TOWARDS A HOLOGRAPHIC BOSE-HUBBARD MODEL

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Based on MF-Harrison-Karch-Meyer-Paquette, JHEP04(2015)068 and MF-Meyer-Tezuka, unpublished work

Motivation

- Broken translation invariance in the real materials
  - Translation invariance broken on the lattice

- Introduction of Holographic lattices
  - Periodic functions of the chemical potential

Horowitz, Santos, D. Tong "12

Horowitz, Santos, "13

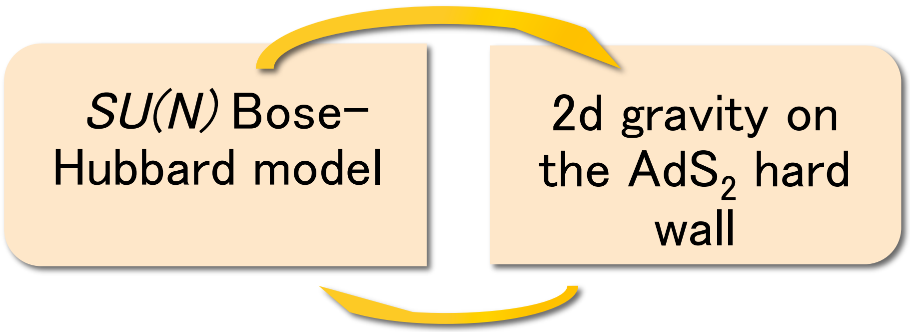
- Holographic lattices of impurities in the probe limit
  - Observing holographic large N dimerization transition

Kachru-Karch-Yaida "09, "10

Motivation

- Bose-Hubbard model as the effective theory on an optical lattice including the hopping term
  - The extension to the *SU(N)* Bose-Hubbard model

    Conjecture of *MF-Harrison-Karch-Meyer-Paquette 2014*



  - To compute the VEV of the hopping term in both sides of the duality concretely and compare them

Boson-Hubbard model

The effective theory of the cold bosons on an optical lattice: Including the hopping term and short-range repulsive interactions U on each a site

$$H = -w\sum_{<ij>}(b_i^\dagger b_j + b_j^\dagger b_i) - \mu\sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1), \quad n_j = b_j^\dagger b_j$$

w : the hopping integral

U(1) symmetry: $b_i \to e^{i\theta} b_i$

Only two phases at *T=0:* *Fisher-Weichman-Grinstein-Fisher, ``89*

U/w>>1: Mott insulator phase (localized bosons)

U/w<<1: Superfluid phase (delocalization)

