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POSITIVE MASS AS A PRINCIPLE ?

Tetsuya Shiromizu

Department of Mathematics/Kobayashi-Maskawa Institute

Nagoya University

Collaborator: Masato Nozawa(Unviversita di Milano)

Based on

Nozawa and Shiromizu, Physical Review D89, 023011(2014)

Nozawa and Shiromizu, Nuclear Physics B887, 380(2014)

KOBAYASHI-MASKAWA INSTITUTE FOR THE ORIGIN OF PARTICLES AND THE UNIVERSE

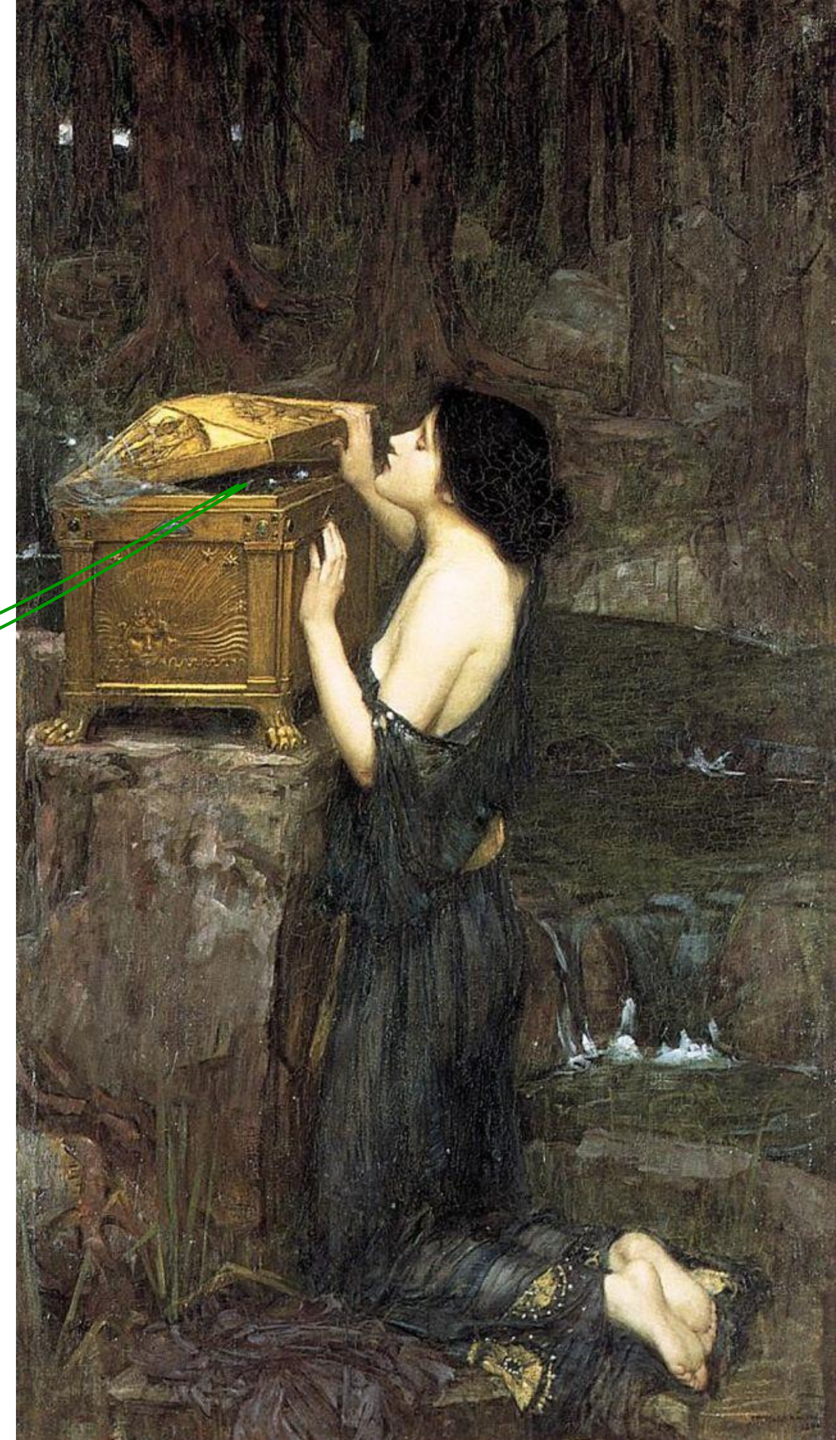
NAGOYA UNIVERSITY

CONTENTS

1. INTRODUCTION

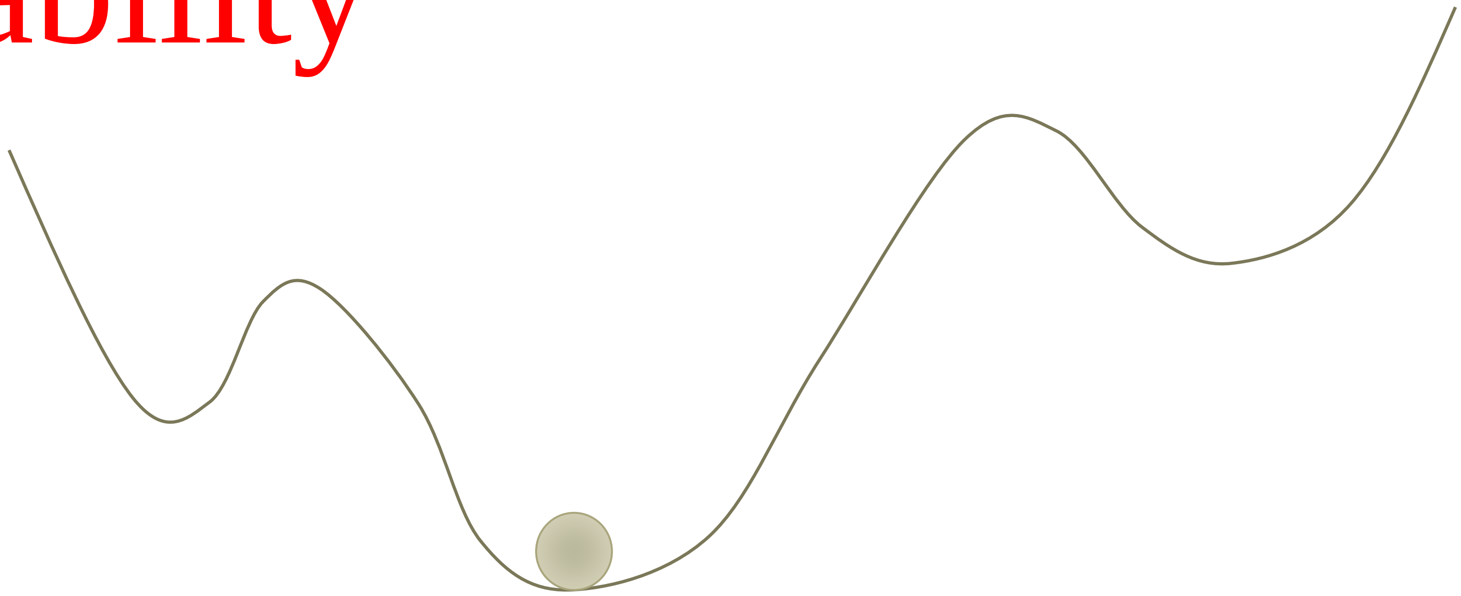
PANDORA'S BOX WAS OPENED?

Cosmological constant, Galileon, Chamaeleon, massive gravity, f(R), higher curvature, Horava-Lifshitz, DGP braneworld, Tracker, K-essence, …



STABILITY SHOULD BE REQUIRED

Stability



A CELEBRATED THEOREM IN GR

POSITIVE MASS THEOREM

Schoen&Yau 1981, Witten 1981

- Asymptotic flat and regular spacetime
- Einstein equation
- (dominant) energy condition(energy density of matter is non-negative)

⇒ Total mass is non-negative $M \geq 0$

Spacetime is flat if and only if $M = 0$

Positive mass theorem guarantees the (kinematical) stability **in non-linear level**

POSITIVE MASS AS A PRINCIPLE ?

Our proposal

Positivity of total mass should hold in relevant theories.

2. A PROPOSAL AND RESULT

A HINT

Scalar potentials consistent with positive mass

Boucher 1984, Townsend 1985

Lagrangian

$$L = R - \frac{1}{2}(\nabla\phi)^2 - U(\phi)$$

Potential for scalar field

$$\Rightarrow \quad U(\phi) = 8\left(\frac{dW(\phi)}{d\phi}\right)^2 - 12\left(W(\phi)\right)^2$$

A DEMONSTRATION

Nozawa & Shiromizu 2014

Starting point

$$S = \int d^4x \sqrt{-g}\left[R + 2K(\phi, X) + L_{matter}\right]$$

$$X = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \sim \dot{\phi}^2$$

Theory compatible with positive mass

$$K = X - U(\phi) = X - 8\left(\frac{dW(\phi)}{d\phi}\right)^2 + 12\left(W(\phi)\right)^2$$

