Black Hole Entropy in Loop Quantum Gravity

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Outline

1. **Introduction Isolated Horizon**

2. **Chen-Simons Theory Description of Isolated Horizon Entropy in LQG**

3. **BF Theory Description of Isolated Horizon Entropy** [arXiv:1401.2967, 1409.0985, 1505.03647]

4. **Concluding Remarks**

Thermodynamics of BH

Law	Thermodynamics	Black holes
Zeroth	T constant throughout body in thermal equilibrium	κ constant over horizon of a stationary black hole
First	$dU = TdS +$ work terms	$dM = \frac{\kappa}{8\pi}da + \Omega_H dJ + \Phi_H dQ$
Second	$\Delta S \geq 0$ in any process	$\Delta a \geq 0$ in any process
Third	Impossible to achieve $T = 0$ by a physical process	Impossible to achieve $\kappa = 0$ by a physical process

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- The three pillars of fundamental physics is brought together by

$$S_{BH} = \frac{k_B c^3 A r_{BH}}{4G\hbar}.$$

Limitation of the global notions in GR

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- The event horizon definition of BH requires knowledge of the entire space-time all the way to future null infinity.
- The use of stationary space-times to derive black hole thermodynamics is not ideal.
- The global nature of event horizon makes it difficult to use in quantum theory. In order for a definition of the horizon of black hole to make sense, one needs to be able to formulate it in terms of phase space functions which can be quantized.
- The global notions of ADM energy and ADM angular momentum are of limited use, because they do not distinguish the mass of black holes from the energy of surrounding gravitational radiation.

Quasi-local notion of Isolated Horizon

- The notion of isolated horizon is defined quasi-locally as a portion of the event horizon which is in equilibrium [Ashtekar, Beetle and Fairhurst, 1998].

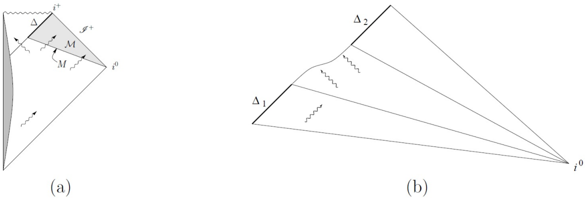

Figure 1: (a) A typical gravitational collapse. The portion Δ of the horizon at late times is isolated. The space-time $\mathcal{M}$ of interest is the triangular region bounded by Δ, $\mathscr{I}^+$ and a partial Cauchy slice M. (b) Space-time diagram of a black hole which is initially in equilibrium, absorbs a small amount of radiation, and again settles down to equilibrium. Portions Δ_1 and Δ_2 of the horizon are isolated.

Quasi-local notion of Isolated Horizon

- (Weakly) Isolated Horizon: A three-dimensional null hypersurface Δ of a space-time $(\mathcal{M}, g_{ab})$ is said to be a weakly isolated horizon if the following conditions hold:
  - (1). Δ is topologically $\mathbb{R} \times S$ with S a compact two-dimensional manifold;
  - (2). The expansion $\theta_{(l)}$ of any null normal l to Δ vanishes;
  - (3). The field equations hold at Δ, and the stress-energy tensor T_{ab} of external matter fields is such that, at Δ, $-T^a{}_b l^b$ is a future-directed and causal vector for any future-directed null normal l^a.
  - (4). An equivalence class $[l]$ of future-directed null normals is equipped with Δ, with $l' \sim l$ if $l' = cl$ ($c > 0$ a constant), such that $\mathcal{L}_l \omega_a \triangleq 0$ for all $l \in [l]$, where ω_a is related to the induced derivative operator D_a on Δ by $D_a l_b \triangleq \omega_a l_b$.

Thermodynamics of Isolated Horizon

- The definition of weakly isolated horizon implies automatically the zeroth law of IH mechanics as the surface gravity $\kappa_{(l)} \equiv \omega_a l^a$ is constant on Δ [Ashtekar, Beetle and Fairhurst, 1998].

