New Einstein-Hilbert type action with nonlinear SUSY and unity of nature

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OUTLINE

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1. Motivation

@ The success of Two SMs, i.e. GR and GWS model.

@ However, many unsolved fundamental problems in SMs: e.g.,

- Gravitational Force,
- Space-time dimension *four*,
- Three generations of quarks and leptons,
- Chiral eigenstates,
- Neutrino mass M_ν
- Dark Matter, Dark enegy; $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda$(cosmological term)?
- SUSY!?, Origin of SUSY breaking, etc.

@ GR describes geometry of space-time.
However, unpleasant differences between GR and SUGRA:

- General Relativity(GR) $\Longleftrightarrow$ Geometry of Riemann space(Physical:$[x^\mu]$, GL(4,R))

While,

- SUGRA $\Longleftrightarrow$ Geometry of superspace (Mathematical:$[x^\mu, \theta_\alpha]$, sPoicaré)

$\Longrightarrow$ **New SUSY paradigm on specific physical space-time.**

As for the particle spectrum based upon linear SUSY representation:

@Group theoretical Observation *(Z.Phys.C18,25(1983),Euro.Phys.J.C7,341(1999))*:

- Among all SO(N) sP,

SM with just 3 generations emerges from a single irreducible rep. of **only** SO(10) sP.

- 10 supercharges $Q^I, (I = 1, 2, \cdots .10)$ are decomposed and assigned as follows:

$\underline{\mathbf{10}}_{SO(10)} = \underline{\mathbf{5}}_{SU(5)} + \underline{\mathbf{5}}^*{}_{SU(5)} \Leftrightarrow \underline{5}_{SU(5)GUT}$ analogue multiplet of supercharges:

$\underline{\mathbf{5}}_{SU(5)} = [\ \underline{\mathbf{3}}^{*c}, \underline{\mathbf{1}}^{ew}, (\frac{e}{3}, \frac{e}{3}, \frac{e}{3}) : Q_a(a = 1, 2, 3)\ ] + [\ \underline{\mathbf{1}}^c, \underline{\mathbf{2}}^{ew}, (-e, 0) : Q_m(m = 4, 5)\ ].$

- **Massless helicity states of gravity multiplet of SO(10) sP with CPT conjugation** are specified by the helicity $h = (2 - \frac{n}{2})$ and the dimension $\underline{d}_{[n]} = \underline{\frac{10!}{n!(10-n)!}}$:

$$|h>= Q^{I_n}Q^{I_{n-1}} \cdots Q^{I_2}Q^{I_1}|2>, \ \ Q^{I_n}\ (n = 0, 1, 2, \cdots, 10): \text{ super charge}$$

$\|h\|$	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
			$\underline{1}_{[0]}$	$\underline{10}_{[1]}$	$\underline{45}_{[2]}$	$\underline{120}_{[3]}$	$\underline{210}_{[4]}$
$\underline{d}_{[n]}$	$\underline{1}_{[10]}$	$\underline{10}_{[9]}$	$\underline{45}_{[8]}$	$\underline{120}_{[7]}$	$\underline{210}_{[6]}$	$\underline{252}_{[5]}$	$\underline{210}_{[4]}$

@ **Spin $\frac{1}{2}$ Dirac particles survivours after *a tentative Higgs-like mechanism:***

$SU(3)$	Q_e	$SU(2)\otimes U(1)$
$\underline{\mathbf{1}}$	0 -1 -2	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$ $(E)\,(M)$
$\underline{\mathbf{3}}$	$5/3$ $2/3$ $-1/3$ $-4/3$	$\begin{pmatrix}u\\ d\end{pmatrix}\begin{pmatrix}c\\ s\end{pmatrix}\begin{pmatrix}t\\ b\end{pmatrix}\begin{pmatrix}h\\ o\end{pmatrix}\begin{pmatrix}a\\ f\end{pmatrix}\begin{pmatrix}g\\ m\end{pmatrix}\begin{pmatrix}r\\ i\\ n\end{pmatrix}$
$\underline{\mathbf{6}}$	$4/3$ $1/3$ $-2/3$	$\begin{pmatrix}P\\ Q\\ R\end{pmatrix}\begin{pmatrix}X\\ Y\\ Z\end{pmatrix}$
$\underline{\mathbf{8}}$	0 -1	$\begin{pmatrix}N_1\\ E_1\end{pmatrix}\begin{pmatrix}N_2\\ E_2\end{pmatrix}$

@ **One SM Higg-doublet survives in the low energy**

• How to write down **N=10 SUSY with gravity** beyond N-G theorem in S-matrix **?!**

• We need
(i) A certain degeneracy of space-time,
(ii) General Relativity principle on physical SUSY space-time
possesing **space-time symmetries** $SO(1,3), SL(2,C), GL(4,R)$.

We show in this talk:

• **N=10 SUSY with gravity** is obtained by the geometrical description of
specific unstable *physical* (Riemann) space-time
possesessing **NLSUSY structure at each point**.

• The nonlinear(NL) SUSY invariant coupling of spin $\frac{1}{2}$ fermion with spin 2 graviton circumvents the no-go theorem for SO(N>8) Linear SUSY.

$$\Downarrow$$

• **New SUSY paragigm beyond the SMs indicating a certain gravitational composite structure for all particles and/or a fundamental fermionic structure.**

A brief review of NLSUSY:

- Take flat space-time specified by x^a and ψ_α.
- Consider one form $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,

κ is an arbitrary constant with the dimension l^{+2}.

- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$

with a global spinor parameter ζ.

- An invariant acction($\sim$ invariant volume) is obtained:

$S = -\frac{1}{2\kappa^2}\int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA}$,

L_{VA} is N=1 Volkov-Akulov model of NLSUSY given by

$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right]$,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,

$t^a{}_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \bar{\psi}\gamma^a\partial_b\psi)$,

which is invariant under N=1 NLSUSY transformation:

$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\zeta}\gamma^a\psi)\partial_a\psi$. $\longleftrightarrow$ NG fermioon for SB SUSY

- ψ is NG fermion (the coset space coordinate) of $\frac{superPoincare}{Poincare}$.
- ψ is quantized canonically in compatible with SUSY algebra.

2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

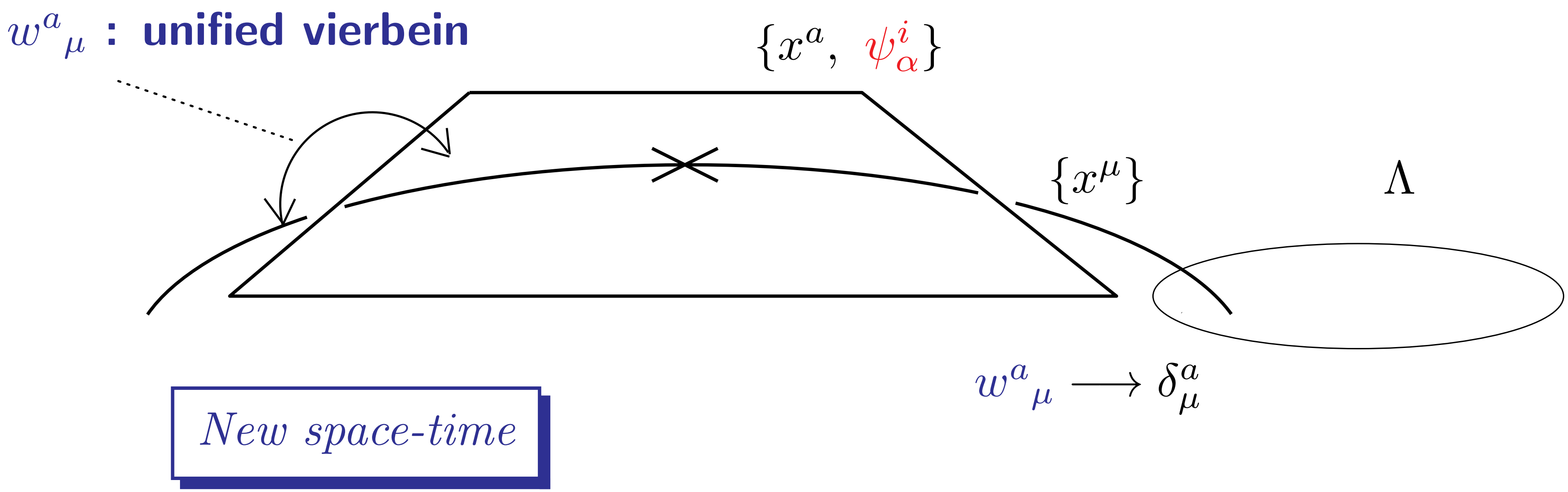
2.1. New Space-time as Ultimate Shape of Nature