U(1) symmetry is broken in SF

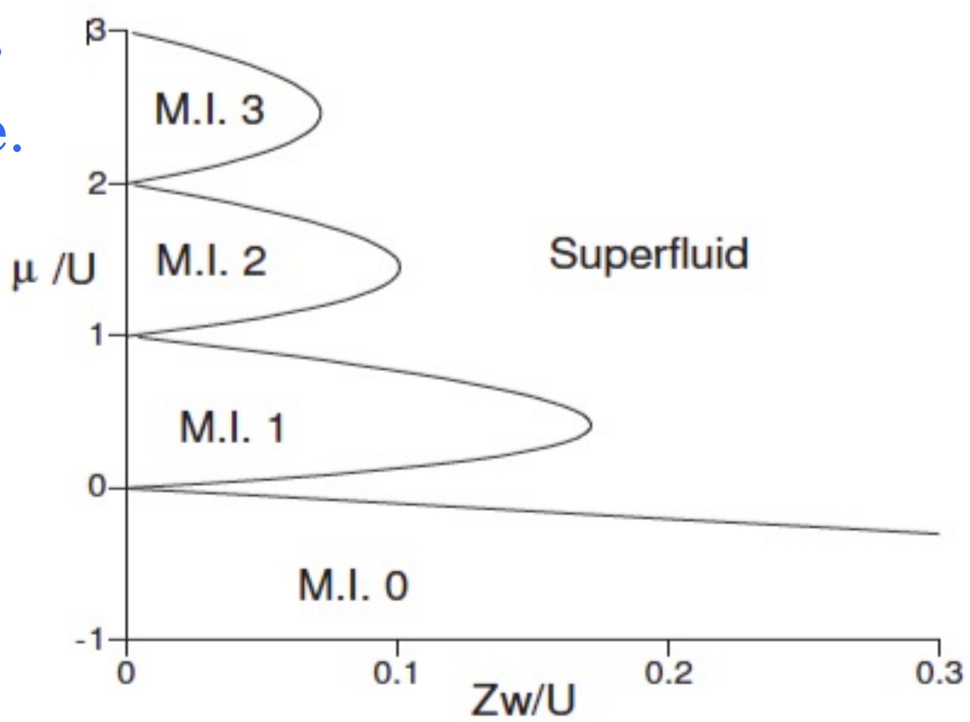
Lobe-shaped structure

Phase structure of the ground state like the lobe-shape

M.I. : the Mott insulator of equally occupied state.

The amplitude of the lobe decreasing as $1/\rho$

(Picture taken from *Sachdev, Quantum Phase transition*)



Derivation of the phase structure (SF/Mott Transition)

How to derive the phase structure in the boson-Hubbard model?

The mean-field approach introducing ψ

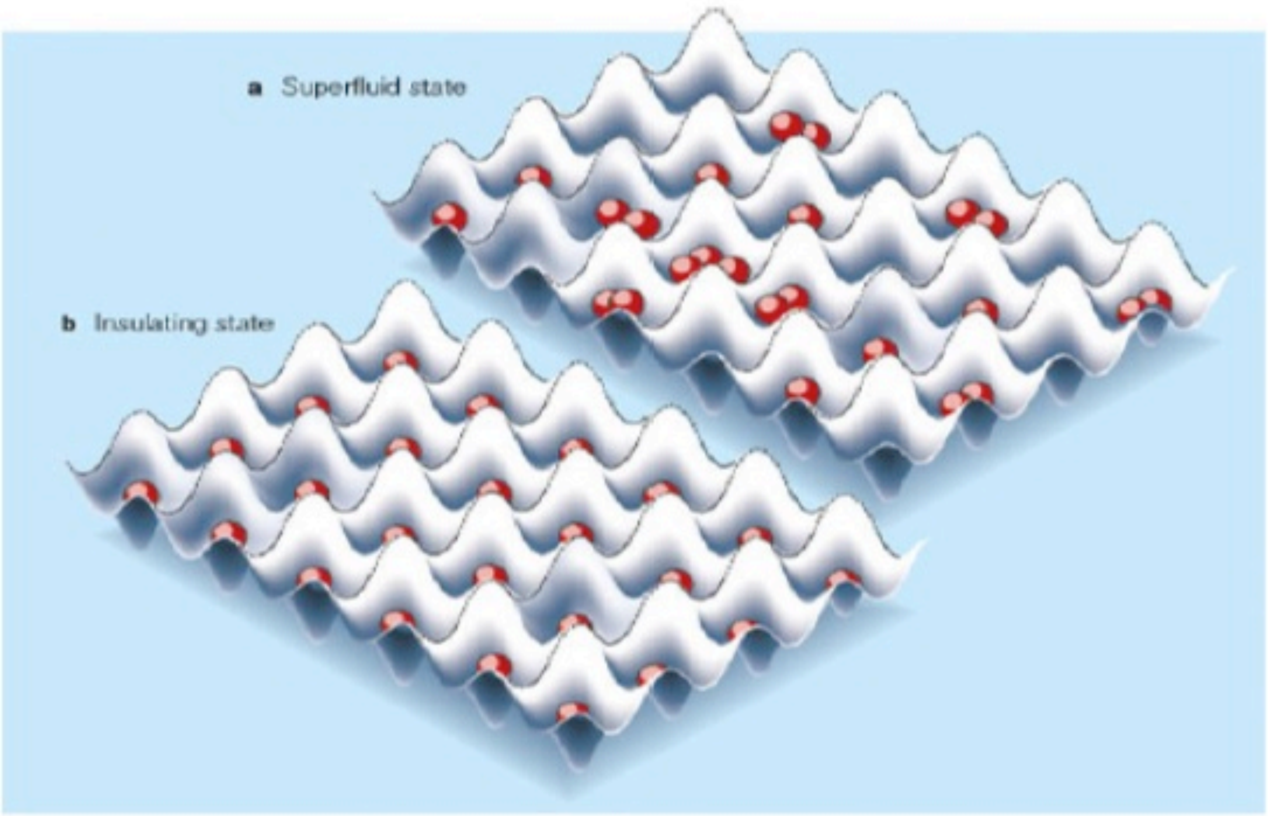
$$S_\infty(\psi) = \beta N \left(\frac{1}{2} r(\mu_b, t_{hop}, T)|\psi|^2 + u(\mu_b, T)|\psi|^4 + O(|\psi|^6) \right)$$

