Institut d'Astrophysique de Paris
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Rencontres du Vietnam
Hot Topics in General Relativity & Gravitation

BIMETRIC GRAVITY AND DARK MATTER

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Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
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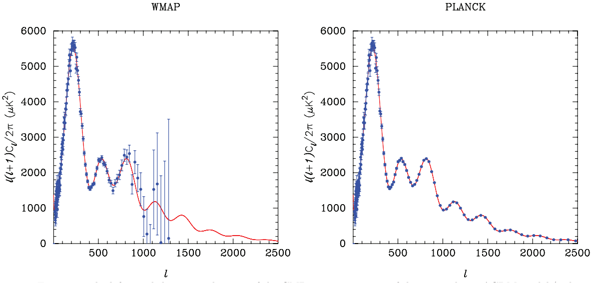
The cosmological concordance model Λ-CDM



This model brilliantly accounts for:

- The mass discrepancy between the dynamical and luminous masses of clusters of galaxies
- The precise measurements of the anisotropies of the cosmic microwave background (CMB)
- The formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
- The fainting of the light curves of distant supernovae

Challenges with CDM at galactic scales

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004, Famaey & McGaugh 2012]

1. Unobserved predictions
   - Numerous but unseen satellites of large galaxies
   - Phase-space correlation of galaxy satellites
   - Generic formation of dark matter cusps in galaxies
   - Tidal dwarf galaxies dominated by dark matter
2. Unpredicted observations
   - Correlation between mass discrepancy and acceleration
   - Surface brightness of galaxies and the Freeman limit
   - Flat rotation curves of galaxies
   - Baryonic Tully-Fisher relation for spirals
   - Faber-Jackson relation for ellipticals

All these challenges are mysteriously solved (sometimes with incredible success) by the MOND empirical formula [Milgrom 1983]

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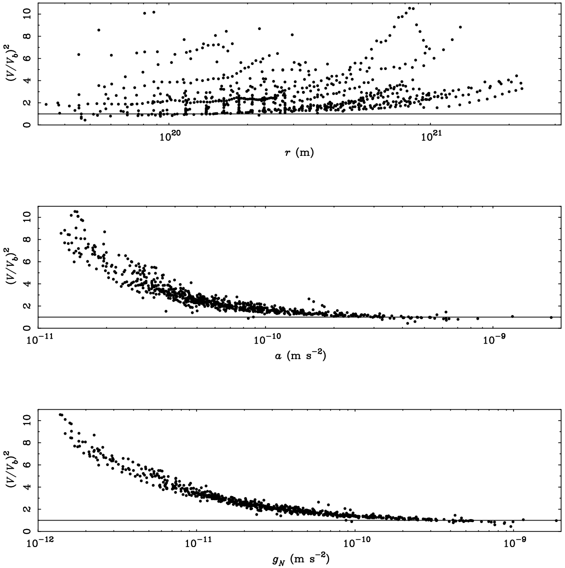
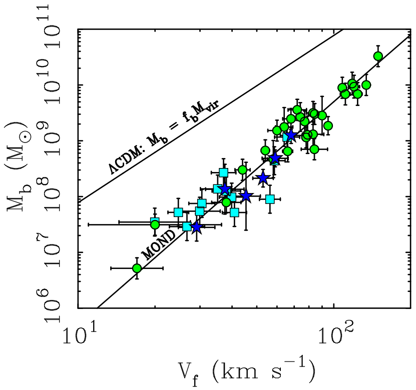
Mass discrepancy versus acceleration



Baryonic Tully-Fisher relation [Tully & Fisher 1977, McGaugh 2011]

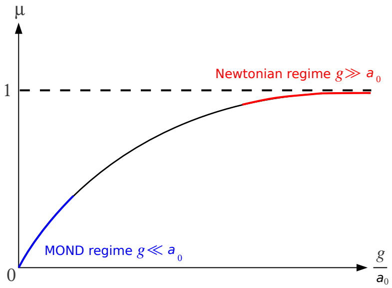


We have approximately $V_f \simeq (G\, M_b\, a_0)^{1/4}$ where $a_0 \simeq 1.2 \times 10^{-10}$m/s^2 is very close (mysteriously enough) to typical cosmological values

$$a_0 \simeq 1.3\, a_\Lambda \quad \text{with} \quad a_\Lambda = \frac{c^2}{2\pi}\sqrt{\frac{\Lambda}{3}}$$