We consider **new (unstable)** ***physical*** **space-time inspired by nonlinear(NL) SUSY:**

The tangent space of new space-time is specified by
SL(2,C) Grassmann coordinates ψ_α **for NLSUSY**
besides the ordinary SO(1,3) Minkowski coordinates x^a,

i.e.,
the coordinate ψ_α of the the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning to the NLSUSY NG fermion (called *superon* hereafter) are attached at every curved space-time point besides x^a.

• **Ultimate shape of nature** $\Longleftrightarrow$ **(empy) unstable space-time:**



(Locally homomorphic non-compact groups SO(1,3) and SL(2,C) for space-time symmetry are analogous to compact groups SO(3) and SU(2) for gauge symmetry of 't Hooft-Polyakov monopole, though SL(2,C) is realized nonlinearly.)

• Note that $SO(1,3) \cong SL(2,C)$ is crucial for NLSUSYGR scenario.

4 dimensional space-time is singled out.

2.2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

We have found that geometrical arguments of Einstein general relativity(GR) can be extended to **new (unstable) space-time**.

- Unified vierbein $w^a{}_\mu(x)$(*ulvierbein*) of new space-time:

(Note: Grassmann d.o.f. induces the imaginary part of $w^a{}_\mu(x)$.)

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi),$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a,$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, .., N)$$

(By conventions the first index A and the second index B of $t^A{}_B$ represent those of γ-matrix and the derivative, respectively.)

- **N-extended NLSUSYGR action of Eienstein-Hilbert(EH)-type for new space-time.** $\Longrightarrow$

N-extended NLSUSY GR action:

(*Phys.Lett.B501,237(2001), Phys.Lett.B507,260(2001).*)

$$L_{\rm NLSUSYGR}(w) = -\frac{c^4}{16\pi G}|w|\{\Omega(w) + \Lambda\}, \quad (1)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \quad (2)$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I\gamma^a\partial_\mu\psi^I - \partial_\mu\bar{\psi}^I\gamma^a\psi^I), (I = 1, 2, .., N) \quad (3)$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$: the vierbein of new space-time(ulvierbein)
- $e^a{}_\mu(x)$: the ordinary vierbein for the local SO(1,3) d.o.f.of GR,
- $t^a{}_\mu(\psi(x))$: the mimic vierbein for the local SL(2,C) d.o.f. composed of

the stress-energy-momentum of NG fermion $\psi(x)^I$(called superons),

- $\Omega(w)$: the scalar curvature of new space-time in terms of $w^a{}_\mu$,
- $s_{\mu\nu} \equiv w^a{}_\mu\eta_{ab}w^b{}_\nu$, $s^{\mu\nu}(x) \equiv w^\mu{}_a(x)\eta^{ab}w^\nu{}_a(x)$: metric tensors of new space-time.
- G : the Newton gravitational constant.
- Λ : cosmological term in new space-time indicating NLSUSY of tangent space.

• Remarkably NLSUSYGR scenario fixes the arbitrary constatnt κ^2 to

$$\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1},$$

with the dimension $(length)^4 \sim (enegy)^{-4}$.

• The sign $\Lambda > 0$ in the action $L_{\rm NLSUSYGR}$ is now fixed uniuely,
(i) which gives the correct sign to the kinetic term of $\psi(x)$
in the energy momentum tensor and
(ii)allows the negative dark energy density interpretation of Λ
in the Einstein equation. ($\rightarrow$ Sec.4).

• No-go theorem for $N > 8$ with gravity has been circumvented by using NLSUSY, i.e. by the vacuum(flat space) degeneracy.

• Note that $SO(1, D-1) \cong SL(d, C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2-1)$ holds for only $D = 4, d = 2$.

NLSUSYGR scenario predicts 4 dimensional space-time.

2.3. Symmetries of NLSUSY GR(N-extended action)

- **Space-time symmetries** $(\sim sP)$**:**

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}] \tag{4}$$

- **Internal symmetries for N-extended NLSUSY GR (N-superons ψ^I $(I = 1, 2, ..N)$):**

$$[\text{global SO(N)}] \otimes [\text{local U}(1)^{\text{N}}] \otimes [\text{chiral}]. \tag{5}$$

Examples:

- **Invariance under the new NLSUSY transformation**;

$$\delta_\zeta \psi^I = \frac{1}{\kappa}\zeta^I - i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_\rho\psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_{[\mu}e^a{}_{\rho]}. \tag{6}$$

(6) induce GL(4,R) transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \tag{7}$$

where ζ is a constant spinor parameter, $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and
$\xi^\rho = -i\kappa\bar{\zeta}^I\gamma^\rho\psi^I$.
Commutators of two new NLSUSY transformations (6) on ψ^I and $e^a{}_\mu$
close to GL(4,R),

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^\mu\partial_\mu\psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho\partial_\rho e^a{}_\mu + e^a{}_\rho\partial_\mu\Xi^\rho, \tag{8}$$

where $\Xi^\mu = 2i\bar{\zeta}^I{}_1\gamma^\mu\zeta^I{}_2 - \xi_1^\rho\xi_2^\sigma e_a{}^\mu\partial_{[\rho}e^a{}_{\sigma]}$.
$q.e.d.$

- **New NLSUSY (6) is the square-root of GL(4,R)**;

$$[\delta_1, \delta_2] = \delta_{\mathrm{GL(4,R)}}, \quad i.e. \quad \delta \sim \sqrt{\delta_{\mathrm{GL(4,R)}}}.$$

c.f. SUGRA

$$[\delta_1, \delta_2] = \delta_P \underline{+\delta_L + \delta_g}$$

- The ordinary local GL(4,R) invariance is manifest by the construction.

- **Invariance under the local Lorentz transformation**;

$$\delta_L \psi^I = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^I\gamma_5\gamma_d\psi^I(\partial_\mu\epsilon_{bc}) \tag{9}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

(9) induce the familiar local Lorentz transformation on $w^a{}_\mu$:

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{10}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra,
e.g., the new form on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu\beta_{bc}), \tag{11}$$

where $\beta_{ab} = -\beta_{ba}$ is given by $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$.
$q.e.d.$

2.4. Big Decay of New Space-Time:

- The Noether's theorem finds the conserved supercurrent:

$$S^{I\mu} = i\frac{c^4\Lambda}{16\pi G}{e_a}^{\mu}\gamma^a\psi^I + \cdots . \quad (12)$$

- The supercurrent couples the graviton and the superon(NG fermion) to the vacuum with the strength $\frac{c^4\Lambda}{16\pi G}$:

$$< 0|{S_\alpha}^{I\mu}|{e_b}^{\nu}{\psi_\beta}^{J} >= i\frac{c^4\Lambda}{16\pi G}\delta^{\mu\nu}\delta^{IJ}(\gamma_b)_{\alpha\beta} \quad (13)$$

- $L_{\rm NLSUSYGR}(w)$ would break down spontaneously**(Big Decay)** to **ordinary Riemann space-time(graviton) and superon(NG fermion):**