POINT

Look at the original proof (Witten's version)

$$\nabla\varepsilon \supset \Gamma\varepsilon \sim \partial g\varepsilon \sim \frac{M}{r^2}\varepsilon$$

ε : spinor

$$M = \int_{S_\infty} dS_i \varepsilon^+ \nabla^i \varepsilon = \int_\Sigma \left(|\nabla_i \varepsilon|^2 + T_{00}^{matter} \right) d\Sigma$$

Gauss theorem
Einstein equation

There is a room to modify the mass formula.

For example, we can replace ∇ by another operator.

3. EINSTEIN-SCALAR SYSTEM

Nozawa & Shiromizu 2014

MODEL

action

$$S=\int d^4x\sqrt{-g}\left[R+2K(\phi,X)+2L_{matter}\right]$$

$$X=-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

Einstein equation

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G(T^{(\phi)}_{\mu\nu}+T^{(matter)}_{\mu\nu})$$

$$T^{(\phi)}_{\mu\nu}=\partial_X K\nabla_\mu\phi\nabla_\nu\phi+Kg_{\mu\nu}$$

In general it does not satisfy energy conditions.

$T^{(matter)}_{\mu\nu}$ is supposed to satisfy energy condition

A ROOM TO MODIFY

Dirac-Witten equation $\gamma^i \nabla_i \varepsilon = 0, \quad \varepsilon : \text{spinor}$

γ^μ : Dirac matrix

$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}$

$$\nabla_\mu \to \hat{\nabla}_\mu$$

$$\hat{\nabla}_\mu \varepsilon = (\nabla_\mu + A_\mu)\varepsilon, \; \gamma^{\mathrm{i}} \hat{\nabla}_i \varepsilon = 0$$

It is supposed that additional terms decays rapidly near infinity so that it dose not contribute to mass

MODIFIED NESTER TENSOR

$$\hat{N}^{\mu\nu} := -i\left(\bar{\varepsilon}\gamma^{\mu\nu\rho}\hat{\nabla}_\rho\varepsilon - \overline{\hat{\nabla}_\rho\varepsilon}\gamma^{\mu\nu\rho}\varepsilon\right)$$

$$\bar{\varepsilon} := i\varepsilon^+\gamma^0$$

$$\gamma_{\mu\nu\alpha} = \gamma_{[\mu}\gamma_\nu\gamma_{\alpha]}$$

$$\nabla_\nu\hat{N}^{\mu\nu} = 2i\overline{\hat{\nabla}_\rho\varepsilon}\gamma^{\mu\nu\rho}\hat{\nabla}_\nu\varepsilon - G^\mu_\nu V^\nu - i\bar{\varepsilon}\gamma^{\mu\nu\rho}F_{\nu\rho}\varepsilon$$

$$-i\bar{\varepsilon}(\bar{A}_\nu\gamma^{\mu\nu\rho} - \gamma^{\mu\nu\rho}A_\nu)\hat{\nabla}_\rho\varepsilon + i\overline{\hat{\nabla}_\rho\varepsilon}(\bar{A}_\nu\gamma^{\mu\nu\rho} - \gamma^{\mu\nu\rho}A_\nu)\varepsilon$$

Sign is not under control

$$V^\mu = i\bar{\varepsilon}\gamma^\mu\varepsilon$$

$$F_{\mu\nu} = 2(\nabla_{[\mu}A_{\nu]} + A_{[\mu}A_{\nu]})$$

$$\bar{A}_\mu = \gamma^0 A^+_\mu\gamma^0$$

- Requirement("positivity condition") $\bar{A}_\nu\gamma^{\mu\nu\rho} = \gamma^{\mu\nu\rho}A_\nu$

- $\nabla_\nu\hat{N}^{\mu\nu} = 2i\hat{\nabla}_\rho\varepsilon\gamma^{\mu\nu\rho}\hat{\nabla}_\nu\varepsilon - G^\mu_\nu V^\nu - i\bar{\varepsilon}\gamma^{\mu\nu\rho}F_{\nu\rho}\varepsilon$

