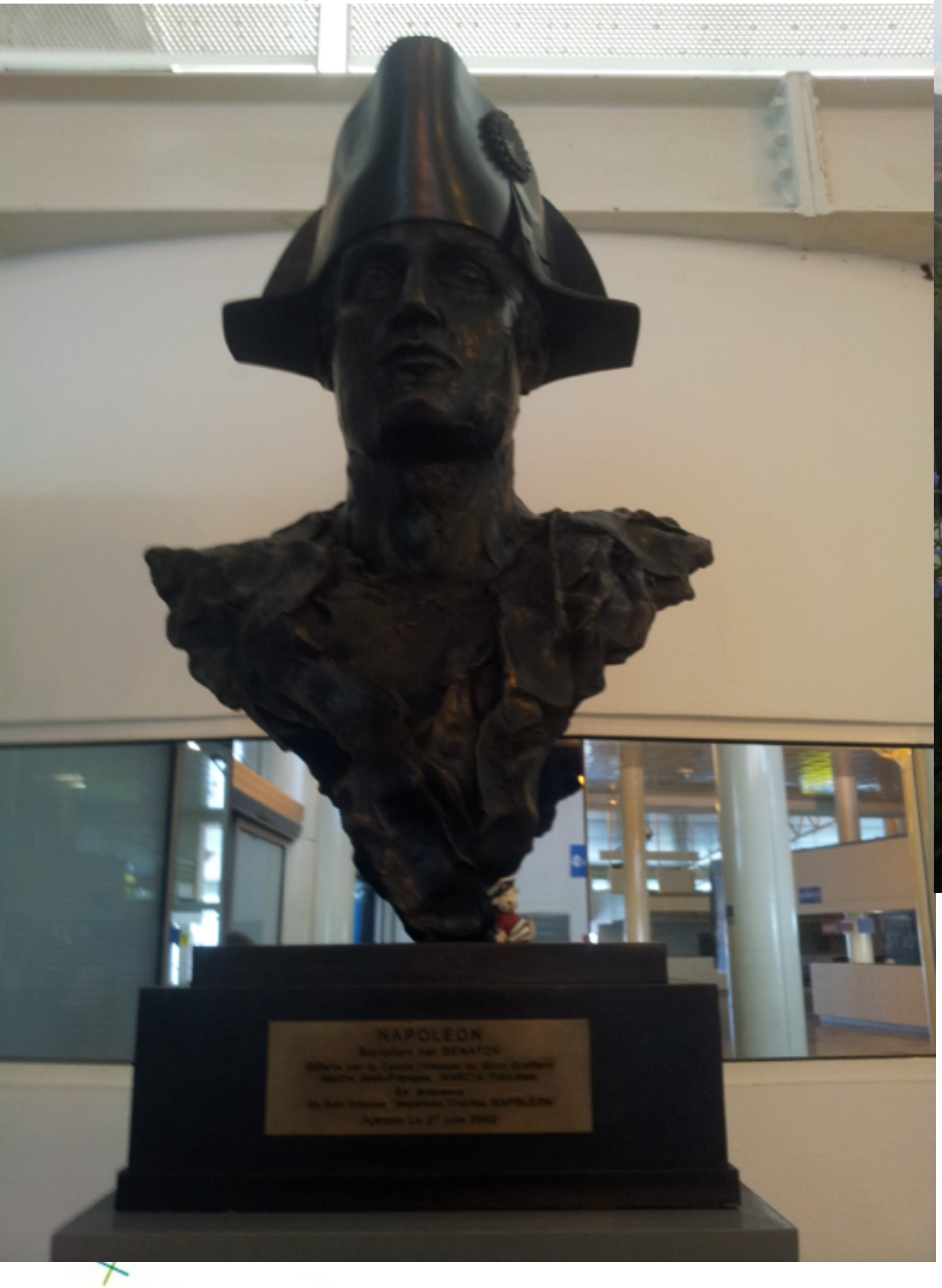
Testing quantum mechanics with cosmology?



Patrick Peter

Institut d'Astrophysique de Paris

GRεCO

Cargèse - 22 Sept. 2014

Quantum mechanics of closed systems

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

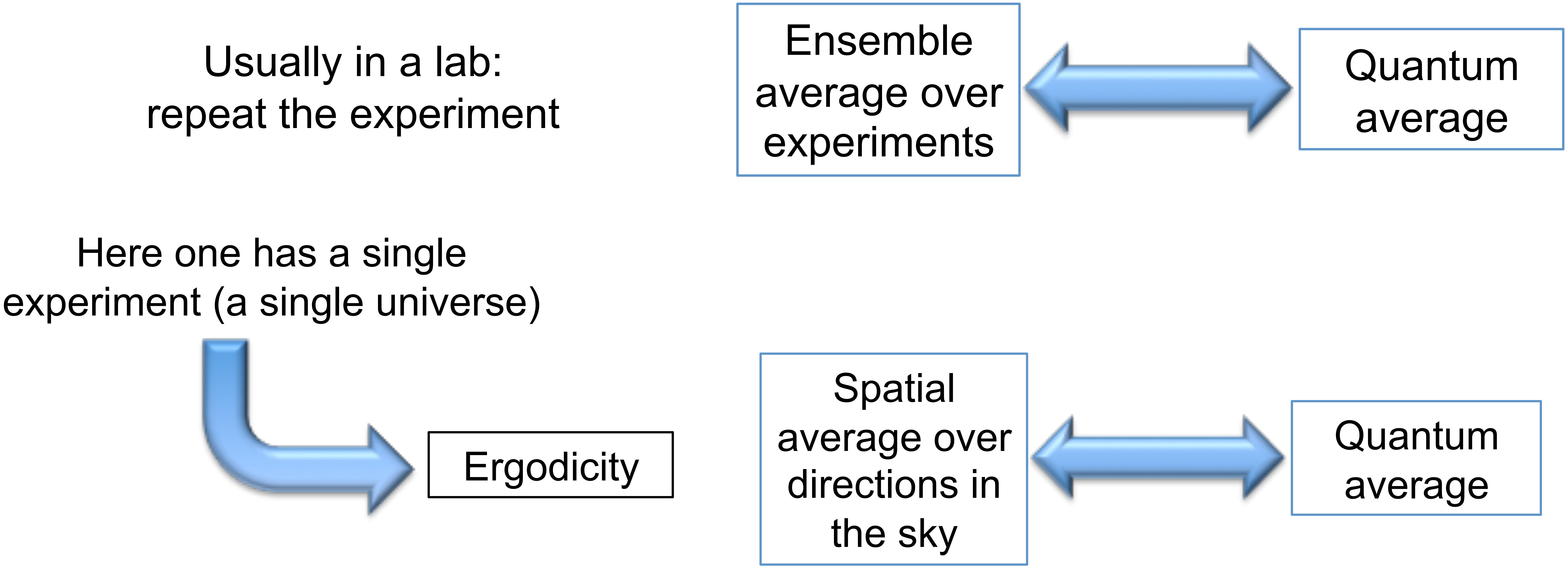
Schrödinger equation = linear (superposition principle) / unitary evolution
Wavepacket reduction = non linear / stochastic

Mutually incompatible

+ External observer

Predictions for quantum theory/cosmology

Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$



Usually in a lab:
repeat the experiment

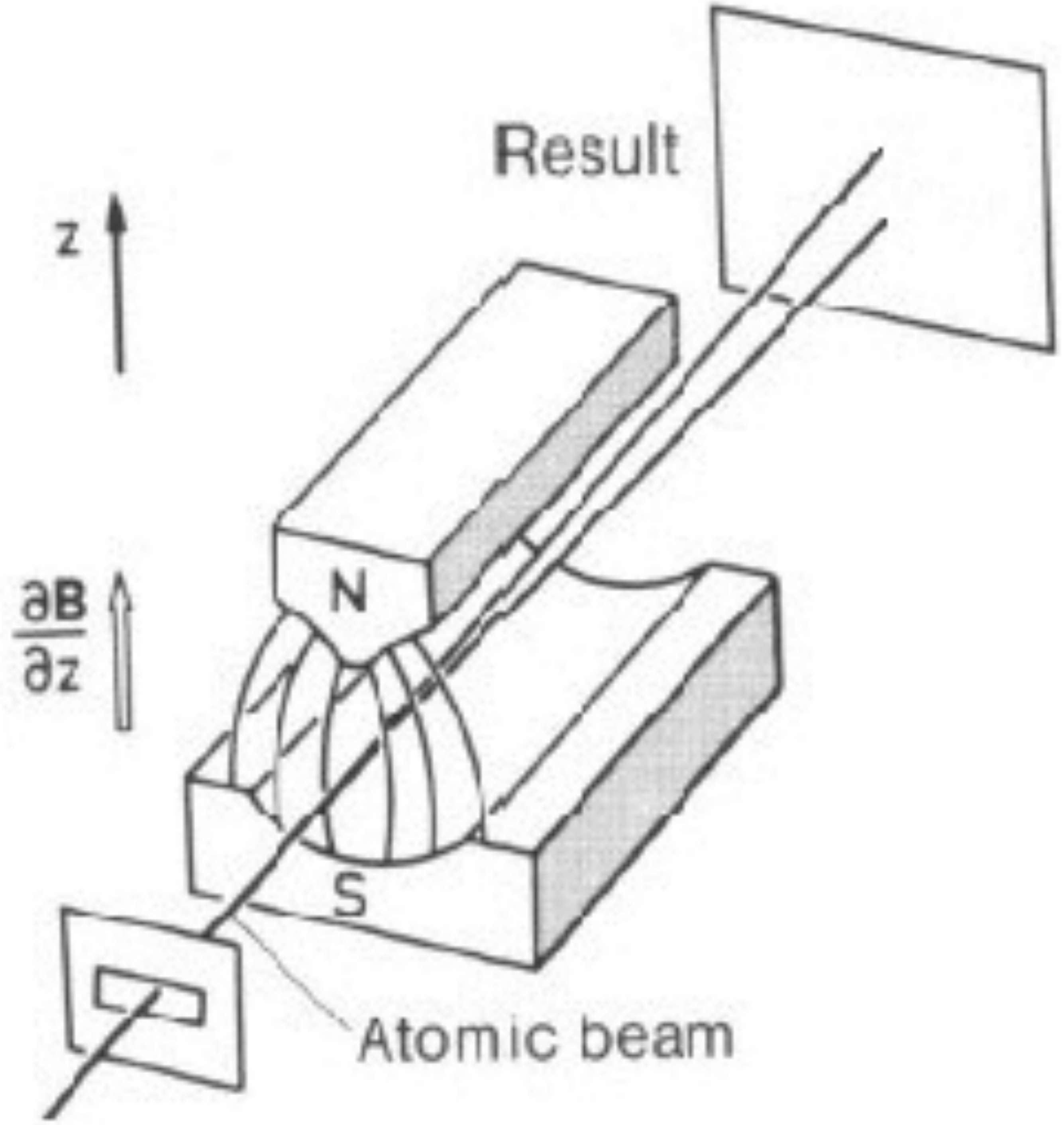
Ensemble average over experiments ⟷ Quantum average

Here one has a single experiment (a single universe)

Ergodicity

Spatial average over directions in the sky ⟷ Quantum average

The measurement problem in quantum mechanics



Stern-Gerlach

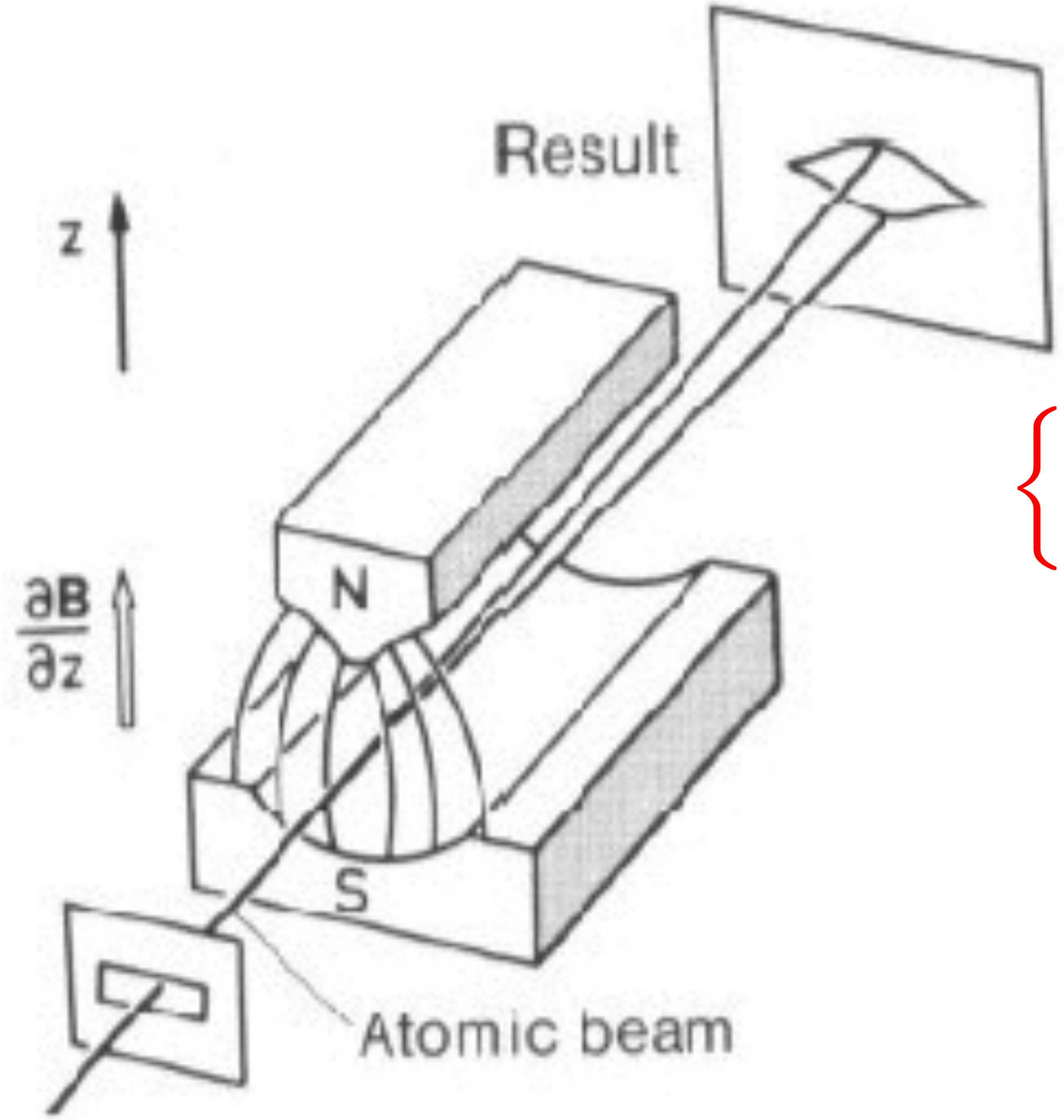
$$|\Psi_{\mathrm{in}}\rangle = \frac{1}{\sqrt{2}}\overbrace{\left(|\uparrow\rangle + |\downarrow\rangle\right)}^{\text{pure state}} \otimes |\mathrm{SG}_{\mathrm{in}}\rangle$$

Unitary, deterministic
Schödinger evolution

$$\begin{aligned} |\Psi_{\mathrm{f}}\rangle &= \exp\left[\int_{t_{\mathrm{in}}}^{t_{\mathrm{f}}} \hat{H}(\tau)\,\mathrm{d}\tau\right] |\Psi_{\mathrm{in}}\rangle \\ &= \frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle + |\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle\right) \end{aligned}$$

Problem: how to reach the actual measurement $|\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle$ or $|\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle$?

The measurement problem in quantum mechanics



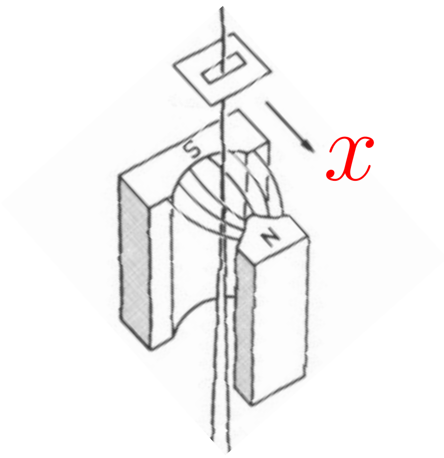
Stern-Gerlach

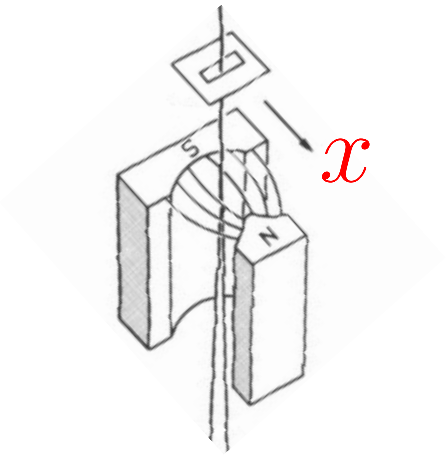
Statistical mixture

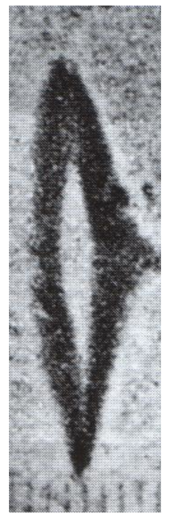
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_\uparrow\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_\downarrow\rangle\right\}$$

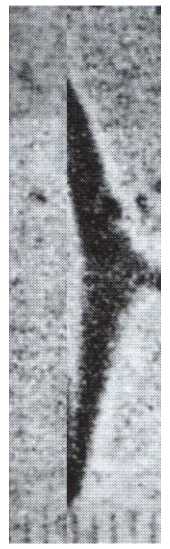
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$

$$\left\{(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$



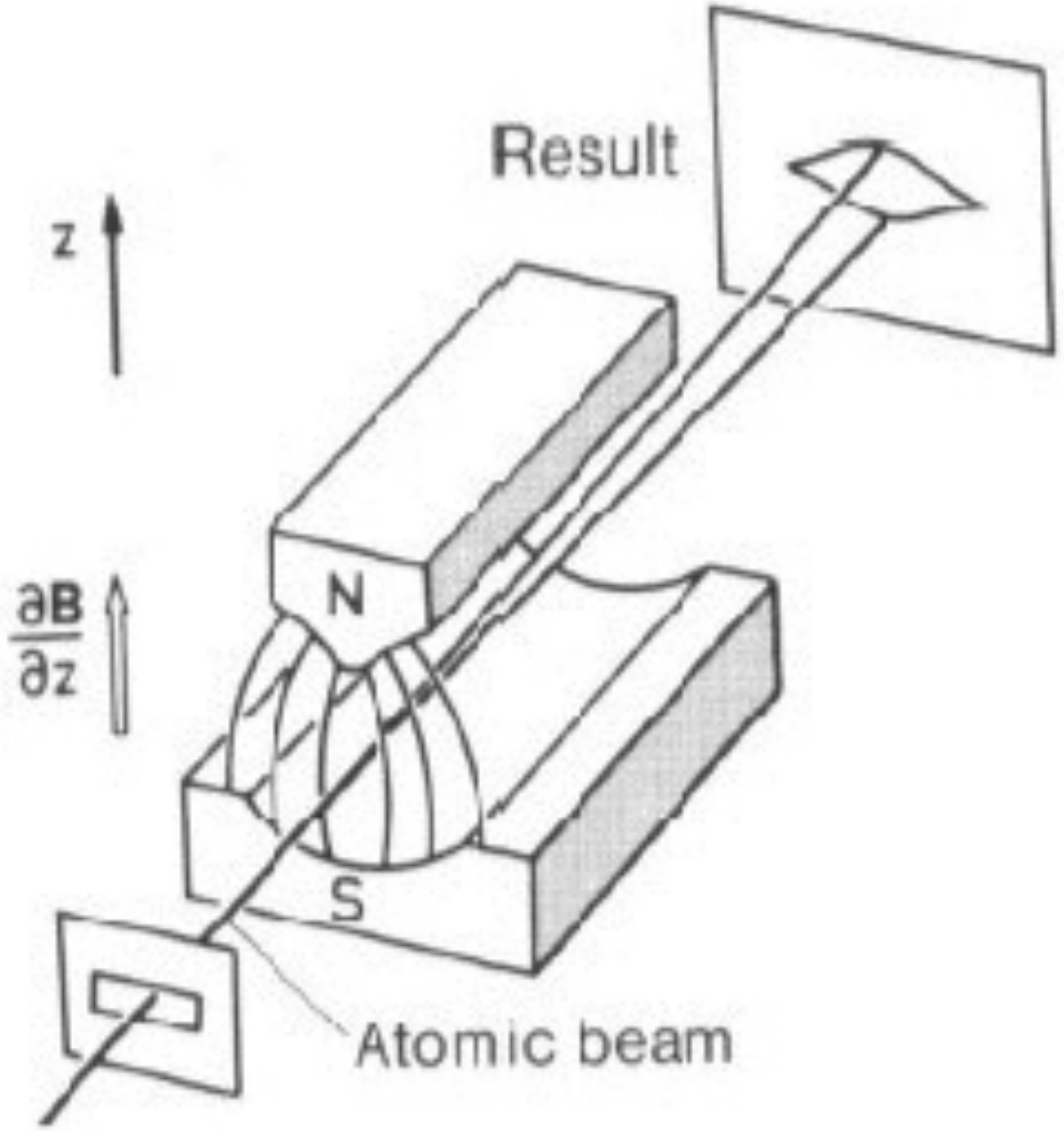






What about situations in which one has only one realization?

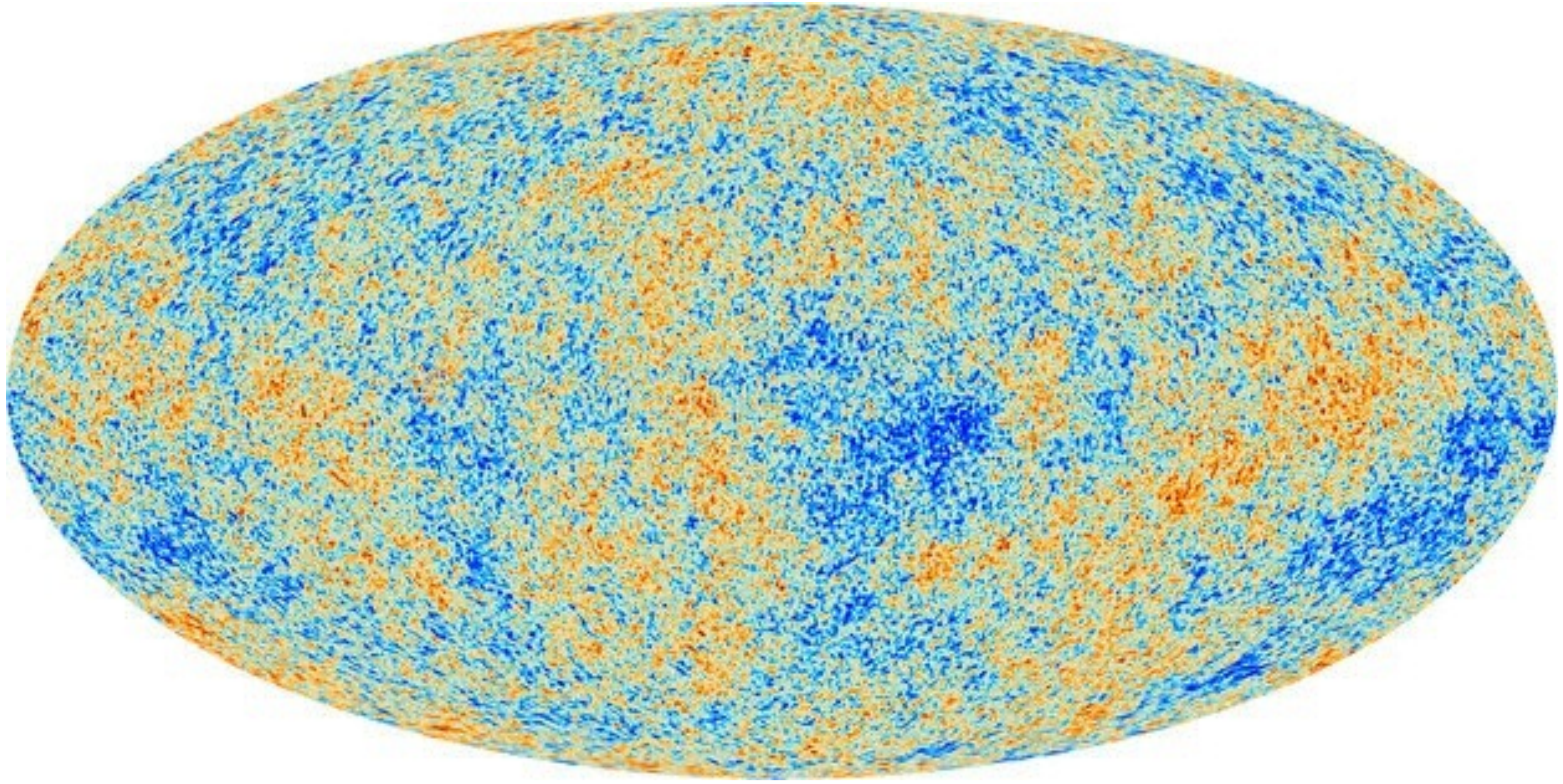
The measurement problem in quantum mechanics



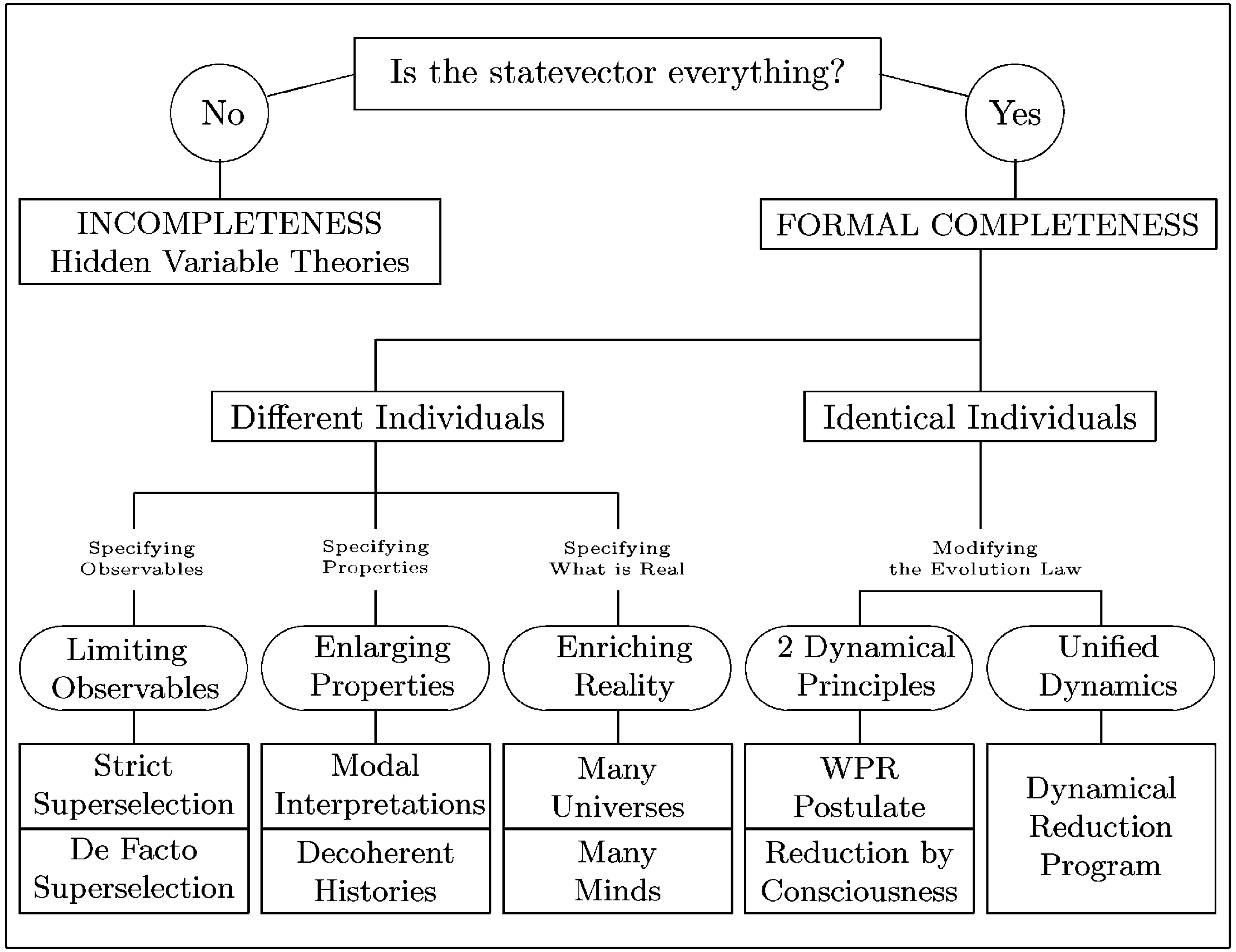
Stern-Gerlach

What about situations in which one has only one realization?

What about the Universe itself?



- Possible solutions and a criterion: the Born rule



Is the statevector everything?

No

INCOMPLETENESS
Hidden Variable Theories

Yes

FORMAL COMPLETENESS

Different Individuals

Specifying Observables
Limiting Observables
Strict Superselection
De Facto Superselection

Specifying Properties
Enlarging Properties
Modal Interpretations
Decoherent Histories

Specifying What is Real
Enriching Reality
Many Universes
Many Minds

Identical Individuals

Modifying the Evolution Law

2 Dynamical Principles
WPR Postulate
Reduction by Consciousness

Unified Dynamics
Dynamical Reduction Program

A. Bassi & G.C. Ghirardi, *Phys. Rep.* **379**, 257 (2003)

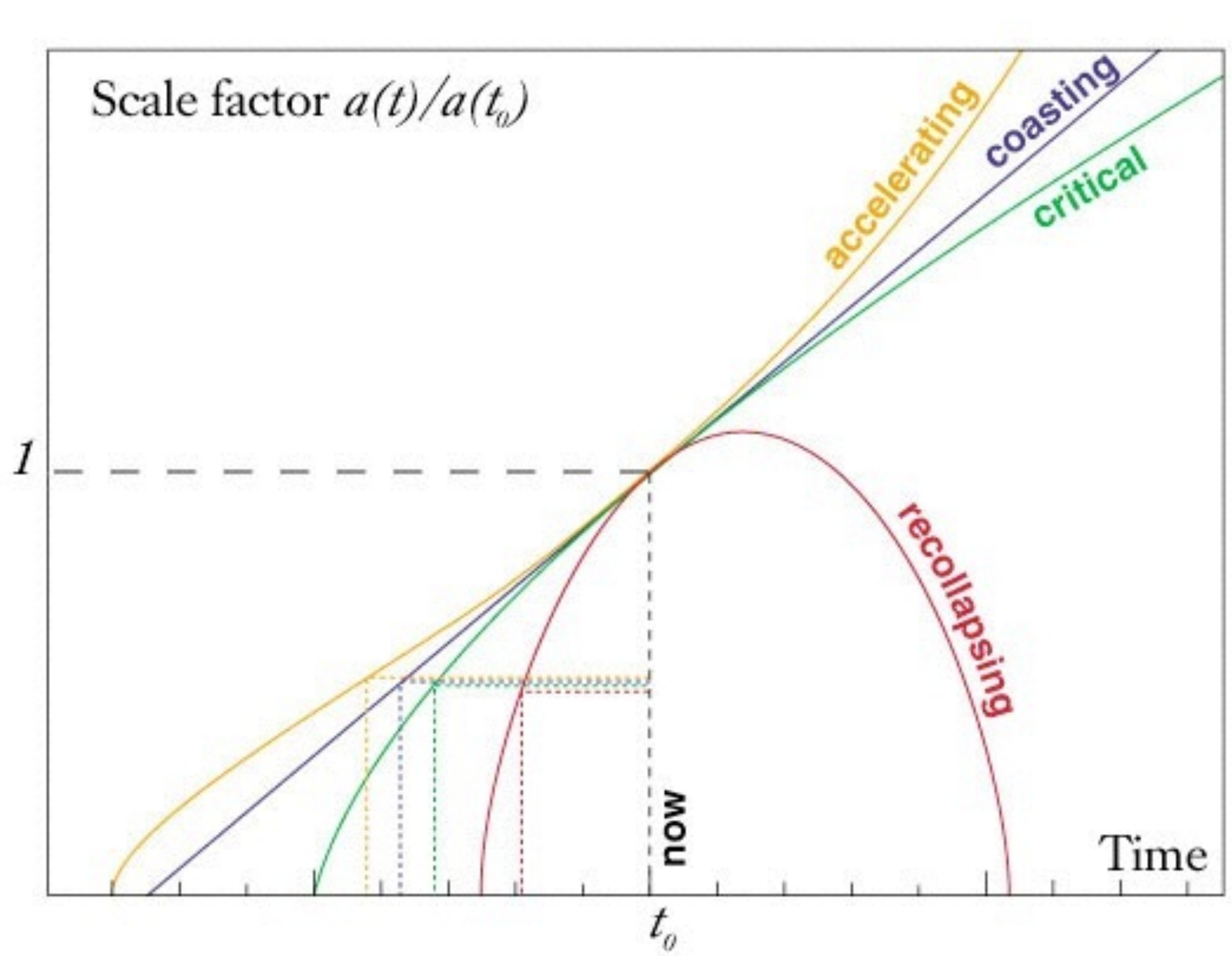
- ***Superselection rules***
- ***Modal interpretation***
- ***Consistent histories***
- ***Many worlds / many minds***

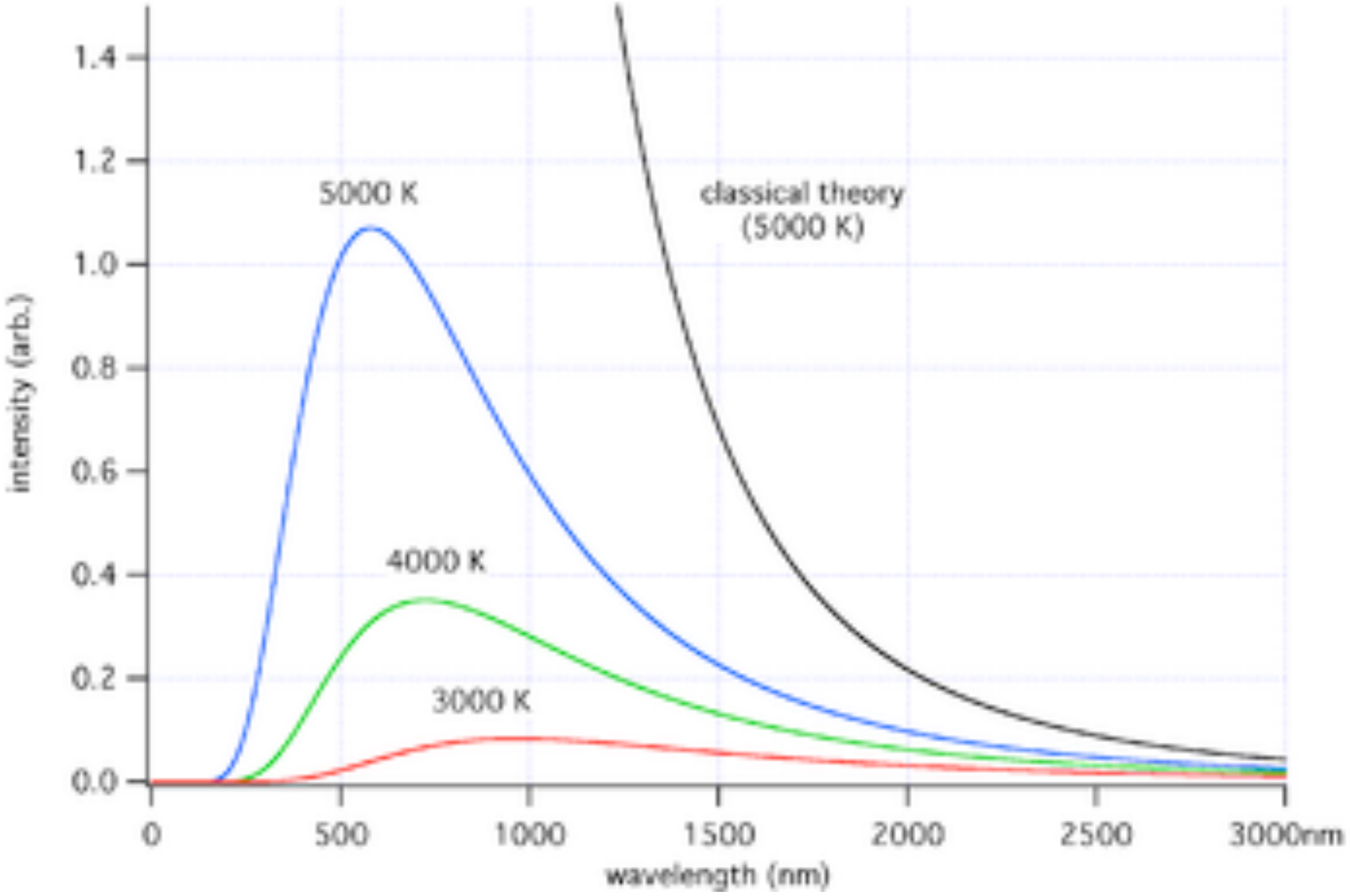
- ***Hidden variables***
- ***Modified Schrödinger dynamics***

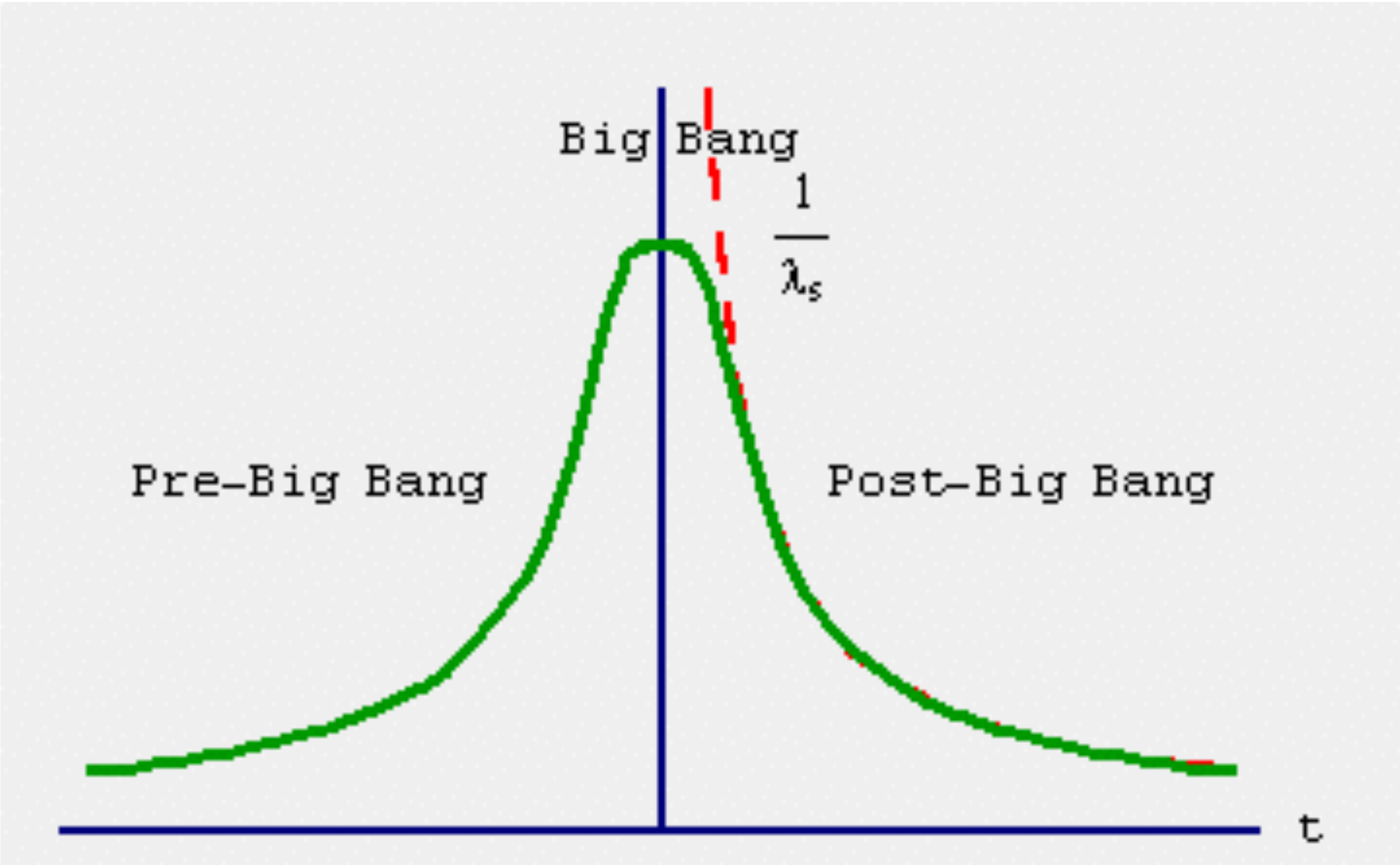
} Born rule not put by hand!

