Some aspects of bimetric gravity

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Based on:

- *MvS, J. Enander, A. Schmidt-May,*
  E. Mörtsell & S. F. Hassan
  arXiv:1111.1655

- *S. F. Hassan, A. Schmidt-May & MvS*
  arXiv:1203.5283, arXiv:1204.5202, arXiv:1208.1515,
  arXiv:1208.1797, arXiv:1212.4525, arXiv:1303.6940
  arXiv:1407.2772

Outline of the talk

- Basic Motivations
- Linear massive spin-2 fields + History
- Bimetric theory, some details
- Bimetric cosmology
- Mass eigenstates
- Higher derivative formulation
- Partial masslessness
- Generalizations
- Summary

Outline of the talk

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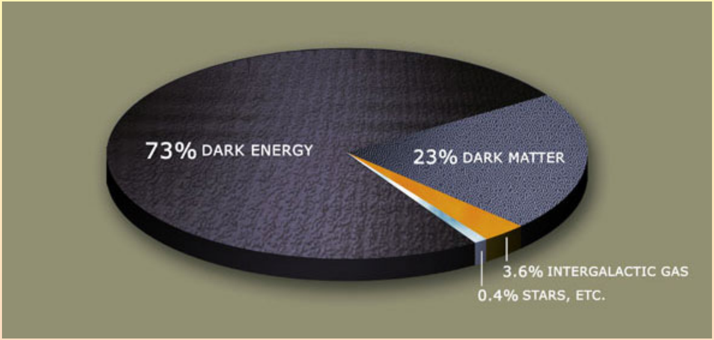
Motivation (Original)

Cosmology and the Dark sector(s)



- Cosmological constant problem. Can a small graviton mass naturally explain the observed smallness of Λ?
- The Dark sectors. Can a modification of Einstein gravity “remedy" the inclusion of unknown energy sources?

Motivation (More general)

Understand spin-2 interactions in field theory

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Good understanding of lower spin theories

- Spin-0: Higgs(?!), Inflaton(??), $\pi^{0,\pm}$ mesons,...
- Spin-1/2: Leptons, quarks, baryons
- Spin-1: Photon, gluons, Z, $W^{\pm}$, vector mesons,...

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  Klein-Gordon: $\left(\Box - m^2\right)\phi = 0$

- Spin-1/2: Leptons, quarks, baryons

  Dirac: $\left(i\partial\!\!\!/ - m\right)\psi = 0$

- Spin-1: Photon, gluons, Z, $W^{\pm}$, vector mesons,...

  Maxwell, Proca: $\partial^{\nu} F_{\nu\mu} - m^2 A_{\mu} = 0$

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  Einstein-Hilbert (massless): $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$

But no consistent theory of massive/interacting spin-2??

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But no consistent theory of massive/interacting spin-2 ...
until recently!!

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Fierz-Pauli theory

The FP equation:

Linear massive spin-2 field $h_{\mu\nu}$ in background $\bar{g}_{\mu\nu}$

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\, h_{\rho\sigma} - \frac{2\Lambda}{d-2}\left(h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h^{\rho}_{\rho}\right) + \frac{m^2_{\mathrm{FP}}}{2}\,\left(h_{\mu\nu} - a\bar{g}_{\mu\nu}h^{\rho}_{\rho}\right) = 0$$

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[Fierz, Pauli (1939)]

$$\left(\Box - m_{\mathrm{FP}}^2\right) h_{\mu\nu} = 0\,, \quad \nabla^{\mu} h_{\mu\nu} = 0\,, \quad \bar{g}^{\mu\nu} h_{\mu\nu} = 0$$

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Problem: Nonlinear completion in terms of $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. Generically removes a constraint, resulting in a propagating ghost-mode. *[Boulware,Deser (1972)]*

Ghost?

Classically: Negative kinetic energy, unbounded Hamiltonian.

Quantum theory: Loss of probability interpretation ($P > 1$)

Historical progress

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- Fully dynamical theory of interacting spin-2 shown to be ghost free nonlinearly *[Hassan, Rosen (2011-2012)]*

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Ghost free bimetric theory

The basic construction:

$$\mathcal{L} = m_g^{d-2}\sqrt{|g|}R(g) - 2m^d\sqrt{|g|}V(S;\beta_n) + m_f^{d-2}\sqrt{|f|}R(f)$$

$$S = \sqrt{g^{-1}f}\,, \quad \sqrt{|g|}V(S;\beta_n) = \sqrt{|f|}V(S^{-1};\beta_{d-n})$$

$$V(S;\beta_n) = \sum_{n=0}^{d}\beta_n e_n(S) = \beta_0 + \sum_{n=1}^{d-1}\beta_n e_n(S) + \sqrt{|g^{-1}f|}\beta_d$$

