Scale-invariance and the origin of dark radiation

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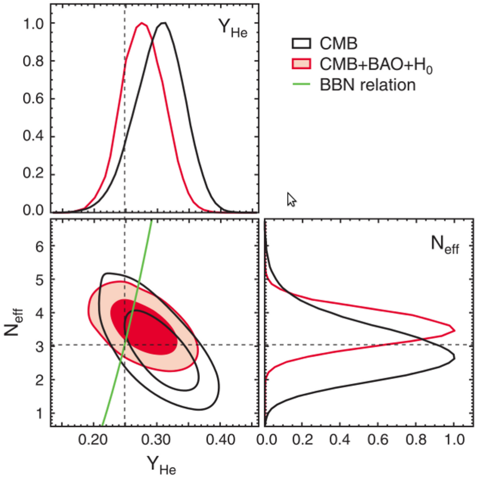
Motivation for scale invariance at high energy

- Idea: change M_P by a vacuum expectation value of new scalar field.
- SM is classically scale invariant at HE
- SI can help to cancel quadratic divergence in quantum corrections to Higgs boson mass

  W. A. Bardeen, "On naturalness in the standard model," FERMILAB-CONF-95-391-T.
- SI provides exponentially flat inflaton potential which is preferred by Planck data
- Some amount of dark radiation?

The hint for the dark radiation



Spontaneously broken scale invariance: dilaton

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R^2-dilaton inflation

- Most common scale-invariant action for scalar field and gravity:

$$S_0 = \int d^4x \sqrt{-g}\,\frac{1}{2}[\beta R^2 + (\partial_\mu X)^2 - \xi X^2 R].$$

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- Let's add a Lagrange multiplier Λ and new scalar $\mathcal{R}$:

$$S = \int d^4x\sqrt{-g}\,\left[\frac{1}{2}(\beta\mathcal{R}^2 + (\partial_\mu X)^2 - \xi X^2\mathcal{R}) - \Lambda\mathcal{R} + \Lambda R\right].$$

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- Integrating out auxiliary field $\mathcal{R} = (\xi X^2/2 + \Lambda)/\beta$ we obtain

$$S = \int d^4x\sqrt{-g}\,\left[\Lambda R + \frac{1}{2}(\partial_\mu X)^2 - \frac{1}{2\beta}(\Lambda + \frac{1}{2}\xi X^2)^2\right].$$

R^2-dilaton inflation

- Go to the Einstein frame:

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \boxed{\Omega^2 = -2\Lambda/M_P^2}$$

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$$V = \frac{M_P^4}{8\beta}\left(1 - \frac{6\xi X^2}{\omega^2}\right)^2, \quad \omega = \sqrt{6}M_P\Omega$$

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- Field redefinition:

$$\omega = r\sin\theta\,,\ \ X = r\cos\theta$$

R^2-dilaton inflation

- Kinetic term and potential:

$$K = \frac{6M_P^2}{2\sin^2\theta}\left((\partial_\mu \log r)^2 + (\partial_\mu\theta)^2\right)\,, \quad V = \frac{M_P^4}{8\beta}\left(1 - 6\xi\cot^2\theta\right)^2\,,$$

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- After canonical normalisation: usual gravity + dilaton ρ + scalaron f:

$$S = \int d^4x\left[-\frac{M_P}{2}R + \frac{1}{2}(\partial_\mu\rho)^2\cosh^2\left(\frac{f_0 - f}{\sqrt{6}M_P}\right) + \frac{1}{2}(\partial_\mu f)^2 - V(f)\right]$$

$$V(f) = \frac{M_P^4}{8\beta}\left(1 - 6\xi\sinh^2\left(\frac{f_0 - f}{\sqrt{6}M_P}\right)\right)^2$$

