A unifying description of dark energy

Filippo Vernizzi - IPhT, CEA Saclay

Based on:

- 1504.05481 with J. Gleyzes M. Mancarella and D. Langlois
- 1411.3712 with J. Gleyzes and D. Langlois
- 1304.4840 with J. Gleyzes, D. Langlois and F. Piazza
- 1210.0201 with G. Gubitosi and F. Piazza

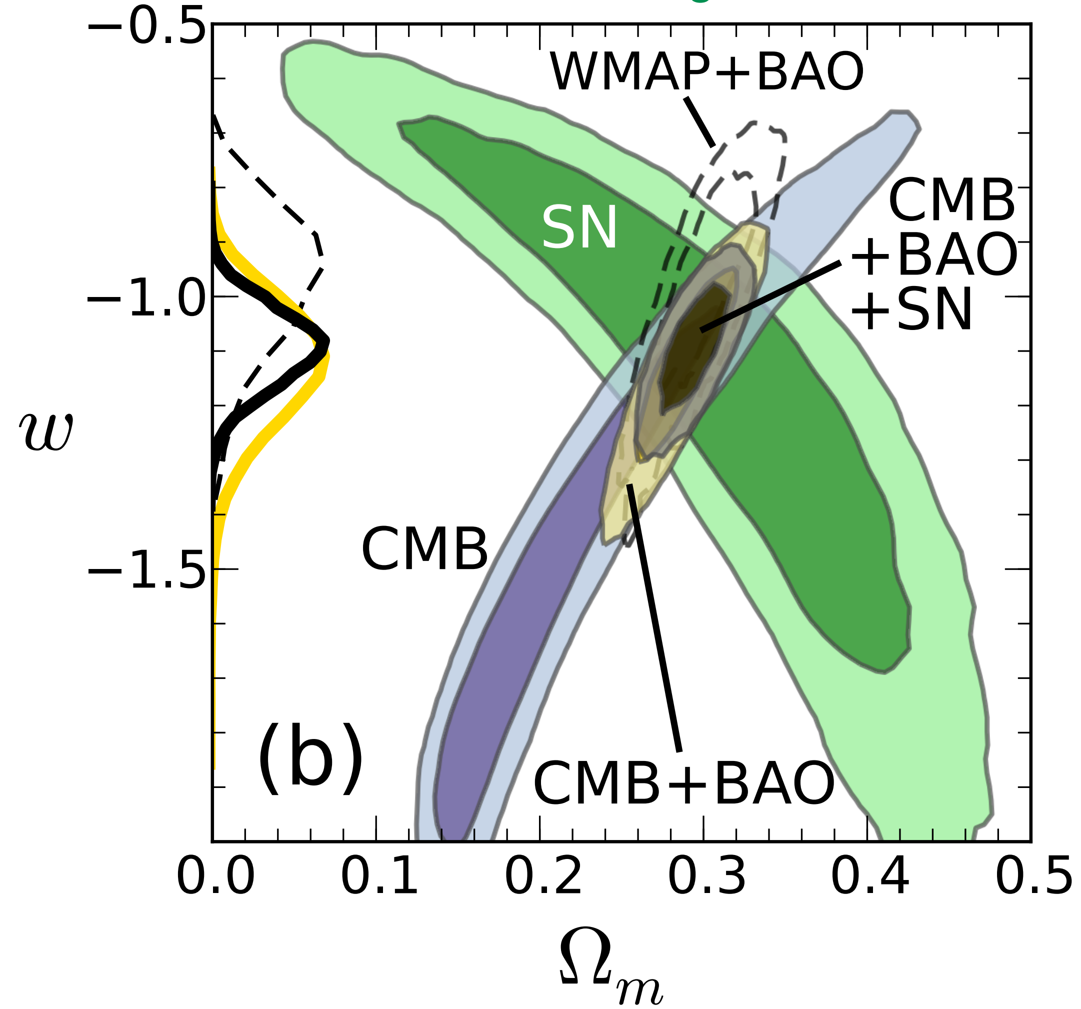
ICISE, Quy Nhon - August 13, 2015

Standar Model: ΛCDM

- Observations well consistent with LCDM

$$w \equiv \frac{P_{\rm DE}}{\rho_{\rm DE}} = -1.019^{+0.075}_{-0.080} \quad (95\%) \quad \text{Planck+BAO+SN}$$

Mortonson, Weinberg, White '13



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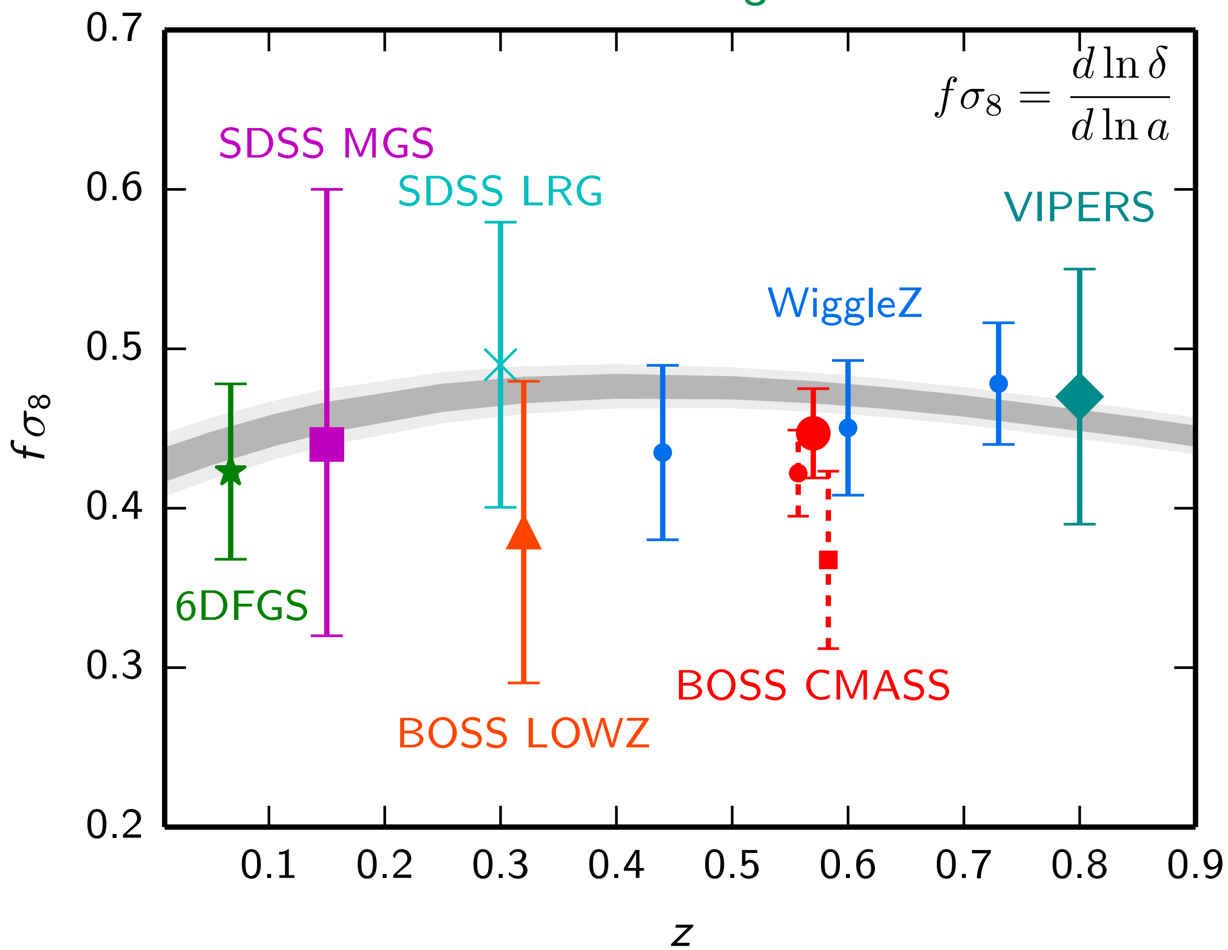
Planck '15 "Cosmological Parameters"