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Elementary symmetric polynomials

$$e_0(\mathbb{X}) = 1\,, \qquad e_1(\mathbb{X}) = [\mathbb{X}]\,, \qquad e_2(\mathbb{X}) = \tfrac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]),$$

$$e_3(\mathbb{X}) = \tfrac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3])\,,$$

$$e_4(\mathbb{X}) = \tfrac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4])\,,$$

$$\vdots$$

$$e_d(\mathbb{X}) = \det(\mathbb{X})$$

$$e_k(\mathbb{X}) = 0 \quad \text{for} \quad k > d\,, \qquad [\; e_n(\mathbb{X}) \sim (\mathbb{X})^n \;]$$

Ghost free bimetric theory, contd.

Equations of motion:

$$R_{\mu\nu}(g) - \tfrac{1}{2} g_{\mu\nu} R(g) + \frac{m^d}{m_g^{d-2}} V^g_{\mu\nu} = \frac{1}{2m_g^{d-2}} T^g_{\mu\nu}$$

$$R_{\mu\nu}(f) - \tfrac{1}{2} f_{\mu\nu} R(f) + \frac{m^d}{m_f^{d-2}} V^f_{\mu\nu} = \frac{1}{2m_f^{d-2}} T^f_{\mu\nu}$$

Bianchi constraints (for conserved sources)**:**

$${}^g\nabla^\mu V^g_{\mu\nu} = 0 = {}^f\nabla^\mu V^f_{\mu\nu}$$

related through the covariance identity

$$\sqrt{|g|}\, {}^g\nabla^\mu V^g_{\mu\nu} = -\sqrt{|f|}\, {}^f\nabla^\mu V^f_{\mu\nu}$$

Outline of the talk

Cosmological solutions

Isotropic & homogeneous ansatz :

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$$

$$f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -X^2(t)\,\mathrm{d}t^2 + Y^2(t)\,\mathrm{d}\vec{x}^2$$

The Bianchi constraint imply $X = \frac{\dot{Y}}{\dot{a}} = \frac{\mathrm{d}Y}{\mathrm{d}a}$.

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The Bianchi constraint imply $X = \frac{\dot{Y}}{\dot{a}} = \frac{\mathrm{d}Y}{\mathrm{d}a}$.
The equations of motion reduce to

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3m_g^2} + \frac{m^4}{3m_g^2}\left(\beta_0 + 3\beta_1\frac{Y}{a} + 3\beta_2\left(\frac{Y}{a}\right)^2 + \beta_3\left(\frac{Y}{a}\right)^3\right)$$

together with the quartic expression

$$\alpha^2\beta_3\left(\frac{Y}{a}\right)^4 + \left(3\alpha^2\beta_2 - \beta_4\right)\left(\frac{Y}{a}\right)^3 + 3\left(\alpha^2\beta_1 - \beta_3\right)\left(\frac{Y}{a}\right)^2 + \left(\frac{\alpha^2\rho}{m^4} + \alpha^2\beta_0 - 3\beta_2\right)\frac{Y}{a} - \beta_1 = 0\,,$$

where $\rho = -T^0{}_0$ is the energy density of the matter fluid.

Cosmological solutions, contd.

Model	B_0	B_1	B_2	B_3	B_4	Ω_m	$\chi^2_{\mathbf{min}}$	**p-value**	**log-evidence**
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B_1, Ω_m^0)	0	free	0	0	0	free	551.60	0.8355	-281.73
(B_2, Ω_m^0)	0	0	free	0	0	free	894.00	< 0.0001	-450.25
(B_3, Ω_m^0)	0	0	0	free	0	free	1700.50	< 0.0001	-850.26
(B_1, B_2, Ω_m^0)	0	free	free	0	0	free	546.52	0.8646	-279.77
(B_1, B_3, Ω_m^0)	0	free	0	free	0	free	542.82	0.8878	-280.10
(B_2, B_3, Ω_m^0)	0	0	free	free	0	free	548.04	0.8543	-280.91
(B_1, B_4, Ω_m^0)	0	free	0	0	free	free	548.86	0.8485	-281.42
(B_2, B_4, Ω_m^0)	0	0	free	0	free	free	806.82	< 0.0001	-420.87
(B_3, B_4, Ω_m^0)	0	0	0	free	free	free	685.30	0.0023	-351.14
$(B_1, B_2, B_3, \Omega_m^0)$	0	free	free	free	0	free	546.50	0.8582	-279.61
$(B_1, B_2, B_4, \Omega_m^0)$	0	free	free	0	free	free	546.52	0.8581	-279.56
$(B_1, B_3, B_4, \Omega_m^0)$	0	free	0	free	free	free	546.78	0.8563	-280.00
$(B_2, B_3, B_4, \Omega_m^0)$	0	0	free	free	free	free	549.68	0.8353	-282.89
$(B_1, B_2, B_3, B_4, \Omega_m^0)$	0	free	free	free	free	free	546.50	0.8515	-279.60
Full bimetric model	free	free	free	free	free	free	546.50	0.8445	-279.82