Singularity problem

Purely classical effect?







Quantum cosmology

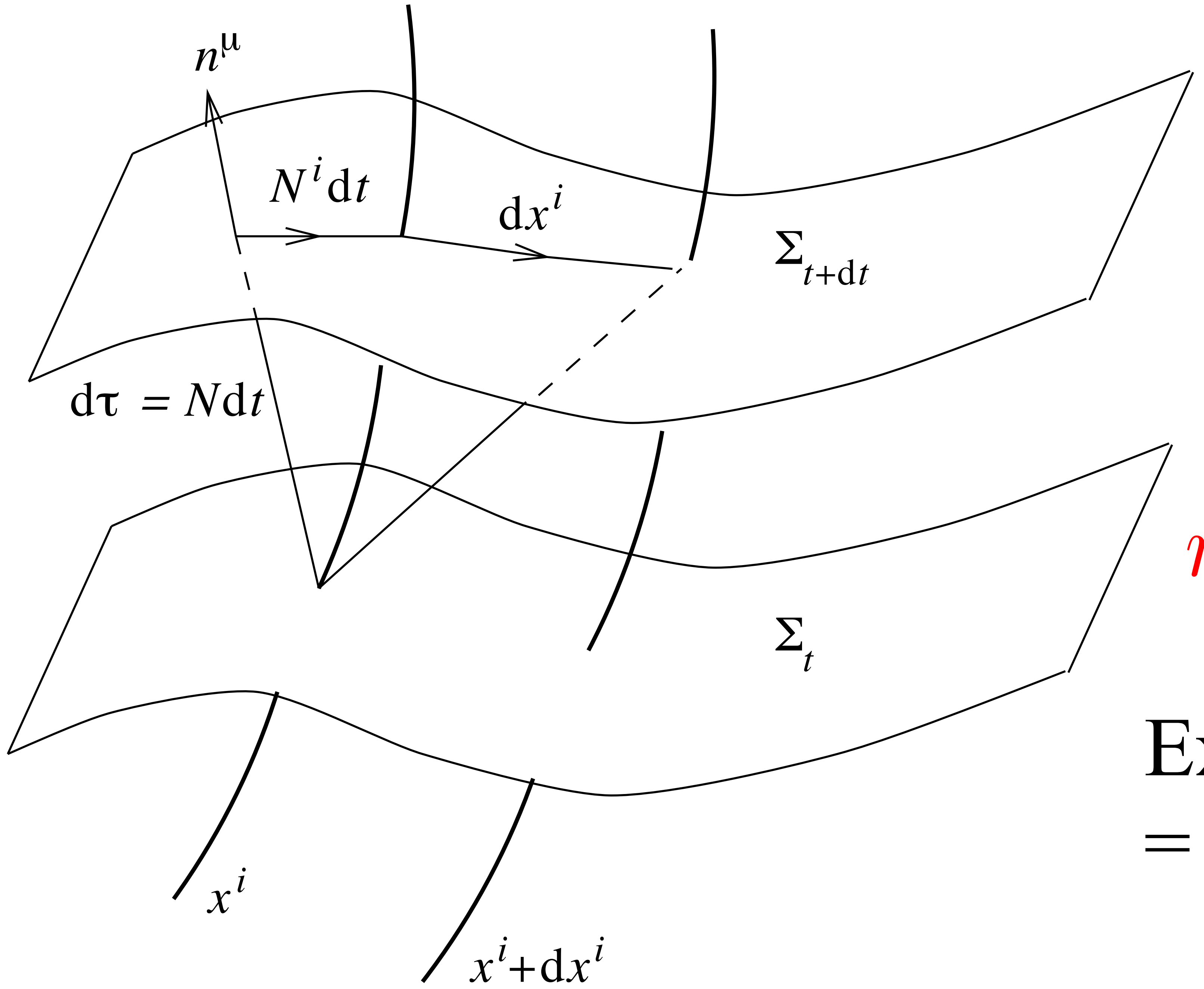
- Hamiltonian GR

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

Lapse function

Intrinsic metric
= first fundamental form

Shift vector



n^μ Normal to Σ_t

Intrinsic curvature tensor ${}^3R^i{}_{jkl}\left(h\right)$

Extrinsic curvature
= second fundamental form

$$K_{ij} \equiv -\nabla_j n_i = -\Gamma^0{}_{ij} n_0$$
$$= \frac{1}{2\mathcal{N}}\left(\nabla_j\mathcal{N}_i + \nabla_i\mathcal{N}_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action: $$\mathcal{S} = \frac{1}{16\pi G_{\mathrm{N}}}\left[\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + 2\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{h}K^i{}_i\right] + \mathcal{S}_{\mathrm{matter}}$$

In 3+1 expansion: $\mathcal{S} \equiv \int \mathrm{d}t L = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}t\mathrm{d}^3x\, \mathcal{N}\sqrt{h}\left(K_{ij}K^{ij} - K^2 + {}^3R - 2\Lambda\right) + \mathcal{S}_{\mathrm{matter}}$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(K^{ij} - h^{ij}K\right)$$

$$\pi_\Phi \equiv \frac{\delta L}{\delta \dot{\Phi}} = \frac{\sqrt{h}}{\mathcal{N}}\left(\dot{\Phi} - \mathcal{N}^i \frac{\partial \Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}} = 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}_i} = 0 \end{aligned}\right\} \text{Primary constraints}$$

Hamiltonian $H \equiv \int \mathrm{d}^3x \left(\pi^0\dot{\mathcal{N}} + \pi^i\dot{\mathcal{N}}_i + \pi^{ij}\dot{h}_{ij} + \pi_\Phi\dot{\Phi}\right) - L = \int \mathrm{d}^3x \left(\pi^0\dot{\mathcal{N}} + \pi^i\dot{\mathcal{N}}_i + \mathcal{N}\mathcal{H} + \mathcal{N}_i\mathcal{H}^i\right)$

Variation wrt lapse $\mathcal{H} = 0$ Hamiltonian constraint

Variation wrt shift $\mathcal{H}^i = 0$ momentum constraint

$\Longrightarrow$ Classical description

- Superspace & canonical quantisation

Relevant configuration space?

matter fields

$$\mathrm{Riem}(\Sigma) \equiv \left\{ h_{ij}\left(x^{\mu}\right), \Phi\left(x^{\mu}\right) \;\middle|\; x \in \Sigma \right\}$$

parameters

GR $\Longrightarrow$ invariance / diffeomorphisms $\Longrightarrow \mathrm{Conf} = \dfrac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}_0(\Sigma)}$ superspace

Wave functional $\Psi\left[h_{ij}(x), \Phi(x)\right]$

Dirac canonical quantisation

$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}} \qquad \pi_{\Phi} \to -i\frac{\delta}{\delta \Phi} \qquad \pi^{0} \to -i\frac{\delta}{\delta \mathcal{N}} \qquad \pi^{i} \to -i\frac{\delta}{\delta \mathcal{N}_i}$$

Primary constraints

$$\hat{\pi}\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}} = 0$$

$$\hat{\pi}^i\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}_i} = 0$$

Momentum constraint $\hat{\mathcal{N}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\frac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\rm N}\hat{T}^{0i}\Psi$

$\Longrightarrow$ Ψ is the same for configurations $\{h_{ij}(x), \Phi(x)\}$ related by a coordinate transformation

Hamiltonian constraint

$$\boxed{\hat{\mathcal{H}}\Psi = \left[-16\pi G_{\rm N}\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G_{\rm N}}\left(-{}^3R + 2\Lambda + 16\pi G_{\rm N}\hat{T}^{00}\right)\right]\Psi = 0}$$

Wheeler - De Witt equation

$$\mathcal{G}_{ijkl} = \frac{1}{2}h^{-1/2}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

DeWitt metric...

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t),\phi(t)\right]$

Conceptual and technical problems:

Infinite number of dof $\longrightarrow$ a few: mathematical consistency?
Freeze momenta? Heisenberg uncertainties?
QM = minisuperspace of QFT

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space $=$ mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x)=\phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t),\phi(t)\right]$

However, one can actually make calculations!

Exemple : Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70)

$$p = p_0 \left[\frac{\dot{\varphi} + \theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$$

$(\varphi, \theta, s) =$ Velocity potentials

canonical transformation: $T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega}$ …

+ rescaling (volume…) + units… : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N \searrow a^{3\omega}$$

Wheeler-De Witt $H\Psi = 0$

$$\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}}$$

space defined by $\chi > 0 \longrightarrow$ constraint $\bar{\Psi}\dfrac{\partial\Psi}{\partial\chi} = \Psi\dfrac{\partial\bar{\Psi}}{\partial\chi}$

Gaussian wave packet

⇨ $$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2 + T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2 + T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $$S = \frac{T\chi^2}{T_0^2 + T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

What do we do with the wave function of the Universe???

Hidden Variable Theories

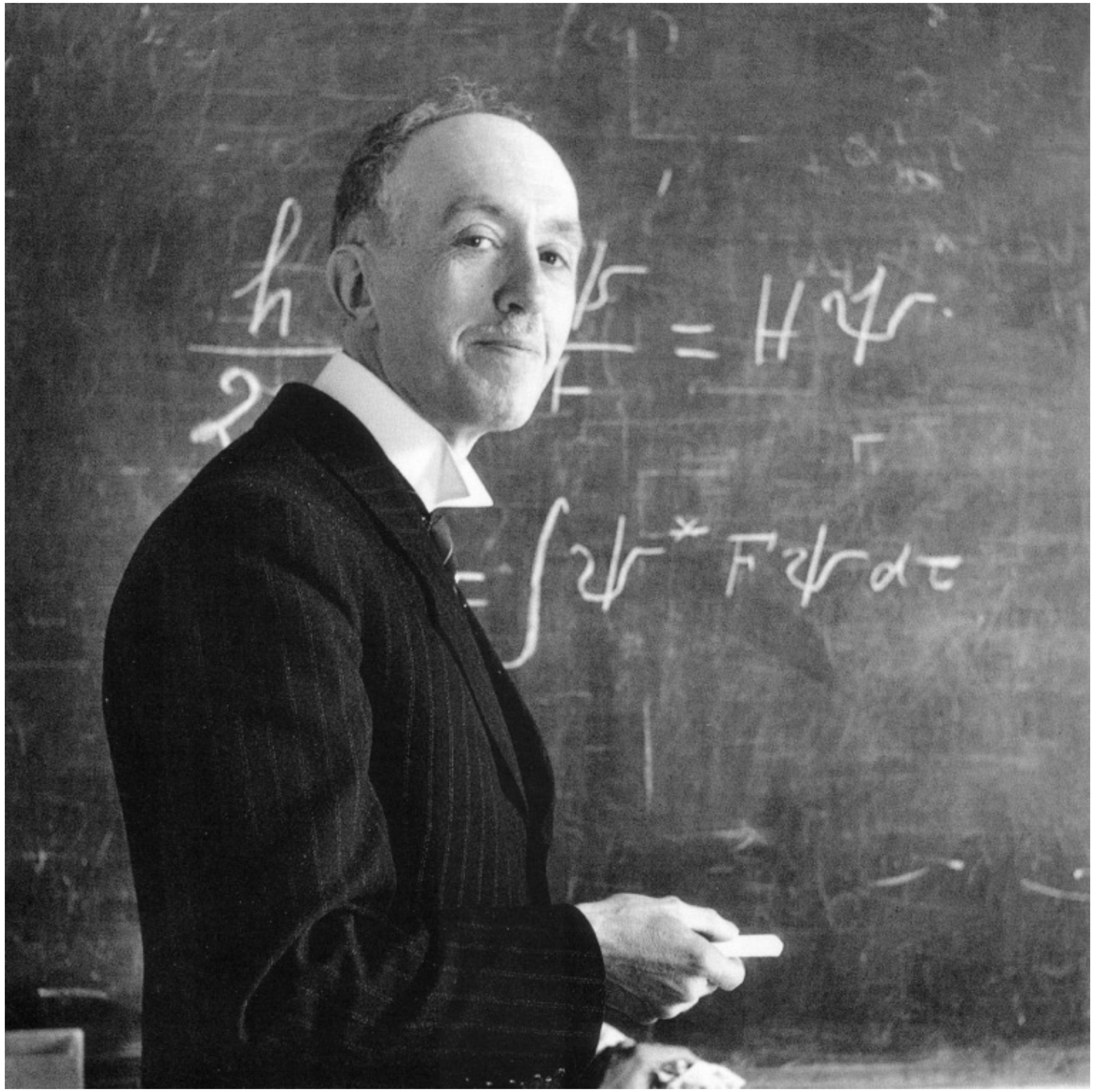
Schrödinger $$i\frac{\partial\Psi}{\partial t}=\left[-\frac{\nabla^2}{2m}+V\left(\boldsymbol{r}\right)\right]\Psi$$

Polar form of the wave function $$\Psi=A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS\left(\boldsymbol{r},t\right)}$$

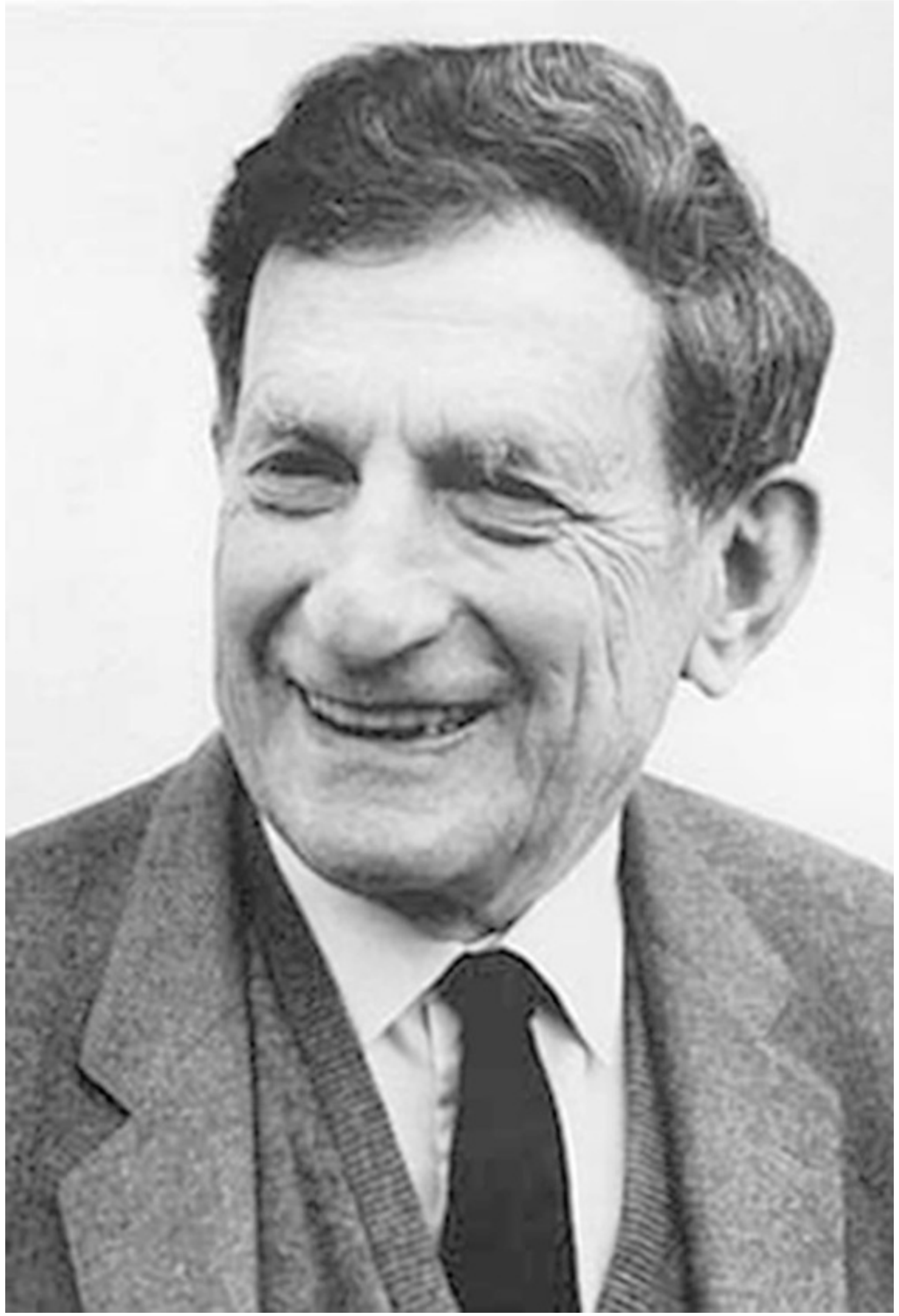
Hamilton-Jacobi $$\frac{\partial S}{\partial t}+\frac{\left(\nabla S\right)^2}{2m}+V\left(\boldsymbol{r}\right)+Q\left(\boldsymbol{r},t\right)=0$$

quantum potential $$\equiv-\frac{1}{2m}\frac{\nabla^2 A}{A}$$

Ontological *formulation* (dBB)



Louis de Broglie (Prince, duke ...)



David Bohm (Communist)

1927 Solvay meeting and von Neuman mistake ... ‘*In 1952, I saw the impossible done*’ (J. Bell)

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists\,\boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^{*}\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^{2}} = -\boldsymbol{\nabla}S$$

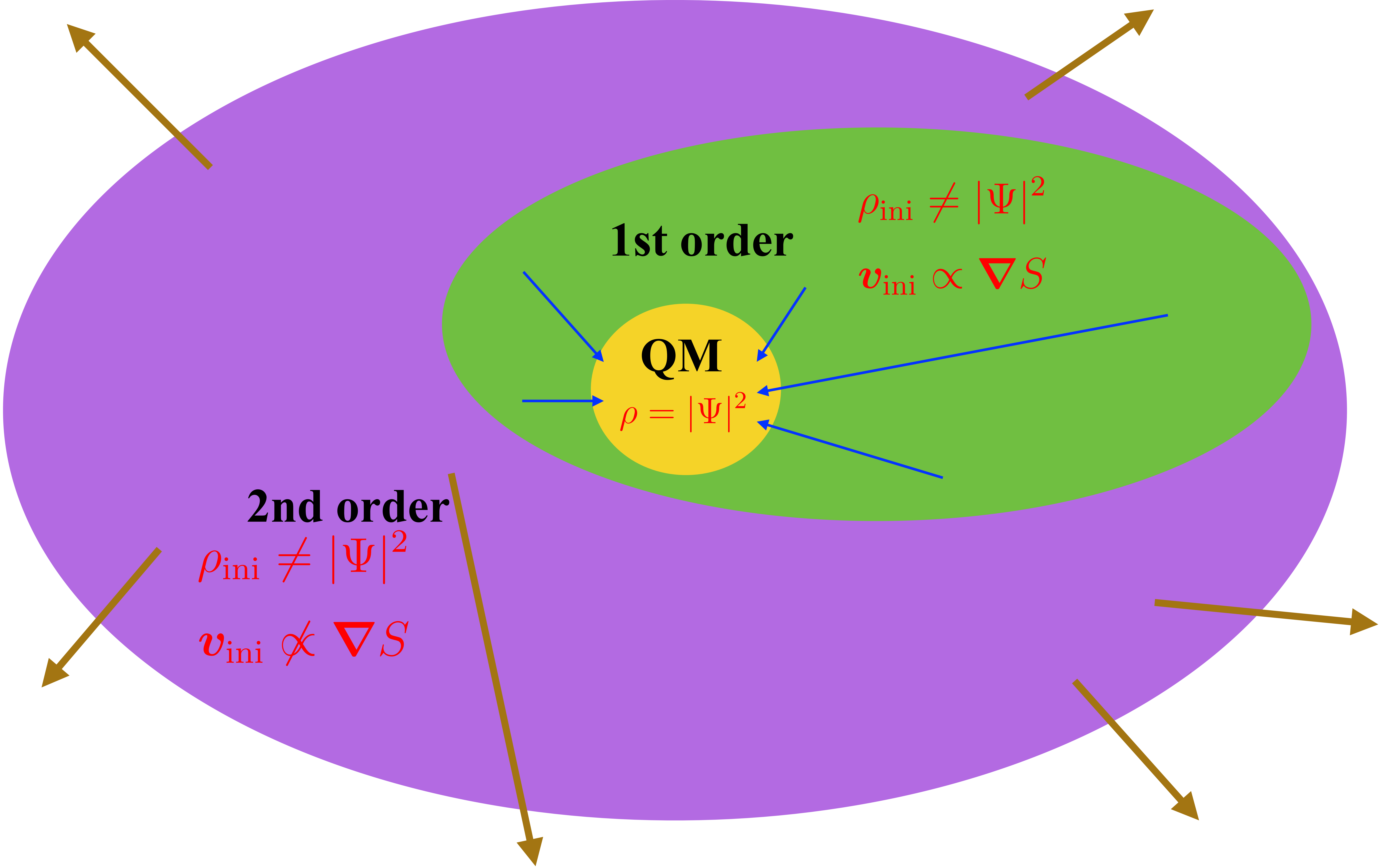
$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (BdB) $\exists\,\boldsymbol{x}(t)$

Trajectories satisfy (Bohm)

$$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V+Q)$$

$$Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$



1st order

$\rho_{\text{ini}} \neq |\Psi|^2$

$\boldsymbol{v}_{\text{ini}} \propto \boldsymbol{\nabla} S$

QM

$\rho = |\Psi|^2$

2nd order

$\rho_{\text{ini}} \neq |\Psi|^2$

$\boldsymbol{v}_{\text{ini}} \not\propto \boldsymbol{\nabla} S$

1st order: can be tested

2nd order: has been tested…

and is ruled out!

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS\left(\boldsymbol{r},t\right)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^{*}\boldsymbol{\nabla}\Psi}{\left|\Psi(\boldsymbol{x},t)\right|^{2}} = -\boldsymbol{\nabla}S$$

Properties:

☺ strictly equivalent to Copenhagen QM

➡ probability distribution (attractor)

$$\exists t_0 ; \rho\left(\boldsymbol{x},t_0\right) = \left|\Psi\left(\boldsymbol{x},t_0\right)\right|^{2}$$

☺ classical limit well defined $Q \longrightarrow 0$

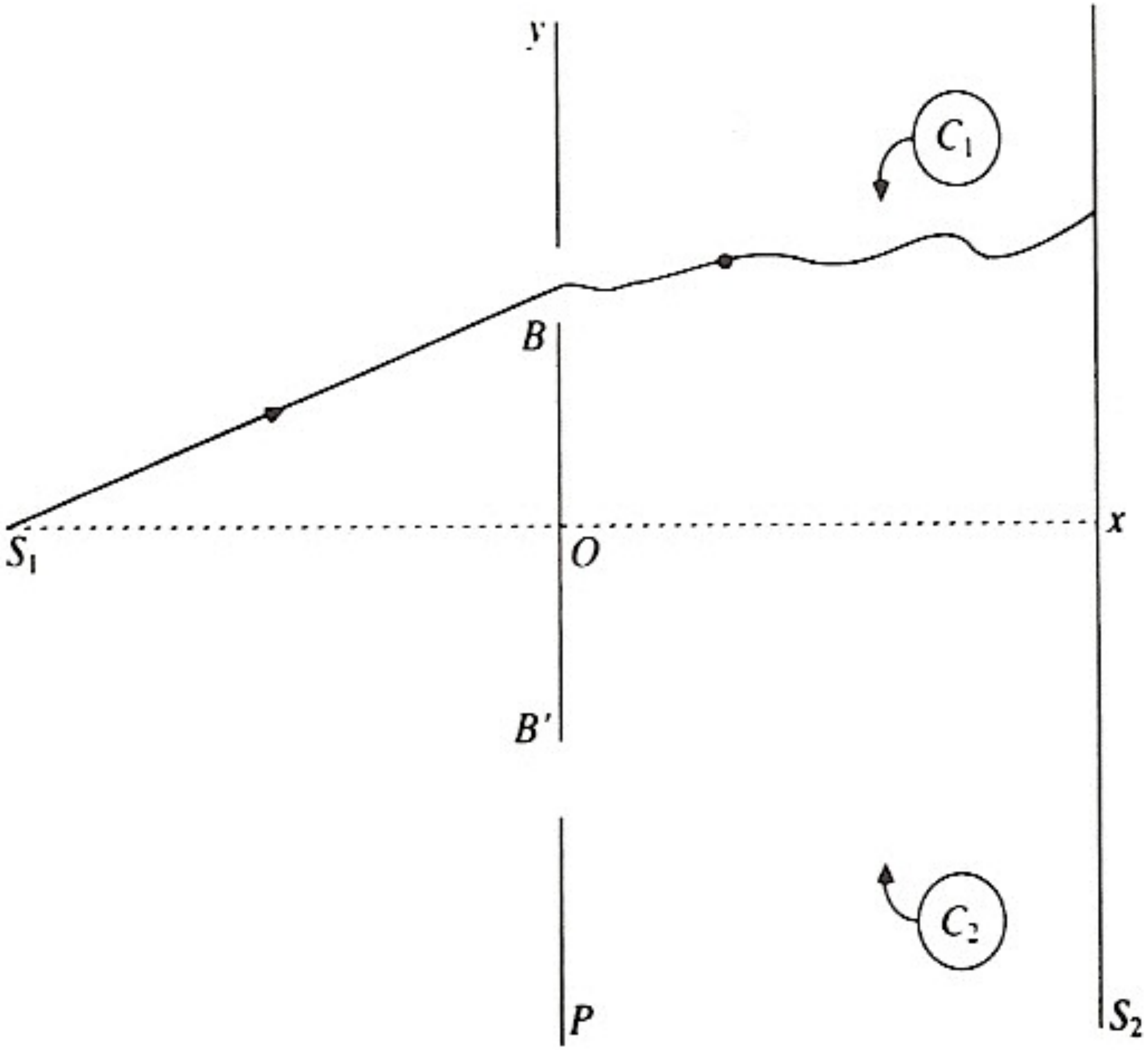
☺ state dependent

☺ $\exists$ intrinsic reality

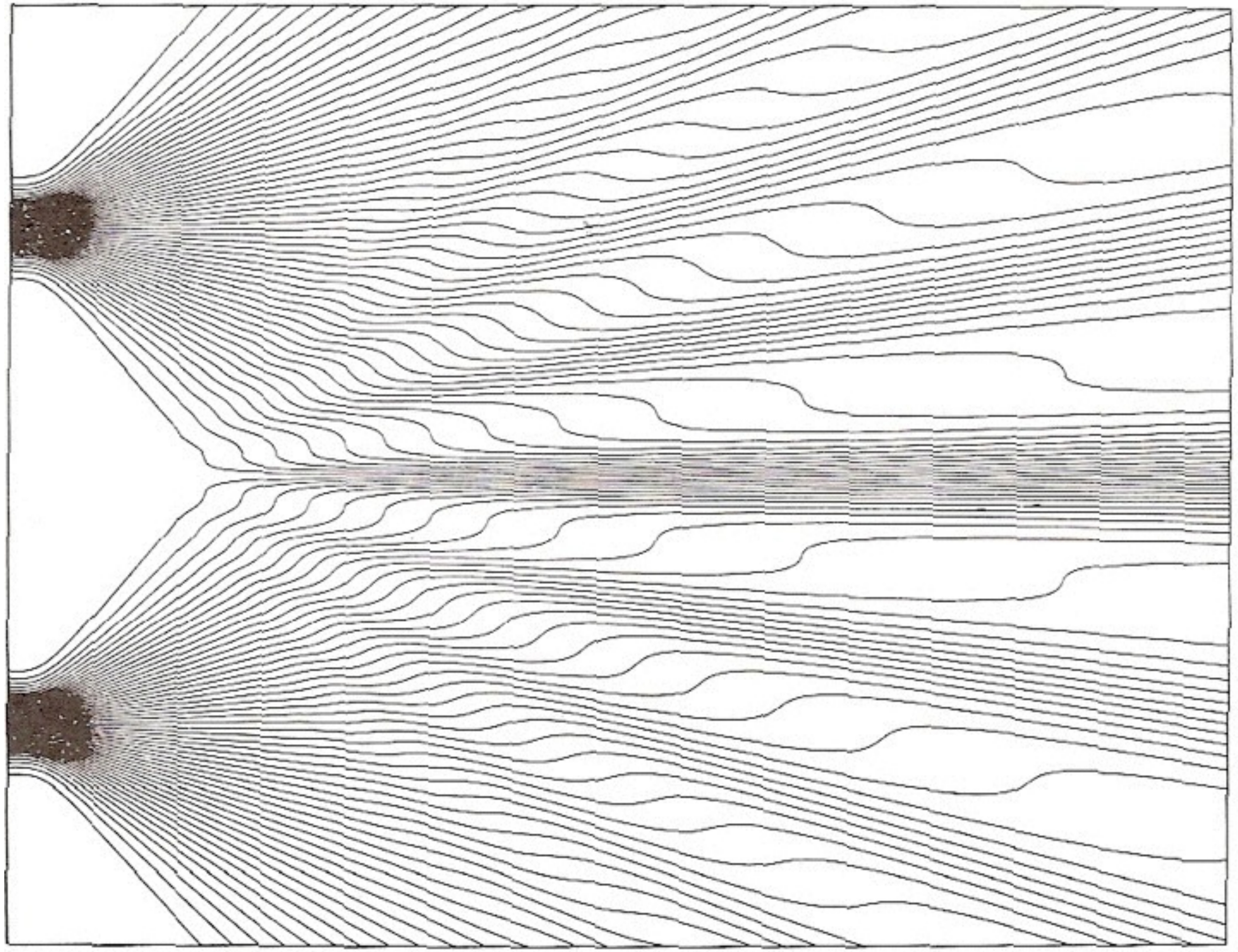
➡ non local …

☺ no need for external classical domain/observer!

The two-slit experiment:



The two-slit experiment:



Surrealistic trajectories?

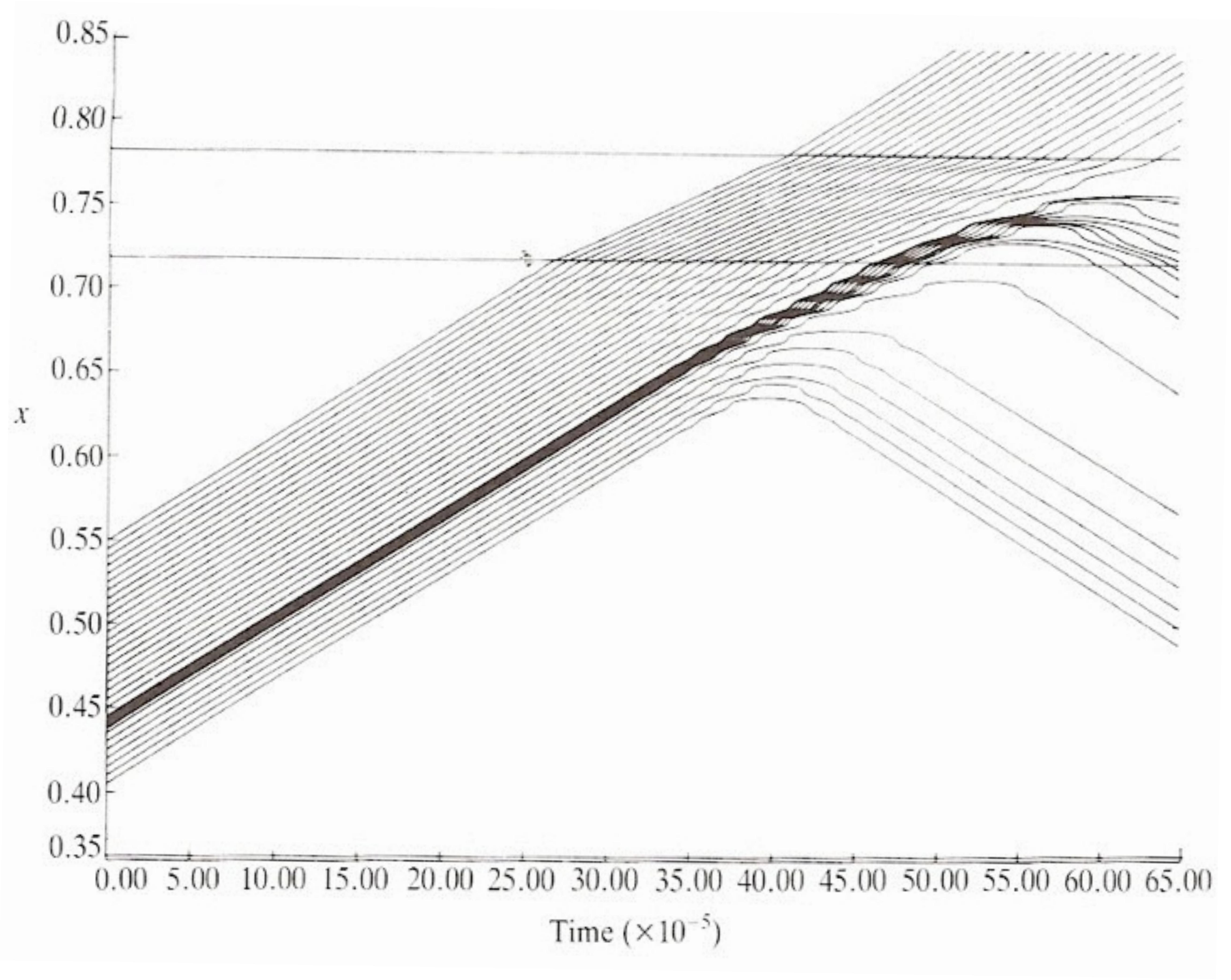
Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(\xcancel{V} + Q\right)$$

> ... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

R. P. Feynman (1961)

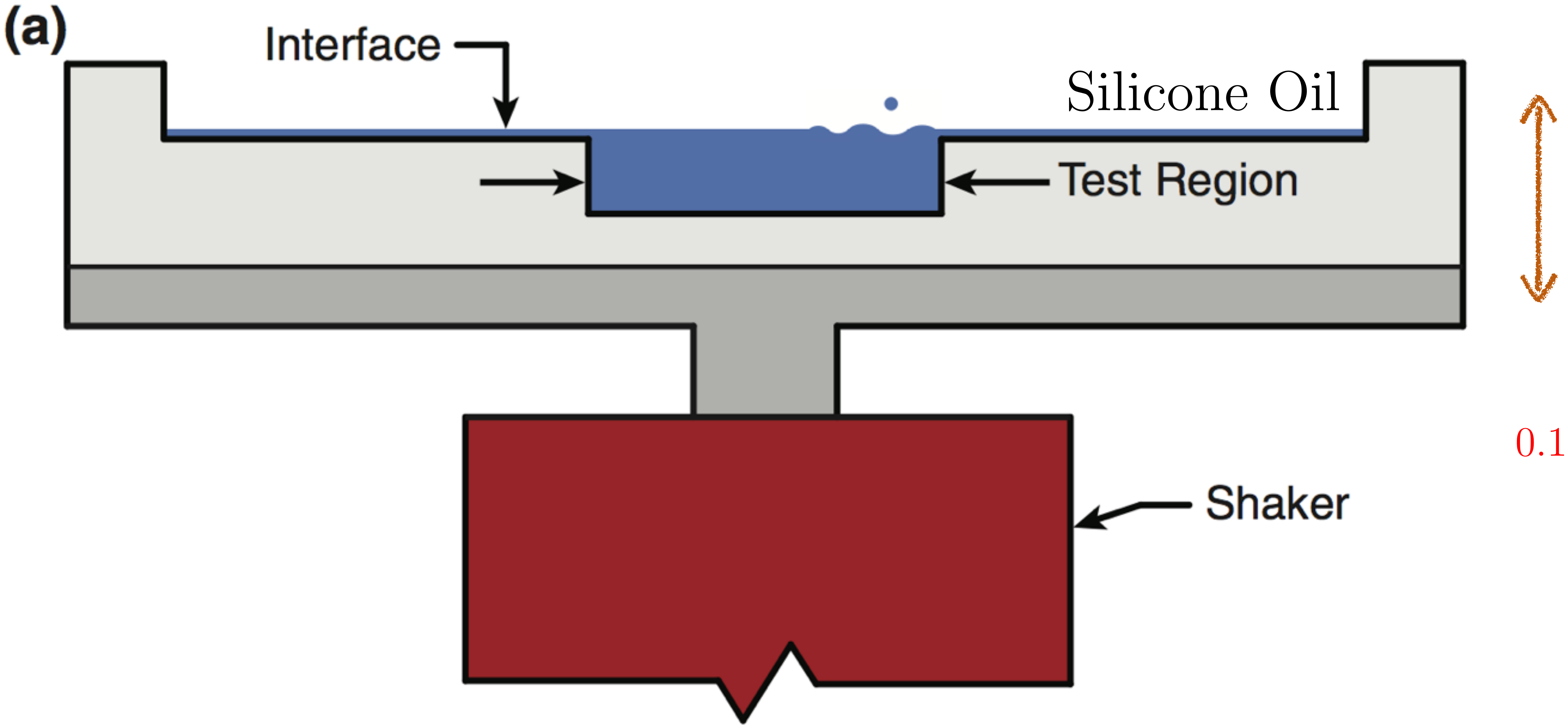
Diffraction by a potential simple understanding of tunnelling ...