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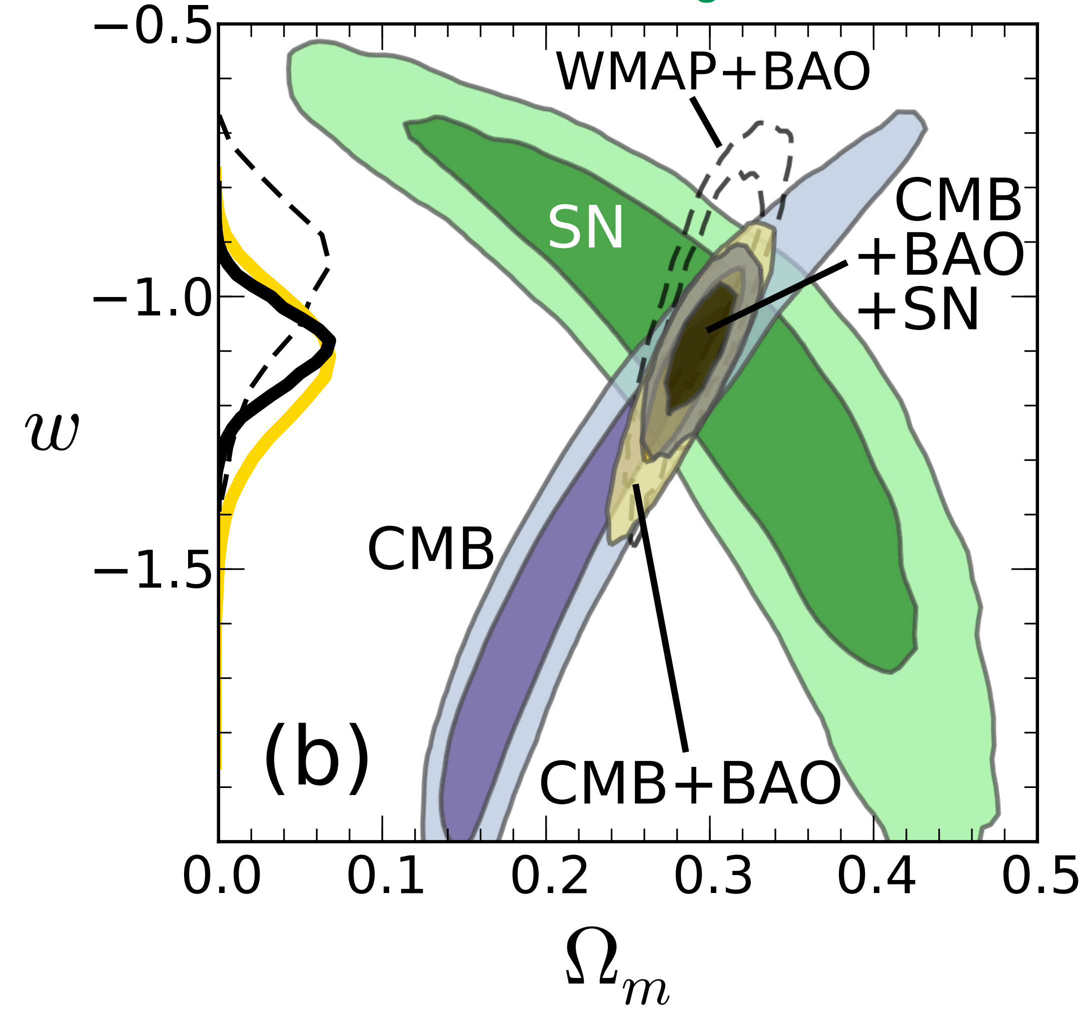

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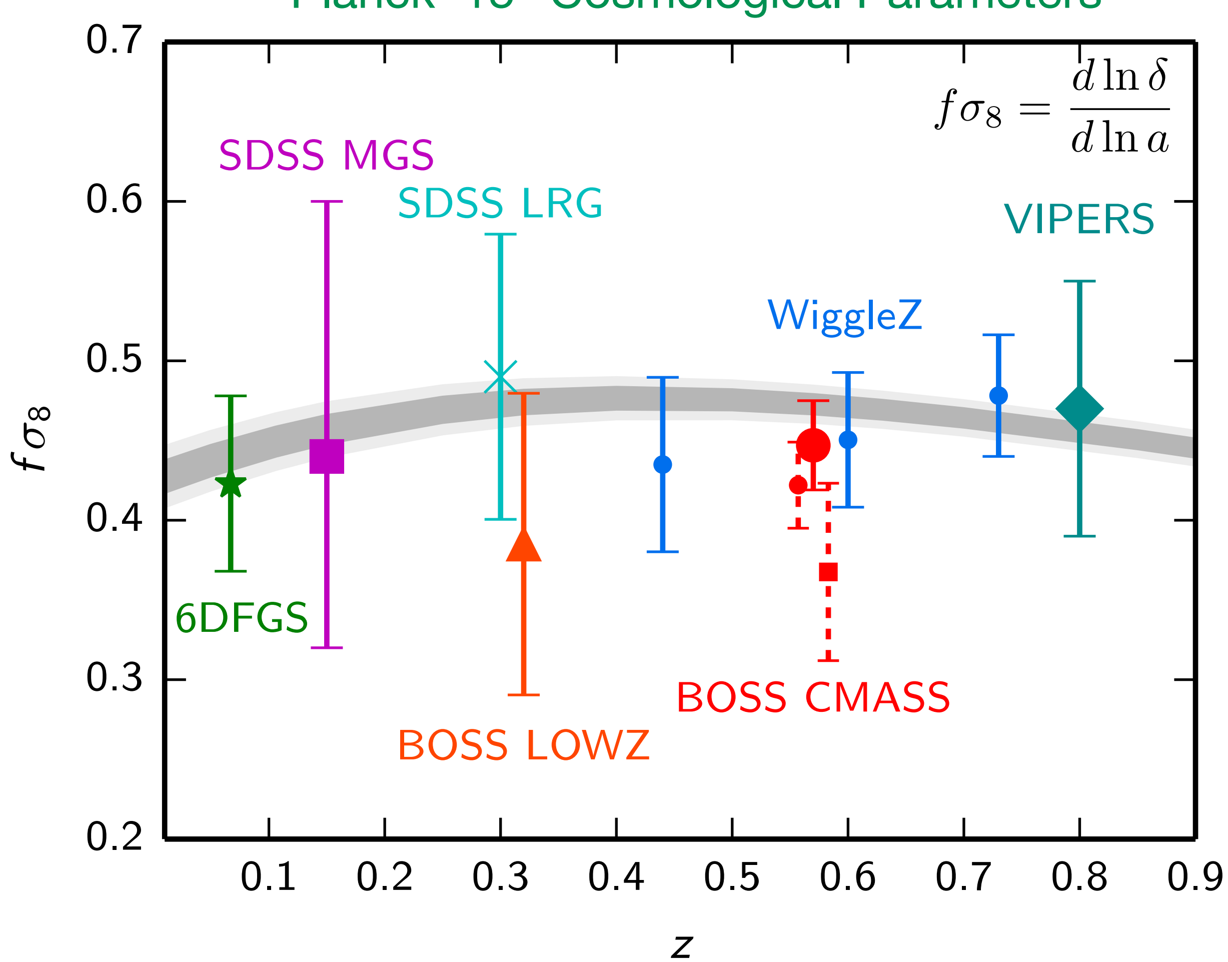

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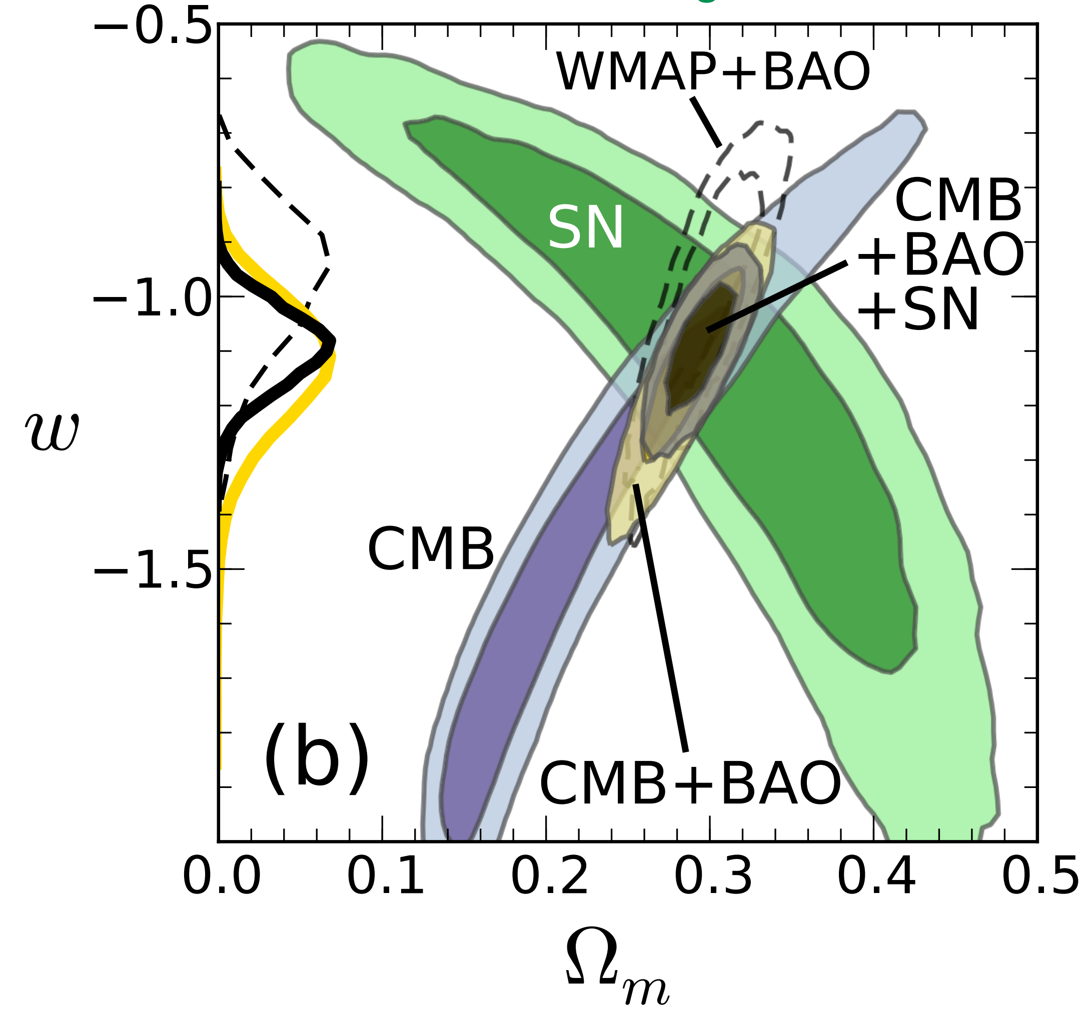

Λ?