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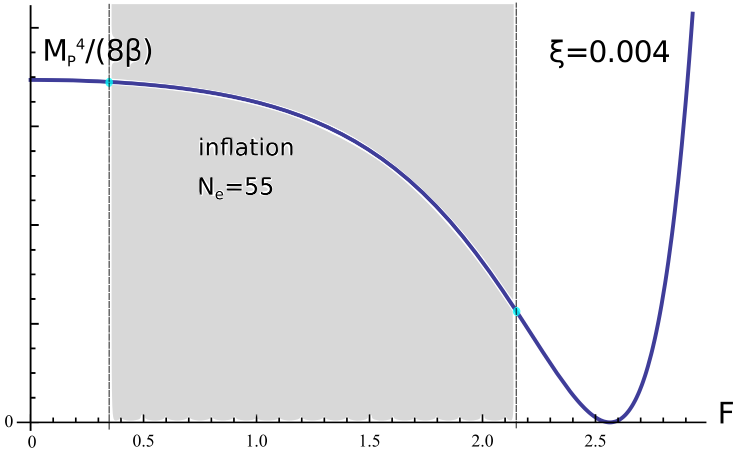
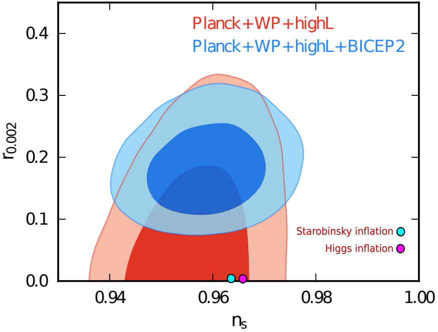
- Dilaton ρ is massless shift-symmetric goldstone boson of spontaneously broken SI.

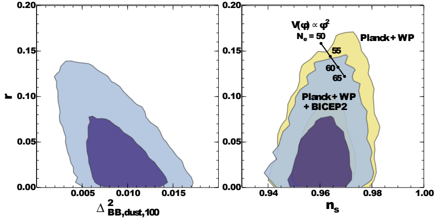
Picture of potential



Inflationary predictions

When $\xi < 0.004$ - very close to Starobinsky inflation.



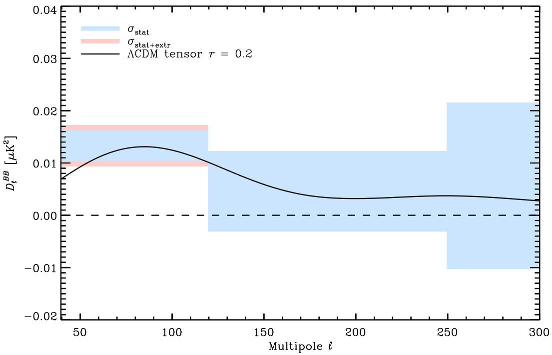


P. A. R. Ade et al. [BICEP2 Collaboration], Phys. Rev. Lett. 112, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]]

M. J. Mortonson and U. Seljak, arXiv:1405.5857 [astro-ph.CO].

Planck intermediate results. XXX. The angular power spectrum of polarized dust emission at intermediate and high Galactic latitudes

Planck Collaboration: Dust polarization at high latitudes



Reheating and producing dilatons

- Inflaton f couples to any Weyl variation of the matter lagrangian:
- to Higgs boson – the main reheating mechanism

$$\Gamma_H = \left(\frac{1}{6\beta}\right)^{3/2} \frac{4M_P}{192\pi}(1+6\xi')^2$$

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- to gauge bosons due to conformal anomaly (gives impact to reheating only when $\xi' \approx -1/6$):

$$\Gamma_{gauge} = \frac{\Sigma b_i^2 \alpha_i^2 N_i}{16\pi^2} \left(\frac{1}{6\beta}\right)^{3/2} \frac{M_P}{48\pi}$$

Number of relativistic degrees of freedom

$$\Delta N_{eff} \simeq 2.85\frac{\rho_\rho}{\rho_H} = 2.85\frac{\Gamma_\rho}{\Gamma_H} = \frac{0.71}{(1+6\xi')^2}$$

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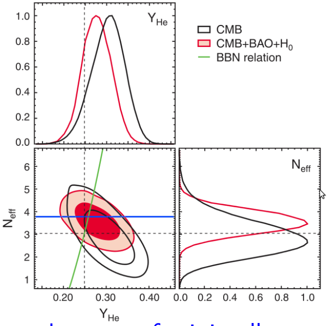
$\Delta N_{eff} < 1.24 \Longrightarrow |\xi' + 1/6| > 0.13$, conformal case is forbidden



The blue line correspond to the case of minimally coupled Higgs boson, $\xi' = 0$

Interplay between R^2-dilaton and Higgs-dilaton inflation

Action:

$$S_0 = \int d^4x\sqrt{-g}\left[\frac{1}{2}[\beta R^2 + (\partial_\mu X)^2 - \xi X^2 R - \xi' h^2 R + (\partial_\mu h)^2] - \frac{\lambda}{4}h^4\right]$$

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Einstein frame action: dilaton ρ, scalaron f, Higgs ϕ