Modified Poisson equation [Milgrom 1983, Bekenstein & Milgrom 1984]

$$\nabla \cdot \left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \right] = -4\pi\, G\, \rho_{\text{baryons}} \quad \text{with} \quad \boldsymbol{g} = \nabla U$$



- The Newtonian regime is recoved when $g \gg a_0$
- In the MOND regime $g \ll a_0$ we have $\boxed{\mu = \frac{g}{a_0} + \mathcal{O}(g^2)}$

Modified gravity theories

1. Generalized Tensor-Scalar theory (RAQUAL) [Bekenstein & Sanders 1994]
2. Tensor-Vector-Scalar theory (TeVeS) [Bekenstein 2004, Sanders 2005]
3. Generalized Einstein-Æther theories [Zlosnik *et al.* 2007, Halle *et al.* 2008]
4. Khronometric theory [Blanchet & Marsat 2011, Sanders 2011, Barausse *et al.* 2015]
5. Bimetric theory (BIMOND) [Milgrom 2012]

- These theories contain non-standard kinetic terms parametrized by an arbitrary function which is linked *in fine* to the MOND function
- In some cases they have stability problems associated with the fact that the Hamiltonian is not bounded from below [Clayton 2001, Bruneton & Esposito-Farèse 2007]
- Generically they have problems to recover the cosmological model Λ-CDM at large scales and the spectrum of CMB anisotropies [Skordis, Mota *et al.* 2006]

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Dielectric analogy of MOND [Blanchet 2007]

- In electrostratics the Gauss equation is modified by the polarization of the dielectric (dipolar) material

$$\nabla\cdot\Big[\underbrace{(1+\chi_e)\boldsymbol{E}}_{\boldsymbol{D}\text{ field}}\Big]=\frac{\rho_e}{\varepsilon_0}\qquad\Longleftrightarrow\qquad\nabla\cdot\boldsymbol{E}=\frac{\rho_e+\rho_e^{\text{polar}}}{\varepsilon_0}$$

- Similarly MOND can be viewed as a modification of the Poisson equation by the polarization of some dipolar medium

$$\nabla\cdot\left[\mu\left(\frac{g}{a_0}\right)\boldsymbol{g}\right]=-4\pi\,G\,\rho_{\mathrm{b}}\qquad\Longleftrightarrow\qquad\nabla\cdot\boldsymbol{g}=-4\pi\,G\left(\rho_{\mathrm{b}}+\underbrace{\rho^{\text{polar}}}_{\text{dark matter}}\right)$$

The MOND function can be written $\mu=1+\chi$ where χ appears as a susceptibility coefficient of some dipolar DM medium

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- Similarly MOND can be viewed as a modification of the Poisson equation by the polarization of some dipolar medium

$$\nabla \cdot \left[\mu\left(\frac{g}{a_0}\right) \boldsymbol{g} \right] = -4\pi\, G\, \rho_{\mathrm{b}} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{g} = -4\pi\, G \Big(\rho_{\mathrm{b}} + \underbrace{\rho^{\text{polar}}}_{\text{dark matter}} \Big)$$

The MOND function can be written $\mu = 1 + \chi$ where χ appears as a susceptibility coefficient of some dipolar DM medium

Dipolar dark matter (DDM) [Blanchet & Le Tiec 2008; 2009]

1. Attempt at implementing in a relativistic way the dielectric analogy of MOND

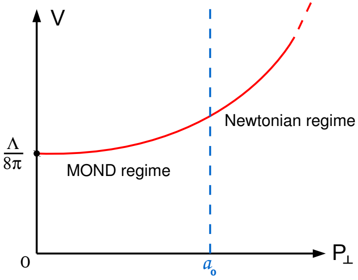
The DDM action in standard general relativity is

$$S_{\text{DDM}} = \int \mathrm{d}^4x \sqrt{-g} \Big[\underbrace{-\rho}_{\text{CDM}} + J^\mu \dot{\xi}_\mu - V(P_\perp) \Big]$$