Motivations

- New dynamics imply time/space deviations w.r.t. GR
  - Time evolution: $\phi = \phi(t)$
  - Spatial fluctuations: $\phi = \phi(t, \vec{x})$. Fluid dynamics, pressure, speed of sound, stresses, etc.

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- Small scales: stringent solar system tests (screening)
- Large scales: Linear regime is applicable. Growth of structure is not unique.

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New physics in time/scale dependent modifications of growth

- Current and future surveys will accurately measure the growth history of LSS. Expected 1-2 order-of-magnitude improvement over larger redshift range.
- Given current models, democratic bridging of theoretical modelling with observations: unifying and effective treatment.

Effective approach

- No redundancies: minimal action. Theories that share
  - same physical degrees of freedom
  - same interactions (e.g. to matter)
  - same regime of validity

are the same

Effective approach

- No redundancies: minimal action. Theories that share
  - same physical degrees of freedom — single scalar field fluctuations
  - same interactions (e.g. to matter) — universal couplings
  - same regime of validity — linear regime, …

are the same

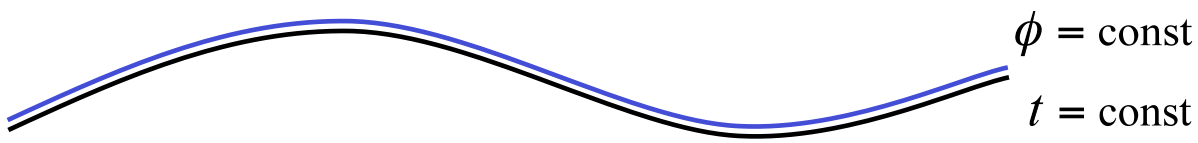
Constructing the action

1. Scalar field breaks time diffs; gravitational action preserves spatial diffs

- ADM (3+1) decomposition in unitary gauge:

Creminelli et al. '06; Cheung et al. `07

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$



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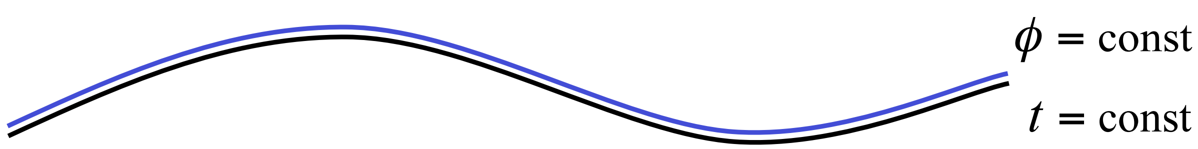

2. Action: all terms that respect spatial diffs in the action (Jordan frame)

$$S = \int d^4x \sqrt{-g} L[t; N, K^i_j, {}^{(3)}R^i_j, \ldots]$$

Lapse	N	*time kinetic energy of scalar*	$\sim \dot{\phi}$
Extrinsic curvature	K_{ij}	*time kinetic energy of metric*	$\sim \partial_t g_{ij}$
Intrinsic 3d curvature	${}^{(3)}R_{ij}$	*spatial kinetic energy of metric*	$\sim \partial_k g_{ij}$

$$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Examples

- General Relativity:

$$L_{\rm GR} = \frac{M_{\rm Pl}^2}{2}\, {}^{(4)}R$$

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$$L_{\rm GR} = \frac{M_{\rm Pl}^2}{2}\underbrace{\left[K_{ij}K^{ij} - K^2 + {}^{(3)}R\right]}_{{}^{(4)}R \text{ by Gauss-Codazzi}}$$

Examples

- General Relativity:

$$L_{\rm GR} = \frac{M_{\rm Pl}^2}{2}\left[K_{ij}K^{ij} - K^2 + {}^{(3)}R\right]$$

- General Relativity + minimal quintessence:

$$L_{\rm GR+Q} = \left[\frac{M_{\rm Pl}^2}{2}\,{}^{(4)}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

Examples

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- General Relativity + minimal quintessence:

$$L_{\rm GR+Q} = \left[\frac{M_{\rm Pl}^2}{2}(K_{ij}K^{ij} - K^2 + {}^{(3)}\!R) + \boxed{\frac{c(t)}{2N^2} - V(t)}\right]$$

Scalar kinetic term $\quad -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \quad \to \quad -\frac{1}{2}g^{00} = \frac{\dot\phi_0^2(t)}{2N^2}$

Scalar potential term $\quad V(\phi) \quad \to \quad V(t)$

Examples

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- *f(R)*:

$$L_{f(R)} = \frac{M_{\rm Pl}^2}{2}\, f\left({}^{(4)}R\right)$$

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- *f(R)*:

$$L_{f(R)} = \frac{M_{\rm Pl}^2}{2} f'(t)\left[K_{ij}K^{ij} - K^2 + {}^{(3)}\!R + \boxed{2\frac{f''(t)}{f'(t)}\frac{K}{N}} + V(t)\right]$$

Scalar kinetic term comes from mixing with metric: **braiding**

Deffayet et al. '10

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$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

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$$S = \int d^4x \sqrt{-g} L[t; N, K^i_j, {}^{(3)}R^i_j, \ldots]$$

3. Expand at quadratic order (i.e. linear theory)

   ▶ 3-d tensors: $\delta N \equiv N - 1\,,\qquad \delta K_{ij} \equiv K_{ij} - H h_{ij}\,,\qquad {}^{(3)}R_{ij}$

$$L(N, K^i_j, R^i_j, \ldots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K^i_j}\delta K^i_j + \frac{\partial L}{\partial R^i_j}\delta R^i_j + L^{(2)} + \ldots$$

Second-order Lagrangian

$$L^{(2)} = \frac{1}{2} L_{NN} \delta N^2 + \frac{1}{2} \frac{\partial^2 L}{\partial K^i_j \partial K^k_l} \delta K^i_j \delta K^k_l + \frac{1}{2} \frac{\partial^2 L}{\partial R^i_j \partial R^k_l} \delta R^i_j \delta R^k_l +$$

$$+ \frac{\partial^2 L}{\partial K^i_j \, \partial R^k_l} \delta K^i_j \delta R^k_l + \frac{\partial^2 L}{\partial N \partial K^i_j} \delta N \delta K^i_j + \frac{\partial^2 L}{\partial N \partial R^i_j} \delta N \delta R^i_j + \ldots$$

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4. Remove higher time and space derivatives and define convenient coefficients (using Bellini & Sawicki notation) 1404.3713 Bellini & Sawicki

$$S^{(2)} = \int d^4x \, a^3 \, \frac{M^2(t)}{2} \Big[\delta K_{ij} \delta K^{ij} - \delta K^2 + \delta_2 \big(\sqrt{h}/a^3 \, {}^{(3)}\!R \big) + \delta N \, {}^{(3)}\!R$$
$$+ \alpha_K(t) H^2(t) \, \delta N^2 + 4 \alpha_B(t) H(t) \, \delta N \delta K + \alpha_T(t) \, \delta_2 \big(\sqrt{h}/a^3 R \big) + \alpha_H(t) \, \delta N \, {}^{(3)}\!R \Big]$$

1304.4840 with Gleyzes, Langlois, Piazza

Most general second-order action without higher (spatial and time) derivatives

Second-order Lagrangian

$$L^{(2)} = \frac{1}{2} L_{NN} \delta N^2 + \frac{1}{2} \frac{\partial^2 L}{\partial K^i_j \partial K^k_l} \delta K^i_j \delta K^k_l + \frac{1}{2} \frac{\partial^2 L}{\partial R^i_j \partial R^k_l} \delta R^i_j \delta R^k_l +$$
$$+ \frac{\partial^2 L}{\partial K^i_j \, \partial R^k_l} \delta K^i_j \delta R^k_l + \frac{\partial^2 L}{\partial N \partial K^i_j} \delta N \delta K^i_j + \frac{\partial^2 L}{\partial N \partial R^i_j} \delta N \delta R^i_j + \dots$$

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1404.3713 Bellini & Sawicki

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1304.4840 with Gleyzes, Langlois, Piazza

Most general second-order action without higher (spatial and time) derivatives

- For $\dot{M} = \alpha_i = 0$ second-order action for General Relativity
- Deviations from GR (LCDM) on linear scales independent of background evol.