In the Mott insulator phase, $\psi=0$

$w=0$ state: an exact eigenstate $|n_0\rangle$ of the total number operator

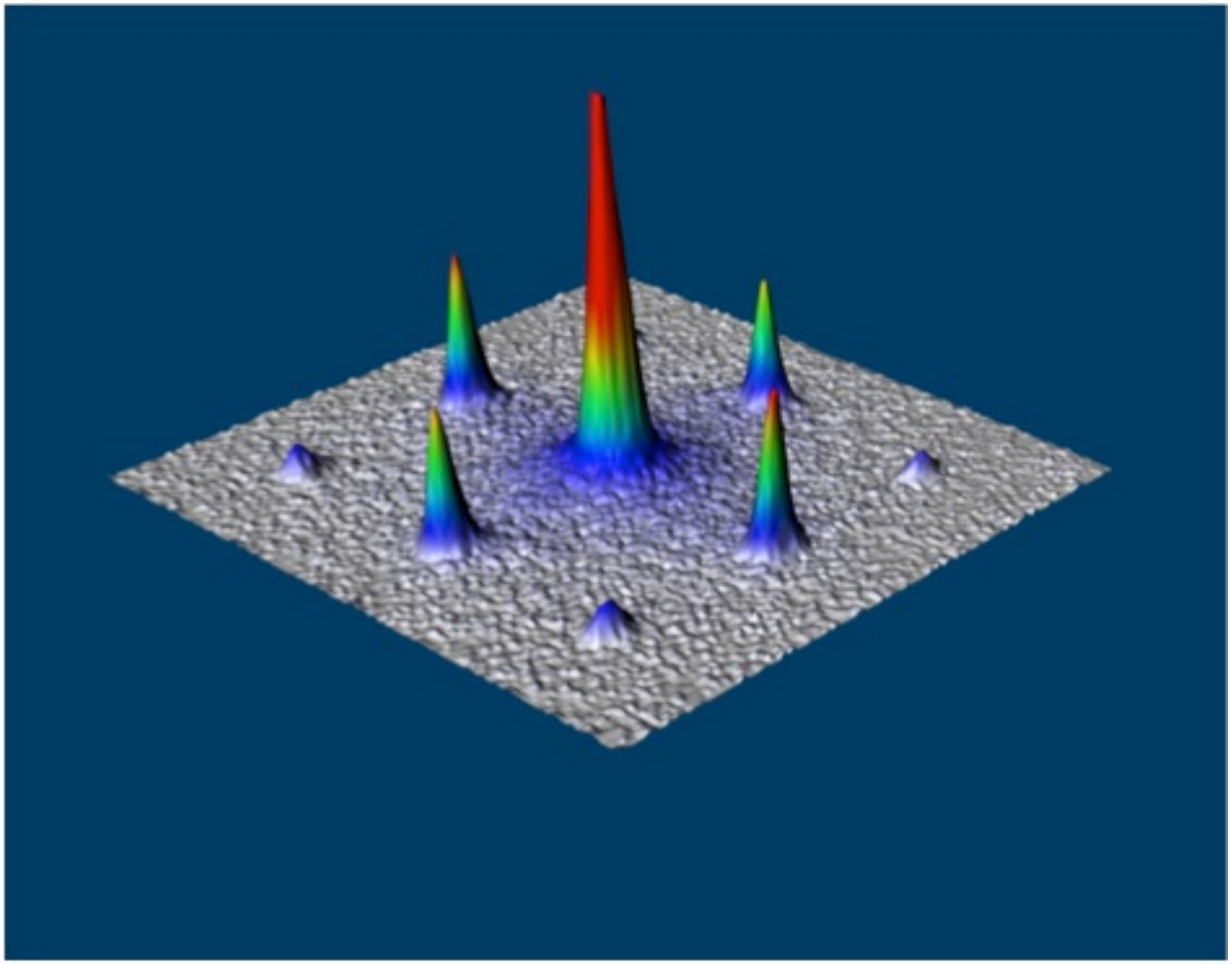
The ground state at non-zero w: not a simple state like $|n_0\rangle$