Aside: a nice hydrodynamical analogy

Faraday waves… (1831)

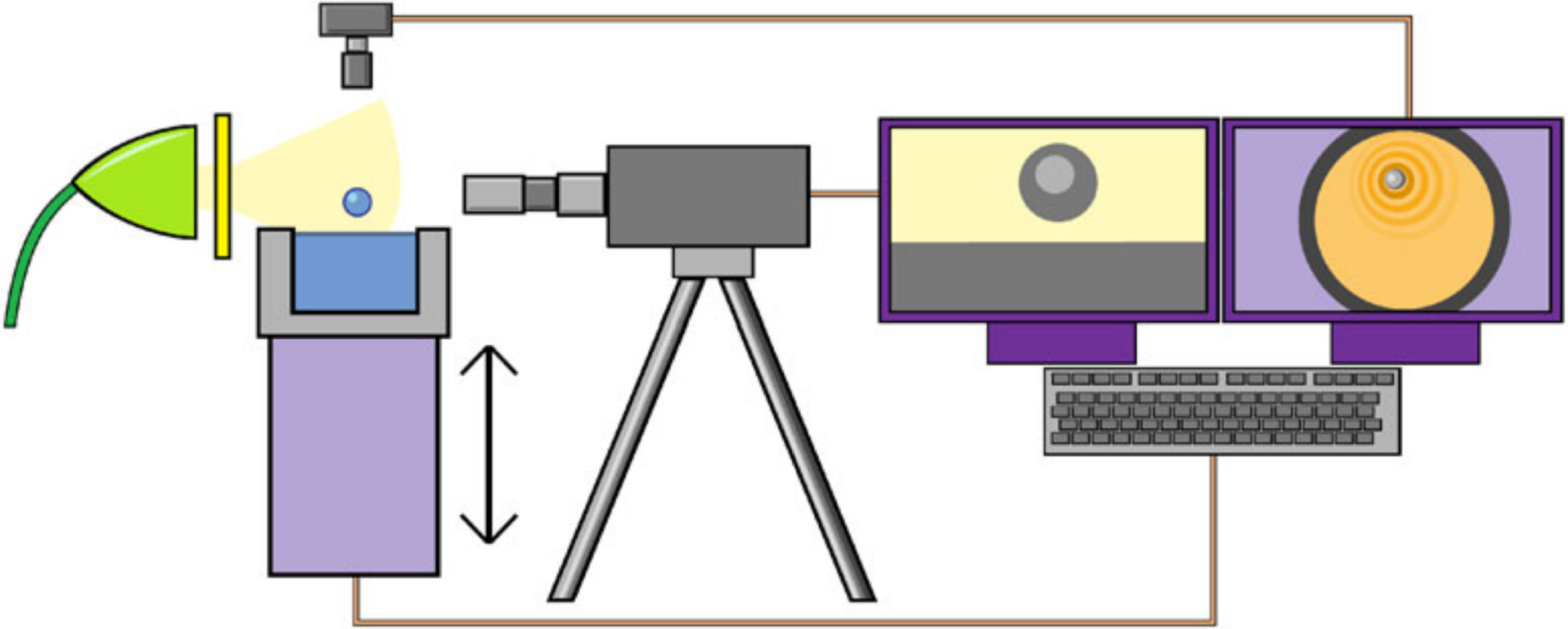
forced standing surface waves



$$\underset{0.19\ \text{mm}}{\underbrace{A_0}} \sin\left(2\pi \underset{70\ \text{Hz}}{\underbrace{f}} t\right)$$

Just above Faraday wave threshold

J. Walker (1978)
Y. Couder et al. (>2006)



Typical values for the experiment

R_0	Drop radius	0.07–0.8 mm
ρ	Silicone oil density	949–960 kg m^{-3}
ρ_a	Air density	1.2 kg m^{-3}
σ	Drop surface tension	20–21 mN m^{-1}
g	Gravitational acceleration	9.81 m s^{-2}
V_{in}	Drop incoming speed	0.1–1 m s^{-1}
V_{out}	Drop outgoing speed	0.01–1 m s^{-1}
μ	Drop dynamic viscosity	10^{-3}–10^{-1} kg m^{-1} s^{-1}
μ_a	Air dynamic viscosity	1.84×10^{-5} kg m^{-1} s^{-1}
ν	Drop kinematic viscosity	10–100 cSt
ν_a	Air kinematic viscosity	15 cSt
T_C	Contact time	1–20 ms
C_R	$= V_{in}/V_{out}$ Coefficient of restitution	0–0.4
f	Bath shaking frequency	40–200 Hz
γ	Peak bath acceleration	0–70 m s^{-2}
ω	$= 2\pi f$ Bath angular frequency	250–1250 rad s^{-}1
ω_D	$= (\sigma/\rho R_0^3)^{1/2}$ Characteristic drop oscillation frequency	300–5000 s^{-1}
We	$= \rho R_0 V_{in}^2/\sigma$ Weber number	0.01–1
Bo	$= \rho g R_0^2/\sigma$ Bond number	10^{-3}–0.4
Oh	$= \mu(\sigma\rho R_0)^{-1/2}$ Drop Ohnesorge number	0.004–2
Oh_a	$= \mu_a(\sigma\rho R_0)^{-1/2}$ Air Ohnesorge number	10^{-4}–10^{-3}
Ω	$= 2\pi f\sqrt{\rho R_0^3/\sigma}$ Vibration number	0–1.4
Γ	$= \gamma/g$ Peak non-dimensional bath acceleration	0–7

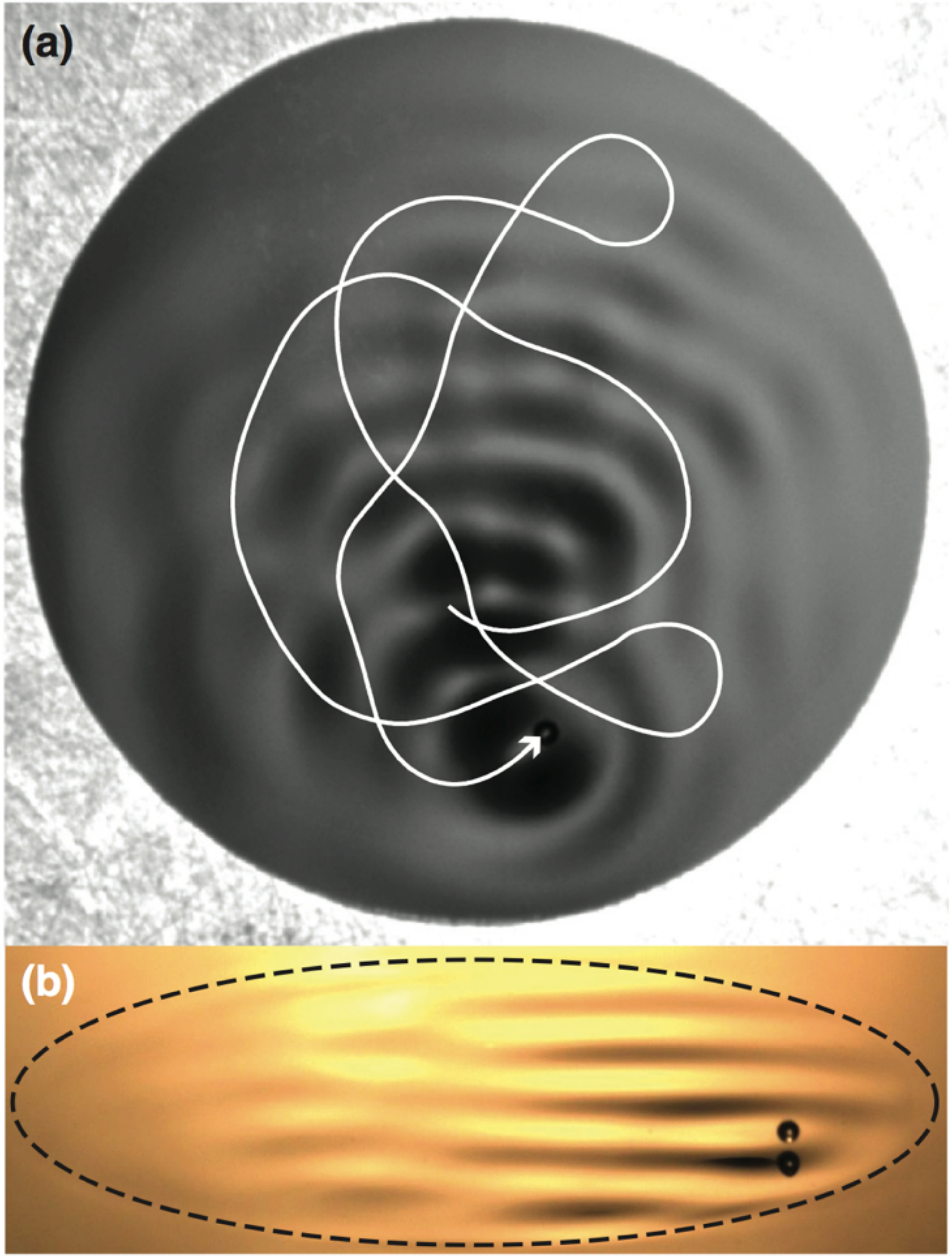
Bouncing droplet…

or bouncing droplets…

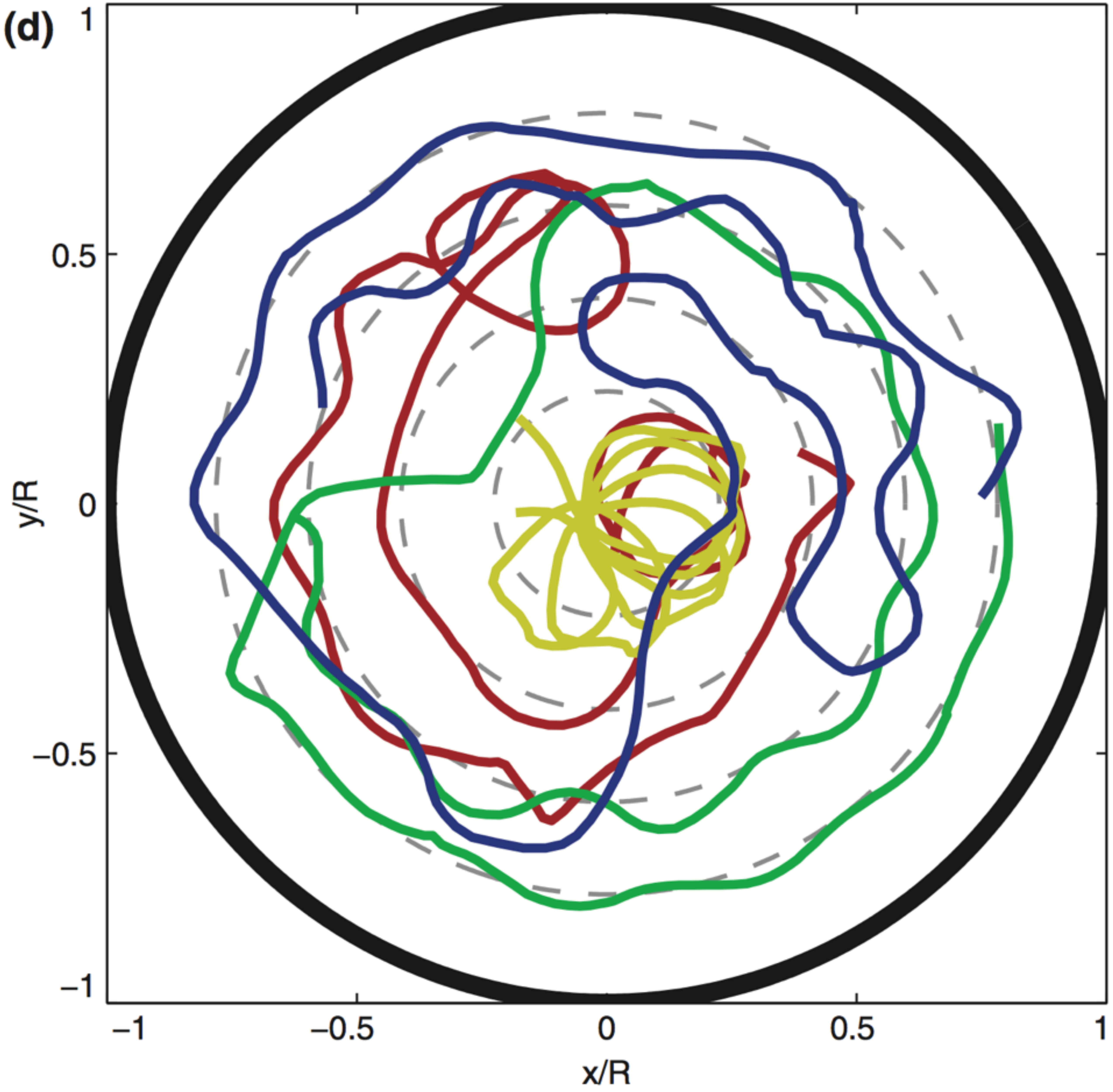
+ subharmonic modulation (larger forcing amplitude) => instability => motion!!!

droplets become walkers…

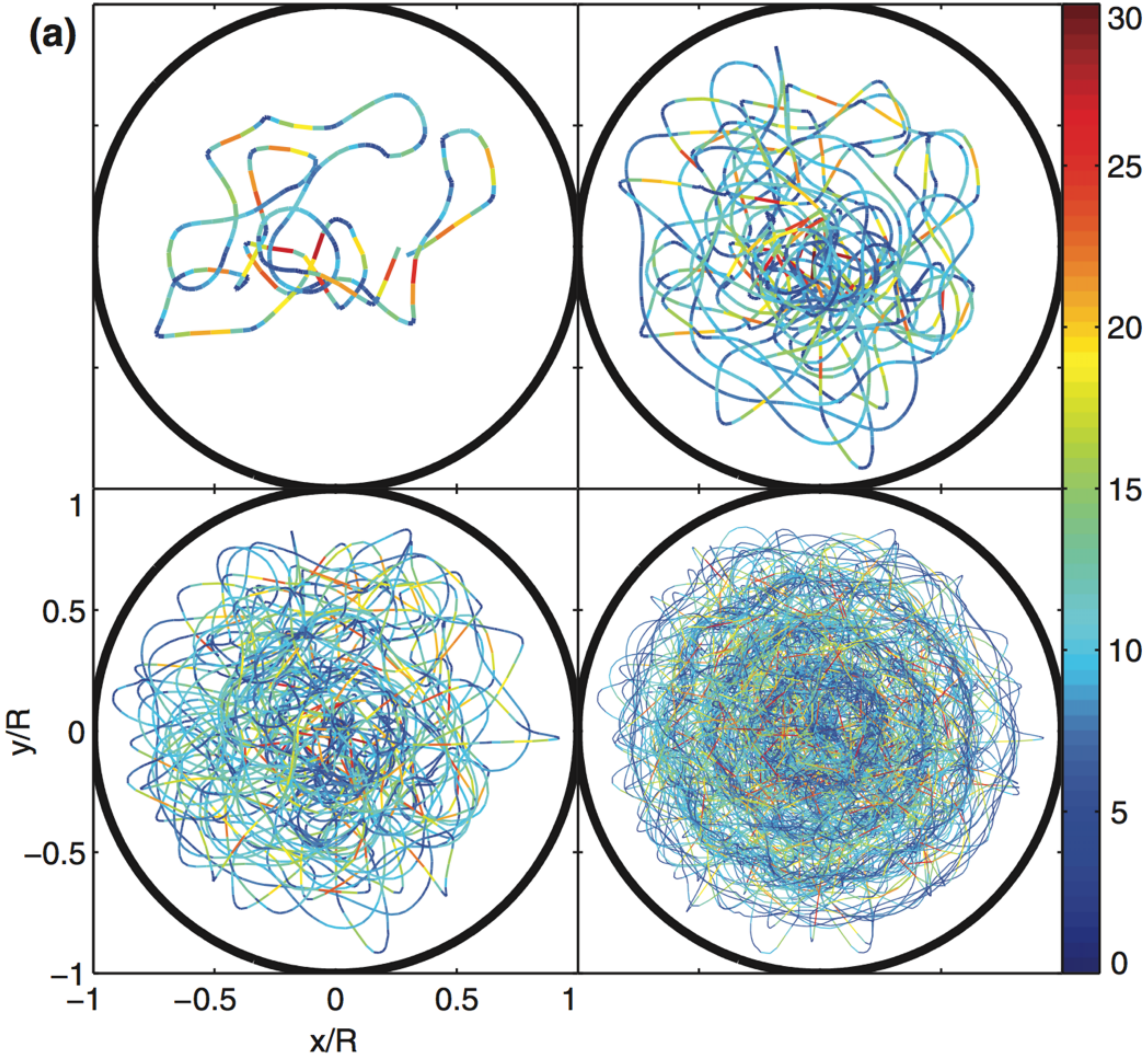
one image per bounce => suppress vertical motion => horizontal mode only



apparent randomness
of the motion…

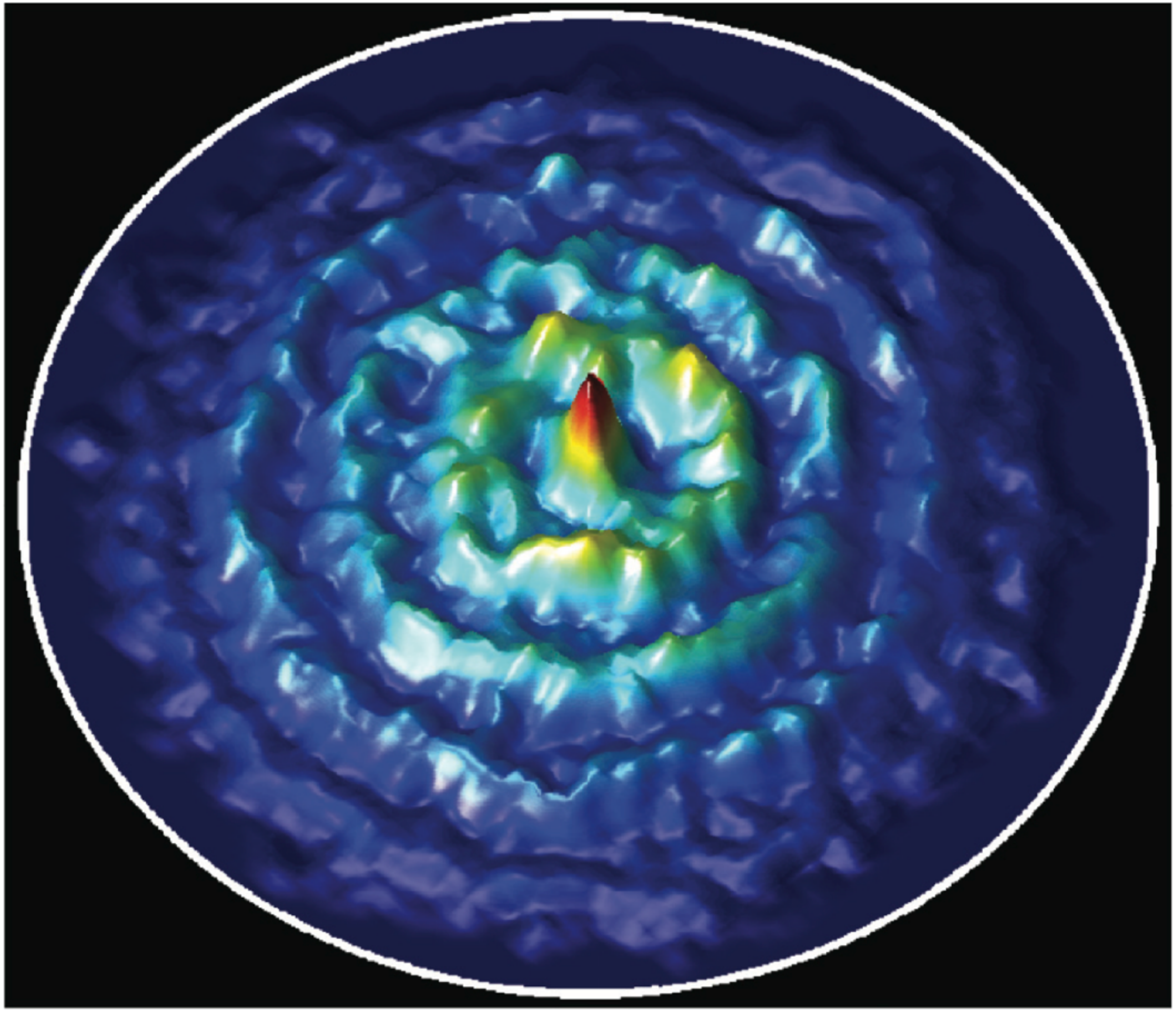


integrate over time…

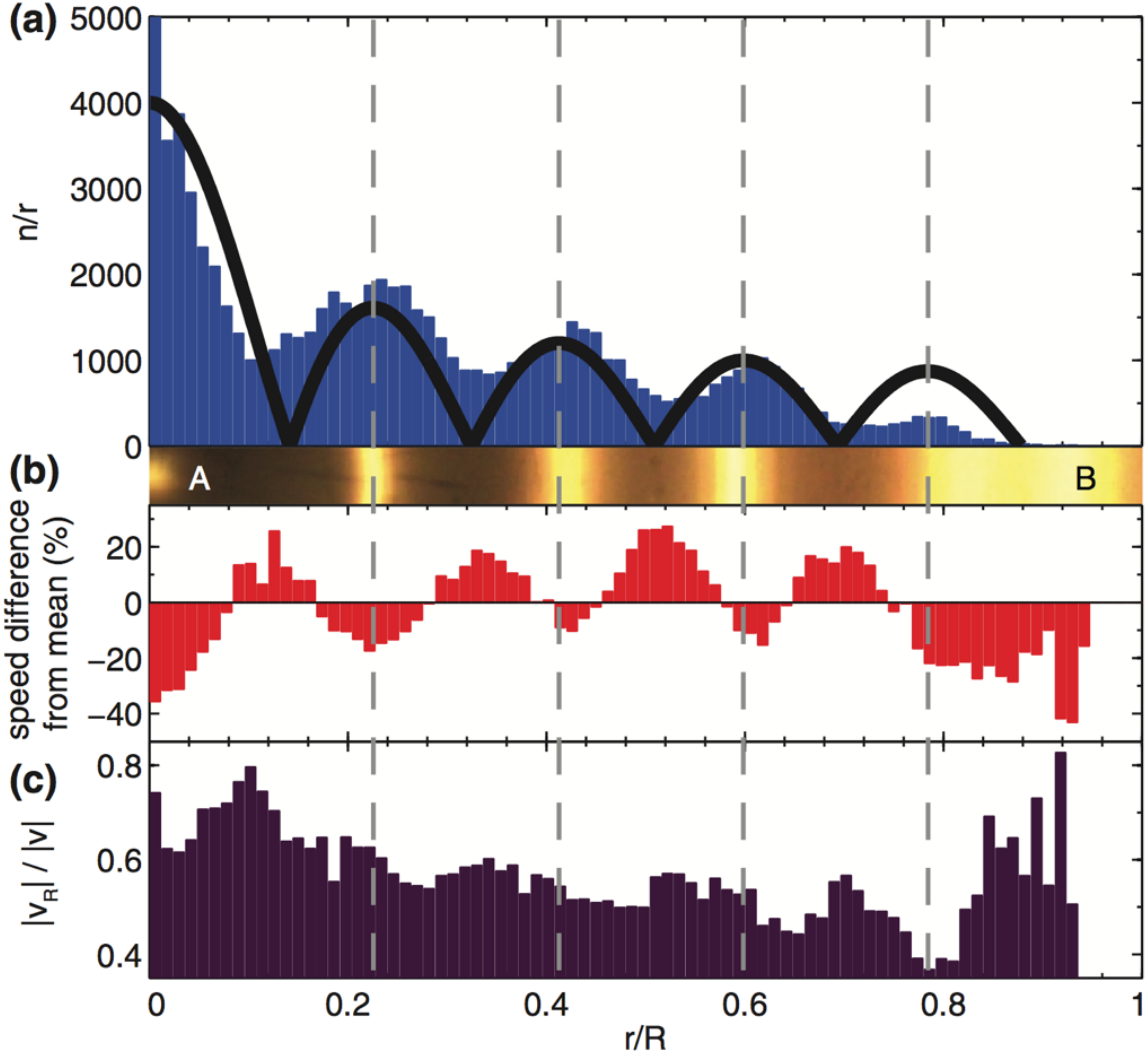


longer times…

and reconstruct the standing wave pattern!

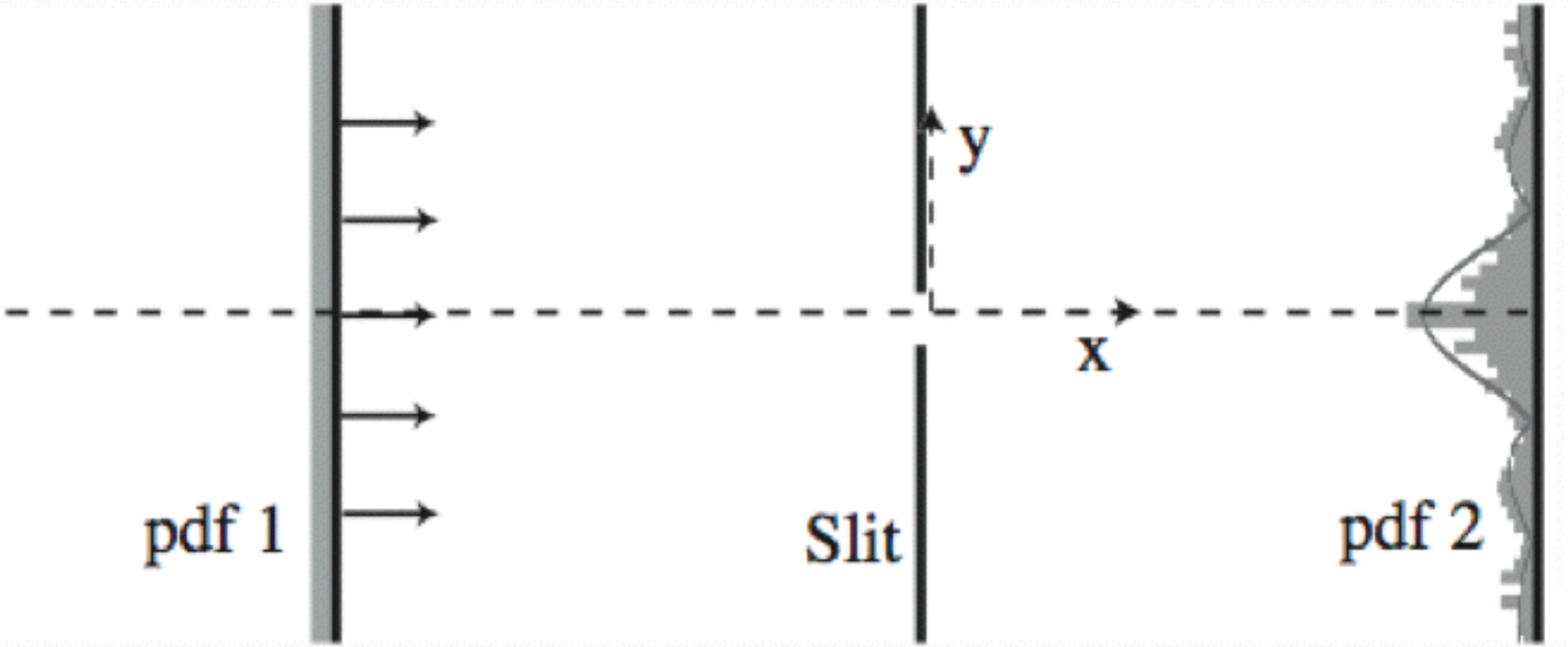


Probability
Distribution
Function

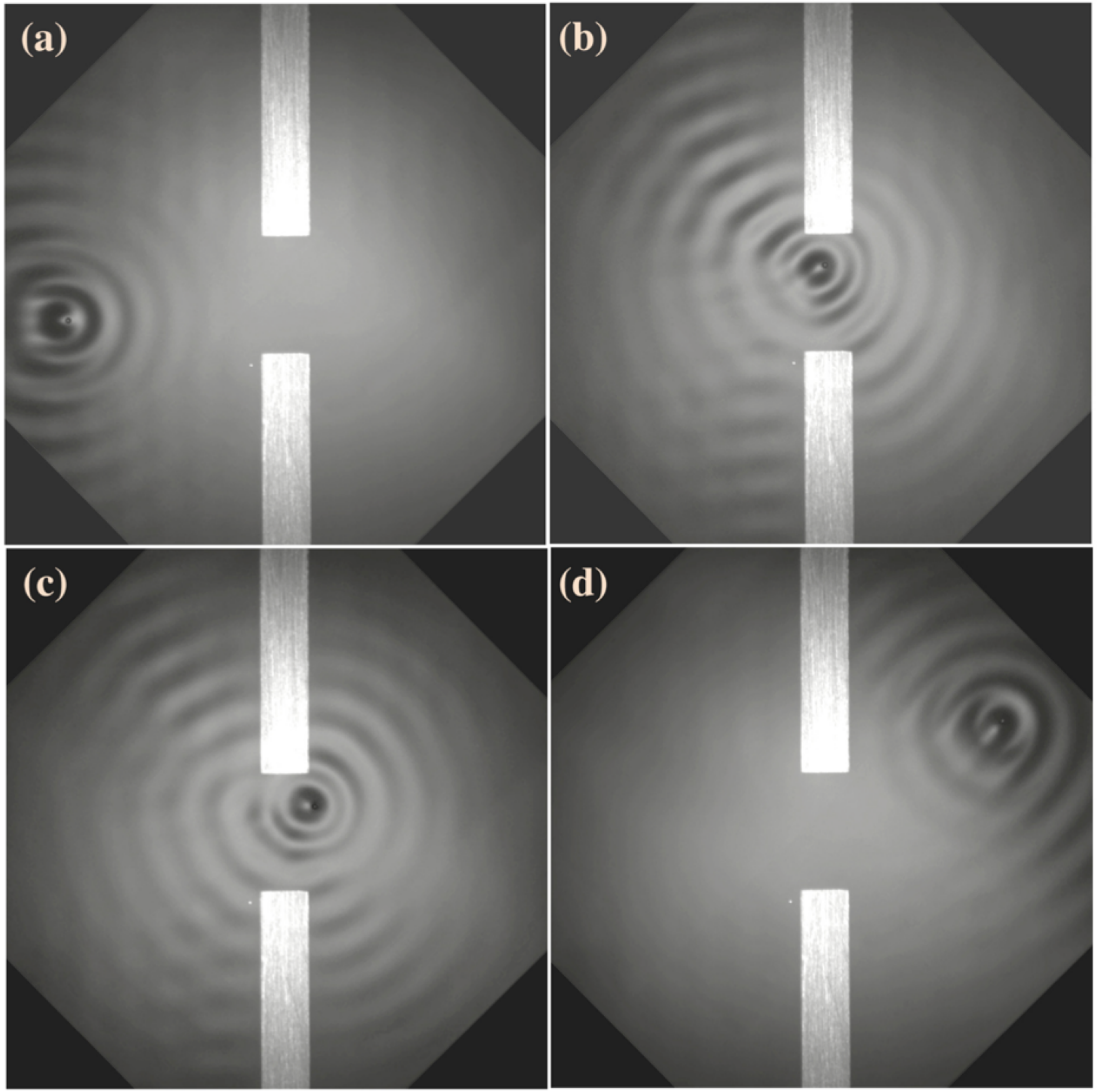


Comparison with actual Faraday wave pattern

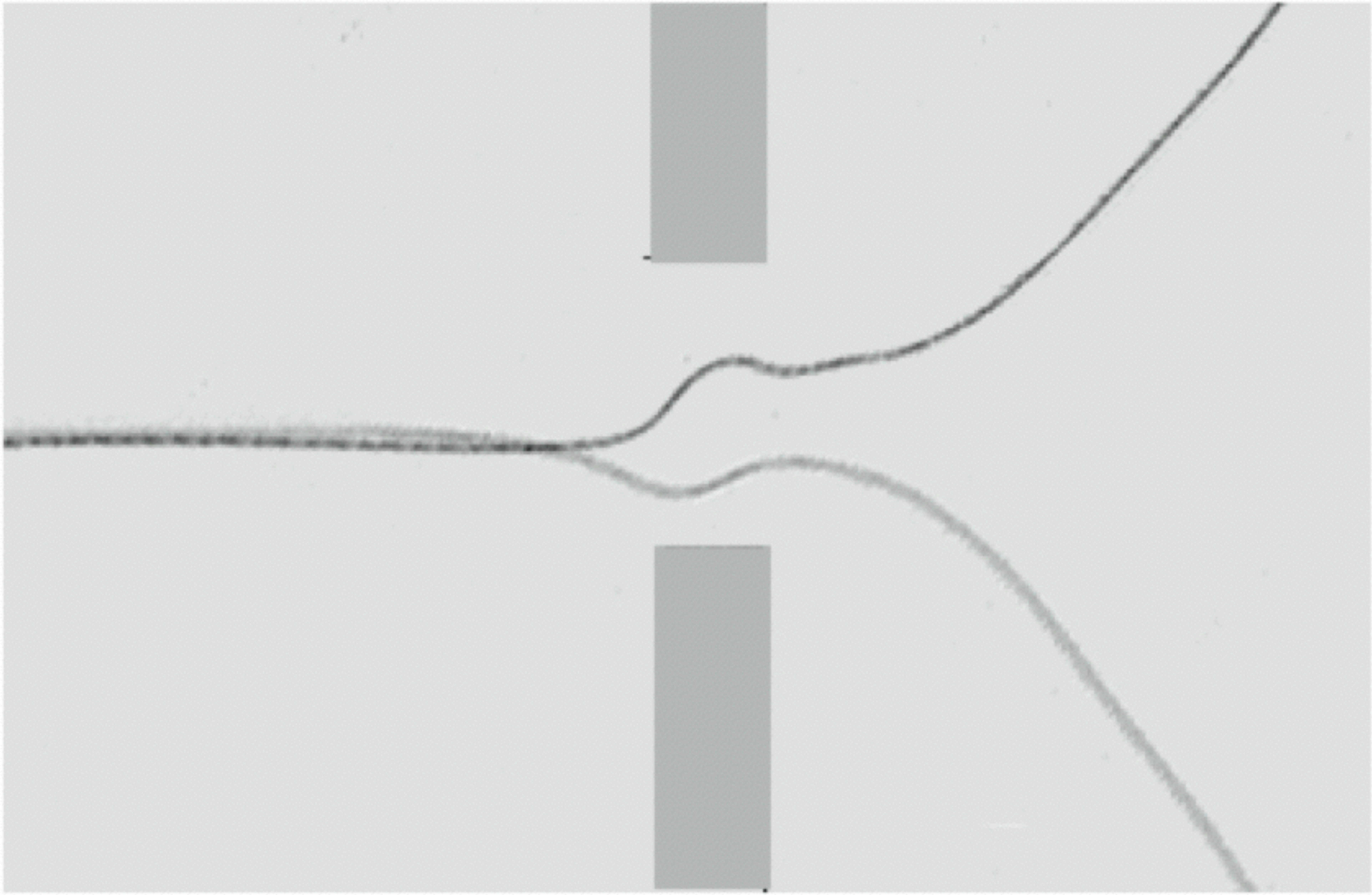
self-interfering classical particle!



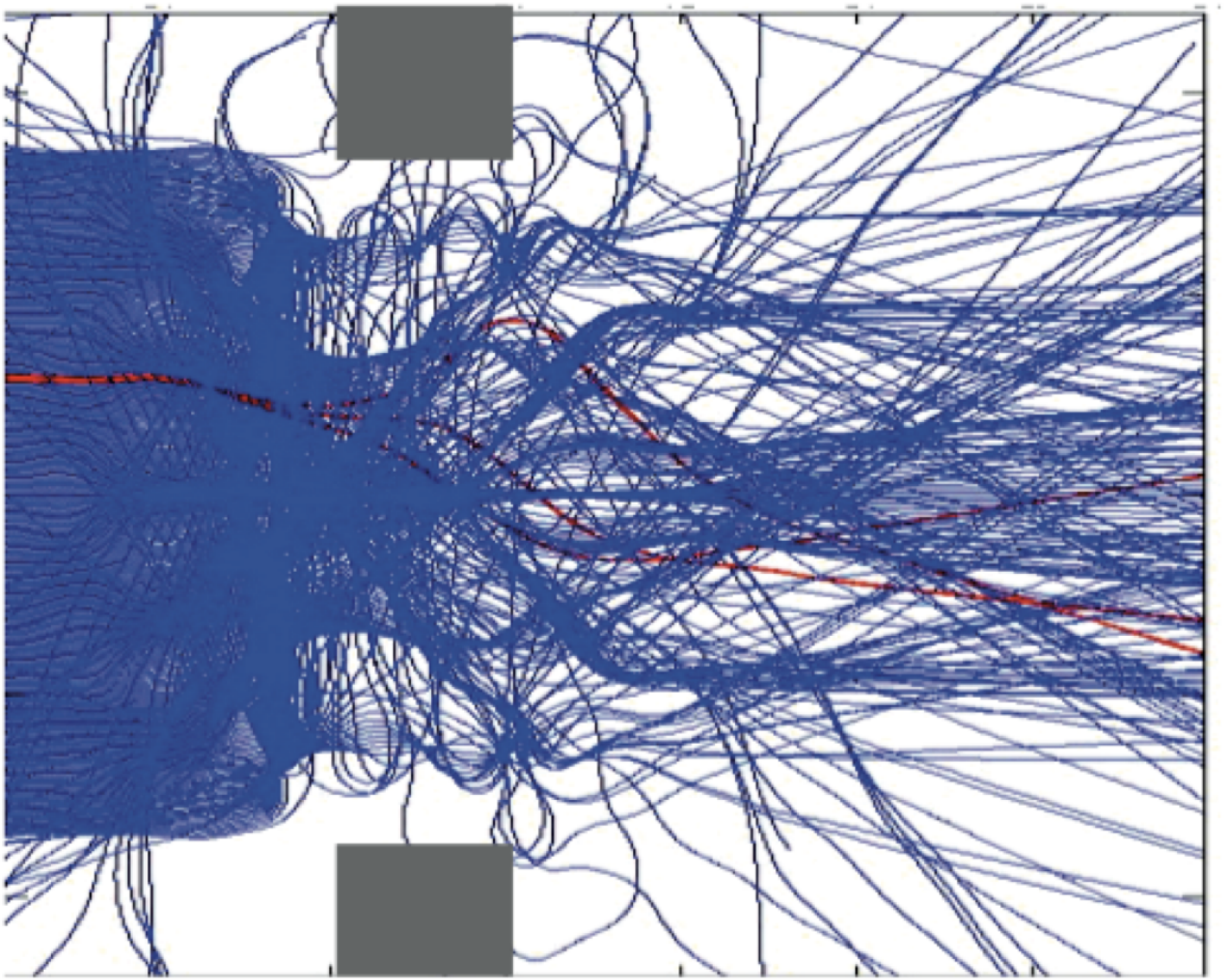
experimental setup



actual snapshots



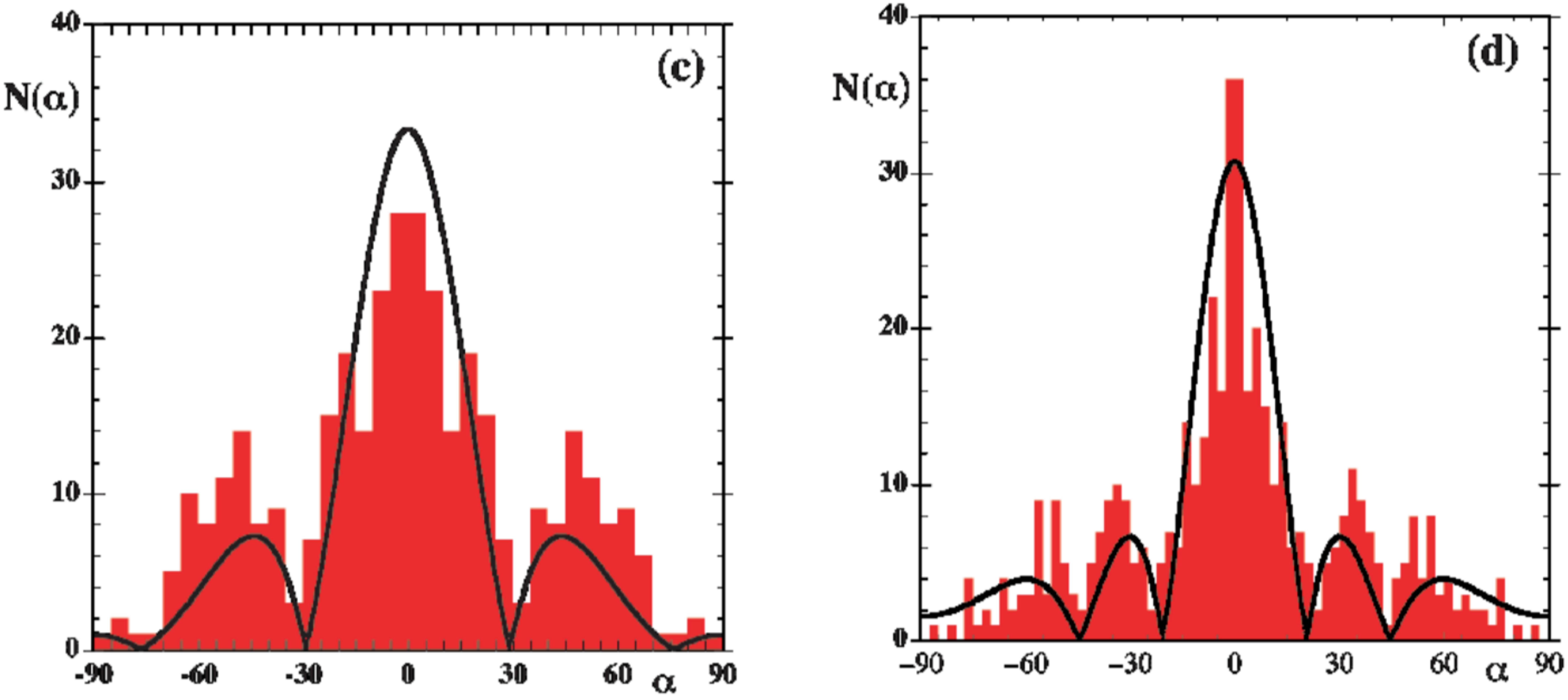
a couple of trajectories…



apparently random again

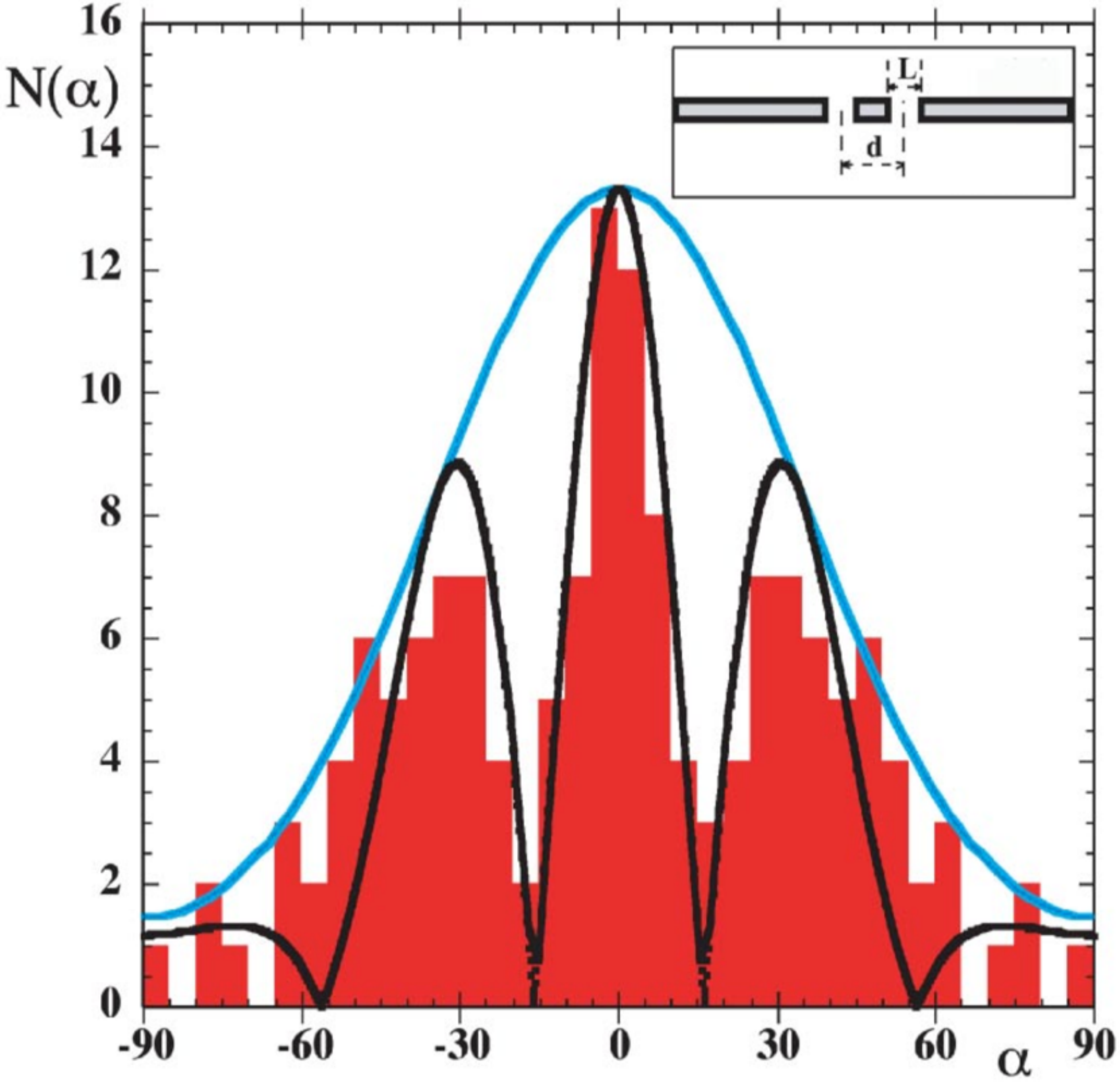
more trajectories!