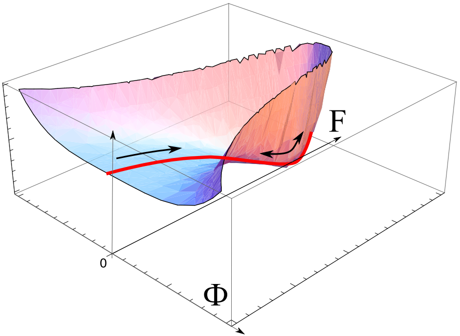
$$L = \frac{1}{2}(\partial_\mu\rho)^2\,\cosh^2(F) + \frac{1}{2}(\partial_\mu\phi)^2\,\sinh^2(F) + \frac{1}{2}(\partial_\mu f)^2 - V(\Phi, F)$$

$$(f_0 - f)/(\sqrt{6}M_P) \equiv F,\;\; \phi = \sqrt{6}M_P\Phi$$

$$V = \frac{M_P^4}{8\beta}\left(1 - 6\sinh^2(F)\left[\xi\cos^2\Phi + \xi'\sin^2\Phi\right]\right)^2 + 9\lambda M_P^4\sinh^4(F)\sin^4\Phi$$

Potential

$\xi' < \xi$



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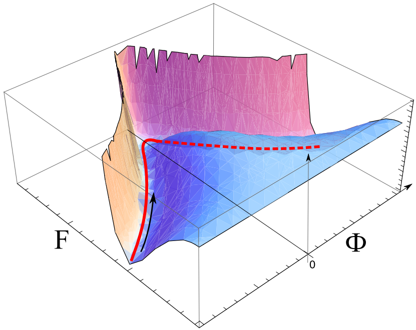
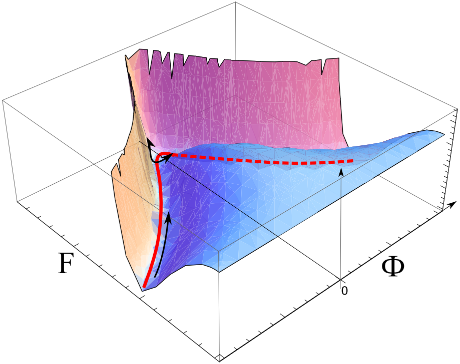


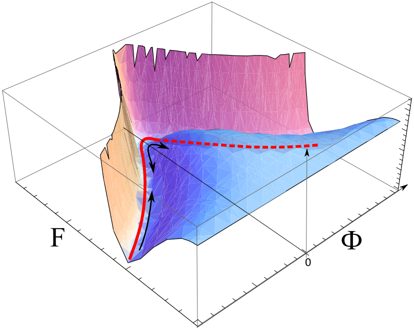
No dilaton production

$\xi' > \xi$

$\xi'^2 \ll 2\beta\lambda$



$\xi'^2 \gg 2\beta\lambda$



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Trajectories

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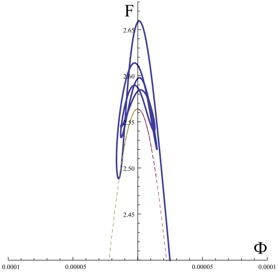
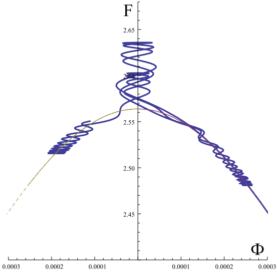


$\xi'^2 \sim 2\beta\lambda$



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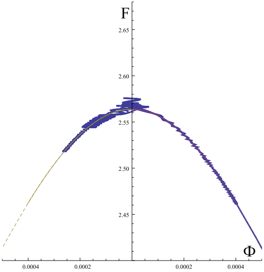

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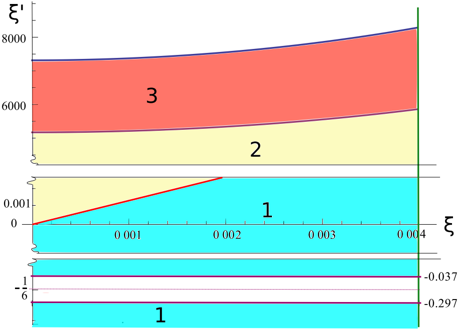
Space of parameters



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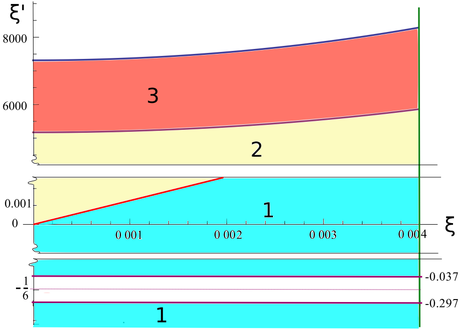