Experimental result: ^{87}Rb cold atoms:
By changing the bottom of the potential, bosons moving around the sites (delocalization).



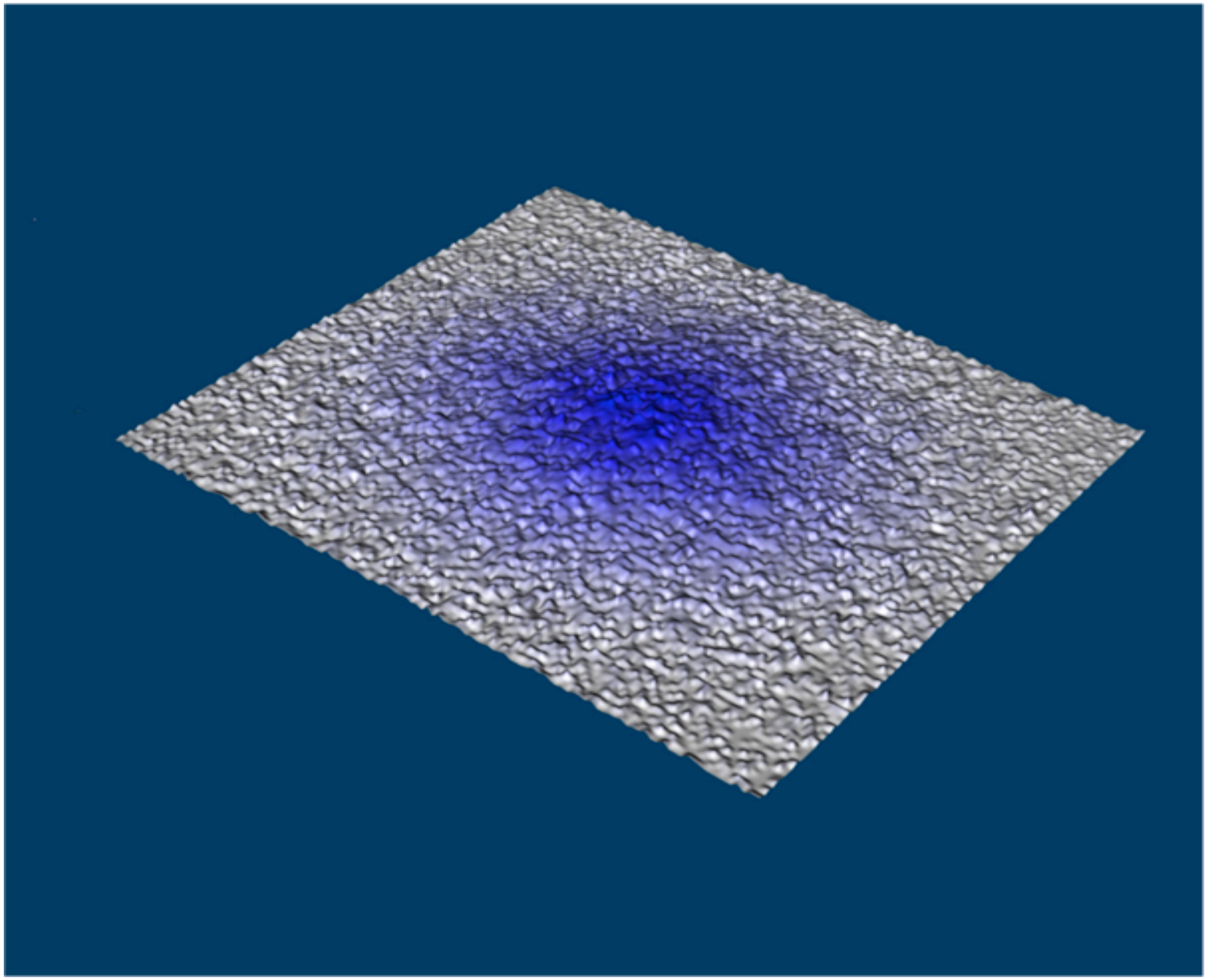
From *H.T.C. Stoof, Nature 415, 25 (2002)*