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- Let us consider an 4-dimensional spacetime region $\mathcal{M}$ with an isolated horizon Δ as an inner boundary.
  The Hamiltonian framework for $\mathcal{M}$ provides an elegant way to define the quasi-local notions of energy E_Δ and angular momentum J_Δ associated to Δ.
- Then the first law of IH mechanics holds as [Ashtekar, Beetle and Lewandowski, 2001]

$$\delta E_\Delta = \frac{\kappa_{(l)}}{8\pi G}\delta a_\Delta + \Phi_{(l)}\delta Q_\Delta + \Omega_{(l)}\delta J_\Delta .$$

Kinematical structure of LQG

- In canonical LQG, the kinematical Hilbert space is spanned by spin network states $|\Gamma, \{j_e\}, \{i_v\} >$, where Γ denotes some graph in the spatial manifold M, each edge e of Γ is labeled by a half-integer j_e and each vertex v is labeled by an intertwinor i_v.

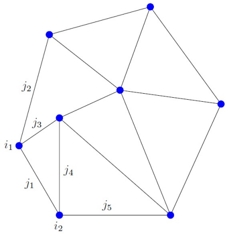

Figure: Dona and Speziale, arXiv:1007.0402.

Quantum isolated horizon

- In the case when M has a boundary H, some edges of spin networks in M may intersect H and endow it a quantum area at each intersection.

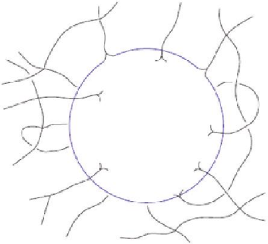

Figure: Ashtekar, Baez and Krasnov, gr-qc/0005126.

Palatini formalism

Consider the Palatini action of GR on $\mathcal{M}$:

$$S[e,A] = -\frac{1}{4\kappa}\int_{\mathcal{M}} \varepsilon_{IJKL} e^I \wedge e^J \wedge F(A)^{KL} + \frac{1}{4\kappa}\int_{\tau_\infty} \varepsilon_{IJKL} e^I \wedge e^J \wedge A^{KL}$$

- For later convenience, we define the solder form $\Sigma^{IJ} \equiv e^I \wedge e^J$.

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- For later convenience, we define the solder form $\Sigma^{IJ} \equiv e^I \wedge e^J$.
- The second-order variation of the Palatini action leads to the conservation identity of the symplectic current as

$$\frac{1}{\kappa}\Big(\int_{M_1} \delta_{[1}(*\Sigma)_{IJ} \wedge \delta_{2]} A^{IJ} - \int_{M_2} \delta_{[1}(*\Sigma)_{IJ} \wedge \delta_{2]} A^{IJ} + \int_{\Delta} \delta_{[1}(*\Sigma)_{IJ} \wedge \delta_{2]} A^{IJ}\Big) = 0,$$

where $(*\Sigma)_{KL} = \frac{1}{2}\varepsilon_{IJKL}\Sigma^{IJ}$, and M_1, M_2 are spacelike boundary of $\mathcal{M}$.

Basic variables in time gauge

- The symplectic flux across the horizon can be expressed as a sum of two terms corresponding to the 2D compact surfaces $H_1 = \Delta \cap M_1$ and $H_2 = \Delta \cap M_2$.

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- The symplectic flux across the horizon can be expressed as a sum of two terms corresponding to the 2D compact surfaces $H_1 = \Delta \cap M_1$ and $H_2 = \Delta \cap M_2$.
- Let the $so(3,1)$ connection A^{IJ} and the cotetrad e^I be in the time-gauge in which e_0^a is normal to the partial Cauchy surface M, reducing the internal local gauge group from $SO(1,3)$ to $SO(3)$.
- The pull-back of the spacetime variables to M can be written in terms of the Ashtekar-Barbero variables as

$$\mathcal{A}^i = \gamma A^{0i} - \frac{1}{2}\epsilon^i{}_{jk}A^{jk}; \qquad \Sigma^i = \epsilon^i{}_{jk}\Sigma^{jk}.$$