$L_{\rm SGM}(e,\psi)$ **(Superon-Graviton Model(SGM)).**

@ Superon-Graviton Model(SGM) after Big Decay:

$$L_{\rm NLSUSYGR}(w) = L_{\rm SGM}(e,\psi) \equiv -\frac{c^4}{16\pi G}|e|\{R(e) + |w_{VA}(\psi^I)|\Lambda + \tilde{T}(e,\psi^I)\}. \quad (14)$$

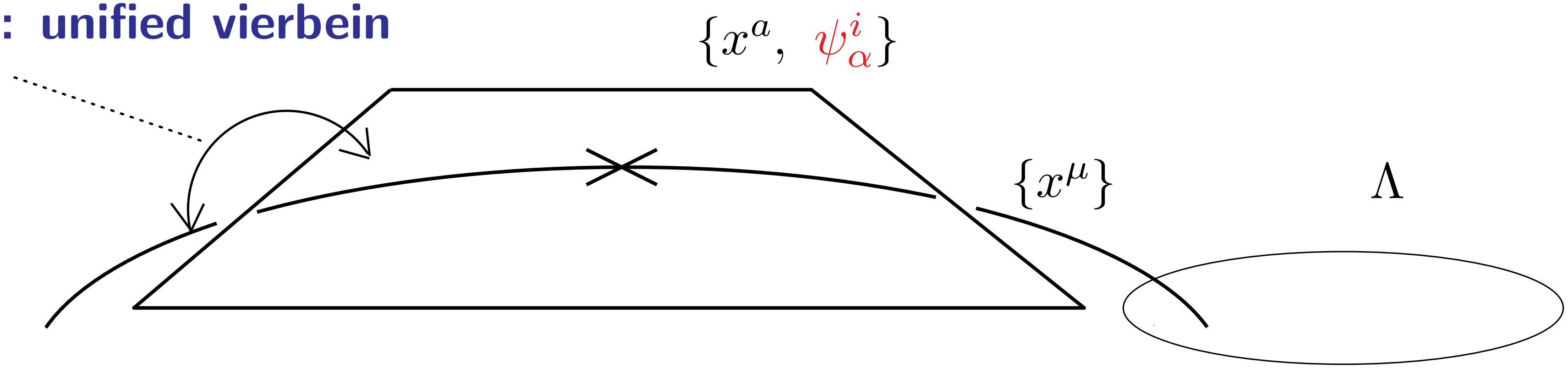
- $R(e)$: the Ricci scalar curvature of ordinary Riemann space-time
- Λ : the cosmological term
- $|w_{VA}(\psi^I)| = \det w^a{}_b = \det \{\delta^a{}_b + t^a{}_b(\psi^I)\}$: NLSUSY action for superon
- $\tilde{T}(e,\psi^I)$: the gravitational interaction of superon

• Big Decay to graviton-superon system induces the spacial expamsion of space-time by the Pauli principle.

• $L_{\rm SGM}(e,\psi^I)$ is anticipated to constitute
gravitational composite massless eigenstates of (broken) SUSY SO(N) sP
followed by the Big Bang SMs scenario.

The ignition of Big Bang proceeding to the true vacuum(SMs).

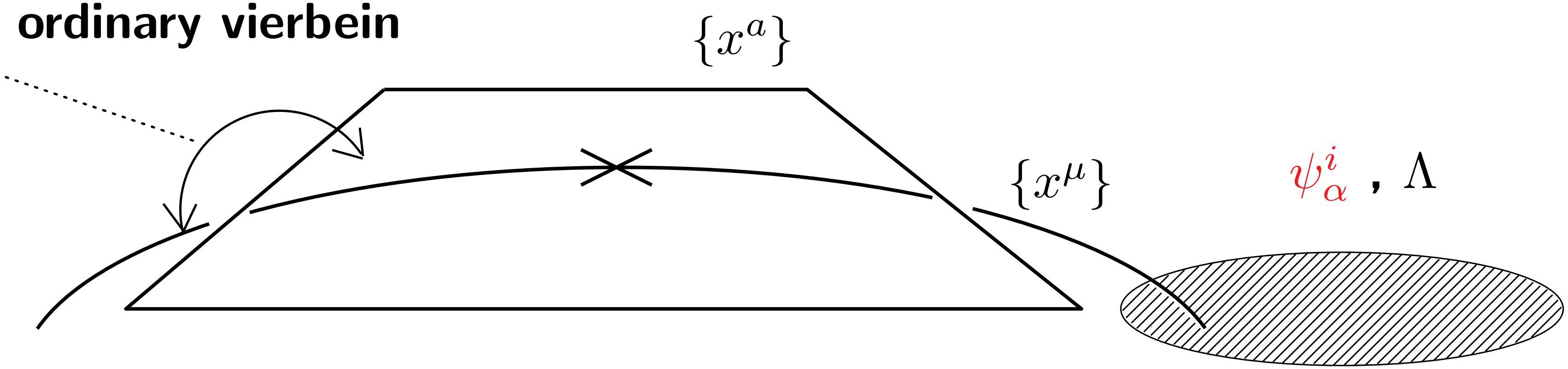
$w^a{}_\mu$: **unified vierbein**



New space-time

$\Downarrow$ **Big Decay**

$e^a{}_\mu$: **ordinary vierbein**



Riemann spacetime $\oplus$ **matter**

(asymptotic)

Ignition of Big Bang towards the true vacuum

3. Vacuum structure of $L_{\rm NLSUSYGR}$ $\Leftrightarrow$ Linearization of NLSUSY

$SO(10)$ sP LSUSY algebra determines
the vacuum particle configuration of $L_{\rm SGM}(e,\psi)$. $\longleftrightarrow$ c.f. $O(4)$ for rel. H-atom

By respecting SUSY algebra throughout we show in *local flat space*:

- N-LSUSY broken theory emerges
in the true vacuum of N-NLSUSY theory $L_{\rm SGM}(e,\psi)$
as massless composites of NG fermions.

$\Longleftrightarrow$ **NL/L SUSY relation(equivalence)** $\longleftrightarrow$ **c.f. BCS/LG**

- These phenomena are the phase transition of NLSUSY $L_{\rm SGM}(e,\psi)$
from the false vacuum with $V_{\rm P.E.} = \Lambda > 0$
towards the true vacuum with $V_{\rm P.E.} = 0$
achieved by forming massless composite states of LSUSY.

3.1. NL/L SUSY relation(equivalence) for N=2 SUSY :

We demonstrate NL/L SUSY relation for N=2 SUSY in ***flat space***.
($N \geq 2$ SUSY for a realistic model building in SGM scenario.)

• $N = 2$ SGM in Riemann-flat (${e^a}_\mu \to {\delta^a}_\mu$) space-time reduces to $N = 2$ NLSUSY in the cosmological term of NLSUSYGR:

$$\underline{L_{\rm NLSUSYGR}(w) = L_{\rm SGM}(e,\psi) \longrightarrow L_{\rm NLSUSY}(\psi)} \leftrightarrow \Lambda \text{ term of NLSUSYGR.}$$

N=2, d=2 NLSUSY model:

$$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \qquad (15)$$

where,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,

$t^a{}_b = -i\kappa^2(\bar\psi_j \gamma^a \partial_b \psi^j - \bar\psi_j \gamma^a \partial_b \psi^j)$, $(j = 1, 2)$,

which is invariant under N=2 NLSUSY transformation,

$\delta_\zeta \psi^j = \frac{1}{\kappa}\zeta^j - i\kappa(\bar\zeta_k \gamma^a \psi^k - \bar\zeta_k \gamma^a \psi^k)\partial_a \psi^j, (j = 1, 2)$.

N=2, d=2 LSUSY Theory (SUSY QED):

- Helicity states of N=2 vector supermultiplet:

$$\left(\begin{array}{c} +1 \\ +\frac{1}{2},\ +\frac{1}{2} \\ 0 \end{array}\right) + [\mathrm{CPT conjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell minimal vector supermultiplet: $(v^a, \lambda^i, A, \phi, D;$i=1,2$)$. in *WZ gauge*. ($A$ and ϕ are two singlets, 0^+ and 0^-, scalar fields.)