- Interaction term couples the matter current $J^\mu = \rho u^\mu$ to a vector field ξ^μ called the dipole moment
- Potential term V built from the norm of the polarization field $P_\perp = \rho\, \xi_\perp$ and projected orthogonally to the four-velocity u^μ

2. The only physical components of the dipole moment are those orthogonal to the four-velocity, hence the dipole moment vector is space-like

Dipolar dark matter (DDM) [Blanchet & Le Tiec 2008; 2009]



- The potential V is phenomenologically determined through third order

$$V = \frac{\Lambda}{8\pi} + 2\pi\, P_\perp^2 + \frac{16\pi^2}{3a_0}\, P_\perp^3 + \mathcal{O}\left(P_\perp^4\right)$$

- The natural order of magnitude of the cosmological constant Λ is comparable with a_0 namely $\Lambda \sim a_0^2$ in good agreement with observations

Agreement with Λ-CDM at cosmological scales

- In a cosmological perturbation around a FLRW background the space-like dipole moment must belong to the first-order perturbation

$$\xi_\perp^\mu = \mathcal{O}(1)$$

- The stress-energy tensor reduces to $T^{\mu\nu} = T^{\mu\nu}_{\text{DE}} + T^{\mu\nu}_{\text{DDM}}$ where the DDM takes the form of a perfect fluid with zero pressure

$$T^{\mu\nu}_{\text{DDM}} = \varepsilon\, u^\mu u^\nu + \mathcal{O}(2)$$

where $\varepsilon = \rho - \nabla_\mu P_\perp^\mu$ is a dipolar energy density

The dipolar fluid is undistinguishable from standard Λ-CDM at the level of first-order cosmological perturbations

Some drawbacks of this model

- The Poisson equation in the weak-field limit is

$$\nabla \cdot \Big[\boldsymbol{g} - 4\pi\, G P_\perp \Big] = -4\pi\, G \big(\rho_\mathsf{b} + \rho \big)$$

The MOND equation follows from an hypothesis of weak clustering of DDM

The DDM does not cluster much in galaxies compared to the baryons, and stays essentially at rest with respect to some mean cosmological background

$$\rho \approx \overline{\rho} \ll \rho_\mathsf{b} \quad \text{and} \quad \boldsymbol{v} \simeq \boldsymbol{0}$$

- The equation of evolution of the dipole moment vector $\xi^\mu_\perp$ involves an instability (although with a very long time scale)
- The model is phenomenological and not related to any microscopic description of the dipole moment

Microscopic description of DDM?

- The DM medium by individual dipole moments $\boldsymbol{p}$ and a polarization field $\boldsymbol{P}$

$$\boldsymbol{P} = n\,\boldsymbol{p} \qquad \text{with} \qquad \boldsymbol{p} = m\,\boldsymbol{\xi}$$

- The polarization is induced by the gravitational field of ordinary masses

$$\boldsymbol{P} = -\frac{\chi}{4\pi\,G}\,\boldsymbol{g} \qquad \rho_{\mathrm{DM}} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}$$

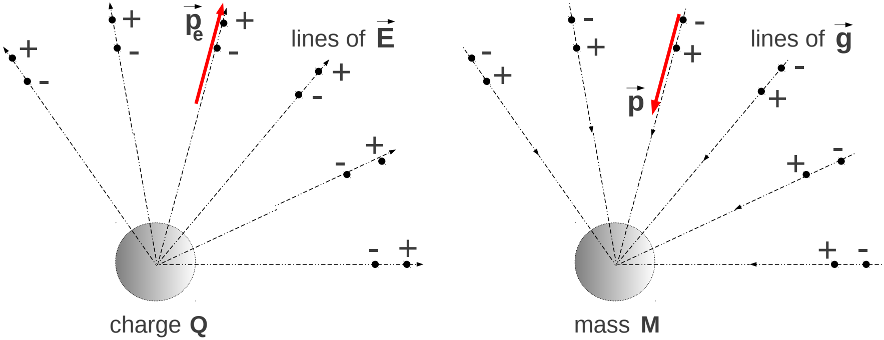
The dipole moments should be made by particles with positive and negative gravitational masses $(m_{\mathrm{i}}, m_{\mathrm{g}}) = (m, \pm m)$