Symplectic structure in time gauge

For spherically symmetric IHs, the symplectic structure can be obtained on M with the inner boundary $H = M \cap \Delta$ as [Engle, Noui, Perez, Pranzetti, 2009]

$$\Omega(\delta_1, \delta_2) = \frac{1}{2\kappa\gamma} \int_M 2\delta_{[1}\Sigma^i \wedge \delta_{2]}\mathcal{A}_i - \frac{1}{\kappa} \frac{a_0}{\pi(1-\gamma^2)} \oint_H 2\delta_{[1}\mathcal{A}_i \wedge \delta_{2]}\mathcal{A}^i .$$

- The symplectic structure consists of a bulk term, the standard symplectic structure used in LQG, and a surface term, the symplectic structure of an $SU(2)$ Chern-Simons theory on H.

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- The symplectic structure consists of a bulk term, the standard symplectic structure used in LQG, and a surface term, the symplectic structure of an $SU(2)$ Chern-Simons theory on H.
- In terms of the Ashtekar-Barbero variables, the isolated horizon boundary conditions take the form

$$\Sigma^i = -\frac{a_0}{\pi(1-\gamma^2)} F^i(\mathcal{A}).$$

Calculation of the entropy for IH

- Employing the spectrum of the area operator in LQG, a detail analysis can estimates the number of Chern-Simons surface states on the punctured horizon consistent with the given area [Ashtekar, Baez, Krasnov, 2000].
- The expression of the entropy agrees with the Hawking-Bekenstein formula by choosing the Barbero-Immirzi parameter $\gamma \approx 0.274$ [Domagala, Lewandowski, 2004].
- The above isolated horizon framework was generalized to arbitrary even-dimensional spacetime [Bodendorfer, Thiemann, Thurn, 2013], where the horizon degrees of freedom are encoded in the $SO(2n)$-Chern-Simons theory.

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- The above isolated horizon framework was generalized to arbitrary even-dimensional spacetime [Bodendorfer, Thiemann, Thurn, 2013], where the horizon degrees of freedom are encoded in the $SO(2n)$-Chern-Simons theory.
- Limitation: The framework is only valid for even-dimensional spacetime, since Chern-Simons theory can only lives on odd-dimensional manifold.
  Is there any way out?

Near horizon coordinates

- In the neighborhood of Δ, we choose the Bondi-like coordinates given by (v, r, x^i), $i = 1, 2$, where the horizon is given by $r = 0$ [Lewandowski, 2000].

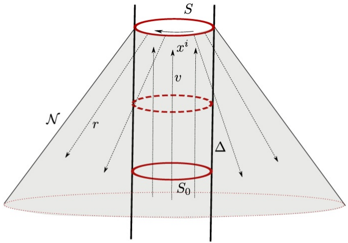

Figure 11: The near horizon coordinates. The isolated horizon is Δ and the transverse null surface is $\mathcal{N}$. The affine parameter along the outgoing null geodesics on $\mathcal{N}$ is r, and v is a coordinate along the null generators on Δ, and x^1 are coordinates on the cross-sections of Δ.

Figure: Krishnan, arXiv:1303.4635.

Gauge choice of the tetrad

- To describe the geometry near the isolated horizon Δ, it is convenient to employ the Newman-Ponrose formalism with the null tetrad $(l, n, m, \bar{m})$ adapted to Δ, such that the real vectors l and n coincide with the outgoing and ingoing future directed null vectors at Δ respectively.

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- We choose an appropriate set of co-tetrad fields which are compatible with the metric as:

$$e^0 = \sqrt{\frac{1}{2}}(\alpha n + \frac{1}{\alpha} l),\ e^1 = \sqrt{\frac{1}{2}}(\alpha n - \frac{1}{\alpha} l),$$
$$e^2 = \sqrt{\frac{1}{2}}(m + \bar{m}),\ e^3 = i\sqrt{\frac{1}{2}}(m - \bar{m}),$$

  where $\alpha(x)$ is an arbitrary function of the coordinates.
- Each choice of $\alpha(x)$ characterizes a local Lorentz frame in the plane $\mathcal{I}$ formed by $\{e^0, e^1\}$.