[Akrami et al (2012)]

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Generalizations

Summary

- What is the physical spectrum of the bimetric theory?
- Solutions close to GR?

Proportional backgrounds

For proportional ansatz $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$:

$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu} R(\bar{g}) + \begin{pmatrix} \Lambda_g \\ \Lambda_f \end{pmatrix} \bar{g}_{\mu\nu} = \frac{1}{2m_g^{d-2}} \begin{pmatrix} \bar{T}^g_{\mu\nu} \\ \alpha^{2-d}\, \bar{T}^f_{\mu\nu} \end{pmatrix}, \quad \alpha = \frac{m_f}{m_g}$$

and as a consequence

$$\Lambda_g = \Lambda_f \,, \quad \bar{T}^g_{\mu\nu} = \alpha^{2-d}\, \bar{T}^f_{\mu\nu}$$

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and as a consequence

$$\Lambda_g = \Lambda_f\,, \quad \bar{T}^g_{\mu\nu} = \alpha^{2-d}\,\bar{T}^f_{\mu\nu}$$

$$\Lambda_g = \frac{m^d}{m_g^{d-2}} \sum_{n=0}^{d-1} \binom{d-1}{n} c^n \beta_n = \frac{m^d}{m_g^{d-2}} (\alpha c)^{2-d} \sum_{n=1}^{d} \binom{d-1}{n-1} c^n \beta_n = \Lambda_f$$

Generically gives $c = c(\alpha, \beta_n)$

Conceptually very important class of solutions

Proportional backgrounds

First a remark; For any covariant bimetric theory it is “straightforward" to find a striking restriction on classical solutions:

If one metric is an Einstein metric, the other metric is also an Einstein metric (proportional to the first metric).

Mass spectrum

Fluctuations on the proportional background:

For the linear modes ($\bar{T}^{g,f} = 0$):

$$\delta M_{\mu\nu} = \frac{1}{2c}\left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right) , \quad \delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \alpha^{d-2} c^{d-4} \delta f_{\mu\nu}\right)$$

The field equations are

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma}\, \delta G_{\rho\sigma} - \frac{2\tilde{\Lambda}_g}{d-2}\left(\delta G_{\mu\nu} - \frac{1}{2}\bar{G}_{\mu\nu}\bar{G}^{\rho\sigma}\delta G_{\rho\sigma}\right) = 0\,,$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma}\, \delta M_{\rho\sigma} - \frac{2\tilde{\Lambda}_g}{d-2}\left(\delta M_{\mu\nu} - \frac{1}{2}\bar{G}_{\mu\nu}\bar{G}^{\rho\sigma}\delta M_{\rho\sigma}\right) + \frac{1}{2}\tilde{m}_{\mathrm{FP}}^2\left(\delta M_{\mu\nu} - \bar{G}_{\mu\nu}\bar{G}^{\rho\sigma}\delta M_{\rho\sigma}\right) = 0$$

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The FP mass of δM:

$$\tilde{m}^2_{\mathrm{FP}} = \tfrac{m^d}{m_g^{d-2}}\,(\alpha c)^{2-d}\sum_{k=1}^{d-1}\binom{d-2}{k-1} c^k \beta_k$$

and $\tilde{\Lambda}_g = \Lambda_g$

Outline of the talk

- Can we solve for one metric ... at least perturbatively?

Higher derivative equations

Solving the $g_{\mu\nu}$ equation for $f_{\mu\nu}$ in a curvature expansion gives the solution:

$$f_{\mu\nu} = a_1 g_{\mu\nu} + \frac{a_2}{m^2} P_{\mu\nu} + \frac{a_3}{m^4} P^2_{\mu\nu} + \frac{a_4}{m^4} P P_{\mu\nu} + \frac{a_5}{m^4} g_{\mu\nu} e_2(P) + \mathcal{O}(R^3/m^6)$$