- Helicity states of N=2 scalar supermultiplet:

$$\left(\begin{array}{c} +\frac{1}{2} \\ 0,\ 0 \\ -\frac{1}{2} \end{array}\right) + [\mathrm{CPT conjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets: $(\chi,\ B^i,\ \nu,\ F^i;\ i=1,2)$, B^i and F^i are complex.

• **The most genaral $N=2, d=2$ SUSYQED action ($m=0$ case)** :

$$L_{\mathrm{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf}, \tag{16}$$

$$\begin{aligned}
L_{V0} &= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D, \\
L'_{\Phi 0} &= \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}|\partial_a B^i|^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}|F^i|^2, \\
L_e &= e\left\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5 \nu \right. \\
&\quad \left. + B^i(\bar{\lambda}^i \chi - \epsilon^{ij}\bar{\lambda}^j \nu) - \frac{1}{2}|B^i|^2 D \right\} + \{h.c.\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)|B^i|^2, \\
L_{Vf} &= f\{A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + (A^2 - \phi^2)D - \epsilon^{ab}A\phi F_{ab}\}
\end{aligned} \tag{17}$$

• Note that
$J=0$ states in the vector multiplet for $N \geq 2$ SUSY induce Yukawa coupling.

$L_{\mathrm{N=2SUSYQED}}$ is invariant under $N = 2$ LSUSY transformation:

- For the minimal vector off-shell supermultiplet:

$$
\begin{aligned}
&\delta_\zeta v^a = -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
&\delta_\zeta \lambda^i = (D - i\partial\!\!\!/ A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\partial\!\!\!/\phi\zeta^j, \\
&\delta_\zeta A = \bar{\zeta}^i\lambda^i, \\
&\delta_\zeta \phi = -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
&\delta_\zeta D = -i\bar{\zeta}^i\partial\!\!\!/\lambda^i.
\end{aligned} \tag{18}
$$

$$
[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{19}
$$

where $\zeta^i, i = 1, 2$ are constant spinors and $\delta_g(\theta)$ is the $U(1)$ gauge transformation for only v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i\, v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- For the two scalar off-shell supermultiplets:

$$\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial\!\!\!/ B^i)\zeta^i - e\epsilon^{ij}V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij}\bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\partial\!\!\!/ B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial\!\!\!/ \chi - i\epsilon^{ij}\bar{\zeta}^j \partial\!\!\!/ \nu \\
&\quad -e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i)B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\chi = \Xi^a \partial_a \chi - e\theta\nu, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]B^i = \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\nu = \Xi^a \partial_a \nu + e\theta\chi, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]F^i = \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{20}$$

with $V^i = iv_a\gamma^a\zeta^i - \epsilon^{ij}A\zeta^j - \phi\gamma_5\zeta^i$ and the U(1) gauge parameter θ.

$N = 2$ NL/L SUSY relation(equivalence):

$$L_{\mathrm{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} = L_{\mathrm{N=2NLSUSY}} + [\text{surface terms}], \qquad (21)$$

is proved by the followings:

(i) Construct SUSY invariant(composite) relations which express component fields of LSUSY supermultiplet as the composites of superons ψ_j of NLSUSY.

(ii) Show that performing NLSUSY transformations of constituent superons ψ^j in SUSY invariant(composite) relations reproduces familiar LSUSY transformations among the LSUSY supermultiplet recasted by SUSY invariant(composite) relations.

(iii) Substituting SUSY invariant (composite) relations into $L_{\mathrm{N=2LSUSYQED}}$, we obtain $L_{\mathrm{N=2NLSUSY}}$ and the NL/L SUSY relation(equivalence) is established.

• SUSY invariant (composite) relationsns for the vector off-shell supermultiplet:

$$
\begin{aligned}
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|, \\
\lambda^i &= \xi\psi^i|w|, \\
A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\
D &= \frac{\xi}{\kappa}|w|,
\end{aligned}
\tag{22}
$$

where ξ is a VEV factor of the auxiliary field D.

• SUSY composite relations for scalar off-shell supermultiplets:

$$\chi = \xi^i \left[\psi^i|w| + \frac{i}{2}\kappa^2\partial_a\{\gamma^a\psi^i\bar{\psi}^j\psi^j|w|\}\right]$$

$$B^i = -\kappa\left(\frac{1}{2}\xi^i\bar{\psi}^j\psi^j - \xi^j\bar{\psi}^i\psi^j\right)|w|,$$

$$\nu = \xi^i\epsilon^{ij}\left[\psi^j|w| + \frac{i}{2}\kappa^2\partial_a\{\gamma^a\psi^j\bar{\psi}^k\psi^k|w|\}\right],$$

$$F^i = \frac{1}{\kappa}\xi^i\left\{|w| + \frac{1}{8}\kappa^3\partial_a\partial^a(\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w|)\right\} - i\kappa\xi^j\partial_a(\bar{\psi}^i\gamma^a\psi^j|w|)$$

$$-\underline{\frac{1}{4}e\kappa^2\xi\xi^i\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w|}. \qquad (23)$$

• The quartic fermion self-interaction term in F^i is the origin of the local $U(1)$ gauge symmetry of LSUSY.

• ξ^i is the VEV factor of F^i.

• SUSY invariant(composite) relations produce a new off-shell commutator algebra which closes on only a translation:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{24}$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}\gamma^a\zeta_{2L} - \bar{\zeta}_{1R}\gamma^a\zeta_{2R}) \tag{25}$$

• Note that the commutator does not induce the U(1) gauge transformation, which is different from the ordinary LSUSY.

• Substituting these SUSY coposite relations into $L_{\rm N=2LSUSYQED}$,
we find NL/L SUSY relation:

$$L_{\rm N=2LSUSYQED} = f(\xi, \xi^i) L_{\rm N=2NLSUSY} + [\text{suface terms}], \tag{26}$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \tag{27}$$

⇒ LSUSY may be regarded as composite eigenstates of (space-time) symmetries.

• NL/L SUSY relation bridges naturally
the cosmology and the low energy particle physics in NLSUSY GR. (⇒ Sec. 4).

• **The direct linearization of highly nonlinear SGM action (14) in curved space remains to be carried out.**

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the cosmological term Λ and achieves the true vacuum of SGM
as massless composites of NG fermion.

Is SM a low energy effective theory of SG/NLSUSYGR?

♣ Systematics of NL/L SUSY relation for $N = 2$ SUSY QED

SUSY invariant(composite) relations: in the superfield formulation.

Linearization of NLSUSY in the $d = 2$ superfield formulation

- General superfields are given for the $N = 2$ vector supermultiplet by

$$\mathcal{V}(x, \theta^i) = C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2}\bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2}\bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}^i \gamma_5 \theta^j \phi(x) - \frac{i}{4}\epsilon^{ij}\bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2}\bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8}\bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x), \quad (28)$$

and for the $N = 2$ scalar supermultiplet by

$$\Phi^i(x, \theta^i) = B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij}\bar{\theta}^j \nu(x) - \frac{1}{2}\bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i\bar{\theta}^i \partial\!\!\!/ B^j(x)\theta^j + \frac{i}{2}\bar{\theta}^j \theta^j (\bar{\theta}^i \partial\!\!\!/ \chi(x) - \epsilon^{ik}\bar{\theta}^k \partial\!\!\!/ \nu(x)) + \frac{1}{8}\bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x). \quad (29)$$

• Take the following ψ^i-dependent supertranslations with $-\kappa\psi(x)$,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{30}$$

and denote the resulting superfields on (x'^a, θ'^i) and their θ-epansions as

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \tag{31}$$

• Hybrid global SUSY transformations $\delta^h = \delta^L(x.\theta) + \delta^{NL}(\psi)$ on (x'^a, θ'^i) give:

$$\delta^h\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \delta^h\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \tag{32}$$

• Therefore, the following conditions, i.e. SUSY invariant constraints

$$\tilde{\varphi}^I_{\mathcal{V}}(x) = \xi^I_{\mathcal{V}}(\text{constant}) \quad \tilde{\varphi}^I_{\Phi}(x) = \xi^I_{\Phi}(\text{constant}), \tag{33}$$

are invariant (conserved quantities) under hybrid supertrasformations,
which provide SUSY invariant relations.