statistical determinacy



One slit + fit

Y. Couder and E. Fort, *Phys. Rev. Lett.* **97**, 154101 (2006)



Two slits + fit

Y. Couder and E. Fort, *Phys. Rev. Lett.* **97**, 154101 (2006)

> … a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.
>
> R. P. Feynman (1961)

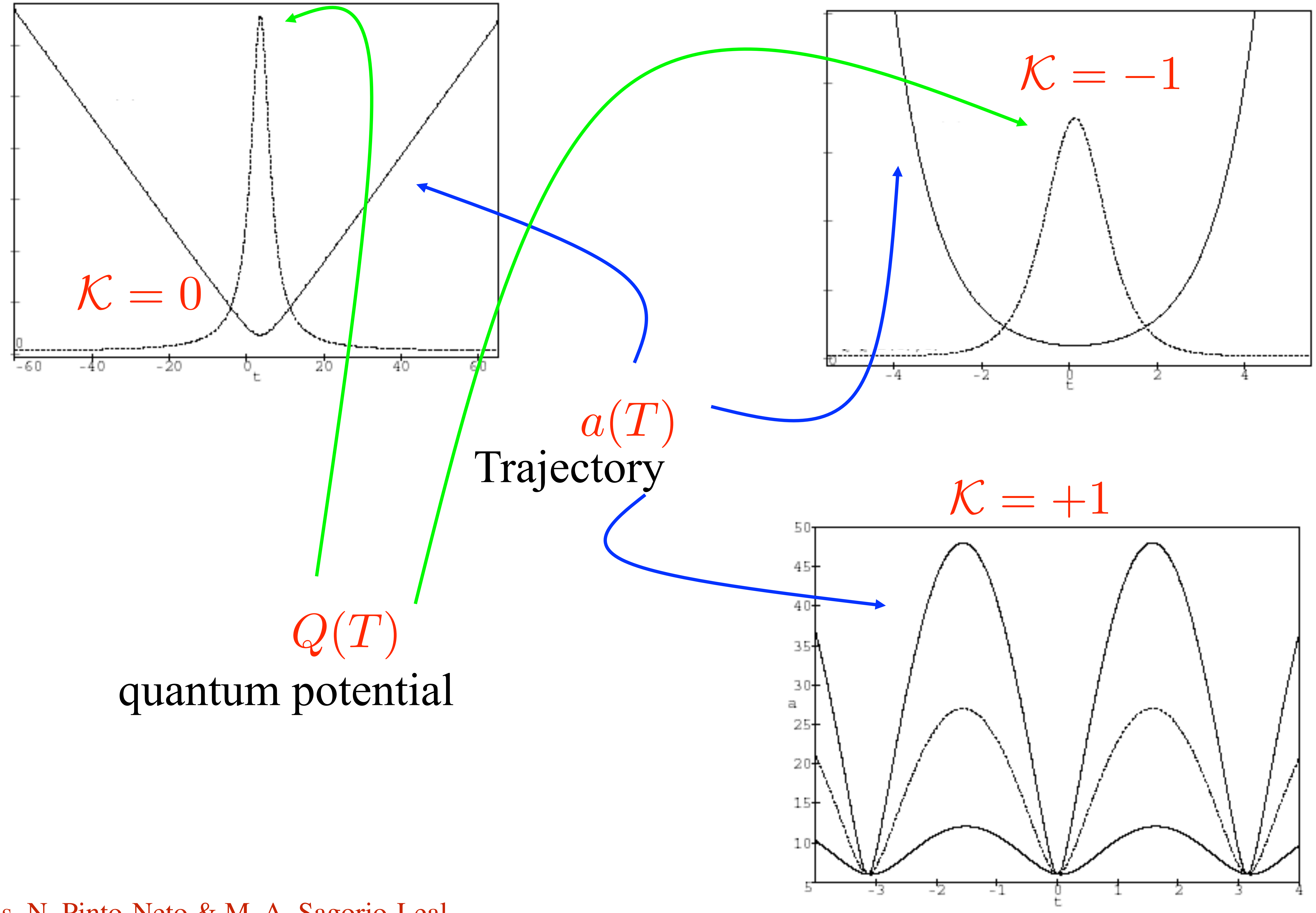
Back to the QC wave function

Gaussian wave packet

$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $$S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

Hidden trajectory $$a = a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$



$\mathcal{K} = 0$

$\mathcal{K} = -1$

$a(T)$

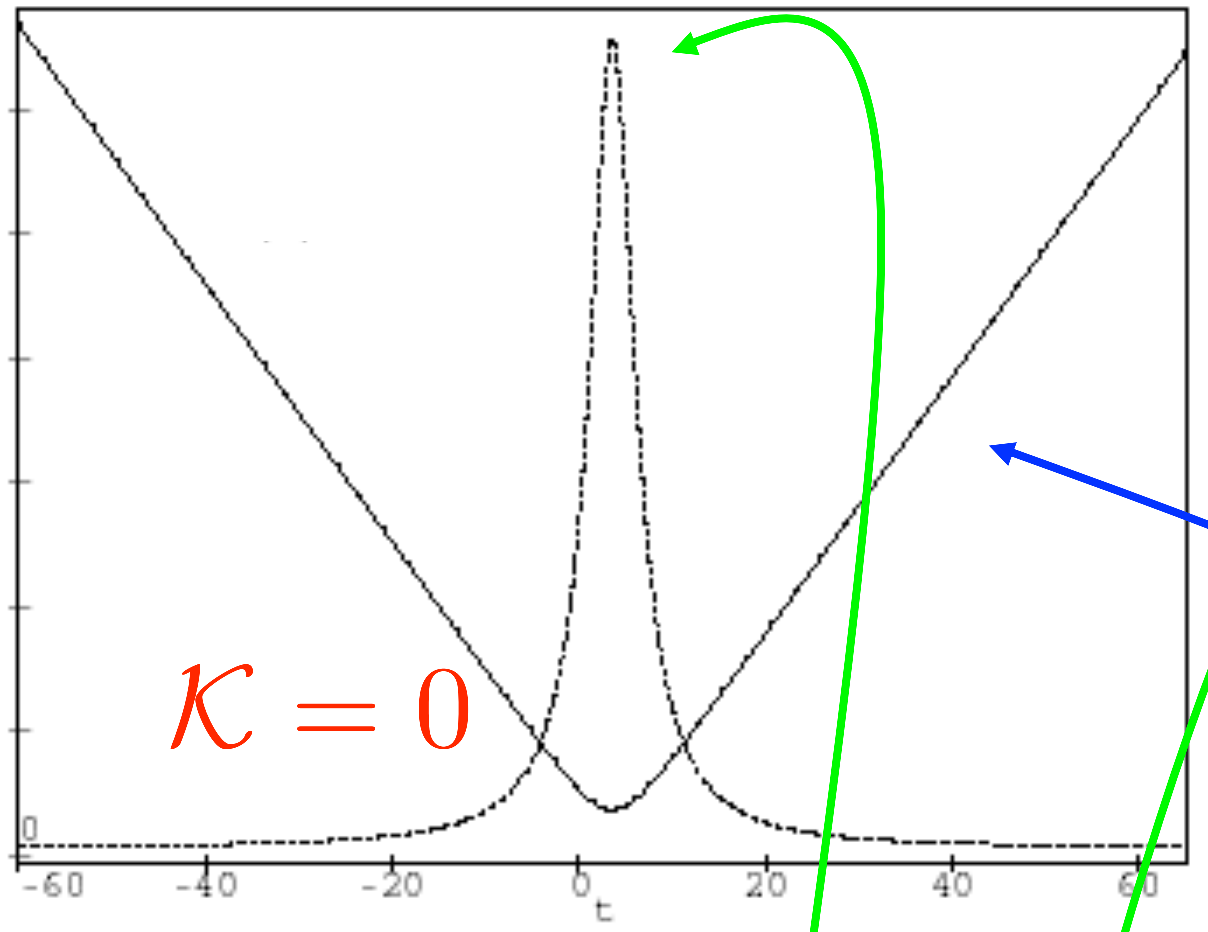
Trajectory

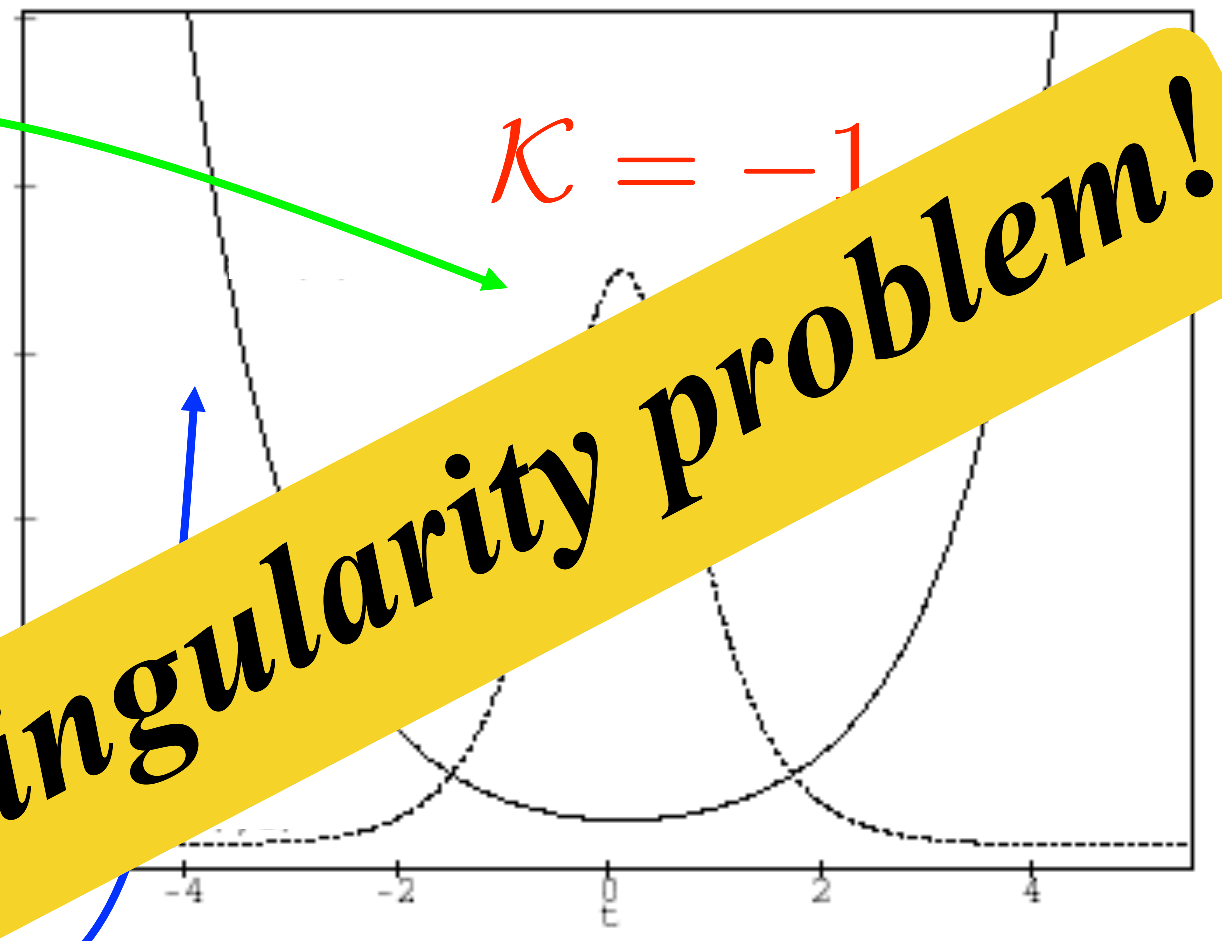
$\mathcal{K} = +1$

$Q(T)$

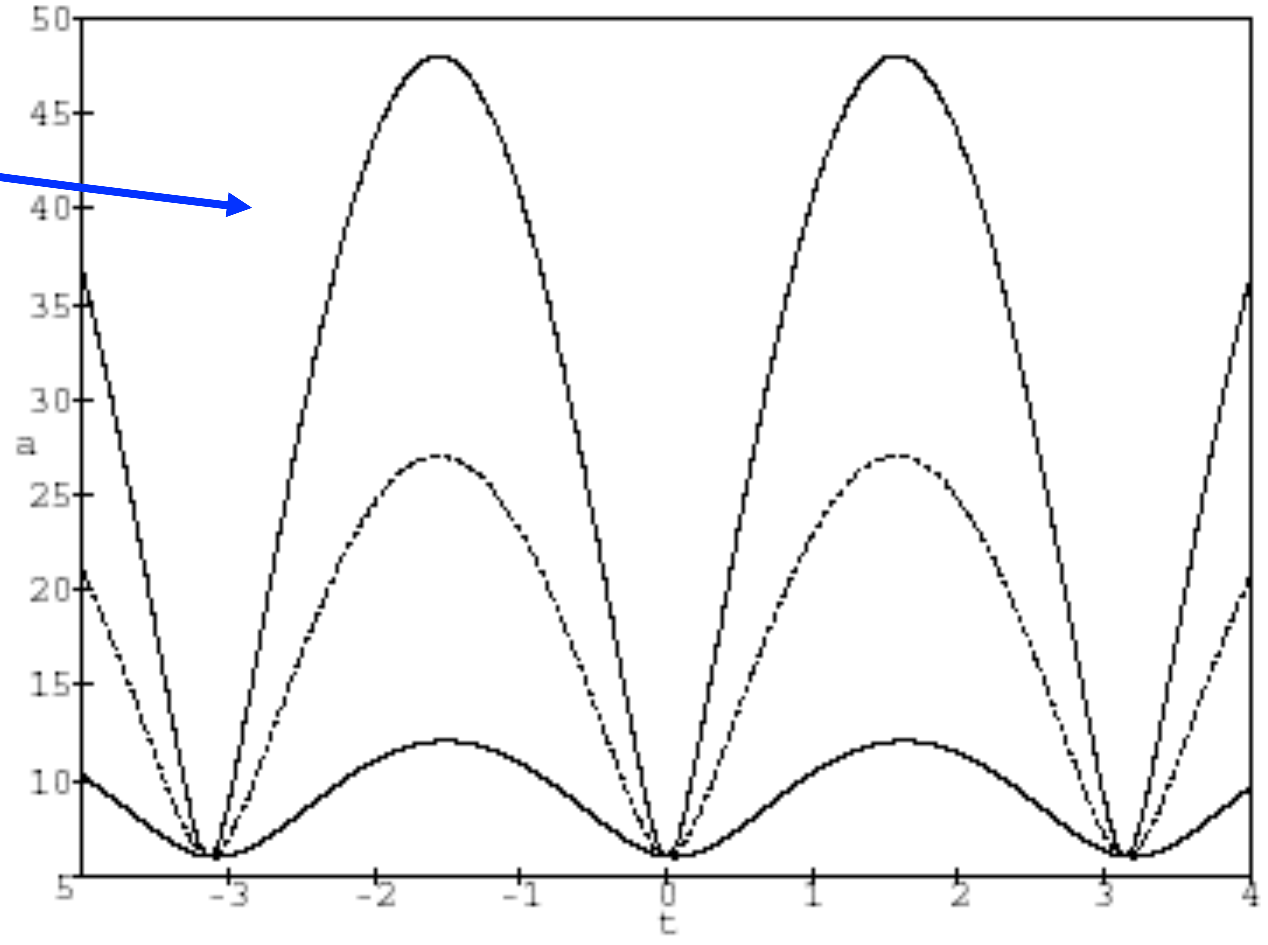
quantum potential

J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,
Phys. Lett. **A241**, 229 (1998)





$\mathcal{K} = +1$



Natural quantum solution to the singularity problem!

(T)

ain potential

J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,
Phys. Lett. **A241**, 229 (1998)

Quantum equilibrium

(Valentini & Westman, 2005)

$$i\frac{\mathrm{d}}{\mathrm{d}t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Particle in a box - 2D

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} - \frac{1}{2}\frac{\partial^2\psi}{\partial y^2} + V\psi$$

infinite square well - size π

Density of actual configurations

$$\rho\left(x,y,t\right) \quad \Longrightarrow \frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho\dot{x}\right) + \frac{\partial}{\partial y}\left(\rho\dot{y}\right) = 0$$

continuity equation

Energy eigenfunctions $\phi_{mn}\left(x,y\right) = \frac{2}{\pi}\sin\left(mx\right)\sin\left(ny\right)$

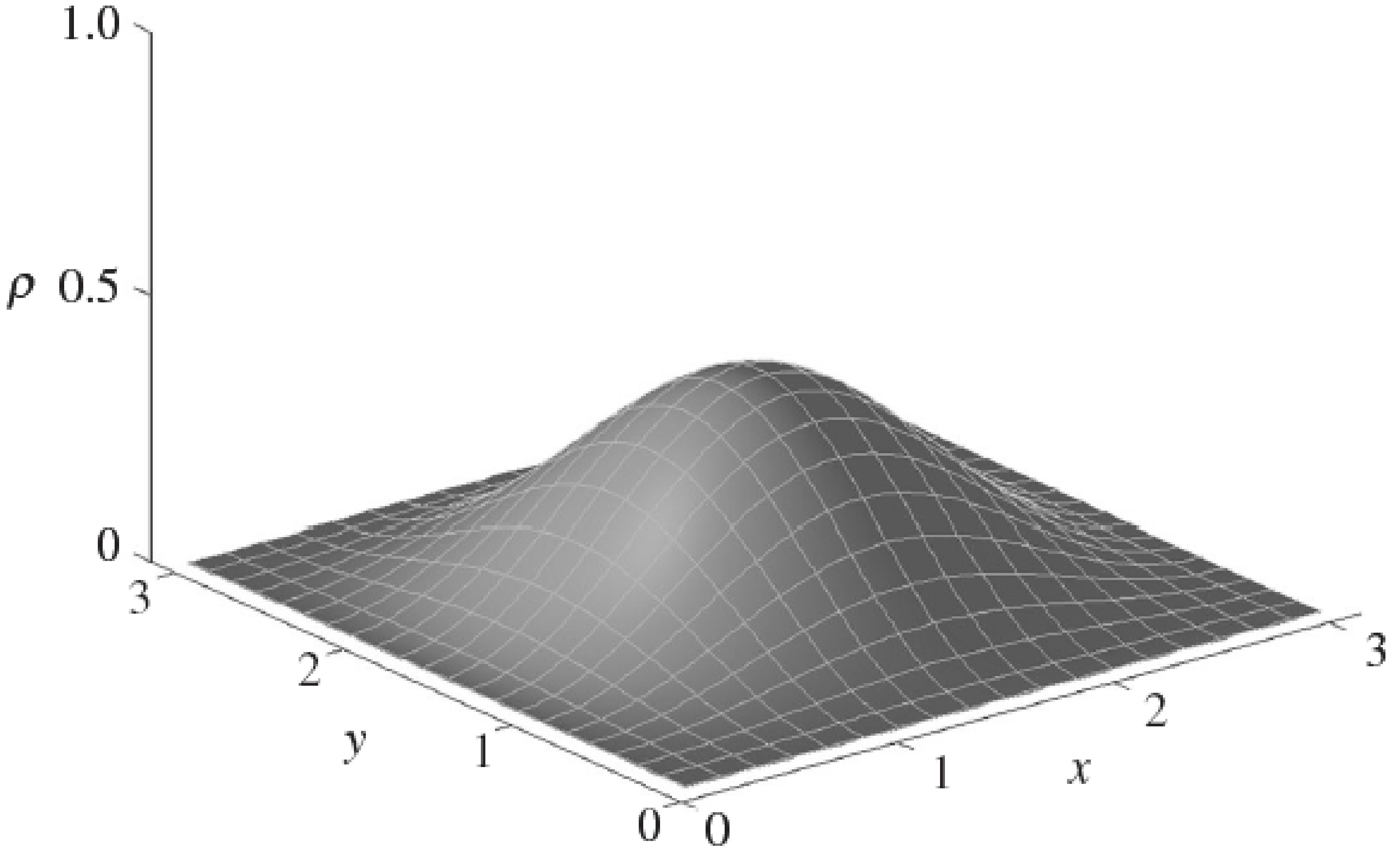
Energy levels $E_{mn} = \frac{1}{2}\left(m^2+n^2\right)$

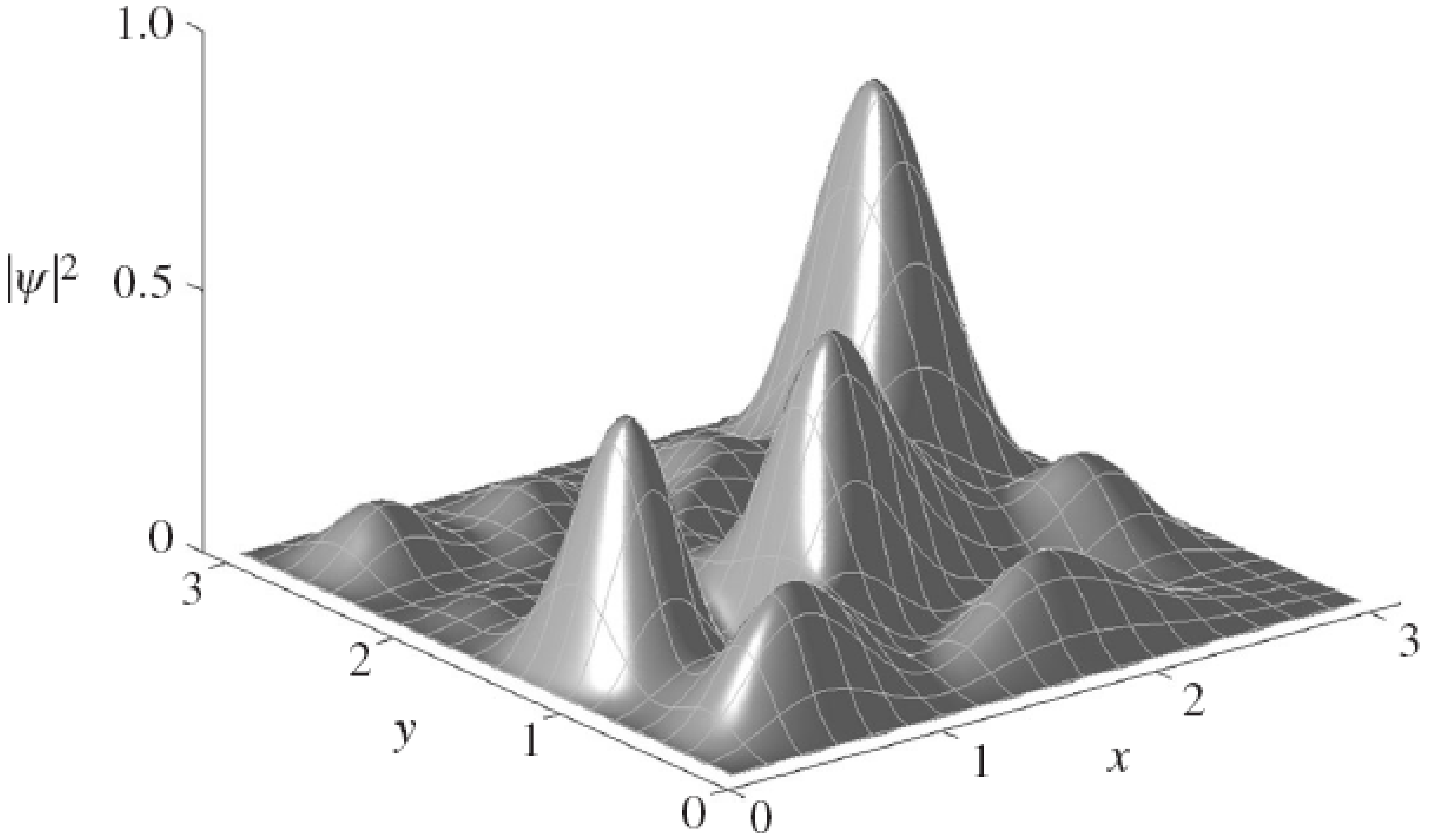
Initial configuration

$$\psi(x,y,0) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp(i\theta_{mn})$$

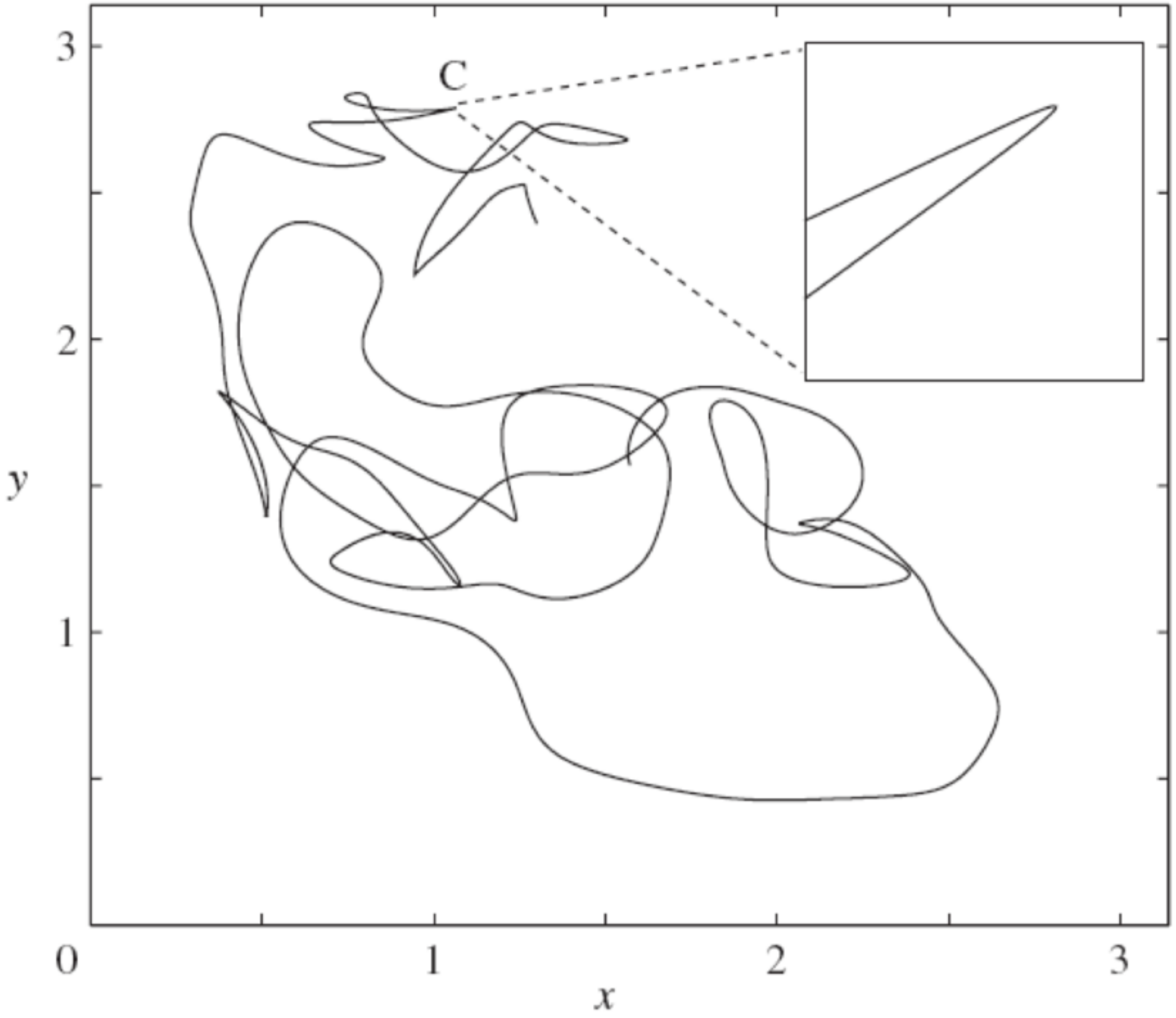
$$\rho(x,y,0) = |\phi_{11}(x,y)|^2$$

$$\psi(x,y,t) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp i(\theta_{mn} - E_{mn}t)$$