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Plugging this solution back in the $f_{\mu\nu}$ equations result in a higher derivative equation for $g_{\mu\nu}$:

$$0 = \Lambda g_{\mu\nu} + \frac{b_1}{m^2} \mathcal{G}_{\mu\nu} + \frac{b_{11}}{m^2} \Lambda P_{\mu\nu} + \frac{c_1}{m^4} B_{\mu\nu} + \frac{c_{11}}{m^4} P^2_{\mu\nu} + \frac{c_{12}}{m^4} P P_{\mu\nu} + \frac{c_{13}}{m^4} g_{\mu\nu} P^{\rho\sigma} P_{\rho\sigma} + \frac{c_{14}}{m^4} g_{\mu\nu} P^2 + \mathcal{O}(R^3/m^6)$$

Here $\mathcal{G}_{\mu\nu}$ is the Einstein tensor, $P_{\mu\nu}$ the Schouten tensor $P_{\mu\nu} = R_{\mu\nu} - \frac{1}{2(d-1)} g_{\mu\nu} R$ and $B_{\mu\nu}$ the Bach tensor

$$-B_{\mu\nu} = \nabla^2 P_{\mu\nu} + \nabla_\mu \nabla_\nu P - \nabla_\rho \nabla_\mu P^\rho{}_\nu - \nabla_\rho \nabla_\nu P^\rho{}_\mu + 2P^2_{\mu\nu} - \frac{1}{2} g_{\mu\nu} P^{\rho\sigma} P_{\rho\sigma}$$

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- Any special corners of the bimetric theory parameter space?
- Enhanced symmetries?

Partial masslessness

In standard linear Fierz-Pauli massive gravity on de Sitter backgrounds the “Higuchi bound" plays a special role

$$m_{\mathrm{FP}}^2 \geq 2\Lambda/(d-1) \quad (\text{unitary})$$

[Higuchi (1989)]

For $m_{\mathrm{FP}}^2 = 2\Lambda/(d-1)$ the massive field acquires a new gauge symmetry under

$$h_{\mu\nu} \to h_{\mu\nu} + \left(\nabla_\mu \partial_\nu + \frac{m_{\mathrm{FP}}^2}{d-2} \bar{g}_{\mu\nu}\right) \xi(x)$$

[Deser, Waldron (2001)]

This symmetry removes one propagating degree of freedom.

Question: Can this gauge symmetry be extended nonlinearly?

Partial masslessness, contd.

Some recent work:

- Cubic PM vertices ($\sim h^3$) in $d = 4$ *[Zinoviev (2006)]*
- Cubic PM vertices exist only in $d = 3, 4$ with 2 derivatives
  For $d > 4$, higher derivative theory needed.
  [Joung, Lopez, Taronna (2012)]

We can identify a nonlinear bimetric theory as a candidate PM theory and verify all of these known results

Partial masslessness, contd.

- Naturally addressed in bimetric theory–dynamical background.

Partial masslessness, contd.

- Naturally addressed in bimetric theory–dynamical background.
- Proportional backgrounds–de Sitter.

Partial masslessness, contd.

- Naturally addressed in bimetric theory–dynamical background.
- Proportional backgrounds–de Sitter.

Recall the fluctuation equations on proportional backgrounds

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma}\,\delta G_{\rho\sigma} - \frac{2\tilde{\Lambda}_g}{d-2}\Big(\delta G_{\mu\nu} - \tfrac{1}{2}\bar{G}_{\mu\nu}\bar{G}^{\rho\sigma}\delta G_{\rho\sigma}\Big) = 0\,,$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma}\,\delta M_{\rho\sigma} - \frac{2\tilde{\Lambda}_g}{d-2}\Big(\delta M_{\mu\nu} - \tfrac{1}{2}\bar{G}_{\mu\nu}\bar{G}^{\rho\sigma}\delta M_{\rho\sigma}\Big) + \tfrac{1}{2}\tilde{m}^2_{\rm FP}\left(\delta M_{\mu\nu} - \bar{G}_{\mu\nu}\bar{G}^{\rho\sigma}\delta M_{\rho\sigma}\right) = 0$$

Nonlinear Partial masslessness in d=4

PM symmetry in linearized bimetric theory:

$$\delta M_{\mu\nu} \to \delta M_{\mu\nu} + \Big(\nabla_\mu \partial_\nu + \frac{m_{\rm FP}^2}{2}\,\bar{G}_{\mu\nu}\Big)\xi(x)\,, \qquad \delta G_{\mu\nu} \to \delta G_{\mu\nu}$$

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Take $\xi = \xi_0$ (constant). Transformation of the original bimetric variables:

$$\delta g_{\mu\nu} \to \delta g_{\mu\nu} + a\,\frac{\Lambda}{3}\,\xi_0\,\bar{g}_{\mu\nu}\,, \qquad \delta f_{\mu\nu} \to \delta f_{\mu\nu} + b\,\frac{\Lambda}{3}\,\xi_0\,\bar{g}_{\mu\nu}$$