- Because like masses attract and unlike ones repel we have anti-screening of ordinary masses by polarization masses

$$\boxed{\chi < 0}$$

which is in agreement with DM and MOND

Anti-screening by polarization masses



Screening by polarization charges

$$\boxed{\chi_e > 0}$$

Anti-screening by polarization masses

$$\boxed{\chi < 0}$$

Need of a non-gravitational internal force

- The constituents of the dipole will repel each other so we need a non-gravitational force to stabilize the dipolar medium

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{\nabla}(U+\phi) \qquad \frac{\mathrm{d}\underline{\boldsymbol{v}}}{\mathrm{d}t} = -\boldsymbol{\nabla}(U+\phi)$$

- The internal force is generated by the gravitational charge *i.e.* the mass

$$\Delta\phi = -\frac{4\pi G}{\chi}(\rho - \underline{\rho})$$

- The DM medium appears as a polarizable plasma of particles $(m, \pm m)$ oscillating at the natural plasma frequency

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2\boldsymbol{\xi} = 2\boldsymbol{g} \qquad \text{with} \qquad \omega = \sqrt{-\frac{8\pi\, G\, \rho_0}{\chi}}$$

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

1. To describe relativistically some microscopic DM particles with positive or negative gravitational masses one needs two metrics
   - $g_{\mu\nu}$ obeyed by ordinary particles (including baryons)
   - $f_{\mu\nu}$ obeyed by "dark" particles
2. In addition the DM particles forming the dipole moment should interact via a non-gravitational force field, *e.g.* a (spin-1) "graviphoton" vector field $\mathcal{A}_\mu$ with field strength $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$
3. One needs to introduce into the action the kinetic terms for all these fields, and to define the interaction between the two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

1. The action of the model involves three sectors

$$S = \int \mathrm{d}^4x \Big\{ \overbrace{\sqrt{-g}\left(\frac{R_g - 2\lambda_g}{32\pi} - \rho_{\text{bar}} - \rho\right)}^{\text{ordinary sector}} + \overbrace{\sqrt{-f}\left(\frac{R_f - 2\lambda_f}{32\pi} - \underline{\rho}\right)}^{\text{dark sector}}$$
$$\underbrace{+ \sqrt{-\mathcal{G}_{\text{eff}}}\left[\frac{\mathcal{R}_{\text{eff}} - 2\lambda_{\text{eff}}}{16\pi\varepsilon} + (\mathcal{J}^\mu_g - \mathcal{J}^\mu_f)\mathcal{A}_\mu + \frac{a_0^2}{8\pi}\mathcal{W}(\mathcal{X})\right]}_{\text{interaction sector}} \Big\}$$

2. The two metrics interact *via* the auxiliary metric

$$\boxed{\mathcal{G}^{\text{eff}}_{\mu\nu} = g_{\mu\rho}X^\rho_\nu = f_{\mu\rho}Y^\rho_\nu}$$

where the square-root matrices are $X = \sqrt{g^{-1}f}$ and $Y = \sqrt{f^{-1}g}$

3. The physics of the model will be obtained when the coupling constant ε is

$$\boxed{\varepsilon \ll 1 \quad \text{i.e.} \quad \varepsilon \ll \frac{G}{c^3}}$$

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

1. The gauge vector field $\mathcal{A}_\mu$ is generated by the DM mass currents

$$J_g^\mu = \rho_g u_g^\mu \qquad J_f^\mu = \rho_f u_f^\mu$$

2. It obeys a non-standard kinetic term $\mathcal{W}(\mathcal{X})$ where

$$\mathcal{X} = -\frac{\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}}{2a_0^2}$$

3. The function $\mathcal{W}$ is phenomenologically adjusted so as to recover
   - MOND in the weak-acceleration regime $\ll a_0$
   - the 1PN limit of GR in the strong-acceleration regime $\gg a_0$

$$\mathcal{W}(\mathcal{X}) = \begin{cases} \mathcal{X} - \frac{2}{3}\mathcal{X}^{3/2} + \mathcal{O}\left(\mathcal{X}^2\right) & \text{when } \mathcal{X} \to 0 \\ A + \frac{B}{\mathcal{X}^b} + o\left(\frac{1}{\mathcal{X}^b}\right) & \text{when } \mathcal{X} \to \infty \end{cases}$$