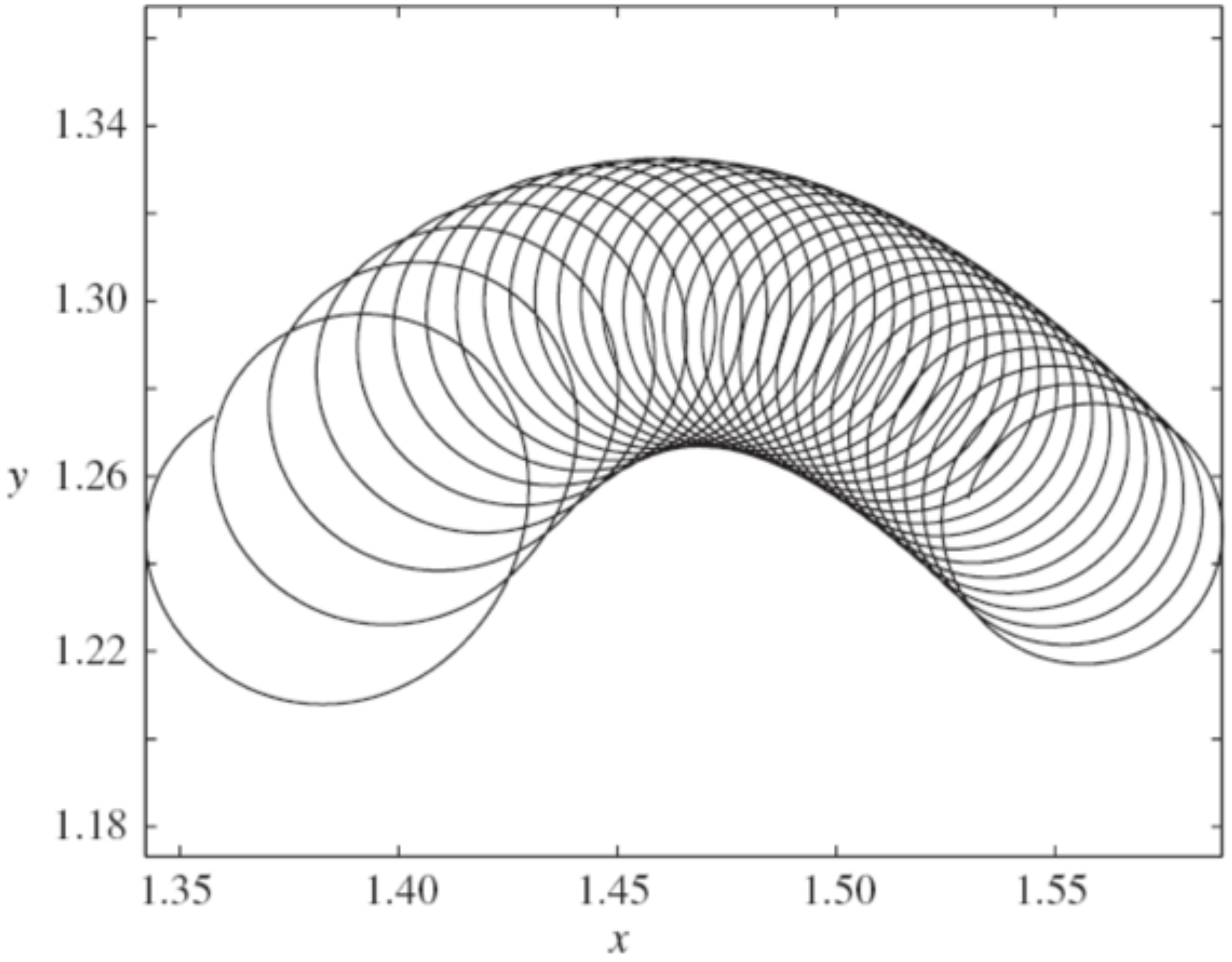


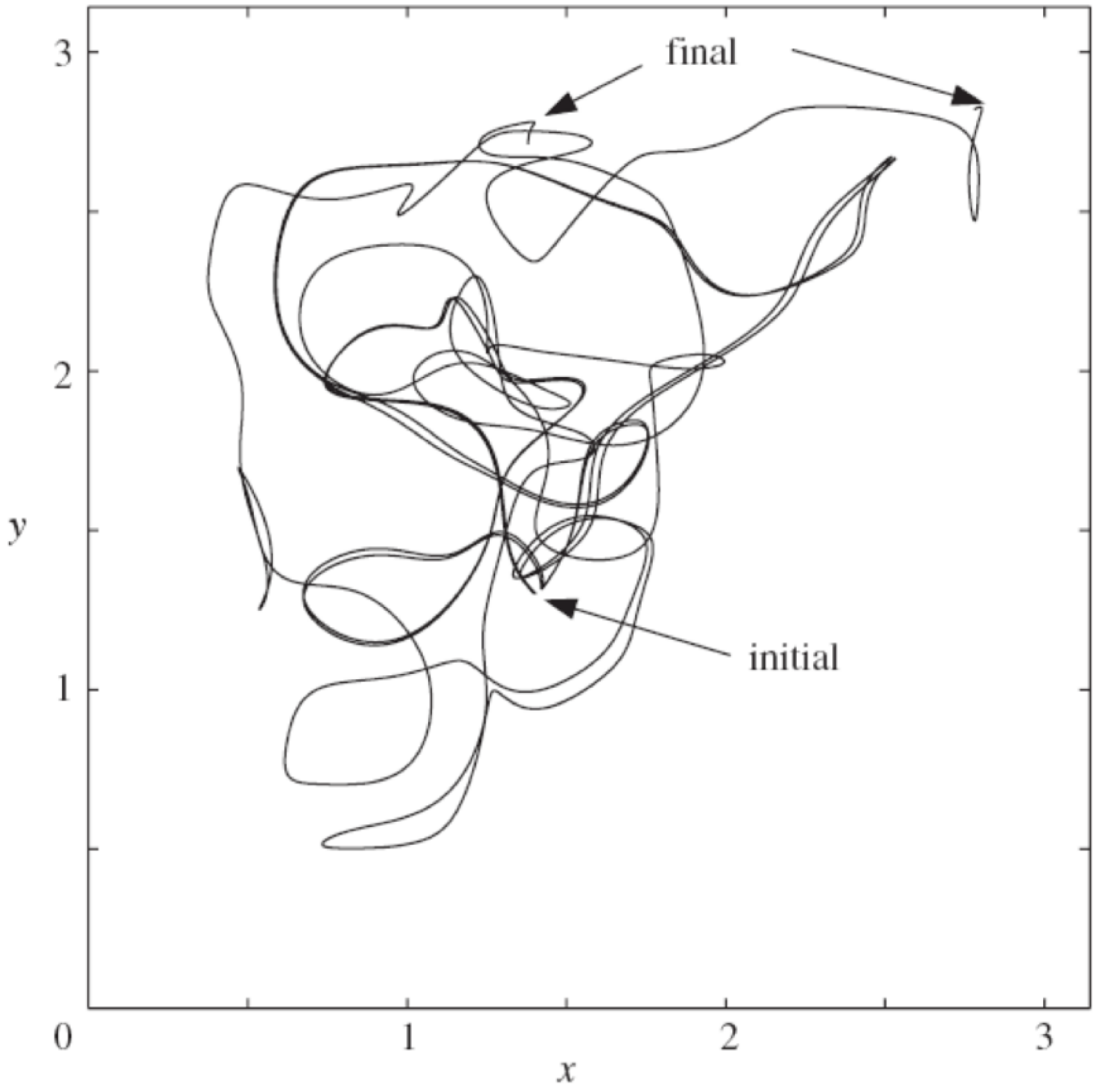


Typical quantum trajectory…



Close-up of a trajectory near a node

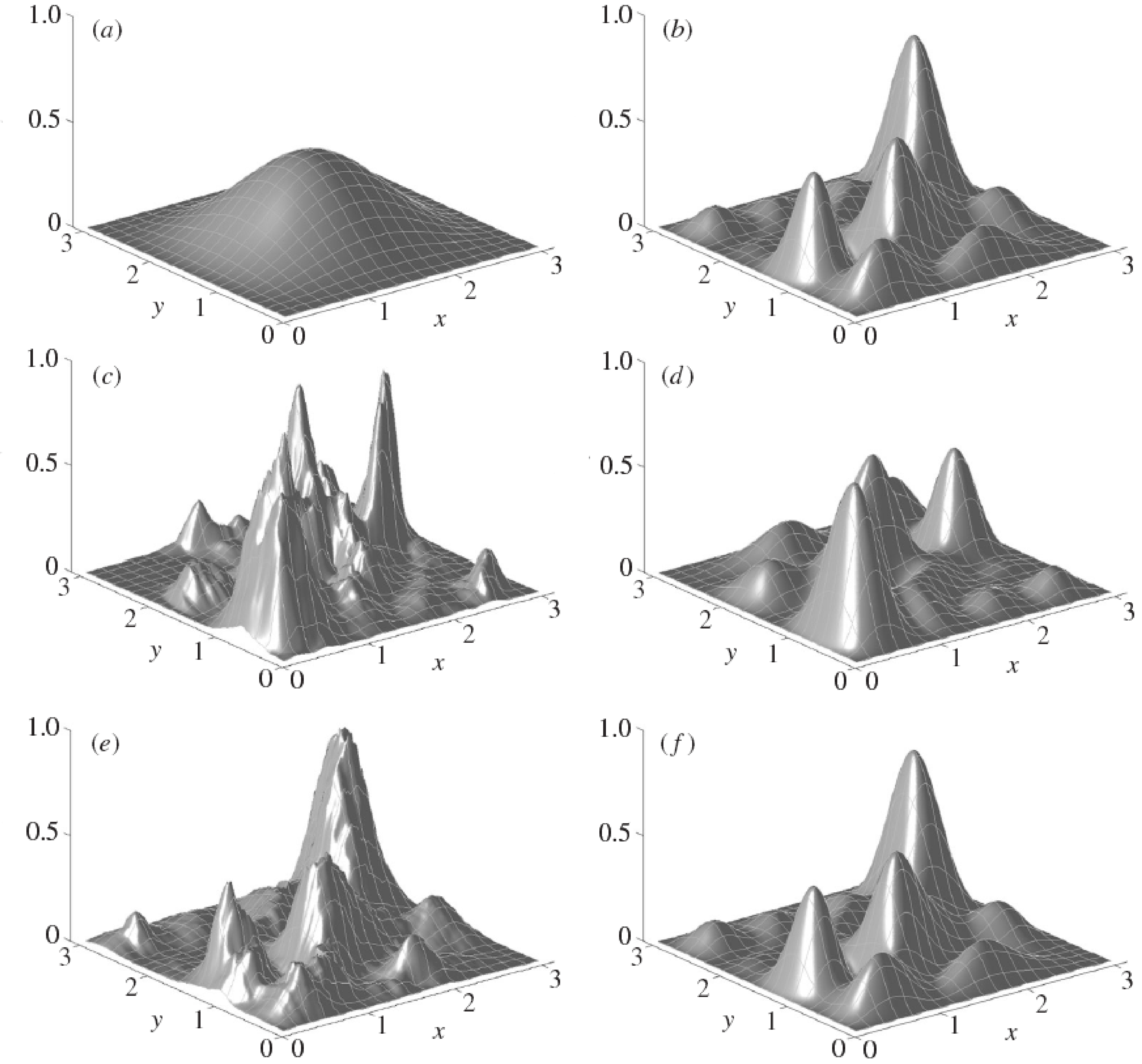




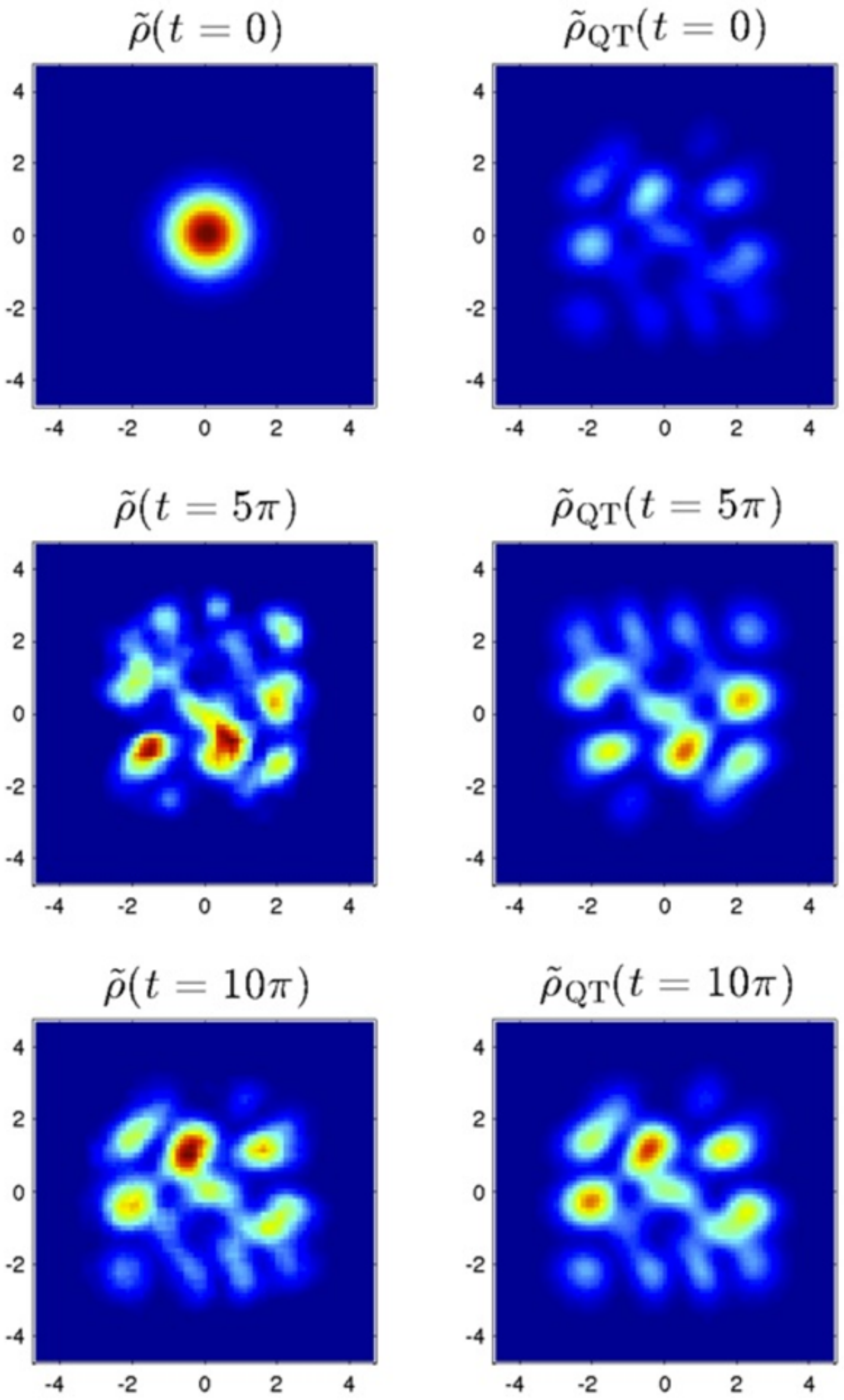
chaotic mixing…

Dynamical evolutions

ρ



$|\Psi|^2$



$\tilde{\rho}(t=0)$ $\tilde{\rho}_{\mathrm{QT}}(t=0)$

$\tilde{\rho}(t=5\pi)$ $\tilde{\rho}_{\mathrm{QT}}(t=5\pi)$

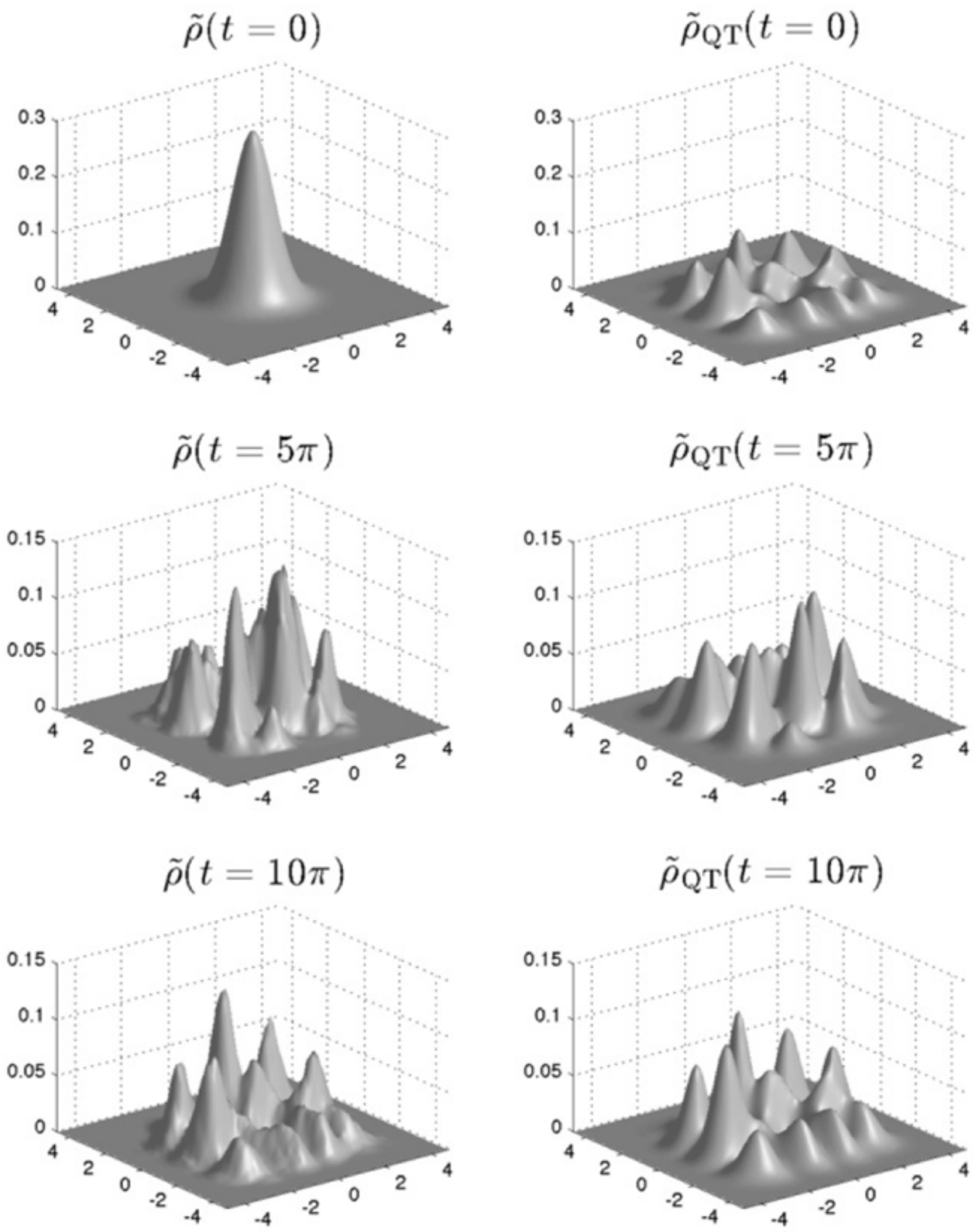
$\tilde{\rho}(t=10\pi)$ $\tilde{\rho}_{\mathrm{QT}}(t=10\pi)$

chaotic mixing…

↓

relaxation towards equilibrium

just like ordinary thermal equilibrium



$\tilde{\rho}(t=0)$

$\tilde{\rho}_{\rm QT}(t=0)$

$\tilde{\rho}(t=5\pi)$

$\tilde{\rho}_{\rm QT}(t=5\pi)$

$\tilde{\rho}(t=10\pi)$

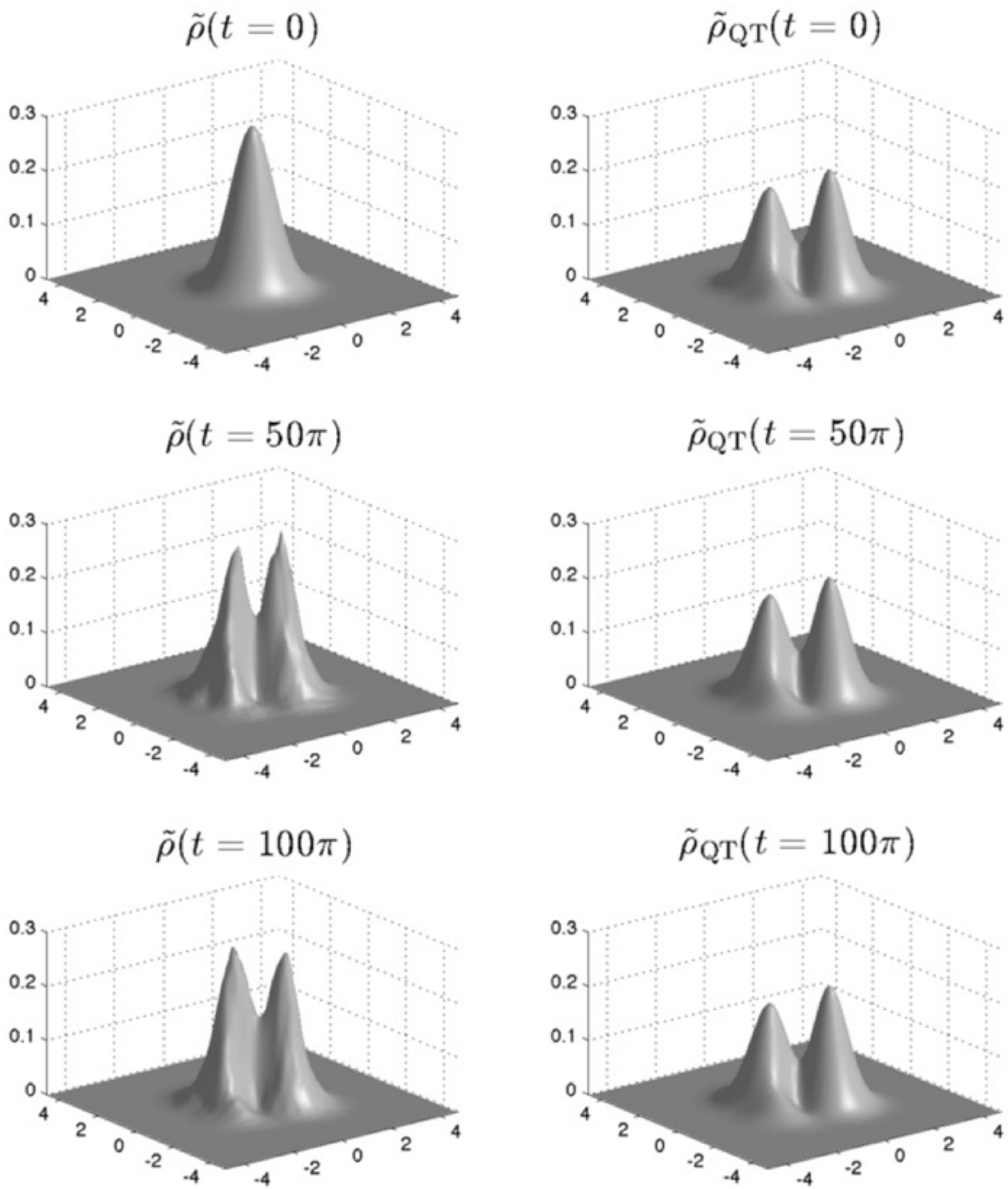
$\tilde{\rho}_{\rm QT}(t=10\pi)$

chaotic mixing…

↓

relaxation towards equilibrium

just like ordinary thermal equilibrium



chaotic mixing…

↓

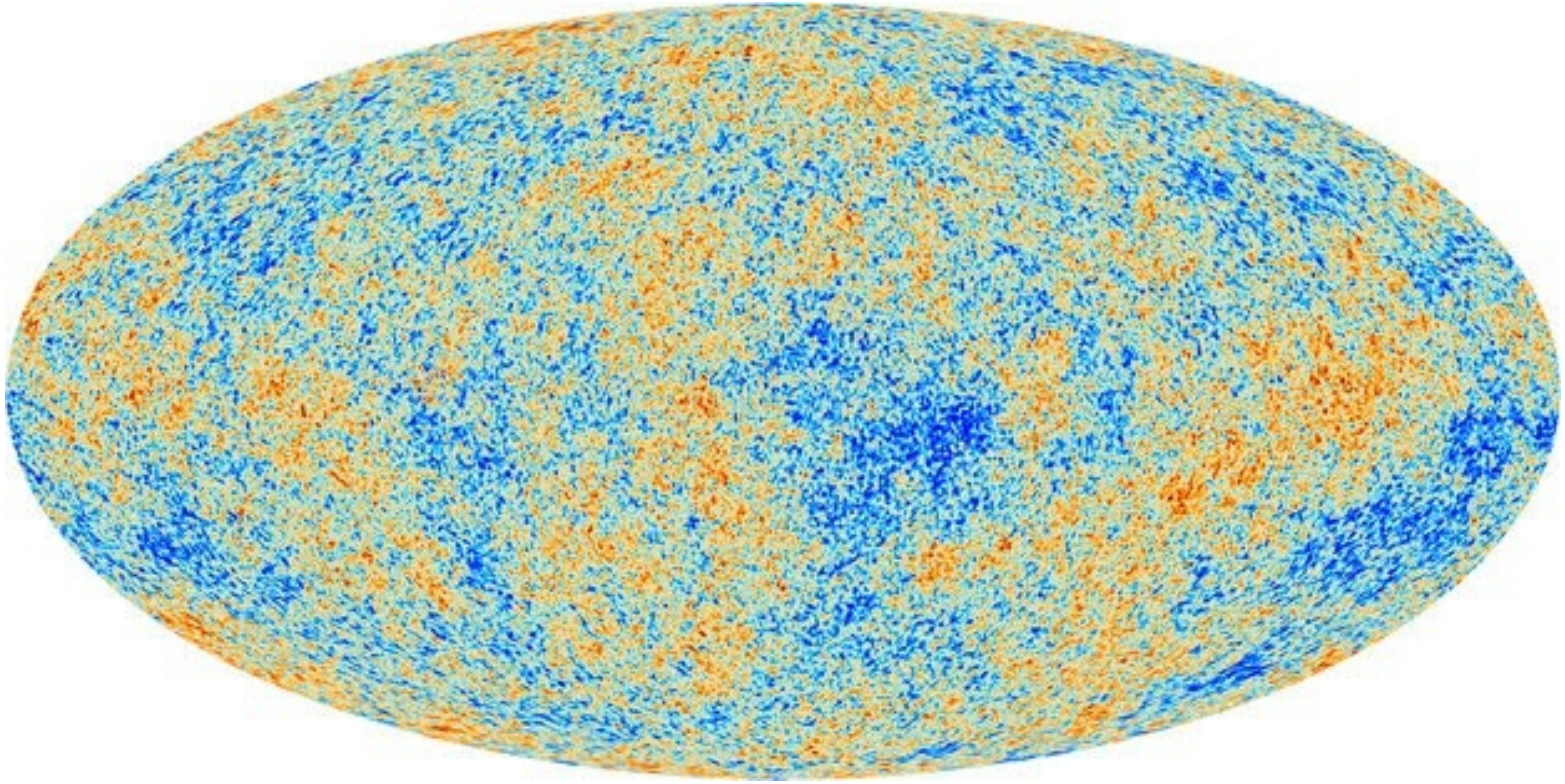
relaxation towards equilibrium

just like ordinary thermal equilibrium

possibly slightly smaller width for low number of modes…

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$



Cargèse / 22 september 2014

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations

$$\frac{\Delta T}{T} \propto v \ \sim \Phi \sim \delta g_{00}$$

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v} \;\sim \Phi \sim \delta g_{00}$$

second order perturbed Einstein action $\;{}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4x \left[\left(v'\right)^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2 \right]$

variable-mass scalar field in Minkowski spacetime

$$\epsilon_1 \;=\; 1 - \mathcal{H}'/\mathcal{H}^2$$

slow-roll parameter

\+ Fourier transform $\;v\left(\eta, \boldsymbol{x}\right) = \frac{1}{\left(2\pi\right)^{3/2}} \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, v_{\boldsymbol{k}}\left(\eta\right) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$\Longrightarrow \quad {}^{(2)}\delta S \;=\; \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{k} \left\{ v_{\boldsymbol{k}}' v_{\boldsymbol{k}}^{*\,\prime} + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^{*} \left[\frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} - k^2 \right] \right\}$$

Lagrangian formulation...

Hamiltonian

$$H=\int \mathrm{d}^3\boldsymbol{k}\left\{p_{\boldsymbol{k}}p_{\boldsymbol{k}}^{*}+v_{\boldsymbol{k}}v_{\boldsymbol{k}}^{*}\left[\overbrace{k^2-\frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2\left(\eta,\boldsymbol{k}\right)}\right]\right\}$$

collection of parametric oscillators with time dependent frequency

factorization of the full wave function

real and imaginary parts

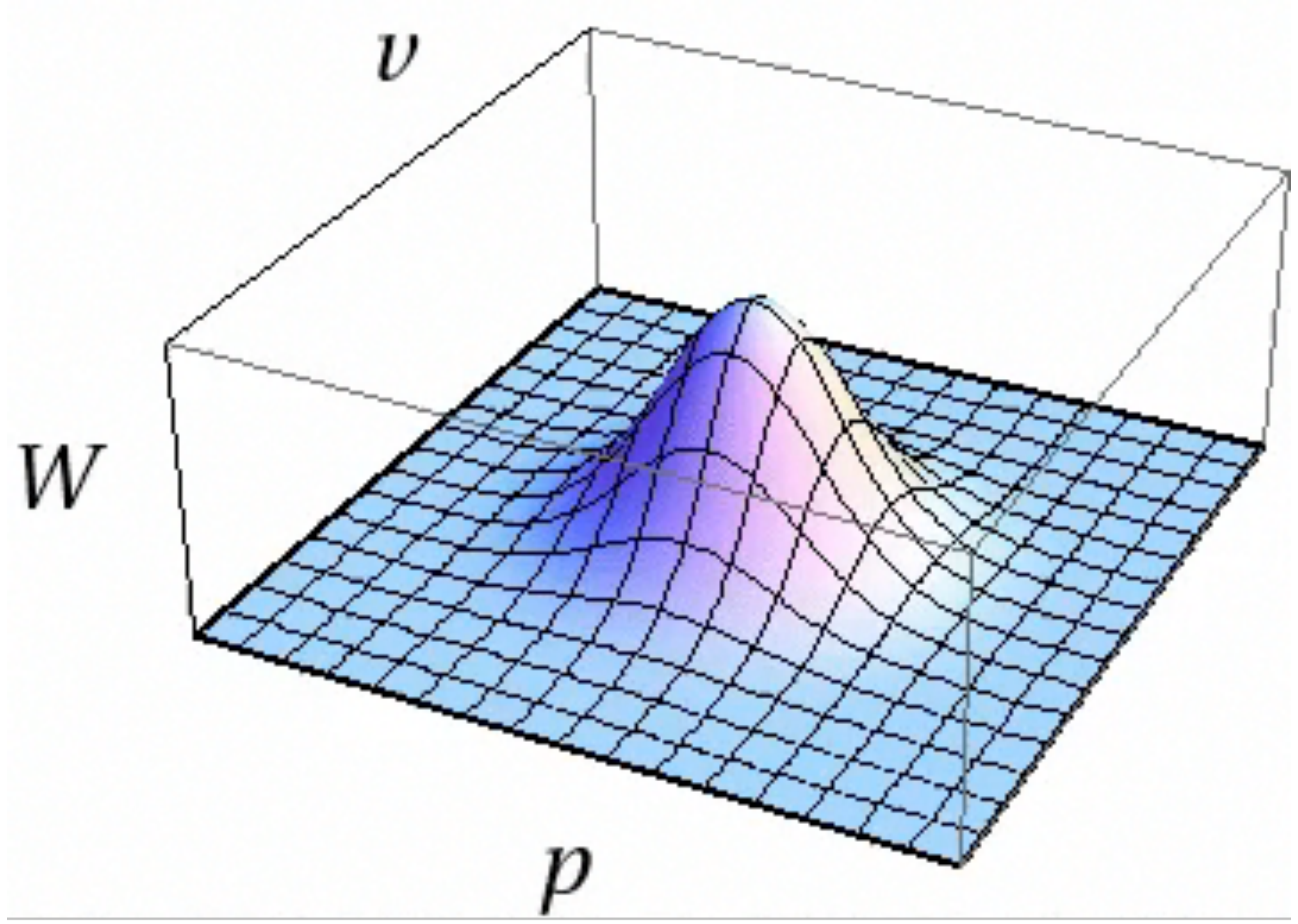
$$\Psi\left[v(\eta,\boldsymbol{x})\right]=\prod_{\boldsymbol{k}}\Psi_{\boldsymbol{k}}\left(v_{\boldsymbol{k}}^{\mathrm{R}},v_{\boldsymbol{k}}^{\mathrm{I}}\right)=\prod_{\boldsymbol{k}}\Psi_{\boldsymbol{k}}^{\mathrm{R}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}\right)\Psi_{\boldsymbol{k}}^{\mathrm{I}}\left(v_{\boldsymbol{k}}^{\mathrm{I}}\right)$$

$$\left.\begin{aligned}
&i\frac{\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}}{\partial\eta}=\hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}}\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}\\
&\hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}}=-\frac{1}{2}\frac{\partial^2}{\partial\left(v_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2}+\frac{1}{2}\omega^2(\eta,\boldsymbol{k})\left(\hat{v}_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2
\end{aligned}\right\}$$

Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right) = \left[\frac{2\Re\mathrm{e}\,\Omega_{\boldsymbol{k}}\left(\eta\right)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

Wigner function $W\left(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}\right) = \int \frac{\mathrm{d}x}{2\pi^2} \Psi^*\left(v_{\boldsymbol{k}} - \frac{x}{2}\right) \mathrm{e}^{-ip_{\boldsymbol{k}}x} \Psi\left(v_{\boldsymbol{k}} + \frac{x}{2}\right)$

large squeezing limit ⇒ $W \propto \delta\left(p_{\boldsymbol{k}} + k \tan\phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

Ergodicity

realization

spatial direction

$$\left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\xi} \simeq \left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\boldsymbol{e}}$$

Primordial Power Spectrum

Standard case

Quantization in the
Schrödinger picture
(functional representation)

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with $$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}$$

$$\hat{p}_{\boldsymbol{k}} = i\frac{\partial}{\partial v_{\boldsymbol{k}}}$$

and $$\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}$$

$$= k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

Power-law inflation example

$$a(\eta) = \ell_0\boxed{(-\eta)^{1+\beta}}$$

$$\beta \lesssim -2$$

(de Sitter: $\beta = -2$)

Parametric Oscillator System

Primordial Power Spectrum

Standard case

Quantization in the
Schrödinger picture
(functional representation)

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta)\, v_{\boldsymbol{k}}^2}$$

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

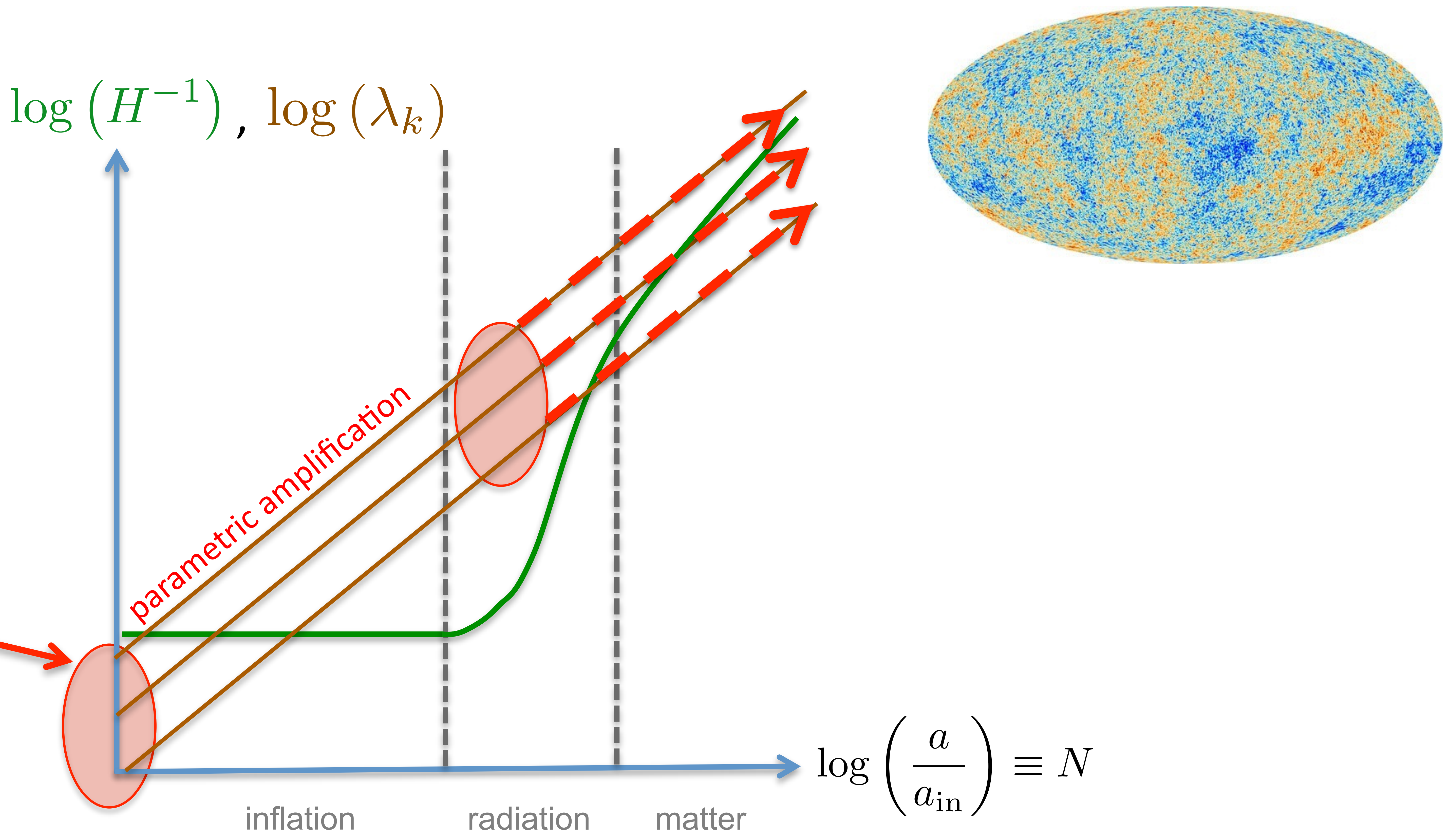
with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k}, \eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta, \boldsymbol{k})$$

$$\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}}$$

$$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k}, \eta) f_{\boldsymbol{k}} = 0}$$



Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Primordial Power Spectrum

Standard case

Two physical scales

Hubble radius $H^{-1} = \frac{a^2}{a'} \underset{\beta\sim-2}{\simeq} \ell_0$

wavelength $\lambda = \frac{a}{k} \underset{\beta\sim-2}{\simeq} \frac{\ell_0}{-k\eta}$

Sub-Hubble regime

$\lambda \ll H^{-1}$

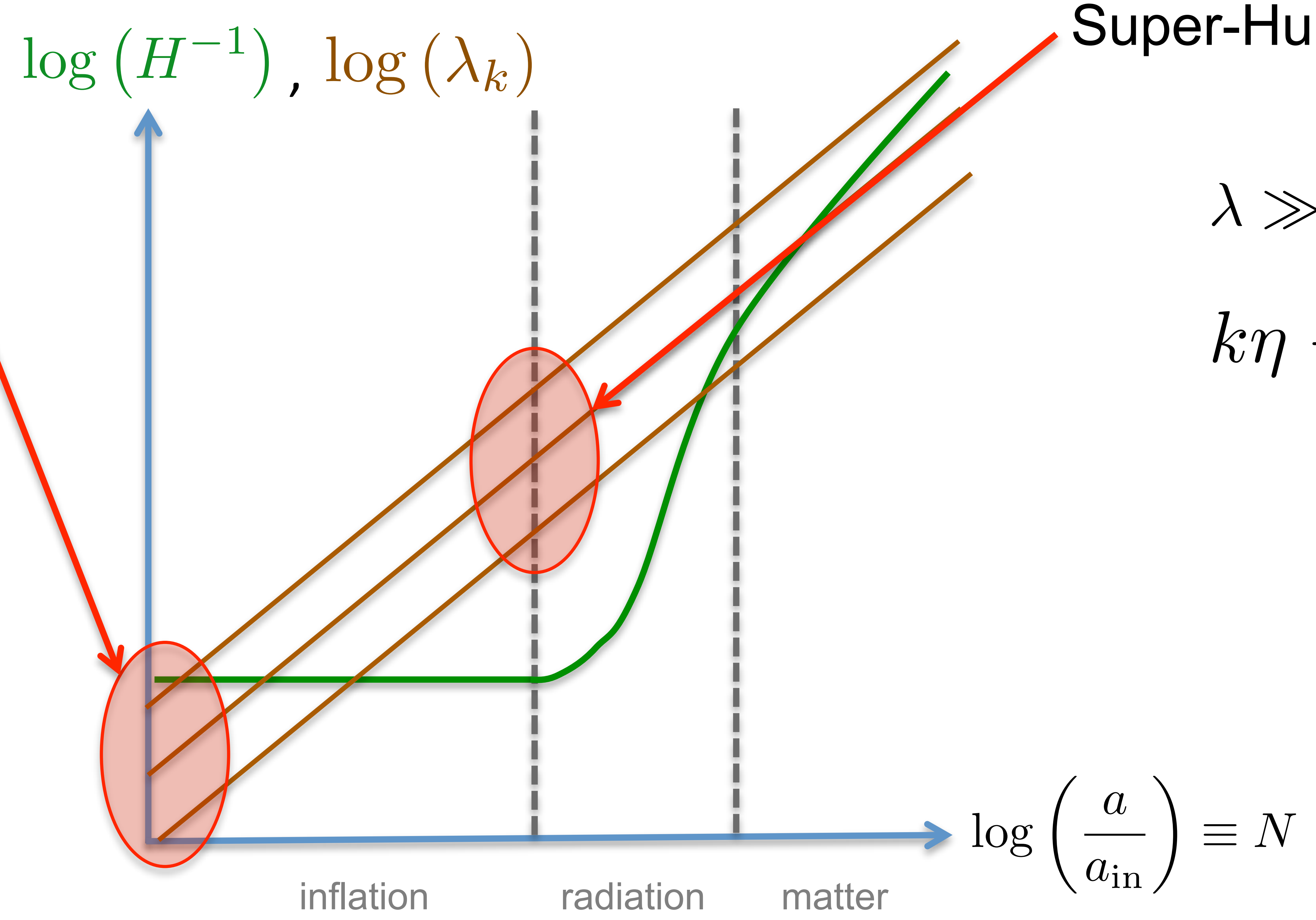
$k\eta \to -\infty$

$\omega \simeq k$

harmonic oscillator

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$

Bunch Davis vacuum



Super-Hubble regime

$\lambda \gg H^{-1}$

$k\eta \to 0^-$

sets initial conditions $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

$$v''_{\boldsymbol{k}} + \left[\boldsymbol{k}^2 - U(\eta)\right] v_{\boldsymbol{k}} = 0$$

Vacuum state

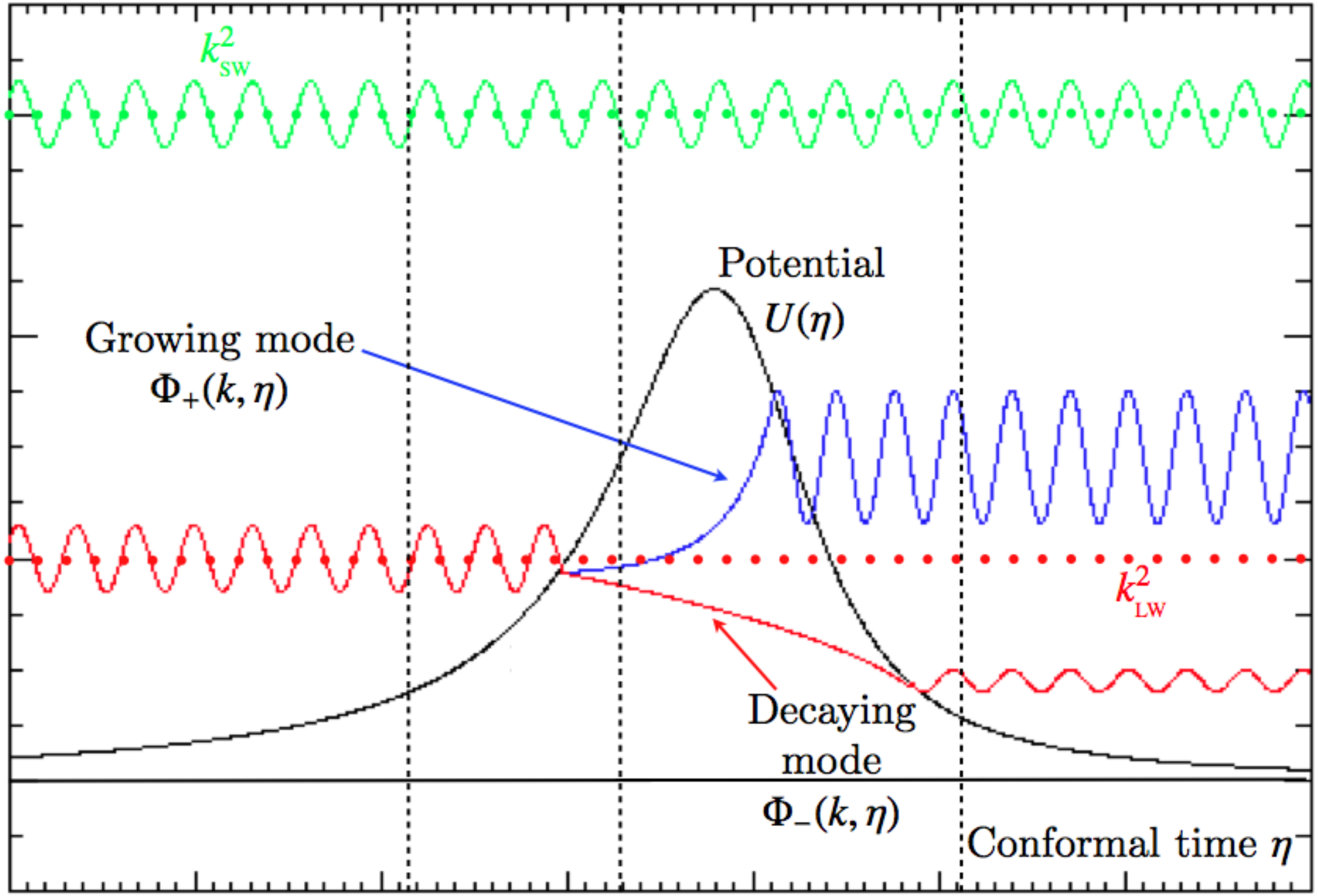
$$v_{\boldsymbol{k}} \xrightarrow[|k\eta|\to\infty]{} \frac{\mathrm{e}^{-ik\eta}}{\sqrt{2k}}$$

Initial conditions fixed!

compare

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + \frac{2m}{\hbar^2}\left[E - U(x)\right]\Psi = 0$$

(time independent
Schrödinger equation)



Transmission & Reflexion coefficients!