Variables restricted to the horizon

- Restricted to the horizon Δ, the co-tetrad fields satisfy

$$e^0 \triangleq e^1 \triangleq \sqrt{1/2}\alpha n$$

- Hence the solder fields Σ^{IJ} restricted to Δ satisfy:

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- We can also get the following properties for the connection restricted to Δ:

$$A^{0i} \triangleq A^{1i}, \forall i = 2, 3,$$
$$A^{01} \triangleq d\beta(x) + \pi m + \bar{\pi}\bar{m},$$

where $\beta(x) = \tilde{\kappa} v + \ln \alpha(x)$, the spin coefficients π and $\bar{\pi}$ are the components of $l^a \nabla_a n$ along $\bar{m}$ and m respectively.

Horizon degrees of freedom

- The horizon integral of the symplectic current can be reduced to
$$\frac{1}{\kappa}\int_{\Delta}\delta_{[1}(*\Sigma)_{IJ}\wedge\delta_{2]}A^{IJ}=\frac{2}{\kappa}\int_{\Delta}\delta_{[1}\Sigma^{23}\wedge\delta_{2]}A^{01}.$$

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- Since the property of isolated horizon ensures that the area element of the slice is unchanged for different v, we have
$$d(*\Sigma)_{01}=d\Sigma^{23}\triangleq 0.$$
Thus Σ^{23} is closed.

- So we can define an 1-form B locally on Δ such that
$$\Sigma^{23}=dB.$$

Horizon gauge freedom and symplectic structure

- Under a SO(1,1) boost on the plane spanned by $\{e^0, e^1\}$ with group element $g = exp(\zeta)$, we get

$$A'^{01} = A^{01} - d\zeta, \qquad \Sigma'_{23} = \Sigma_{23}.$$

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- Hence A^{01} is a SO(1,1) connection, and Σ^{23} is in its adjoint representation.
- Let $\tilde{A}^{01} := \pi m + \bar{\pi}\bar{m}$. Then $A^{01} = d\beta + \tilde{A}^{01}$, and it turns out

$$\int_{\Delta} \delta_{[1}\Sigma^{23} \wedge \delta_{2]}\tilde{A}^{01} = 0$$

- In terms of Ashtekar-Barbero variables, the full symplectic structure can be obtained as

$$\Omega(\delta_1, \delta_2) = \frac{1}{2\kappa\gamma}\int_M 2\delta_{[1}\Sigma^i \wedge \delta_{2]}\mathcal{A}_i + \frac{1}{\kappa}\oint_H 2\delta_{[2}B \wedge \delta_{1]}A$$

Quantum BF theory with sources

- To adapt the structure of LQG in the bulk, the boundary BF theory is intersected by the spin networks, and satisfies

$$F = dA = 0, \quad dB = \frac{\Sigma^1}{2\kappa}$$

Quantum *BF* theory with sources

- To adapt the structure of LQG in the bulk, the boundary *BF* theory is intersected by the spin networks, and satisfies

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- Let's assume that the graph Γ underling a spin network state intersects H by n intersections: $\mathcal{P} = \{p_i | i = 1, \cdots, n\}$. For every intersection p_i we associated a small enough bounded neighborhood s_i. Then the physical degrees of freedom of our sourced BF theory are encoded in

$$f_i = \int_{s_i} dB = \oint_{\partial s_i} B$$

- We can obtain the quantum Hilbert space of the *BF* theory with n intersections as: $\mathcal{H}_H^{\mathcal{P}} = L^2(\mathbb{R}^n)$.

Quantum horizon boundary condition

- Consider the bulk kinematical Hilbert space $\mathcal{H}_M^{\mathcal{P}}$ defined on a graph $\Gamma \subset M$ with $\mathcal{P}$ as the set of its end points on H. $\mathcal{H}_M^{\mathcal{P}}$ can be spanned by the spin network states $|\mathcal{P}, \{j_p, m_p\}; \cdots >$, where j_p and m_p are respectively the spin labels and magnetic numbers of the edge e_p with $p \in \mathcal{P}$.