Plasma-like solution for the DDM medium

- Field equation for the graviphoton

$$\nabla^{\text{eff}}_{\nu}\Big[\mathcal{W}'\,\mathcal{F}^{\mu\nu}\Big] = 4\pi\left(\mathcal{J}^{\mu}_{g} - \mathcal{J}^{\mu}_{f}\right)$$

- The two DM fluids differ by small displacements y^{μ}_{g} and y^{μ}_{f} from a common equilibrium configuration

$$\begin{aligned}\mathcal{J}^{\mu}_{g} &= \mathcal{J}^{\mu}_{0} + \nabla^{\text{eff}}_{\nu}\left(\mathcal{J}^{\nu}_{0}\,y^{\mu}_{g} - \mathcal{J}^{\mu}_{0}\,y^{\nu}_{g}\right) + \mathcal{O}\left(2\right)\,,\\ \mathcal{J}^{\mu}_{f} &= \mathcal{J}^{\mu}_{0} + \nabla^{\text{eff}}_{\nu}\left(\mathcal{J}^{\nu}_{0}\,y^{\mu}_{f} - \mathcal{J}^{\mu}_{0}\,y^{\nu}_{f}\right) + \mathcal{O}\left(2\right)\end{aligned}$$

- The plasma-like solution for the internal field is

$$\boxed{\mathcal{W}'\,\mathcal{F}^{\mu\nu} = -4\pi\left(\mathcal{J}^{\mu}_{0}\,\xi^{\nu}_{\perp} - \mathcal{J}^{\nu}_{0}\,\xi^{\mu}_{\perp}\right) + \mathcal{O}\left(2\right)}$$

where $\xi^{\mu}_{\perp} = \perp^{\mu}_{\nu}\left(y^{\nu}_{g} - y^{\nu}_{f}\right)$ is the relative displacement vector or dipole vector

Non-relativistic limit of the model

1. The fluids of DM particles slightly differ from an equilibrium configuration

$$\rho_g = \rho_0 - \frac{1}{2}\,\boldsymbol{\nabla}\cdot\boldsymbol{P} \qquad \rho_f = \rho_0 + \frac{1}{2}\,\boldsymbol{\nabla}\cdot\boldsymbol{P}$$

   where the polarization $\boldsymbol{P} = \rho_0\,\boldsymbol{\xi}$ is proportional to the internal force $\boldsymbol{\nabla}\phi$

2. The two Newtonian potentials U_g and U_f obey when $\varepsilon \ll 1$

$$\boxed{U_g + U_f = 0}$$

3. The remaining Poisson equation in the ordinary sector reduces to

$$\Delta U_g = -4\pi\Big[\rho_{\text{bar}} + \underbrace{\rho_g - \rho_f}_{\text{dark matter}}\Big] \quad \text{with} \quad \boxed{\rho_{\text{DDM}} = \rho_g - \rho_f = -\boldsymbol{\nabla}\cdot\boldsymbol{P}}$$

4. In the limit $\varepsilon \ll 1$ there is a mechanism of gravitational polarization and the MOND equation is recovered in all dynamical situations

Post-Newtonian limit in the Solar System

1. Parametrize the two metrics at 1PN order by standard 1PN potentials

$$g_{\mu\nu}^{\text{1PN}}\left[V_g, V_g^i\right] \quad \text{and} \quad f_{\mu\nu}^{\text{1PN}}\left[V_f, V_f^i\right]$$

2. Solve the algebraic equation defining the effective metric to obtain

$$\left(\mathcal{G}_{\mu\nu}^{\text{eff}}\right)^{\text{1PN}}\left[\frac{V_g + V_f}{2}, \frac{V_g^i + V_f^i}{2}\right]$$

3. When $\varepsilon \ll 1$ the potentials V_g and V_f^i in the ordinary sector obey the same standard 1PN equations as in GR