Building blocks of dark energy

$$S^{(2)} = \int d^4x \, a^3 \, \frac{M^2(t)}{2} \Big[\delta K_{ij}\delta K^{ij} - \delta K^2 + \delta_2\big(\sqrt{h}/a^3 \, {}^{(3)}R\big) + \delta N \, {}^{(3)}R$$

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* General Relativity (LCDM)

	α_K	α_B	α_M	α_T	α_H

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✳ Standard kinetic term: quintessence, *k*-essence $P\big(\phi, (\partial\phi)^2\big)$

α_K parametrizes **kineticity** of dark energy ~ (1+w) Ω_{DE} / c_s^2

	kineticity α_K	α_B	α_M	α_T	α_H
quintessence, k-essence	✓				

Building blocks of dark energy

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* Kinetic braiding: DGP, KGB $\quad K\big(\phi, (\partial\phi)^2\big)\Box\phi$

α_B parametrizes **braiding** (kinetic mixing with gravity)

	kineticity α_K	kinetic braiding α_B	α_M	α_T	α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			

Building blocks of dark energy

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* Non-minimal couplings: Brans-Dicke, f(R) $\quad f(\phi)R\,,\ f(R)\,,\ f(G)$

$$\alpha_M = \frac{d\ln M^2}{H\,dt}$$

parametrizes **non-minimal coupling** to *R* (ex: $\alpha_M=-2\alpha_B$ in *f*(*R*))

	kineticity α_K	kinetic braiding α_B	non-minimal coupling α_M	α_T	α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		

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* Enhanced tensor sound speed: all Horndeski theories

α_T parametrizes deviation from **tensor sound-speed** = *c*

	kineticity α_K	kinetic braiding α_B	non-minimal coupling α_M	tensor sound-speed α_T	α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	

Building blocks of dark energy

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✳ Kinetic mixing with matter: beyond Horndeski theories

α_H parametrizes **extensions of Horndeski** theories

	kineticity α_K	kinetic braiding α_B	non-minimal coupling α_M	tensor sound-speed α_T	kinetic mixing with matter α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	
Beyond Horndeski	✓	✓	✓	✓	✓

Building blocks of dark energy

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✳ Kinetic mixing with matter: beyond Horndeski theories

α_H parametrizes **extensions of Horndeski** theories

▶ Consistent nonlinear theories **beyond Horndeski**: two extra functions of ϕ and X

1404.6495 & 1408.1952 with Gleyzes, Langlois, Piazza

confirmed by 1408.0670 Lim, Mukohyama, Namba, Saitou
and 1506.01974 Deffayet, Esposito-Farese, Steer

New unexplored territory!

Building blocks of dark energy

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α_H parametrizes **extensions of Horndeski** theories

- Stability conditions:

	Scalar	Tensor
No ghosts	$\alpha_K + 6\alpha_B^2 > 0$	$M^2 > 0$
No gradient instability	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

Theoretical restriction on the parameter space

Universal couplings

- Horndeski case ($\alpha_H = 0$):

$$S_{\rm gravity} = \int d^4x \mathcal{L}_g(g_{\mu\nu}; \alpha_K, \alpha_B, \alpha_M, \alpha_T)$$

- Equivalence Principle. All species are coupled to the same metric:

For each species: $$S_{\rm m} = \int d^4x \sqrt{-g}\mathcal{L}(g_{\mu\nu}, \psi_{\rm m})$$

Non-universal couplings

1504.05481 with Gleyzes, Langlois, Mancarella

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$$S_{\rm gravity} = \int d^4x \mathcal{L}_g(g_{\mu\nu}; \alpha_K, \alpha_B, \alpha_M, \alpha_T)$$

- ~~Equivalence Principle~~. Species are coupled to different metrics:

For each species: $$S_{\rm m} = \int d^4x \sqrt{-g} \mathcal{L}(\tilde{g}_{\mu\nu}, \psi_{\rm m})$$

$$\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi) \partial_\mu \phi \partial_\nu \phi$$

Two new parameters per species: $$\alpha_C \equiv \frac{1}{2} \frac{d \ln C}{d \ln a} \qquad \alpha_D \equiv \frac{D}{C - D}$$

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Two new parameters per species: $$\alpha_C \equiv \frac{1}{2}\frac{d\ln C}{d\ln a} \qquad \alpha_D \equiv \frac{D}{C-D}$$

- Structure of Horndeski invariant under the above metric transformation

Bettoni and Liberati '12

Parameter-space rotation

1504.05481 with Gleyzes, Langlois, Mancarella

- Total of 4 + 2 N_S parameters:

$$S_{\text{gravity}} = \int d^4x \mathcal{L}_g(g_{\mu\nu}; \alpha_K, \alpha_B, \alpha_M, \alpha_T)$$

$$S_{\text{matter}} = \sum_I^{\mathrm{N_S}} \int d^4x \sqrt{-g} \mathcal{L}_I \left(g_{\mu\nu}; \alpha_{C,I}, \alpha_{D,I}; \psi_I\right)$$

- With a rotation in parameter space, $\tilde{\alpha}_i = \mathcal{F}_i(\alpha_j)$, we can choose a base where one of the species is minimally coupled: 4 + 2 N_S -2 = 2 (N_S + 1)

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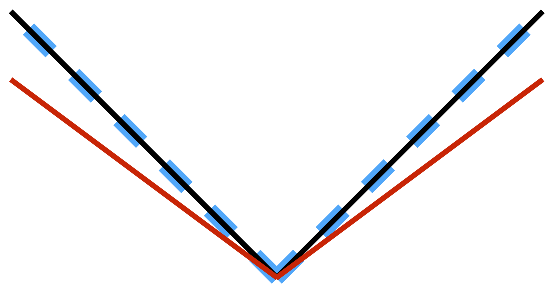
- Ghost and gradient stability conditions are invariant under rotation in par. space



- Observables invariant. Example: $$\frac{\tilde{c}_I^2}{\tilde{c}_J^2} = \frac{c_I^2}{c_J^2}$$

- Inflation: no matter (N_S = 0). We have 2 independent parameters, ex. α_K and α_B

δN^2 , $\delta N \delta K$

1407.8439 with Creminelli, Gleyzes, Noreña

Parameter-space rotation

1504.05481 with Gleyzes, Langlois, Mancarella

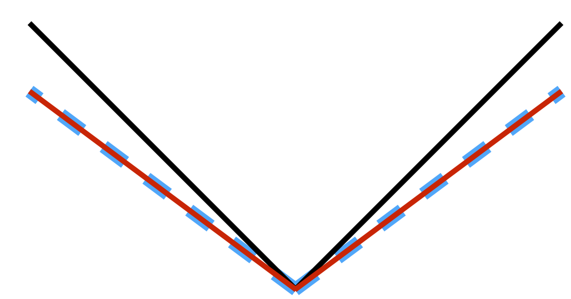
- Total of 4 + 2 N_S parameters:

$$S_{\text{gravity}} = \int d^4x \mathcal{L}_g(g_{\mu\nu}; \alpha_K, \alpha_B, \alpha_M, \alpha_T)$$

$$S_{\text{matter}} = \sum_I^{\mathrm{N_S}} \int d^4x \sqrt{-g} \mathcal{L}_I \left(g_{\mu\nu}; \alpha_{C,I}, \alpha_{D,I}; \psi_I\right)$$

- With a rotation in parameter space, $\tilde{\alpha}_i = \mathcal{F}_i(\alpha_j)$, we can choose a base where one of the species is minimally coupled: 4 + 2 N_S -2 = 2 (N_S + 1)

- Ghost and gradient stability conditions are invariant under rotation in par. space



- Observables invariant. Example: $\dfrac{\tilde{c}_I^2}{\tilde{c}_J^2} = \dfrac{c_I^2}{c_J^2}$

- Inflation: no matter (N_S = 0). We have 2 independent parameters, ex. α_K and α_B

δN^2 , $\delta N \delta K$

1407.8439 with Creminelli, Gleyzes, Noreña

Constraining dark energy

- Can we constrain these parameters?

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- Undo unitary gauge: $t \to t + \pi(t, \vec{x})$
- Newtonian gauge:
  - Scalar fluctuations: $dt^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)d\vec{x}^2$

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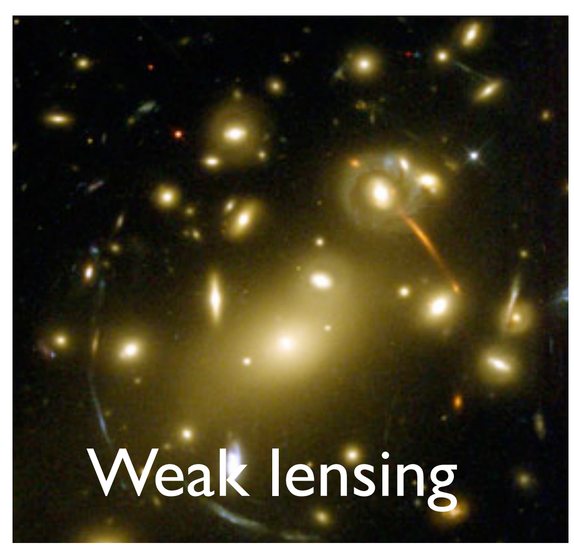
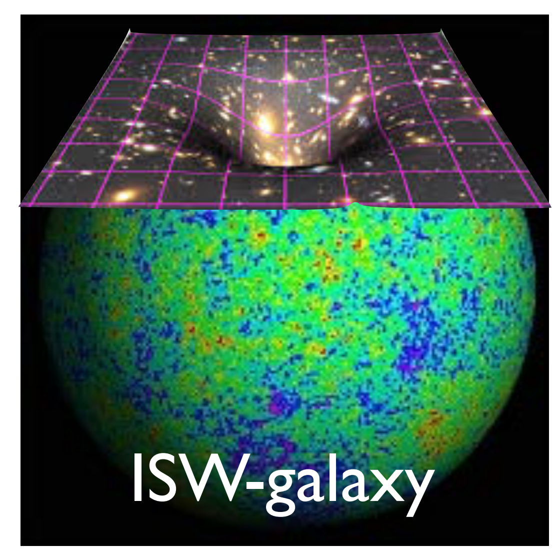
- Quasi-static approximations — valid on scales $k \gg aHc_s^{-1}$. Sawicki, Bellini '15

E.g., for surveys such as Euclid $c_s \gtrsim 0.1$.