Velocity distribution of cold atoms in Superfluid



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39 (2002).

Velocity distribution of cold atoms in Mott insulator



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39 (2002).

A HOLOGRAPHIC CONSTRUCTION OF LARGE N BOSON-HUBBARD MODEL

To realize the lobe-shaped phase structure of the boson-Hubbard model

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However, large N limit necessary to cause the phase transition in the finite volume system.

The AdS/CFT correspondence (Summary)

The large N bose hubbard model side	Gravity side
Occupation number per a site $n_i = b_i^{a\dagger} b_{ia}$	$U(1)^n$ gauge fields A_i
Chemical potential μ_{i}	
Hopping parameter t_{hop}	Bi-fundamental scalar $\phi_{i,j}$
Bi-local condensate ($i \neq j$) $b_i^{a\dagger} b_{ja}$	
Coulomb repulsive parameter U	IR cutoff u_{h}
Spin indices: $a=1,\ldots,N$	Focusing on two-site model later ($i,j=1,2$).

Holographic construction 2

An IR potential in the lagrangian

- Affecting the phase structure of the holographic model

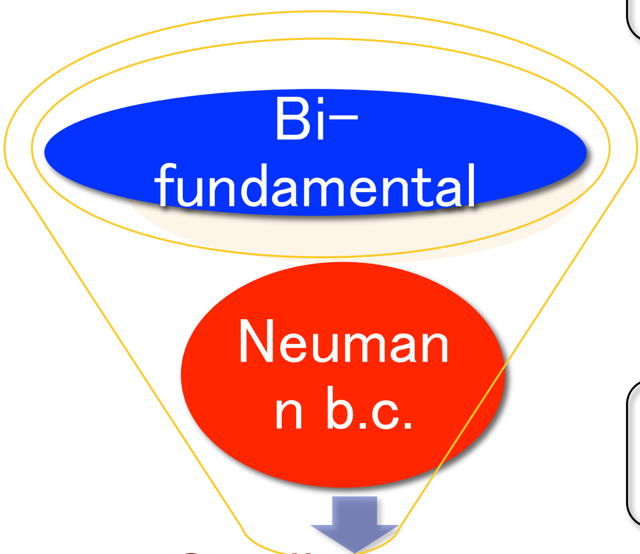
AdS_2 hard wall with cutoff $u_h/t_{hop} \gg 1$

- Appearance of instable modes above the energy scale greater than u_h
  Maldacena-Michelson-Strominger, 1998

Dirac quantization of charges (occupation number)

- Quantization of the coefficient of F1 and D-brane interactions in terms of the string theory