• Putting in general constants as follows:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \tag{34}$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{35}$$

where mass dimensions of constants (or constant spinors) in $d = 2$ are defined by (-1, $\frac{1}{2}$, 0, 0, 0, $-\frac{1}{2}$) for (ξ_c, ξ_Λ^i, ξ_M^{ij}, ξ_ϕ, ξ_v^a, ξ_λ^i), (0, $-\frac{1}{2}$, $-\frac{1}{2}$) for (ξ_B^i, ξ_χ, ξ_ν) and 0 for ξ^i for convenience.

• we obtain straightforwardly the following SUSY invariant relations $\varphi_{\mathcal{V}}^I = \varphi_{\mathcal{V}}^I(\psi)$ for the vector supermultiplet

$$\begin{aligned} C &= \xi_c + \kappa \bar{\psi}^i \xi_\Lambda^i + \frac{1}{2}\kappa^2 (\xi_M^{ij} \bar{\psi}^i \psi^j - \xi_M^{ii} \bar{\psi}^j \psi^j) + \frac{1}{4}\xi_\phi \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - \frac{i}{4}\xi_v^a \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \\ &\quad -\frac{1}{2}\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \xi_\lambda^j - \frac{1}{8}\xi \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j, \\ \Lambda^i &= \xi_\Lambda^i + \kappa (\xi_M^{ij} \psi^j - \xi_M^{jj} \psi^i) + \frac{1}{2}\xi_\phi \kappa \epsilon^{ij} \gamma_5 \psi^j - \frac{i}{2}\xi_v^a \kappa \epsilon^{ij} \gamma_a \psi^j \end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{2}\xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2}\kappa^2 (\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma^a \xi_\lambda^j) \\
& -\frac{1}{2}\xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i\kappa \partial\!\!\!/ C(\psi)\psi^i, \\
M^{ij} &= \xi_M^{ij} + \kappa \bar{\psi}^{(i} \xi_\lambda^{j)} + \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^j + i\kappa\epsilon^{(i|k|}\epsilon^{j)l}\bar{\psi}^k \partial\!\!\!/ \Lambda^l(\psi) - \frac{1}{2}\kappa^2\epsilon^{ik}\epsilon^{jl}\bar{\psi}^k\psi^l\partial^2 C(\psi), \\
\phi &= \xi_\phi - \kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\xi_\lambda^j - \frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5 \partial\!\!\!/ \Lambda^j(\psi) + \frac{1}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2 C(\psi), \\
v^a &= \xi_v^a - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\xi_\lambda^j - \frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j - \kappa\epsilon^{ij}\bar{\psi}^i \partial\!\!\!/ \gamma^a\Lambda^j(\psi) + \frac{i}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j\partial^2 C(\psi) \\
& -i\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^b\psi^j\partial^a\partial_b C(\psi), \\
\lambda^i &= \xi_\lambda^i + \xi\psi^i - i\kappa \partial\!\!\!/ M^{ij}(\psi)\psi^j + \frac{i}{2}\kappa\epsilon^{ab}\epsilon^{ij}\gamma_a\psi^j\partial_b\phi(\psi) \\
& -\frac{1}{2}\kappa\epsilon^{ij}\left\{\psi^j\partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab}\gamma_5\psi^j F_{ab}(\psi)\right\} \\
& -\frac{1}{2}\kappa^2\{\partial^2\Lambda^i(\psi)\bar{\psi}^j\psi^j - \partial^2\Lambda^j(\psi)\bar{\psi}^i\psi^j - \gamma_5\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma_5\psi^j
\end{aligned}
$$

$$
\begin{aligned}
& -\gamma_a \partial^2 \Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j + 2\partial\!\!\!/ \partial_a \Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j\} - \frac{i}{2}\kappa^3 \partial\!\!\!/ \partial^2 C(\psi)\psi^i\bar{\psi}^j\psi^j, \\
D \;=\; & \frac{\xi}{\kappa} - i\kappa\bar{\psi}^i \partial\!\!\!/ \lambda^i(\psi) \\
& + \frac{1}{2}\kappa^2 \left\{ \bar{\psi}^i\psi^j\partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2\phi(\psi) \right. \\
& \left. + \frac{i}{2}\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial^2 v^a(\psi) - i\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial_a\partial_b v^b(\psi) \right\} \\
& - \frac{i}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j \partial\!\!\!/ \partial^2\Lambda^j(\psi) + \frac{1}{8}\kappa^4\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j\partial^4 C(\psi),
\end{aligned} \tag{36}
$$

and the following SUSY invariant relations for the vector multiplet $\varphi^I_\Phi = \varphi^I_\Phi(\psi)$:

$$
\begin{aligned}
B^i \;=\; & \xi^i_B + \kappa(\bar{\psi}^i\xi_\chi - \epsilon^{ij}\bar{\psi}^j\xi_\nu) - \frac{1}{2}\kappa^2\{\bar{\psi}^j\psi^j F^i(\psi) - 2\bar{\psi}^i\psi^j F^j(\psi) + 2i\bar{\psi}^i \partial\!\!\!/ B^j(\psi)\psi^j\} \\
& - i\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i \partial\!\!\!/ \chi(\psi) - \epsilon^{ik}\bar{\psi}^k \partial\!\!\!/ \nu(\psi)\} + \frac{3}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 B^i(\psi), \\
\chi \;=\; & \xi_\chi + \kappa\{\psi^i F^i(\psi) - i\partial\!\!\!/ B^i(\psi)\psi^i\}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\chi(\psi)\bar{\psi}^i\psi^i - \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\nu(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\nu(\psi)\}] \\
& +\frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \frac{i}{2}\kappa^3\partial\!\!\!/ F^i(\psi)\psi^i\bar{\psi}^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\nu \ =\ & \xi_\nu - \kappa\epsilon^{ij}\{\psi^i F^j(\psi) - i\partial\!\!\!/ B^i(\psi)\psi^j\} \\
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\nu(\psi)\bar{\psi}^i\psi^i + \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\chi(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\chi(\psi)\}] \\
& +\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar{\psi}^k\psi^k\partial^2 B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\partial\!\!\!/ F^i(\psi)\psi^j\bar{\psi}^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
F^i \ =\ & \frac{\xi^i}{\kappa} - i\kappa\{\bar{\psi}^i\partial\!\!\!/\chi(\psi) + \epsilon^{ij}\bar{\psi}^j\partial\!\!\!/\nu(\psi)\} \\
& -\frac{1}{2}\kappa^2\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \kappa^2\bar{\psi}^i\psi^j\partial^2 B^j(\psi) + i\kappa^2\bar{\psi}^i\partial\!\!\!/ F^j(\psi)\psi^j \\
& +\frac{1}{2}\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial^2\chi(\psi) + \epsilon^{ik}\bar{\psi}^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 F^i(\psi). \qquad (37)
\end{aligned}
$$

• Choosing the following simple SUSY invariant constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \tilde{D} = \frac{\xi}{\kappa}, \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{38}$$

give previous simple SUSY invariant relations.

Actions in the $d = 2$, $N = 2$ NL/L SUSY relation

By changing the integration variables $(x^a, \theta^i) \to (x'^a, \theta'^i)$, we can confirm systematically that LSUSY actions reduce to NLSUSY representation.