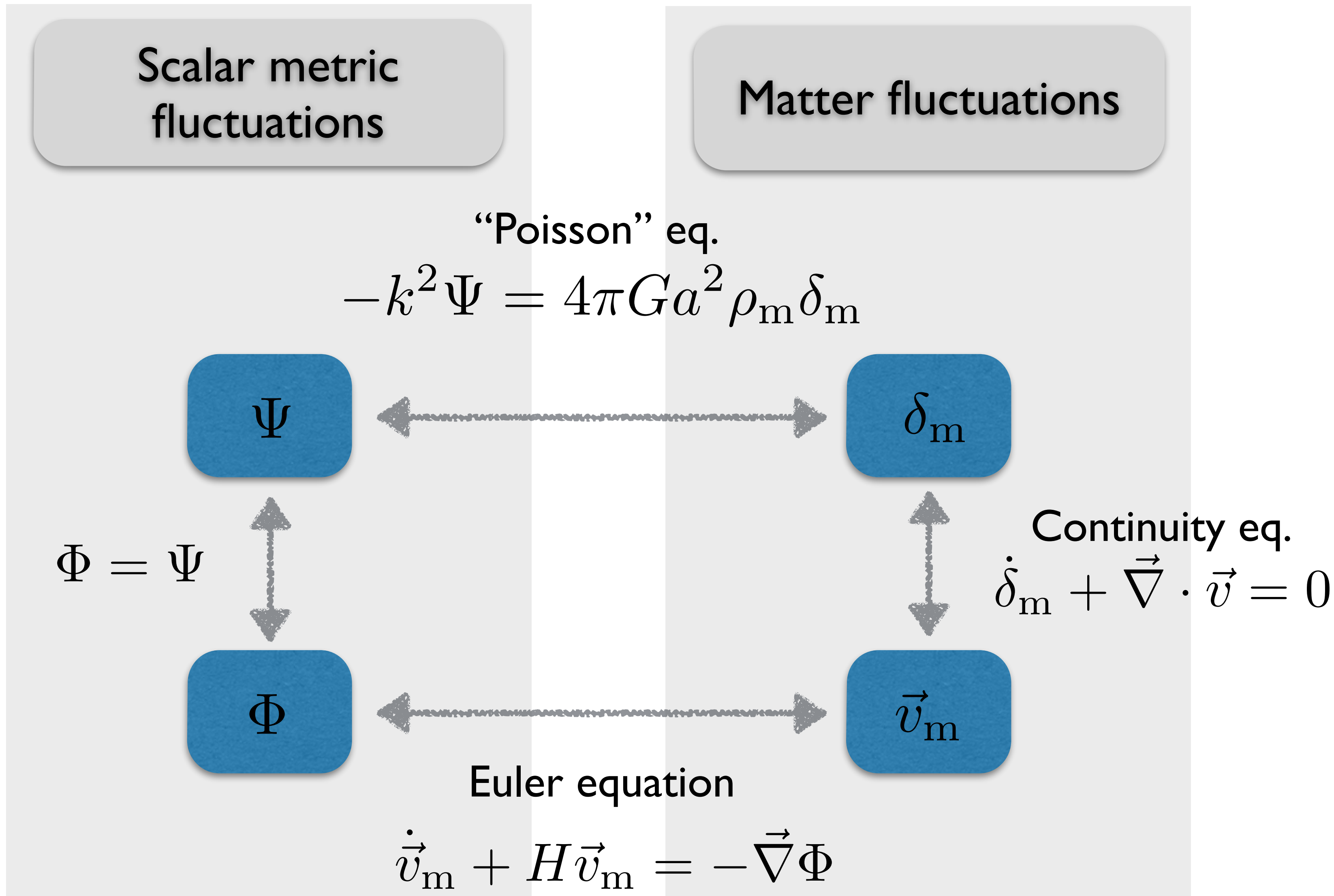




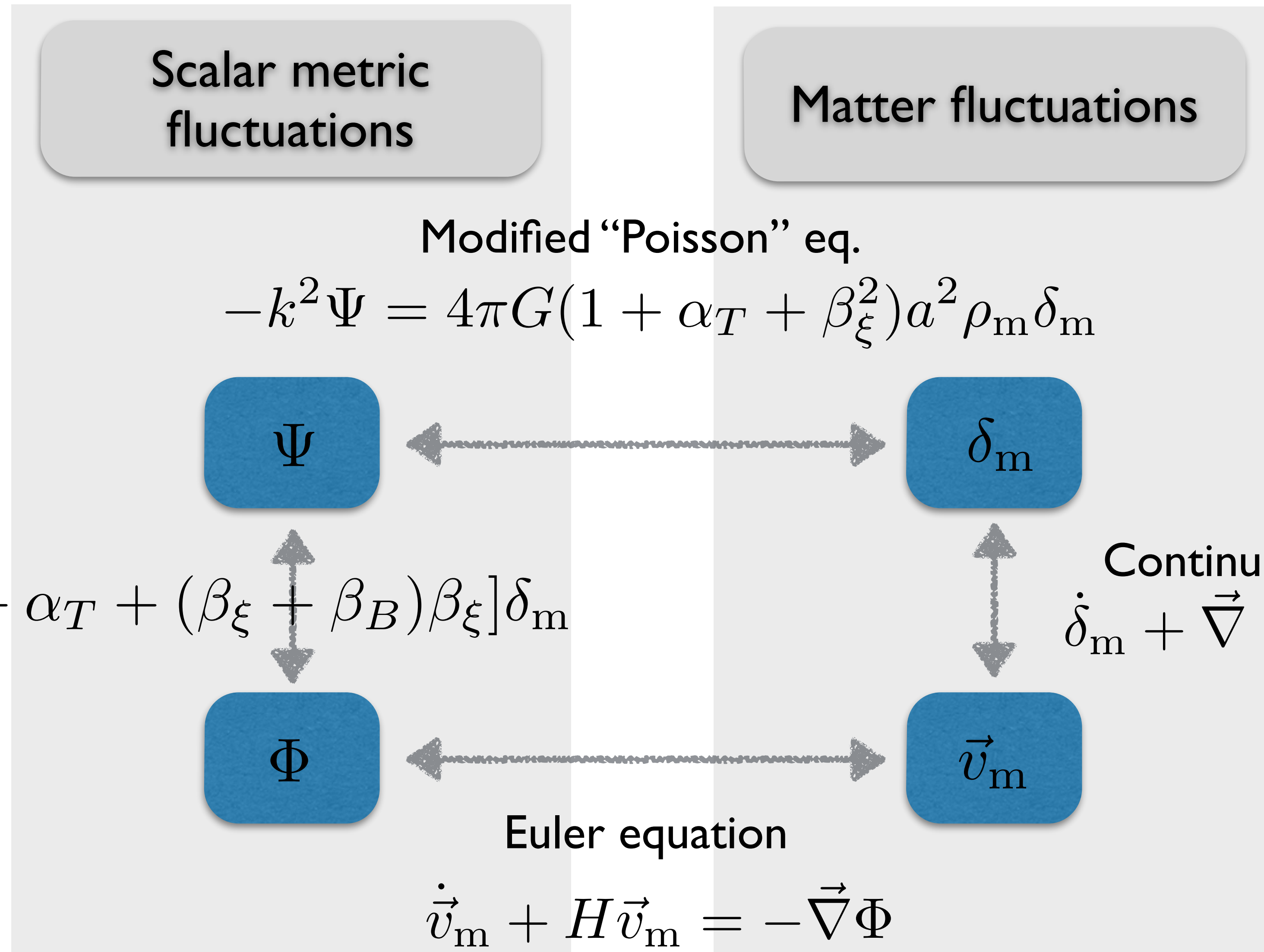
Standard case

Lensing:

$$\Phi + \Psi \propto \frac{a^2 H^2}{k^2} \delta_{\rm m}$$



Modified gravity



Modified "Poisson" eq.

$$-k^2\Psi = 4\pi G(1+\alpha_T+\beta_\xi^2)a^2\rho_{\rm m}\delta_{\rm m}$$

Continuity eq.

$$\dot{\delta}_{\rm m} + \vec{\nabla}\cdot\vec{v} = 0$$

Euler equation

$$\dot{\vec{v}}_{\rm m} + H\vec{v}_{\rm m} = -\vec{\nabla}\Phi$$

Lensing:

$$\Phi+\Psi \propto \frac{a^2H^2}{k^2}[2+\alpha_T+(\beta_\xi+\beta_B)\beta_\xi]\delta_{\rm m}$$

$$\beta_B \equiv \frac{\alpha_B}{\mathcal{A}}\,, \quad \beta_\xi \equiv \frac{1}{\mathcal{A}}\left[\alpha_B(1+\alpha_T)+\alpha_T-\alpha_M\right]$$