Homogeneous phase or Non-homogeneous phase



Small vacuum expectation value of hopping term

Homogeneous phase : $A_t^{(1)} = A_t^{(2)}$

IR b.c. : Dirichlet type

Corresponding Mott insulator phase

Localized bosons

Non-homogeneous phase : $A_t^{(1)} \neq A_t^{(2)}$

IR b.c. : Neumann type

SF (Large kinetic energy, delocalization)

c.f. Axial vector of hard/soft wall AdS/QCD

Lobe-shaped phase structure of the holographic model

Realizing the lobe-shaped structure in the large *N* limit

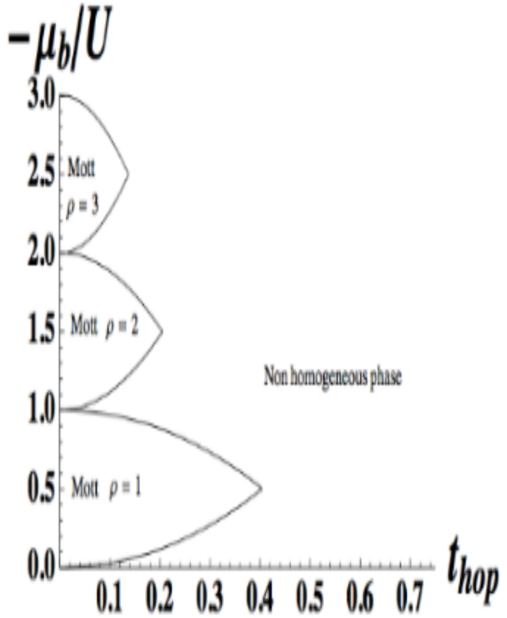
$t_{hop}/U >> 1$ non-homogenous phase favored

Generated VEV of bi-local

$t_{hop}/U << 1$ Mott phase favored

The amplitude of the lobe $t_{hop} \approx \frac{1}{\rho}$

Cusp: particle-hole symmetric point



Lobe-shaped phase structure of the holographic model

- Except for μ_{b} axis, Symmetry breaking U(1)xU(1) -> U(1)
  - Spontaneous symmetry breaking at μ_{b} axis
  - Appearance of a Goldstone mode at μ_{b} axis

- Large N first order phase transition except at μ_{b} axis
- 2nd order phase transition at μ_{b} axis

- Order parameter of the phase transition

$$\delta n = n_1 - n_2 \qquad dF / dt_{hop} \approx \left\langle b_i^{\dagger} b_j \right\rangle$$

Perturbative Spectrum at small hopping (Dirichlet bc at hard wall):

Always gapped in Mott insulator phase

$$\Delta = \pi U n \; (n\text{: Integer, } n>0)$$

Existence of the zero mode in non-homogeneous phase

Background:

$$\phi^{cl} \approx t_{hop} u^{-\frac{2}{5}} \quad A_A^{cl} \approx \delta\rho u \approx u$$

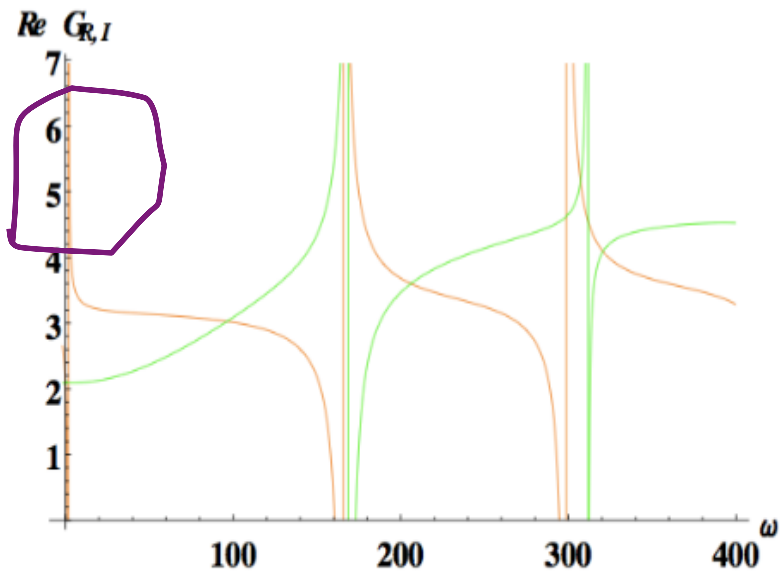
Perturbations

$$\delta\phi_R \approx \phi_R^{as} u^{-\frac{2}{5}} + \phi_R^{(2)as} u^{-\frac{3}{5}} + \dots$$