The model has the same post-Newtonian limit as general relativity and is thus viable in the Solar System (in particular $\beta^{\text{PPN}} = \gamma^{\text{PPN}} = 1$)

Cosmological perturbations

1. Start from isotropic and homogeneous background solutions

$$\left.\begin{array}{ll} g_{\mu\nu}^{\text{FLRW}} & \text{scale factor } a_g \\ f_{\mu\nu}^{\text{FLRW}} & \text{scale factor } a_f \end{array}\right\} \Longrightarrow \left(\mathcal{G}_{\mu\nu}^{\text{eff}}\right)^{\text{FLRW}} \quad \text{scale factor } \sqrt{a_g a_f}$$

2. Adjust a_g and a_f to the different matter contents in the two backgrounds and relate the cosmological constants in the action to the observed cosmological constant Λ in the ordinary sector
3. Compute the first-order perturbations using the standard SVT formalism and define effective gauge invariant DM variables in first-order perturbations as seen in the ordinary sector

The model is undistinguishable from standard Λ-CDM at the level of first-order cosmological perturbations

Ghosts in the gravitational sector [Blanchet & Heisenberg 2015]

1. The presence of the square root of the determinant $\propto \sqrt{-\mathcal{G}_{\text{eff}}}$ in the action corresponds to ghostly potential interactions. The ghost is a very light degree of freedom, at the scale

$$m^2 M_{\text{Pl}}^2 \sqrt{-\mathcal{G}_{\text{eff}}} \sim \frac{m^2 M_{\text{Pl}}^2 (\Box\pi)^2}{\Lambda_3^6} = \frac{(\Box\pi)^2}{m^2}$$

and the theory cannot be used as an effective field theory

2. Another source of ghostly interactions is originated in the presence of three kinetic terms

$$\sqrt{-g}R_g \qquad \sqrt{-f}R_f \qquad \sqrt{-\mathcal{G}_{\text{eff}}}\mathcal{R}_{\text{eff}}$$

which has been checked by studying the model in the minisuperspace, where the Hamiltonian is highly non-linear in the lapses N_g and N_f, signalling the presence of the Boulware-Deser ghost

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DDM via massive bigravity theory [Blanchet & Heisenberg 2015ab]

1. The gravitational sector of the model is based on massive bigravity theory [de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012]

$$S = \int \mathrm{d}^4x \Big\{ \sqrt{-g}\left(\frac{M_g^2}{2}R_g - \rho_{\text{bar}} - \rho_g\right) + \sqrt{-f}\left(\frac{M_f^2}{2}R_f - \rho_f\right) + \sqrt{-g_{\text{eff}}}\left[\frac{m^2}{4\pi} + \mathcal{A}_\mu\left(j_g^\mu - \frac{\alpha}{\beta}j_f^\mu\right) + \frac{a_0^2}{8\pi}\mathcal{W}(X)\right]\Big\}$$

2. The ghost-free potential interactions take the particular form of the square root of the determinant of the effective metric [de Rham, Heisenberg & Ribeiro 2014]

$$\boxed{g_{\mu\nu}^{\text{eff}} = \alpha^2 g_{\mu\nu} + 2\alpha\beta\,\mathcal{G}_{\mu\nu}^{\text{eff}} + \beta^2 f_{\mu\nu}}$$

3. The matter sector is the same as in the previous model

General structure of the model [Blanchet & Heisenberg 2015ab]

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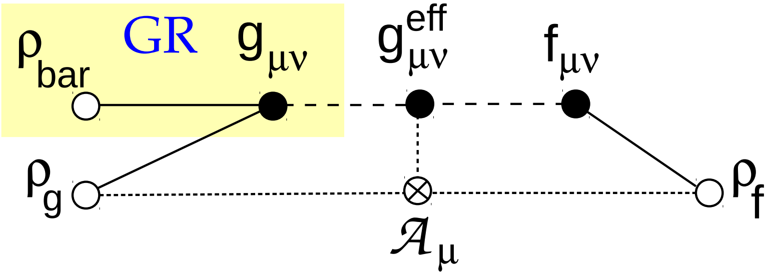

