\omega_{\rm BD} = -\frac{3}{2} + \frac{6}{n(n-3)}\frac{\Omega_{bm,0}}{\Omega_{bm,0}-\Omega_{M,0}}, \quad \omega_{\rm BD} = -\frac{3}{2} - \frac{24}{9+4n^2}\frac{\Omega_{bm,0}}{\Omega_{bm,0}-\Omega_{M,0}}.$$

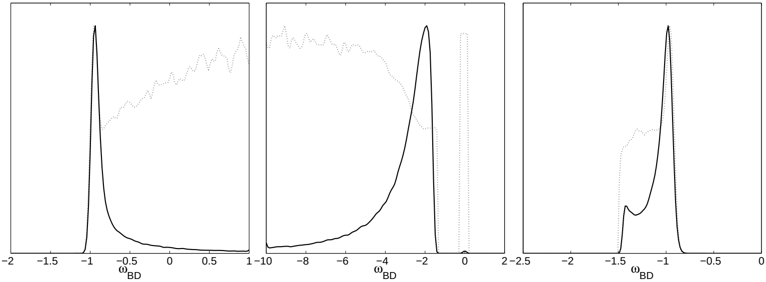


Table : Values of the Brans-Dicke parameter ω_{BD} calculated for the mean of marginalized posterior PDF with 68% confidence level and the best fit parameters of the models.

Union2.1+H(z)+AP+BAO		
	ω_{BD}	
	mean	best fit
model 1a	$-0.8681^{+0.1407}_{-0.0948}$	-0.9782
model 1b	$-1.9499^{+0.0988}_{-0.6576}$	-1.7817
model 2	$-1.2219^{+0.1478}_{-0.0450}$	-1.0646

Conclusions

- We have translated the geometrical approach to the dark energy and dark matter problems in to the substantial approach which in order can be used to test and select the cosmological models by astronomical data.
- In the vicinity of the critical point corresponding to the deSitter state, for carefully chosen initial conditions, we have obtained the corresponding form of the Hubble function which is indistinguishable from the standard cosmological ΛCDM model.
- We shown that in the models with the de Sitter state in the form of a stable node or a sink type critical point vales of the $\omega_{\rm BD}$ parameter close to the value suggested by the low-energy limit of the bosonic string theory are favored.