Modifications of gravity

+ Nonminimal coupling

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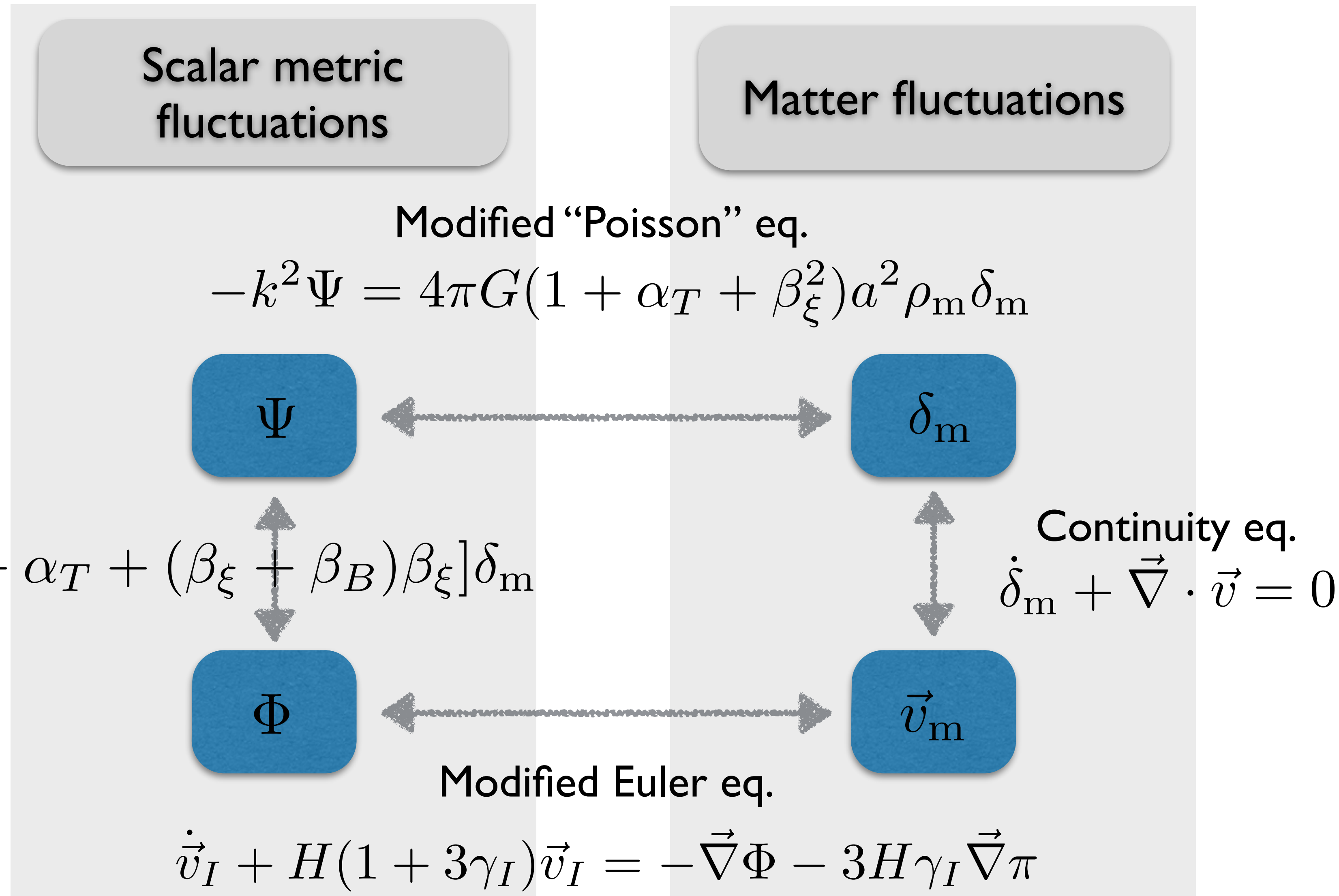


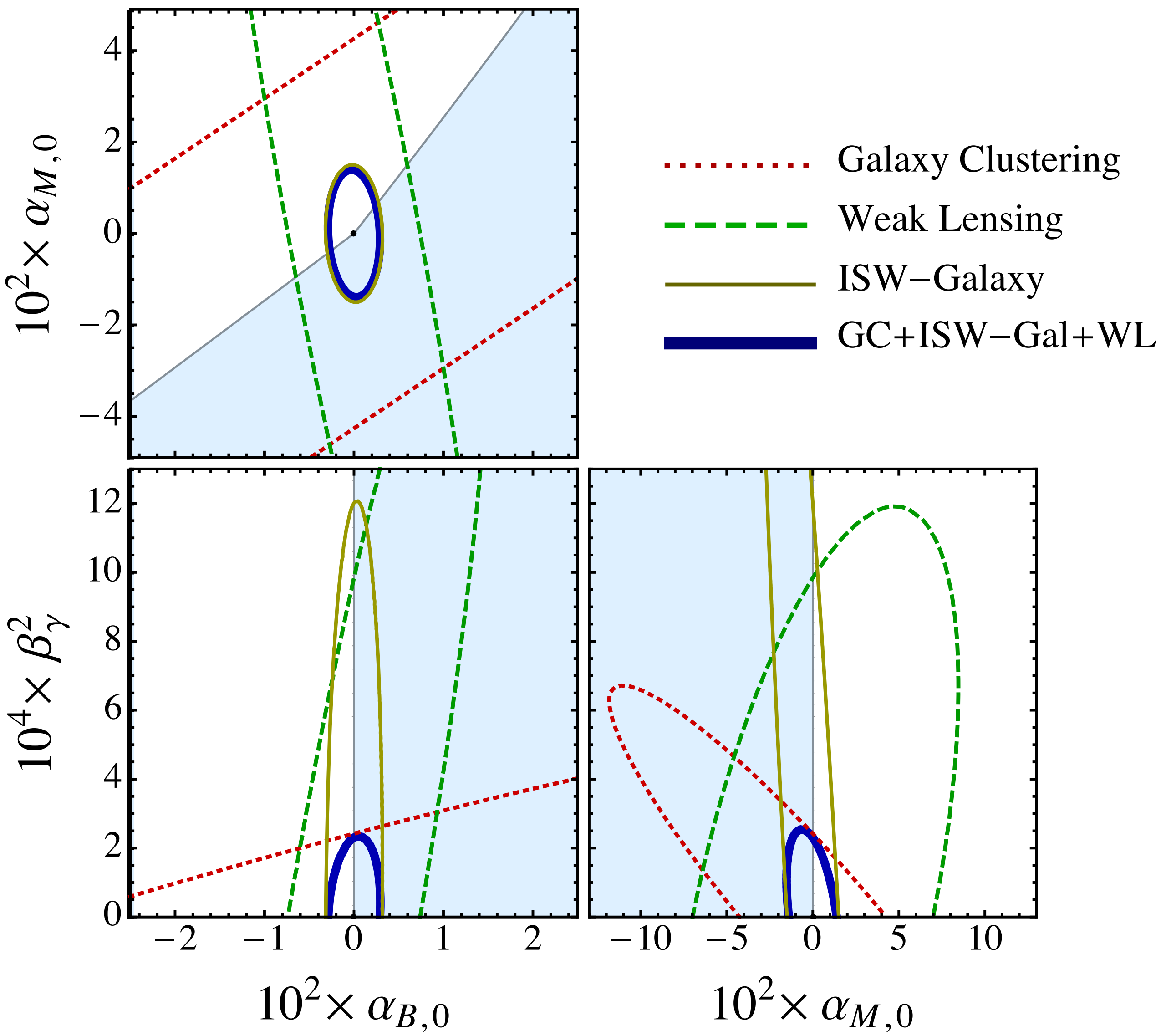
$$\beta_B \equiv \frac{\alpha_B}{\mathcal{A}}\,, \quad \beta_\xi \equiv \frac{1}{\mathcal{A}}\left[\alpha_B(1+\alpha_T) + \alpha_T - \alpha_M\right]\,, \qquad \beta_{\gamma I} \equiv \frac{3\gamma_I}{\mathcal{A}}$$

Modifications of gravity

non-minimal coupling

Baryons + coupled CDM

- Fisher matrix analysis, Euclid-like specifications
- **Fiducial I: LCDM.** Unmarginalized 1σ contours:



in preparation with
Gleyzes, Langlois, Mancarella

$$\alpha_B = \alpha_{B,0}\frac{1-\Omega_{\rm m}}{1-\Omega_{\rm m,0}}$$

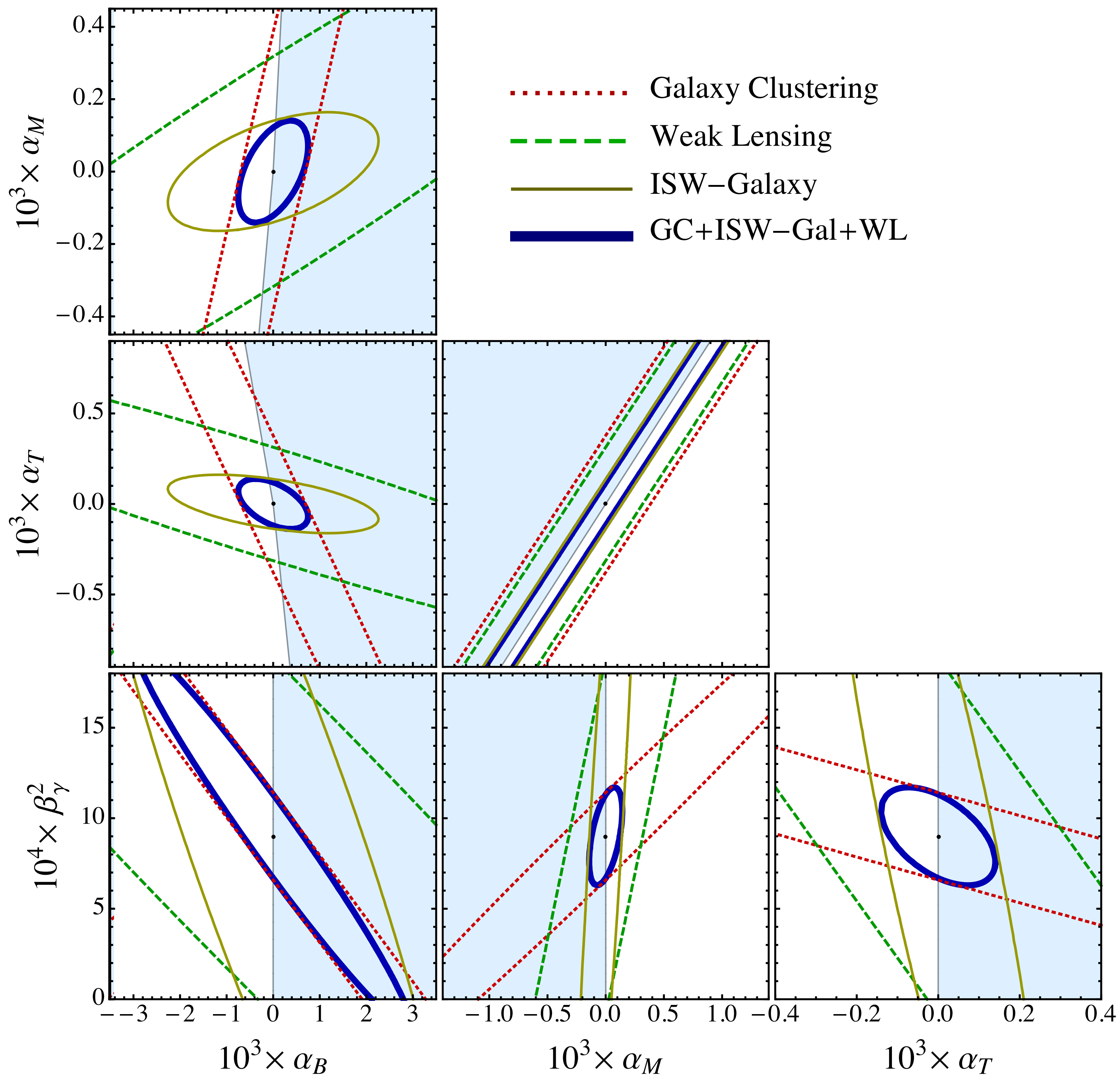
$$\alpha_M = \alpha_{M,0}\frac{1-\Omega_{\rm m}}{1-\Omega_{\rm m,0}}$$

$$\beta_\gamma^2 = \text{const.}$$

Baryons + coupled CDM

in preparation with
Gleyzes, Langlois, Mancarella

- Fisher matrix analysis, Euclid-like specifications
- **Fiducial II: Interacting.** Unmarginalized 1σ contours:



$$\alpha_B = \alpha_{B,0}\frac{1-\Omega_{\rm m}}{1-\Omega_{\rm m,0}}$$

$$\alpha_M = \alpha_{M,0}\frac{1-\Omega_{\rm m}}{1-\Omega_{\rm m,0}}$$

$$\alpha_T = \alpha_{T,0}\frac{1-\Omega_{\rm m}}{1-\Omega_{\rm m,0}}$$

$$\beta_\gamma^2 = \text{const.}$$

Conclusions

* ✳ General description of linear perturbations in scalar-tensor theories of gravity
* ✳ Systematic way to address stability and explore new theories
* ✳ Efficient (minimal) way to parametrize observations on large scales (linear regime)
* ✳ Forecasts: unmarginalized error ~ 10^{-3} on parameters describing modifications of gravity. Degeneracies and dependence on the fiducial model.
* ✳ Future: Relax assumptions (beyond linear regime, more degrees of freedom, etc…), explore phenomenology and forecasts beyond the quasi-static approximation.

Conclusions

* General description of linear perturbations in scalar-tensor theories of gravity

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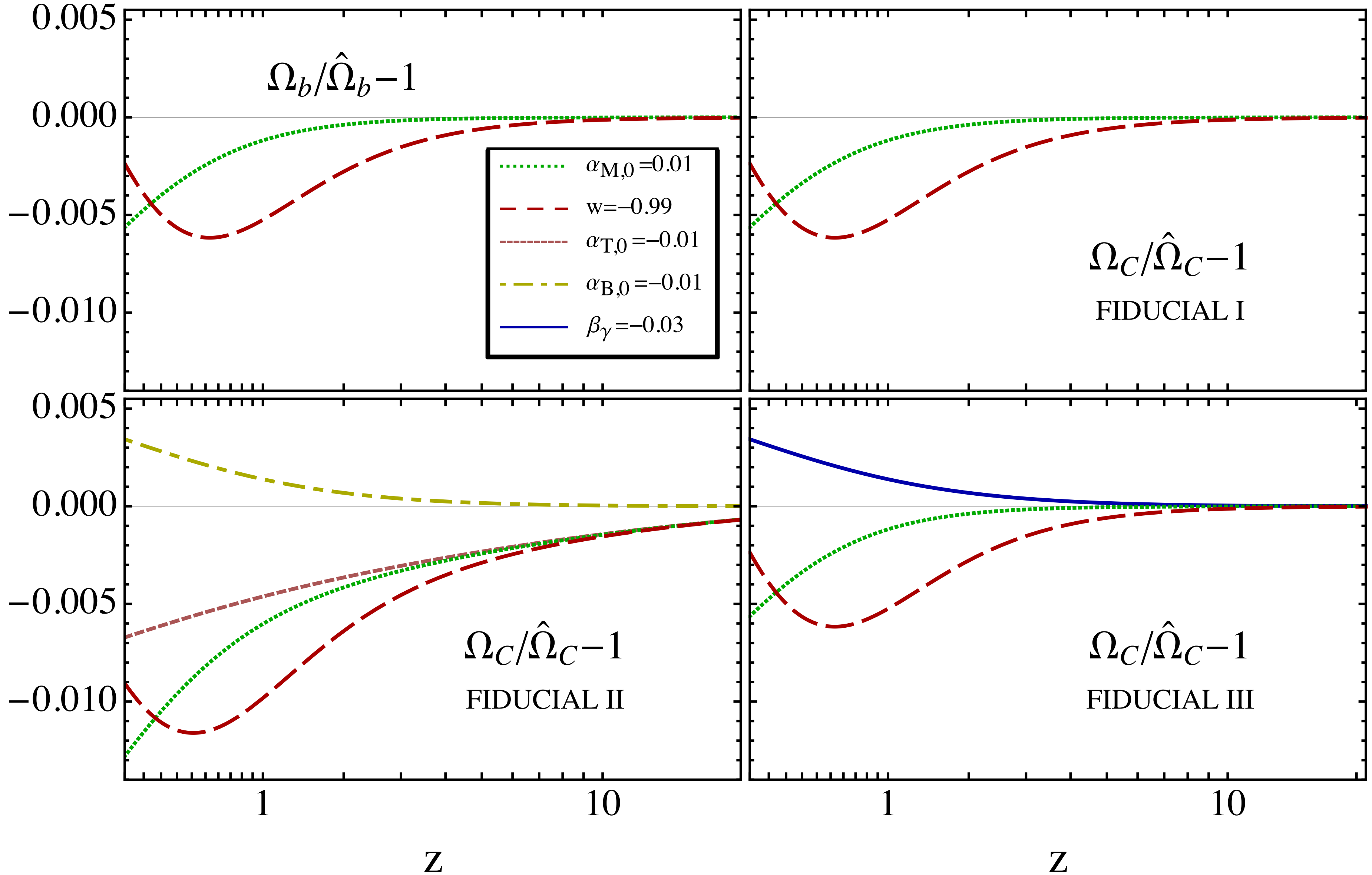
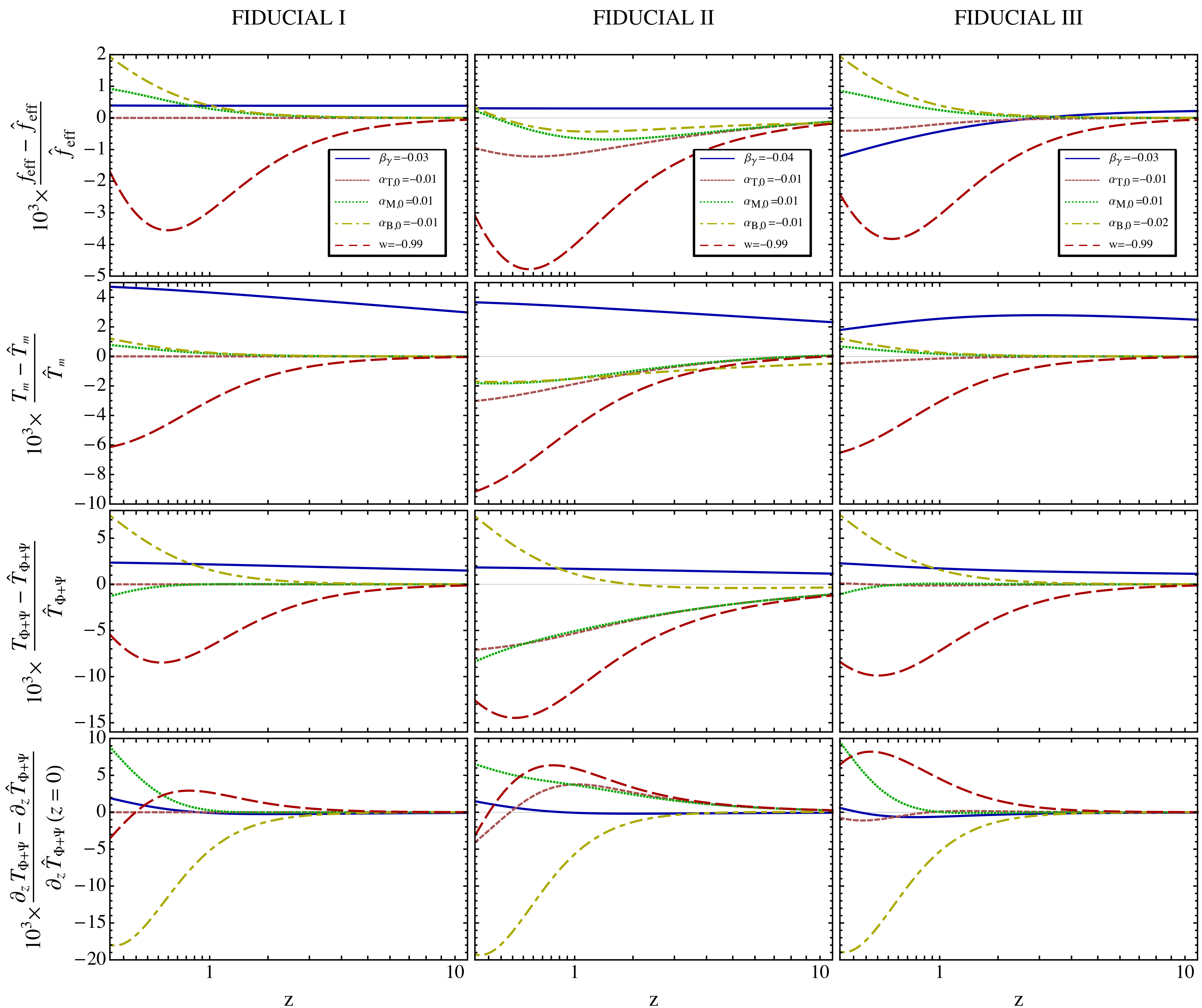
See Shinji Tsujikawa's talk!