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- The integral $\Sigma^1(H) = \int_H \Sigma^1$ can be promoted as an operator:

$$\hat{\Sigma}^1(H)|\mathcal{P}, \{j_p, m_p\}; \cdots >= 16\pi\gamma l_{Pl}^2 \sum_{p \in \Gamma \cap H} m_p |\mathcal{P}, \{j_p, m_p\}; \cdots > .$$

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- The equations of the boundary BF theory motive us to input the quantum version of the horizon boundary condition as

$$(Id \otimes \hat{f}_i(s_i) - \frac{\hat{\Sigma}^1(s_i)}{2\kappa} \otimes Id)(\Psi_v \otimes \Psi_b) = 0,$$

where $\Psi_v \in \mathcal{H}_M^{\mathcal{P}}$ and $\Psi_b \in \mathcal{H}_H^{\mathcal{P}}$.

Solving the quantum boundary condition

- The space of kinematical states on a fixed Γ, satisfying the boundary condition, can be written as

$$\mathcal{H}_\Gamma = \bigoplus_{\{j_p, m_p\}_{p\in\Gamma\cap H}} \mathcal{H}_M^{\mathcal{P}}(\{j_p, m_p\}) \otimes \mathcal{H}_H^{\mathcal{P}}(\{m_p\}),$$

where $\mathcal{H}_H^{\mathcal{P}}(\{m_p\})$ denotes the subspace corresponds to the spectrum $\{m_p\}$ in the spectral decomposition of $\mathcal{H}_H^{\mathcal{P}}$ with respect to the operators $\hat{f}_p$ on the boundary.

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  where $\mathcal{H}_H^{\mathcal{P}}(\{m_p\})$ denotes the subspace corresponds to the spectrum $\{m_p\}$ in the spectral decomposition of $\mathcal{H}_H^{\mathcal{P}}$ with respect to the operators $\hat{f}_p$ on the boundary.
- The imposition of the diffeomorphism constraint implies that one only needs to consider the diffeomorphism equivalence class of quantum states.
  Hence, in the following states counting, we will only take account of the number of intersections on H, while the possible positions of intersections are irrelevant.

Area constraint

- For the bulk Hilbert space $\mathcal{H}_M^{\mathcal{P}}$ with a horizon boundary H, the flux-area operator $\hat{a}_H^{flux}$ corresponding to the classical area $\int_H |dB|$ of H can also be naturally defined as [Barbero, Lewandowski, Villasenor, 2009]

$$\hat{a}_H^{flux}|\mathcal{P}, \{j_p, m_p\}; \cdots >= 8\pi\gamma l_{Pl}^2 (\sum_{p=1}^{n} |m_p|)|\mathcal{P}, \{j_p, m_p\}; \cdots > .$$

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- We have the area constraint:

$$\sum_{p\in\mathcal{P}}|m_p|=a, \quad m_p\in\mathbb{N}/2,$$

where $a=\dfrac{a_H}{8\pi\gamma l_{Pl}^2}$.

States counting

- For a given horizon area a_H, the horizon states satisfying the boundary condition are labeled by sequences $(m_1, \cdots, m_n)$ subject to area constraint, where $2m_i$ are integers.
- We assume that for each given ordering sequence $(m_1, \cdots, m_n)$, there exists at least one state in the bulk Hilbert space of LQG, which is annihilated by the Hamiltonian constraint.

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- We assume that for each given ordering sequence $(m_1, \cdots, m_n)$, there exists at least one state in the bulk Hilbert space of LQG, which is annihilated by the Hamiltonian constraint.
- The dimension of the horizon Hilbert space compatible with the given macroscopic horizon area can be calculated as:

$$\mathcal{N} = \sum_{n=0}^{n=2a-1} C^n_{2a-1} 2^{n+1} = 2 \times 3^{2a-1},$$

where C^j_i are the binomial coefficients.