$$\delta\phi_R \approx \phi_R^{as} u^{-\frac{2}{5}} + \phi_R^{(2)as} u^{-\frac{3}{5}} + \dots$$

$$\delta A_A \approx A^{as} u + \dots$$

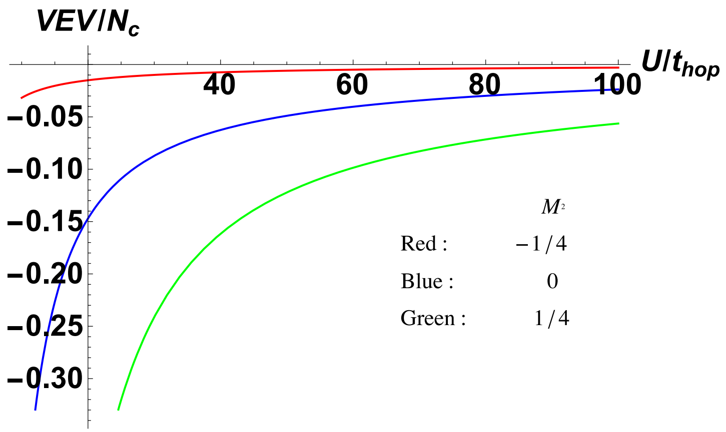
Green's function: $G_{R,I} = -\frac{\phi_{R,I}^{(2)as}}{\phi_{R,I}^{as}}.$

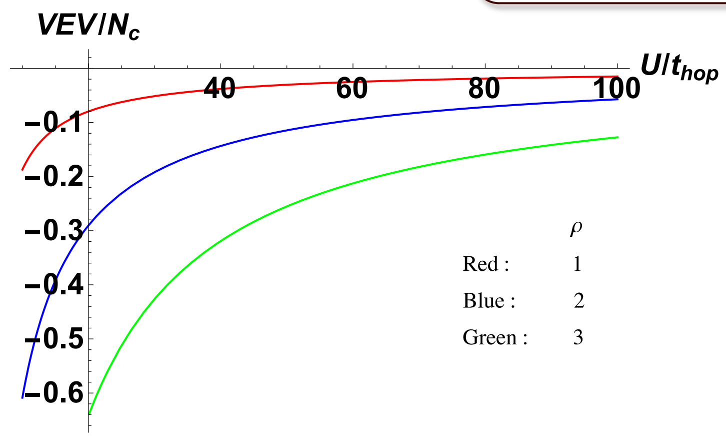


The effective hopping parameter in the gravity dual (homogeneous phase)

Effective hopping in the Mott insulator phase

$$\left\langle b_i^{a\dagger} b_{ja} \right\rangle \equiv dF / dt_{hop}$$





(N_c=170, Left : fixed occupation number Right : fixed bulk mass)

Bi-local's VEV approaching to 0 in large U limit (Mott insulator)

Bulk mass M contributing to dependence on N_c

The effective hopping parameter in the gravity dual (non-homogeneous phase)