(a) The kinetic (free) action with the Fayet-Iliopoulos (FI) D term for the $N = 2$ vector supermultiplet $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$S_{\mathcal{V}\mathrm{free}} = \int d^2x \left\{ \int d^2\theta^i \, \frac{1}{32} (\overline{D^i \mathcal{W}^{jk}} D^i \mathcal{W}^{jk} + \overline{D^i \mathcal{W}_5^{jk}} D^i \mathcal{W}_5^{jk}) + \int d^4\theta^i \, \frac{\xi}{2\kappa} \mathcal{V} \right\}_{\theta^i = 0}$$
$$= \xi^2 S_{N=2\mathrm{NLSUSY}}, \qquad (39)$$

where

$$\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \qquad (40)$$

(Note) The FI D term gives the correct sign of the NLSUSY action.

(b) Yukawa interaction terms for $\mathcal{V}$ vanish,

i.e.

$$
\begin{aligned}
S_{\mathcal{V}f} &= \frac{1}{8}\int d^2x\ f\left[\int d^2\theta^i\ \mathcal{W}^{jk}(\mathcal{W}^{jl}\mathcal{W}^{kl} + \mathcal{W}_5^{jl}\mathcal{W}_5^{kl}) \right. \\
&\quad \left. + \int d\bar{\theta}^i d\theta^j\ 2\{\mathcal{W}^{ij}(\mathcal{W}^{kl}\mathcal{W}^{kl} + \mathcal{W}_5^{kl}\mathcal{W}_5^{kl}) + \mathcal{W}^{ik}(\mathcal{W}^{jl}\mathcal{W}^{kl} + \mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\}\right]_{\theta^i=0} \\
&= 0,
\end{aligned}
\tag{41}
$$

by means of cancellations among four NG-fermion self-interaction terms.

[Note]

- General mass terms for $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$ vanish as well. $\rightarrow$ Chirality is encoded in the vacuum.

(c) The *most general* gauge invariant action for Φ^i coupled with $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$\begin{aligned} S_{\mathrm{gauge}} &= -\frac{1}{16}\int d^2x \int d^4\theta^i e^{-4e\mathcal{V}}(\Phi^j)^2 \\ &= -(\xi^i)^2 S_{N=2\mathrm{NLSUSY}}. \end{aligned} \tag{42}$$

• Here $U(1)$ gauge interaction terms with the gauge coupling constant e produce

four NG-fermion self-interaction terms as

$$S_e(\text{for the minimal off shell multiplet}) = \int d^2x \left\{ \frac{1}{4} e\kappa\xi(\xi^i)^2 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \right\}, \tag{43}$$

which are absorbed in the SUSY invariant relation of the auxiliary field $F^i = F^i(\psi)$ by adding four NG-fermion self-interaction terms as (23):

$$F^i(\psi) \longrightarrow F^i(\psi) - \frac{1}{4} e\kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w_{VA}|. \tag{44}$$

Therefore,

• under SUSY invariant relations,
the $N = 2$ NLSUSY action $S_{N=2\mathrm{NLSUSY}}$ is related to $N = 2$ SUSY QED action:

$$f(\xi, \xi^i) S_{N=2\mathrm{NLSUSY}} = S_{N=2\mathrm{SUSYQED}} \equiv S_{\mathcal{V}\mathrm{free}} + S_{\mathcal{V}f} + S_{\mathrm{gauge}} \tag{45}$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$. $\Longrightarrow$ NL/L SUSY relation gives the relation between

the cosmology and the low energy particle physics in NLSUSY GR scenario(in Sec. 4).

- **SGM scenario predicts the magnitude of the bare gauge coupling constant.**

More general SUSY invariant constraints, i.e. NLSUSY vev of 0^+ auxiliary field:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (46)$$

produce

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \ e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \quad (47)$$

where e is the bare gauge coupling constant.

- This mechanism is natural and favorable for SGM scenario as a theory of everything.

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

3.2. $N = 3$ NL/L SUSY relation and SUSY Yang-MIlls theory

- Physical helicity states of $N = 3$ LSUSY vector supermultiplet:

$$\left[\underline{1}(+1), \underline{3}\left(+\frac{1}{2}\right), \underline{3}(0), \underline{1}\left(-\frac{1}{2}\right) \right] + [\text{CPT conjugate}], \tag{48}$$

where $\underline{n}(\lambda)$ means the dimension $\underline{n}$ and the helicity λ, are accomodated in $N = 3$ off-shell vector supermultiplet($d = 2$):

- $N = 3$ superYang-Mills(SUSYYM) minimal off-shell gauge multiplet,

$$\{v^{aI}(x), \lambda^{iI}(x), A^{iI}(x), {\chi_\alpha}^I(x), \phi^I(x), D^{iI}(x)\}, \quad (I = 1, 2, \cdots, dim.G) \tag{49}$$

Each component field belongs to the adjoint representation of the YM gauge group G: $[T^I, T^J] = if^{IJK}T^K$ and denoted as $\varphi^i = \varphi^{iI}T^I$, etc..

- $N = 3$ (pure) SUSYYM action:

$$S_{\rm SYM} = \int d^2x \ {\rm tr}\left\{-\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{D}\lambda^i + \frac{1}{2}(D_a A^i)^2 + \frac{i}{2}\bar{\chi}\not{D}\chi + \frac{1}{2}(D_a\phi)^2 + \frac{1}{2}(D^i)^2 \right.$$
$$-ig\{\epsilon^{ijk}A^i\bar{\lambda}^j\lambda^k - [A^i, \bar{\lambda}^i]\chi + \phi(\bar{\lambda}^i\gamma_5\lambda^i + \bar{\chi}\gamma_5\chi)\}$$
$$\left. +\frac{1}{4}g^2([A^i, A^j]^2 + 2[A^i, \phi]^2)\right\}, \tag{50}$$

where g is the gauge coupling constant, D_a and F_{ab} are the covariant derivative and the YM gauge field strength defined as

$$D_a\varphi = \partial_a\varphi - ig[v_a, \varphi],$$
$$F_{ab} = \partial_a v_b - \partial_b v_a - ig[v_a, v_b]. \tag{51}$$

• SUSYYM action is invariant under $N=3$ LSUSY transformations:

$$\begin{aligned}
\delta_\zeta v^a &= i\bar{\zeta}^i\gamma^a\lambda^i, \\
\delta_\zeta \lambda^i &= \epsilon^{ijk}(D^j - i\partial\!\!\!/ A^j)\zeta^k + \frac{1}{2}\epsilon^{ab}F_{ab}\gamma_5\zeta^i - i\gamma_5\partial\!\!\!/\phi\zeta^i \\
&\quad +ig([A^i, A^j]\zeta^j + \epsilon^{ijk}[A^j,\phi]\gamma_5\zeta^k), \\
\delta_\zeta A^i &= \epsilon^{ijk}\bar{\zeta}^j\lambda^k - \bar{\zeta}^i\chi, \\
\delta_\zeta \chi &= (D^i + i\partial\!\!\!/ A^i)\zeta^i + ig(\epsilon^{ijk}A^iA^j\zeta^k - [A^i,\phi]\gamma_5\zeta^i), \\
\delta_\zeta \phi &= \bar{\zeta}^i\gamma_5\lambda^i, \\
\delta_\zeta D^i &= -i\epsilon^{ijk}\bar{\zeta}^j\partial\!\!\!/\lambda^k - i\bar{\zeta}^i\partial\!\!\!/\chi + ig(\bar{\zeta}^i[\lambda^j, A^j] + \bar{\zeta}^j[\lambda^i, A^j] - \bar{\zeta}^j[\lambda^j, A^i] \\
&\quad -\epsilon^{ijk}\bar{\zeta}^j[\chi, A^k] + \epsilon^{ijk}\bar{\zeta}^j\gamma_5[\lambda^k,\phi] + \bar{\zeta}^i\gamma_5[\chi,\phi]),
\end{aligned} \tag{52}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a) + \delta_G(\theta) + \delta_g(\theta), \tag{53}$$