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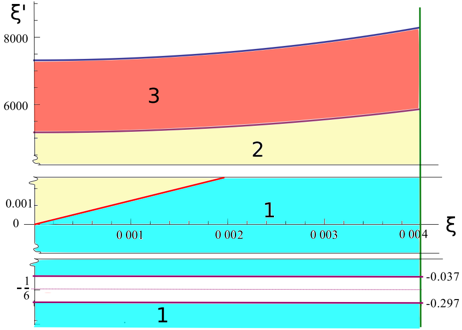

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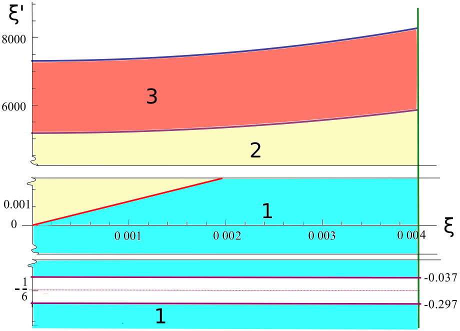

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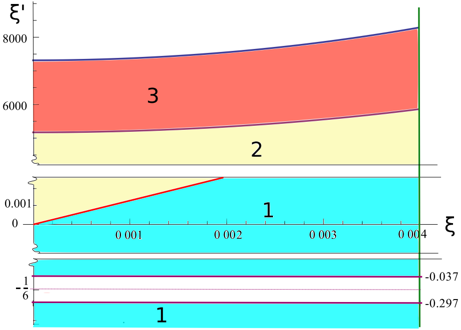

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- In all these cases parameter β is defined by curvature perturbation amplitude $\Delta \simeq 5 \times 10^{-5}$, the e-folding number is $N_e = 55$ and $\lambda = 0.01$.

Scale-invariant inflation

Inflaton ϕ and dilaton X:

$$L = -\frac{1}{2}\xi X^2 R + \frac{1}{2}(\partial_\mu X)^2 + \frac{1}{2}(\partial_\mu \phi)^2 - X^4\, V\left(\frac{\phi}{X}\right)$$

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$$\Downarrow$$

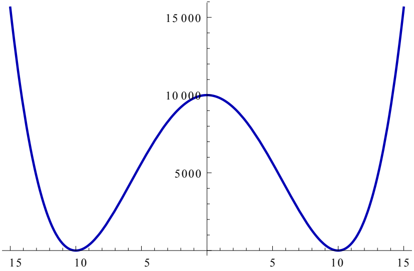
$$L = \frac{1}{2}(\partial f)^2 + \frac{1}{2}(\partial \rho)^2 \cosh^2 \tilde{f} - \frac{M_P^4}{\xi^2}\, V(\sqrt{1+6\xi}\,\sinh \tilde{f}).$$

$$\tilde{f} \equiv f/\zeta M_P, \quad \zeta \equiv \sqrt{\frac{1+6\xi}{\xi}}$$

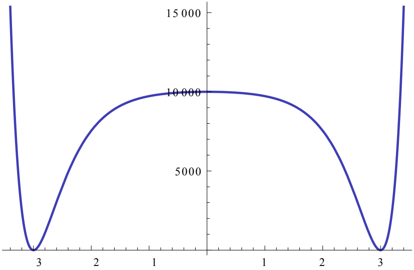
Flattening the potential

$$V = \lambda(\phi^2 - \alpha X^2)^2$$

$$V = \frac{\lambda M_P^4}{\xi^2}((\sqrt{1+6\xi}\sinh(\tilde{f}))^2 - \alpha)$$



$\Rightarrow$



Reheating

- Inflaton decay to dilatons:

$$\Gamma_\rho = \frac{m^3 \tanh^2 \tilde{f}_0}{32\pi\zeta^2 M_P^2}, \quad m - -\text{inflaton mass}$$

- No dilaton overproduction $\Rightarrow$

$$T_{reh} > \frac{1.87}{\sqrt{\Delta N_{max}}} g_*^{-1/4} \sqrt{\Gamma_\rho M_P}, \quad \Delta N_{max} = N_{eff} - 3.04 \simeq 1$$

- Natural reheating – inflaton decay to Higgs boson.

$$\Gamma_H / \Gamma_\rho = 4(1 + 6\xi')^2$$

! The conformal Higgs with $\xi' = -1/6$ is also forbidden

Conclusions and problems

- We studied scale-invariant extension of the Starobinsky model of inflation
- Due to low reheating temperature dilaton can be produced at reheating and give impact to the dark radiation which is in a good agreement with present data
- Quantum level?

Thanks for your attention!