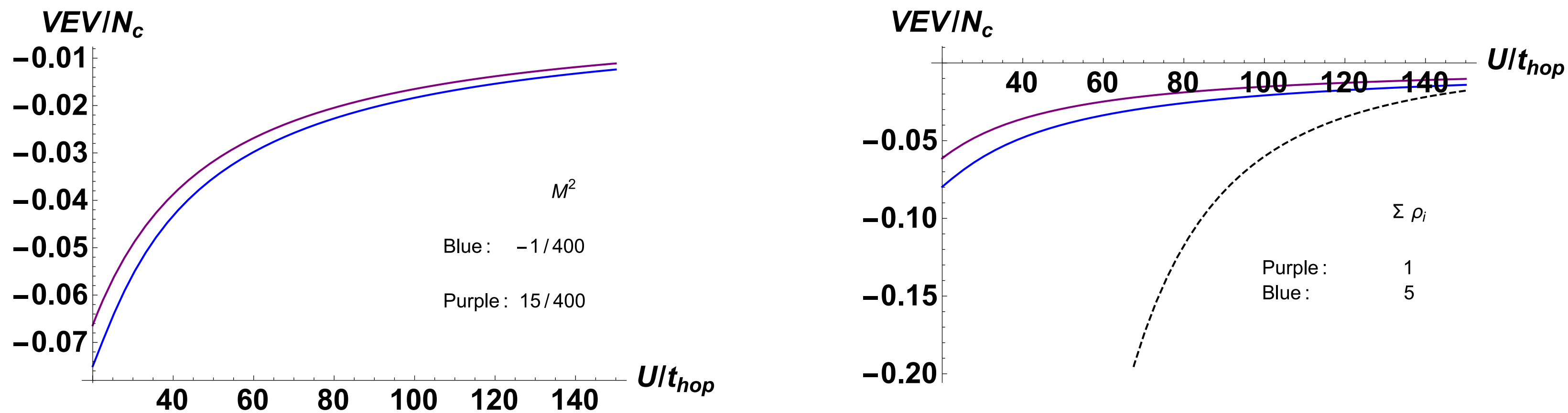


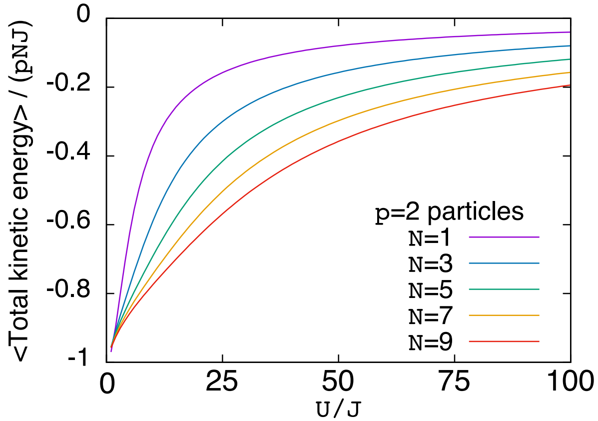
Figure: a Finite gauge coupling $q = \frac{\sqrt{6}}{5}$

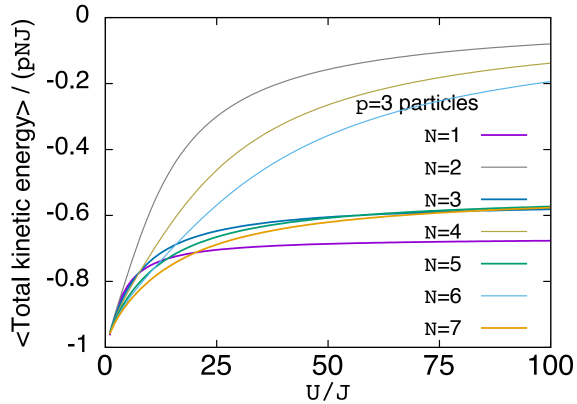
- Numerical approach failed for large mass M region above the BF bound $M^2_{BF} = -\frac{1}{4}$
- Similar results in the weak coupling $q << 1$

Large N Bose-Hubbard model side

Effective hopping in the large N ***Bose-Hubbard model***

Numerical result of VEV of hopping term fixing the particle number





Left（*p=2*): Mott insulator phase in even # of particle system at large *U*（VEV～*1/U*)

Right (*p=3*): VEV non-zero at large *U* in the system of odd # of particles and odd # of components ⇔ b.c. φ=const in 2d gravity?

Discussion

- Realizing the lobe shape of the phase structure of the boson-Hubbard model

- The Mott/non-homogeneous phase transition as 1st order by using dF/dt_{hop} as the order parameter
  - Comparision of dF/dt_{hop} : $SU(N_c)$ Bose-Hubbard model fitting with the gravity dual when $N_c \sim 170$

- A top down model: a D3/D5/D7 system where N D3 are replaced by the AdS_5 soliton
  - Non-Abelian D5s wrapping $AdS_2 x S^4$ Dual to the effective theory on the lattice

THANK YOU!