where $\delta_G(\theta)$ means $\delta_G(\theta)\varphi = ig[\theta,\varphi]$ and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ijk}\bar{\zeta}_1^i\zeta_2^j A^k - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- SUSY invariant(composite) relations for $N = 3$ YM off-shell gauge supermultiplet

$$
\begin{aligned}
v^{aI} &= -\frac{i}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma^a\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) + \frac{1}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_b(\bar{\psi}^j\gamma_5\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
\lambda^{iI} &= \epsilon^{ijk}\xi^{jI}\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) \\
&\quad +\frac{i}{2}\kappa^2\xi^{jI}\partial_a\{\epsilon^{ijk}\gamma^a\psi^k\bar{\psi}^l\psi^l + \epsilon^{ab}\epsilon^{jkl}(\gamma_b\psi^i\bar{\psi}^k\gamma_5\psi^l - \gamma_5\psi^i\bar{\psi}^k\gamma_b\psi^l)\} + \mathcal{O}(\kappa^4), \\
A^{iI} &= \kappa\left(\frac{1}{2}\xi^{iI}\bar{\psi}^j\psi^j - \xi^{jI}\bar{\psi}^i\psi^j\right)(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k) - \frac{i}{2}\kappa^3\xi^{iI}\partial_a(\bar{\psi}^i\gamma^a\psi^j\bar{\psi}^k\psi^k) + \mathcal{O}(\kappa^5), \\
\chi^I &= \xi^{iI}\psi^i(1 - i\kappa^2\bar{\psi}^j\partial\!\!\!/\psi^j) + \frac{i}{2}\kappa^2\xi^{iI}\partial_a(\gamma^a\psi^i\bar{\psi}^j\psi^j) + \mathcal{O}(\kappa^4), \\
\phi^I &= -\frac{1}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma_5\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) - \frac{i}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_a(\bar{\psi}^j\gamma_b\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
D^{iI} &= \frac{1}{\kappa}\xi^{iI}|w| - i\kappa\xi^{jI}\partial_a\{\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k)\} \\
&\quad -\frac{1}{8}\kappa^3\partial_a\partial^a\{(\xi^{iI}\bar{\psi}^j\psi^j - 4\xi^{jI}\bar{\psi}^i\psi^j)\bar{\psi}^k\psi^k\} + \mathcal{O}(\kappa^5),
\end{aligned}
\tag{54}
$$

• Arbitrary real constants ξ^{iI} of auxirially fields D^{iI} bridge
$N = 3$ SUSY and the YM gauge group G.

• Substituting (54) into the SYM action (50),
we can show the NL/L SUSY relation for $N = 3$ SUSY:

$$S_{\rm SUSYYM}(\psi) = -(\xi^{iI})^2 S_{\rm NLSUSY} + [\text{surface terms}]. \tag{55}$$

4. Significances of NLSUSYGR for Low energy particle physics and Cosmology

The variation of SGM action $L_{\rm N=2SGM}(e,\psi)$ with respect to ${e^a}_\mu$ yields the equation of motion for ${e^a}_\mu$ in Riemann space-time:

$$R_{\mu\nu}(e) - \frac{1}{2} g_{\mu\nu} R(e) = \frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e,\psi) - g_{\mu\nu}\frac{c^4\Lambda}{16\pi G}\}, \qquad (56)$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

• Note that $-\frac{c^4\Lambda}{16\pi G}$ can be interpreted as the negative energy density of space-time, i.e. the dark energy density ρ_D.
(The negative sign in r.h.s is unique.)

4.1. Low Energy Particle Physics of NLSUSY GR :

We have seen in the preceding section that
$N = 2$ SGM is essentially N=2 NLSUSY action in Riemann-flat (tangent) space-time.
We focus on **N=2 NLSUSY action**.

- The low energy theorem for NLSUSY gives

the following superon(massless NG fermion)-vacuum coupling

$$< \psi^{j}{}_{\alpha}(x)|J^{k\mu}{}_{\beta}|0 >= i\sqrt{\frac{c^4\Lambda}{16\pi G}}(\gamma^{\mu})_{\alpha\beta}\delta^{jk} + \cdots , \tag{57}$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{16\pi G}}\gamma^{\mu}\psi^{k} + \cdots$ is the conserved supercurrent.

$\sqrt{\frac{c^4\Lambda}{16\pi G}} = \frac{1}{\sqrt{2}\kappa}$ **is the coupling constant (**g_{sv}**) of superon with the vacuum**.

- **NLSUSYGR/SGM scenario explains the chiral symmetry of SM**:

The variation of NLSUSY action with respect to $\bar{\psi}$ gives the following equation of motion for the real four component spinor ψ;

$$\partial\!\!\!/\psi - i\kappa^2 \left\{ T^a{}_a \partial\!\!\!/\psi - T^a{}_b \gamma^b \partial_a \psi + \frac{1}{2}(\partial_a T^b{}_b - \partial_b T^b{}_a)\gamma^a \psi \right\}$$
$$-\frac{1}{2}(-i\kappa^2)^2 \epsilon_{abcd}\epsilon^{efgd}(T^a{}_e T^b{}_f \gamma^c \partial_g \psi + T^a{}_e \partial_g T^b{}_f \gamma^c \psi) = 0, \quad (58)$$

where $T^a{}_b = i\kappa^{-2} t^a{}_b = \bar{\psi}\gamma^a \partial_b \psi$. Considering NLSUSY as a whole describes NG fermion ψ, the equation of motion should allow the free massless spin $\frac{1}{2}$ case as is usual in the local field theory :

$$\partial\!\!\!/\psi(x) = 0. \quad (59)$$

Thi is the case, ***provided*** ψ ***is chiral, i.e. ,***

<u>NLSUSY higher order self-interactions constrain the chirality of NG fermion</u>, <u>therefore superons, ,quarks and leptons in NLSUSYGR scenario.</u>

For extracting the low energy particle physics of $N = 2$ SGM (NLSUSY GR) we consider in Riemann-flat space-time, where NL/L SUSY relation(equivalence) gives:

$$L_{N=2\mathrm{SGM}} \longrightarrow L_{N=2\mathrm{NLSUSY}} + [\text{suface terms}] = L_{N=2\mathrm{SUSYQED}}. \tag{60}$$

- We study vacuum structures of $N = 2$ LSUSY QED action in stead of $N = 2$ SGM.

The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\mathrm{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2 \right\} D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \tag{61}$$

• Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A,\phi,B^i) = \frac{1}{2}f^2\left\{A^2 - \phi^2 - \frac{e}{2f}|B^i|^2 - \frac{\xi}{f\kappa}\right\}^2 + \frac{1}{2}e^2(A^2+\phi^2)|B^i|^2 \geq 0. \qquad (62)$$

• Two different types of vacua $V = 0$ exist in (A, ϕ, B^i)-space:

$$(I) \quad A = \phi = 0, \quad |\tilde{B}^i|^2 = -k^2 \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa}\right) \qquad (63)$$

and

$$(II) \quad |\tilde{B}^i|^2 = 0, \quad A^2 - \phi^2 = k^2. \quad \left(\quad k^2 = \frac{\xi}{f\kappa}\right) \qquad (64)$$

• Expansions of $A, \phi, \tilde{B}^i$ around vacuum values give the low energy particle content in the true vacuum which is represented by **the field with the hat symbol**.

- For the type (I) vacuum with $SO(2)$ symmetry for $(\tilde{B}^1, \tilde{B}^2)$, $\quad e\xi > 0$,

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{|\partial_a \hat{B}^1|^2 - 2(-ef)k^2|\hat{B}^1|^2\} \\
& + \frac{1}{2}\{(\partial_a \hat{A})^2 + (\partial_a \hat{\phi})^2 - 2(-ef)k^2(\hat{A}^2 + \hat{\phi}^2)\} \\
& + \frac{1}{2}|\partial_a \hat{B}^2|^2 \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2 v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \sqrt{-2ef}(\bar{\lambda}^1 \chi - \bar{\lambda}^2 \nu) + \cdots,
\end{aligned}
\tag{65}
$$

and following mass spectra

$$
\begin{aligned}
& m^2_{\hat{B}^1} = m^2_{\hat{A}} = m^2_{\hat{\phi}} = m^2_{v_a} = 2(-ef)k^2 = -\frac{2\xi e}{\kappa}, \\
& m_{\lambda^i} = m_{\chi} = m_{\nu} = m_{\hat{B}^2} = 0.
\end{aligned} \tag{66}
$$

- The vacuum breaks both SUSY and the local $U(1)(O(2))$ spontaneously.