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For dynamical backgrounds, this is equivalent to

$$\bar{g}'_{\mu\nu} = \bar{g}_{\mu\nu} + a\frac{\Lambda}{3}\xi_0\,\bar{g}_{\mu\nu}\,, \quad \bar{f}'_{\mu\nu} = \bar{f}_{\mu\nu} + b\frac{\Lambda}{3}\xi_0\,\bar{g}_{\mu\nu}$$

$$\bar{f}' = c'^2(\xi_0)\,\bar{g}' \qquad c' \neq c$$

Not a valid background solution! No PM symmetry??

Nonlinear Partial masslessness in d=4

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Not a valid background solution! No PM symmetry?? (caveat!)

Nonlinear Partial masslessness in d=4

$\bar{f}'$ and $\bar{g}'$ are solutions only if c is not determined by $\Lambda_g = \Lambda_f$,

$$\beta_1 + \left(3\beta_2 - \alpha^2\beta_0\right) c + \left(3\beta_3 - 3\alpha^2\beta_1\right) c^2 + \left(\beta_4 - 3\alpha^2\beta_2\right) c^3 - \alpha^2\beta_3 c^4 = 0$$

c is undetermined for,

$$\alpha^2\beta_0 = 3\beta_2\,, \qquad 3\alpha^2\beta_2 = \beta_4\,, \qquad \beta_1 = \beta_3 = 0$$

This gives the unique candidate nonlinear PM theory. Has been verified that this global symmetry exist fully nonlinearly.

Nonlinear Partial masslessness in d=4

Side remark and further support:

A quartic equation in a cosmological setup

$$\alpha^2\beta_3\left(\frac{Y}{a}\right)^4+\left(3\alpha^2\beta_2-\beta_4\right)\left(\frac{Y}{a}\right)^3+3\left(\alpha^2\beta_1-\beta_3\right)\left(\frac{Y}{a}\right)^2 \\ +\left(\alpha^2\beta_0-3\beta_2\right)\frac{Y}{a}-\beta_1=0$$

leaves the function $Y(t)/a(t)$ undetermined for PM parameters!

Solutions with a cosmological gauge symmetry!

Conformal gravity & Partial masslessness

Even more compelling:

The HD equation for PM parameters is given to lowest order in curvature by

$$B_{\mu\nu} = 0$$

- To lowest order in curvature the PM candidate has a Weyl symmetry!
- Establishes a gauge symmetry also close to flat space, away from de Sitter!

Nonlinear PM theory

Checks:

- For $d = 2, 3, 4$ we find that $m_{\mathrm{FP}}^2 = \frac{2\Lambda_g}{d-1}$
- For $d > 4$, $\beta_n = 0$. Nonlinear PM exist only for $d = 3, 4$.
- Higher dimensions need higher derivatives, works out.
- Realization of the ξ_0 global gauge transformation in the nonlinear theory.
- Physical parameters independent of gauge parameter ξ_0.
- Full Gauge symmetry of the nonlinear theory? Weyl symmetry in low curvature limit supports its existence, but not yet found.

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Outline of the talk

Basic Motivations

Linear massive spin-2 fields + History

Bimetric theory, some details

Bimetric cosmology

Mass eigenstates

Higher derivative formulation

Partial masslessness

Generalizations

Summary

- Higher derivative extension and PM in higher dimensions
- Interactions of several massive spin-2 fields
- Vielbein formulation

Outline of the talk

- Basic Motivations
- Linear massive spin-2 fields + History
- Bimetric theory, some details
- Bimetric cosmology
- Mass eigenstates
- Higher derivative formulation
- Partial masslessness
- Generalizations
- Summary

Summary

- The ghost free bimetric theory has solutions indistuingishable from ΛCDM at the background level.
- It describes massive and massless spin-2 fields. Alternatively, it describes a massive spin-2 field coupled to gravity and extends to multi-spin-2 considerations.
- Fluctuations with FP masses exist around $\bar{f} = c^2 \bar{g}$ backgrounds. Covers all GR background solutions.
- The nonlinear PM candidate leaves c undetermined and has a global scaling symmetry. Can exist only in $d = 3$ and $d = 4$, in 2-derivative theories. But can exist in higher dimensions with more than 2 derivatives. Consistent with all known results.
- Higher derivative single metric formulaton exist. The PM subset coincides with Conformal gravity in a derivative expansion and support existence of an extra gauge symmetry.

The end is only the beginning . . .

Thanks for your attention!