MASS FORMULA

$$\hat{\nabla}_\mu \varepsilon = (\nabla_\mu + A_\mu)\varepsilon, \ \gamma^{\mathrm{i}} \hat{\nabla}_i \varepsilon = 0 \qquad \bar{A}_\nu \gamma^{\mu\nu\rho} = \gamma^{\mu\nu\rho} A_\nu$$

$$8\pi GM = \int_\Sigma d\Sigma \left[-2i\overline{\hat{\nabla}_\rho \varepsilon} \gamma^{\mu\nu\rho} \hat{\nabla}_\nu \varepsilon + G^\mu_\nu V^\nu - S^\mu \right] u_\mu$$

$$V^\mu = i\bar{\varepsilon}\gamma^\mu \varepsilon$$

$$S^\mu := -i\bar{\varepsilon}\gamma^{\mu\nu\rho} F_{\mu\nu} \varepsilon$$

$$F_{\mu\nu} := 2(\nabla_{[\mu} A_{\nu]} + A_{[\mu} A_{\nu]})$$

u^μ is the unit normal vector to Σ

STRATEGY

$$\gamma^i \hat{\nabla}_i \varepsilon = 0$$

$$M \sim \int_{S_\infty} dS_i \varepsilon^+ \hat{\nabla}^i \varepsilon = \int_\Sigma \nabla_i (\varepsilon^+ \hat{\nabla}^i \varepsilon) d\Sigma$$

$$T^{(\phi)}_{\mu\nu} = \partial_X K \nabla_\mu \phi \nabla_\nu \phi + K g_{\mu\nu}$$

$$= \int_\Sigma \left(|\hat{\nabla} \varepsilon|^2 + T^{matter}_{00} + T^{(\phi)}_{00} + S^0 \right) d\Sigma$$

Einstein equation

Try to construct a theory/fix Aμ so that this part will be non-negative

$$S^\mu := -i \bar{\varepsilon} \gamma^{\mu\nu\rho} F_{\mu\nu} \varepsilon$$

$$F_{\mu\nu} := 2(\nabla_{[\mu} A_{\nu]} + A_{[\mu} A_{\nu]})$$

CANDIDATE FOR A_μ

$$M = \int_\Sigma \left(|\hat{\nabla}\varepsilon|^2 + T_{00}^{matter} + T_{00}^{(\phi)} + S^0 \right) d\Sigma$$

$$\phi, \nabla\phi \in T_{\mu\nu}^{(\phi)}$$

$$\partial A, A^2 \in S^\mu$$

As almost unique candidate satisfying the positivity condition $\bar{A}_\nu \gamma^{\mu\nu\rho} = \gamma^{\mu\nu\rho} A_\nu$

$$A_\mu = W(\phi)\gamma_\mu$$

EXPRESSION FOR S^{μ}

$$M = \int_{\Sigma} \left(|\hat{\nabla}\varepsilon|^2 + T_{00}^{matter} + T_{00}^{(\phi)} + S^0 \right) d\Sigma$$

$$A_{\mu} = W(\phi)\gamma_{\mu}$$

$$\delta\lambda := \frac{1}{\sqrt{2}}\left(f(\phi, X)\gamma^{\mu}\nabla_{\mu}\phi - 4 f^{-1}(\phi, X)\frac{dW(\phi)}{d\phi} \right)\varepsilon$$

$$\begin{aligned} S^{\mu} &= -i\bar{\varepsilon}\gamma^{\mu\nu\rho}F_{\mu\nu}\varepsilon \\ &= -4i\bar{\varepsilon}\gamma^{\mu\nu}\varepsilon\nabla_{\nu}\phi\partial_{\phi}W + 12V^{\mu}W^2 \\ &= i\overline{\delta\lambda}\gamma^{\mu}\delta\lambda + V^{\nu}\left[f^2\nabla^{\mu}\phi\nabla_{\nu}\phi + \delta_{\nu}^{\mu}\left(-\frac{1}{2} f^2(\nabla\phi)^2 - 8 f^{-2}(\partial_{\phi}W)^2 + 12W^2 \right) \right] \end{aligned}$$

$$T_{\mu\nu}^{(\phi)} = \partial_X K \nabla_{\mu}\phi\nabla_{\nu}\phi + Kg_{\mu\nu}$$