Primordial Power Spectrum

Standard case

$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0}$ with $\omega^2(\boldsymbol{k},\eta) = k^2 - \dfrac{\beta(\beta+1)}{\eta^2}$ and $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

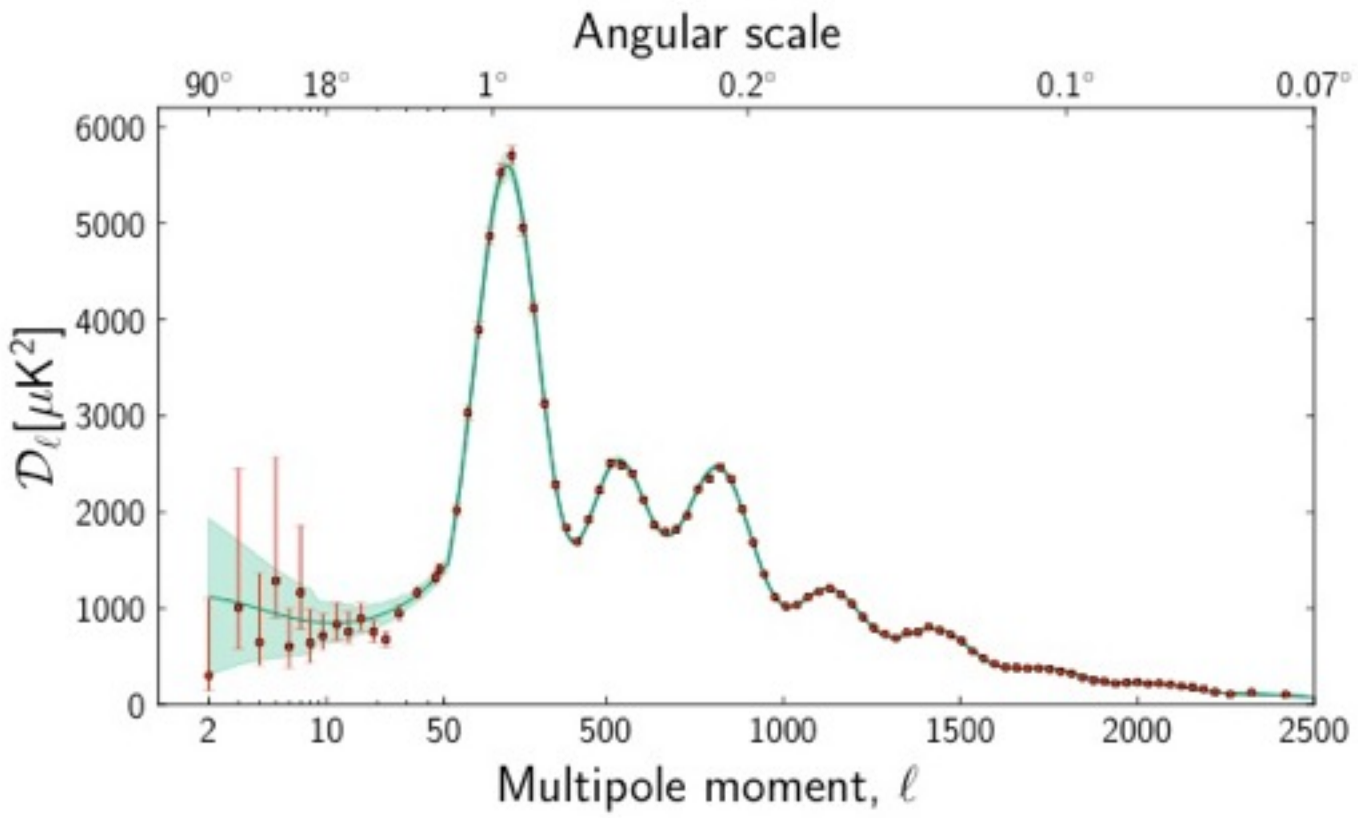
Uniquely determines $f_{\boldsymbol{k}}$ $\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}}$ $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation$(k\eta \to 0^-)$, this gives $P_v(k) = \dfrac{k^3}{2\pi^3}\left(\langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

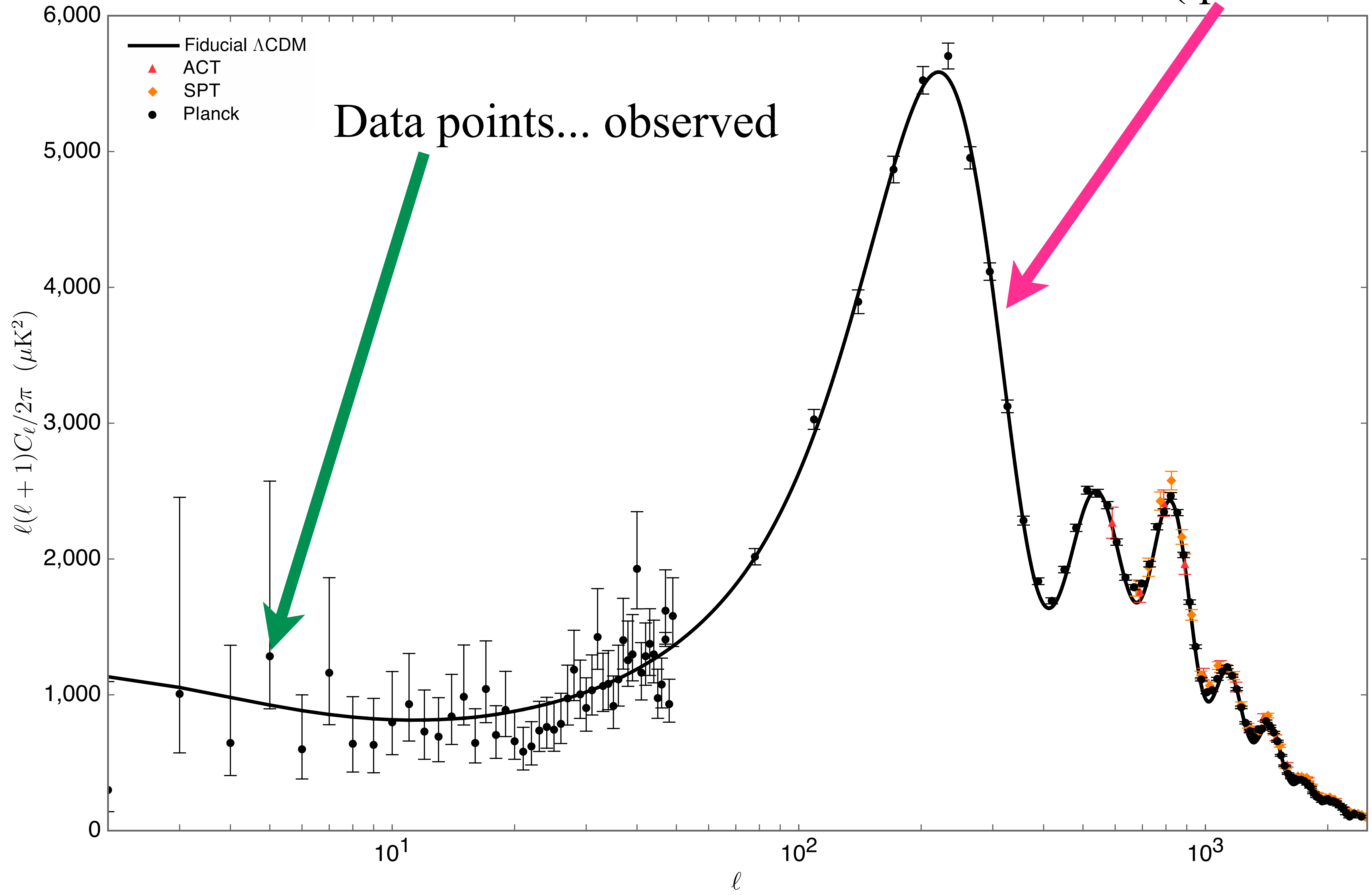
and eventually $P_\zeta(k) = \dfrac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1} P_v(k) = A_S k^{n_\mathrm{S}-1}$

with $n_\mathrm{S} = 2\beta + 5 \underset{\beta\sim -2}{\simeq} 1$

Planck: $1 - n_\mathrm{s} = 0.0389 \pm 0.0054$



Planck + ACT + SPT data



Theoretical prediction
(quantum vacuum fluctuations)

Data points... observed

Recall: Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p^*_{\boldsymbol{k}} + v_{\boldsymbol{k}} v^*_{\boldsymbol{k}} \left[k^2 \overbrace{- \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta, \boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

Simpler model: spectator scalar field in an expanding and finite size Universe

$$\phi_{\cancel{\boldsymbol{k}}} = \frac{\sqrt{V}}{(2\pi)^{3/2}} \left(q_{\cancel{\boldsymbol{k}}1} + i q_{\cancel{\boldsymbol{k}}2} \right) \qquad H = \sum_{\boldsymbol{k},\ r=1,2} \frac{1}{2a^3} \pi^2_{\cancel{\boldsymbol{k}}r} + \frac{1}{2} a k^2 q^2_{\cancel{\boldsymbol{k}}r}$$

$$a^3 \to m$$

$$k/a \to \omega$$

$$i \frac{\partial \psi}{\partial t} = \sum_{r=1}^{2} \left(-\frac{\partial_r^2}{2m} + \frac{1}{2} m \omega^2 q_r^2 \right)$$

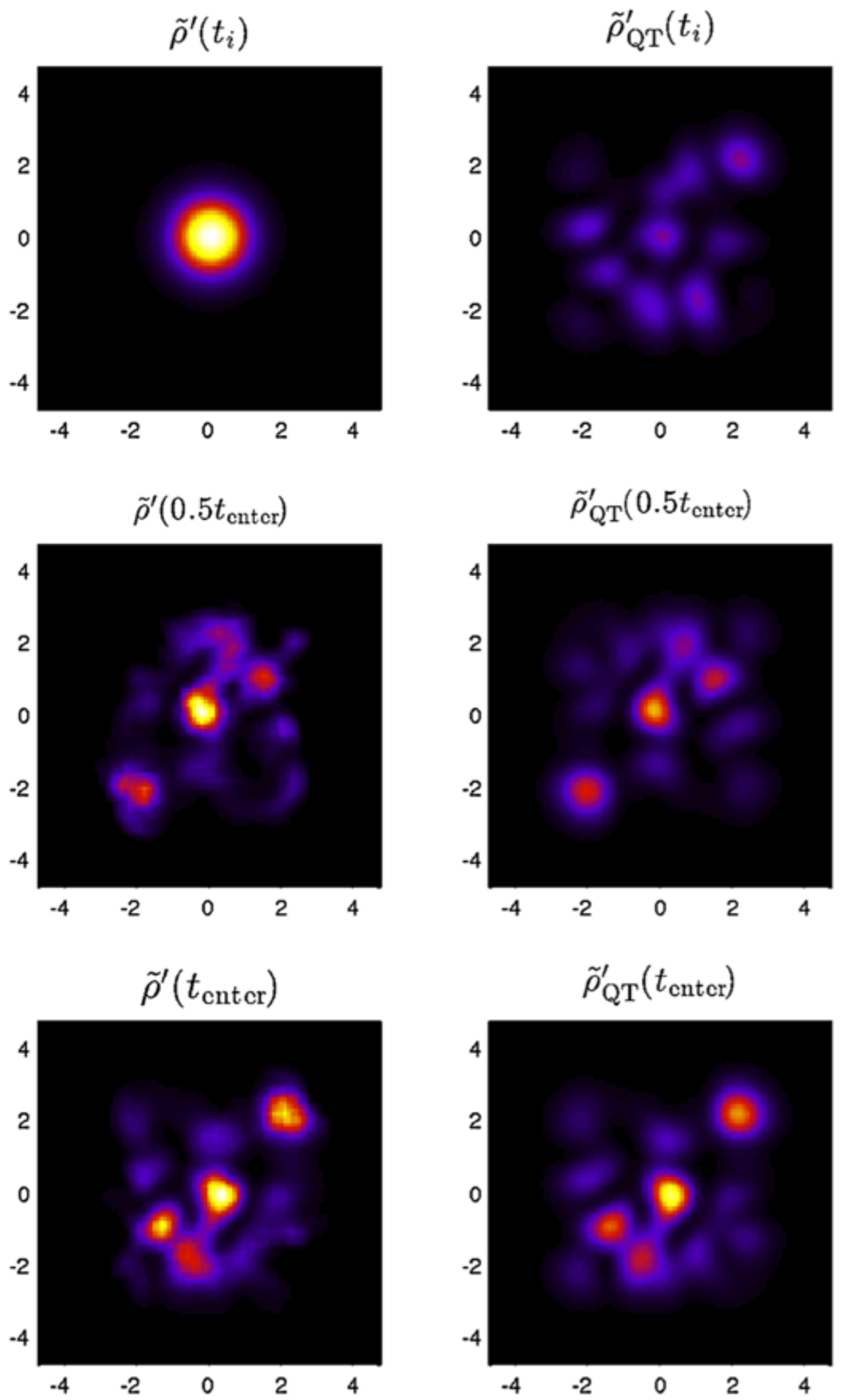
dBB trajectory of the field component $\dot{q}_r = m^{-1}\Im\mathrm{m}\,\frac{\partial_r\psi}{\psi}$

Statistical distribution $\frac{\partial\rho}{\partial t} + \sum_r \partial_r\left(\frac{\rho}{m}\Im\mathrm{m}\,\frac{\partial_r\psi}{\psi}\right)$

$$i\frac{\partial\psi}{\partial t} = \sum_{r=1}^{2}\left(-\frac{\partial_r^2}{2m}+\frac{1}{2}m\omega^2 q_r^2\right)$$

Relaxation of a 2D harmonic oscillator

(time dependent mass & frequency)

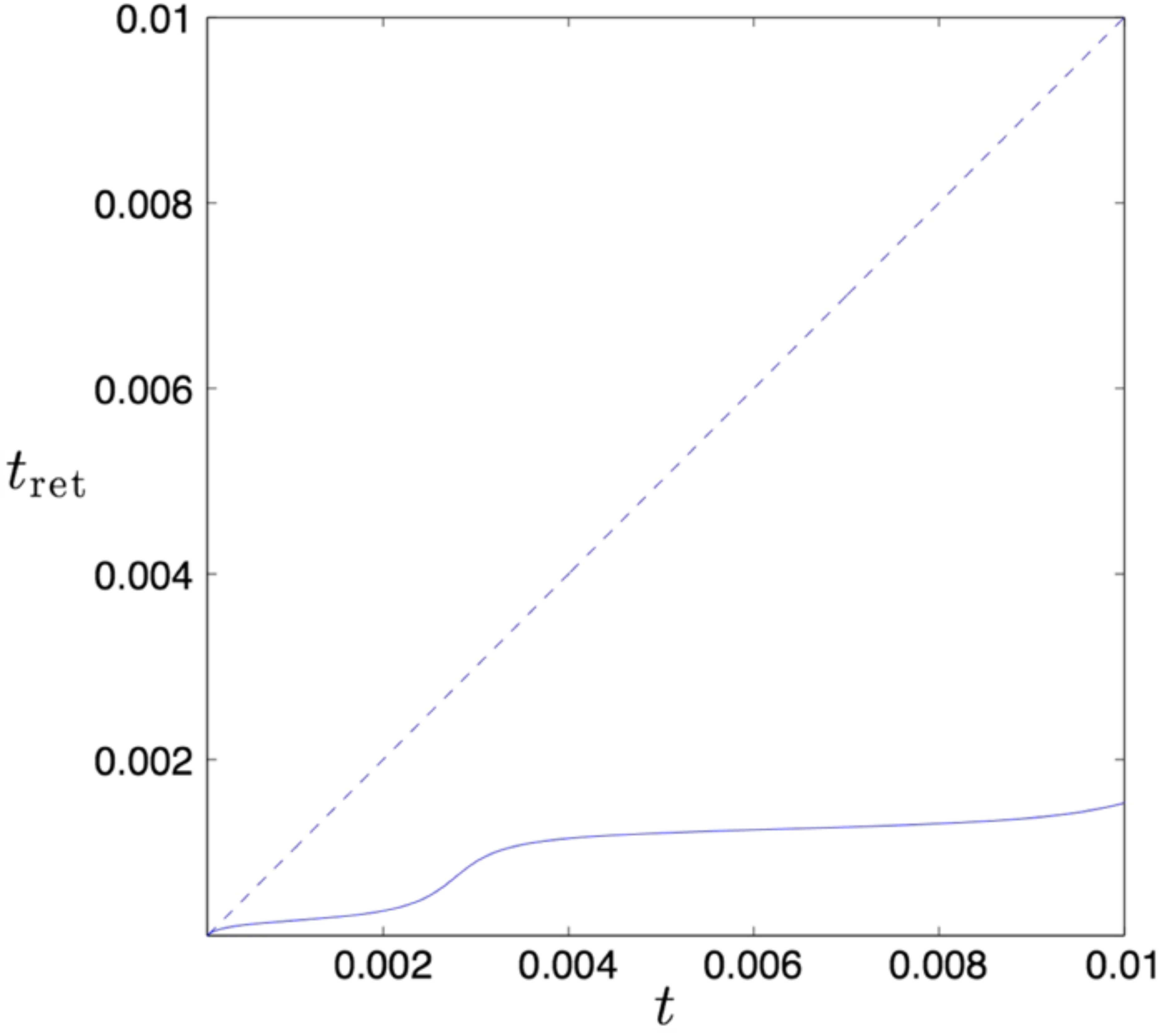


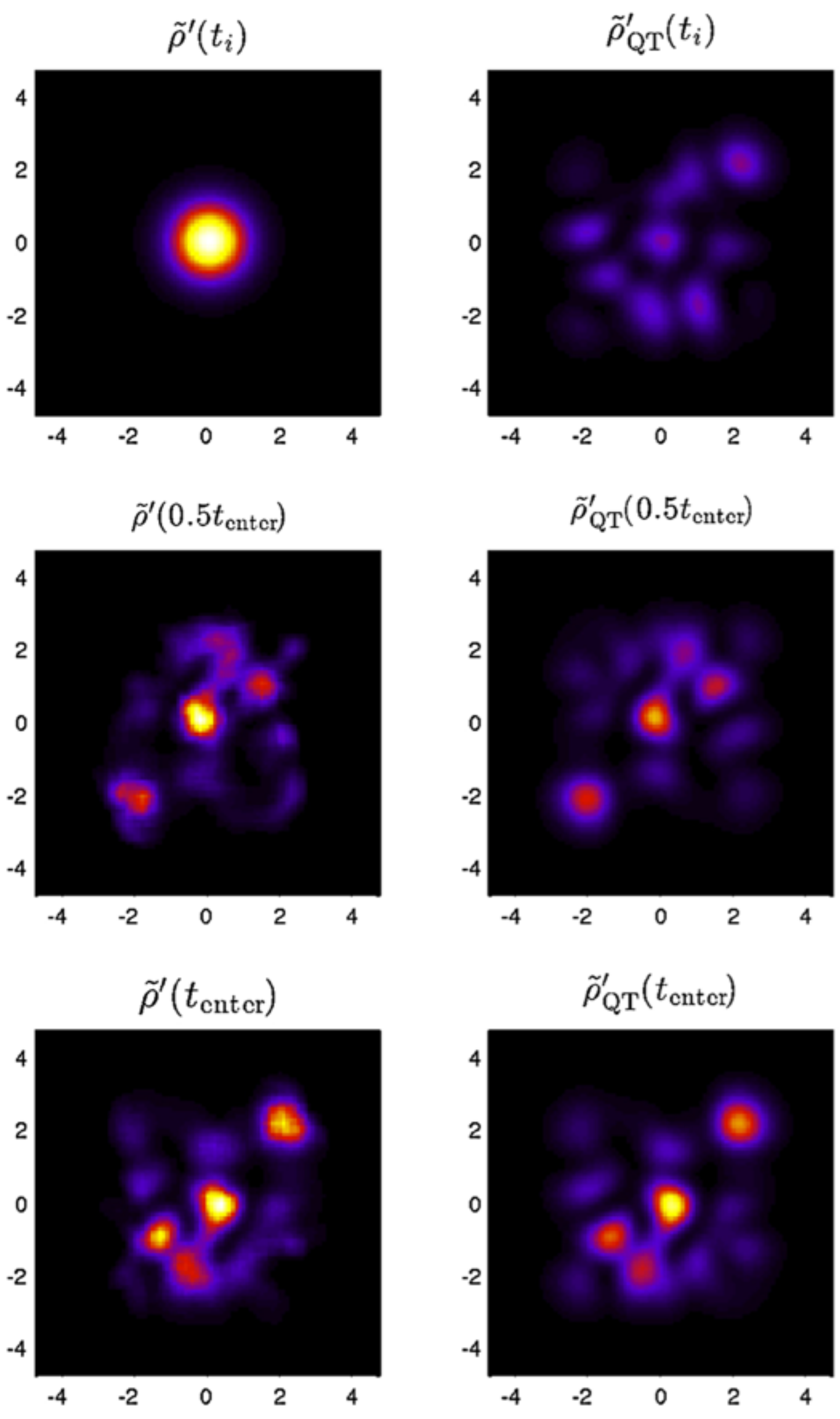
(constant mass & frequency)

Out-of-equilibrium time evolution

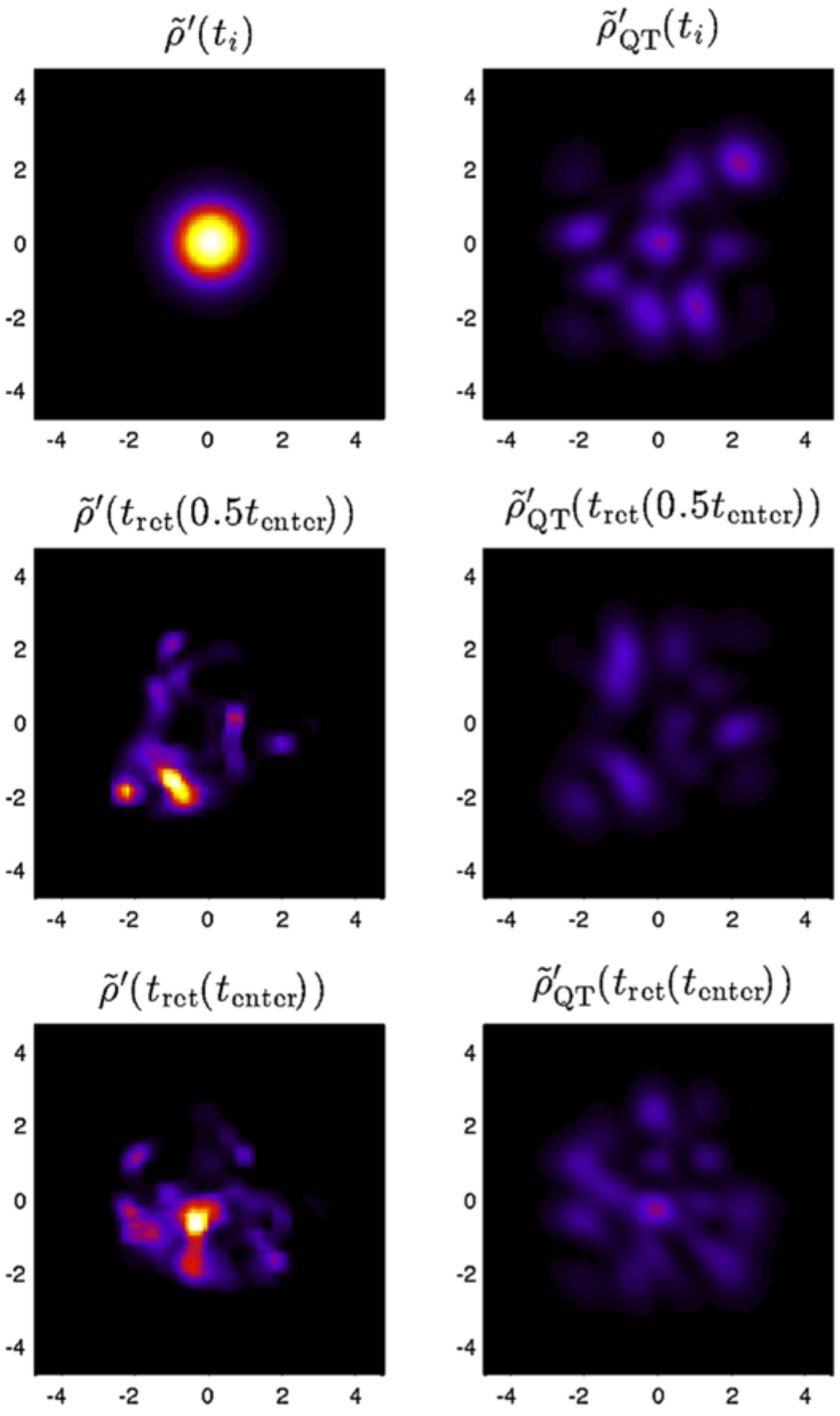
- Usual behaviour = evolves towards equilibrium (Minkowski or slowly expanding Universe)

- expansion: there is a retarded time…





without expansion

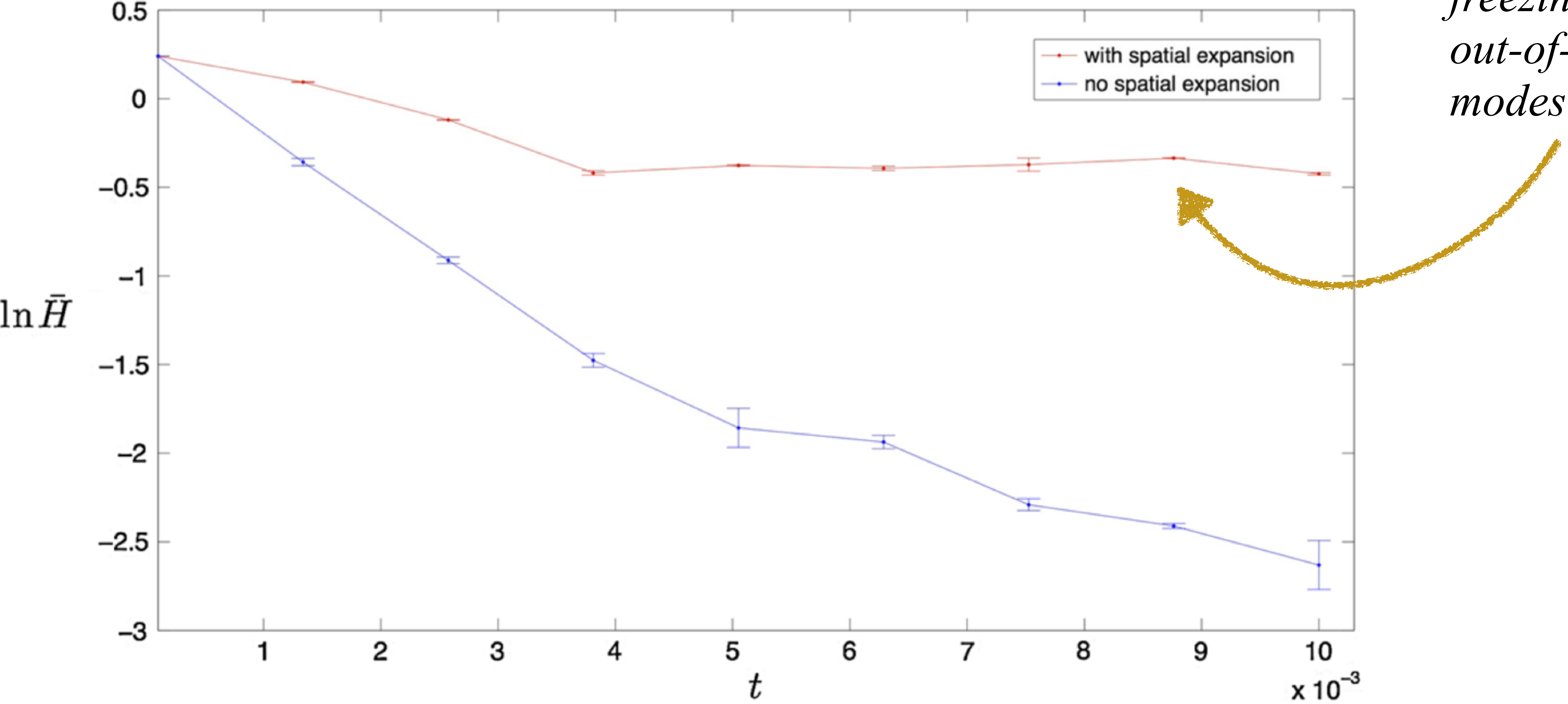


with expansion

S. Colin & A. Valentini, *Phys. Rev.* **D88** 103515 (2013)

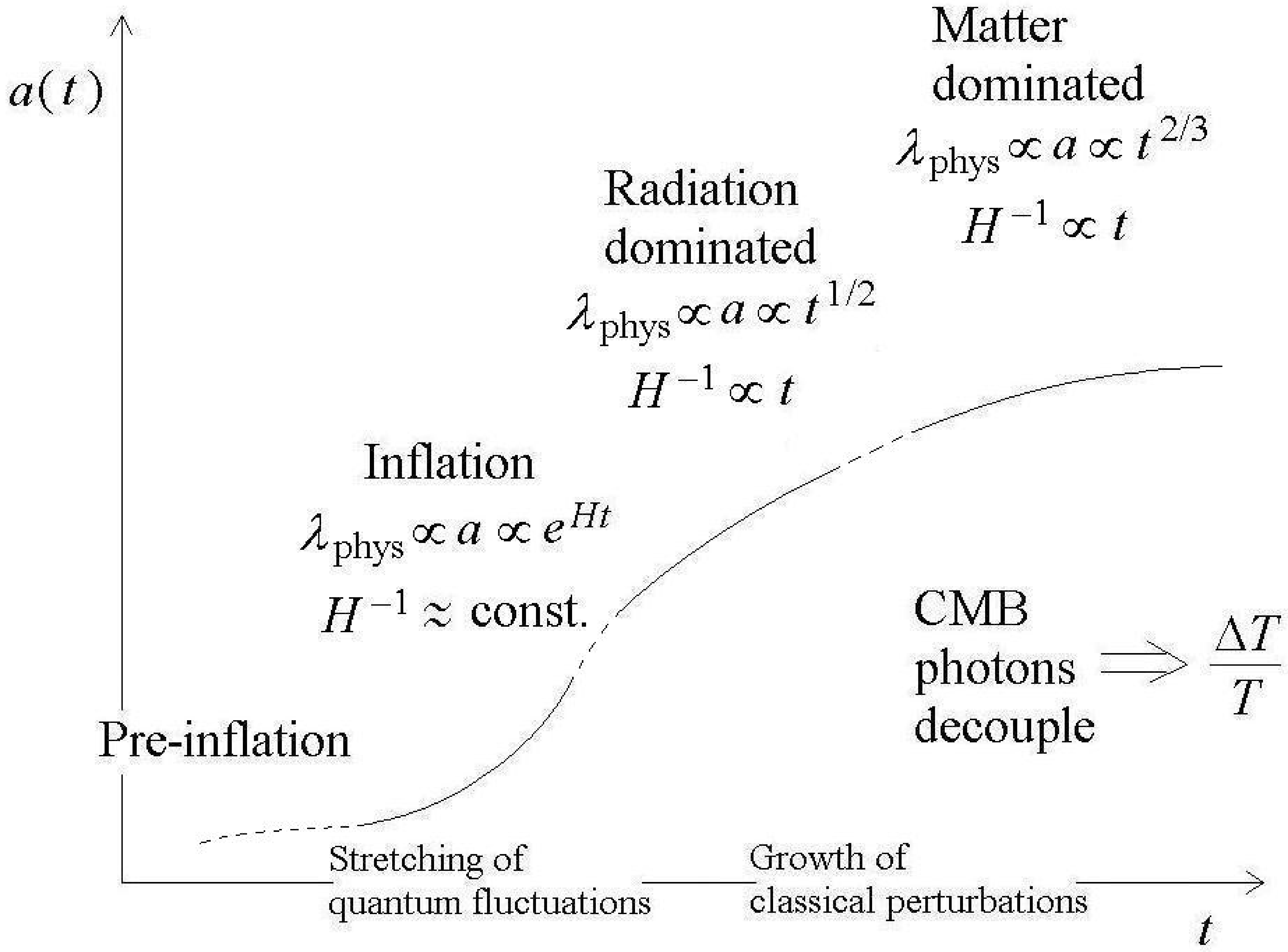
$$H \equiv \int \mathrm{d}q\, \rho \ln\left(\frac{\rho}{|\Psi|^2}\right)$$

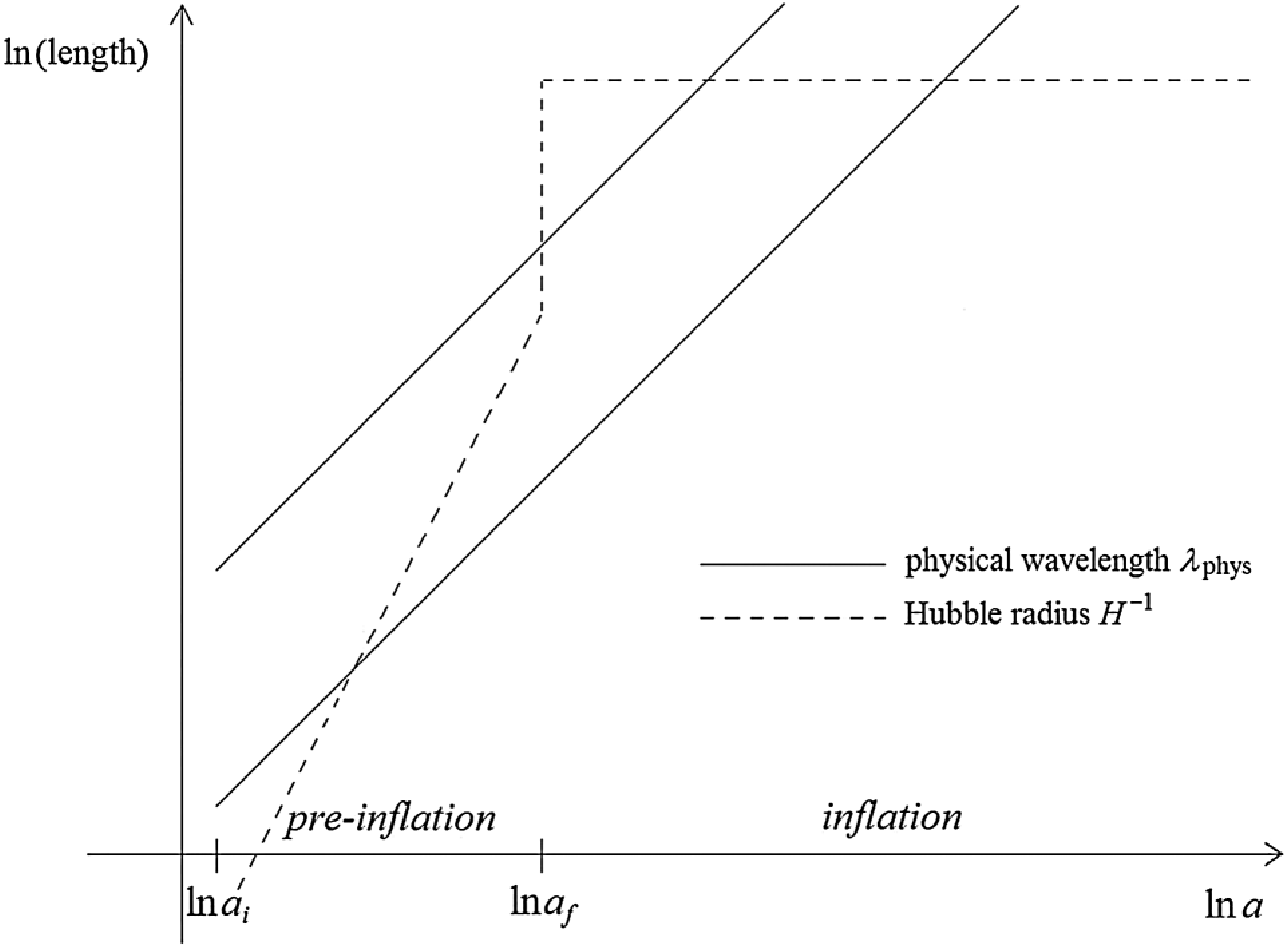
measures “out-of-equilibrium-ness”



freezing of
out-of-equilibrium
modes

A simplified model




ln(length)
physical wavelength $\lambda_{\rm phys}$
Hubble radius H^{-1}
pre-inflation
inflation
$\ln a_i$
$\ln a_f$
$\ln a$

Very long wavelengths

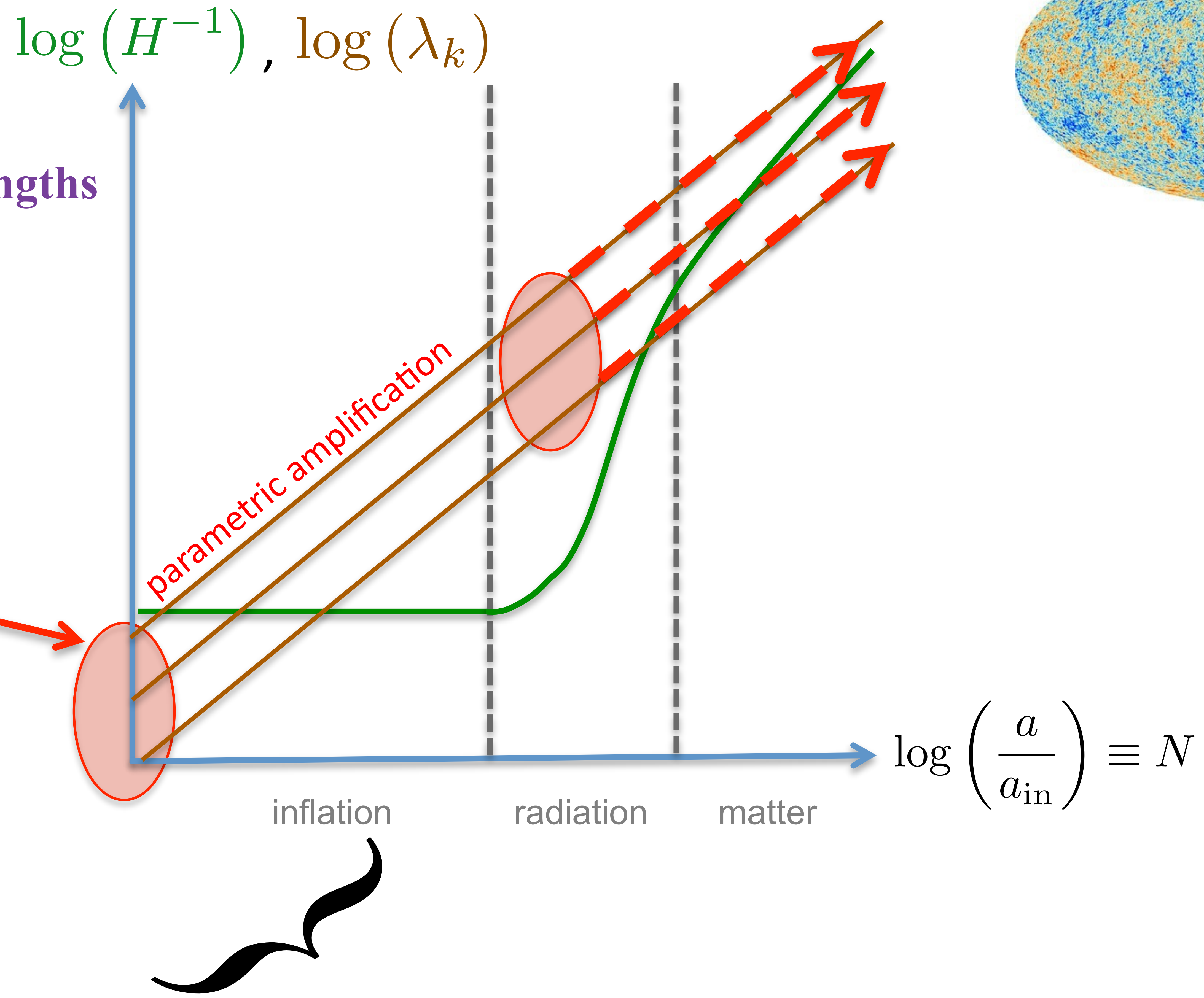
no time
to equilibrate

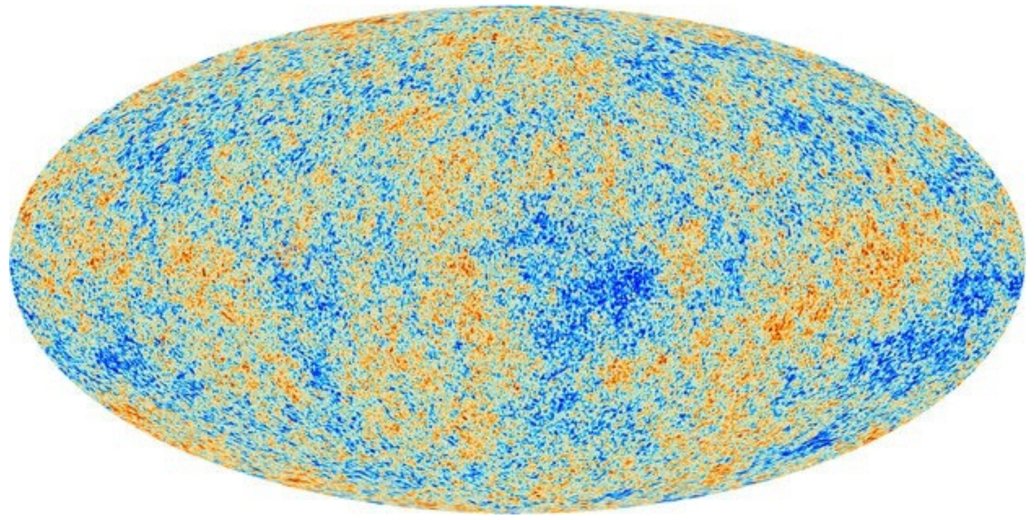
less power

Harmonic oscillator
fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Out-of-Equilibrium
initial density:
less quantum noise





enough time
to equilibrate

Small wavelengths

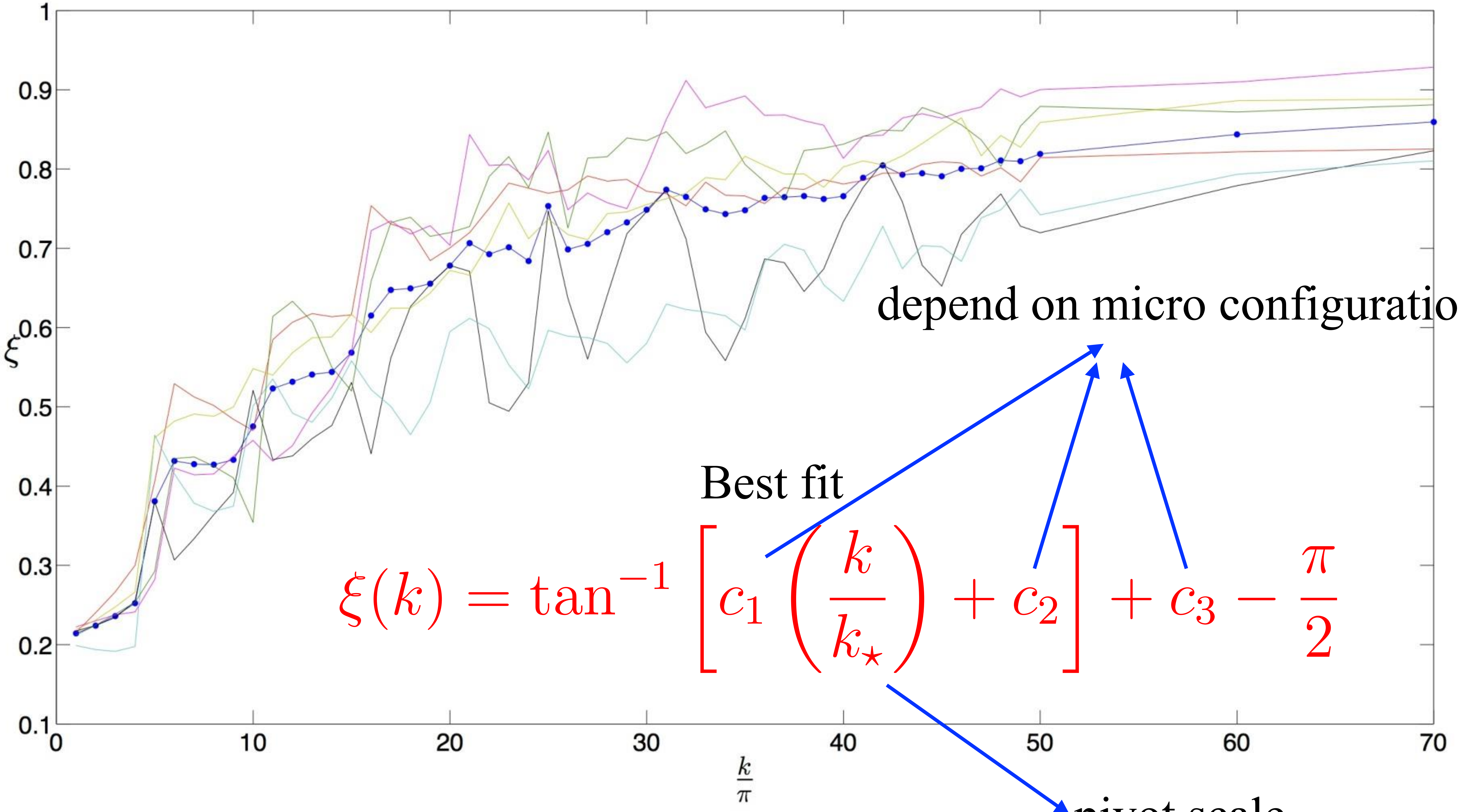
usual spectrum

Initial out-of-equilibrium conditions

S. Colin & A. Valentini, arXiv:1407.8262

$$\mathcal{P}(k) = \mathcal{P}(k)_{\mathrm{QE}}\xi(k)$$

width deficit

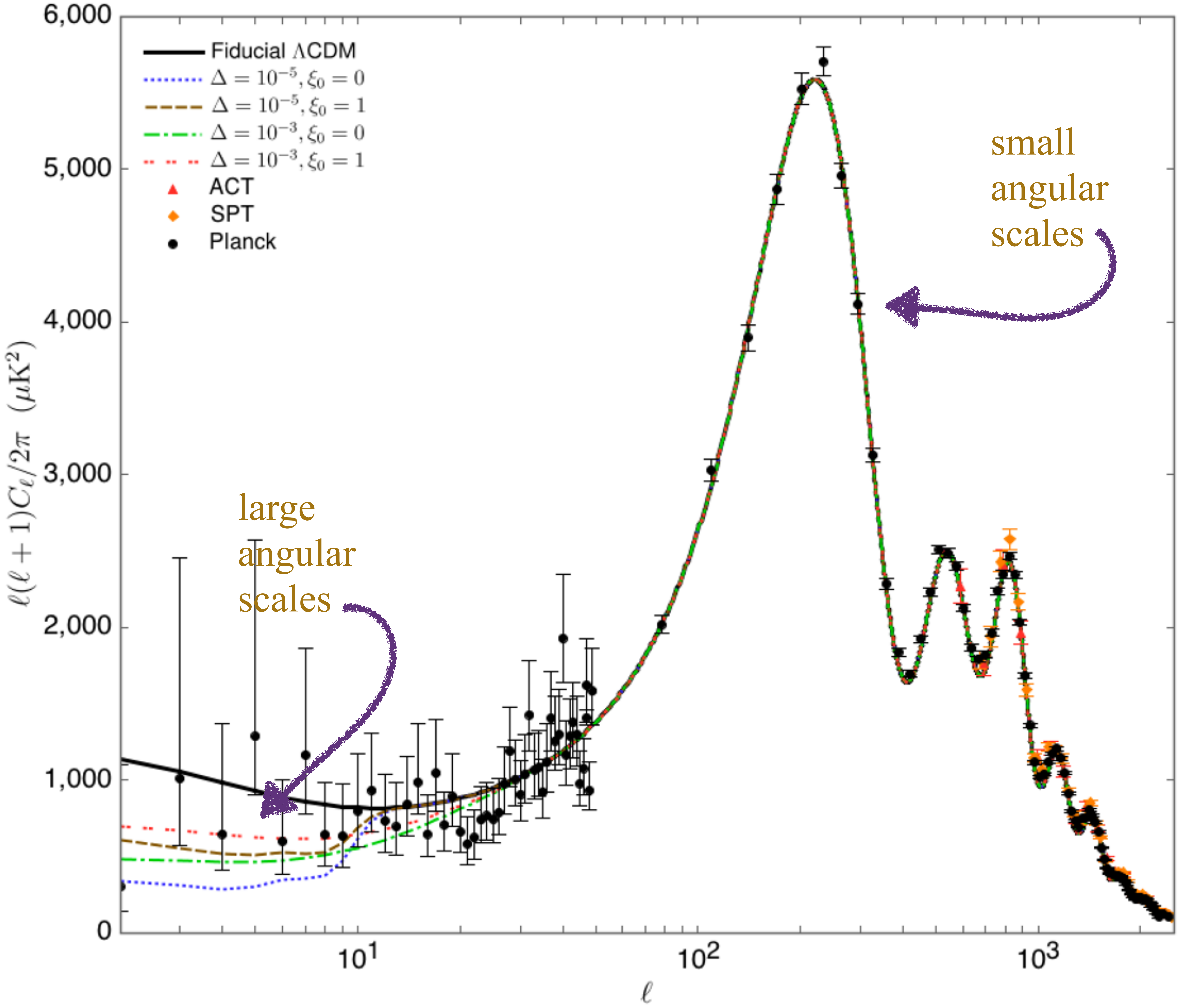


depend on micro configuration

Best fit

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

pivot scale



Better fit???