Entropy of IH

- The entropy for an isolated horizon is given by [Wang, YM, Zhao, 2014]

$$S = \ln \mathcal{N} = (2 \ln 3) a + \ln \frac{2}{3} = \frac{\ln 3}{\pi \gamma} \frac{a_H}{4 l_{Pl}^2} + \ln \frac{2}{3}.$$

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- If we fix the value of the Barbero-Immirzi parameter as $\gamma = \frac{\ln 3}{\pi}$, which is different from its value predicted in the Chern-Simons approach, the Bekenstein-Hawking area law can be obtained.
- The quantum correction to the Bekenstein-Hawking area law in our approach is a constant $\ln(2/3)$ rather than a logarithmic term.

Generalization to Arbitrary Dimensions

- The above BF theory approach admits extension to arbitrary dimensional horizons [Wang, Huang, 2014].
- While the boundary theory is still $SO(1,1)$ BF theory with sources, the bulk theory would be LQG based on $SO(D)$ connections [Bodendorfer, Thiemann, Thurn, 2011].
- In the bulk theory, one possible choice is to implement the simplicity constraint on the edges of a spin network by restricting the representations of $SO(D)$ to be of class 1, so that their highest weight vector is determined by a single non-negative integer λ [Freidel, Krasnov, Puzio, 1999].

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- As the source of the boundary BF theory, the integration of the bulk field Σ^{01} becomes an operator as

$$\hat{\Sigma}^{01}(H)|\mathcal{P}, \{\lambda_p, m_p\}; \cdots >= 16\pi\gamma l_{Pl}^{D-2} \sum_{p \in \Gamma \cap H} m_p |\mathcal{P}, \{\lambda_p, m_p\}; \cdots >$$

Arbitrary dimensional case

- With the flux-area operator, the area constraint beomes:

$$\sum_{p\in\mathcal{P}} |m_p| = a, \quad m_p \in \mathbb{N}$$

- The compatible dimension of the horizon Hilbert space:

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- The entropy for an arbitrary dimensional IH reads

$$S = \ln \mathcal{N} = (\ln 3)a + \ln \frac{2}{3} = \frac{\ln 3}{2\pi\gamma} \frac{a_H}{4 l_{Pl}^2} + \ln \frac{2}{3}.$$

- The value of the Barbero-Immirzi parameter is fixed as $\gamma = (\ln 3)/(2\pi)$

Summary and Remarks

- The quasilocal notion of isolated horizon lays down a suitable framework to study black hole entropy by quantum gravity.
- In the Chern-Simons theory description of the horizon, the boundary degrees of freedom are encoded in the Chern-Simons connection,
  while in the BF theory description, the connection becomes pure gauge, and the non-trivial degrees of freedom of the horizon are all encoded in the B field.

Summary and Remarks

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  while in the BF theory description, the connection becomes pure gauge, and the non-trivial degrees of freedom of the horizon are all encoded in the B field.
- The BF theory explanation of isolated horizon entropy in LQG is applicable to general IHs in arbitrary dimensions.
- The approach grasps the most important internal symmetry $SO(1,1)$ for IHs, but ignores the remaining symmetries.

Summary and Remarks

- The value for the Barbero-Immirzi parameter, $\gamma = (\ln 3)/\pi$, based on $SU(2)$ connection in 4d spacetime coincides with its value obtained in a particular case in [Barbero, Lewandowski, Villasenor, 2009] by employing the same flux-area operator but still in the approach of Chern-Simons theory.

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- The value for the Barbero-Immirzi parameter, $\gamma = (\ln 3)/\pi$, based on $SU(2)$ connection in 4d spacetime coincides with its value obtained in a particular case in [Barbero, Lewandowski, Villasenor, 2009] by employing the same flux-area operator but still in the approach of Chern-Simons theory.
- In the generalization to arbitrary dimensional spacetime based on $SO(D)$ connections, the value for the Barbero-Immirzi parameter, $\gamma = (\ln 3)/(2\pi)$, is dimension independent.
- In 4d spacetime, the different choices of connection formulations imply different values for the Barbero-Immirzi parameter by the entropy calculations. This provides the possibility to determine the internal gauge group of LQG from other considerations or experiments.

!Thanks!