Physical effects: background



Physical effects: perturbations



Horndeski theories

- Most general LI scalar-tensor theory with at most second-order equations of motions

(Horndeski '73, Deffayet et al.'11)

$$L_H = G_2(\phi, X) + G_3(\phi, X)\Box\phi +$$
$$+ G_4(\phi, X)\,{}^{(4)}R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$
$$+ G_5(\phi, X)\,{}^{(4)}G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\,\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\nu\lambda}\phi_{;\lambda}^{\ ;\mu}\right]$$

$$X \equiv \phi_{;\mu}\phi^{;\mu} \equiv \nabla_\mu\phi\nabla^\mu\phi$$

- Unitary gauge formulation:

1304.4840 with Gleyzes, Langlois, Piazza

$$L_H = A_2(t, N) + A_3(t, N)K +$$
$$+ B_4(t, N)\,{}^{(3)}R + A_4(t, N)(K^2 - K_{ij}K^{ij})$$
$$+ B_5(t, N)\,{}^{(3)}G^{ij}K_{ij} + A_5(t, N)(K^3 - 3KK_{ij}K^{ij} + 2K_{ij}K^{ik}K^j{}_k)$$

with
$$A_4 = -B_4 + 2XB_{4,X}$$
$$A_5 = -XB_{5,X}/3$$

Background

$$L(N, K^i_j, R^i_j, \dots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K^i_j}\delta K^i_j + \frac{\partial L}{\partial R^i_j}\delta R^i_j + L^{(2)} + \dots$$

- FRW metric: $$ds^2 = -N_0^2(t)dt^2 + a^2(t)d\vec{x}^2$$

- All background solutions are given in terms of only 3 functions:

$$S^{(0)} = \int d^3x dt a^3 N_0 L_0 \left(N_0, {K^i}_j = \frac{\dot{a}}{N_0 a}, {R^i}_j = 0 \right)$$

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- Matter action: $$\delta S_{\rm m} = \frac{1}{2}\int d^4x \sqrt{-g}\, T^{\mu\nu}\delta g_{\mu\nu}$$
- Friedmann equations:

$$H^2 = \frac{1}{3M^2}(\rho_{\rm m} + \rho_{\rm DE})$$

$$\dot{H} = -\frac{1}{2M^2}(\rho_{\rm m} + p_{\rm m} + \rho_{\rm DE} + p_{\rm DE})$$

$$\rho_{\rm DE} = c + \Lambda - 3H(M^2)_{,t}$$

$$p_{\rm DE} = c - \Lambda + 2H(M^2)_{,t} + (M^2)_{,tt}$$

First-order Lagrangian

$$L(N, K^i_j, R^i_j, \dots) = \bar{L} + \underbrace{L_N \delta N + \frac{\partial L}{\partial K^i_j}\delta K^i_j + \frac{\partial L}{\partial R^i_j}\delta R^i_j}_{=0} + L^{(2)} + \dots$$

$= 0$ by the bkgd EOM

Stability

$$\mathcal{L} = +\dot{\varphi}^2 - c_s^2(\nabla\varphi)^2$$

positive kinetic energy
= absence of ghosts

positive sound speed = absence
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$$\mathcal{L} = M^2(t)\left\{\left(\alpha_K(t) + 6\alpha_B^2(t)\right)\left[\dot{\zeta}^2 - c_s^2(\nabla\zeta)^2\right] + \left[\dot{\gamma}_{ij}^2 - \left(1+\alpha_T(t)\right)(\nabla\gamma_{ij})^2\right]\right\}$$

No higher time (and space) derivatives

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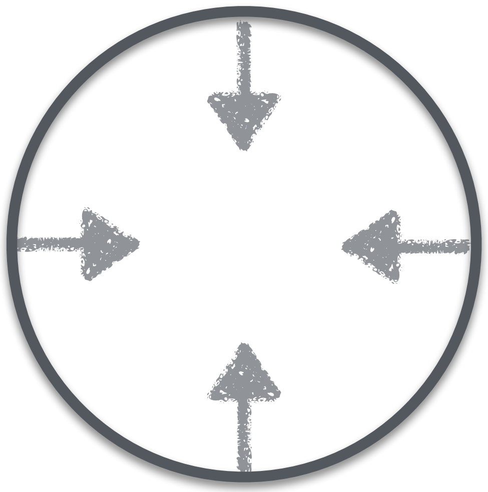
No higher time (and space) derivatives

	Scalar	Tensor
No ghosts	$\alpha_K + 6\alpha_B^2 > 0$	$M^2 > 0$
No gradient instability	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

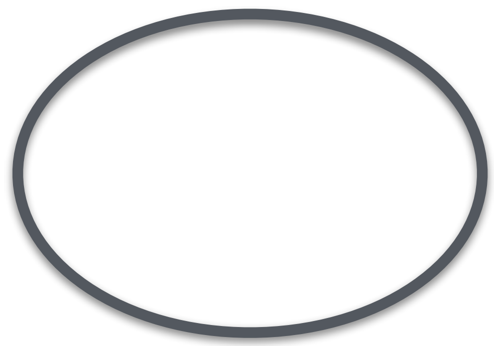
- Theoretical restriction on the parameter space

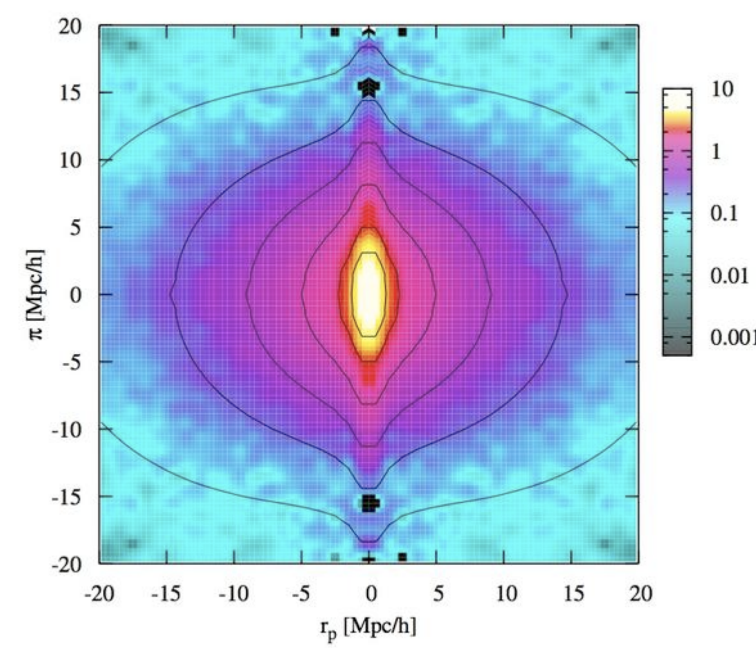
Growth of structures

Real space



Redshift space





$$\delta^z_{\rm gal} = \delta_{\rm gal} + \cos^2\alpha \frac{\vec{\nabla}\cdot\vec{v}_{\rm gal}}{H} \longrightarrow \vec{\nabla}\cdot\vec{v}_{\rm gal} \approx \vec{\nabla}\cdot\vec{v}_{\rm m} \longrightarrow \nabla^2\Phi$$

$$\delta_{\rm gal} = b\delta_{\rm m}\ , \qquad \ddot{\delta}_{\rm m} + 2H\dot{\delta}_{\rm m} = \nabla^2\Phi$$

Weak lensing

$$M_{ij} = \int_{z_s}^{0} w(z, z_s)\, \partial_i \partial_j (\Phi + \Psi)\, dz$$