↓

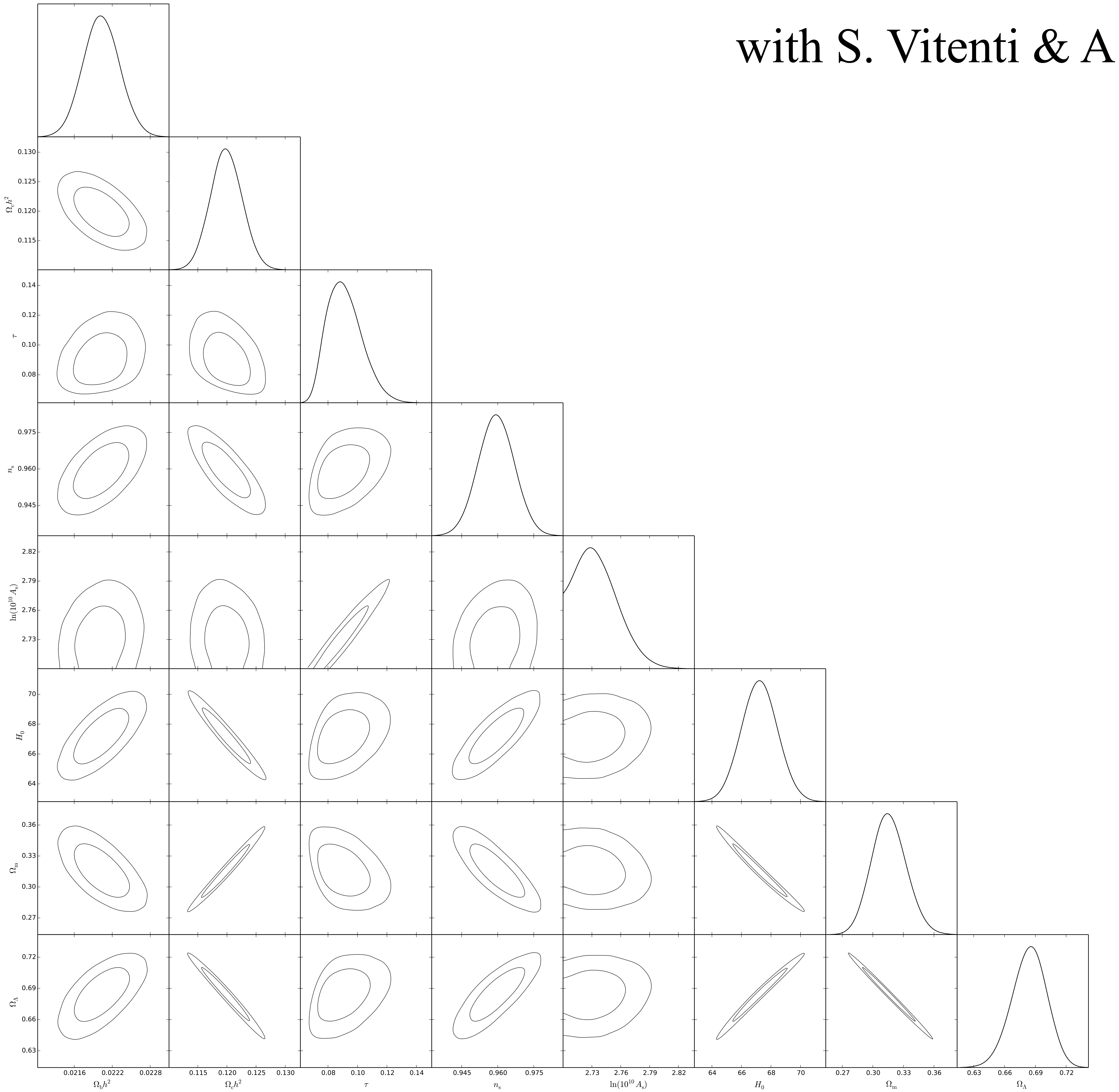
CosmoMC chains

Results…

work in progress!

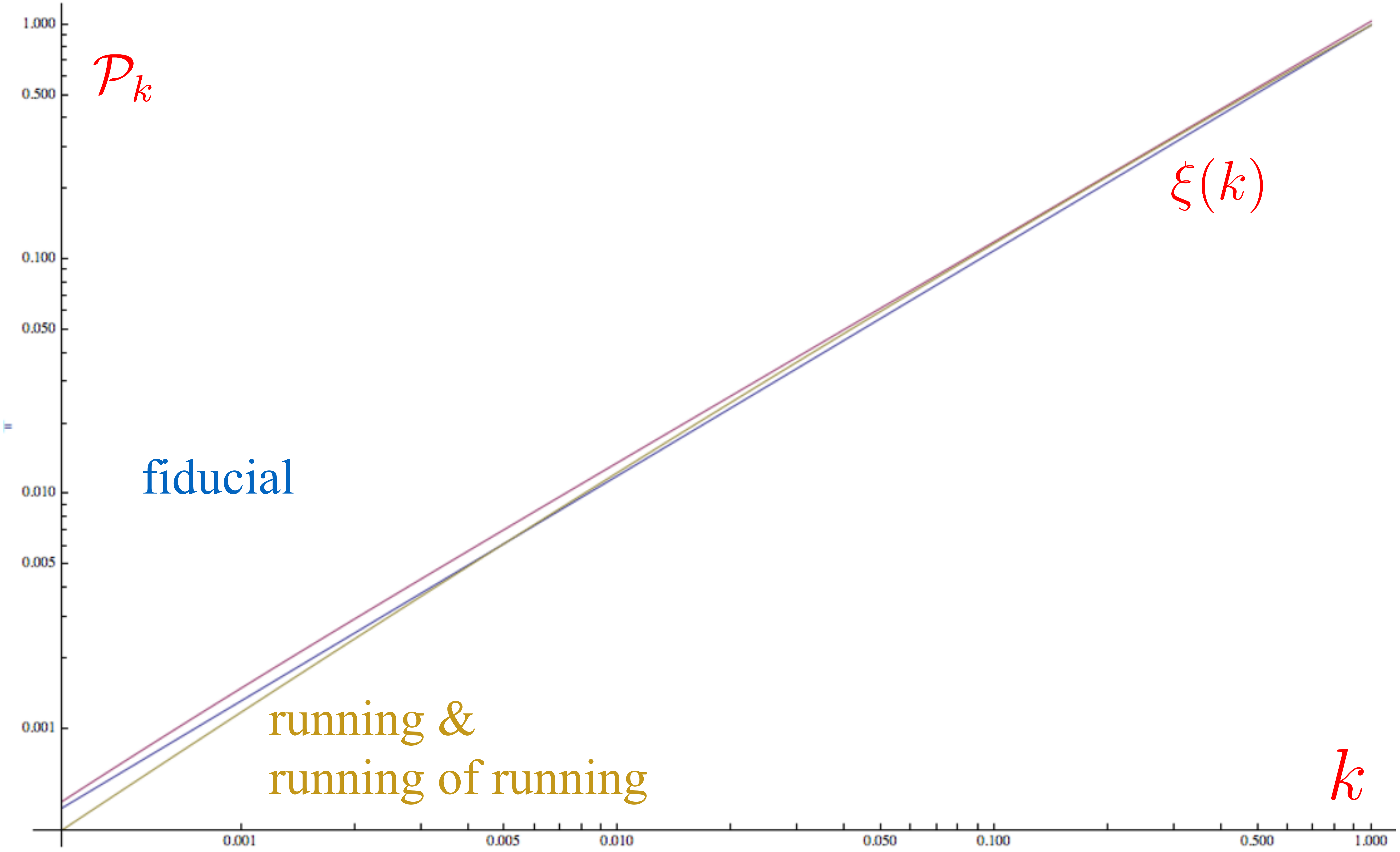
with S. Vitenti & A. Valentini

Usual Planck best-fit



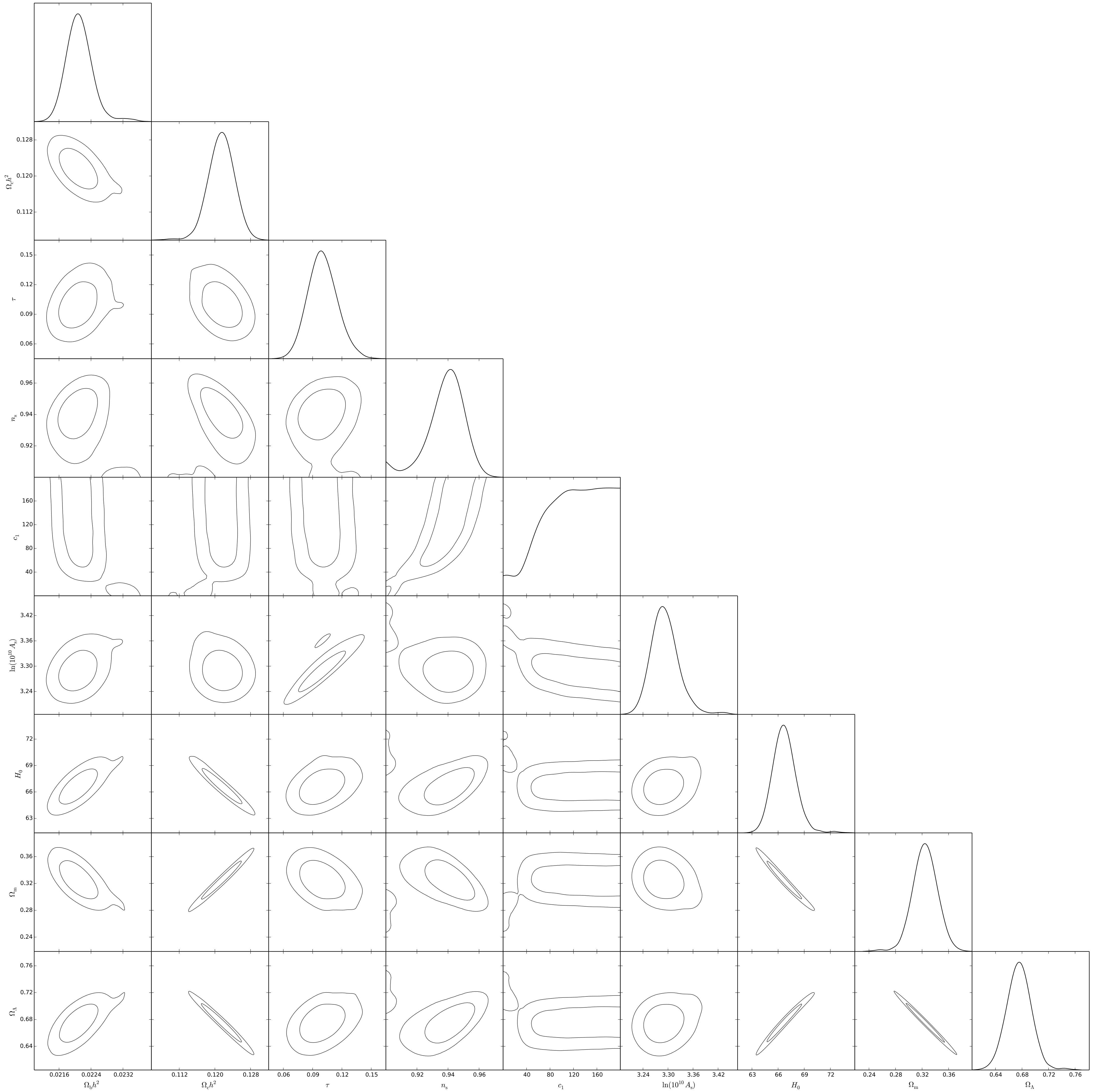
Cargèse / 22 september 2014

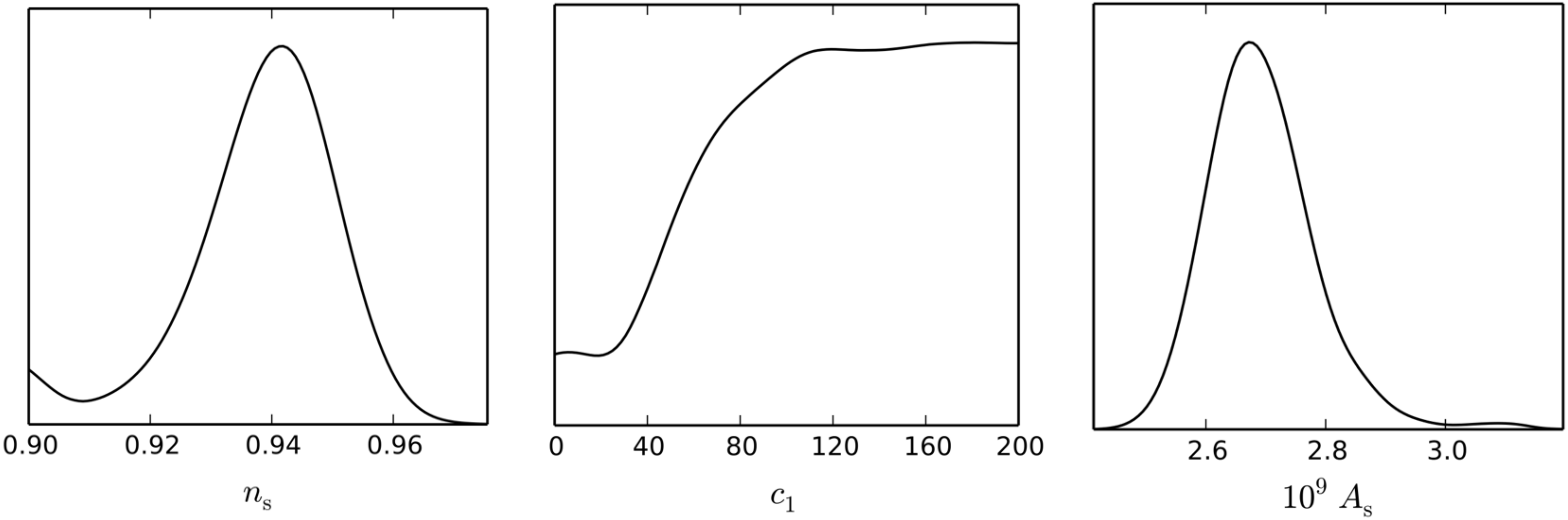
with only one parameter added, others held fixed: $\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + \underset{\substack{\downarrow\\ 3.0}}{c_2}\right] + \underset{\substack{\downarrow\\ 0.85}}{c_3} - \frac{\pi}{2}$



with only one parameter added, others held fixed: $\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$

c_2 ↓ 3.0 c_3 ↓ 0.85

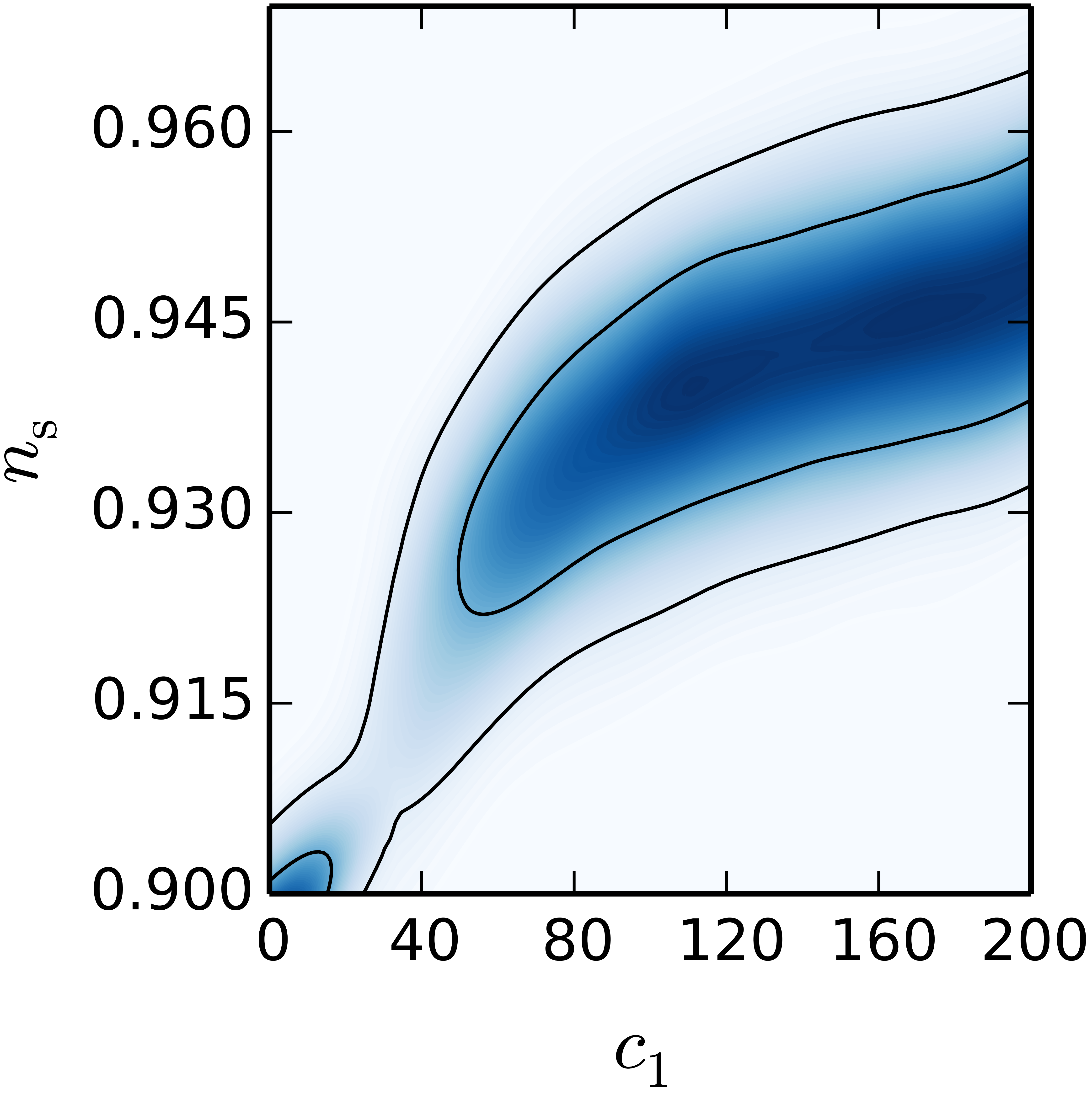




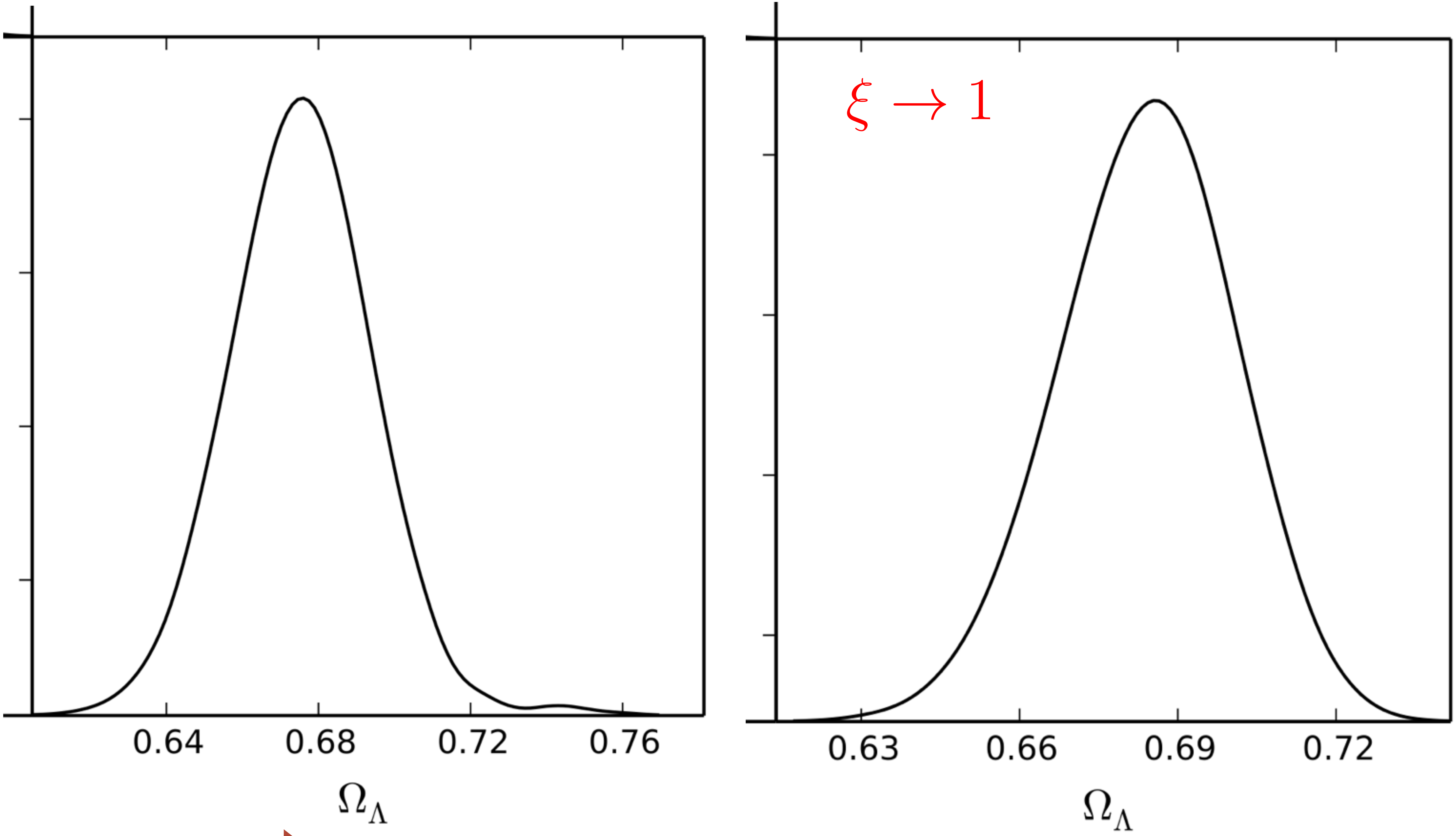
redder…

not conclusive

degeneracy



convergence???



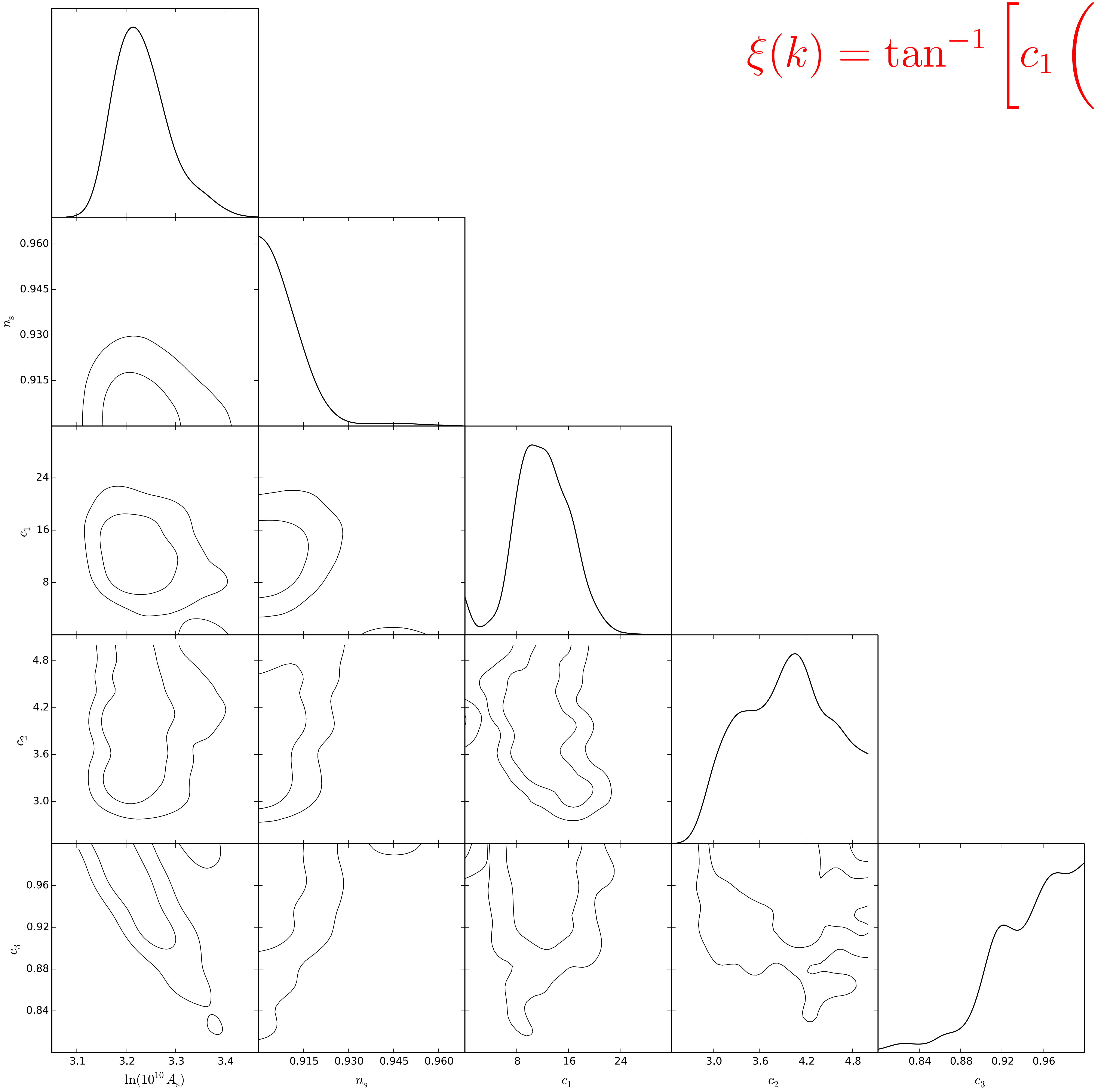
compatibility with SN Ia data?

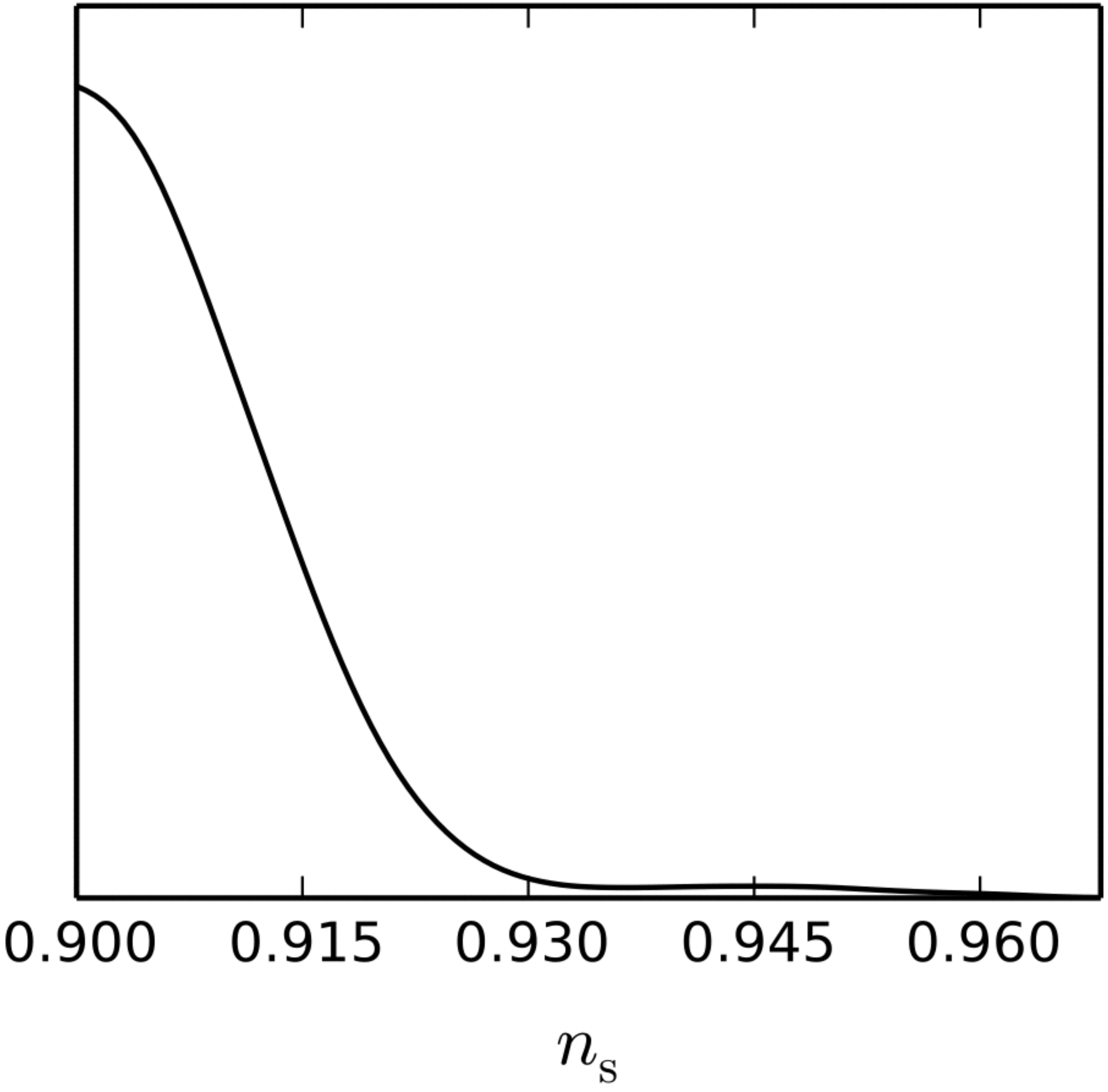
Constrained model

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

prior

$c_3 < 1$

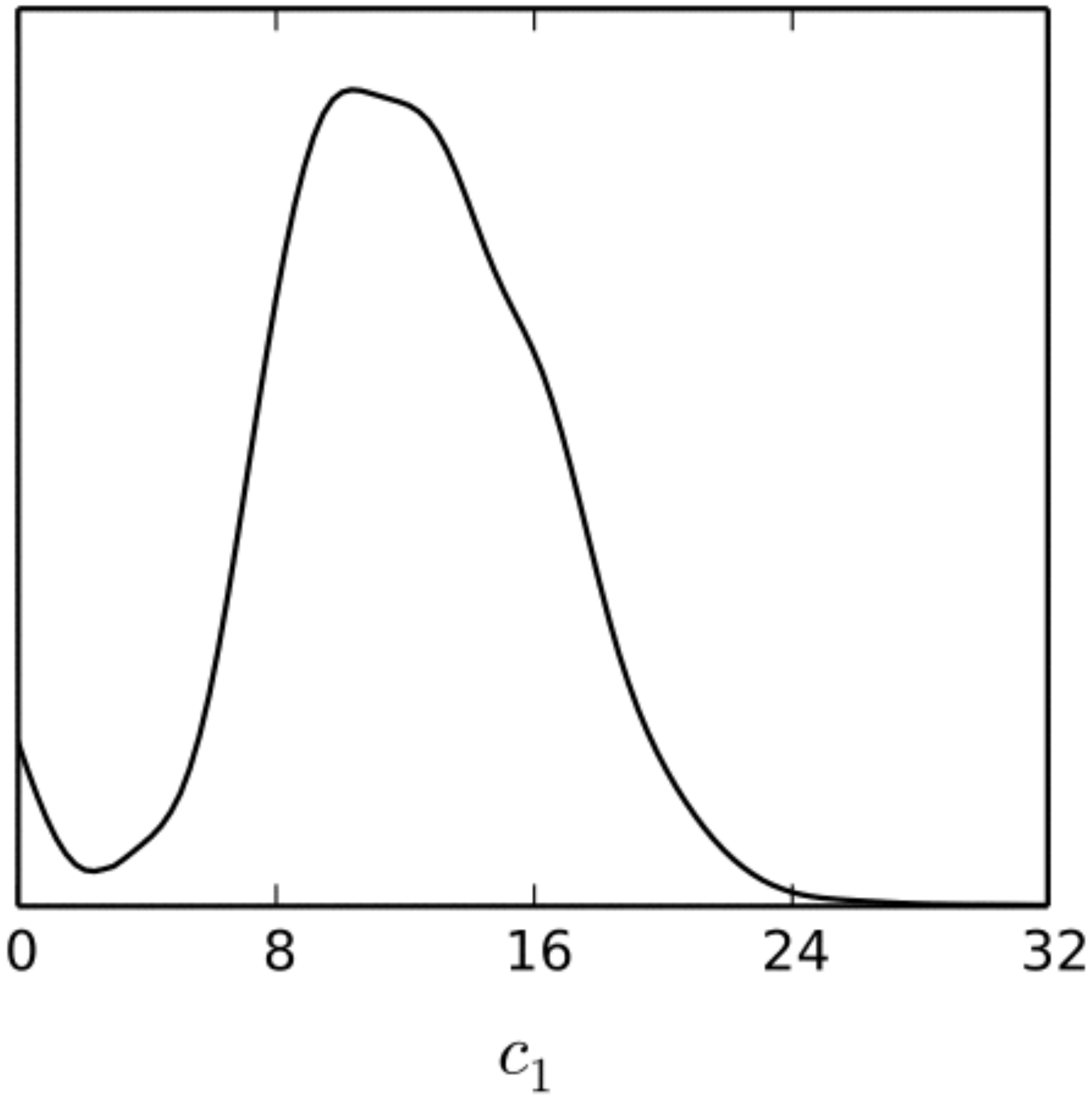


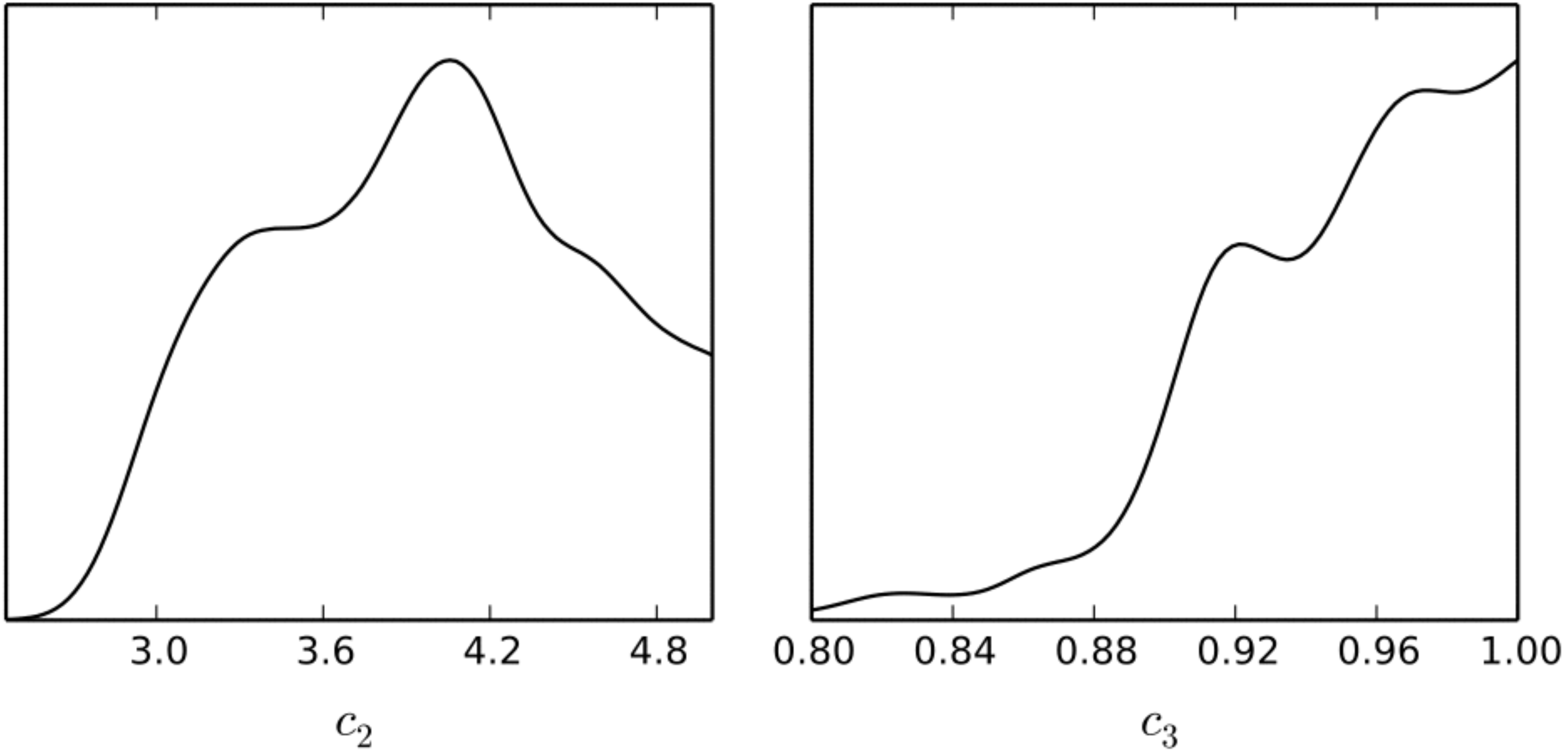


demands a very
red primordial spectrum

$$\xi(k) = \tan^{-1}\left[\underset{\big\downarrow}{c_1}\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