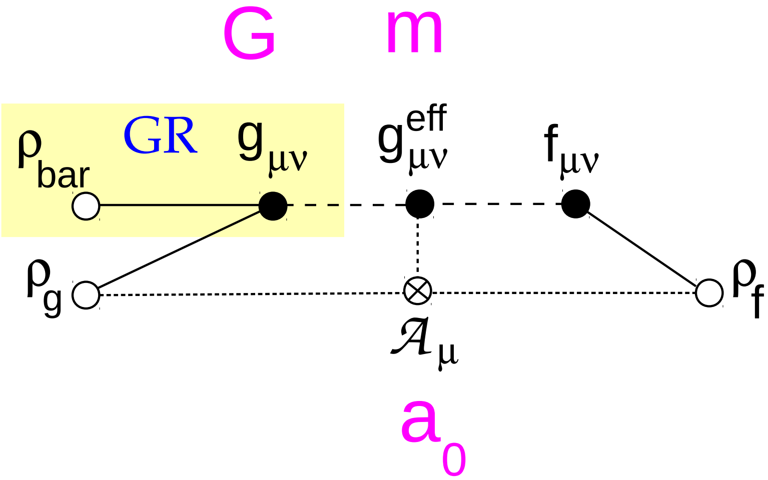
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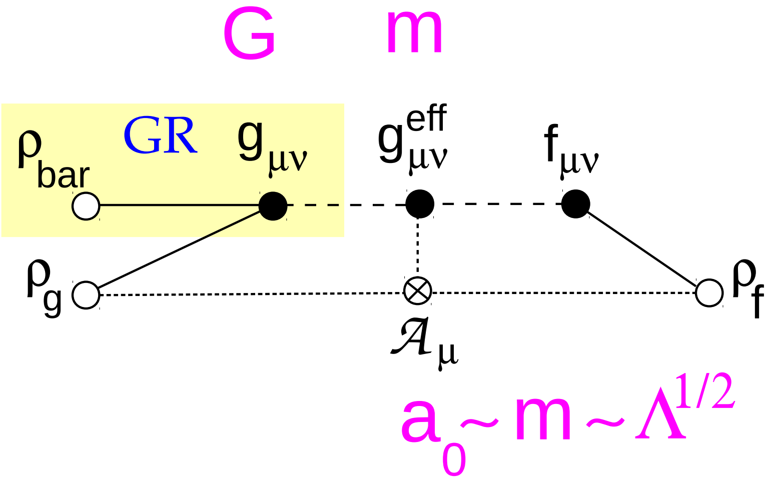
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ρ_f
$a_0 \sim m \sim \Lambda^{1/2}$

Gravitational polarization & MOND

1. Equations of motion of DM particles in the non-relativistic limit $c \to \infty$

$$\frac{\mathrm{d}\boldsymbol{v}_g}{\mathrm{d}t} = \boldsymbol{\nabla}\left(U_g + \phi\right) \qquad \frac{\mathrm{d}\boldsymbol{v}_f}{\mathrm{d}t} = \boldsymbol{\nabla}\left(U_f - \frac{\alpha}{\beta}\phi\right)$$

2. With massive bigravity the two g and f sectors are linked together by a constraint equation coming from the Bianchi identities

$$\boxed{\boldsymbol{\nabla}\left(\alpha U_g + \beta U_f\right) = 0}$$

showing that α/β is the ratio between gravitational and inertial masses of f particles with respect to g metric

3. The DM medium is at equilibrium when the Coulomb force annihilates the gravitational force, $\boldsymbol{\nabla} U_g + \boldsymbol{\nabla}\phi = 0$, at which point the polarization is aligned with the gravitational field

$$\boxed{\boldsymbol{P} = \frac{1}{4\pi}\mathcal{W}'\,\boldsymbol{\nabla} U_g}$$

Gravitational polarization & MOND

1. From the massless combination of the two metrics combined with the Bianchi identity we get a Poisson equation for the ordinary Newtonian potential U_g

$$\Delta U_g = -4\pi\Big(\rho_{\text{bar}} + \underbrace{\rho_g - \frac{\alpha}{\beta}\rho_f}_{\text{DDM}}\Big)$$