($\hat{B}^2$ is the NG boson for the spontaneous breaking of $U(1)$ symmetry and totally gauged away by the Higgs-Kibble mechanism for the $U(1)$ gauge.)

- All bosons have the same mass, and remarkably all fermions remain massless.

- λ^i are not NG fermions of LSUSY. $\leftarrow < \delta\lambda > \sim < D >= 0$

- Off-diagonal mass terms

$\sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) = \sqrt{-2ef}(\bar{\chi}_{\rm D}\lambda + \bar{\lambda}\chi_{\rm D})$ would induce mixings of fermions. $\Rightarrow$ pathological?

• For the type (II) vacuum with $SO(1,1)$ symmetry for (A,ϕ), e.g. $f\xi > 0$,

$$
\begin{aligned}
L_{N=2\text{SUSYQED}} = \; & \frac{1}{2}\{(\partial_a \hat{A})^2 - 4f^2k^2\hat{A}^2\} \\
& + \frac{1}{2}\{|\partial_a \hat{B}^1|^2 + |\partial_a \hat{B}^2|^2 - e^2k^2(|\hat{B}^1|^2 + |\hat{B}^2|^2)\} \\
& + \frac{1}{2}(\partial_a \hat{\phi})^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \partial\!\!\!/ \lambda^i - 2fk\bar{\lambda}^i\lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/\chi + \bar{\nu}\partial\!\!\!/\nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots . \qquad (67)
\end{aligned}
$$

and following mass spectra:

$$
\begin{aligned}
m_{\hat{A}}^2 &= m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \\
m_{\hat{B}^1}^2 &= m_{\hat{B}^2}^2 = m_{\chi}^2 = m_{\nu}^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \\
m_{v_a} &= m_{\hat{\phi}} = 0,
\end{aligned} \tag{68}
$$

which produces **mass hierarchy** by the factor $\frac{e}{f}$ **independent of** κ. $(\kappa^{-2} = \frac{c^4\Lambda}{16\pi G})$

• The vacuum breaks both SUSY and $SO(1,1)$ for (A, ϕ)
and restores(maintains) $SO(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$, spontaneously,

which produces NG-Boson $\hat{\phi}$ and massless photon v_a
and gives soft masses $< A >$ to λ^i.

• We have shown explicitly that

N=2 LSUSY QED, i.e. the matter sector(Λ term) of $N = 2$ SGM (in flat-space), possesses a true vacuum type (II).

• The resulting model describes:

one massive charged Dirac fermion ($\psi_D{}^c \sim \chi + i\nu$),
one massive neutral Dirac fermion ($\lambda_D{}^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) (v_a),
one charged scalar ($\hat{B}^1 + i\hat{B}^2$),
one neutral complex scalar ($\hat{A} + i\hat{\phi}$),

which are composites of superons.

• Remakably, the lepton-Higgs sector of SM analogue $SU(2)_{gl} \times U(1)$ appears without superpartners.

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
as massless composites of NG fermion
from
the NLSUSY cosmological constant of SGM.

4.2 Cosmological meanings of SGM scenario:

• In the composite SGM view of $N = 2$ LSUSY QED,
the vacuum (II) explains naturally observed mysterious (numerical) relations:

(dark) energy density of the universe $\sim {m_\nu}^4 \sim (10^{-12} GeV)^4 \sim {g_{sv}}^2$,

provided ${\lambda_D}^0$ is identified with neutrino [in $d = 4$ as well],
which gives a new insight into the origin of mass.

• Big Decay(BD) induces spontaneous expansion of space-time
due to the quantum mechanical exclusion principle for superon(NG fermion)
and simultaneously forms the gravitational composite massless states of SO(10) sP,
which continues to Big Bang(BB) SM scenario.

6. Summary

NLSUSY GR(SGM) scenario:

• Ultimate entity; **New unstable** $d = 4$ **space-time** U:$[x^a, {\psi_\alpha}^N; x^\mu]$ described by $[L_{\rm NLSUSYGR}(\tilde{e})]$: NLSUSY GR on New space-time with $\Lambda > 0$
• Mach principle is encoded geometrically

$\Longrightarrow$ **Big Decay** (due to false vacuum $V_{\rm P.E.} = \Lambda > 0$) **to** $[L_{\rm SGM}(e.\psi)]$;

• The creation of Riemann space-time $[x^a; x^\mu]$ and massless fermionic matter $[{\psi_\alpha}^{\bf N}]$
$[L_{\rm SGM} = L_{\rm EH}(e) - \Lambda + T(\psi.e)]$: Einstein GR with $V_{\rm P.E.} = \Lambda > 0$ and N superon

$\Longrightarrow$ Formation of gravitational masless composite states:L_{LSUSY}
$\Longrightarrow$ **Ignition of Big Bang Universe**

• Phase transition towards the true vacuum $V_{\rm P.E} = 0$,
achieved by forming composite massless LSUSY and
subsequent oscilations around the true vacuum. $\Longrightarrow$ (MS)**SM**

• In flat space-time, broken N-LSUSY theory emerges from the N-NLSUSY cosmological term of $L_{\rm SGM}(e, \psi)$ [NL/L SUSY relation]. $\longleftrightarrow$ BCS vs GL

The cosmological constant is the origin of everything!

Predictions and Conjectures:

@ Group theory of SO(10) sP with $\underline{10} = \underline{5} + \underline{5}^*$ and
superon-quintet(SQ) hypothesis with $\underline{5} = \underline{5}_{SU(5)GUT}$

- Spin-$\frac{3}{2}$ lepton-type doublet (Γ^-, ν_Γ); Doubly charged spin 1/2 particles $E^{2\pm}$
- neutral $J^P = 1^-$ boson S.
- Proton decay diagrams in GUTs are forbidden by selection rules. $\Rightarrow$ stable proton
- Neutrino problems(mass and oscillation) are gravitational(composite) origin.

@Field theory via Linearization:

- Chirality in SM may be a NLSUSY higher order self interaction effrect.
- NLSUSY GR(SGM) scenario predicts **4 dimensional space-time**.
- The bare gauge coupling constant is determined.
- N-LSUSY from N-NLSUSY $\Longleftrightarrow$ SQ hypothesis for all particles
- Superfluidity of space-time.

cosmological term $\leftrightarrow$ dark energy density $\leftrightarrow$ SUSY Br. $\rightarrow m_\nu$

Many Open Questions ! e.g.,

- Direct linearization of SGM action in curved space-time.
- Superfield systematics of NL/L SUSY relation for SGM action.
- What is the broken SUGRA-like(?) equivalent theory?
- Complete the detour of No-Go Th.! (High-spin fields in the linearized $N = 10$ theory.)

- Revisit unsolved problems of SMs and GUT **from SQM composite viewpoints**.

e.g., (e,ν_e): $\epsilon^{lm} Q_l Q_m Q^*{}_n$, (u,d): $\epsilon^{abc} Q_b Q_c Q_m$, (c,s): $\epsilon^{lm} Q_l Q_m \epsilon^{abc} Q_b Q_c Q^*{}_n, \cdots$

- SGM scenario suggests $N \geq 2$ low energy MSSM, SUSY GUT, without R-Parity?
- Effects of colored exotic particles in the low energy physics

- Superfluidity of sapce-time and matter?

- Equivalence principle and NLSUSYGR.

- The role of duality.

- Physical consequences of spin $\frac{3}{2}$ NLSUSYGR.