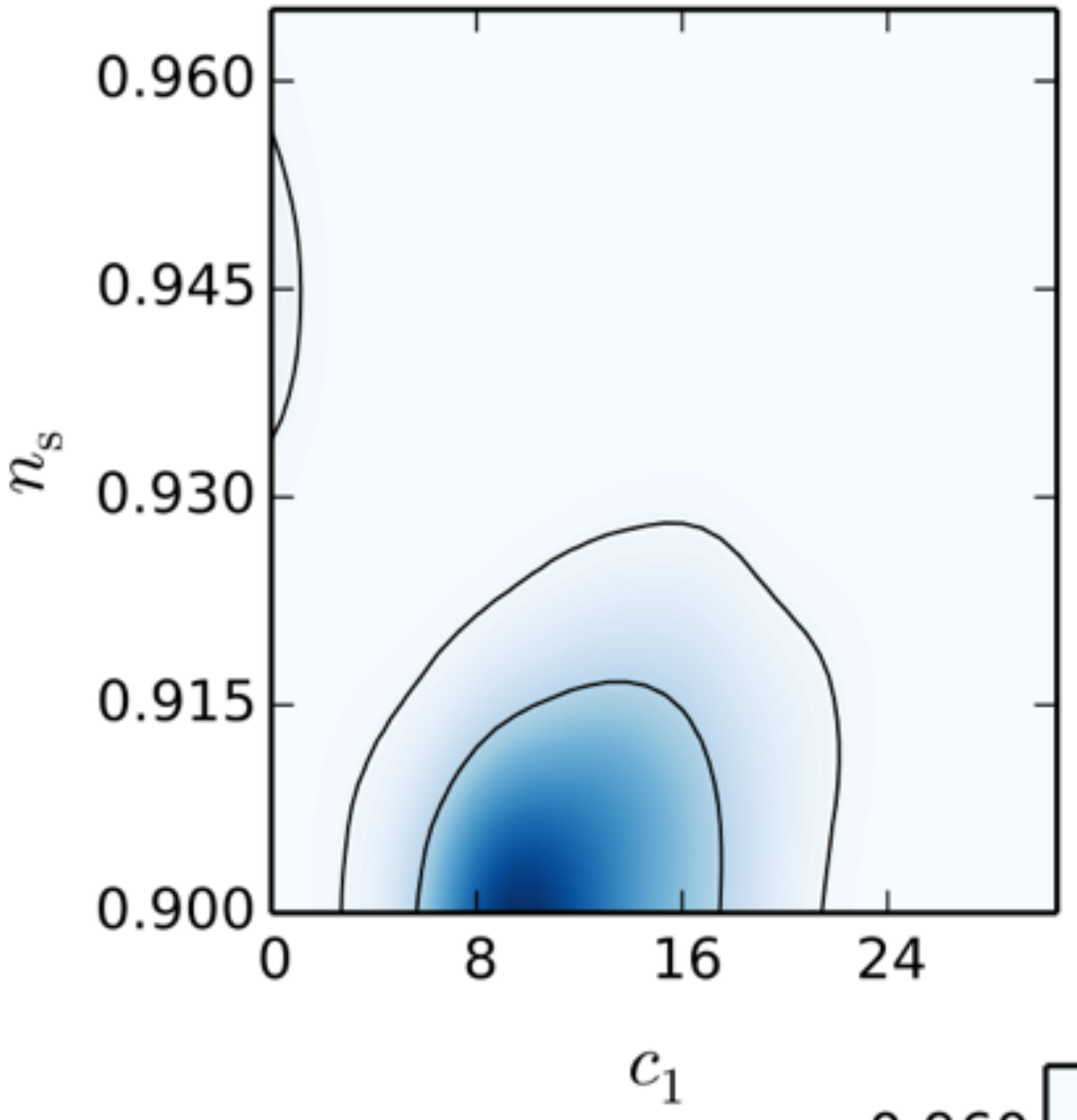
much smaller quantum scale

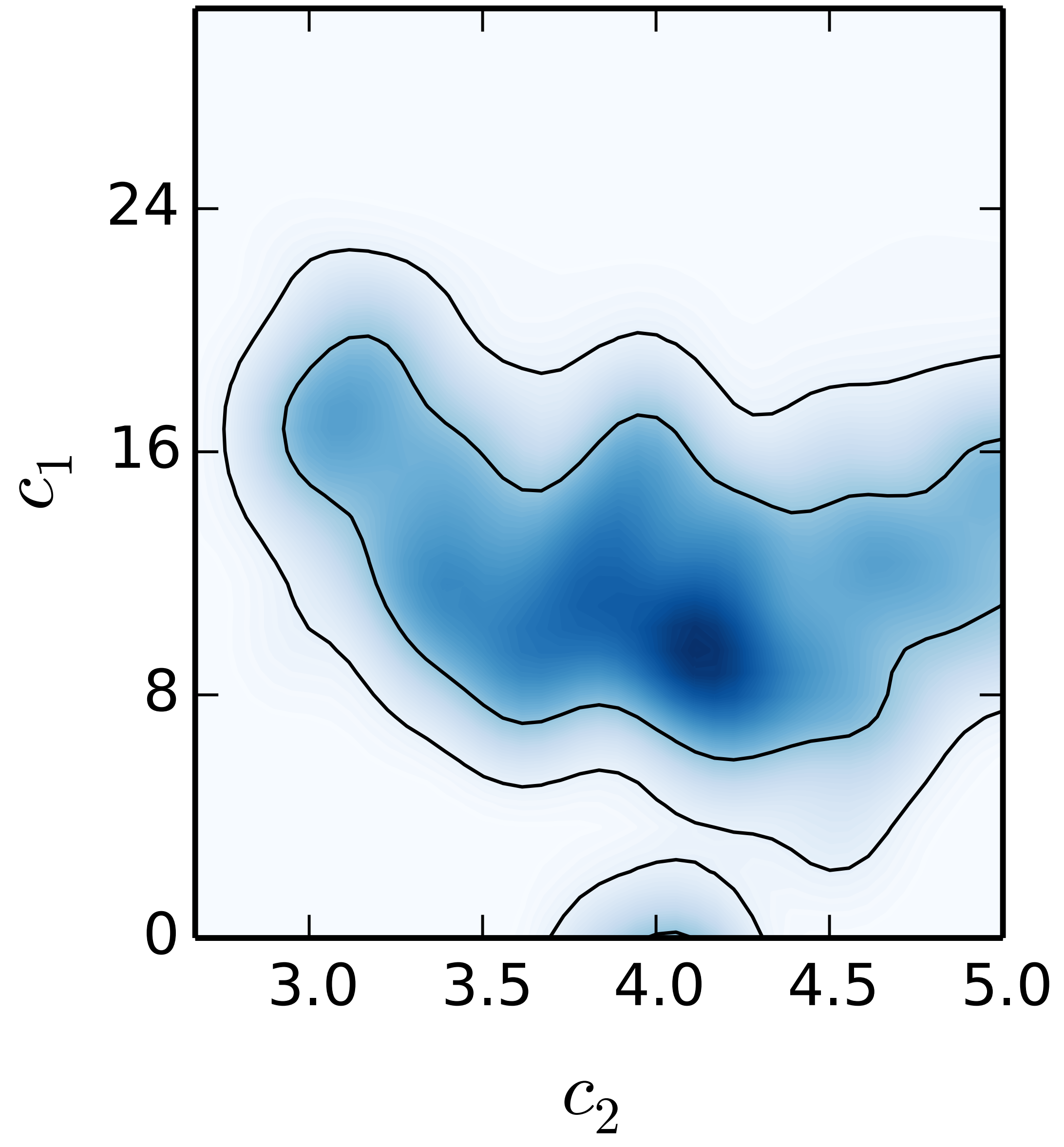


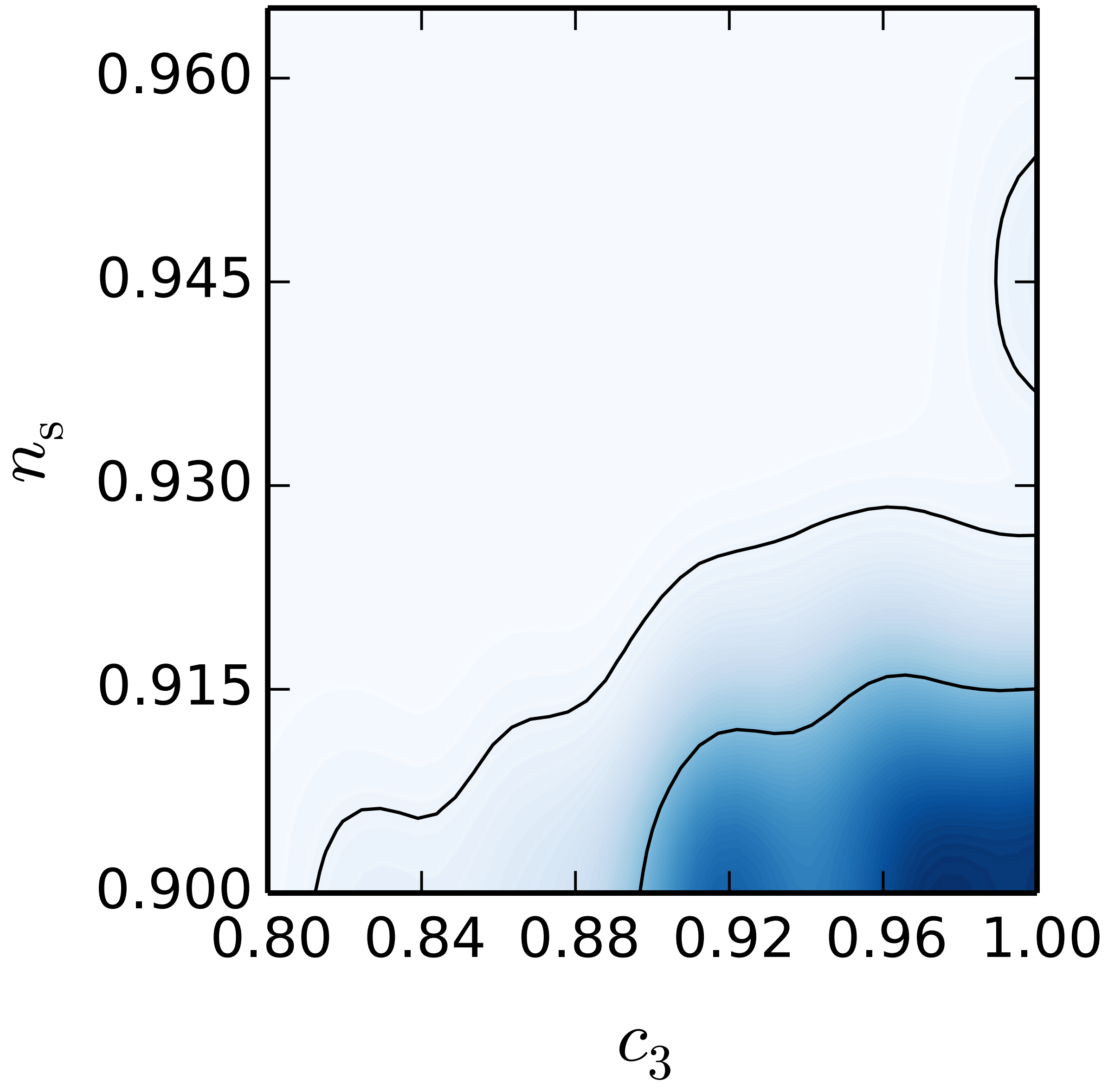


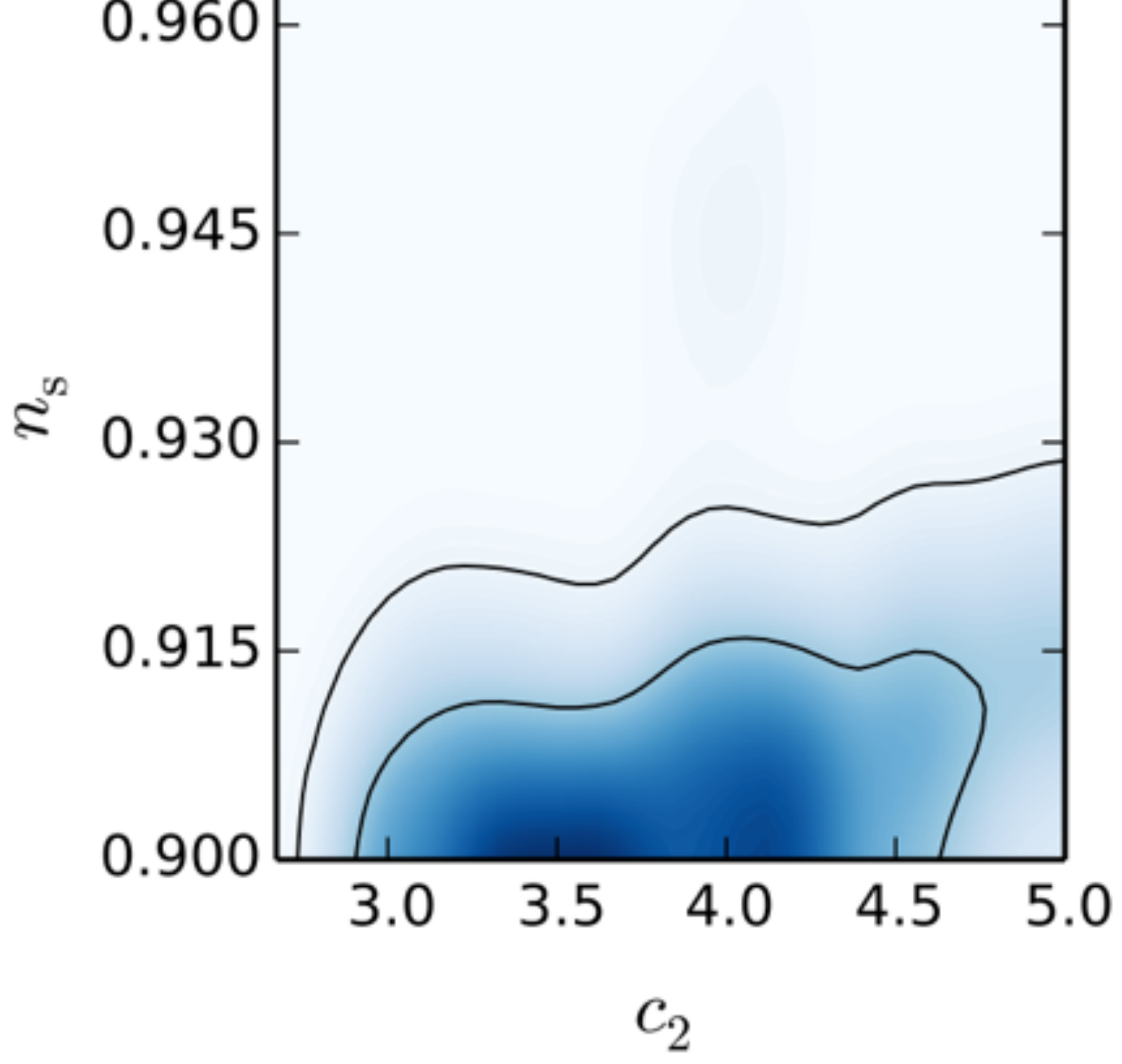
not very conclusive, but seems to favor $c_3 \geq 1$

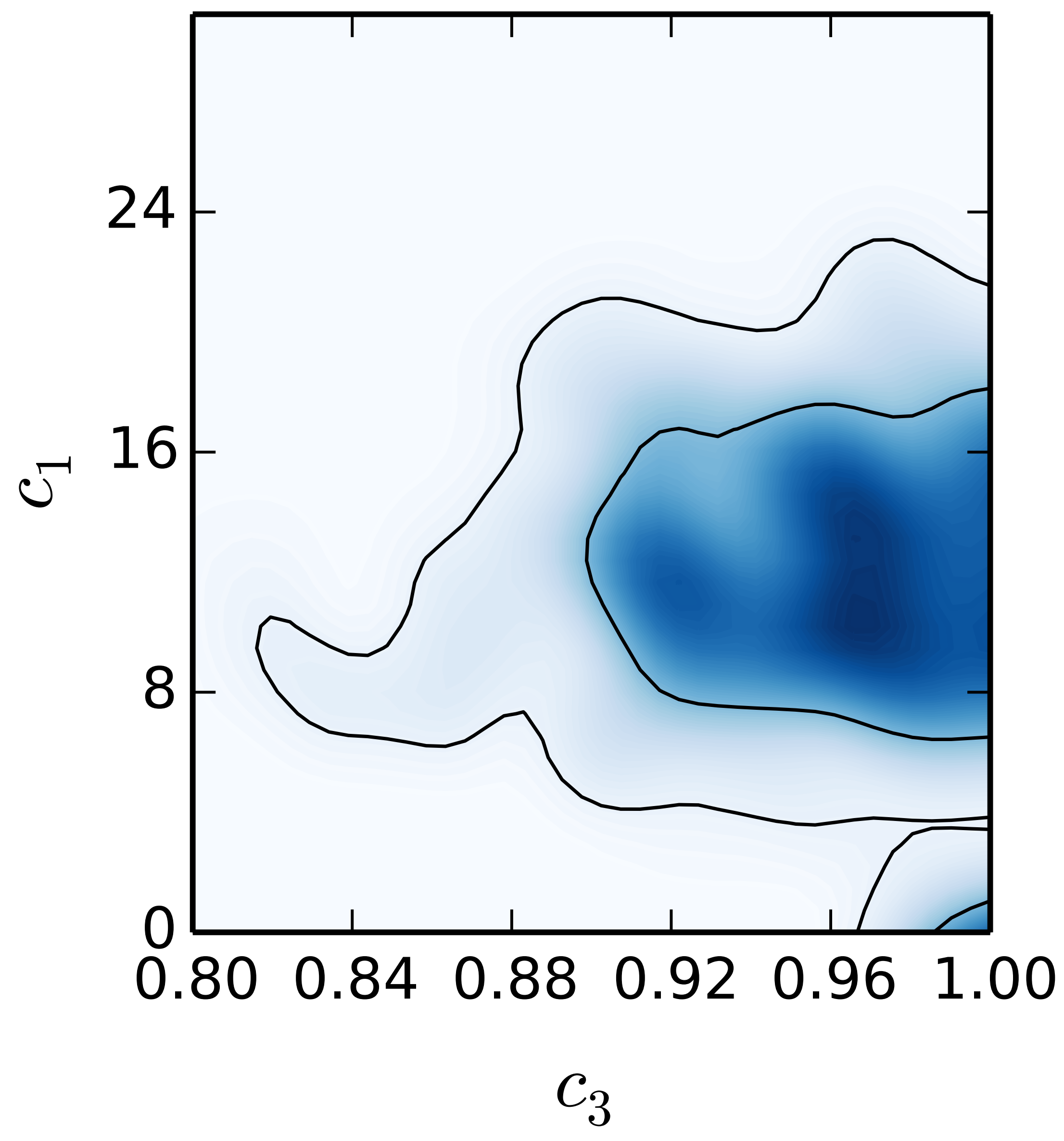
summary for the constrained model:

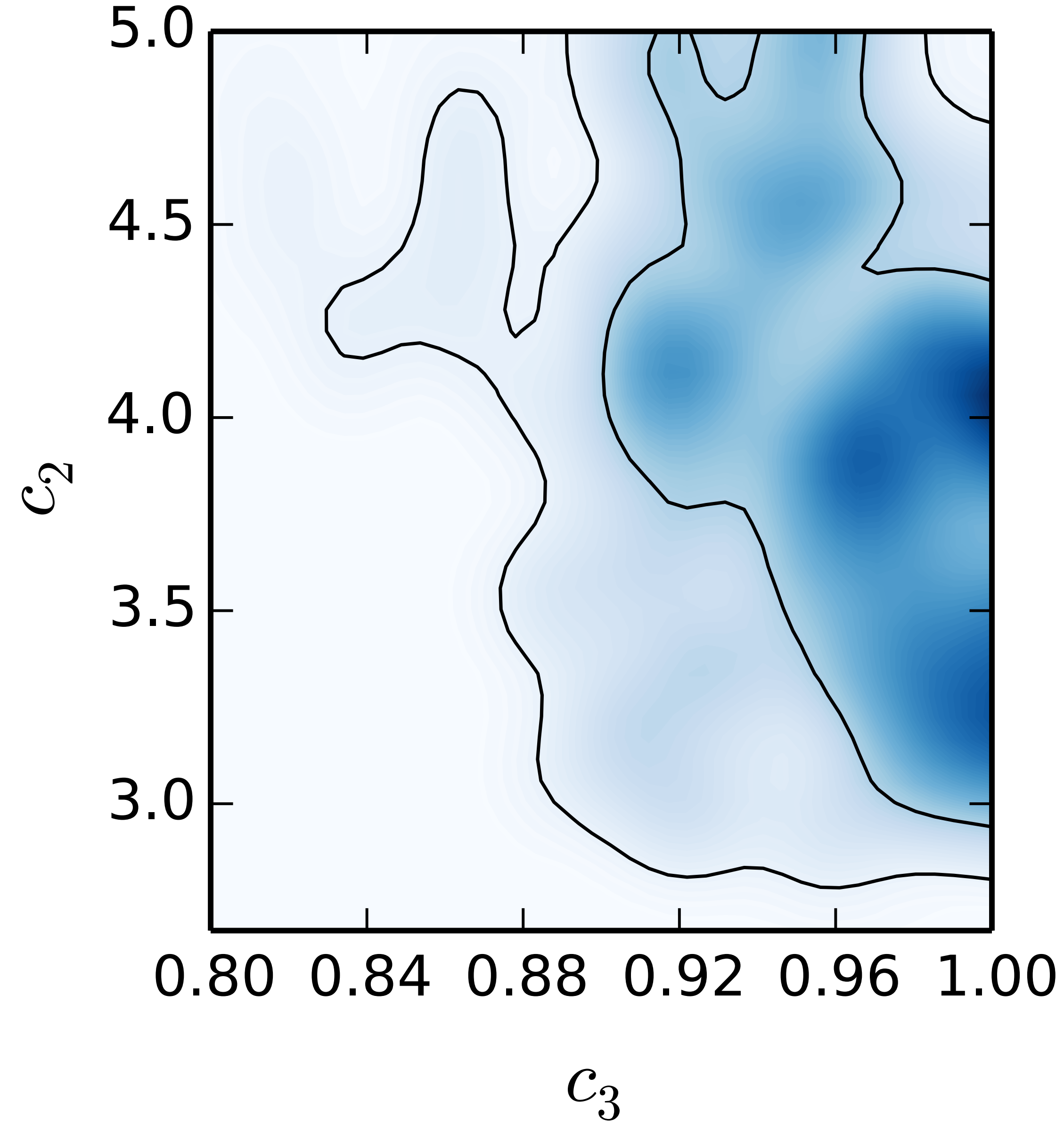








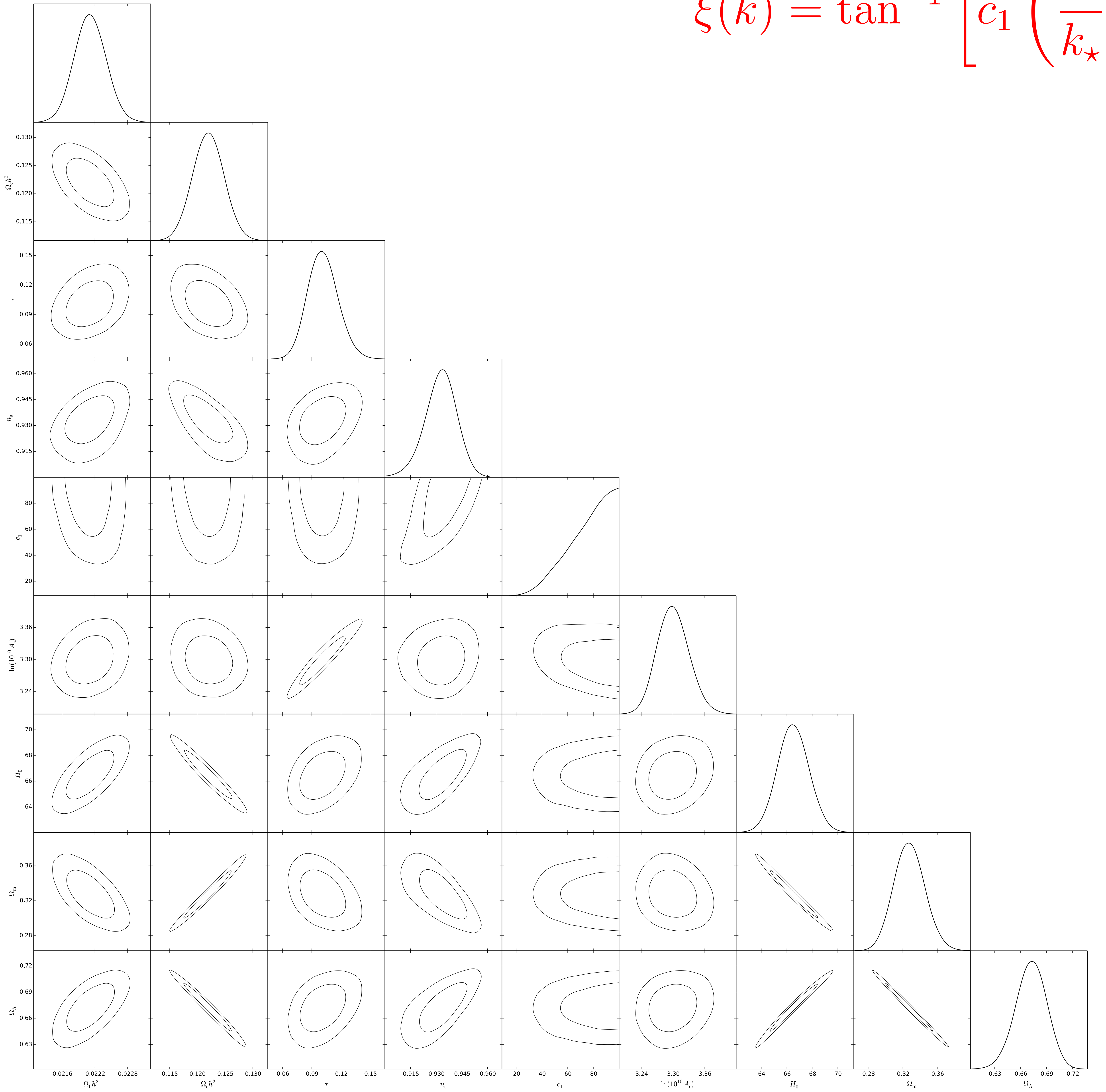


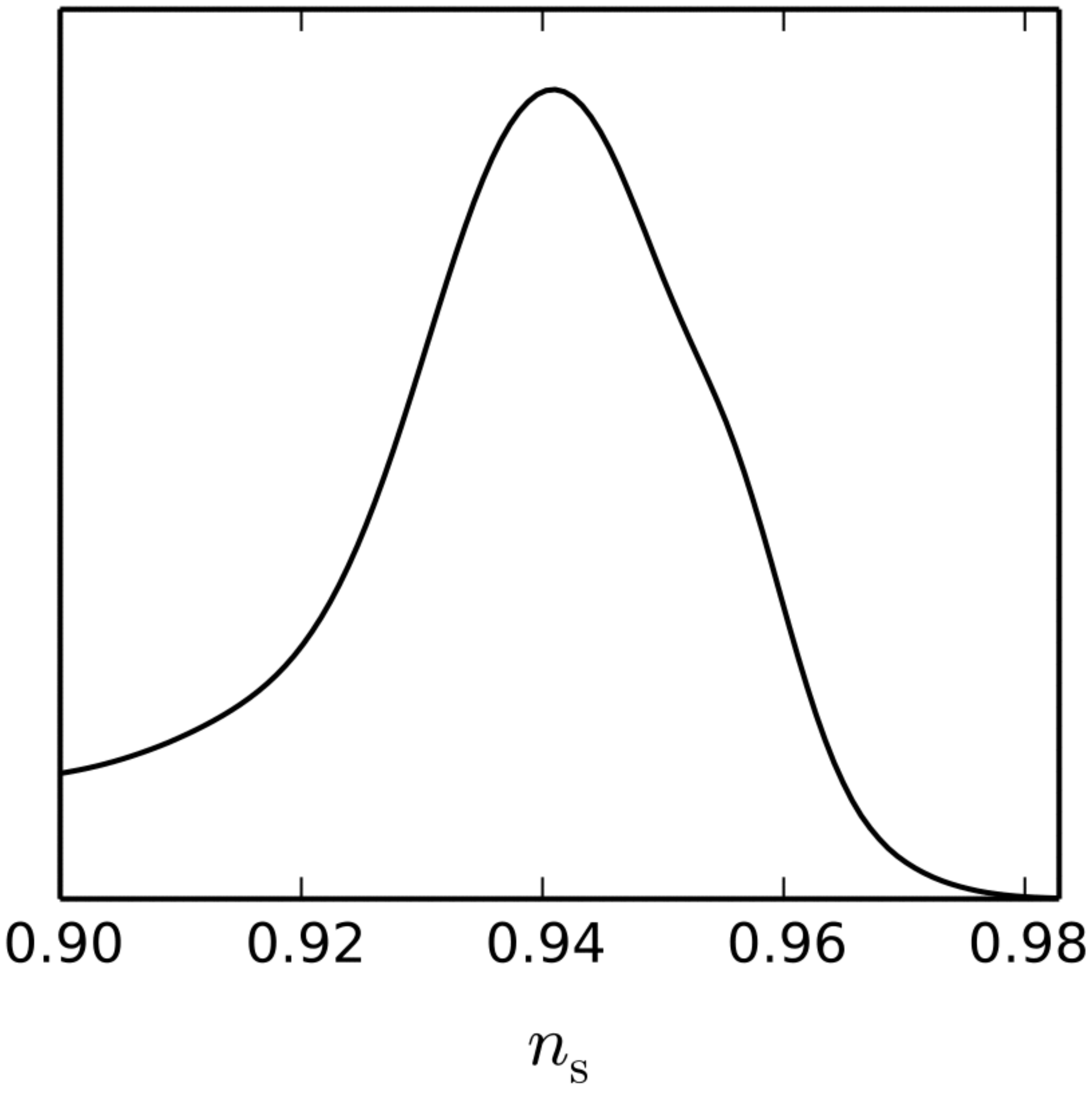


Full model

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

no prior

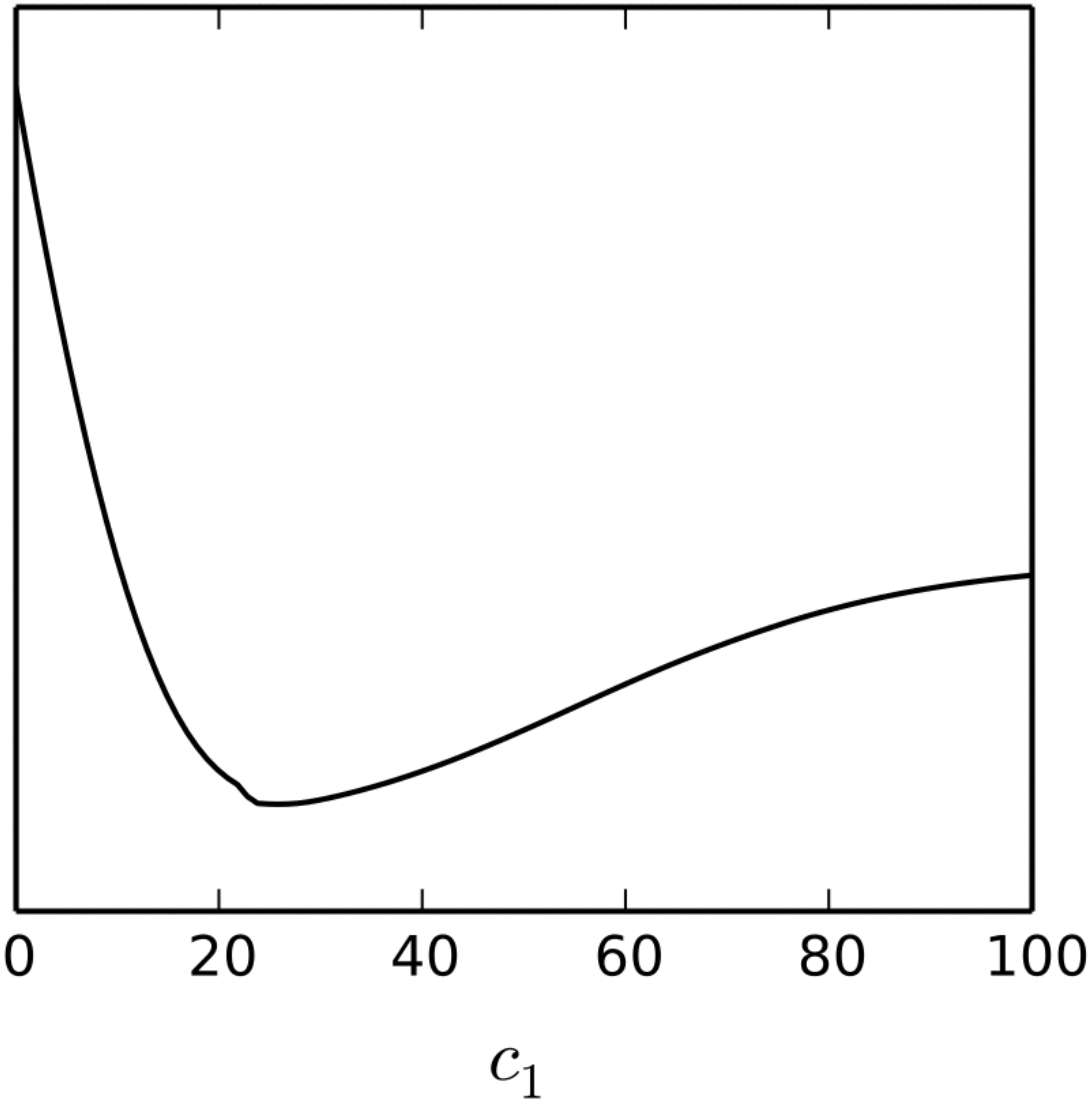


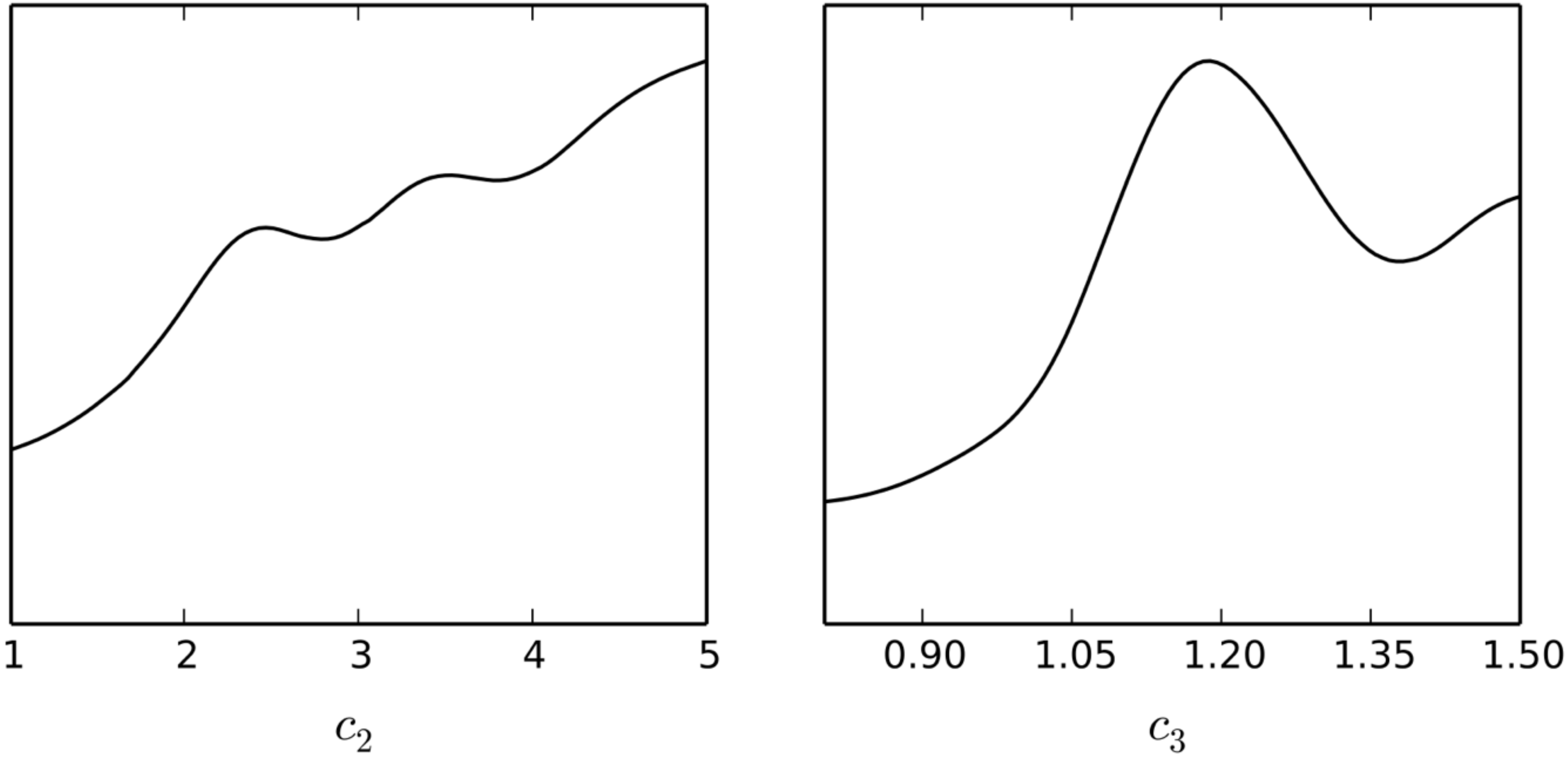


still redder primordial
spectrum, but converging!

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

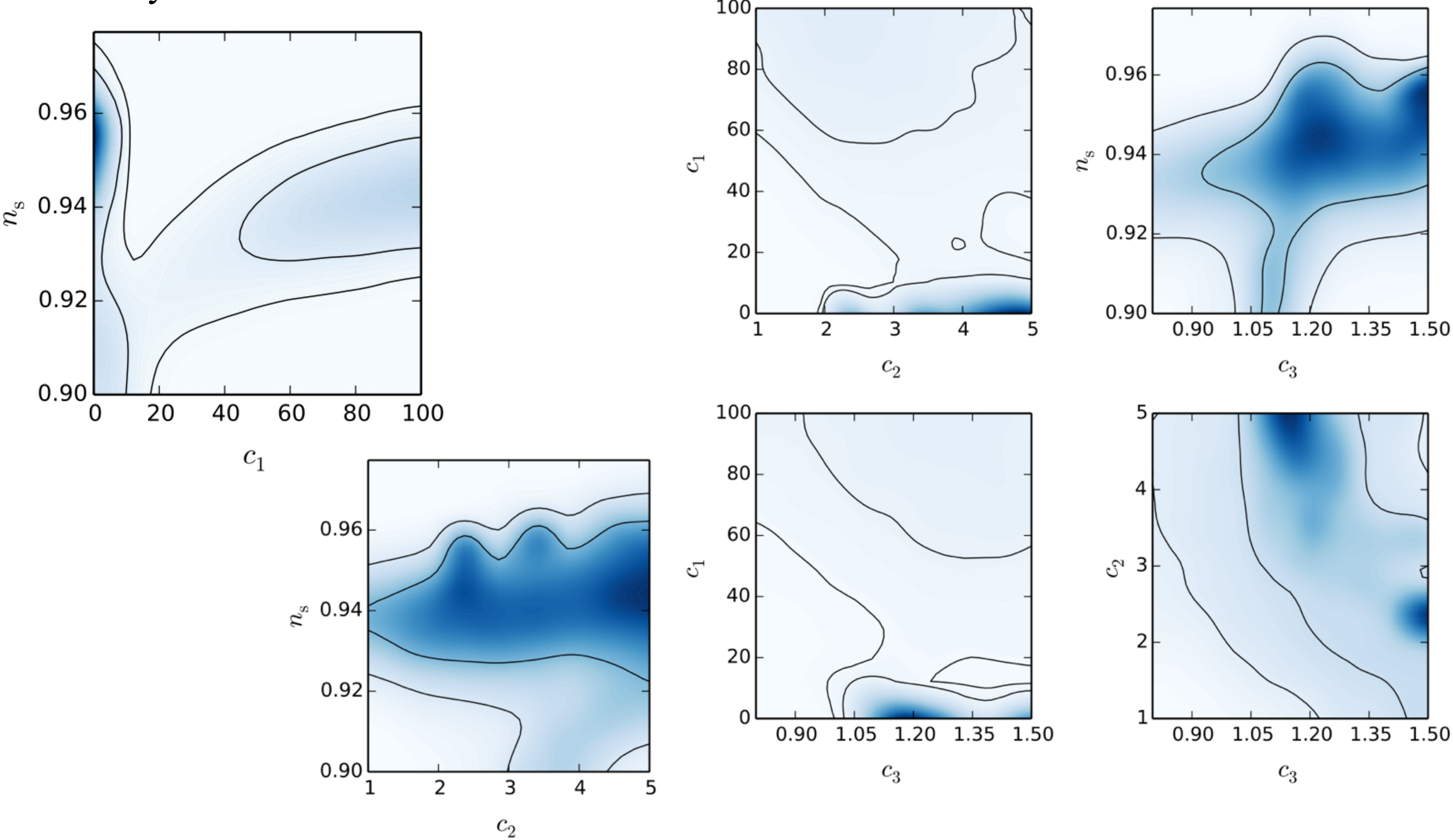
2 possible options:
very large & small
quantum scale





still not very conclusive, but definitely favors $c_3 \geq 1$

summary for the constrained model:



Conclusion

Cosmology can be used to test different formulations/extensions of quantum mechanics

more work still needs be done
(other modifications of QM can be tested…)