Look for K such that they are cancelled out each other

REQUIREMENT 2

$$\partial_X K = f^2$$

$$K = f^2 X - 8 f^{-2}\left(\frac{dW(\phi)}{d\phi}\right)^2 + 12\big(W(\phi)\big)^2$$

$$\Rightarrow \quad T_{00}^{(\phi)} + S^0 = |\delta\lambda|^2 \geq 0$$

CONSTRAINT FOR THEORY

$$\begin{cases} K_X := \partial_X K = f^2 \\ K = f^2 X - 8 f^{-2} \left(\dfrac{dW(\phi)}{d\phi} \right)^2 + 12 \left(W(\phi) \right)^2 \end{cases}$$

$$\Rightarrow \quad X K_X - K - \frac{8W_\phi^2}{K_X} = -12 W(\phi)^2$$

$$\Rightarrow \quad \partial_X \left(X K_X - K - \frac{8W_\phi^2}{K_X} \right) = K_{XX} \left(X + \frac{8W_\phi^2}{K_X^2} \right) = 0, \quad W_\phi := \frac{dW}{d\phi}, K_{XX} := \partial_X^2 K$$

$$\Rightarrow \quad (i) K_{XX} = 0 \quad \Rightarrow \quad K = X - U(\phi) = X - 8 \left(\frac{dW(\phi)}{d\phi} \right)^2 + 12 \left(W(\phi) \right)^2$$

$$(ii) X + \frac{8W_\phi^2}{K_X^2} = 0 \quad \Rightarrow \quad K = 4\sqrt{2} \frac{dW(\phi)}{d\phi} (-X)^{1/2} + 12 \left(W(\phi) \right)^2$$

CASE (II)

$$K = 4\sqrt{2}\,\frac{dW(\phi)}{d\phi}(-X)^{1/2} + 12\big(W(\phi)\big)^2$$

Homogeneous-isotropic universe

$$\phi = \phi(t) \quad \Rightarrow X = \dot{\phi}^2/2 > 0$$

factor $(-X)^{1/2}$ is pure imaginary

Case (ii) does not work for cosmology

SUMMARY

Nozawa & Shiromizu 2014

action

$$S = \int d^4x\sqrt{-g}\left[R + 2K(\phi, X) + L_{matter}\right] \qquad X = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

(i)

$$K = X - U(\phi) = X - 8\left(\frac{dW(\phi)}{d\phi}\right)^2 + 12\left(W(\phi)\right)^2$$

Canonical form with “superpotential”

(ii)

$$K = 4\sqrt{2}\,\frac{dW(\phi)}{d\phi}(-X)^{1/2} + 12\left(W(\phi)\right)^2$$

No cosmological solution

4. SUMMARY AND DISCUSSION

OUR PROPOSAL FOR DARK ENERGY

Positive mass may play as one of principle to determine the model for dark energy etc.

REMAINING ISSUES

- unique way? $\bar{A}_\nu \gamma^{\mu\nu\rho} = \gamma^{\mu\nu\rho} A_\nu, A_\mu = W(\phi)\gamma_\mu$
- more general cases for dark energy?
- modified gravity？Non-linear massive gravity,….
- What happens if we employ Schoen-Yau’s proof?
- positive mass for asymptotically deSitter spacetime?

AN EXTENSION

Elder, Jyoce, Khoury & Tolley, PRD91(2015)

$$M = \int_\Sigma \left(|\hat{\nabla}\varepsilon|^2 + T_{00}^{matter} + |\delta\lambda|^2 + |\delta\lambda_2|^2 \right) d\Sigma$$

✔

$$\delta\lambda_2 = G(X,\phi)\varepsilon$$

$$K \to K(\phi, X) = f^2 X - 8 f^{-2} \left(\frac{dW(\phi)}{d\phi} \right)^2 + 12 (W(\phi))^2 - (G(X,\phi))^2$$

✔

Our subsequent argument does not work in general…

AN EXTENSION: BACK TO BOUCHER & TOWNSEND

Elder, Jyoce, Khoury & Tolley, PRD91(2015)

Embedded into multi field

Boucher 1984, Townsend 1985

$$L = -\frac{1}{2} f_{IJ}(\phi)\partial_\mu \phi^I \partial^\mu \phi^J - V(\phi^I)$$

f_{IJ} : positive definite, $V = 8 f^{IJ} W_I W_J - 12W^2$

$\Rightarrow$ Positive mass theorem holds

$$L = -\frac{1}{2} P_\chi (\partial\phi)^2 - \frac{1}{2} Z^2 (\partial\chi)^2 - \chi P_\chi + P(\phi, \chi)$$

Comparison to BT $\Rightarrow$ $-P + \chi P_\chi = 8\frac{W_\phi^2}{P_\chi} + 8\frac{W_\chi^2}{Z^2} - 12W^2$

Turn off $\Rightarrow$ $Z \to 0, W = w(\phi) + \frac{Z}{2\sqrt{2}} G(\chi,\phi) + O(Z^2), \chi = X = -(\partial\phi)^2/2$