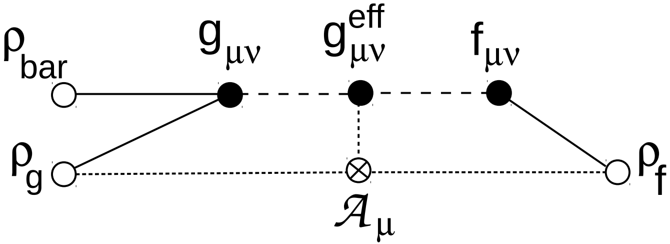
2. With the plasma-like solution for the internal force and the mechanism of gravitational polarization this yields the MOND equation

$$\boxed{\nabla\cdot\Big[\underbrace{\big(1-\mathcal{W}'\big)}_{\text{MOND function}}\nabla U_g\Big] = -4\pi\rho_{\text{bar}}}$$

3. Finally the DM medium undergoes stable plasma-like oscillations in linear perturbations around the equilibrium

Ghost in the DM sector

- By construction the model is safe in the gravitational sector



- The matter fields $\rho_{\rm bar}$, ρ_g, ρ_f and internal vector field $\mathcal{A}_\mu$ are directly coupled to one and only one metric in agreement with [de Rham, Heisenberg & Ribeiro 2014]
- However the indirect coupling of the DM fields ρ_g, ρ_f to the effective metric $g^{\rm eff}_{\mu\nu}$ through their interaction with $\mathcal{A}_\mu$ generates a ghost in the decoupling limit in the DM sector

Ghost in the DM sector

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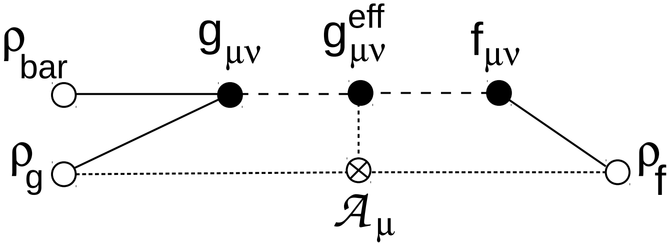

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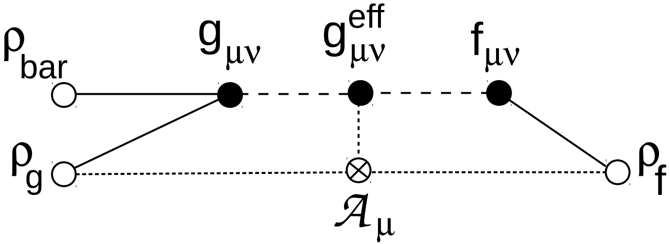
- However the indirect coupling of the DM fields ρ_g, ρ_f to the effective metric $g^{\text{eff}}_{\mu\nu}$ through their interaction with $\mathcal{A}_\mu$ generates a ghost in the decoupling limit in the DM sector

Ghost in the DM sector

- By construction the model is safe in the gravitational sector



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Summary

1. The aim is to reproduce within a single relativistic framework
   - The concordance cosmological model Λ-CDM and its tremendous successes at cosmological scales and notably the fit of the CMB
   - The phenomenology of MOND which is a basic set of phenomena relevant to galaxy dynamics and DM distribution at galactic scales
2. In the present approach
   - The phenomenology of MOND is explained by a physical mechanism of gravitational polarization
   - The DM appears to be a diffuse medium polarizable in the field of ordinary matter and undergoing stable plasma-like oscillations
3. The most promising and elegant route in this approach is within the framework of massive bigravity theories

Future works

1. The cosmology of the latest model based on massive bigravity should be investigated and the agreement with Λ-CDM checked
2. The strong field regime in the Solar System and the PPN parameters are still to be computed (or should a Vainshtein mechanism be invoked?)
3. The status of the remaining ghost in the DM sector is unclear
   - since it appears only at second-order perturbation is it physically harmfull?
   - could it be eliminated by order reduction of the DM equations of motion?
   - or does it simply kills the model?
4. The internal vector field could be replaced by a non-Abelian Yang-Mills vector field based on SU(2) or SU(3) to avoid the need of introducing an arbitrary function in the action